Some Set Properties Underlying Geometry and Physics

George M. Van Treeck
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Proving the Euclidean volume equation and some distance equations, including the inner product and Euclidean distance, are instances of abstract sets of ordered combinations (n-tuples) provides a foundation for: 1) a proof that another combinatorial (permutation) property of distance limits distance to 3 dimensions (other dimensions are members of different sets), 2) short and simple derivations of gravity, charge, electromagnetic, relativity, and quantum physics equations and constants, 3) adding quantum extensions to classical and relativity equations. All the proofs are verified in Rocq.

[1]

CONTENTS

Keywords: mathematical physics, combinatorics, set theory, distance measure, inner product, gravity, charge, electromagnetism, relativity, quantum physics.

INTRODUCTION

Many math physics equations either assume Euclidean space (for example, Newton's gravity force, Coulomb's charge force, electromagnetism, and Schrödenger's equation) [2][3], or assume that the space near each local coordinate point is Euclidean (Riemann and pseudo-Riemann spaces, special and general relativity) [4][5][6]. Although Euclidean geometry permeates math and physics, mathematical analysis defines the Euclidean volume and distance equations [7][8] rather deriving the equations from an abstract set and limit-based foundation (without using notions of interval, line, side, and angle).

Proving the Euclidean volume equation and some distance equations, including the inner product and Euclidean distance, are instances of abstract sets of ordered combinations (n-tuples) provides a foundation for: 1) a proof that another combinatorial (permutation) property of distance limits distance to 3 dimensions (other dimensions are members of different sets), 2) short and simple derivations of what have been empirical gravity, charge, and electromagnetic equations and constants, 3) shorter and simpler derivations of some well-known relativity and quantum physics equations, and 4) simple addition of quantum extensions to classical and relativity equations.

The proofs in this article have been verified using using the Rocq proof verification system [9]. The formal proofs are in the Rocq files, "euclidrelations.v" and "threed.v," which are included as ancillary files.

Let $|x_i|$ be the cardinal of (number of elements in) the countable set, x_i. And let v_c be the integer number of ordered combinations (n-tuples) of the members of $x_1, \cdots, x_n$. The number of n-tuples, v_c, will be proved imply the Euclidean volume equation:

$$\forall\, v_c, d_c, |x_i| \in \{0,\ \mathbb{N}\},\ \ x_i \in \{x_1, \cdots, x_n\}, \\ v_c = \textstyle\prod_{i=1}^n |x_i| \quad \Rightarrow \quad v = \prod_{i=1}^n s_i,\ \ s_i, v \in \mathbb{R}. \quad (1)$$

For all $n > 1$, there are an infinite number of possible domain values, $s_1, \cdots, s_n$, that multiplied yield the same range value, v. Inferring a domain value, d, from v, requires an inverse (bijective) function, $d = f_n^{-1}(v)$ and $v = f_n(d)$. The simplest bijective case, for all n, extends the $n = 1$ case, $v_c = |x_1| = d_c^1$:

$$\exists\, d_c, v_c, |x_i| \in \{0, \mathbb{N}\} :\ v_c = \textstyle\prod_{i=1}^n |x_i| = \prod_{i=1}^n d_c = d_c^n. \quad (2)$$

A set of n-tuples being the union of disjoint subsets of n-tuples implies that the domain value, d_c, is also the inverse function of the sum of n-tuples:

$$d_c^n = \textstyle\sum_{i=1}^m v_{c_i} = \sum_{i=1}^m (\prod_{j=1}^n |x_{i,j}|) \\ \Rightarrow \quad d^n = \textstyle\sum_{i=1}^m (\prod_{j=1}^n s_{i,j}). \quad (3)$$

The $n = 2$ case is the inner product.

Where each v_{c_i} is also the bijective function, $v_{c_i} = d_{c_i}^n$:

$$d_c^n = \textstyle\sum_{i=1}^m d_{c_i}^n \quad \Rightarrow \quad d^n = \sum_{i=1}^m d_i^n. \quad (4)$$

$|d|$ is the p-norm (Minkowski distance) [10], which will be proved to imply the metric space properties [8]. The $n = 2$ case is the Euclidean distance.

Volume and distance are derived from sets of ordered combinations (n-tuples). Volume and distance have another combinatorial (permutation) property.

All union and intersection operatiosn on sets of n-tuples are commutative. Therefore, the addition, subtraction, multiplication, and division operations in calculating the number of n-tuples in a volume or distance are also commutative. The commutative property requires being able to operate on a set of n number of members in any one of $n!$ permutated sequences.

Reliably re-sequencing a set of members in the same order requires assigning a sequential order to the members. Further, the *only* sequential order that allows starting with any set member and sequencing in a repeatable order, is a cyclic order.

Reliably re-sequencing of a cyclic set in all $n!$ permutations, is a symmetry, where every set member is either an *immediate* cyclic successor or an *immediate* cyclic predecessor to every other set member, which is, herein, referred to as an "immediate symmetric" cyclic set (ISCS). First-order logic will be used to prove an ISCS has $n \leq 3$ members.

Application to physics uses the following 3 hypotheses:

1. **ISCS:** Physical distance is an ISCS of 3 dimensions, $\{r_1, r_2, r_3\}$, and $\{t\ (\mathit{time}), m\ (\mathit{mass}), q\ (\mathit{charge})\}$ is the ISCS of "non-distance" dimensions, each dimension $\subseteq \mathbb{R}$. Physical space is 6-dimensional: r_1-r_2-r_3-t-m-q.
2. **Cartesian:** Each local coordinate point is the origin of a Cartesian grid (the space near each local coordinate point is Euclidean), where for each Cartesian axis unit interval length, r_p, of distance, there is a constant Cartesian axis unit interval length: t_p of time; m_p of mass; and q_p of charge, such that: $r = (r_p/t_p)t = (r_p/m_p)m = (r_p/q_p)q$, where $r_p/t_p = c_t$, $r_p/m_p = c_m$, and $r_p/q_p = c_q$.
3. **Maximum ratios** The Cartesian axis unit ratios, c_t, c_m, and c_q are the largest ratios. For example, the speed of light is limited to c_t.

A consequence of these hypotheses is that all equations derived from combining the constant ratios are the same equations at each local coordinate point, which is the reason the laws of physics are same at each local coordinate point.

The Newton's gravity [3] and Coulomb's charge force [2] equations will be derived from the ratios, where:

$$F = (c_m c_t^2) m_1 m_2 / r^2 = G m_1 m_2 / r^2 \quad \text{and} \quad (5)$$
$$F = (c_q^2 c_t^2 / c_m) q_1 q_2 / r^2 = k_e q_1 q_2 / r^2. \quad (6)$$

The ratio, c_m, is calculated from the empirical values of G, and c_t (the speed of light) in Newton's equation. Next, the ratio, c_q, is calculated from the values of k_e, c_t, and c_m in Coulomb's equation. The derivations from the ratios will show that G, k_e, ε_0, μ_0, and $\hbar$ are **not** fundamental (atomic) constants.

Algebraic manipulation of the 3 direct proportion ratios yields 3 inverse proportion ratios, $r = t_p r_p / t = m_p r_p / m = q_p r_p / q$, where $k_t = t_p r_p$, $k_m = m_p r_p$, and $k_q = q_p r_p$. The combination of the direct and inverse proportion ratios are used to derive the Planck relation and the reduced Planck constant, $\hbar = k_m c_t$. The values of k_t, k_m, and k_q are calculated from the values of $\hbar$, c_t, c_m, and c_q.

r_p, t_p, m_p, and q_p are the Planck units, which are calculated from G, k_e, c_t, and $\hbar$. The fine structure electron coupling constant, α, is derived, in this article, as the ratio of two forces that reduces to the ratio of subtypes: $\alpha = q_e^2/q_p^2$, which is much simpler and more elucidating than the standard equation, $\alpha = q_e^2/4\pi\varepsilon_0\hbar c$ [11].

The proofs and the 3 direct proportion ratios are used to provide simple derivations of: the gravitational constant, G, the Newton, Gauss, and Poisson gravity equations, Coulomb's charge force and charge constant, k_e, the special relativity equations, the Schwarzschild time dilation and black hole metric equations (pointing to a simplified method of finding solutions to Einstein's general relativity equations), the Gauss, Lorentz, and Faraday electromagnetic equations, the vacuum permittivity, ε_0, and vacuum permeability, μ_0, constants.

The ratios and Planck relation are used to derive the Compton wavelength, the position-space Schrödenger, and the Dirac wave equations. And, finally, the inverse proportion ratios are also used to add quantum extensions to some general relativity and classical physics equations.

RULER MEASURE AND CONVERGENCE

Definition .1. Ruler measure, $M = \sum_{i=1}^{p} \kappa = p\kappa$, where $\forall\, s, \kappa \in \mathbb{R}$,
$0 < \kappa \leq 1, \quad (p = floor(s/\kappa) \quad \vee \quad p = ceiling(s/\kappa))$.

Theorem .2. *Ruler convergence:* $M = \lim_{\kappa \to 0} p\kappa = s$.

The formal proof, "limit_c_0_M_eq_exact_size," is in the file, euclidrelations.v.

Proof. (epsilon-delta proof)
By definition of the floor function, $floor(x) = max(\{y : y \leq x,\ y \in \mathbb{Z},\ x \in \mathbb{R}\})$:

$$p = floor(s/\kappa) \quad \wedge \quad 0 \leq |floor(s/\kappa) - s/\kappa| < 1 \\ \Rightarrow \quad |p - s/\kappa| < 1. \quad (7)$$

Multiply both sides of inequality 7 by κ:

$$\forall\, 0 < \kappa \leq 1, \quad |p - s/\kappa| < 1 \\ \Rightarrow \quad |p\kappa - s| < |\kappa| = |\kappa - 0|. \quad (8)$$

$$\forall\, \epsilon = \delta \quad \wedge \quad |p\kappa - s| < |\kappa - 0| < \delta \\ \Rightarrow \quad |\kappa - 0| < \delta \ \wedge\ |p\kappa - s| < \delta = \epsilon \\ := \quad M = \lim_{\kappa \to 0} p\kappa = s. \quad \square \quad (9)$$

The following is an example of ruler convergence for the interval, $[0, \pi]$: $s = \pi - 0$, and $p = floor(s/\kappa) \Rightarrow p \cdot \kappa = 3.1_{\kappa=10^{-1}},\ 3.14_{\kappa=10^{-2}},\ 3.141_{\kappa=10^{-3}}, ..., \pi_{\lim_{\kappa\to 0}}$.

Lemma .3. $\forall\ n \geq 1, \quad 0 < \kappa \leq 1 \Rightarrow \lim_{\kappa \to 0} \kappa^n = \lim_{\kappa \to 0} \kappa$.

Proof. The formal proof , "lim_c_to_n_eq_lim_c," is in the Rocq file, euclidrelations.v.

$$n \geq 1 \quad \wedge \quad 0 < \kappa \leq 1 \quad \Rightarrow \quad 0 < \kappa^n < \kappa \\ \Rightarrow \quad |\kappa - \kappa^n| < |\kappa| = |\kappa - 0|. \quad (10)$$

$$\forall\, \epsilon = \delta \quad \wedge \quad |\kappa - \kappa^n| < |\kappa - 0| < \delta \\ \Rightarrow \quad |\kappa - 0| < \delta \quad \wedge \quad |\kappa - \kappa^n| < \delta = \epsilon \\ := \quad \lim_{\kappa \to 0} \kappa^n = 0. \quad (11)$$

$$\lim_{\kappa \to 0} \kappa^n = 0 \quad \wedge \quad \lim_{\kappa \to 0} \kappa = 0 \\ \Rightarrow \quad \lim_{\kappa \to 0} \kappa^n = \lim_{\kappa \to 0} \kappa. \quad \square \quad (12)$$

VOLUME

Euclidean volume

Theorem .4. *Euclidean volume,*

$$\forall\, v_c, d_c, |x_i| \in \{0,\ \mathbb{N}\},\ \ x_i \in \{x_1, \cdots, x_n\}, \\ v_c = \prod_{i=1}^{n} |x_i| \quad \Rightarrow \quad v = \prod_{i=1}^{n} s_i, \quad s_i, v \in \mathbb{R}. \quad (13)$$

The formal proof, "Euclidean_volume," is in the Rocq file, euclidrelations.v.

Proof.

$$v_c = \prod_{i=1}^{n} |x_i| \quad \Leftrightarrow \quad v_c\kappa = (\prod_{i=1}^{n} |x_i|)\kappa \\ \Leftrightarrow \quad \lim_{\kappa \to 0} v_c\kappa = \lim_{\kappa \to 0}(\prod_{i=1}^{n} |x_i|)\kappa. \quad (14)$$

Apply the ruler (.1) and ruler convergence (.2) to equation 14:

$$\exists\, v, \kappa \in \mathbb{R} : v_c = floor(v/\kappa) \quad \Rightarrow \quad v = \lim_{\kappa \to 0} v_c\kappa \quad \wedge \\ \lim_{\kappa \to 0} v_c\kappa = \lim_{\kappa \to 0}(\prod_{i=1}^{n} |x_i|)\kappa \\ \Rightarrow \quad v = \lim_{\kappa \to 0}(\prod_{i=1}^{n} |x_i|)\kappa. \quad (15)$$

Apply lemma .3 to equation 15:

$$v = \lim_{\kappa \to 0}(\prod_{i=1}^{n} |x_i|)\kappa \quad \wedge \quad \lim_{\kappa \to 0} \kappa^n = \lim_{\kappa \to 0} \kappa \\ \Rightarrow \quad v = \lim_{\kappa \to 0}(\prod_{i=1}^{n} |x_i|)\kappa^n = \lim_{\kappa \to 0}(\prod_{i=1}^{n} |x_i|\kappa). \quad (16)$$

Apply the ruler (.1) and ruler convergence (.2) to s_i:

$$\exists\, s_i, \kappa \in \mathbb{R} : floor(s_i/\kappa) = |x_i| \quad \Rightarrow \quad \lim_{\kappa \to 0}(|x_i|\kappa) = s_i. \quad (17)$$

$$v = \lim_{\kappa \to 0}(\prod_{i=1}^{n} |x_i|\kappa) \quad \wedge \quad \lim_{\kappa \to 0}(|x_i|\kappa) = s_i \\ \Leftrightarrow \quad v = \prod_{i=1}^{n} s_i \quad \square \quad (18)$$

Sum of volumes

Lemma .5. *The number of n-tuples, v_c, is the sum of the number of n-tuples, v_{c_i}, in each subset of n-tuples, implies a volume is the sum of volumes,*

$$v_c = \sum_{i=1}^{m} v_{c_i} \quad \Rightarrow \quad v = \sum_{i=1}^{m} v_i, \quad v, v_i \in \mathbb{R}.$$

The formal proof, "sum_of _volumes," is in the Rocq file, euclidrelations.v.

Proof. From the condition of this theorem:

$$v_c = \sum_{i=1}^{m} v_{c_i} \quad \Leftrightarrow \quad \lim_{\kappa \to 0} v_c\kappa = \lim_{\kappa \to 0} \sum_{j=1}^{m}(v_{c_i}\kappa). \quad (19)$$

Apply lemma .3 to equation 19:

$$\lim_{\kappa\to 0} v_c\kappa = \lim_{\kappa\to 0}(\textstyle\sum_{j=1}^{m} v_{c_i})\kappa \quad \wedge \\ \lim_{\kappa\to 0} \kappa^n = \lim_{\kappa\to 0} \kappa \\ \Leftrightarrow \ \lim_{\kappa\to 0} v_c\kappa = \lim_{\kappa\to 0} \textstyle\sum_{j=1}^{m}(v_{c_i}\kappa). \quad (20)$$

Apply the ruler (.1) and ruler convergence theorem (.2) to equation 20:

$$\exists\, v, v_i:\ v = \lim_{\kappa\to 0} v_c\kappa \quad \wedge \\ \lim_{\kappa\to 0} v_c\kappa = \lim_{\kappa\to 0} \textstyle\sum_{j=1}^{m}(v_{c_i}\kappa) \\ \Rightarrow \quad v == \lim_{\kappa\to 0} \textstyle\sum_{j=1}^{m}(v_{c_i}\kappa). \quad (21)$$

Apply the ruler (.1) and ruler convergence theorem (.2) to equation 21:

$$v == \lim_{\kappa\to 0} \textstyle\sum_{j=1}^{m}(v_{c_i}\kappa) \quad \wedge \\ \exists\, v_i, v_{c_i}:\ v_i = \lim_{\kappa\to 0} v_{c_i}\kappa \\ \Rightarrow \quad v = \textstyle\sum_{j=1}^{m} v_i^n. \quad \square \quad (22)$$

DISTANCE

Definition .6. Countable distance,

$$\forall\, v_c, d_c, |x_i| \in \{0,\ \mathbb{N}\},\ x_i \in \{x_1, \cdots, x_n\}, \\ v_c = \textstyle\prod_{i=1}^{n} |x_i| = \prod_{i=1}^{n} d_c = d_c^n. \quad (23)$$

Sum of volumes distance

Theorem .7. *Sum of volumes distance:*

$$d_c^n = v_c = \textstyle\sum_{i=1}^{m} v_{c_i} \quad \Rightarrow \quad d^n = \sum_{i=1}^{m}(\prod_{j=1}^{n} s_{i_j}).$$

The formal proof, "sum_of _volumes _distance," is in the Rocq file, euclidrelations.v.

Proof. From lemma .5 and the Euclidean volume theorem .4:

$$d_c^n = \textstyle\sum_{i=1}^{m} v_{c_i} \quad \Rightarrow \quad d^n = \sum_{i=1}^{m} v_i, \quad \wedge \\ v_i = \textstyle\prod_{j=1}^{n} s_{i_j} \quad \Rightarrow \quad d^n = \sum_{i=1}^{m}(\prod_{j=1}^{n} s_{i_j}). \quad \square \quad (24)$$

Minkowski distance (p-norm)

Theorem .8. *Minkowski distance (p-norm):*

$$d_c^n = v_c = \textstyle\sum_{i=1}^{m} v_{c_i} = \sum_{i=1}^{m} d_{c_i}^n \quad \Leftrightarrow \quad d^n = \sum_{i=1}^{m} d_i^n.$$

The formal proof, "Minkowski_distance," is in the Rocq file, euclidrelations.v.

Proof. From theorem .7 and the Euclidean volume theorem .4:

$$d_c^n = v_c = \textstyle\sum_{i=1}^{m} v_{c_i} \quad \Rightarrow \quad d^n = v = \sum_{i=1}^{m} v_i \quad \wedge \\ v_i = \textstyle\prod_{j=1}^{n} d_i = d_i^n \quad \Rightarrow \quad d^n = \sum_{i=1}^{m} d_i^n \quad \square \quad (25)$$

Distance inequality

The formal proof, distance_inequality, is in the Rocq file, euclidrelations.v.

Theorem .9. *Distance inequality*

$$\forall\, n \in \mathbb{N},\ v_a, v_b \geq 0:\ (v_a + v_b)^{1/n} \leq v_a^{1/n} + v_b^{1/n}.$$

Proof. Expand $(v_a^{1/n} + v_b^{1/n})^n$ using the binomial expansion:

$$\forall\, v_a, v_b \geq 0: \quad v_a + v_b \leq v_a + v_b + \\ \textstyle\sum_{i=1}^{n}\binom{n}{k}(v_a^{1/n})^{n-k}(v_b^{1/n})^k + \\ \textstyle\sum_{i=1}^{n}\binom{n}{k}(v_a^{1/n})^{k}(v_b^{1/n})^{n-k} = (v_a^{1/n} + v_b^{1/n})^n. \quad (26)$$

Take the n^{th} root of both sides of the inequality 26:

$$\forall\, v_a, v_b \geq 0,\ n \in \mathbb{N}: \quad v_a + v_b \leq (v_a^{1/n} + v_b^{1/n})^n \\ \Rightarrow \quad (v_a + v_b)^{1/n} \leq v_a^{1/n} + v_b^{1/n}. \quad \square \quad (27)$$

Distance sum inequality

The formal proof, distance_sum_inequality, is in the Rocq file, euclidrelations.v.

Theorem .10. *Distance sum inequality*

$$\forall\, m, n \in \mathbb{N},\ a_i, b_i \geq 0: \\ (\textstyle\sum_{i=1}^{m}(a_i^n + b_i^n))^{1/n} \leq (\sum_{i=1}^{m} a_i^n)^{1/n} + (\sum_{i=1}^{m} b_i^n)^{1/n}.$$

Proof. Apply the distance inequality (.9):

$$\forall\, m, n \in \mathbb{N},\ v_a, v_b \geq 0:\ v_a = \textstyle\sum_{i=1}^{m} a_i^n \quad \wedge \\ v_b = \textstyle\sum_{i=1}^{m} b_i^n \quad \wedge \quad (v_a + v_b)^{1/n} \leq v_a^{1/n} + v_b^{1/n} \\ \Rightarrow \quad ((\textstyle\sum_{i=1}^{m} a_i^n) + (\sum_{i=1}^{m} b_i^n))^{1/n} = (\sum_{i=1}^{m}(a_i^n + b_i^n))^{1/n} \leq \\ (\textstyle\sum_{i=1}^{m} a_i^n)^{1/n} + (\sum_{i=1}^{m} b_i^n)^{1/n}. \quad \square \quad (28)$$

Metric Space

All Minkowski distances (p-norms) imply the metric space properties. The formal proofs: triangle_inequality, symmetry, non_negativity, and identity_of_indiscernibles are in the Rocq file, euclidrelations.v.

Theorem .11. *Triangle Inequality:*
$d(s_1, s_2) = (\sum_{i=1}^{2} s_i^p)^{1/p} \Rightarrow d(u, w) \leq d(u, v) + d(v, w)$.

Proof. $\forall\, p \geq 1, \quad k > 1, \quad u = s_1, \quad w = s_2, \quad v = w/k$:

$$(u^p + w^p)^{1/p} \leq ((u^p + w^p) + 2v^p)^{1/p} = \\ ((u^p + v^p) + (v^p + w^p))^{1/p}. \quad (29)$$

Apply the distance inequality (.9) to the inequality 29:

$$
\begin{aligned}
&(u^p+w^p)^{1/p} \leq ((u^p+v^p)+(v^p+w^p))^{1/p} \quad \wedge \\
&\qquad (v_a+v_b)^{1/n} \leq v_a^{1/n}+v_b^{1/n} \\
&\qquad \wedge \quad v_a = u^p+v^p \quad \wedge \quad v_b = v^p+w^p \\
&\Rightarrow \quad (u^p+w^p)^{1/p} \leq ((u^p+v^p)+(v^p+w^p))^{1/p} \leq \\
&\qquad (u^p+v^p)^{1/p}+(v^p+w^p)^{1/p} \\
&\qquad \Rightarrow \quad d(u,w) = (u^p+w^p)^{1/p} \leq \\
&(u^p+v^p)^{1/p}+(v^p+w^p)^{1/p} = d(u,v)+d(v,w). \quad \square
\end{aligned} \tag{30}
$$

Theorem .12. *Symmetry:*
$d(s_1,s_2) = (\sum_{i=1}^{2} s_i^p)^{1/p} \quad \Rightarrow \quad d(u,v) = d(v,u)$.

Proof. By the commutative law of addition:

$$
\begin{aligned}
&\forall\, p \,:\, p \geq 1, \quad d(s_1,s_2) = (\textstyle\sum_{i=1}^{2} s_i^p)^{1/p} = (s_1^p+s_2^p)^{1/p} \\
&\Rightarrow \quad d(u,v) = (u^p+v^p)^{1/p} = (v^p+u^p)^{1/p} = d(v,u). \quad \square
\end{aligned} \tag{31}
$$

Theorem .13. *Non-negativity:*
$d(s_1,s_2) = (\sum_{i=1}^{2} s_i^p)^{1/p} \Rightarrow d(u,w) \geq 0$.

Proof. By definition, the length of an interval is always ≥ 0:

$$
\begin{aligned}
&\forall\, [a_1,b_1],\ [a_2,b_2], \quad u = b_1-a_1,\ v = b_2-a_2, \\
&\qquad \Rightarrow \quad u \geq 0,\ v \geq 0.
\end{aligned} \tag{32}
$$

$$
p \geq 1,\ \ u,v \geq 0 \quad \Rightarrow \quad d(u,v) = (u^p+v^p)^{1/p} \geq 0. \tag{33}
$$

$\square$

Theorem .14. *Identity of Indiscernibles:* $d(u,u) = 0$.

Proof. From the non-negativity property (.13):

$$
\begin{aligned}
&d(u,w) \geq 0 \quad \wedge \quad d(u,v) \geq 0 \quad \wedge \quad d(v,w) \geq 0 \\
&\qquad \Rightarrow \quad \exists\, d(u,w) = d(u,v) = d(v,w) = 0.
\end{aligned} \tag{34}
$$

$$
d(u,w) = d(v,w) = 0 \quad \Rightarrow \quad u = v. \tag{35}
$$

$$
d(u,v) = 0 \quad \wedge \quad u = v \quad \Rightarrow \quad d(u,u) = 0. \tag{36}
$$

$\square$

Properties limiting a set to at most 3 members

The following definitions and proof use first order logic. A Horn clause-like expression is used, here, to make the proof easier to read. By convention, the proof goal is on the left side and supporting facts are on the right side of the implication sign ($\leftarrow$). The formal proofs in the Rocq file threed.v are:

`Lemmas: adj111, adj122, adj212, adj123, adj133, adj233, adj213, adj313, adj323, and not_all_mutually_adjacent_gt_3.`

Definition .15. Immediate Cyclic Successor of m is n:

$$
\begin{aligned}
&\forall\, x_m, x_n \in \{x_1, \cdots, x_{setsize}\} : \\
&\qquad Successor(m,n,setsize) \\
&\leftarrow \quad (m = setsize \wedge n = 1) \quad \vee \quad (n = m+1 \leq setsize).
\end{aligned} \tag{37}
$$

Definition .16. Immediate Cyclic Predecessor of m is n:

$$
\begin{aligned}
&\forall\, x_m, x_n \in \{x_1, \cdots, x_{setsize}\} : \\
&\qquad Predecessor(m,n,setsize) \\
&\leftarrow \quad (m = 1 \wedge n = setsize) \quad \vee \quad (n = m-1 \geq 1).
\end{aligned} \tag{38}
$$

Definition .17. Adjacent: Member m is sequentially adjacent to member n if the immediate cyclic successor of m is n or the immediate cyclic predecessor of m is n. Notionally:

$$
\begin{aligned}
&\forall\, x_m, x_n \in \{x_1, \cdots, x_{setsize}\} : Adjacent(m,n,setsize) \\
&\leftarrow Successor(m,n,setsize) \vee Predecessor(m,n,setsize).
\end{aligned} \tag{39}
$$

Definition .18. Immediate Symmetric (every set member is sequentially adjacent to every other member):

$$
\forall\, x_m, x_n \in \{x_1, \cdots, x_{setsize}\} : \quad Adjacent(m,n,setsize). \tag{40}
$$

Theorem .19. *An immediate symmetric cyclic set is limited to at most 3 members.*

Proof.
Every member is adjacent to every other member, where $setsize \in \{1,\ 2,\ 3\}$:

$$
\begin{aligned}
&Adjacent(1,1,1) \leftarrow Successor(1,1,1) \leftarrow \\
&\qquad (m = setsize \wedge n = 1).
\end{aligned} \tag{41}
$$

$$
\begin{aligned}
&Adjacent(1,2,2) \leftarrow Successor(1,2,2) \leftarrow \\
&\qquad (n = m+1 \leq setsize).
\end{aligned} \tag{42}
$$

$$
\begin{aligned}
&Adjacent(1,2,3) \leftarrow Successor(1,2,3) \leftarrow \\
&\qquad (n = m+1 \leq setsize).
\end{aligned} \tag{43}
$$

$$
\begin{aligned}
&Adjacent(2,1,3) \leftarrow Predecessor(2,1,3) \leftarrow \\
&\qquad (n = m-1 \geq 1).
\end{aligned} \tag{44}
$$

$$Adjacent(3,1,3) \leftarrow Successor(3,1,3) \leftarrow (n = setsize \wedge m = 1). \quad (45)$$

$$Adjacent(1,3,3) \leftarrow Predecessor(1,3,3) \leftarrow (m = 1 \wedge n = setsize). \quad (46)$$

$$Adjacent(2,3,3) \leftarrow Successor(2,3,3) \leftarrow (n = m + 1 \leq setsize). \quad (47)$$

$$Adjacent(3,2,3) \leftarrow Predecessor(3,2,3) \leftarrow (n = m - 1 \geq 1). \quad (48)$$

Member 2 is the only immediate successor of member 1 for all $setsize \geq 3$, which implies member 3 is not ($\neg$) an immediate successor of member 1 for all $setsize \geq 3$:

$$\neg Successor(1,3,setsize \geq 3) \leftarrow Successor(1,2,setsize \geq 3) \leftarrow (n = m + 1 \leq setsize). \quad (49)$$

Member $n = setsize > 3$ is the only immediate predecessor of member 1, which implies member 3 is not ($\neg$) an immediate predecessor of member 1 for all $setsize > 3$:

$$\neg Predecessor(1,3,setsize \geq 3) \leftarrow Predecessor(1,setsize,setsize > 3) \leftarrow (m = 1 \wedge n = setsize > 3). \quad (50)$$

For all $setsize > 3$, some elements are not ($\neg$) sequentially adjacent to every other element (not immediate symmetric):

$$\neg Adjacent(1,3,setsize > 3) \leftarrow \neg Successor(1,3,setsize > 3) \quad \wedge \quad \neg Predecessor(1,3,setsize > 3). \quad \Box \quad (51)$$

The Symmetric goal matches Adjacent goals 41 through 48 and fails for all "setsize" greater than three.

APPLICATIONS TO PHYSICS

Where distance is an immediate symmetric cyclic set (ISCS) of dimensions, the 3D proof (.19) requires more dimensions to have non-distance types (members of other sets). Let $\tau = \{t\ (time),\ m\ (mass),\ q\ (charge)\}$ be the ISCS of type "non-distance" dimensions, where for each Cartesian axis unit length, r_p, of distance, r, there are Cartesian axis unit lengths: t_p of time, t; m_p of mass, m; and q_p of charge, q, such that:

$$r = (r_p/t_p)t = (r_p/m_p)m = (r_p/q_p)q, \quad (52)$$

where c_t, c_m, and c_q are the maximum ratios:

$$c_t = r_p/t_p, \quad c_m = r_p/m_p, \quad c = c_t = r_p/q_p. \quad (53)$$

Ratio-derived G, Newton, Gauss, and Poisson gravity laws

From equation 53:

$$r = c_m m \quad \wedge \quad r = c_t t \quad \Rightarrow \quad r/(c_t t)^2 = c_m m/r^2 \quad \Rightarrow \quad r/t^2 = (c_m c_t^2)m/r^2 = Gm/r^2, \quad (54)$$

where $G = c_m c_t^2$, conforms to the SI units: $m^3 \cdot kg^{-1} \cdot s^{-2}$ [3].

Newton's law follows from multiplying both sides of equation 54 by m:

$$r/t^2 = Gm/r^2 \;\Leftrightarrow\; F := mr/t^2 = Gm^2/r^2. \quad (55)$$

$$F = Gm^2/r^2 \wedge \forall\, m \in \mathbb{R} : \exists\, m_1, m_2 \in \mathbb{R} : m_1 m_2 = m^2 \quad \Rightarrow \quad F = Gm_1 m_2/r^2. \quad (56)$$

In this article, the following rationale for Gauss's and Poisson's laws for gravity are presented: Equation 54 relates linear acceleration, r/t^2, to mass and distance. Gauss's gravity field, $\mathbf{g}$, and Poisson's gravity field, $-\nabla\Phi(r,t)$, relates orbital acceleration, $2\pi r/t^2$, to mass and distance. Multiplying both sides of equation 54 by 2π and differentiating yields Gauss's and Poison's laws [2]:

$$\mathbf{g} = -\nabla\Phi(\vec{r},t) = 2\pi r/t^2 = 2\pi Gm/r^2 \quad \Rightarrow \quad \nabla \cdot \mathbf{g} = \nabla^2\Phi(\vec{r},t) = -4\pi Gm/r^3. \quad (57)$$

$$\nabla \cdot \mathbf{g} = \nabla^2\Phi(\vec{r},t) = -4\pi Gm/r^3 \quad \wedge \quad \rho = m/r^3 \quad \Rightarrow \quad \nabla \cdot \mathbf{g} = \nabla^2\Phi(\vec{r},t) = -4\pi G\rho. \quad (58)$$

Ratio-derived k_e and Coulomb's charge law

[2] From equation 53:

$$r = c_q q \quad \Rightarrow \quad r^2 = c_q^2 q^2 \quad \Rightarrow \quad c_q^2 q^2/r^2 = 1. \quad (59)$$

$$r = c_m m = c_t t \quad \Rightarrow \quad mr = ((1/c_m)r)(c_t t) = (c_t^2/c_m)t^2 \quad \Rightarrow \quad (c_m/c_t^2)mr/t^2 = 1. \quad (60)$$

$$c_q^2 q^2/r^2 = 1 \quad \wedge \quad (c_m/c_t^2)mr/t^2 = 1 \quad \Rightarrow \quad F := mr/t^2 = (c_q^2 c_t^2/c_m)q^2/r^2 = k_e q^2/r^2, \quad (61)$$

where $k_e = c_q^2 c_t^2/c_m$, conforms to the SI units: $kg \cdot m^3 \cdot s^{-2} \cdot C^{-2} = N \cdot m^2 \cdot C^{-2}$ [2].

$$\exists\, q_1, q_2 \in \mathbb{R} :\; q_1 q_2 = q^2 \quad \wedge \quad F = k_e q^2/r^2 \quad \Rightarrow \quad F = k_e q_1 q_2/r^2. \quad (62)$$

3 direct proportion ratios: c_t, c_m, and c_q

$$c_t = r_p/t_p \approx 2.99792458 \cdot 10^8 m\ s^{-1}. \quad (63)$$

$$G = c_m c_t^2 = c_m c_t^2 \Rightarrow \quad c_m = r_p/m_p \approx 7.4261602691 \cdot 10^{-28} m\ kg^{-1}. \quad (64)$$

$$k_e = c_q^2 c_t^2/c_m \Rightarrow \quad c_q = r_p/q_p \approx 8.6175172023 \cdot 10^{-18} m\ C^{-1}. \quad (65)$$

3 inverse proportion ratios: k_t, k_m, and k_q

$$\begin{aligned} &r/t = r_p/t_p, \quad r/m = r_p/m_p \\ &\Rightarrow \quad (r/t)/(r/m) = (r_p/t_p)/(r_p/m_p) \\ &\Rightarrow \quad (mr)/(tr) = (m_p r_p)/(t_p r_p) \\ &\Rightarrow \quad mr = m_p r_p = k_m, \quad tr = t_p r_p = k_t. \quad (66) \end{aligned}$$

$$\begin{aligned} &r/t = r_p/t_p, \quad r/q = r_p/q_p \\ &\Rightarrow \quad (r/t)/(r/q) = (r_p/t_p)/(r_p/q_p) \\ &\Rightarrow (qr)/(tr) = (q_p r_p)/(t_p r_p) \\ &\Rightarrow \quad qr = q_p r_p = k_q, \quad tr = t_p r_p = k_t. \quad (67) \end{aligned}$$

Ratio-derived $\hbar$, h, and Planck relation

[12] Applying both the direct proportion ratio (63), and inverse proportion ratio (66):

$$r = ct \quad \wedge \quad m = k_m/r \Rightarrow \quad m(ct)^2 = (k_m/r)r^2 = k_m r. \quad (68)$$

$$\begin{aligned} &m(ct)^2 = k_m r \quad \wedge \quad r/t = c \quad \Rightarrow \\ &E := mc^2 = k_m r/t^2 = (k_m c)(1/t) \\ &= \hbar\omega = \hbar\omega(2\pi/2\pi) = hf, \quad (69) \end{aligned}$$

where the reduced Planck constant, $\hbar = k_m c$, angular frequency, $\omega = 1/t$, the full Planck constant, $h = 2\pi\hbar$, and the cycles per second frequency (Hertz), $f = 1/2\pi t$.

$$k_m = m_p r_p = \hbar/c \approx 3.5176729162 \cdot 10^{-43}\ kg\ m. \quad (70)$$

$$k_t = t_p r_p = k_m c_m/c_t \approx 8.7136291599 \cdot 10^{-79}\ s\ m. \quad (71)$$

$$k_q = q_p r_p = k_t c_t/c_q \approx 3.0313607071 \cdot 10^{-52}\ C\ m. \quad (72)$$

4 quantum (Planck) units: r_p, t_p, m_p, q_p

:

$$r_p = \sqrt{r_p^2} = \sqrt{c_t k_t} = \sqrt{c_m k_m} = \sqrt{c_q k_q} \approx 1.6162550244 \cdot 10^{-35}\ m. \quad (73)$$

$$t_p = r_p/c_t \approx 5.3912464472 \cdot 10^{-44}\ s. \quad (74)$$

$$m_p = r_p/c_m \approx 2.176434343 \cdot 10^{-8}\ kg. \quad (75)$$

$$q_p = r_p/c_q \approx 1.875546038 \cdot 10^{-18}\ C. \quad (76)$$

Ratio-derived fine structure constant, α

The ratios of two subtypes of force implies ratios of the form: $\alpha_\tau = \frac{F_{\tau_1}}{F_{\tau_2}} = \frac{K\tau_1^2/r^2}{K\tau_2^2/r^2} = \frac{\tau_1^2}{\tau_2^2}$. For example, where q_e is the elementary (electron) charge ($1.60217663 \cdot 10^{-19}\ C$), and q_p is Planck charge unit, the fine structure electron coupling constant is:

$$\alpha_q = q_e^2/q_p^2 \approx 0.0072973526. \quad (77)$$

Ratio-derived Space-time-mass-charge

Let r be an Euclidean distance. Then by the Minkowski distance theorem (.8), $r^2 = \sum_{i=1}^m r_i^2$. Let, $r' = r_1$ and $r_v^2 = (\sum_{i=2}^m r_i^2)$. From the 3D theorem (.19) and Cartesian hypothesis (2):

$$\begin{aligned} &\forall\, \tau \in \{t, m, q\},\ r^2 = r'^2 + r_v^2,\ \exists\, \mu, \nu :\ r = \mu\tau \quad \wedge \\ &r_v = \nu\tau \quad \Rightarrow \quad (\mu\tau)^2 = r'^2 + (\nu\tau)^2 \\ &\Rightarrow \quad r' = \sqrt{(\mu\tau)^2 - (\nu\tau)^2} \ = \ \mu\tau\sqrt{1 - (\nu/\mu)^2}. \quad (78) \end{aligned}$$

Local frame distance, r', contracts relative to a distant observer frame distance, r, as $\nu \to \mu$:

$$r' = \mu\tau\sqrt{1 - (\nu/\mu)^2} \quad \wedge \quad \mu\tau = r \Rightarrow \quad r' = r\sqrt{1 - (\nu/\mu)^2}. \quad (79)$$

A distant observer frame type, τ, dilates relative to the local observer frame type, τ', as $\nu \to \mu$:

$$\mu\tau = r'/\sqrt{1 - (\nu/\mu)^2} \quad \wedge \quad r' = \mu\tau' \Rightarrow \quad \tau = \tau'/\sqrt{1 - (\nu/\mu)^2}. \quad (80)$$

Where τ is type, time, the space-like flat Minkowski spacetime event interval is:

$$\begin{aligned} &\mathrm{d}r^2 = \mathrm{d}r'^2 + \mathrm{d}r_v^2 \quad \wedge \quad \mathrm{d}r_v^2 = \mathrm{d}r_1^2 + \mathrm{d}r_2^2 + \mathrm{d}r_3^2 \quad \wedge \\ &\mathrm{d}(\mu\tau) = \mathrm{d}r \ \Rightarrow \ \mathrm{d}r'^2 = \mathrm{d}(\mu\tau)^2 - \mathrm{d}r_1^2 - \mathrm{d}r_2^2 - \mathrm{d}r_3^2. \quad (81) \end{aligned}$$

Ratio-derived Schwarzschild's time dilation and black hole metric

From equations 79 and 52:

$$\sqrt{1-(v^2/c^2)} = \sqrt{1-(v^2/c^2)} \quad \wedge \quad c_m m/r = 1$$
$$\Rightarrow \quad \sqrt{1-(v^2/c^2)} = \sqrt{1-(c_m m)v^2/rc^2}. \quad (82)$$

Where v_{escape} is the escape velocity:

$$\sqrt{1-(v^2/c^2)} = \sqrt{1-(c_m m)v^2/rc^2} \quad \wedge$$
$$KE = mv^2/2 = mv_{escape}^2$$
$$\Rightarrow \quad \sqrt{1-(v^2/c^2)} = \sqrt{1-2c_m mv_{escape}^2/rc^2}. \quad (83)$$

$$\sqrt{1-(v^2/c^2)} = \lim_{v_{escape}\to c} \sqrt{1-2c_m mv_{escape}^2/rc^2}$$
$$= \sqrt{1-2c_m mc^2/rc^2}. \quad (84)$$

Combining equation 84 with the derivation of G (56):

$$c_m c^2 = G \quad \wedge \quad \sqrt{1-(v^2/c^2)} = \sqrt{1-2c_m mc^2/rc^2}$$
$$\Rightarrow \quad \sqrt{1-(v^2/c^2)} = \sqrt{1-2Gm/rc^2}. \quad (85)$$

Combining equation 85 with equation 80 yields Schwarzschild's gravitational time dilation [13] [14]:

$$\sqrt{1-(v^2/c^2)} = \sqrt{1-2Gm/rc^2} \quad \wedge$$
$$t' = t\sqrt{1-(v^2/c^2)} \quad \Rightarrow \quad t' = t\sqrt{1-2Gm/rc^2}. \quad (86)$$

Schwarzschild defined the black hole event horizon radius, $r_s := 2Gm/c^2$. From equations 79 and 85:

$$r' = r\sqrt{1-(v/c)^2} \quad \wedge$$
$$\sqrt{1-(v/c)^2} = \sqrt{1-2Gm/rc^2} \quad \wedge \quad r_s := 2Gm/c^2$$
$$\Rightarrow \quad r' = r\sqrt{1-2Gm/rc^2} = r\sqrt{1-r_s/r}. \quad (87)$$

Applying equation 87 to the time-like spacetime interval equation 81:

$$r' = r\sqrt{1-r_s/r} \quad \wedge \quad \mathrm{d}s^2 = \mathrm{d}r'^2 - \mathrm{d}r^2 \quad \Rightarrow$$
$$\mathrm{d}s^2 = (\sqrt{1-r_s/r}\mathrm{d}r)^2 - (\mathrm{d}r'/\sqrt{1-r_s/r})^2$$
$$= (1-r_s/r)\mathrm{d}r^2 - (1-r_s/r)^{-1}\mathrm{d}r'^2. \quad (88)$$

General relativity does not have a special frame of reference, so let $r' = r$.

$$\mathrm{d}s^2 = (1-r_s/r)\mathrm{d}r^2 - (1-r_s/r)^{-1}\mathrm{d}r^2 \quad \wedge$$
$$\mathrm{d}r = \mathrm{d}(ct) \quad \wedge \quad c = 1$$
$$\Rightarrow \quad \mathrm{d}s^2 = (1-r_s/r)\mathrm{d}t^2 - (1-r_s/r)^{-1}\mathrm{d}r^2. \quad (89)$$

Using spherical coordinates to translate from 2D to 4D yields Schwarzschild's black hole metric [13] [14]:

$$\mathrm{d}s^2 = (1-r_s/r)\mathrm{d}t^2 - (1-r_s/r)^{-1}\mathrm{d}r^2 = f(r,t)$$
$$\Rightarrow \quad \mathrm{d}s^2 = (1-r_s/r)\mathrm{d}t^2 - (1-r_s/r)^{-1}\mathrm{d}r^2 -$$
$$r^2(\mathrm{d}\theta^2 + sin^2\theta\mathrm{d}\phi^2)$$
$$\Rightarrow \quad g_{\mu,\nu} = diag[1-r_s/r),\ (1-r_s/r)^{-1},\ r^2(\mathrm{d}\theta^2),$$
$$r^2(sin^2\theta\mathrm{d}\phi^2)]. \quad (90)$$

Simplified general relativity solutions

Step 1) Use the ratios to define functions returning scalar values for each component of the metric, $g_{\nu,\mu}$, in Einstein's field equations [4] [6]: All functions derived from the ratios and special relativity are valid metrics, for example, the previous Schwarzschild black hole metric derivation using the ratios ().

Step 2) Express the Einstein field equation as 2D tensors: As shown in equation 90, the Schwarzschild metric was first derived as a 2D metric and then expanded to a 4D metric. Further, the 4D flat spacetime interval equation (81) is an instance of the 2D equation, $\mathrm{d}r'^2 = \mathrm{d}(ct)^2 - \mathrm{d}r_v^2$.

The 2D metric tensor allows using the much simpler 2D Ricci curvature and scalar curvature. And the 2D tensors reduce the number of independent equations to solve, which can be used to set constraints on the solutions in the 4D tensors.

Step 3) One simple method to translate from 2D to 4D is to use spherical coordinates, where r and t remain unchanged and two added dimensions are the angles, ϕ, and θ. For example, the 2D Schwarzschild metric was translated to 4D using this method in equation 90.

Ratio-relativity-derived μ_0 and Lorentz's law

In this article, the following rationale for Gauss's electric field is presented: Coulomb's charge force equation 61 relates linear acceleration, r/t^2, to charge and distance. Gauss's electric field, $\mathbf{E}$, relates orbital (or rotational) acceleration, $2\pi r/t^2$ to charge and distance:

$$F_C = mr/t^2 = k_e q^2/r^2$$
$$\Rightarrow \quad \exists\ F_E \in \mathbb{R} : F_E = m(2\pi r/t^2) = 2\pi k_e q^2/r^2. \quad (91)$$

Applying the distance contraction equation 79 to equation 91, where r is the distant observer frame of reference and r' is moving particle local frame of reference:

$$r = r'/\sqrt{1-v^2/c^2} \quad \wedge \quad F = 2\pi k_e q^2/r^2$$
$$\Rightarrow \quad F = 2\pi k_e q^2(1-v^2/c^2)/r'^2. \quad (92)$$

$$E := 2\pi k_e q/r'^2 \;\;\Rightarrow\;\; F = q(E - v^2(2\pi k_e/c^2)q/r'^2). \quad (93)$$

$$B := (2\pi k_e/c^2)vq/r'^2 \quad \Rightarrow \quad F = q(E - vB). \quad (94)$$

$$F = q(E - vB) \quad \Rightarrow \quad \mathbf{F} = q(\mathbf{E} - \vec{\mathbf{v}} \times \mathbf{B}), \quad (95)$$

which is Lorentz law in the rest frame of reference. And

$$\mathbf{F} = q(\mathbf{E} + \vec{\mathbf{v}} \times \mathbf{B}), \quad (96)$$

is Lorentz law in the distant observer frame of reference. The direction of rotation (curl) depends on your frame of reference.

The electric field, $E := 2\pi k_e q/r'^2$, conforms to the SI units $kg \cdot m \cdot s^{-2} \cdot C^{-1} = N \cdot C^{-1}$ and the magnetic field, $B = (2\pi k_e/c^2)vq/r'^2$, conforms to the base SI units: $kg \cdot s^{-1} \cdot C^{-1} = kg \cdot s^{-2} \cdot A^{-1} = T$.

$$B := (2\pi k_e/c^2)vq/r'^2 \quad \wedge \quad B := \mu_0 H \quad \wedge \\ \mu_0 := 4\pi k_e/c^2 \quad \Rightarrow \quad H = vq/2r'^2, \quad (97)$$

where $\mu_0 = 4\pi k_e/c^2$ conforms to the SI units $kg \cdot m \cdot C^{-2} = kg \cdot m \cdot s^{-2} A^{-2}$ and $H = vq/2r'^2$ conforms to the SI units $C \cdot s^{-1} \cdot m^{-1} = A \cdot m^{-1}$.

Ratio-derived ε_0 and Gauss's electric field law

From equation 93:

$$E = 2\pi k_e q/r^2 \quad \Leftrightarrow \quad \mathbf{E} = 2\pi k_e q/\vec{\mathbf{r}}^2 \\ \Rightarrow \quad \nabla \cdot \mathbf{E} = -4\pi k_e q/\vec{\mathbf{r}}^3. \quad (98)$$

$$\nabla \cdot \mathbf{E} = -4\pi k_e q/\vec{\mathbf{r}}^3 \quad \wedge \quad \varepsilon_0 := 1/4\pi k_e \quad \wedge \quad \rho = q/\vec{\mathbf{r}}^3 \\ \Rightarrow \quad \nabla \cdot \mathbf{E} = -\rho/\varepsilon_0, \quad (99)$$

which is Gauss's electric field law [2].

Ratio-derived Faraday's law

From the magnetic field equation 94, where the electric and magnetic fields are propagating at the speed, $v = c$:

$$B = (2\pi k_e/c^2)qv/r^2 \quad \wedge \quad v = c \quad \wedge \quad r = ct \\ \Rightarrow \quad B = (2\pi k_e/c^3)q/t^2. \quad (100)$$

$$B = (2\pi k_e/c^3)q/t^2 \quad \Rightarrow \quad \partial B/\partial t = -(4\pi k_e/c^3)q/t^3. \quad (101)$$

$$\partial B/\partial t = -(4\pi k_e/c^3)q/t^3 \quad \wedge \quad r = ct \\ \Rightarrow \quad \partial B/\partial t = -4\pi k_e q/r^3. \quad (102)$$

From equation 98:

$$\mathbf{E} = 2\pi k_e q/\vec{\mathbf{r}}^2 \quad \Rightarrow \quad \nabla \times \mathbf{E} = 4\pi k_e q/\vec{\mathbf{r}}^3. \quad (103)$$

Combining equations 103 and 102 yields Faraday's law [2]:

$$\nabla \times \mathbf{E} = 4\pi k_e q/\vec{\mathbf{r}}^3 \quad \wedge \quad \partial \mathbf{B}/\partial t = -4\pi k_e q/\vec{\mathbf{r}}^3 \\ \Rightarrow \quad \nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t. \quad (104)$$

Ratio-derived Compton wavelength, λ

[12] From equations 66 and 69:

$$r = k_m/m \quad \wedge \quad h = 2\pi k_m c \\ \Rightarrow \quad \lambda = 2\pi r = 2\pi k_m/m = (2\pi k_m/m)(c/c) = h/mc. \quad (105)$$

Ratio-derived Schrödenger's position-space equation

Start with the previously derived Planck relation 69 and multiply the kinetic energy component by mc/mc:

$$mc^2 = \hbar\omega = \hbar/t \quad \Rightarrow \quad \exists V(r,t) : \hbar/t = \hbar/2t + V(r,t) \\ \Rightarrow \quad \hbar/t = \hbar mc/2mct + V(r,t). \quad (106)$$

And from the distance-to-time (speed of light) ratio (63):

$$\hbar/t = \hbar mc/2mct + V(r,t) \quad \wedge \quad r = ct \\ \Rightarrow \quad \hbar/t = \hbar mc^2/2mcr + V(r,t). \quad (107)$$

$$\hbar/t = \hbar mc^2/2mcr + V(r,t) \;\; \wedge \;\; \hbar/t = mc^2 \\ \Rightarrow \quad \hbar/t = \hbar^2/2mcrt + V(r,t). \quad (108)$$

$$\hbar/t = \hbar^2/2mcrt + V(r,t) \quad \wedge \quad r = ct \\ \Rightarrow \quad \hbar/t = \hbar^2/2mr^2 + V(r,t). \quad (109)$$

Multiply both sides of equation 109 by a function, $\Psi(r,t)$.

$$\hbar/t = \hbar^2/2mr^2 + V(r,t) \\ \Rightarrow \quad (\hbar/t)\Psi(r,t) = (\hbar^2/2mr^2)\Psi(r,t) + V(r,t)\Psi(r,t). \quad (110)$$

$$(\hbar/t)\Psi(r,t) = (\hbar^2/2mr^2)\Psi(r,t) + V(r,t)\Psi(r,t) \quad \wedge \\ \forall\ \Psi(r,t) :\ \partial^2\Psi(r,t)/\partial r^2 = (-1/r^2)\Psi(r,t) \quad \wedge \\ \partial\Psi(r,t)/\partial t = (i/t)\Psi(r,t) \\ \Rightarrow \quad i\hbar\partial\Psi(r,t)/\partial t = -(\hbar^2/2m)\partial^2\Psi(r,t)/\partial r^2 + V(r,t)\Psi(r,t), \quad (111)$$

which is the one-dimensional position-space Schrödenger's equation [15][12].

$$i\hbar\partial\Psi(r,t)/\partial t = -(\hbar^2/2m)\partial^2\Psi(r,t)/\partial r^2 + V(r,t)\Psi(r,t) \quad \wedge \quad ||\mathbf{\vec{r}}|| = r \quad \Rightarrow \quad \exists\, \mathbf{\vec{r}} : \, i\hbar\partial\Psi(\mathbf{\vec{r}},t)/\partial t = -(\hbar^2/2m)\partial^2\Psi(\mathbf{\vec{r}},t)/\partial \mathbf{\vec{r}}^2 + V(\mathbf{\vec{r}},t)\Psi(\mathbf{\vec{r}},t), \quad (112)$$

which is the 3-dimensional position-space Schrödenger's equation [15] [12].

Ratio-relativity-derived Dirac's wave equation

Using the derived Planck relation 69:

$$mc^2 = \hbar/t \quad \Rightarrow \quad \exists\, V(r,t) : \, mc^2/2 + V(r,t) = \hbar/t \quad \Rightarrow \quad 2\hbar/t - 2V(r,t) = mc^2. \quad (113)$$

$$\forall\, V(r,t) : \, V(r,t) = i\hbar/t \quad \wedge \quad r = ct \quad \wedge \quad 2\hbar/t - 2V(r,t) = mc^2 \;\Rightarrow\; 2\hbar/t - i2\hbar c/r = mc^2. \quad (114)$$

Use the ratios, $r = c_q q$, and, $r = ct$. to multiply each term on the left side of equation 114 by 1:

$$qc_q/r = qc_q/ct = 1 \quad \wedge \quad 2\hbar/t - i2\hbar c/r = mc^2 \quad \Rightarrow \quad 2\hbar(qc_q/c)/t^2 - i2\hbar((qc_q/c)/r^2)c = mc^2. \quad (115)$$

Applying a quantum amplitude equation in complex form to equation 116:

$$A_0 = (c_q/c)((1/t) - i(1/r)) \quad \wedge \quad 2\hbar(qc_q/c)/t^2 - i2\hbar((qc_q/c)/r^2)c = mc^2 \;\Rightarrow\; 2\hbar\partial(-qA_0)/\partial t - i2\hbar(\partial(-qA_0)/\partial r)c = mc^2. \quad (116)$$

Translating equation 116 to moving (rest frame) coordinates via the Lorentz factor, $\gamma_0 = 1/\sqrt{1-(v/c)^2}$:

$$2\hbar\partial(-qA_0)/\partial t - i\hbar h(\partial(-qA_0)/\partial r)c = mc^2 \;\Rightarrow\; \gamma_0 2\hbar\partial(-qA_0)/\partial t - \gamma_0 i2\hbar(\partial(-qA_0)/\partial r)c = mc^2. \quad (117)$$

Multiplying both sides of equation 117 by $\Psi(r,t)$:

$$\gamma_0 2\hbar\partial(-qA_0)/\partial t - \gamma_0 i2\hbar(\partial(-qA_0)/\partial r)c = mc^2 \quad \Rightarrow \quad \gamma_0 2\hbar(\partial(-qA_0)/\partial t)\Psi(r,t) - \gamma_0 i2\hbar(\partial(-qA_0)/\partial r)c\Psi(r,t) = mc^2\Psi(r,t). \quad (118)$$

Applying the vectors to equation 118:

$$\gamma_0 2\hbar(\partial(-qA_0)/\partial t)\Psi(r,t) - \gamma_0 i2\hbar(\partial(-qA_0)/\partial r)c\Psi(r,t) = mc^2\Psi(r,t) \quad \wedge \quad ||\mathbf{\vec{r}}|| = r \quad \wedge \quad ||\mathbf{\vec{A}}|| = A_0 \quad \wedge \quad ||\vec{\gamma}|| = \gamma_0 \quad \Leftrightarrow \quad \exists\, \mathbf{\vec{r}}, \mathbf{\vec{A}}, \vec{\gamma} : \, \gamma_0 2\hbar(\partial(-qA_0)/\partial t)\Psi(r,t) - \vec{\gamma}\cdot i2\hbar(\partial(-q\mathbf{\vec{A}})/\partial r)c\Psi(\mathbf{\vec{r}},t) = mc^2\Psi(\mathbf{\vec{r}},t). \quad (119)$$

Adding a $\frac{1}{2}$ spin to equation 116 yields Dirac's wave equation [16] [12]:

$$\gamma_0 2\hbar(\partial(-qA_0)/\partial t)\Psi(r,t) - \vec{\gamma}\cdot i2\hbar(\partial(-q\mathbf{\vec{A}})/\partial r)c\Psi(\mathbf{\vec{r}},t) = mc^2\Psi(\mathbf{\vec{r}},t) \quad \wedge \quad A_0 = \frac{1}{2}(c_q/c)((1/t) - i(1/r)) \quad \Rightarrow \quad \gamma_0\hbar(\partial(-qA_0)/\partial t)\Psi(r,t) - \vec{\gamma}\cdot i\hbar(\partial(-q\mathbf{\vec{A}})/\partial r)c\Psi(\mathbf{\vec{r}},t) = mc^2\Psi(\mathbf{\vec{r}},t). \quad (120)$$

Total mass

The total mass of a particle is $m = \sqrt{m_0^2 + m_{ke}^2}$, where m_0 is the rest mass and m_{ke} is the kinetic energy-equivalent mass. Applying both the direct (63) and inverse proportion ratios (66):

$$m_0 = r/c_m \quad \wedge \quad m_{ke} = k_m/r \quad \wedge \quad m = \sqrt{m_0^2 + m_{ke}^2} \quad \Rightarrow \quad m = \sqrt{(r/c_m)^2 + (k_m/r)^2}. \quad (121)$$

Quantum extension to general relativity

The simplest way to demonstrate how to add quantum physics to general relativity is by extending Schwarzschild's time dilation equation and black hole metric (). Start by changing equation 82 in the Schwarzschild derivation:

$$\sqrt{1-(v^2/c^2)} = \sqrt{1-(v^2/c^2)(r/r)} \quad \wedge \quad r = \sqrt{(c_m m)^2 + (k_m/m)^2} = Q_m \quad \Rightarrow \quad \sqrt{1-(v^2/c^2)} = \sqrt{1 - Q_m v^2/rc^2}. \quad (122)$$

$$\sqrt{1-(v^2/c^2)} = \sqrt{1 - Q_m v^2/rc^2} \quad \wedge \quad KE = mv^2/2 = mv_{escape}^2 \quad \Rightarrow \quad \sqrt{1-(v^2/c^2)} = \sqrt{1 - 2Q_m v_{escape}^2/rc^2}. \quad (123)$$

$$\sqrt{1-(v^2/c^2)} = \lim_{v_{escape}\to c}\sqrt{1-2Q_m v_{escape}^2/rc^2}$$
$$\Rightarrow \quad \sqrt{1-(v^2/c^2)} = \sqrt{1-2Q_m c^2/rc^2} = \sqrt{1-2Q_m/r}. \tag{124}$$

Combining equation 124 with equation 80 yields Schwarzschild's gravitational time dilation with a quantum mass effect:

$$\sqrt{1-(v^2/c^2)} = \sqrt{1-2Q_m/r} \quad \wedge \quad t' = t\sqrt{1-(v^2/c^2)}$$
$$\Rightarrow \quad t' = t\sqrt{1-2Q_m/r}. \tag{125}$$

Schwarzschild defined the black hole event horizon radius, $r_s := 2Gm/c^2$. The radius with the quantum extension is $r_s := 2Q_m$. At this point the exact same equations 87 through 90 yield what looks like the same Schwarzschild black hole metric.

Quantum extension to Newton's gravity force

The quantum mass effect is easier to understand in the context Newton's gravity equation than in general relativity, because the metric equations and solutions in the EFEs are much more complex. From equations 121 and 52:

$$m/\sqrt{(r/c_m)^2+(k_m/r)^2} = 1 \quad \wedge \quad r^2/(ct)^2 = 1$$
$$\Rightarrow \quad r^2/(ct)^2 = m/\sqrt{(r/c_m)^2+(k_m/r)^2}$$
$$\Rightarrow \quad r^2/t^2 = c^2 m/\sqrt{(r/c_m)^2+(k_m/r)^2}. \tag{126}$$

$$r^2/t^2 = c^2 m/\sqrt{(r/c_m)^2+(k_m/r)^2}$$
$$\Rightarrow \quad (m/r)(r^2/t^2) = (m/r)(c^2 m/\sqrt{(r/c_m)^2+(k_m/r)^2})$$
$$\Rightarrow \quad F := mr/t^2 = c^2 m^2/\sqrt{(r^4/c_m^2)+k_m^2}. \tag{127}$$

$$F = c^2 m^2/\sqrt{(r^4/c_m^2)+k_m^2} \quad \wedge$$
$$\forall\, m \in \mathbb{R},\ \exists\, m_1, m_2 \in \mathbb{R}:\ m_1 m_2 = m^2$$
$$\Rightarrow \quad F = c^2 m_1 m_2/\sqrt{(r^4/c_m^2)+k_m^2}. \tag{128}$$

Quantum extension to Coulomb's force

$$q^2/((r/c_q)^2+(k_q/r)^2) = 1 \quad \wedge \quad r^2/(ct)^2 = 1$$
$$\Rightarrow \quad r^2/(ct)^2 = q^2/((r/c_q)^2+(k_q/r)^2)$$
$$\Rightarrow \quad r^2/t^2 = c^2 q^2/((r/c_q)^2+(k_q/r)^2). \tag{129}$$

$$(1/r)(r^2/t^2) = (1/r)(c^2 q^2/((r/c_q)^2+(k_q/r)^2))$$
$$\Rightarrow \quad r/t^2 = c^2 q^2/(r^3/c_q^2+k_q^2/r). \tag{130}$$

$$\forall\, q \in \mathbb{R}:\ \exists\, q_1, q_2 \in \mathbb{R}:\ q_1 q_2 = q^2 \quad \wedge$$
$$r/t^2 = c^2 q^2/(r^3/c_q^2+k_q^2/r)$$
$$\Rightarrow \quad \exists\, q_1, q_2 \in \mathbb{R}:\ r^2/t^2 = c^2 q_1 q_2/(r^3/c_q^2+k_q^2/r). \tag{131}$$

$$r^2/t^2 = c^2 q_1 q_2/(r^3/c_q^2+k_q^2/r) \quad \wedge \quad m = r/c_m$$
$$\Rightarrow \quad F := mr/t^2 = (c^2/c_m) q_1 q_2/(r^2/c_q^2+k_q^2/r^2). \tag{132}$$

INSIGHTS AND IMPLICATIONS

1. The ruler measure (.1) and convergence theorem (.2) were shown to be useful tools for proving that a countable sets of n-tuples imply a corresponding real-valued equation.
2. Defining all Euclidean and non-Euclidean distance measures as the inverse function of the sum of subset volumes:
$$\forall\, n,\ d: \quad d = f_n^{-1}(v) = f_n^{-1}(\textstyle\sum_{i=1}^m v_i): \tag{133}$$
   (a) shows the intimate relation between distance and volume that definitions, like inner product space and metric space, ignore [6] [7] [8];
   (b) is a more simple and concise definition of a distance measure that includes the properties of inner product space and metric space [6] [7] [8].
3. The left side of the distance sum inequality (.10),
$$(\textstyle\sum_{i=1}^m (a_i^n + b_i^n))^{1/n} \le (\sum_{i=1}^m a_i^n)^{1/n} + (\sum_{i=1}^m b_i^n)^{1/n}, \tag{134}$$
   differs from the left side of Minkowski's sum inequality [10]:
$$(\textstyle\sum_{i=1}^m (a_i^n + b_i^n)^{\mathbf{n}})^{1/n} \le (\sum_{i=1}^m a_i^n)^{1/n} + (\sum_{i=1}^m b_i^n)^{1/n}. \tag{135}$$
   (a) The two inequalities are only the same where $n = 1$.
   (b) The distance sum inequality (.10) is a more fundamental inequality because the proof does not require the convexity and Hölder's inequality assumptions required to prove the Minkowski sum inequality [10].
   (c) The distance sum inequality term, $\forall\ n > 1$, $v_i^n = a_i^n + b_i^n$: $d = v^{1/n} = (\sum_{i=1}^m v_i^n)^{1/n}$, is the Minkowski distance, which makes it directly related to geometry. But the Minkowski sum inequality term, $\forall\ n > 1$, $v > 0$: $d = v^{1/n} = (\sum_{i=1}^m ((v_i^n)^{\mathbf{n}}))^{1/n} = (\sum_{i=1}^m v_i^{\mathbf{n^2}})^{1/n}$, is *not* a Minkowski distance.

(d) The distance sum inequality might be applicable to machine learning.

4. **Combinatorics**, the set of ordered combinations of countable, disjoint sets (n-tuples), $v_c = \prod_{i=1}^{n} |x_i|$, was proven to imply: the Euclidean volume equation (.4), the sum of volumes equation (.7) (which includes the inner product), and the Minkowski distance equation (.8) (which includes the Manhattan and Euclidean distance equations), without relying on the geometric primitives and relations in Euclidean geometry [17] [18], axiomatic geometry [19] [20] [21] [22] [23], trigonometry [24] [25] calculus [26] [24] [27], and vector analysis [6].

5. **Combinatorics**, repeatable sequencing through an ordered set of n number of members to yield all $n!$ permutations of its members (without jumping around) was proved to be an immediate symmetric cyclic set (ISCS) having $n \leq 3$ members (.19). Higher dimensions must have different types (members of different sets).

   (a) For example, the vector inner product space can only be extended beyond 3 dimensions if and only if the higher dimensions have non-distance types, for example, dimensions time, mass, and charge.

   (b) As shown in the special relativity section (), there is 6-dimensional space-time-mass-charge.

   (c) If each type of quantum state is an ISCS, then there are at most 3 states of the same type: 3 orientations per dimension of space, 3 quark color charges, {red, green, blue}, 3 quark anti-color charges, and so on.

   (d) If the states are not ordered (a bag of states), then a state value is undetermined (or superimposed) until observed (like Schrödenger's poisoned cat being both alive and dead until the box is opened [15]).

   (e) A discrete (point) value has measure 0 (zero-length interval size). The ratio of a time or distance interval length to zero is undefined, which is the reason quantum entangled (discrete) state values exist independent of time and distance.

6. For each Cartesian axis unit, r_p, of a 3-dimensional distance interval having a length, r, there are Cartesian axis units of other types of intervals forming unit ratios (): $c_t = r_p/t_p$, $c_m = r_p/m_p$, $c_q = r_p/q_p \Leftrightarrow$ the inverse proportion ratios (): $k_t = r_p t_p$, $k_m = r_p m_p$, $k_t = r_p q_p$, where r_p, t_p, m_p, and q_p are the Planck units.

7. Empirical laws *describe* relations. Deriving empirical laws from the ratios *explains* the relations. Further, all the derivations of the physics equations from the ratios were much shorter and simpler than other derivations, which shows that the ratios are an important tool for physicists and engineers.

8. As shown in subsection the derivation of the Schwarzschild's time dilation (), and black hole metric () [13] [14] using ratios exposed a way of simplifying the finding of solutions to Einstein's field equations.

9. The speed of light ratio, c_t, is a component of the constants: $G = c_m c_t^2$, $k_e = c_q^2 c_t^2/c_m$, $\varepsilon_0 = 1/4\pi k_e = 1/4\pi(c_q^2 c_t^2/c_m)$, $\hbar = k_m c_t$.

   The only constant that does not contain c_t is vacuum permeability: $\mu_0 = 4\pi k_e/c_t^2 = 4\pi c_q^2/c_m$.

10. Using the quantum (Planck) units, r_p and t_p: $r_p/t_p^2 \approx 5.5607262989 \cdot 10^{51}\ m\ s^{-2}$, which suggests a maximum acceleration for masses. And $2\pi r_p/t_p^2$ would be maximum orbital or rotational acceleration.

11. The simplification of μ_0 into the quantum units shows two interesting relationships:

$$\mu_0 = 4\pi\frac{k_e}{c_t^2} = 4\pi\frac{c_q^2}{c_m} = 4\pi\frac{(r_p/q_p)^2}{r_p/m_p} = 4\pi\frac{m_p r_p}{q_p^2} = 4\pi\frac{k_m}{q_p^2}$$
$$\approx 4\pi\frac{3.5176729162 \cdot 10^{-43}}{3.5176729162 \cdot 10^{-35}} = 4\pi \cdot 10^{-7}\ kg\ m\ C^{-2}$$
$$= 4\pi \cdot 10^{-7}\ H\ m^{-1}. \quad (136)$$

   (a) The first time $k_m = m_p r_p$ appears is in the derivation of the Planck relation and Planck constant, $\hbar = k_m c$ (), the second time in the Compton wavelength, $r = k_m/m$ (). And now, k_m appears as a components of k_e, ε_0, and μ_0.

   (b) It is an open question why $\frac{k_m}{q_p^2}$ seems to equal $1.0 \cdot 10^{-7}$ exactly.

12. The fine structure constant, α was derived from the ratio of two forces of two subtypes that reduces to ratio of the square of the subtypes $\alpha = q_e^2/q_p^2 \approx 0.0072973526$ (), which is the empirical CODATA value [11].

   (a) The CODATA electron coupling version of the fine structure constant, α is defined as: $\alpha = q_e^2/4\pi\varepsilon_0\hbar c) = q_e^2/2\varepsilon_0 hc$ [11]. The following steps show that the CODATA definition reduces to the ratio-derived equation:

$$\varepsilon_0 := 1/4\pi k_e = 1/(4\pi(c_q^2 c_t^2/c_m) \quad \wedge \quad \hbar = k_m c_t \quad \wedge \quad h = 2\pi\hbar$$
$$\Rightarrow \quad \varepsilon_0 hc = 2\pi k_m c_t^2/(4\pi(c_q^2/c_m)c_t^2) = k_m/(2(c_q^2/c_m))$$
$$= m_p r_p/(2((r_p/q_p)^2/(r_p/m_p))) = q_p^2/2. \quad (137)$$

$$\alpha = q_e^2/2\varepsilon_0 hc \quad \wedge \quad \varepsilon_0 hc = q_p^2/2$$
$$\Rightarrow \quad \alpha = q_e^2/2(q_p^2/2) = q_e^2/q_p^2. \quad (138)$$

(b) Other fine structure constants can also be expressed more simply as the ratios of two subtypes of fields, for example, an electron gravity coupling constant can be expressed as the ratio of the rest electron mass to a Planck mass unit: $\alpha_{G_m} = m_e^2/m_p^2$.

13. Empirical and hypothesized laws of physics use an *opaque* constant, K, that is defined to make an equation, where the units balance, $g = Kf(r, t, \cdots)$. The opacity has led to the *incorrect* assumptions of those constants, K, being fundamental (atomic) constants.

    In this article, some opaque constants are derived directly from (composed of) the ratios: gravity, $G = c_m c_t^2$ (56), charge, $k_e = c_q^2 c_t^2/c_m$ (61), and Planck $h = k_m c_t$ (69). $\varepsilon_0 = 1/4\pi k_e = 1/4\pi c_m/((c_q^2/c_m)c_t^2)$ (99) and $\mu_0 = 4\pi k_e/c_t^2 = 4\pi c_q^2/c_m$ (96).

    And the quantum extensions to: Schwarzschild's time dilation (124) Newton's gravity force (128), and Coulomb's charge force show, that where the quantum effects become measurable, the constants G, k_e, ε_0, and μ_0 no longer exist (are no longer valid).

    Therefore, G, k_e, ε_0, μ_0, and h are **not** fundamental (atomic) constants.

14. The derivations of: $\nabla \cdot \mathbf{g} = -4\pi G\rho$ from $\mathbf{g} = 2\pi Gm/r^2$ (57), $\nabla \cdot \mathbf{E} = -\rho/\varepsilon_0$ from $\mathbf{E} = 2\pi k_e q/r^2$ (99), and $\partial\mathbf{B}/\partial t = -\mu_0\rho$ from $\mathbf{B} = 2\pi k_e q/r^2$ (102), show that the use of mass and charge density, ρ, are unnecessary complications that obfuscates the pattern, $\nabla \cdot f(x, y, r) = -2k_{x,y}y/r^3$, being derived from the inverse square pattern, $f(x, y, r) = k_{x,y}y/r^2$. And the energy density in the stress-energy tensor, $T_{\mu,\nu}$, in Einstein's field equations [6] also obfuscates the inverse square assumption.

15. Einstein's relativity equations assume the Lorentz transformations, assume that the laws of physics are same at each coordinate point, assume the notion of light, and assume that the speed of light is the same at each coordinate point [4] [6]. The derivations, in this article, were made without those assumptions (does even require the notion of light). Assuming Cartesian coordinates at each coordinate point, creates unit ratios, where all equations (laws) derived from the unit ratios must be the same at each coordinate point. Deriving numeric values for the ratios assumes that the ratio, c_t, is the maximum speed.

16. The derivation of the magnetic field from the ratios and special relativity (93) shows that magnetic field, $\mathbf{B}$, is the spacetime bend (curl) of the electric field, $\mathbf{E}$ due to relativistic velocities. The magnetic force is a torque caused by spacetime bending of the radial charge force.

    A charged particle's spin (angular momentum) axis has an orientation. "Paired spins" is where the orientations of two valence electrons are in opposite directions. The opposite spacetime bending due to relativistic spins cancel each other. Materials with unpaired spins that have net aligned orientations is a permanent magnet.

    A current in a conductor is where electrons are moving in the same direction with the orientations aligned in that same direction. Applying electromagnetic radiation to a thin, conductive film containing unpaired spins aligned in the same direction will create a current with near 100% efficiency minus electrical resistance. Such a low-resistance film would be several times more efficient than current solar panels.

    True elementary particles do not have a half or fractional spin. A "half-spin" is a π radians rotation of the spin axis orientation. Positive and negative charges with orientations in the same direction have opposite spins (opposite angular momenta).

17. The quantum extensions to: Schwarzschild's time dilation (124) black hole metric (90), Newton's gravity force (128), and Coulomb's charge force (131) make quantifiable predictions:

    (a) The gravitation and charge forces peak at finite amounts as $r \to 0$: for gravity, $\lim_{r\to 0} F = c^2 m_1 m_2/k_m$, and for charge, $\lim_{r\to 0} F = 0$. Finite maximum gravity and charge forces: 1) allows radioactivity, finite sloped energy well walls; and 2) eliminates the problem of forces going to infinity as $r \to 0$, which might eliminate the need to hypothesize the existence of a weak force and strong force.

    (b) The quantum-extended Schwarzschild time dilation and metric, gravity, and charge equations reduce to the classic equations, where the distance between masses and charges is sufficiently large or the masses and charges sufficiently large that the quantum effect is not measurable. **Note** that G, k_e, ε_0, μ_0, and κ (Einstein's constant, which contains G) do not exist (are not valid), where the quantum effects becomes measurable.

(c) And the covariant tensor components, in Einstein's field equations, that had the units $1/distance^2$, will now have the more complex units, $1/\sqrt{(distance^4/c_m^2) + k_m^2}$.

(d) $1/\sqrt{(distance^4/c_m^2) + k_m^2}$ implies that as distance $\to$ 0, spacetime curvature peaks at a finite amount, which might imply that black holes have sizes > 0 (might not be singularities). Black hole evaporation might be possible. If there was a "big bang," then it might not have originated from a singularity.

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