Some Set Properties Underlying Geometry and Physics

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Euclidean volume and some distance equations are instances of abstract sets of ordered combinations (n-tuples). The same sets of n-tuples include another combinatorial property limiting a set to 3 members, e.g., a set of 3 distance dimensions and a set of 3 non-distance dimensions (time, mass, and charge). Where the space near each local coordinate point is 6D Euclidean, each Cartesian axis distance unit corresponds 1-1 to axis units of time, mass, and charge. Unit ratios allows short and simple derivations of gravity, charge, electromagnetic, relativity, and quantum physics equations and constants. The ratios also allow simple quantum extensions to classical and relativity equations. All the proofs are verified in Rocq.

[1]

CONTENTS

Keywords: mathematical physics, combinatorics, set theory, distance measure, inner product, gravity, charge, electromagnetism, relativity, quantum physics.

INTRODUCTION

Classical physics equations assume Euclidean space. And relativity assumes a pseudo-Riemann geometry (inner product space), where the space near each local coordinate point is Euclidean[4][5]. But mathematical analysis defines Euclidean volume, distance, and inner product, where the definitions are justified by finger-pointing to Euclidean and Cartesian geometry[2][3].

In this article, the Euclidean volume function and some distance functions, including the inner product and Euclidean distance, are proved to be instances of abstract sets of ordered combinations (n-tuples). Deriving Euclidean volume and distance equations without using the geometry notions of: interval, line, side, angle, length/distance, area, or volume provides an analytic perspective (set, sequence, limit, and combinatorial perspective), which will be shown to have applications to geometry and physics.

The rest of this introduction is an overview. The proofs, in this article, have been verified using using the Rocq proof verification system [6]. The formal proofs are in the Rocq files, "euclidrelations.v" and "threed.v," which are included as ancillary files.

Let $|x_i|$ be the cardinal of (number of elements in) the countable set, x_i. And let v_c be the integer number of ordered combinations (n-tuples) of the members of $x_1, \cdots, x_n$. The number of n-tuples, v_c, will be proved imply the Euclidean volume equation:

$$\forall\, v_c, d_c, |x_i| \in \{0,\ \mathbb{N}\},\ \ x_i \in \{x_1, \cdots, x_n\},$$
$$v_c = \textstyle\prod_{i=1}^{n} |x_i| \quad \Rightarrow \quad v = \prod_{i=1}^{n} s_i,\ \ s_i, v \in \mathbb{R}. \quad (1)$$

For all $n > 1$, there are an infinite number of possible domain values, $s_1, \cdots, s_n$, that multiplied yield the same range value, v. Inferring a domain value, d, from v, requires an inverse (bijective) function, $d = f_n^{-1}(v)$ and $v = f_n(d)$. For n-tuples, there is one bijective function that satisfies the $n = 1$ case: $v_c = |x_1| = d_c^1$. Generalizing to all n:

$$\exists\, d_c, v_c, |x_i| \in \{0, \mathbb{N}\} : \; v_c = \prod_{i=1}^{n} |x_i| = \prod_{i=1}^{n} d_c = d_c^n. \tag{2}$$

A set of n-tuples being the union of disjoint subsets of n-tuples implies that the domain value, d_c, is also the inverse function of the sum of n-tuples, where it will be proved that:

$$d_c^n = \sum_{i=1}^{m} v_{c_i} = \sum_{i=1}^{m} (\prod_{j=1}^{n} |x_{i,j}|) \quad \Rightarrow \quad d^n = \sum_{i=1}^{m} (\prod_{j=1}^{n} s_{i,j}). \tag{3}$$

Where each $s_{i,j}$ is $\pm$-signed, the $n = 2$ case is the vector inner product.

Where each v_{c_i} is also the bijective function, $v_{c_i} = d_{c_i}^n$:

$$d_c^n = \sum_{i=1}^{m} d_{c_i}^n \quad \Rightarrow \quad d^n = \sum_{i=1}^{m} d_i^n. \tag{4}$$

$|d|$ is the p-norm (Minkowski distance) [7], which will be proved to imply the metric space properties [3]. The $n = 2$ case is the Euclidean distance.

If the proofs are valid, then volume and distance are instances of sets of ordered combinations (n-tuples). But these same sets of n-tuples include another combinatorial (permutation) property. The union, intersection, multiplication, and addition operations defining the total number of n-tuples and the corresponding volume and distance equations are commutative.

The commutative property allows sequencing n number of set members in any one of $n!$ possible sequences. Repeatable sequencing requires defining a sequential order on the set. Further, the *only* sequence that allows starting with any set member and sequencing through all n set members is a cyclic sequence.

Repeatable sequencing of a cyclic sequence in any one of $n!$ possible sequences, is a symmetry, where every set member is either an *immediate* cyclic successor or an *immediate* cyclic predecessor (sequentially adjacent) to every other set member, which is, herein, referred to as an "immediate symmetric" cyclic sequence (ISCS). An ISCS will be proved to have $n \leq 3$ members.

Application to physics uses the following 3 hypotheses:

1. **ISCS:** Physical distance is an ISCS of 3 dimensions, $\{r_1, r_2, r_3\}$, and $\{t\ (time), m\ (mass), q\ (charge)\}$ is the ISCS of "non-distance" dimensions, each dimension $\subseteq \mathbb{R}$. Physical space is 6-dimensional: r_1-r_2-r_3-t-m-q.

2. **Cartesian:** The space near each local coordinate point is Euclidean, where each local coordinate point is the origin of a Cartesian grid and for each Cartesian axis unit interval length, r_p, of distance, r, there is a constant Cartesian axis unit interval length: t_p of time, t; m_p of mass, m; and q_p of charge, q, such that: $r = (r_p/t_p)t = (r_p/m_p)m = (r_p/q_p)q$. And let: $r_p/t_p = c_t$, $r_p/m_p = c_m$, and $r_p/q_p = c_q$.

3. **Maximum ratios** The Cartesian axis unit ratios, c_t, c_m, and c_q are the largest ratios. For example, the speed of light is limited to c_t.

The proofs and the 3 direct proportion ratios, c_t, c_m, and c_q, are used to provide simple derivations of: the gravitational constant, $G = c_m c_t^2$, the Newton, Gauss, and Poisson gravity equations [8][9], Coulomb's charge force [9] and charge constant, $k_e = c_q^2 c_t^2 / c_m$, the special relativity equations[4][5], the Schwarzschild time dilation and black hole metric equations [10][11] (illustrating a simplified method of finding solutions to Einstein's general relativity equations), the Gauss, Lorentz, and Faraday electromagnetic equations, the vacuum permittivity, ε_0, and vacuum permeability, μ_0, constants [9]. The derivations from the ratios will show that G, k_e, ε_0, μ_0, and $\hbar$ are **not** fundamental (atomic) constants.

Algebraic manipulation of the 3 direct proportion ratios yields 3 inverse proportion ratios, $r = t_p r_p / t = m_p r_p / m = q_p r_p / q$, letting $k_t = t_p r_p$, $k_m = m_p r_p$, and $k_q = q_p r_p$. The direct and inverse proportion ratios are used to derive the Planck relation [9][12] and the reduced Planck constant, $\hbar = k_m c_t$. The values of k_t, k_m, and k_q are derived from $\hbar$, c_t, c_m, and c_q.

The values of r_p, t_p, m_p, and q_p are calculated from the ratios and found to be the Planck units. The fine structure electron coupling constant, α, is derived, in this article, as the ratio of two subtypes of force that reduces to the ratio, $\alpha = q_e^2/q_p^2$, which is simpler and more elucidating than the standard equation, $\alpha = q_e^2/4\pi\varepsilon_0\hbar c$ [13].

The ratios and Planck relation are used to derive the Compton wavelength, the position-space Schrödenger, and the Dirac wave equations [9][14][15]. And, finally, the inverse proportion ratios are used to add quantum extensions to some general relativity and classical physics equations.

RULER MEASURE AND CONVERGENCE

Definition .1. Ruler measure, $M = \sum_{i=1}^{p} \kappa = p\kappa$, where $\forall\, s, \kappa \in \mathbb{R}$, $\quad 0 < \kappa \leq 1, \quad (p = floor(s/\kappa) \quad \vee \quad p = ceiling(s/\kappa))$.

Theorem .2. *Ruler convergence:* $M = \lim_{\kappa \to 0} p\kappa = s$.

The formal proof, "limit_c_0_M_eq_exact_size," is in the file, euclidrelations.v.

Proof. (epsilon-delta proof)
By definition of the floor function, $floor(x) = max(\{y : y \leq x,\ y \in \mathbb{Z},\ x \in \mathbb{R}\})$:

$$p = floor(s/\kappa) \quad \wedge \quad 0 \leq |floor(s/\kappa) - s/\kappa| < 1 \\ \Rightarrow \quad |p - s/\kappa| < 1. \quad (5)$$

Multiply both sides of inequality 5 by κ:

$$\forall\, 0 < \kappa \leq 1, \quad |p - s/\kappa| < 1 \\ \Rightarrow \quad |p\kappa - s| < |\kappa| = |\kappa - 0|. \quad (6)$$

$$\forall\, \epsilon = \delta \quad \wedge \quad |p\kappa - s| < |\kappa - 0| < \delta \\ \Rightarrow \quad |\kappa - 0| < \delta \ \wedge\ |p\kappa - s| < \delta = \epsilon \\ := \quad M = \lim_{\kappa \to 0} p\kappa = s. \quad \square \quad (7)$$

The following is an example of ruler convergence where, $s = \pi \Rightarrow p = floor(s/\kappa) \Rightarrow p\kappa = 3.1_{\kappa=10^{-1}}$, $3.14_{\kappa=10^{-2}}$, $3.141_{\kappa=10^{-3}}, ..., \pi_{\lim_{\kappa\to 0}}$.

Lemma .3. $\forall\, n \geq 1,\ 0 < \kappa \leq 1 : \lim_{\kappa\to 0} \kappa^n = \lim_{\kappa \to 0} \kappa$.

Proof. The formal proof , "lim_c_to_n_eq_lim_c," is in the Rocq file, euclidrelations.v.

$$n \geq 1 \quad \wedge \quad 0 < \kappa \leq 1 \quad \Rightarrow \quad 0 < \kappa^n < \kappa \\ \Rightarrow \quad |\kappa - \kappa^n| < |\kappa| = |\kappa - 0|. \quad (8)$$

$$\forall\, \epsilon = \delta \quad \wedge \quad |\kappa - \kappa^n| < |\kappa - 0| < \delta \\ \Rightarrow \quad |\kappa - 0| < \delta \quad \wedge \quad |\kappa - \kappa^n| < \delta = \epsilon \\ := \quad \lim_{\kappa\to 0} \kappa^n = 0. \quad (9)$$

$$\lim_{\kappa\to 0} \kappa^n = 0 \quad \wedge \quad \lim_{\kappa \to 0} \kappa = 0 \\ \Rightarrow \quad \lim_{\kappa\to 0} \kappa^n = \lim_{\kappa\to 0} \kappa. \quad \square \quad (10)$$

VOLUME

Euclidean volume

Theorem .4. *Euclidean volume,*

$$\forall\, v_c, d_c, |x_i| \in \{0,\ \mathbb{N}\},\ \ x_i \in \{x_1, \cdots, x_n\}, \\ v_c = \textstyle\prod_{i=1}^n |x_i| \quad \Rightarrow \quad v = \prod_{i=1}^n s_i, \quad s_i, v \in \mathbb{R}. \quad (11)$$

The formal proof, "Euclidean_volume," is in the Rocq file, euclidrelations.v.

Proof.

$$v_c = \textstyle\prod_{i=1}^n |x_i| \quad \Leftrightarrow \quad v_c\kappa = (\prod_{i=1}^n |x_i|)\kappa \\ \Leftrightarrow \quad \lim_{\kappa\to 0} v_c\kappa = \lim_{\kappa\to 0}(\prod_{i=1}^n |x_i|)\kappa. \quad (12)$$

Apply the ruler (.1) and ruler convergence (.2) to equation 12:

$$\exists\, v, \kappa \in \mathbb{R} : v_c = floor(v/\kappa) \quad \Rightarrow \quad v = \lim_{\kappa\to 0} v_c\kappa \quad \wedge \\ \lim_{\kappa\to 0} v_c\kappa = \lim_{\kappa\to 0}(\textstyle\prod_{i=1}^n |x_i|)\kappa \\ \Rightarrow \quad v = \lim_{\kappa\to 0}(\prod_{i=1}^n |x_i|)\kappa. \quad (13)$$

Apply lemma .3 to equation 13:

$$v = \lim_{\kappa\to 0}(\textstyle\prod_{i=1}^n |x_i|)\kappa \quad \wedge \quad \lim_{\kappa\to 0} \kappa^n = \lim_{\kappa\to 0} \kappa \\ \Rightarrow \quad v = \lim_{\kappa\to 0}(\prod_{i=1}^n |x_i|)\kappa^n = \lim_{\kappa\to 0}(\prod_{i=1}^n(|x_i|\kappa)). \quad (14)$$

Apply the ruler (.1) and ruler convergence (.2) to s_i:

$$\exists\, s_i, \kappa \in \mathbb{R} : floor(s_i/\kappa) = |x_i| \quad \Rightarrow \quad \lim_{\kappa\to 0}(|x_i|\kappa) = s_i. \quad (15)$$

$$v = \lim_{\kappa\to 0}(\textstyle\prod_{i=1}^n |x_i|\kappa) \quad \wedge \quad \lim_{\kappa\to 0}(|x_i|\kappa) = s_i \\ \Leftrightarrow \quad v = \prod_{i=1}^n s_i \quad \square \quad (16)$$

Sum of volumes

Lemma .5. *The number of n-tuples, v_c, is the sum of the number of n-tuples, v_{c_i}, in each subset of n-tuples, implies a volume is the sum of volumes,*

$$v_c = \textstyle\sum_{i=1}^m v_{c_i} \quad \Rightarrow \quad v = \sum_{i=1}^m v_i, \quad v, v_i \in \mathbb{R}.$$

The formal proof, "sum_of _volumes," is in the Rocq file, euclidrelations.v.

Proof. From the condition of this theorem:

$$v_c = \textstyle\sum_{i=1}^m v_{c_i} \quad \Leftrightarrow \quad \lim_{\kappa\to 0} v_c\kappa = \lim_{\kappa\to 0} \sum_{j=1}^m (v_{c_i}\kappa). \quad (17)$$

Apply lemma .3 to equation 17:

$$\lim_{\kappa\to 0} v_c\kappa = \lim_{\kappa\to 0}(\textstyle\sum_{j=1}^m v_{c_i})\kappa \quad \wedge \\ \lim_{\kappa\to 0} \kappa^n = \lim_{\kappa\to 0} \kappa \\ \Leftrightarrow \quad \lim_{\kappa\to 0} v_c\kappa = \lim_{\kappa\to 0} \sum_{j=1}^m (v_{c_i}\kappa). \quad (18)$$

Apply the ruler (.1) and ruler convergence theorem (.2) to equation 18:

$$\exists\, v, v_i :\ v = \lim_{\kappa\to 0} v_c\kappa \quad \wedge \\ \lim_{\kappa\to 0} v_c\kappa = \lim_{\kappa\to 0} \textstyle\sum_{j=1}^m (v_{c_i}\kappa) \\ \Rightarrow \quad v == \lim_{\kappa\to 0} \sum_{j=1}^m (v_{c_i}\kappa). \quad (19)$$

Apply the ruler (.1) and ruler convergence theorem (.2) to equation 19:

$$v == \lim_{\kappa\to 0} \textstyle\sum_{j=1}^m (v_{c_i}\kappa) \quad \wedge \\ \exists\, v_i, v_{c_i} :\ v_i = \lim_{\kappa\to 0} v_{c_i}\kappa \\ \Rightarrow \quad v = \sum_{j=1}^m v_i. \quad \square \quad (20)$$

DISTANCE

Definition .6. Bijective, countable domain value, d_c:

$$\forall\, v_c, d_c, |x_i| \in \{0,\ \mathbb{N}\},\ \ x_i \in \{x_1, \cdots, x_n\}, \quad v_c = \prod_{i=1}^{n} |x_i| = \prod_{i=1}^{n} d_c = d_c^n. \quad (21)$$

Sum of volumes distance

Theorem .7. *Sum of volumes distance:*

$$d_c^n = v_c = \sum_{i=1}^{m} v_{c_i} \quad \Rightarrow \quad d^n = \sum_{i=1}^{m} (\prod_{j=1}^{n} s_{i_j}).$$

The formal proof, "sum_of _volumes _distance," is in the Rocq file, euclidrelations.v.

Proof. From the sum of volumes lemma .5 and the Euclidean volume theorem .4:

$$d_c^n = \sum_{i=1}^{m} v_{c_i} \quad \Rightarrow \quad d^n = \sum_{i=1}^{m} v_i \quad \wedge \quad v_i = \prod_{j=1}^{n} s_{i_j} \quad \Rightarrow \quad d^n = \sum_{i=1}^{m} (\prod_{j=1}^{n} s_{i_j}). \quad \square \quad (22)$$

Minkowski distance (p-norm)

Theorem .8. *Minkowski distance (p-norm):*

$$d_c^n = v_c = \sum_{i=1}^{m} v_{c_i} = \sum_{i=1}^{m} d_{c_i}^n \quad \Leftrightarrow \quad d^n = \sum_{i=1}^{m} d_i^n.$$

The formal proof, "Minkowski_distance," is in the Rocq file, euclidrelations.v.

Proof. From the sum of volumes distance theorem .7 and the Euclidean volume theorem .4:

$$d_c^n = v_c = \sum_{i=1}^{m} v_{c_i} \quad \Rightarrow \quad d^n = v = \sum_{i=1}^{m} v_i \quad \wedge \quad v_i = \prod_{j=1}^{n} d_i = d_i^n \quad \Rightarrow \quad d^n = \sum_{i=1}^{m} d_i^n \quad \square \quad (23)$$

Distance inequality

The formal proof, distance_inequality, is in the Rocq file, euclidrelations.v.

Theorem .9. *Distance inequality*

$$\forall\, n \in \mathbb{N},\ \ v_a, v_b \geq 0 :\ \ (v_a + v_b)^{1/n} \leq v_a^{1/n} + v_b^{1/n}.$$

Proof. Expand $(v_a^{1/n} + v_b^{1/n})^n$ using the binomial expansion:

$$\forall\, v_a, v_b \geq 0 : \quad v_a + v_b \leq v_a + v_b + \sum_{i=1}^{n} \binom{n}{k} (v_a^{1/n})^{n-k} (v_b^{1/n})^k + \sum_{i=1}^{n} \binom{n}{k} (v_a^{1/n})^k (v_b^{1/n})^{n-k} = (v_a^{1/n} + v_b^{1/n})^n. \quad (24)$$

Take the n^{th} root of both sides of the inequality 24:

$$\forall\, v_a, v_b \geq 0,\ \ n \in \mathbb{N} : \quad v_a + v_b \leq (v_a^{1/n} + v_b^{1/n})^n \quad \Rightarrow \quad (v_a + v_b)^{1/n} \leq v_a^{1/n} + v_b^{1/n}. \quad \square \quad (25)$$

Distance sum inequality

The formal proof, distance_sum_inequality, is in the Rocq file, euclidrelations.v.

Theorem .10. *Distance sum inequality*

$$\forall\, m, n \in \mathbb{N},\ \ a_i, b_i \geq 0 : \quad (\sum_{i=1}^{m} (a_i^n + b_i^n))^{1/n} \leq (\sum_{i=1}^{m} a_i^n)^{1/n} + (\sum_{i=1}^{m} b_i^n)^{1/n}.$$

Proof. Apply the distance inequality (.9):

$$\forall\, m, n \in \mathbb{N},\ \ v_a, v_b \geq 0 :\ \ v_a = \sum_{i=1}^{m} a_i^n \quad \wedge \quad v_b = \sum_{i=1}^{m} b_i^n \quad \wedge \quad (v_a + v_b)^{1/n} \leq v_a^{1/n} + v_b^{1/n}$$
$$\Rightarrow \quad ((\sum_{i=1}^{m} a_i^n) + (\sum_{i=1}^{m} b_i^n))^{1/n} = (\sum_{i=1}^{m} (a_i^n + b_i^n))^{1/n} \leq (\sum_{i=1}^{m} a_i^n)^{1/n} + (\sum_{i=1}^{m} b_i^n)^{1/n}. \quad \square \quad (26)$$

Metric Space

All Minkowski distances (p-norms) imply the metric space properties. The formal proofs: triangle_inequality, symmetry, non_negativity, and identity_of_indiscernibles are in the Rocq file, euclidrelations.v.

Theorem .11. *Triangle Inequality:*
$d(s_1, s_2) = (\sum_{i=1}^{2} s_i^p)^{1/p} \Rightarrow d(u, w) \leq d(u, v) + d(v, w).$

Proof. $\forall\, p \geq 1, \quad k > 1, \quad u = s_1, \quad w = s_2, \quad v = w/k$:

$$(u^p + w^p)^{1/p} \leq ((u^p + w^p) + 2v^p)^{1/p} = ((u^p + v^p) + (v^p + w^p))^{1/p}. \quad (27)$$

Apply the distance inequality (.9) to the inequality 27:

$$(u^p + w^p)^{1/p} \leq ((u^p + v^p) + (v^p + w^p))^{1/p} \quad \wedge \quad (v_a + v_b)^{1/n} \leq v_a^{1/n} + v_b^{1/n} \quad \wedge \quad v_a = u^p + v^p \quad \wedge \quad v_b = v^p + w^p$$
$$\Rightarrow \quad (u^p + w^p)^{1/p} \leq ((u^p + v^p) + (v^p + w^p))^{1/p} \leq (u^p + v^p)^{1/p} + (v^p + w^p)^{1/p}$$
$$\Rightarrow \quad d(u, w) = (u^p + w^p)^{1/p} \leq (u^p + v^p)^{1/p} + (v^p + w^p)^{1/p} = d(u, v) + d(v, w). \quad \square \quad (28)$$

Theorem .12. *Symmetry:*
$d(s_1, s_2) = (\sum_{i=1}^{2} s_i^p)^{1/p} \quad \Rightarrow \quad d(u, v) = d(v, u).$

Proof. By the commutative law of addition:

$$\forall\, p\ :\ p \geq 1, \quad d(s_1, s_2) = (\sum_{i=1}^{2} s_i^p)^{1/p} = (s_1^p + s_2^p)^{1/p} \quad \Rightarrow \quad d(u, v) = (u^p + v^p)^{1/p} = (v^p + u^p)^{1/p} = d(v, u). \quad \square \quad (29)$$

Theorem .13. *Non-negativity:*
$d(s_1, s_2) = (\sum_{i=1}^{2} s_i^p)^{1/p} \Rightarrow d(u, w) \geq 0.$

Proof. By definition, the length of an interval is always ≥ 0:

$$\forall\ [a_1, b_1],\ [a_2, b_2], \quad u = b_1 - a_1,\ v = b_2 - a_2, \\ \Rightarrow \quad u \geq 0,\ v \geq 0. \quad (30)$$

$$p \geq 1,\ \ u, v \geq 0 \quad \Rightarrow \quad d(u, v) = (u^p + v^p)^{1/p} \geq 0. \quad (31)$$

□

Theorem .14. *Identity of Indiscernibles:* $d(u, u) = 0$.

Proof. From the non-negativity property (.13):

$$d(u, w) \geq 0 \quad \wedge \quad d(u, v) \geq 0 \quad \wedge \quad d(v, w) \geq 0 \\ \Rightarrow \quad \exists\, d(u, w) = d(u, v) = d(v, w) = 0. \quad (32)$$

$$d(u, w) = d(v, w) = 0 \quad \Rightarrow \quad u = v. \quad (33)$$

$$d(u, v) = 0 \quad \wedge \quad u = v \quad \Rightarrow \quad d(u, u) = 0. \quad (34)$$

□

Properties limiting a set to at most 3 members

The following definitions and proof use first order logic. A Horn clause-like expression is used, here, to make the proof easier to read. By convention, the proof goal is on the left side and supporting facts are on the right side of the implication sign ($\leftarrow$). The formal proofs in the Rocq file threed.v are:

`Lemmas: adj111, adj122, adj212, adj123, adj133, adj233, adj213, adj313, adj323, and not_all_mutually_adjacent_gt_3.`

Definition .15. Immediate Cyclic Successor of m is n:

$$\forall\ x_m, x_n \in \{x_1, \cdots, x_{setsize}\} : \\ Successor(m, n, setsize) \\ \leftarrow \quad (m = setsize \wedge n = 1) \quad \vee \quad (n = m{+}1 \leq setsize). \\ (35)$$

Definition .16. Immediate Cyclic Predecessor of m is n:

$$\forall\ x_m, x_n \in \{x_1, \cdots, x_{setsize}\} : \\ Predecessor(m, n, setsize) \\ \leftarrow \quad (m = 1 \wedge n = setsize) \quad \vee \quad (n = m - 1 \geq 1). \\ (36)$$

Definition .17. Adjacent: Member m is sequentially adjacent to member n if the immediate cyclic successor of m is n or the immediate cyclic predecessor of m is n. Notionally:

$$\forall\ x_m, x_n \in \{x_1, \cdots, x_{setsize}\} : Adjacent(m, n, setsize) \\ \leftarrow Successor(m, n, setsize) \vee Predecessor(m, n, setsize). \\ (37)$$

Definition .18. Immediate Symmetric (every set member is sequentially adjacent to every other member):

$$\forall\ x_m, x_n \in \{x_1, \cdots, x_{setsize}\} : \quad Adjacent(m, n, setsize). \\ (38)$$

Theorem .19. *An immediate symmetric cyclic set is limited to at most 3 members.*

Proof.
Every member is adjacent to every other member, where $setsize \in \{1,\ 2,\ 3\}$:

$$Adjacent(1, 1, 1) \leftarrow Successor(1, 1, 1) \leftarrow \\ (m = setsize \wedge n = 1). \quad (39)$$

$$Adjacent(1, 2, 2) \leftarrow Successor(1, 2, 2) \leftarrow \\ (n = m + 1 \leq setsize). \quad (40)$$

$$Adjacent(1, 2, 3) \leftarrow Successor(1, 2, 3) \leftarrow \\ (n = m + 1 \leq setsize). \quad (41)$$

$$Adjacent(2, 1, 3) \leftarrow Predecessor(2, 1, 3) \leftarrow \\ (n = m - 1 \geq 1). \quad (42)$$

$$Adjacent(3, 1, 3) \leftarrow Successor(3, 1, 3) \leftarrow \\ (n = setsize \wedge m = 1). \quad (43)$$

$$Adjacent(1, 3, 3) \leftarrow Predecessor(1, 3, 3) \leftarrow \\ (m = 1 \wedge n = setsize). \quad (44)$$

$$Adjacent(2, 3, 3) \leftarrow Successor(2, 3, 3) \leftarrow \\ (n = m + 1 \leq setsize). \quad (45)$$

$$Adjacent(3, 2, 3) \leftarrow Predecessor(3, 2, 3) \leftarrow \\ (n = m - 1 \geq 1). \quad (46)$$

Member 2 is the only immediate successor of member 1 for all $setsize \geq 3$, which implies member 3 is not ($\neg$) an immediate successor of member 1 for all $setsize \geq 3$:

$$\neg Successor(1, 3, setsize \geq 3) \leftarrow \\ Successor(1, 2, setsize \geq 3) \leftarrow (n = m + 1 \leq setsize). \\ (47)$$

Member $n = setsize > 3$ is the only immediate predecessor of member 1, which implies member 3 is not ($\neg$) an immediate predecessor of member 1 for all $setsize > 3$:

$$\neg Predecessor(1, 3, setsize \geq 3) \leftarrow \\ Predecessor(1, setsize, setsize > 3) \leftarrow \\ (m = 1 \wedge n = setsize > 3). \quad (48)$$

For all $setsize > 3$, some elements are not ($\neg$) sequentially adjacent to every other element (not immediate symmetric):

$$\neg Adjacent(1, 3, setsize > 3) \leftarrow \\ \neg Successor(1, 3, setsize > 3) \quad \wedge \\ \neg Predecessor(1, 3, setsize > 3). \quad \square \quad (49)$$

The Symmetric goal matches Adjacent goals 39 through 46 and fails for all "setsize" greater than three.

APPLICATIONS TO PHYSICS

Where distance is an immediate symmetric cyclic sequence (ISCS) of dimensions, the 3D proof (.19) requires more dimensions to have non-distance types (members of other sets). Let $\tau = \{t\ (time),\ m\ (mass),\ q\ (charge)\}$ be the ISCS of type "non-distance" dimensions, where for each Cartesian axis unit length, r_p, of distance, r, there are Cartesian axis unit lengths: t_p of time, t; m_p of mass, m; and q_p of charge, q, such that:

$$r = (r_p/t_p)t = (r_p/m_p)m = (r_p/q_p)q, \quad (50)$$

where c_t, c_m, and c_q are the maximum ratios:

$$c_t = r_p/t_p, \quad c_m = r_p/m_p, \quad c = c_t = r_p/q_p. \quad (51)$$

Derivation of G, and the Newton, Gauss, and Poisson gravity laws

From equation 51:

$$r = c_m m \quad \wedge \quad r = c_t t \quad \Rightarrow \quad r/(c_t t)^2 = c_m m/r^2 \\ \Rightarrow \quad r/t^2 = (c_m c_t^2)m/r^2 = Gm/r^2, \quad (52)$$

where $G = c_m c_t^2$, has the SI units: $m^3 \cdot kg^{-1} \cdot s^{-2}$ [8].

Newton's law follows from multiplying both sides of equation 52 by m:

$$r/t^2 = Gm/r^2 \ \Leftrightarrow \ F := mr/t^2 = Gm^2/r^2. \quad (53)$$

$$F = Gm^2/r^2 \wedge \forall\, m \in \mathbb{R} : \exists\, m_1, m_2 \in \mathbb{R} : m_1 m_2 = m^2 \\ \Rightarrow \quad F = Gm_1 m_2/r^2. \quad (54)$$

In this article, the following rationale for Gauss's and Poisson's laws for gravity is presented: Equation 52 relates linear (straight line) acceleration, r/t^2, to mass and distance. Gauss's gravity field, $\mathbf{g}$, and Poisson's gravity field, $-\nabla\Phi(r,t)$, relates angular (orbital) acceleration, $2\pi r/t^2$, to mass and distance. Multiplying both sides of equation 52 by 2π and differentiating yields Gauss's and Poison's gravity laws [9]:

$$\mathbf{g} = -\nabla\Phi(\vec{r}, t) = 2\pi r/t^2 = 2\pi Gm/r^2 \\ \Rightarrow \quad \nabla \cdot \mathbf{g} = \nabla^2\Phi(\vec{r}, t) = -4\pi Gm/r^3. \quad (55)$$

$$\nabla \cdot \mathbf{g} = \nabla^2\Phi(\vec{r}, t) = -4\pi Gm/r^3 \quad \wedge \quad \rho = m/r^3 \\ \Rightarrow \quad \nabla \cdot \mathbf{g} = \nabla^2\Phi(\vec{r}, t) = -4\pi G\rho. \quad (56)$$

Derivation of k_e and Coulomb's charge law

[9] From equation 51:

$$r = c_q q \quad \Rightarrow \quad r^2 = c_q^2 q^2 \quad \Rightarrow \quad c_q^2 q^2/r^2 = 1. \quad (57)$$

$$r = c_m m = c_t t \\ \Rightarrow \quad mr = ((1/c_m)r)(c_t t) = ((1/c_m)(c_t t))(c_t t) = (c_t^2/c_m)t^2 \\ \Rightarrow \quad (c_m/c_t^2)mr/t^2 = 1. \quad (58)$$

$$c_q^2 q^2/r^2 = 1 \quad \wedge \quad (c_m/c_t^2)mr/t^2 = 1 \\ \Rightarrow \quad F := mr/t^2 = (c_q^2 c_t^2/c_m)q^2/r^2 = k_e q^2/r^2, \quad (59)$$

where $k_e = c_q^2 c_t^2/c_m$, conforms to the SI units: $kg \cdot m^3 \cdot s^{-2} \cdot C^{-2} = N \cdot m^2 \cdot C^{-2}$ [9].

$$\exists\, q_1, q_2 \in \mathbb{R} :\ q_1 q_2 = q^2 \quad \wedge \quad F = k_e q^2/r^2 \\ \Rightarrow \quad F = k_e q_1 q_2/r^2. \quad (60)$$

3 direct proportion ratios: c_t, c_m, and c_q

$$c = c_t \quad \wedge \quad c \approx 2.99792458 \cdot 10^8 m\ s^{-1} \quad \Rightarrow \\ c_t \approx 2.99792458 \cdot 10^8 m\ s^{-1}. \quad (61)$$

$$G = c_m c_t^2 \quad \wedge \quad G \approx 6.67418478 \cdot 10^{-11} m^3/kg/s^2 \quad \Rightarrow \\ c_m = G/c_t^2 \approx 7.42603211 \cdot 10^{-28} m\ kg^{-1}. \quad (62)$$

$$k_e = c_q^2 c_t^2/c_m \quad \wedge \quad k_e \approx 8.9875517923 \cdot 10^9 Nm^2/C^2 \quad \Rightarrow \\ c_q = \sqrt{k_e c_m/c_t^2} \approx 8.61744282 \cdot 10^{-18} m\ C^{-1}. \quad (63)$$

3 inverse proportion ratios: k_t, k_m, and k_q

$$\begin{aligned} &r/t = r_p/t_p \quad \wedge \quad r/m = r_p/m_p \\ &\quad\Rightarrow \quad (r/t)/(r/m) = (r_p/t_p)/(r_p/m_p) \\ &\quad\Rightarrow \quad (mr)/(tr) = (m_p r_p)/(t_p r_p) \\ &\quad\Rightarrow \quad mr = m_p r_p = k_m, \quad tr = t_p r_p = k_t. \end{aligned} \tag{64}$$

$$\begin{aligned} &r/t = r_p/t_p \quad \wedge \quad r/q = r_p/q_p \\ &\quad\Rightarrow \quad (r/t)/(r/q) = (r_p/t_p)/(r_p/q_p) \\ &\quad\Rightarrow (qr)/(tr) = (q_p r_p)/(t_p r_p) \\ &\quad\Rightarrow \quad qr = q_p r_p = k_q, \quad tr = t_p r_p = k_t. \end{aligned} \tag{65}$$

Derivation of $\hbar$, h, and the Planck relation

[9][12] Applying both the direct proportion ratio (61), and inverse proportion ratio (64):

$$\begin{aligned} &r = ct \quad \wedge \quad m = k_m/r \\ &\quad\Rightarrow \quad m(ct)^2 = (k_m/r)r^2 = k_m r. \end{aligned} \tag{66}$$

$$\begin{aligned} &m(ct)^2 = k_m r \quad \wedge \quad r/t = c \quad \Rightarrow \\ &\quad E := mc^2 = k_m r/t^2 = (k_m c)(1/t) \\ &\quad\quad = \hbar\omega = \hbar\omega(2\pi/2\pi) = hf, \end{aligned} \tag{67}$$

where the reduced Planck constant, $\hbar = k_m c$, angular frequency, $\omega = 1/t$, the full Planck constant, $h = 2\pi\hbar$, and the cycles per second frequency (Hertz), $f = \omega/2\pi$. Using $\hbar \approx 1.054571817 \cdot 10^{-34}$:

$$k_m = \hbar/c_t \approx 3.51767294 \cdot 10^{-43}\ kg\ m. \tag{68}$$

$$k_t = k_m c_m/c_t \approx 8.71347873 \cdot 10^{-79}\ s\ m. \tag{69}$$

$$k_q = k_t c_t/c_q \approx 3.03133454 \cdot 10^{-53}\ C\ m. \tag{70}$$

4 quantum (Planck) units: r_p, t_p, m_p, q_p

:

$$\begin{aligned} r_p &= \sqrt{r_p^2} = \sqrt{c_t k_t} = \sqrt{c_m k_m} = \sqrt{c_q k_q} \\ &\approx 1.61624107 \cdot 10^{-35}\ m. \end{aligned} \tag{71}$$

$$t_p = r_p/c_t \approx 5.39119991 \cdot 10^{-44}\ s. \tag{72}$$

$$m_p = r_p/c_m \approx 2.17645313 \cdot 10^{-8}\ kg. \tag{73}$$

$$q_p = r_p/c_q \approx 1.87554604 \cdot 10^{-18}\ C. \tag{74}$$

Derivation of the fine structure constant, α

The ratios of two subtypes of force implies ratios of the form: $\alpha_\tau = \frac{F_{\tau_1}}{F_{\tau_2}} = \frac{K\tau_1^2/r^2}{K\tau_2^2/r^2} = \frac{\tau_1^2}{\tau_2^2}$. For example, where q_e is the elementary (electron) charge ($1.60217663 \cdot 10^{-19}\ C$), and q_p is Planck charge unit, the fine structure electron coupling constant is:

$$\alpha_q = q_e^2/q_p^2 \approx 0.0072973526. \tag{75}$$

Derivation of space-time-mass-charge

Let r be an Euclidean distance. Then by the Minkowski distance theorem (.8), $r^2 = \sum_{i=1}^m r_i^2$. Let, $r' = r_1$ and $r_v^2 = (\sum_{i=2}^m r_i^2)$. From the 3D theorem (.19) and Cartesian hypothesis (2):

$$\begin{aligned} &\forall\, \tau \in \{t, m, q\},\ r^2 = r'^2 + r_v^2,\ \exists\, \mu, \nu :\ r = \mu\tau \quad \wedge \\ &\quad r_v = \nu\tau \quad \Rightarrow \quad (\mu\tau)^2 = r'^2 + (\nu\tau)^2 \\ &\quad\Rightarrow \quad r' = \sqrt{(\mu\tau)^2 - (\nu\tau)^2} \;=\; \mu\tau\sqrt{1 - (\nu/\mu)^2}. \end{aligned} \tag{76}$$

Local frame distance, r', contracts relative to a distant observer frame distance, r, as $\nu \to \mu$:

$$\begin{aligned} &r' = \mu\tau\sqrt{1 - (\nu/\mu)^2} \quad \wedge \quad \mu\tau = r \\ &\quad\Rightarrow \quad r' = r\sqrt{1 - (\nu/\mu)^2}. \end{aligned} \tag{77}$$

A distant observer frame type, τ, dilates relative to the local observer frame type, τ', as $\nu \to \mu$:

$$\begin{aligned} &\mu\tau = r'/\sqrt{1 - (\nu/\mu)^2} \quad \wedge \quad r' = \mu\tau' \\ &\quad\Rightarrow \quad \tau = \tau'/\sqrt{1 - (\nu/\mu)^2}. \end{aligned} \tag{78}$$

Where τ is type, time, the space-like flat Minkowski spacetime event interval is:

$$\begin{aligned} &\mathrm{d}r^2 = \mathrm{d}r'^2 + \mathrm{d}r_v^2 \quad \wedge \quad \mathrm{d}r_v^2 = \mathrm{d}r_1^2 + \mathrm{d}r_2^2 + \mathrm{d}r_3^2 \quad \wedge \\ &\mathrm{d}(\mu\tau) = \mathrm{d}r \;\Rightarrow\; \mathrm{d}r'^2 = \mathrm{d}(\mu\tau)^2 - \mathrm{d}r_1^2 - \mathrm{d}r_2^2 - \mathrm{d}r_3^2. \end{aligned} \tag{79}$$

Derivation of Schwarzschild's time dilation and black hole metric

From equations 77 and 50:

$$\begin{aligned} &\sqrt{1 - (v^2/c^2)} = \sqrt{1 - (1)(v^2/c^2)} \quad \wedge \quad c_m m/r = 1 \\ &\quad\Rightarrow \quad \sqrt{1 - (v^2/c^2)} = \sqrt{1 - c_m(mv^2)/rc^2}. \end{aligned} \tag{80}$$

Where v_{escape} is the escape velocity:

$$\begin{aligned} &\sqrt{1 - (v^2/c^2)} = \sqrt{1 - c_m(mv^2)/rc^2} \quad \wedge \\ &\quad KE = mv^2/2 = mv_{escape}^2 \\ &\quad\Rightarrow \quad \sqrt{1 - (v^2/c^2)} = \sqrt{1 - 2c_m m v_{escape}^2/rc^2}. \end{aligned} \tag{81}$$

For a photon, the escape velocity, $v_{escape} = c$.

$$\sqrt{1-(v^2/c^2)} = \sqrt{1-2c_m m v_{escape}^2/rc^2}. \wedge v_{escape} = c$$
$$\Rightarrow \quad \sqrt{1-(v^2/c^2)} = \sqrt{1-2c_m mc^2/rc^2}. \quad (82)$$

Combining equation 82 with the derivation of G (54):

$$c_m c^2 = G \quad \wedge \quad \sqrt{1-(v^2/c^2)} = \sqrt{1-2c_m mc^2/rc^2}$$
$$\Rightarrow \quad \sqrt{1-(v^2/c^2)} = \sqrt{1-2Gm/rc^2}. \quad (83)$$

Combining equation 83 with equation 78 yields Schwarzschild's gravitational time dilation [10] [11]:

$$\sqrt{1-(v^2/c^2)} = \sqrt{1-2Gm/rc^2} \quad \wedge$$
$$t' = t\sqrt{1-(v^2/c^2)} \quad \Rightarrow \quad t' = t\sqrt{1-2Gm/rc^2}. \quad (84)$$

Schwarzschild defined the black hole event horizon radius, $\alpha := 2Gm/c^2$. From equations 77 and 83:

$$r' = r\sqrt{1-(v/c)^2} \quad \wedge$$
$$\sqrt{1-(v/c)^2} = \sqrt{1-2Gm/rc^2} \quad \wedge \quad \alpha := 2Gm/c^2$$
$$\Rightarrow \quad r' = r\sqrt{1-2Gm/rc^2} = r\sqrt{1-\alpha/r}. \quad (85)$$

Applying equation 85 to the time-like spacetime interval equation 79:

$$r' = r\sqrt{1-\alpha/r} \quad \wedge \quad \mathrm{d}s^2 = \mathrm{d}r'^2 - \mathrm{d}r^2 \quad \Rightarrow$$
$$\mathrm{d}s^2 = (\sqrt{1-\alpha/r}\mathrm{d}r)^2 - (\mathrm{d}r'/\sqrt{1-\alpha/r})^2$$
$$= (1-\alpha/r)\mathrm{d}r^2 - (1-\alpha/r)^{-1}\mathrm{d}r'^2. \quad (86)$$

In general relativity, $r' = r$.

$$\mathrm{d}s^2 = (1-\alpha/r)\mathrm{d}r^2 - (1-\alpha/r)^{-1}\mathrm{d}r^2 \quad \wedge$$
$$\mathrm{d}r = \mathrm{d}(ct) \quad \wedge \quad c = 1 \quad \wedge \quad r' = r$$
$$\Rightarrow \quad \mathrm{d}s^2 = (1-\alpha/r)\mathrm{d}t^2 - (1-\alpha/r)^{-1}\mathrm{d}r^2. \quad (87)$$

Using spherical coordinates to translate from 2D to 4D yields the $+ - --$ form of Schwarzschild's black hole metric [10] [11]:

$$\mathrm{d}s^2 = (1-\alpha/r)\mathrm{d}t^2 - (1-\alpha/r)^{-1}\mathrm{d}r^2$$
$$\Rightarrow \quad \mathrm{d}s^2 = (1-\alpha/r)\mathrm{d}t^2 - (1-\alpha/r)^{-1}\mathrm{d}r^2 -$$
$$r^2(\mathrm{d}\theta^2 + sin^2\theta\mathrm{d}\phi^2)$$
$$\Rightarrow \quad g_{\mu,\nu} = diag[(1-\alpha/r),\ -(1-\alpha/r)^{-1},\ -r^2(\mathrm{d}\theta^2),$$
$$-r^2(sin^2\theta\mathrm{d}\phi^2)]. \quad (88)$$

Simple method to find general relativity solutions

Einstein's field equation is:

$$G_{\mu,\nu} = \mathbf{R} + \frac{1}{2}Rg_{\mu,\nu} = \kappa T_{\mu,\nu}, \quad (89)$$

where: $G_{\mu,\nu}$ is Einstein's tensor defined as $\mathbf{R}+\frac{1}{2}Rg_{\mu,\nu}$, $\mathbf{R}$ is the Ricci curvatature, R is scalar curvature, $g_{\mu,\nu}$ is the metric tensor, $\kappa = 8\pi G/c^4$, and $T_{\mu,\nu}$ is the stress-energy tensor [4][5].

The goal of the field equation is to determine the geodesic path and acceleration of a particle caused by the distribution of mass and energy as specified in the stress-energy tensor, $T_{\mu,\nu}$. This requires solving the equation for the metric tensor, $g_{\mu,\nu}$. But the metric tensor has a complex nonlinear relation to the Ricci and scalar curvature, which makes it complicated to determine the metric. Often there are no exact solutions.

In this article, the metric, $g_{\mu,\nu}$, is determined independent of Einstein's field equation. In this case the metric tensor is the independent variable and the distribution of mass and energy in the stress-energy tensor, $T_{\mu,\nu}$, are the dependent variables.

The infinitesimal space near every coordinate point on a pseudo-Riemann surface is Euclidean. In this case, the Cartesian unit ratios (61) (64) apply, where: all physics equations derived from the ratios and special relativity equations (76) are valid around the infinitesimal space at each coordinate point.

This leads to the following steps to solve for the metric tensor, $g_{\mu,\nu}$, independent of Einstein's field equation.

Step 1) Use the ratios and relativity equations to define functions returning scalar values for each component of the metric, $g_{\nu,\mu}$, in Einstein's field equations [4][5]: All functions derived from the ratios and relativity are valid metrics. An example is the previous Schwarzschild black hole metric derivation using the ratios and special relativity equations (80).

The diagonal components the metric, $g_{\mu,\nu}$, are determined as a generalization of the flat Minkowski spacetime interval equation: $\mathrm{d}s^2 = g_{1,1}\mathrm{d}x_1^2 + \cdots + g_{4,4}\mathrm{d}x_4^2$. $\mathrm{d}s$ is an element of the geodesic line (the affine connection between geodesics). $x_1, \cdots, x_4$ are the dimension of time and the 3 dimensions of space.

Step 2) Express the Einstein field equation as 2D tensors: As shown in equation 88, the Schwarzschild metric was first derived as a 2D metric and then expanded to a 4D metric. Further, the 4D flat spacetime interval equation (79) is an instance of the 2D equation, $\mathrm{d}r'^2 = \mathrm{d}(ct)^2 - \mathrm{d}r_v^2$.

(Optional) The 2D metric tensor allows using the much simpler 2D Ricci curvature and scalar curvature. And the 2D tensors reduce the number of independent equations to solve, which can be used to set constraints on the solutions in the 4D tensors.

Step 3) One simple method to translate from 2D to 4D is to use spherical coordinates, where r and t remain unchanged and two added dimensions are the angles, ϕ, and θ. For example, the 2D Schwarzschild metric was translated to 4D using this method in equation 88. The spherical coordinates can then be translated to other types of coordinates.

Derivation of E, B, and Lorentz's law

In this article, the following rationale for Gauss's electric field is presented: Coulomb's charge force equation (59) relates linear acceleration, r/t^2, to charge and distance. Gauss's electric field, $\mathbf{E}$, relates angular (orbital or rotational) acceleration, $2\pi r/t^2$ to charge and distance:

$$F_C = mr/t^2 = k_e q^2/r^2 \\ \Rightarrow \quad \exists\, F_E \in \mathbb{R} : F_E = m(2\pi r/t^2) = 2\pi k_e q^2/r^2. \quad (90)$$

Applying the distance contraction equation, 77, to equation 90, where r is the distant observer frame of reference and r' is moving particle local frame of reference:

$$r = r'/\sqrt{1 - v^2/c^2} \quad \wedge \quad F = 2\pi k_e q^2/r^2 \\ \Rightarrow \quad F = 2\pi k_e q^2(1 - v^2/c^2)/r'^2. \quad (91)$$

$$E := 2\pi k_e q/r'^2 \;\Rightarrow\; F = q(E - (((2\pi k_e/c^2)q/r'^2))v^2). \quad (92)$$

$$B := (2\pi k_e/c^2)vq/r'^2 \quad \Rightarrow \quad F = q(E - Bv). \quad (93)$$

$$F = q(E - Bv) \quad \Rightarrow \quad \mathbf{F} = q(\mathbf{E} - \mathbf{B} \times \vec{\mathbf{v}}). \quad (94)$$

$$\mathbf{B} \times \vec{\mathbf{v}} = -(\vec{\mathbf{v}} \times \mathbf{B})) \quad \wedge \quad \mathbf{F} = q(\mathbf{E} - \mathbf{B} \times \vec{\mathbf{v}}) \\ \Rightarrow \quad \mathbf{F} = q(\mathbf{E} + \vec{\mathbf{v}} \times \mathbf{B}), \quad (95)$$

which is Lorentz law in the rest (observer on the moving particle) frame of reference.

The electric field, $E := 2\pi k_e q/r'^2$, conforms to the SI units $kg \cdot m \cdot s^{-2} \cdot C^{-1} = N \cdot C^{-1}$ and the magnetic field, $B = (2\pi k_e/c^2)vq/r'^2$, conforms to the base SI units: $kg \cdot s^{-1} \cdot C^{-1} = kg \cdot s^{-2} \cdot A^{-1} = T$.

$$B := (2\pi k_e/c^2)vq/r'^2 \quad \wedge \quad B := \mu_0 H \quad \wedge \\ \mu_0 := 4\pi k_e/c^2 \quad \Rightarrow \quad H = vq/2r'^2, \quad (96)$$

where $\mu_0 = 4\pi k_e/c^2$ conforms to the SI units $kg \cdot m \cdot C^{-2} = kg \cdot m \cdot s^{-2} A^{-2}$ and $H = vq/2r'^2$ conforms to the SI units $C \cdot s^{-1} \cdot m^{-1} = A \cdot m^{-1}$.

Derivation of ε_0 and Gauss's electric field law

From equation 92:

$$E = 2\pi k_e q/r^2 \quad \Leftrightarrow \quad \mathbf{E} = 2\pi k_e q/\vec{\mathbf{r}}^2 \\ \Rightarrow \quad \nabla \cdot \mathbf{E} = -4\pi k_e q/|\vec{\mathbf{r}}|^3. \quad (97)$$

$$\nabla \cdot \mathbf{E} = -4\pi k_e q/|\vec{\mathbf{r}}|^3 \quad \wedge \quad \varepsilon_0 := 1/4\pi k_e \quad \wedge \quad \rho = q/|\vec{\mathbf{r}}|^3 \\ \Rightarrow \quad \nabla \cdot \mathbf{E} = -\rho/\varepsilon_0, \quad (98)$$

which is Gauss's electric field law [9].

Derivation of Faraday's law

From the magnetic field equation 93, where the electric and magnetic fields are propagating at the speed, $v = c$:

$$B = (2\pi k_e/c^2)qv/r^2 \quad \wedge \quad v = c \quad \wedge \quad r = ct \\ \Rightarrow \quad B = (2\pi k_e/c^3)q/t^2. \quad (99)$$

$$B = (2\pi k_e/c^3)q/t^2 \quad \Rightarrow \quad \partial B/\partial t = -(4\pi k_e/c^3)q/t^3. \quad (100)$$

$$\partial B/\partial t = -(4\pi k_e/c^3)q/t^3 \quad \wedge \quad r = ct \\ \Rightarrow \quad \partial B/\partial t = -4\pi k_e q/r^3. \quad (101)$$

From equation 97:

$$\mathbf{E} = 2\pi k_e q/|\vec{\mathbf{r}}|^2 \quad \Rightarrow \quad \nabla \times \mathbf{E} = 4\pi k_e q/|\vec{\mathbf{r}}|^3. \quad (102)$$

Combining equations 102 and 101 yields Faraday's law [9]:

$$\nabla \times \mathbf{E} = 4\pi k_e q/|\vec{\mathbf{r}}|^3 \quad \wedge \quad \partial \mathbf{B}/\partial t = -4\pi k_e q/|\vec{\mathbf{r}}|^3 \\ \Rightarrow \quad \nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t. \quad (103)$$

Derivation of the Compton wavelength, λ

[9][12] From equations 64 and 67:

$$r = k_m/m \quad \wedge \quad h = 2\pi k_m c \quad \Rightarrow \\ \lambda = 2\pi r = 2\pi k_m/m = (2\pi k_m/m)(c/c) = h/mc. \quad (104)$$

Derivation of Schrödenger's position-space wave equation

Start with the previously derived Planck relation 67 and multiply the kinetic energy component by mc/mc:

$$mc^2 = \hbar\omega = \hbar/t \quad \Rightarrow \quad \exists\, V(r,t) : \hbar/t = \hbar/2t + V(r,t) \\ \Rightarrow \quad \hbar/t = \hbar mc/2mct + V(r,t). \quad (105)$$

And from the distance-to-time (speed of light) ratio (61):

$$\hbar/t = \hbar mc/2mct + V(r,t) \quad \wedge \quad r = ct \\ \Rightarrow \quad \hbar/t = \hbar mc^2/2mcr + V(r,t). \quad (106)$$

$$\hbar/t = \hbar mc^2/2mcr + V(r,t) \;\wedge\; \hbar/t = mc^2 \\ \Rightarrow \quad \hbar/t = \hbar^2/2mcrt + V(r,t). \quad (107)$$

$$\hbar/t = \hbar^2/2mcrt + V(r,t) \quad \wedge \quad r = ct \\ \Rightarrow \quad \hbar/t = \hbar^2/2mr^2 + V(r,t). \quad (108)$$

Multiply both sides of equation 108 by a function, $\Psi(r,t)$.

$$\hbar/t = \hbar^2/2mr^2 + V(r,t) \quad \Rightarrow \quad (\hbar/t)\Psi(r,t) = (\hbar^2/2mr^2)\Psi(r,t) + V(r,t)\Psi(r,t). \quad (109)$$

$$(\hbar/t)\Psi(r,t) = (\hbar^2/2mr^2)\Psi(r,t) + V(r,t)\Psi(r,t) \quad \wedge \quad \forall\, \Psi(r,t) :\ \partial^2\Psi(r,t)/\partial r^2 = (-1/r^2)\Psi(r,t) \quad \wedge \quad \partial\Psi(r,t)/\partial t = (i/t)\Psi(r,t) \quad \Rightarrow \quad i\hbar\partial\Psi(r,t)/\partial t = -(\hbar^2/2m)\partial^2\Psi(r,t)/\partial r^2 + V(r,t)\Psi(r,t), \quad (110)$$

which is the one-dimensional position-space Schrödenger's equation [14][12].

$$i\hbar\partial\Psi(r,t)/\partial t = -(\hbar^2/2m)\partial^2\Psi(r,t)/\partial r^2 + V(r,t)\Psi(r,t) \quad \wedge \quad ||\vec{\mathbf{r}}|| = r \quad \Rightarrow \quad \exists\, \vec{\mathbf{r}} :\ i\hbar\partial\Psi(\vec{\mathbf{r}},t)/\partial t = -(\hbar^2/2m)\partial^2\Psi(\vec{\mathbf{r}},t)/\partial\vec{\mathbf{r}}^2 + V(\vec{\mathbf{r}},t)\Psi(\vec{\mathbf{r}},t), \quad (111)$$

which is the 3-dimensional position-space Schrödenger's equation [14] [12].

Derivation of Dirac's wave equation

Using the derived Planck relation 67:

$$mc^2 = \hbar/t \quad \Rightarrow \quad \exists\, V(r,t) :\ mc^2/2 + V(r,t) = \hbar/t \quad \Rightarrow \quad 2\hbar/t - 2V(r,t) = mc^2. \quad (112)$$

$$\forall\, V(r,t) :\ V(r,t) = i\hbar/t \quad \wedge \quad r = ct \quad \wedge \quad 2\hbar/t - 2V(r,t) = mc^2 \ \Rightarrow\ 2\hbar/t - i2\hbar c/r = mc^2. \quad (113)$$

Use the ratios, $r = c_q q$, and, $r = ct$. to multiply each term on the left side of equation 113 by 1:

$$qc_q/r = qc_q/ct = 1 \quad \wedge \quad 2\hbar/t - i2\hbar c/r = mc^2 \quad \Rightarrow \quad 2\hbar(qc_q/c)/t^2 - i2\hbar((qc_q/c)/r^2)c = mc^2. \quad (114)$$

Applying a quantum amplitude equation in complex form to equation 115:

$$A_0 = (c_q/c)((1/t) - i(1/r)) \quad \wedge \quad 2\hbar(qc_q/c)/t^2 - i2\hbar((qc_q/c)/r^2)c = mc^2 \quad \Rightarrow \quad 2\hbar\partial(-qA_0)/\partial t - i2\hbar(\partial(-qA_0)/\partial r)c = mc^2. \quad (115)$$

Translating equation 115 to moving (rest frame) coordinates via the Lorentz factor, $\gamma_0 = 1/\sqrt{1-(v/c)^2}$:

$$2\hbar\partial(-qA_0)/\partial t - i\hbar h(\partial(-qA_0)/\partial r)c = mc^2 \quad \Rightarrow \quad \gamma_0 2\hbar\partial(-qA_0)/\partial t - \gamma_0 i2\hbar(\partial(-qA_0)/\partial r)c = mc^2. \quad (116)$$

Multiplying both sides of equation 116 by $\Psi(r,t)$:

$$\gamma_0 2\hbar\partial(-qA_0)/\partial t - \gamma_0 i2\hbar(\partial(-qA_0)/\partial r)c = mc^2 \quad \Rightarrow \quad \gamma_0 2\hbar(\partial(-qA_0)/\partial t)\Psi(r,t) - \gamma_0 i2\hbar(\partial(-qA_0)/\partial r)c\Psi(r,t) = mc^2\Psi(r,t). \quad (117)$$

Applying the vectors to equation 117:

$$\gamma_0 2\hbar(\partial(-qA_0)/\partial t)\Psi(r,t) - \gamma_0 i2\hbar(\partial(-qA_0)/\partial r)c\Psi(r,t) = mc^2\Psi(r,t) \quad \wedge \quad ||\vec{\mathbf{r}}|| = r \quad \wedge \quad ||\vec{\mathbf{A}}|| = A_0 \quad \wedge \quad ||\vec{\gamma}|| = \gamma_0 \quad \Leftrightarrow \quad \exists\, \vec{\mathbf{r}}, \vec{\mathbf{A}}, \vec{\gamma} :\ \gamma_0 2\hbar(\partial(-qA_0)/\partial t)\Psi(r,t) - \vec{\gamma}\cdot i2\hbar(\partial(-q\vec{\mathbf{A}})/\partial r)c\Psi(\vec{\mathbf{r}},t) = mc^2\Psi(\vec{\mathbf{r}},t). \quad (118)$$

Adding a $\frac{1}{2}$ spin to equation 115 yields Dirac's wave equation [15] [12]:

$$\gamma_0 2\hbar(\partial(-qA_0)/\partial t)\Psi(r,t) - \vec{\gamma}\cdot i2\hbar(\partial(-q\vec{\mathbf{A}})/\partial r)c\Psi(\vec{\mathbf{r}},t) = mc^2\Psi(\vec{\mathbf{r}},t) \quad \wedge \quad A_0 = \frac{1}{2}(c_q/c)((1/t) - i(1/r)) \quad \Rightarrow \quad \gamma_0\hbar(\partial(-qA_0)/\partial t)\Psi(r,t) - \vec{\gamma}\cdot i\hbar(\partial(-q\vec{\mathbf{A}})/\partial r)c\Psi(\vec{\mathbf{r}},t) = mc^2\Psi(\vec{\mathbf{r}},t). \quad (119)$$

Total of a type

Applying both the direct (61) and inverse proportion ratios (65):

$$r = \sqrt{(c_\tau\tau)^2 + (k_\tau/\tau)^2},\ \tau \in \{t, m, q\} \quad \Leftrightarrow \quad \tau = \sqrt{(r/c_\tau)^2 + (k_\tau/r)^2}. \quad (120)$$

Quantum extension to general relativity

The simplest way to demonstrate how to add quantum physics to general relativity is by extending Schwarzschild's time dilation equation and black hole metric. Apply the total of a type equation 120 to the derivation of Schwarzschild's time dilation and metric (80):

$$\sqrt{1-(v^2/c^2)} = \sqrt{1-(v^2/c^2)(r/r)} \quad \wedge \quad r = \sqrt{(c_m m)^2 + (k_m/m)^2} = Q_m \quad \Rightarrow \quad \sqrt{1-(v^2/c^2)} = \sqrt{1 - Q_m v^2/rc^2}. \quad (121)$$

$$\sqrt{1-(v^2/c^2)} = \sqrt{1 - Q_m v^2/rc^2} \quad \wedge \quad KE = mv^2/2 = mv_{escape}^2 \quad \Rightarrow \quad \sqrt{1-(v^2/c^2)} = \sqrt{1 - 2Q_m v_{escape}^2/rc^2}. \quad (122)$$

For a photon, the escape velocity, $v_{escape} = c$.

$$\sqrt{1-(v^2/c^2)} = \sqrt{1-2Q_m v_{escape}^2/rc^2}. \wedge\ v_{escape} = c$$
$$\Rightarrow \quad \sqrt{1-(v^2/c^2)} = \sqrt{1-2Q_m c^2/rc^2} = \sqrt{1-2Q_m/r}. \tag{123}$$

Combining equation 123 with equation 78 yields Schwarzschild's gravitational time dilation with a quantum mass effect:

$$\sqrt{1-(v^2/c^2)} = \sqrt{1-2Q_m/r} \quad \wedge \quad t' = t\sqrt{1-(v^2/c^2)}$$
$$\Rightarrow \quad t' = t\sqrt{1-2Q_m/r}. \tag{124}$$

Schwarzschild defined the black hole event horizon radius, $\alpha := 2Gm/c^2$. The radius with the quantum extension is $\alpha := 2Q_m$. At this point the exact same equations 85 through 88 yield what looks like the same Schwarzschild black hole metric.

Quantum extension to Newton's gravity force

The quantum mass effect is easier to understand in the context Newton's gravity equation than in general relativity, because the metric equations and solutions in the EFEs are much more complex. From equations 120 and 50:

$$m/\sqrt{(r/c_m)^2+(k_m/r)^2} = 1 \quad \wedge \quad r^2/(ct)^2 = 1$$
$$\Rightarrow \quad r^2/(ct)^2 = m/\sqrt{(r/c_m)^2+(k_m/r)^2}$$
$$\Rightarrow \quad r^2/t^2 = c^2 m/\sqrt{(r/c_m)^2+(k_m/r)^2}. \tag{125}$$

$$r^2/t^2 = c^2 m/\sqrt{(r/c_m)^2+(k_m/r)^2}$$
$$\Rightarrow \quad (m/r)(r^2/t^2) = (m/r)(c^2 m/\sqrt{(r/c_m)^2+(k_m/r)^2})$$
$$\Rightarrow \quad F := mr/t^2 = c^2 m^2/\sqrt{(r^4/c_m^2)+k_m^2}. \tag{126}$$

$$F = c^2 m^2/\sqrt{(r^4/c_m^2)+k_m^2} \quad \wedge$$
$$\forall\, m \in \mathbb{R},\ \exists\, m_1, m_2 \in \mathbb{R}:\ m_1 m_2 = m^2$$
$$\Rightarrow \quad F = c^2 m_1 m_2/\sqrt{(r^4/c_m^2)+k_m^2}. \tag{127}$$

Quantum extension to Coulomb's force

$$q^2/((r/c_q)^2+(k_q/r)^2) = 1 \quad \wedge \quad r^2/(ct)^2 = 1$$
$$\Rightarrow \quad r^2/(ct)^2 = q^2/((r/c_q)^2+(k_q/r)^2)$$
$$\Rightarrow \quad r^2/t^2 = c^2 q^2/((r/c_q)^2+(k_q/r)^2). \tag{128}$$

$$(1/r)(r^2/t^2) = (1/r)(c^2 q^2/((r/c_q)^2+(k_q/r)^2))$$
$$\Rightarrow \quad r/t^2 = c^2 q^2/(r^3/c_q^2+k_q^2/r). \tag{129}$$

$$\forall\, q \in \mathbb{R}:\ \exists\, q_1, q_2 \in \mathbb{R}:\ q_1 q_2 = q^2 \quad \wedge$$
$$r/t^2 = c^2 q^2/(r^3/c_q^2+k_q^2/r) \quad \Rightarrow$$
$$\exists\, q_1, q_2 \in \mathbb{R}:\ r/t^2 = c^2 q_1 q_2/(r^3/c_q^2+k_q^2/r). \tag{130}$$

$$r/t^2 = c^2 q_1 q_2/(r^3/c_q^2+k_q^2/r) \quad \wedge \quad m = r/c_m \quad \Rightarrow$$
$$F := mr/t^2 = (c^2/c_m) q_1 q_2/(r^2/c_q^2+k_q^2/r^2). \tag{131}$$

INSIGHTS AND IMPLICATIONS

1. The ruler measure (.1) and convergence theorem (.2) were shown to be useful tools for proving that a countable sets of n-tuples imply a corresponding real-valued equation.
2. Defining all Euclidean and non-Euclidean distance measures as the inverse function of the sum of subset volumes:
   $$\forall\, n,\ d: \quad d = f_n^{-1}(v) = f_n^{-1}(\textstyle\sum_{i=1}^m v_i): \tag{132}$$
   (a) shows the intimate relation between distance and volume that definitions, like inner product space and metric space, ignore [5][2][3];
   (b) is a more simple and concise definition of a distance measure that includes the properties of inner product space and metric space [5][2][3].
3. The left side of the distance sum inequality (.10),
   $$(\textstyle\sum_{i=1}^m (a_i^n + b_i^n))^{1/n} \leq (\sum_{i=1}^m a_i^n)^{1/n} + (\sum_{i=1}^m b_i^n)^{1/n}, \tag{133}$$
   differs from the left side of Minkowski's sum inequality [7]:
   $$(\textstyle\sum_{i=1}^m (a_i^n + b_i^n)^{\mathbf{n}})^{1/n} \leq (\sum_{i=1}^m a_i^n)^{1/n} + (\sum_{i=1}^m b_i^n)^{1/n}. \tag{134}$$
   (a) The two inequalities are only the same where $n = 1$.
   (b) The distance sum inequality (.10) is a more fundamental inequality because the proof does not require the convexity and Hölder's inequality assumptions required to prove the Minkowski sum inequality [7].
   (c) The distance sum inequality term, $\forall\ n > 1$, $v_i^n = a_i^n + b_i^n$: $d = v^{1/n} = (\sum_{i=1}^m v_i^n)^{1/n}$, is the Minkowski distance, which makes it directly related to geometry. But the Minkowski sum inequality term, $\forall\ n > 1$, $v > 0$: $d = v^{1/n} = (\sum_{i=1}^m ((v_i^n)^{\mathbf{n}}))^{1/n} = (\sum_{i=1}^m v_i^{\mathbf{n^2}})^{1/n}$, is *not* a Minkowski distance.

(d) The distance sum inequality might be applicable to machine learning.

4. **Combinatorics**. The the number of elements (n-tuples) in a set of ordered combinations of countable, disjoint sets, $v_c = \prod_{i=1}^{n} |x_i|$, was proven to imply: the Euclidean volume equation (.4), the inverse function of the sum of volumes equation (.7) (which includes the inner product) and the Minkowski distance equation (.8) (which includes the Manhattan and Euclidean distance equations), without relying on the geometric primitives and relations in Euclidean geometry [16][17], axiomatic geometry [18][19][20] [21][22], trigonometry [23] [24] calculus [25][23] [26], and vector analysis [5].

5. **Combinatorics**. The commutative property of the operations defining volume and distance equations requires a combinatorial (permutation) property limiting a set to $n \leq 3$ members (.19). Higher dimensions must have different types (members of different sets).

   (a) For example, the vector inner product space can only be extended beyond 3 dimensions if and only if the higher dimensions have non-distance types, for example, dimensions of time, mass, and charge.

   (b) As shown in the special relativity section (76), there is 6-dimensional space-time-mass-charge.

   (c) If each type of quantum state is an ISCS, then there are at most 3 states of the same type: 3 orientations per dimension of space, 3 quark color charges, {red, green, blue}, 3 quark anti-color charges, and so on.

   (d) If the states are not ordered (a bag of states), then a state value is undetermined (or superimposed) until observed (like Schrödenger's poisoned cat being both alive and dead until the box is opened [14]).

   (e) A discrete (point) value has measure 0 (zero-length interval). The ratio of a time or distance interval length to zero is undefined, which is the reason quantum entangled (discrete) state values exist independent of time and distance.

6. For each Cartesian axis unit, r_p, of a 3-dimensional distance interval having a length, r, there are Cartesian axis units of other types of intervals forming unit ratios (61): $c_t = r_p/t_p$, $c_m = r_p/m_p$, $c_q = r_p/q_p \Leftrightarrow$ the inverse proportion ratios (64): $k_t = r_p t_p$, $k_m = r_p m_p$, $k_t = r_p q_p$, where r_p, t_p, m_p, and q_p are the Planck units (71).

7. Empirical laws *describe* relations. Deriving empirical laws from the ratios *explains* the relations. Further, all the derivations of the physics equations from the ratios were much shorter and simpler than other derivations, which shows that the ratios are an important tool for physicists and engineers.

8. As shown in the subsection deriving the Schwarzschild's time dilation and black hole metric (80) [10][11] using ratios illustrates a way of simplifying the finding of solutions to Einstein's field equations.

9. The speed of light ratio, c_t, is a component of the ratio-derived constants: $G = c_m c_t^2$, $k_e = c_q^2 c_t^2/c_m$, $\varepsilon_0 = 1/4\pi k_e = 1/4\pi(c_q^2 c_t^2/c_m)$, $\hbar = k_m c_t$.

   The only ratio-derived constant, in this article, that does not contain c_t is vacuum permeability: $\mu_0 = 4\pi k_e/c_t^2 = 4\pi c_q^2/c_m$.

10. In the derivation of the Planck relation (67), one could start with $k_m c = h$ (the full Planck constant) instead of $k_m c = \hbar$ (the reduced Planck constant). But, that would require: 1) the derived quantum units (71) to not be the Planck units (that is, a quantum unit would = Planck unit $\times\sqrt{2\pi}$), 2) make the derivation of the fine structure constant (75) more complicated, 3) make the derivation of Schrödenger (111) and Dirac (119) wave equations more complicated and finding solutions to those wave equations more complicated.

11. Using the quantum (Planck) units, r_p and t_p: r_p/t_p^2, suggests a maximum linear acceleration for masses. And $2\pi r_p/t_p^2$ suggests a maximum orbital or rotational acceleration.

12. The simplification of μ_0 into the quantum units shows two interesting relationships:

$$\mu_0 = 4\pi\frac{k_e}{c_t^2} = 4\pi\frac{c_q^2}{c_m} = 4\pi\frac{(r_p/q_p)^2}{r_p/m_p} = 4\pi\frac{m_p r_p}{q_p^2} = 4\pi\frac{k_m}{q_p^2}$$
$$\approx 4\pi\frac{3.5176729162 \cdot 10^{-43}}{3.5176729162 \cdot 10^{-35}} = 4\pi \cdot 10^{-7}\ kg\ m\ C^{-2}$$
$$= 4\pi \cdot 10^{-7}\ H\ m^{-1}. \quad (135)$$

   (a) The first time $k_m = m_p r_p$ appears is in the derivation of the Planck relation and Planck constant, $\hbar = k_m c$ (66), the second time in the Compton wavelength, $r = k_m/m$ (104). And now, k_m appears as a components of k_e, ε_0, and μ_0.

   (b) It is an open question why $\frac{k_m}{q_p^2}$ seems to equal $1.0 \cdot 10^{-7}$ exactly.

13. The fine structure constant, α was derived from the ratio of two forces of two subtypes that reduces

to ratio of the square of the subtypes $\alpha = q_e^2/q_p^2 \approx 0.0072973526$ (75), which is the empirical CODATA value [13].

(a) The CODATA electron coupling version of the fine structure constant, α is defined as: $\alpha = q_e^2/4\pi\varepsilon_0\hbar c) = q_e^2/2\varepsilon_0 hc$ [13]. The following steps show that the CODATA definition reduces to the ratio-derived equation:

$$\varepsilon_0 := 1/4\pi k_e = 1/(4\pi(c_q^2 c_t^2/c_m) \quad \wedge \quad \hbar = k_m c_t \quad \wedge \quad h = 2\pi\hbar$$
$$\Rightarrow \quad \varepsilon_0 hc = 2\pi k_m c_t^2/(4\pi(c_q^2/c_m)c_t^2) = k_m/(2(c_q^2/c_m)) = m_p r_p/(2((r_p/q_p)^2/(r_p/m_p))) = q_p^2/2. \quad (136)$$

$$\alpha = q_e^2/2\varepsilon_0 hc \quad \wedge \quad \varepsilon_0 hc = q_p^2/2 \quad \Rightarrow \quad \alpha = q_e^2/2(q_p^2/2) = q_e^2/q_p^2. \quad (137)$$

(b) The fine structure electron coupling constant is the ratio of electron static charge force to the propagating quantum charge wave force, where a moving charged particle causes a charge (photon/electromagnetic) wave to propagate through space at the speed of light.

(c) Other fine structure constants can also be expressed more simply as the ratios of two subtypes of fields, for example, an electron gravity coupling constant can be expressed as the ratio of the rest electron mass to a Planck mass unit: $\alpha_{G_m} = m_e^2/m_p^2$.

14. Empirical and hypothesized laws of physics use an *opaque* constant, K, that is defined to make an equation, where the units balance, $g = Kf(r, t, \cdots)$. The opacity has led to the *incorrect* assumptions of those constants, K, being fundamental (atomic) constants.

    In this article, some opaque constants are derived directly from (composed of) the ratios: gravity, $G = c_m c_t^2$ (54), charge, $k_e = c_q^2 c_t^2/c_m$ (59), and Planck $h = k_m c_t$ (67). $\varepsilon_0 = 1/4\pi k_e = 1/4\pi c_m/((c_q^2/c_m)c_t^2)$ (98) and $\mu_0 = 4\pi k_e/c_t^2 = 4\pi c_q^2/c_m$ (95).

    And the quantum extensions to: Schwarzschild's time dilation (123) Newton's gravity force (127), and Coulomb's charge force show, that where the quantum effects become measurable, the constants G, k_e, ε_0, and μ_0 no longer exist (are no longer valid).

    Therefore, G, k_e, ε_0, μ_0, and h are **not** fundamental (atomic) constants.

15. The derivations of: $\nabla \cdot \mathbf{g} = -4\pi G\rho$ from $\mathbf{g} = 2\pi Gm/r^2$ (55), $\nabla \cdot \mathbf{E} = -\rho/\varepsilon_0$ from $\mathbf{E} = 2\pi k_e q/r^2$ (98), and $\partial\mathbf{B}/\partial t = -\mu_0\rho$ from $\mathbf{B} = 2\pi k_e q/r^2$ (101), show that the use of mass and charge density, ρ, are unnecessary complications that obfuscates the pattern, $\nabla \cdot f(x, y, r) = -2k_{x,y}y/r^3$, being derived from the inverse square pattern, $f(x, y, r) = k_{x,y}y/r^2$. And the energy density in the stress-energy tensor, $T_{\mu,\nu}$, in Einstein's field equations [5] also obfuscates the inverse square assumption.

16. Einstein's relativity equations: assume the Lorentz transformations, assume the laws of physics are same at each coordinate point, assume the notion of light, and assume that the speed of light is the same at each coordinate point [4][5].

    The derivations, in this article, were made without those assumptions (does even require the notion of light). Assuming Cartesian coordinates at each coordinate point, creates unit ratios, where all equations (laws) derived from the unit ratios must be the same at each coordinate point. Deriving numeric values for the ratios assumes that the ratio, c_t, is the maximum speed.

17. The derivation of the magnetic field from the ratios and special relativity (92) shows that magnetic field, $\mathbf{B}$, is the spacetime bend of the electric field, $\mathbf{E}$ due to relativistic velocities. The magnetic force is a torque caused by spacetime bending of Coulomb's radial charge force.

    A charged particle's spin (angular momentum) axis has an orientation. "Paired spins" is where the orientations of two valence electrons are in opposite directions. The opposite spacetime bending due to relativistic spins cancel each other. Materials with unpaired spins that have net aligned orientations is a permanent magnet.

    A current in a conductor is where electrons are moving in the same direction with the orientations aligned in that same direction. Applying electromagnetic radiation to a thin, conductive film containing unpaired spins aligned in the same direction will create a current with $1 - \alpha \approx 99\%$ efficiency minus electrical resistance (where α is the fine structure electron coupling constant). Such a low-resistance film would be several times more efficient than current solar panels.

    True elementary particles do not have a half or fractional spin. A "half-spin" is a π radians rotation of the spin axis orientation. Positive and negative charges with spin axis orientations in the same direction have opposite spins (opposite angular momenta).

18. The quantum extensions to: Schwarzschild's time dilation (123) black hole metric (88), Newton's

gravity force (127), and Coulomb's charge force (130) make quantifiable predictions:

(a) For gravity, $\lim_{r\to 0} F = c^2 m_1 m_2/k_m$, and for charge, $\lim_{r\to 0} F = 0$. Finite maximum gravity and charge forces: 1) allows radioactivity, finite sloped energy well walls, which might eliminate the need to hypothesize the existence of a weak force and strong force; and 2) eliminates the problem of forces going to infinity as $r \to 0$.

(b) The quantum-extended Schwarzschild time dilation and metric, gravity, and charge equations reduce to the classic equations, where the distance between masses and charges is sufficiently large or the masses and charges sufficiently large that the quantum effect is not measurable. **Note** that G, k_e, ε_0, μ_0, and κ (Einstein's constant, which contains G) do not exist (are not valid), where the quantum effects becomes measurable.

(c) And the covariant tensor components, in Einstein's field equations, that had the units $1/distance^2$, will now have the more complex units, $1/\sqrt{(distance^4/c_\tau^2) + k_\tau^2}$, where $\tau \in \{t, m\}$.

(d) $1/\sqrt{(distance^4/c_\tau^2) + k_\tau^2}$ implies that as distance $\to 0$, spacetime curvature peaks at the Planck distance, r_p (71). The ratio of Planck units, m_p/r_p^3, might indicate a maximum mass density. A finite force (spacetime curvature) and finite mass density would imply that black holes have sizes > 0 (might not be singularities). Black hole evaporation might be possible. If there was a "big bang," then it might not have originated from a singularity.

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