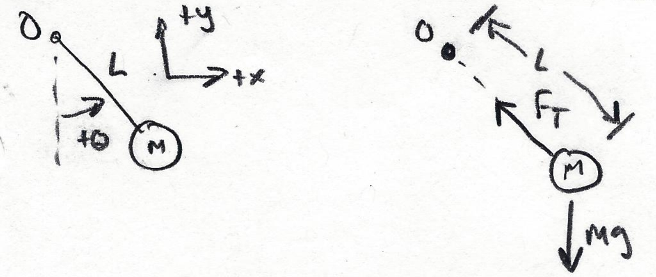
DISCRITIZING THE DAMPED PENDULUM

REF: A PRIMER ON SCIENTIFIC PROGRAMMING WITH PYTHON. H.P. LANGTANGEN. SPRINGER. APPENDIX A.



$$\sum \underline{M}_0 = \dot{\underline{H}}_0 \quad (\text{"0" IS FIXED, PLANAR MOTION})$$

$$-MgL\sin\theta - c\dot{\theta} = ML^2\ddot{\theta}$$

$$\boxed{\ddot{\theta} + \frac{c}{ML^2}\dot{\theta} + \frac{g}{L}\sin\theta = 0}$$

BREAK INTO TWO 1ST ORDER SYSTEMS:

$$\frac{d}{dt}(\theta(t)) = p(t)$$

$$\frac{d}{dt}(p(t)) = \frac{-c}{ML^2}p(t) - \frac{g}{L}\sin(\theta(t))$$

NOW TURN DERIVATIVES INTO DIFFERENCES. FIRST DERIVE FWD. DIFF.

TAYLOR: ABOUT $t = t_0$ $\quad f(t) = \sum_{n=0}^{\infty} \frac{f^n(t_0)}{n!}(t-t_0)^n$

$$f(t) = f(t_0) + f'(t_0)(t-t_0) + \tfrac{1}{2}f''(t_0)(t-t_0)^2 + \tfrac{1}{6}f'''(t_0)(t-t_0)^3 + \ldots$$

EVAL @ $t = t_0 + \Delta t$

$$f(t_0+\Delta t) = f(t_0) + \underbrace{f'(t_0)}_{\text{SOLVE FOR THIS!}}(\Delta t) + \underbrace{\tfrac{1}{2}f''(t_0)(\Delta t)^2 + \tfrac{1}{6}f'''(t_0)(\Delta t)^3 + \ldots}_{O(\Delta t^2)}$$

$$f'(t_0) = \frac{f(t_0+\Delta t) - f(t_0)}{\Delta t} + O(\Delta t)$$

$$\boxed{f'_i \approx \frac{f_{i+1} - f_i}{\Delta t}} \quad \text{FORWARD DIFFERENCE}$$

APPLY TO OUR SYSTEM (TWICE):

$$\frac{\theta_{i+1} - \theta_i}{\Delta t} = p_i \implies \boxed{\theta_{i+1} = \theta_i + (\Delta t)p_i} \quad (1)$$

$$\frac{p_{i+1} - p_i}{\Delta t} = \frac{-c}{ML^2}p_i - \frac{g}{L}\sin(\theta_i) \implies p_{i+1} = \left(p_i - \frac{c\Delta t}{ML^2}p_i\right) - \frac{g\Delta t}{L}\sin(\theta_i)$$

$$\boxed{p_{i+1} = \left(1 - \frac{c\Delta t}{ML^2}\right)p_i - \frac{g\Delta t}{L}\sin(\theta_i)} \quad (2)$$

$$\underline{X} = \begin{bmatrix}\theta \\ \dot{\theta}\end{bmatrix} \quad \therefore \quad \underline{X}_{i+1} = \begin{bmatrix} X_i(1) + \Delta t\, X_i(2) \\ \left(1 - \frac{c\Delta t}{ML^2}\right)X_i(2) - \frac{g\Delta t}{L}\sin(X_i(1)) \end{bmatrix}$$

$\Leftarrow$ NONLINEAR DIFFERENCE EQUATION