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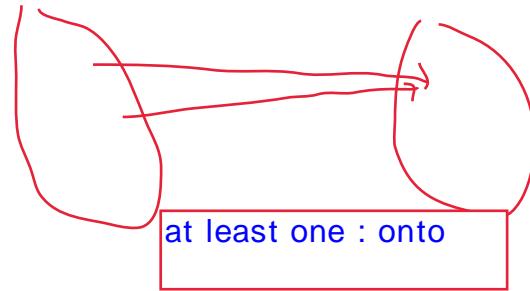
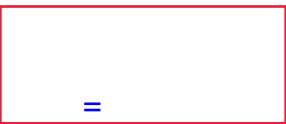
# Linear Algebra

주재걸

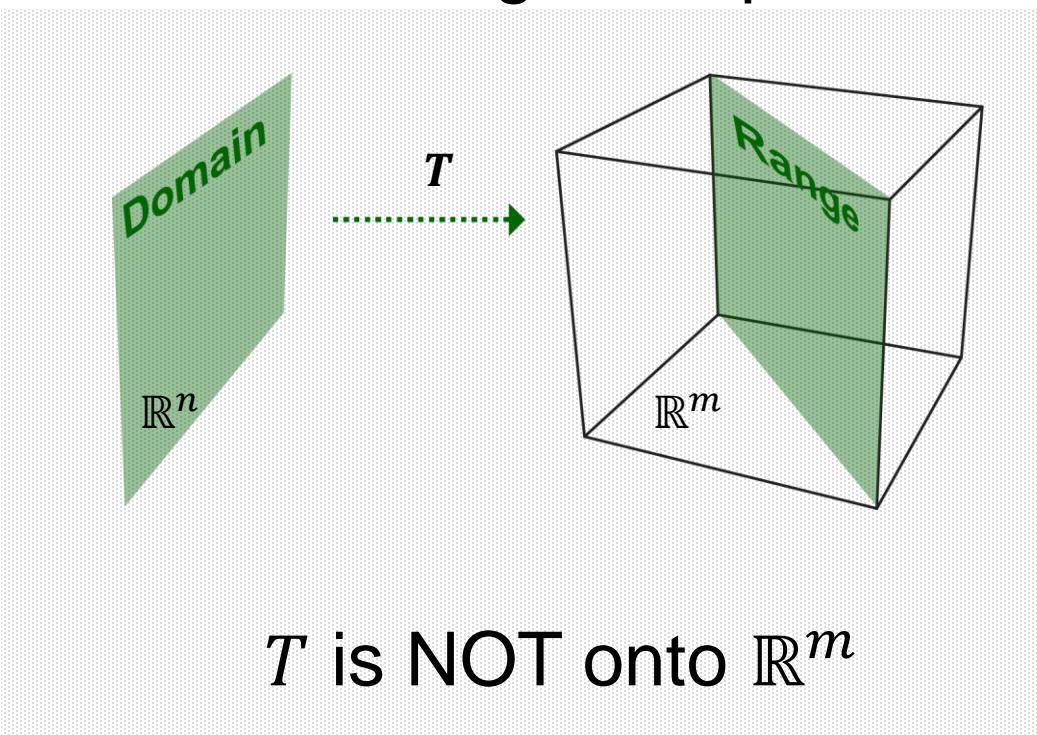
고려대학교 컴퓨터학과



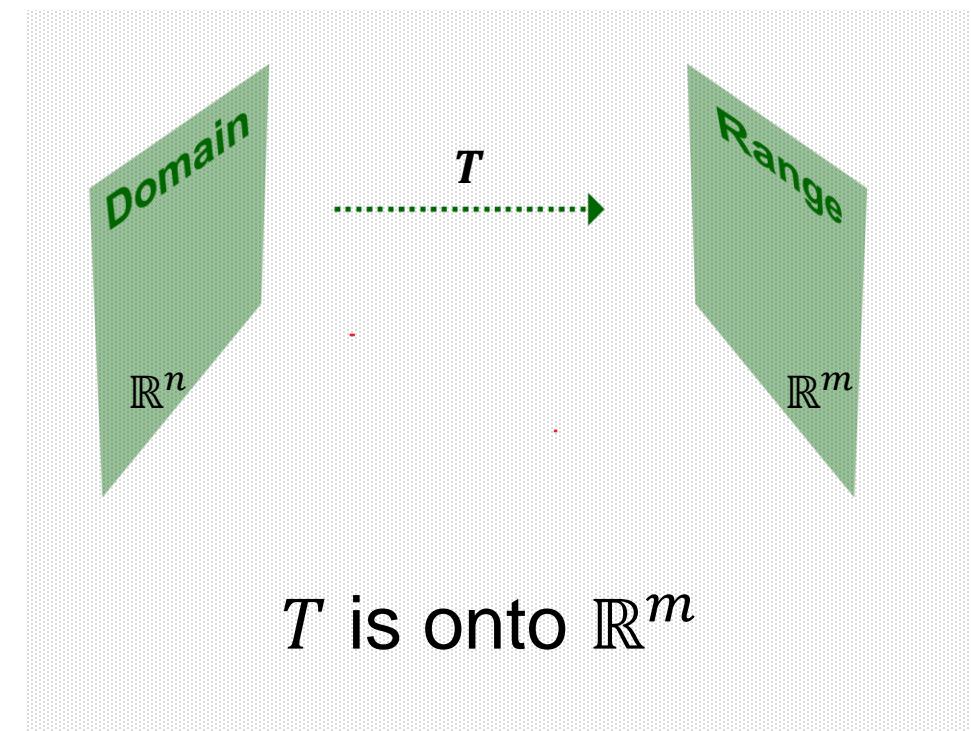
# ONTO and ONE-TO-ONE



- **Definition:** A mapping  $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  is said to be **onto**  $\mathbb{R}^m$  if each  $b \in \mathbb{R}^m$  is the image of **at least** one  $x \in \mathbb{R}^n$ . That is, the range is equal to the co-domain.



$T$  is NOT onto  $\mathbb{R}^m$

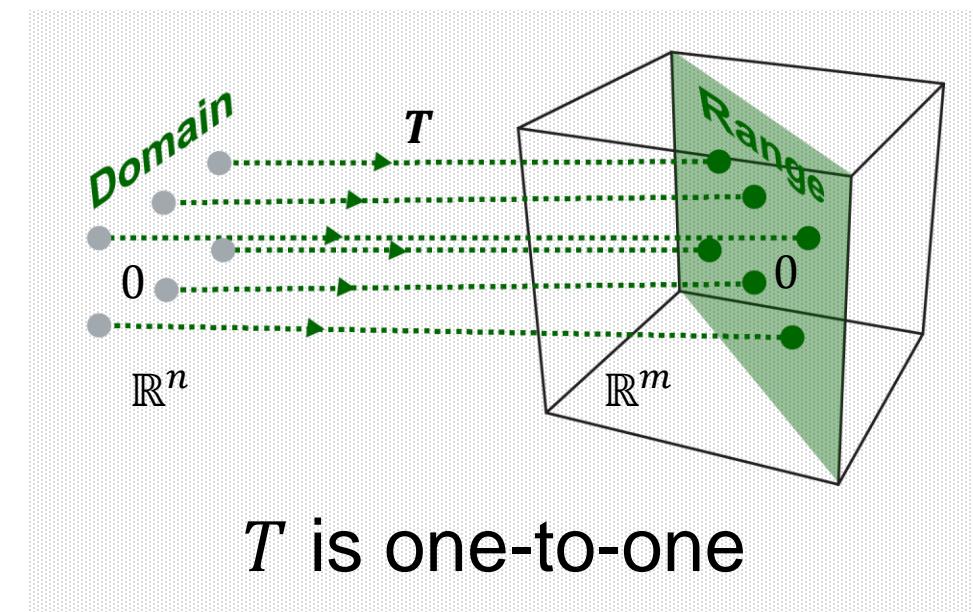
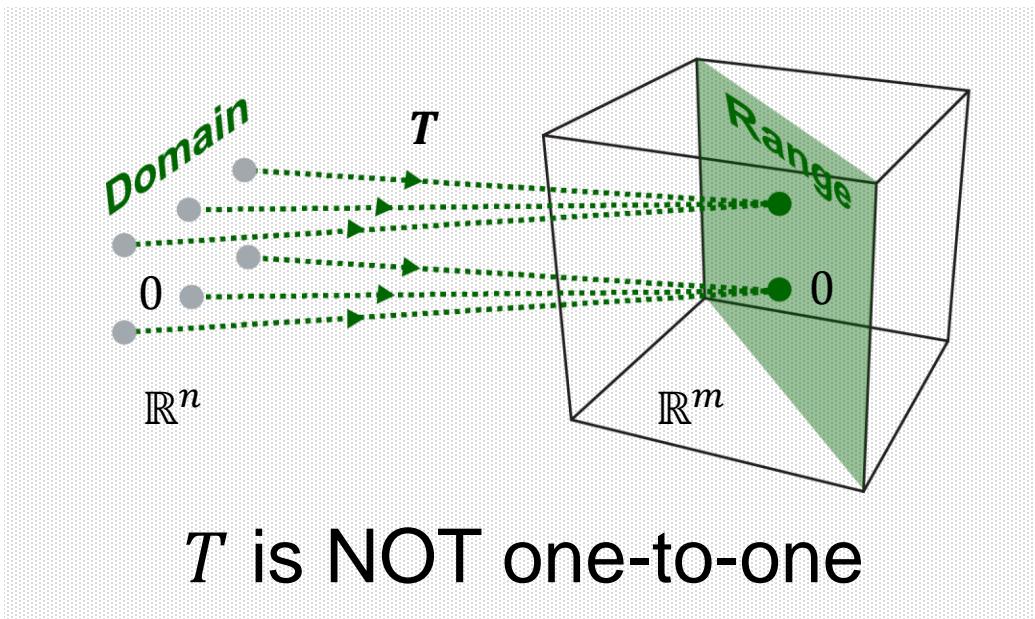


$T$  is onto  $\mathbb{R}^m$

# ONTO and ONE-TO-ONE

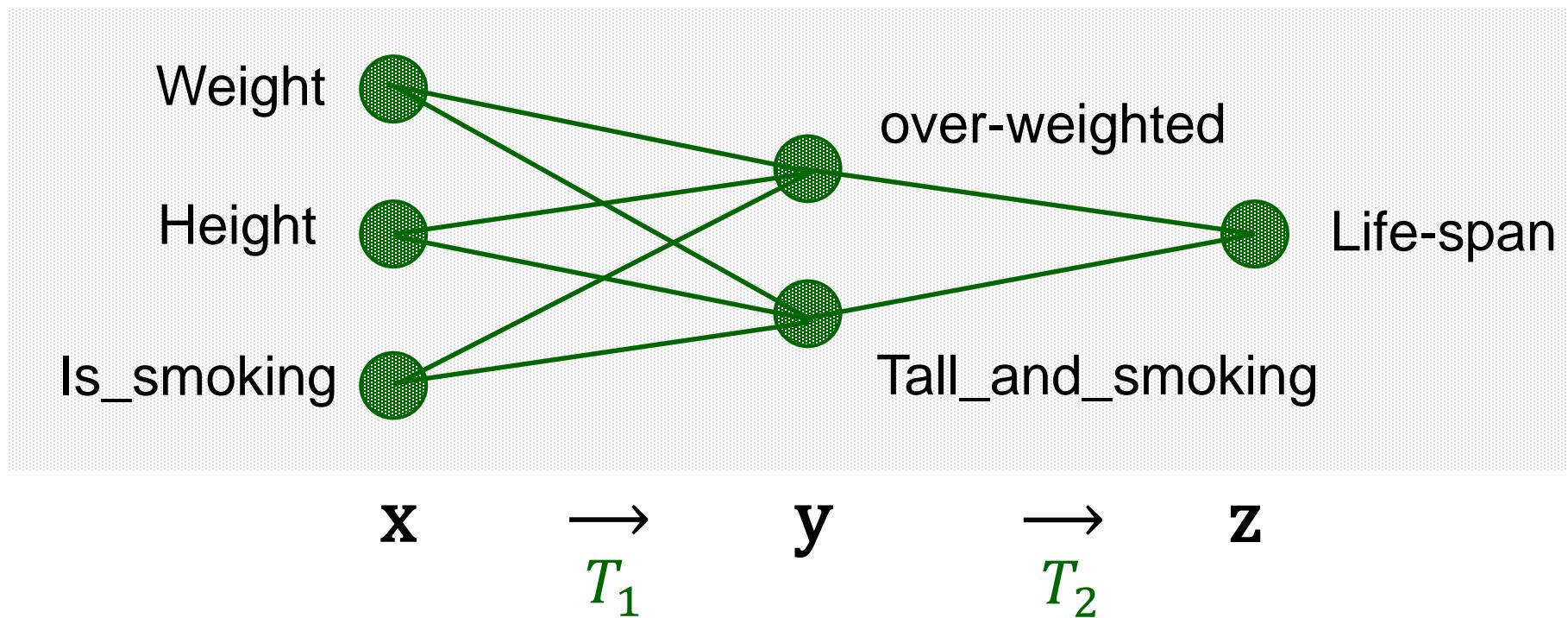


- **Definition:** A mapping  $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  is said to be **one-to-one** if each  $b \in \mathbb{R}^m$  is the image of **at most** one  $x \in \mathbb{R}^n$ . That is, each output vector in the range is mapped by only one input vector, no more than that.



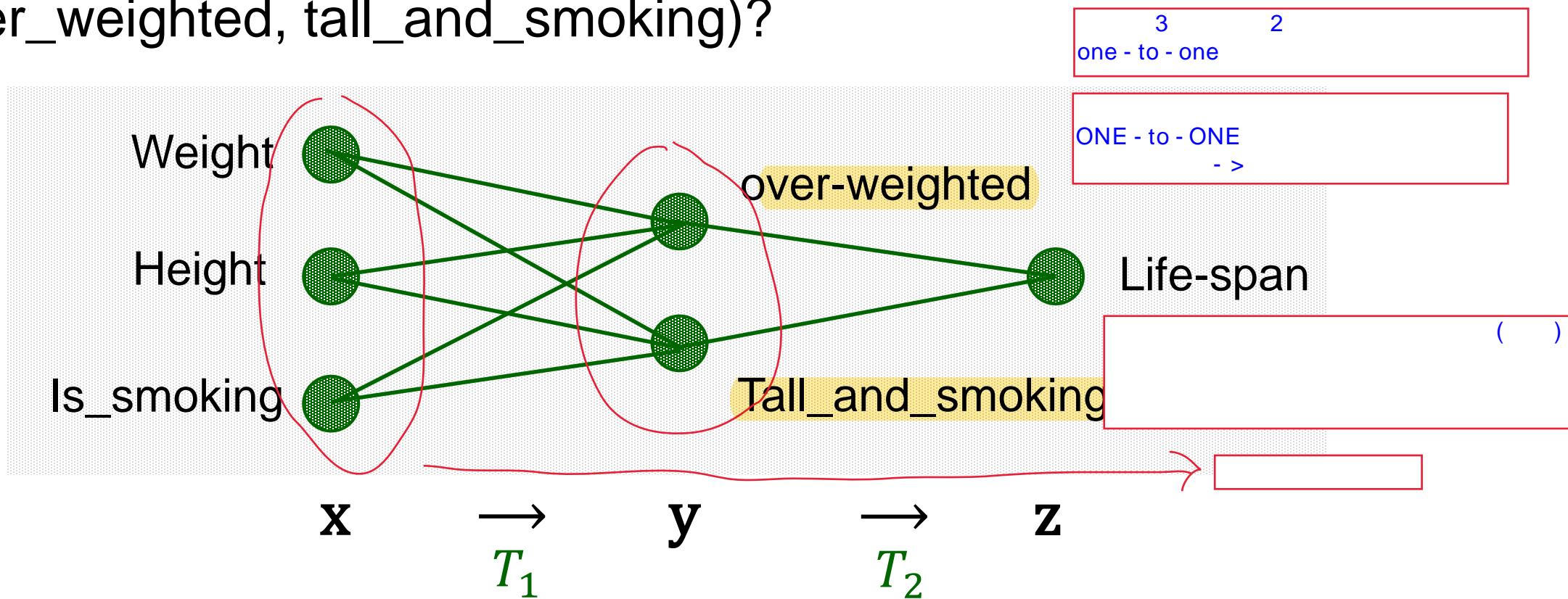
# Neural Network Example

- Fully-connected layers



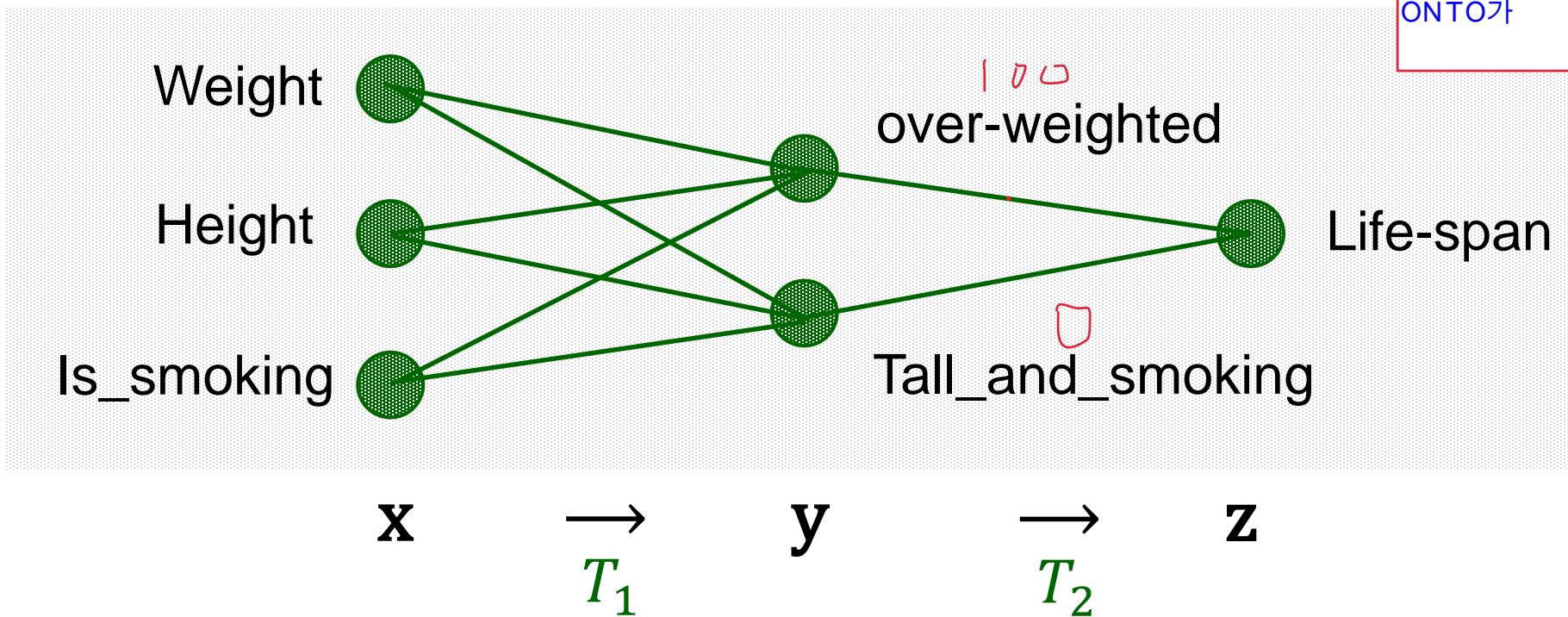
# Neural Network Example: ONE-TO-ONE

- Will there be many (or unique) people mapped to the same (over\_weighted, tall\_and\_smoking)?



# Neural Network Example: ONTO

- Is there any (over\_weighted, tall\_and\_smoking) that does not exist at all?



|                  |       |         |
|------------------|-------|---------|
| 3                | 2     | ONTO    |
| 가                | ONTO가 | 가       |
| Tall_and_smoking | 0     | ONTO가 . |



# ONTO and ONE-TO-ONE

- Let  $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  be a linear transformation, i.e.,
$$T(\mathbf{x}) = A\mathbf{x} \text{ for all } \mathbf{x} \in \mathbb{R}^n.$$
- $T$  is **one-to-one** if and only if the columns of  $A$  are **linearly independent**.
- $T$  maps  $\mathbb{R}^n$  **onto**  $\mathbb{R}^m$  if and only if the columns of  $A$  **span**  $\mathbb{R}^m$ .



# ONTO and ONE-TO-ONE

- Example:

$$\text{Let } T(\mathbf{x}) = A\mathbf{x} = \begin{bmatrix} 2 & 0 \\ -1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$\begin{matrix} 2 & \rightarrow & 3 \\ \text{---} & & \text{---} \\ 0 & & 0 \\ 0 & & 0 \end{matrix}$

- Is  $T$  one-to-one? Yes
- Does  $T$  map  $\mathbb{R}^2$  onto  $\mathbb{R}^3$ ? Yes

# ONTO and ONE-TO-ONE

- Example:

$$\text{Let } T(\mathbf{x}) = A\mathbf{x} = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 3 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$3 \rightarrow 2$   
○  
○

- Is  $T$  one-to-one?  $\checkmark$
- Does  $T$  map  $\mathbb{R}^3$  onto  $\mathbb{R}^2$ ?  $\times$



# Further Study

- Gaussian elimination, row reduction, echelon form
  - Lay Ch1.2,
- LU factorization: efficiently solving linear systems
  - Lay Ch2.5
- Computing invertible matrices
  - Lay Ch2.2
- Invertible matrix theorem for square matrices
  - Lay Ch2.3, Ch2.9