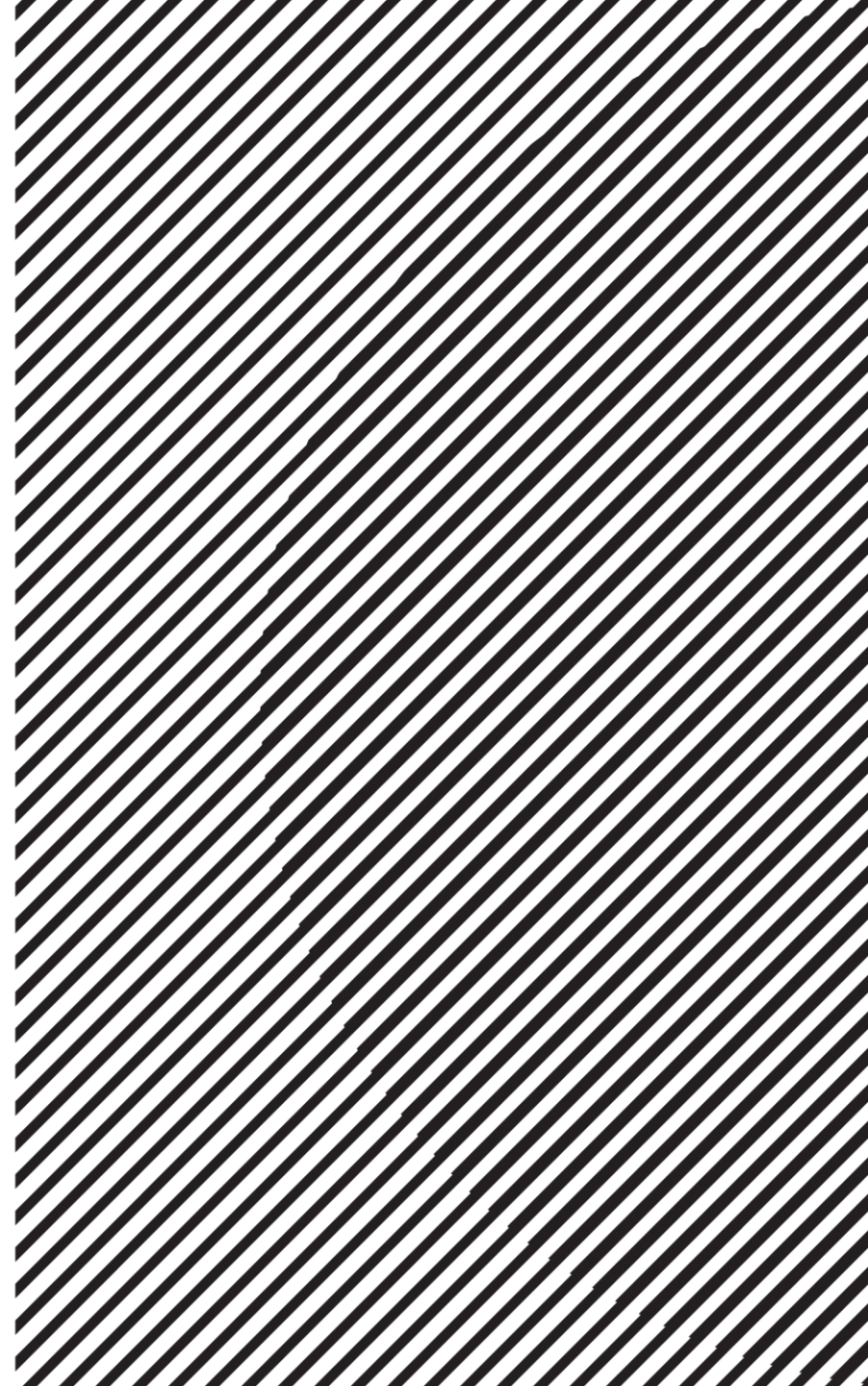


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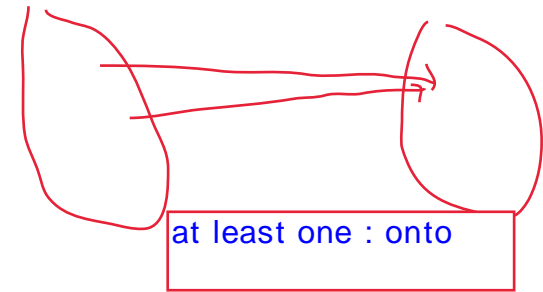
# Linear Algebra

주재걸  
고려대학교 컴퓨터학과

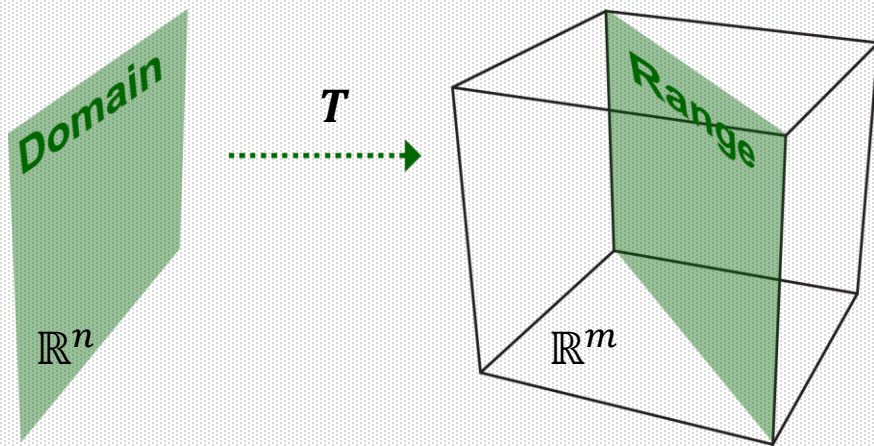


# ONTO and ONE-TO-ONE

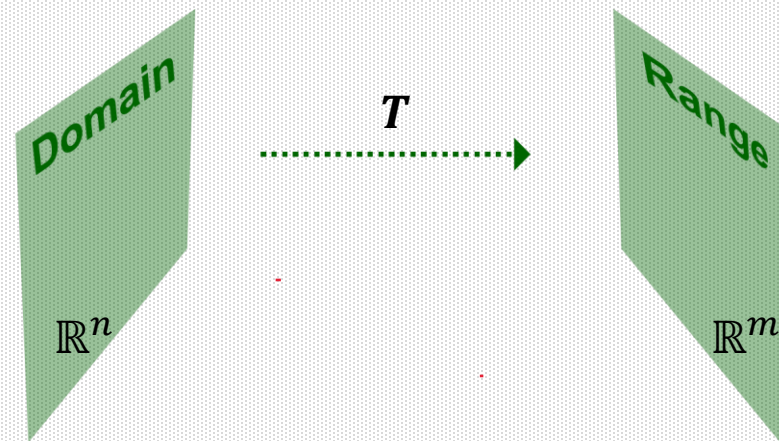
=



- **Definition:** A mapping  $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  is said to be **onto**  $\mathbb{R}^m$  if each  $\mathbf{b} \in \mathbb{R}^m$  is the image of **at least** one  $\mathbf{x} \in \mathbb{R}^n$ . That is, the range is equal to the co-domain.



$T$  is NOT onto  $\mathbb{R}^m$



$T$  is onto  $\mathbb{R}^m$

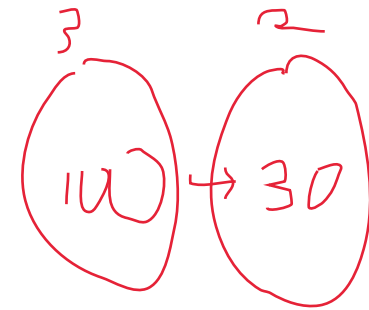
# ONTO and ONE-TO-ONE



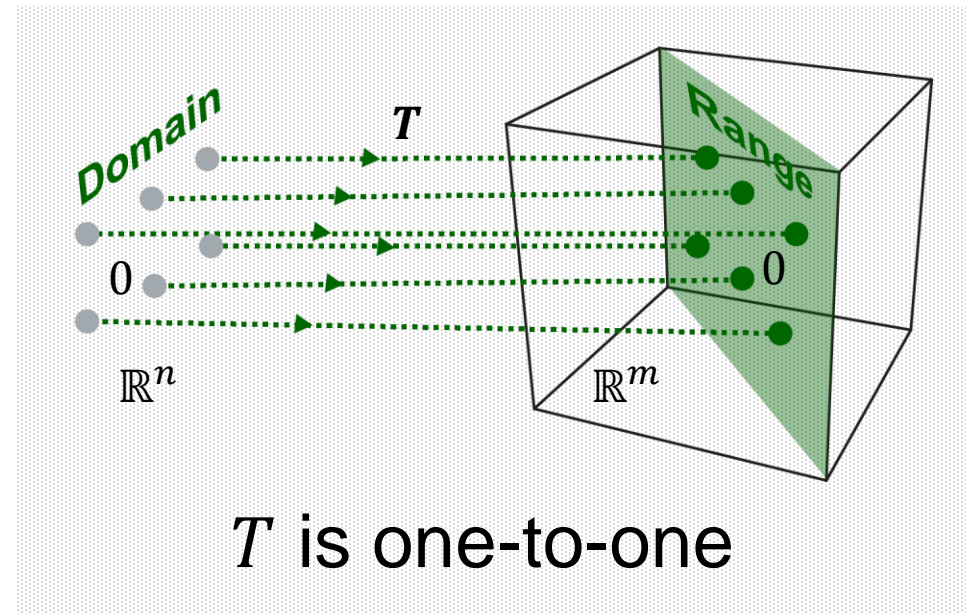
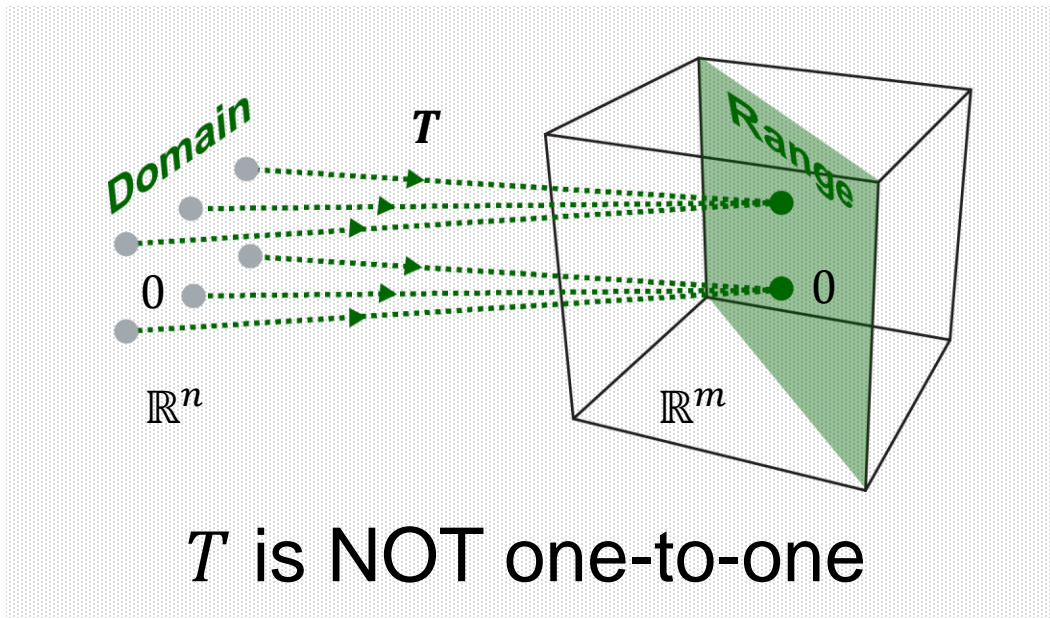
ONTO  
3 --> 2

2 --> 3  
ONTO가

3 --> 2  
ONE-to-ONE가

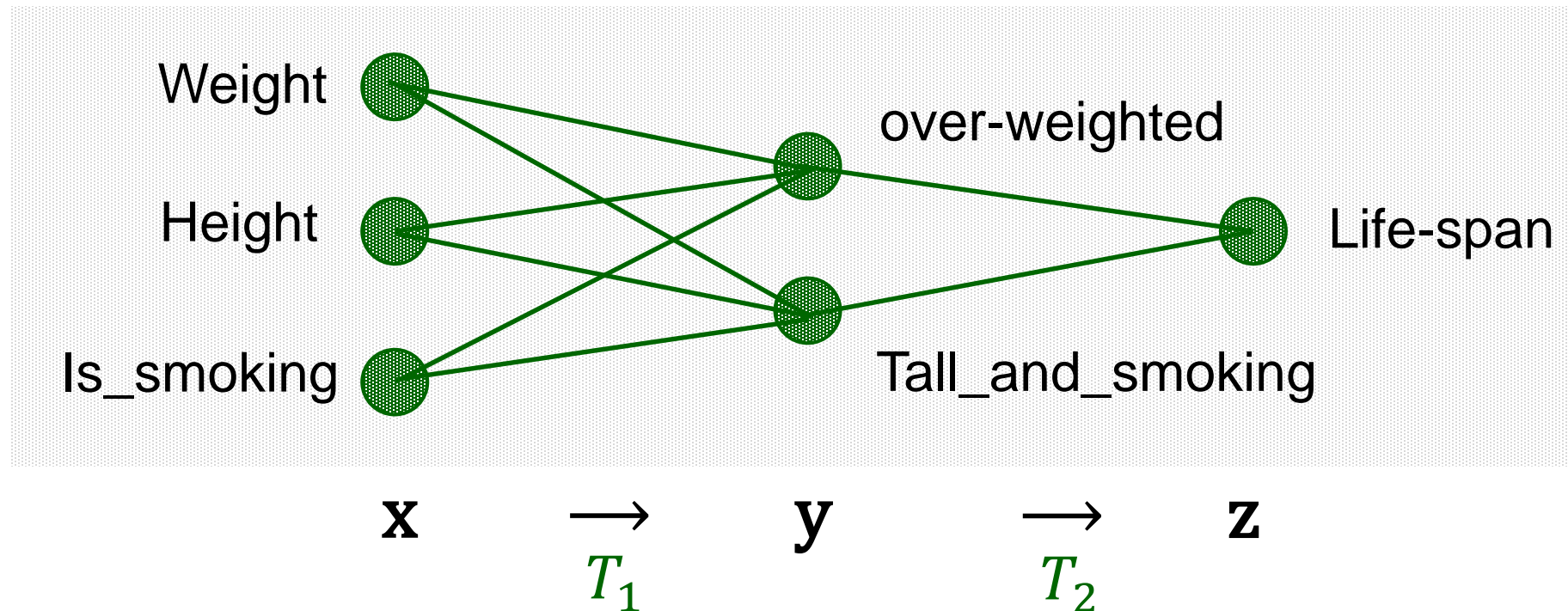


- Definition:** A mapping  $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  is said to be **one-to-one** if each  $\mathbf{b} \in \mathbb{R}^m$  is the image of **at most** one  $\mathbf{x} \in \mathbb{R}^n$ .  
 That is, each output vector in the range is mapped by only one input vector, no more than that.



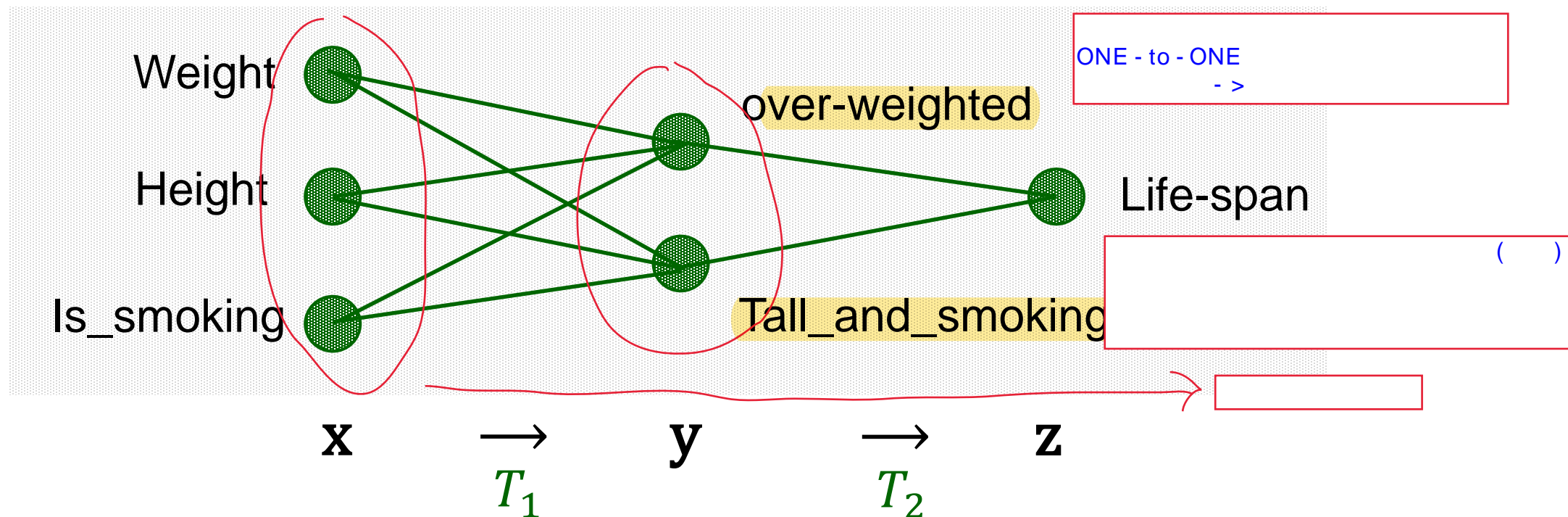
# Neural Network Example

- Fully-connected layers



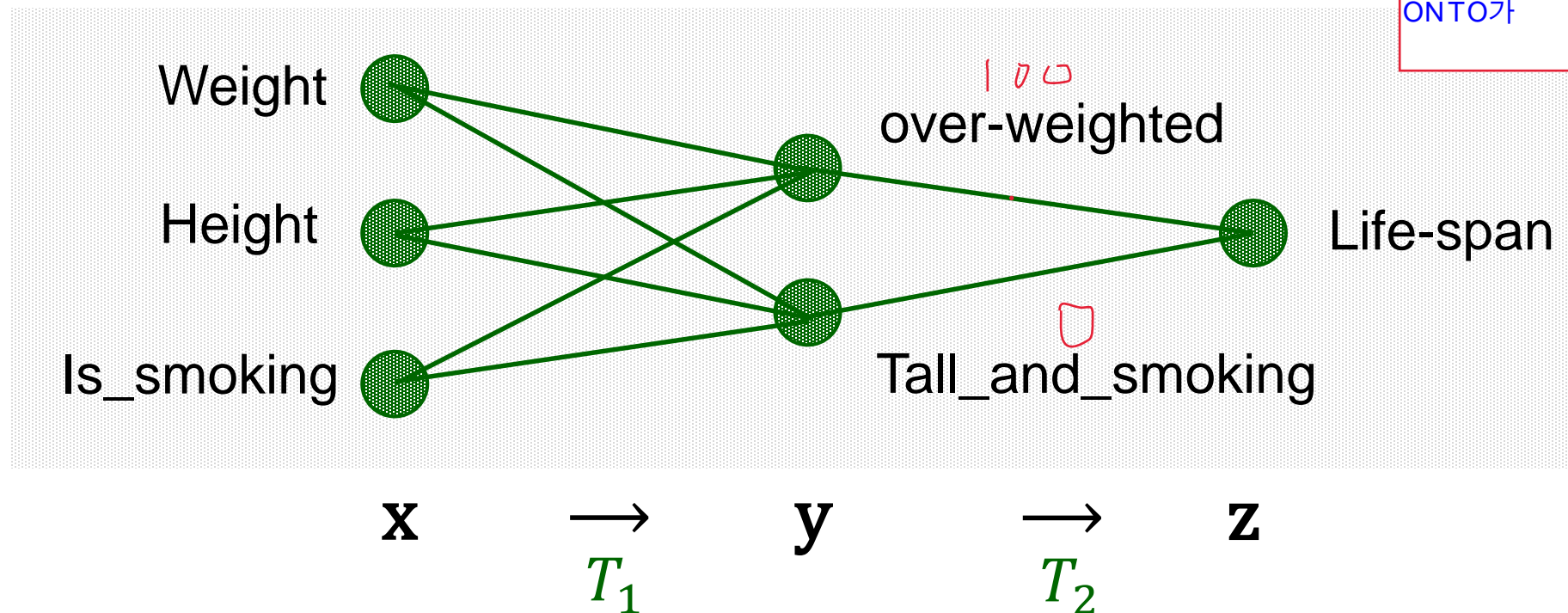
# Neural Network Example: ONE-TO-ONE

- Will there be many (or unique) people mapped to the same (over\_weighted, tall\_and\_smoking)?



# Neural Network Example: ONTO

- Is there any (over\_weighted, tall\_and\_smoking) that does not exist at all?



3 2  
ONTO

ONTO가  
가

Tall\_and\_smoking 0  
ONTO가 .



# ONTO and ONE-TO-ONE

- Let  $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  be a linear transformation, i.e.,

$$T(\mathbf{x}) = A\mathbf{x} \text{ for all } \mathbf{x} \in \mathbb{R}^n.$$

- $T$  is **one-to-one** if and only if the columns of  $A$  are **linearly independent**.
- $T$  maps  $\mathbb{R}^n$  **onto**  $\mathbb{R}^m$  if and only if the columns of  $A$  **span**  $\mathbb{R}^m$ .

# ONTO and ONE-TO-ONE

- **Example:**

$$\text{Let } T(\mathbf{x}) = A\mathbf{x} = \begin{bmatrix} 2 & 0 \\ -1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Handwritten notes:  $2 \rightarrow 3$  (above the matrix), a red line through the 2 in the first row, first column, and three red 0s to the right of the matrix.

- Is  $T$  one-to-one? Yes
- Does  $T$  map  $\mathbb{R}^2$  onto  $\mathbb{R}^3$ ? Yes



# ONTO and ONE-TO-ONE

- **Example:**

$$\text{Let } T(\mathbf{x}) = A\mathbf{x} = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 3 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Handwritten annotations: A red arrow points from the element 5 in the first row of the matrix to the element 2 in the second row. To the right of the vector  $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ , there are two red circles, one next to  $x_2$  and one next to  $x_3$ . Above the matrix, the text  $3 \rightarrow 2$  is written in red.

- Is  $T$  one-to-one?  $\checkmark$
- Does  $T$  map  $\mathbb{R}^3$  onto  $\mathbb{R}^2$ ?  $\gamma$



# Further Study

- Gaussian elimination, row reduction, echelon form
  - Lay Ch1.2,
- LU factorization: efficiently solving linear systems
  - Lay Ch2.5
- Computing invertible matrices
  - Lay Ch2.2
- Invertible matrix theorem for square matrices
  - Lay Ch2.3, Ch2.9