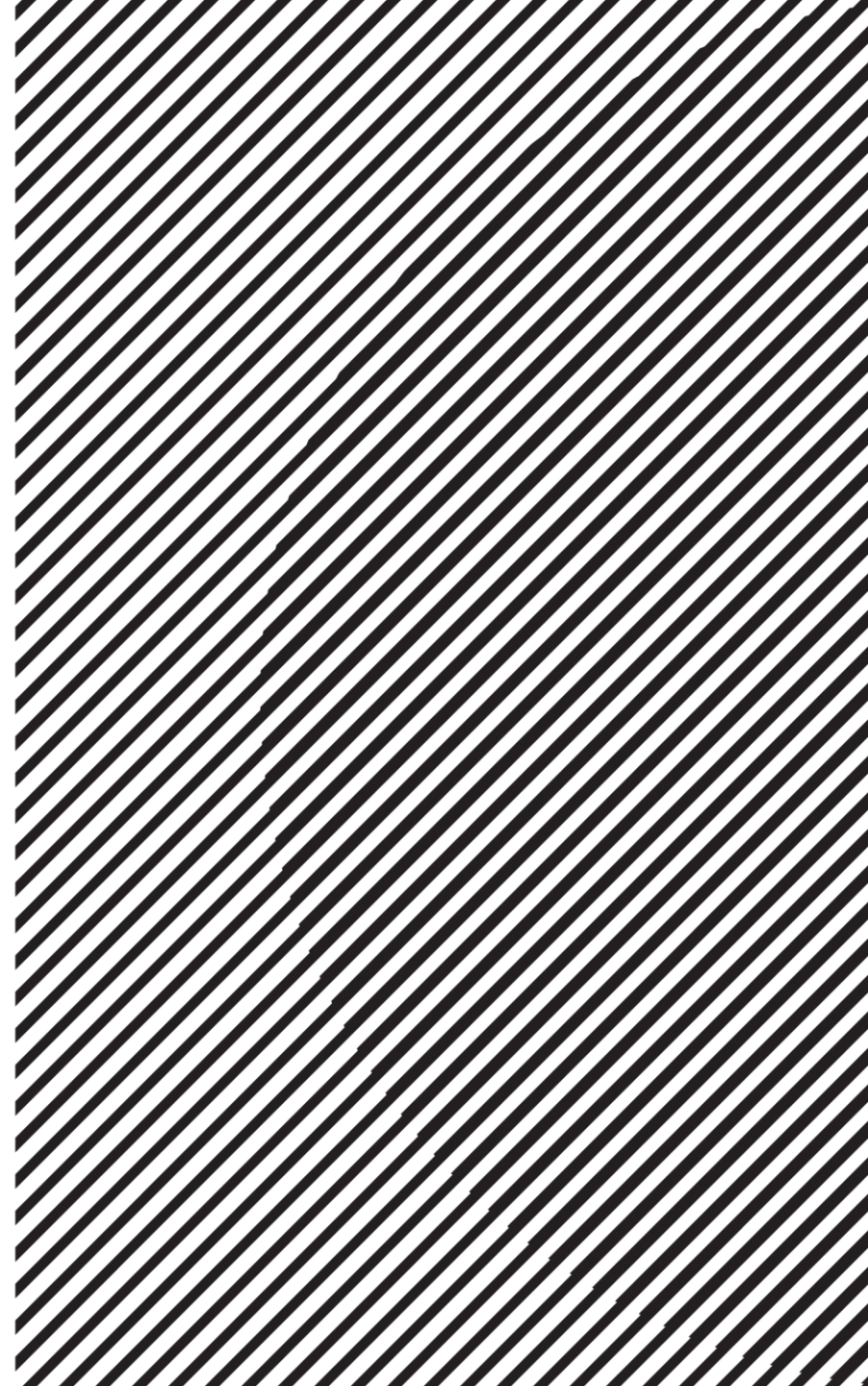
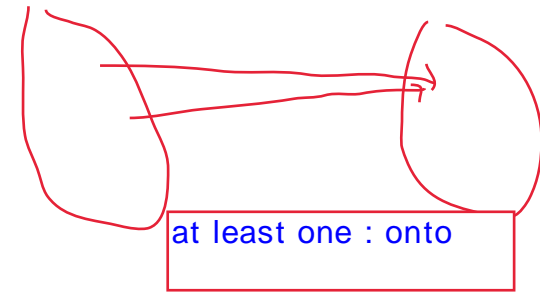

Linear Algebra

주재걸
고려대학교 컴퓨터학과

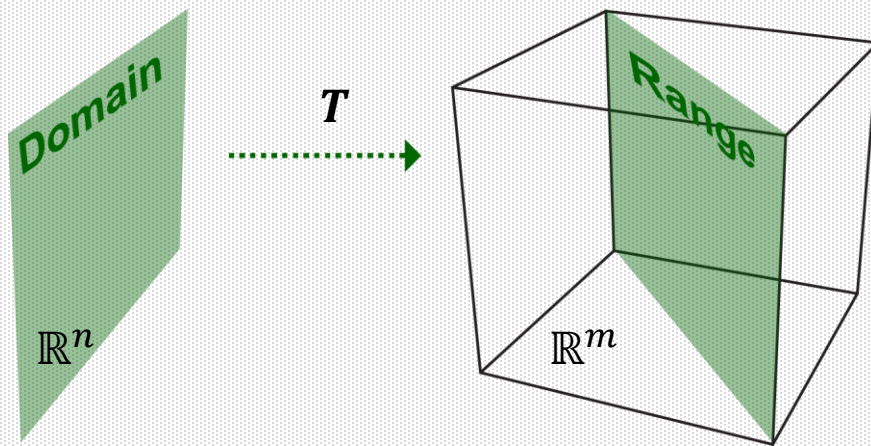


ONTO and ONE-TO-ONE

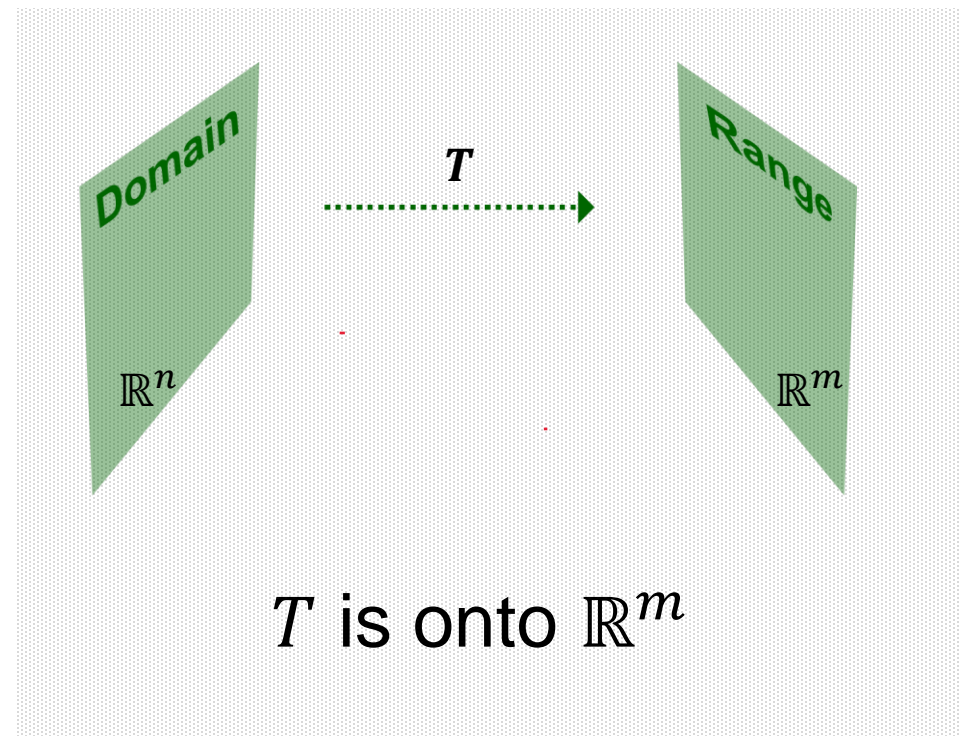
=



- **Definition:** A mapping $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is said to be **onto** \mathbb{R}^m if each $\mathbf{b} \in \mathbb{R}^m$ is the image of **at least** one $\mathbf{x} \in \mathbb{R}^n$. That is, the range is equal to the co-domain.



T is NOT onto \mathbb{R}^m



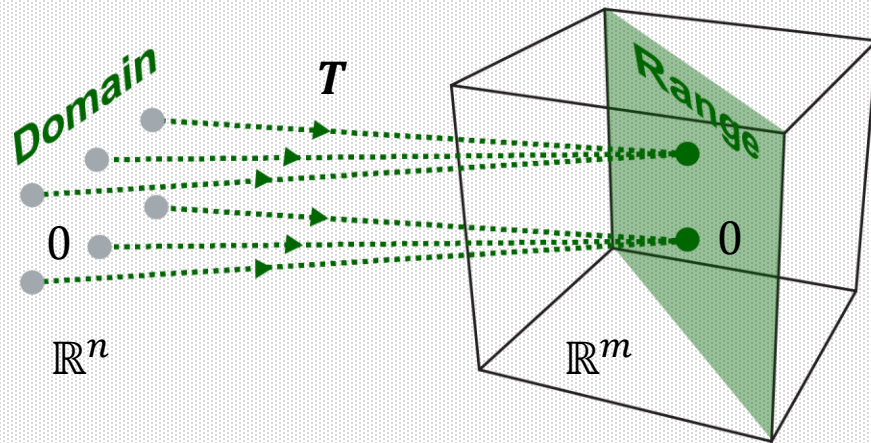
T is onto \mathbb{R}^m

ONTO and ONE-TO-ONE

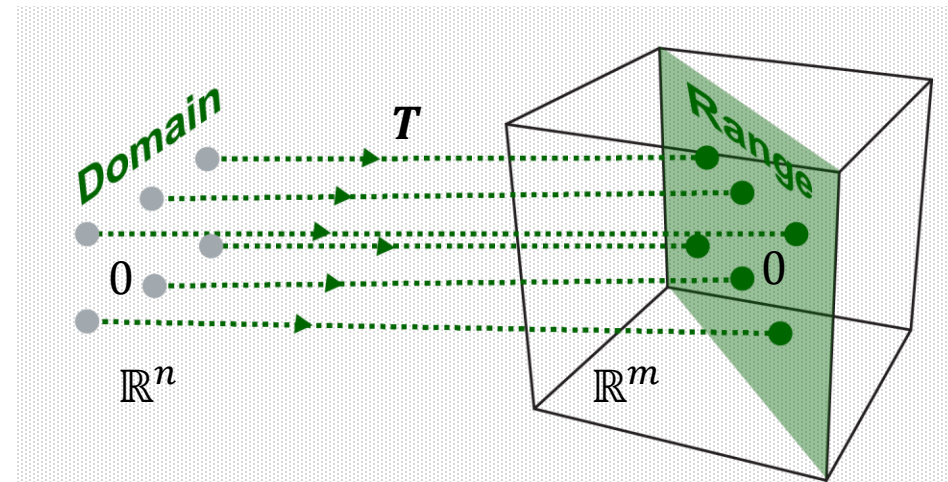
2 \rightarrow 3 one-to-one

3 \rightarrow 2

- Definition:** A mapping $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is said to be **one-to-one** if each $\mathbf{b} \in \mathbb{R}^m$ is the image of **at most** one $\mathbf{x} \in \mathbb{R}^n$. That is, each output vector in the range is mapped by only one input vector, no more than that.



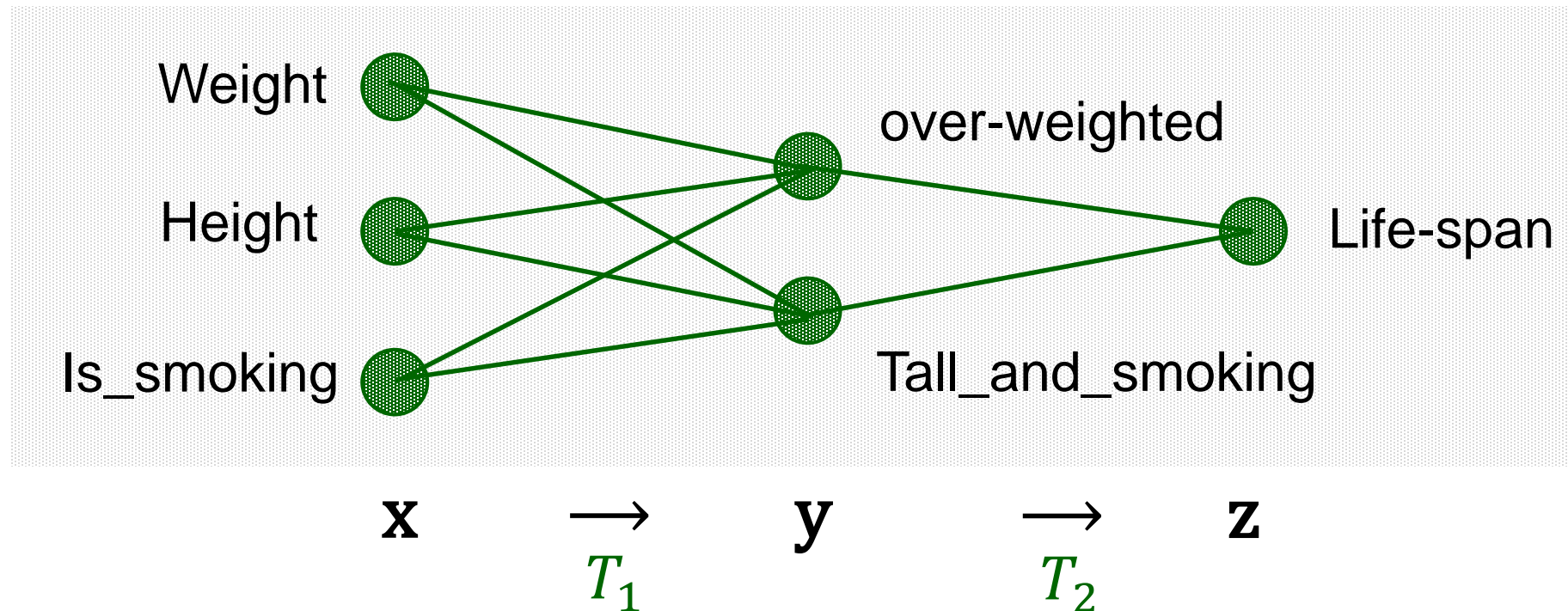
T is NOT one-to-one



T is one-to-one

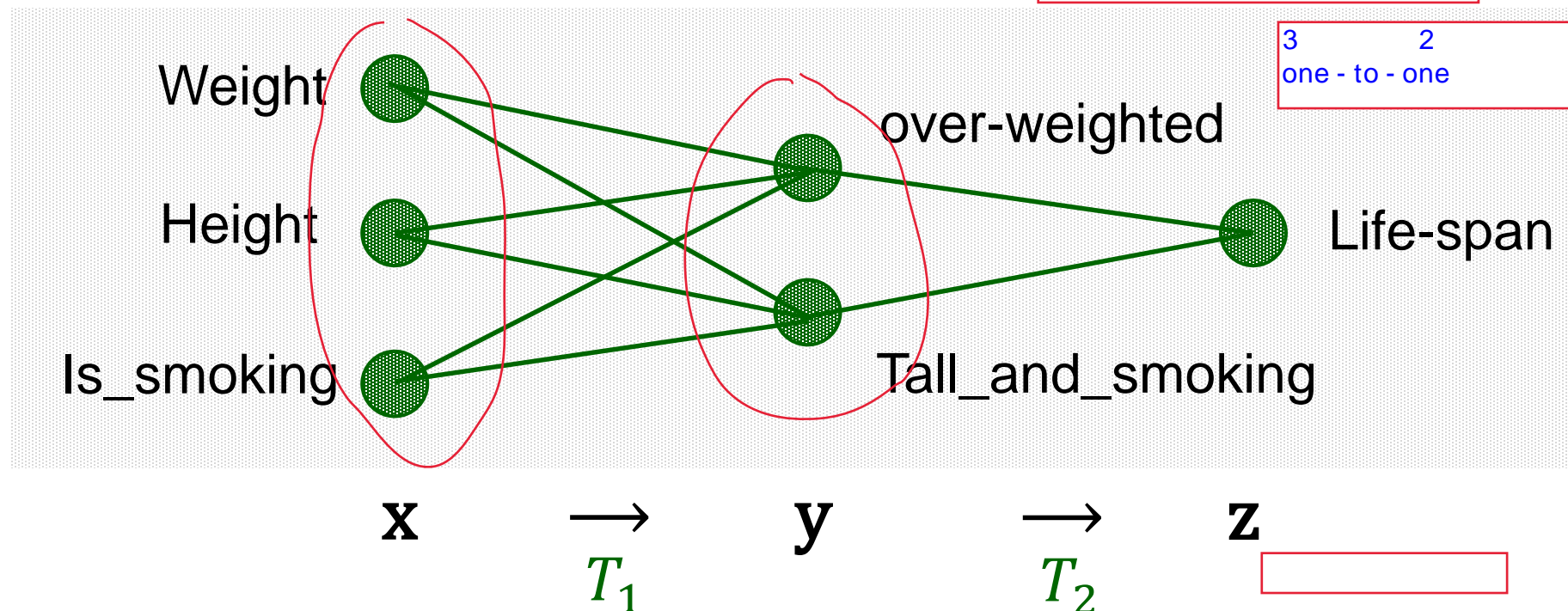
Neural Network Example

- Fully-connected layers



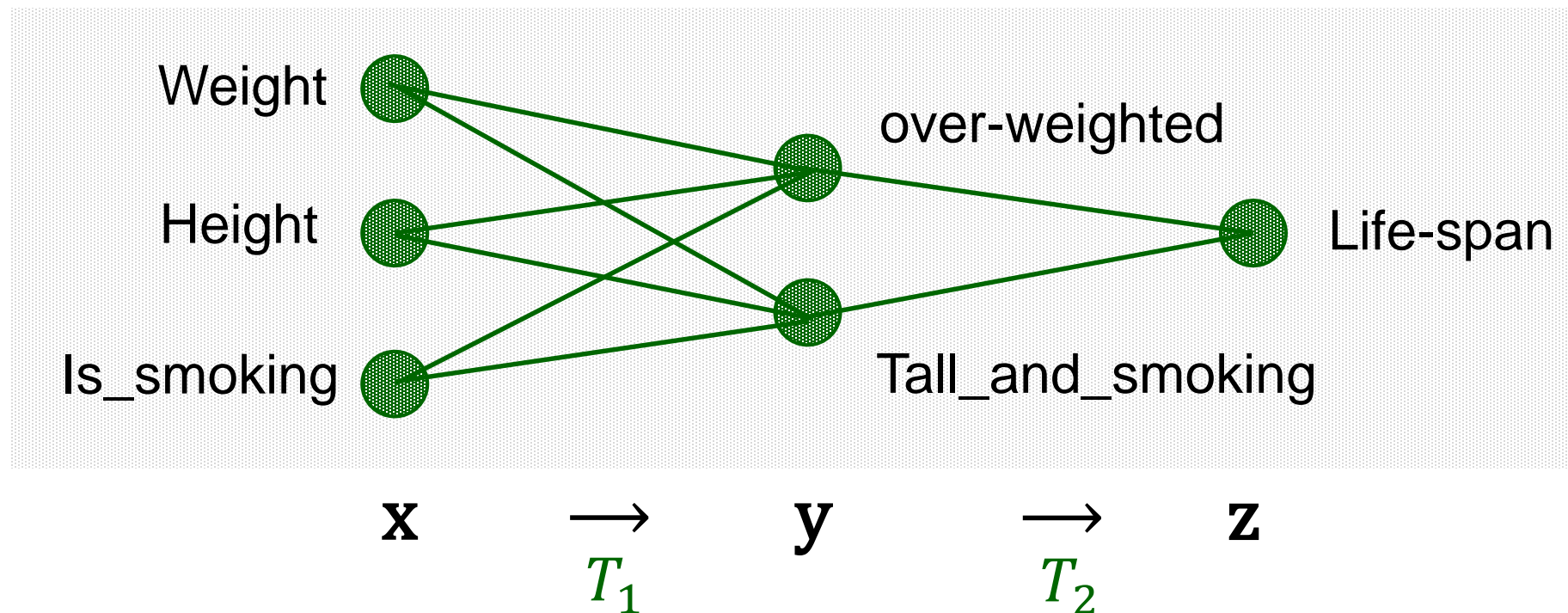
Neural Network Example: ONE-TO-ONE

- Will there be many (or unique) people mapped to the same (over_weighted, tall_and_smoking)?



Neural Network Example: ONTO

- Is there any (over_weighted, tall_and_smoking) that does not exist at all?





ONTO and ONE-TO-ONE

- Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear transformation, i.e.,

$$T(\mathbf{x}) = A\mathbf{x} \text{ for all } \mathbf{x} \in \mathbb{R}^n.$$

- T is **one-to-one** if and only if the columns of A are **linearly independent**.
- T maps \mathbb{R}^n **onto** \mathbb{R}^m if and only if the columns of A **span** \mathbb{R}^m .



ONTO and ONE-TO-ONE

- **Example:**

$$\text{Let } T(\mathbf{x}) = A\mathbf{x} = \begin{bmatrix} 2 & 0 \\ -1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

- Is T one-to-one?
- Does T map \mathbb{R}^2 onto \mathbb{R}^3 ?



ONTO and ONE-TO-ONE

- **Example:**

$$\text{Let } T(\mathbf{x}) = A\mathbf{x} = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 3 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

- Is T one-to-one?
- Does T map \mathbb{R}^3 onto \mathbb{R}^2 ?



Further Study

- Gaussian elimination, row reduction, echelon form
 - Lay Ch1.2,
- LU factorization: efficiently solving linear systems
 - Lay Ch2.5
- Computing invertible matrices
 - Lay Ch2.2
- Invertible matrix theorem for square matrices
 - Lay Ch2.3, Ch2.9