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Finite-time resilient fault-tolerant investment policy scheme for chaotic nonlinear finance system

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ABSTRACT

The problem of unpredictable and irregular fluctuations in investment policy scheme for the chaotic nonlinear finance system with undesirable changes such as wrong economy policy, share loss and natural disaster which is described by the well-known nonlinear differential equation is concerned in this work. A resilient fault-tolerant guaranteed cost controller with delay is designed to tackle the fluctuations in investment policy scheme with minimum guaranteed cost bound and also to achieve finite-time boundedness. Specifically, a new delay-dependent sufficient constraints is derived with the aid of suitable Lyapunov–Krasovskii functional to obtain the required result with minimum disturbance attenuation level through extended passivity performance index. Based on the addressed algorithm, the required investment policy scheme is determined by resorting the obtained constraints into MATLAB LMI toolbox. At last, the theoretical results are validated by presenting numerical simulations.

1. Introduction

1.1. Background and motivation

In recent years, the study of chaotic systems has emerged as an essential framework to model various real world problems in science, economics, finance and engineering [1,2]. Generally, chaotic systems are very sensitive in nature, that is an arbitrarily small change or disruption may cause significant transition in future behavior of systems. Thus the stabilization of chaotic systems receive much attention among research communities [3–7]. During the past few decades, the chaotic nonlinear finance system have received considerable attention due to the complex nature and inherent randomness in economic factors. In this connection, the economic system is mathematically modeled as a chaotic nonlinear differential equation to represent the dynamics of financial issues wherein the design of investment policy plays a crucial role to reduce the risk of economic factors [8–14]. Further, the authors in [8], studied the basic dynamical behaviors of hyper chaotic finance system and also designed a feedback controller to stabilize the system to its equilibrium point. Active control strategy is proposed for fractional chaotic finance systems in the work of Huang and Cao [9]. For stabilizing the finance system, many control strategies such as hybrid feedback control [11], time-delayed feedback control [12] and sliding mode control [14] are addressed in the existing literature. Based on the above discussed studies, the stability of the finance systems is ensured via various control strategies in an infinite period of time. Moreover, it is essential to predict and ensure the stability of the financial system within a finite period of time to regulate the economic crisis, which is discussed in this work.

1.2. Literature review

Moreover in practice, the economic factors have undesirable changes, such as wrong economy policy, natural disaster and international share loss and it may drastically affects the economy. To tackle the above mentioned undesirable changes and to improve the economic performance fault-tolerant investment policy scheme is imperative. In this connection, many researchers reported the fault-tolerant approach for distinct dynamical systems (see [15–17] and the references therein). Lee et al.[15] developed a robust fault-tolerant control scheme for a class of linear time-varying systems to assure the asymptotic stability even in the presence of parametric uncertainties. In [16], a robust fault-tolerant control strategy is developed for the chaotic systems. By utilizing

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an adaptive output feedback fault-tolerant control, Yang et al.[17] reduced the fault effects in switched fuzzy systems and also ensured the asymptotic stability of the system. On the other hand, fluctuations and inaccuracies in the implementation of the investment policy scheme lead to undesirable changes in chaotic nonlinear finance system. Thus, it is significant to develop an appropriate investment policy scheme to deal these unpredictable changes during the implementation [18,19]. The authors in [18] developed a resilient observer-based H_∞ controller via delay-dependent criterion to restraint the stability of stochastic systems. Furthermore, Ma and Chen [19] proposed a robust non-fragile dissipative control design for the class of T-S fuzzy systems with time-varying delay and parameter uncertainties to ensure the stability of the system dynamics in a precise period of time.

In addition, it is more important to find the degradation range incurred by unpredictable and undesirable changes in economy which motivates the study of guaranteed cost control scheme to ensure the guaranteed bound of unpredictable issue to be less than the cost bound [20–24]. The authors in [20] presented a set of sufficient conditions via state feedback control law to assure the asymptotic stability for the uncertain nonlinear systems with minimum cost bound. It is worth noting that in practice, finite-time stability of the finance systems play an imperative role to retain its economic policies [25-31]. Xin et al.[25] employed Jacobian predictor-corrector approach to design a finite-time controller to stabilize the fractional-order chaotic finance system within a precise period of time. In the existing literature, extended passivity based control has been widely investigated to eliminate unpredictable events and to handle fickle investment policy that can be regarded as external disturbances [32–36]. Recently for singular systems Chen et al.[32] developed an efficient extended passivity performance based static output feedback controller and in [36] a sufficient conditions are derived to ensure the asymptotic stability in mean-square sense for the stochastic nonlinear systems by using extended passivity control. To the best of authors knowledge, in the existing literature no work has been reported regarding the finite-time resilient fault-tolerant guaranteed cost investment policy scheme for the chaotic nonlinear finance system.

1.3. Contributions

The main contributions of this work are highlighted as follows

- The nonlinear mathematical model of a finance system with unpredictable and irregular fluctuations in investment policy scheme is formulated.
- By utilizing Lyapunov stability theory and convex optimization technique, a set of novel delay-dependent criterion is developed to tackle the undesirable changes and fluctuations in economic factors over a finite period of time.
- The consistent gain matrices for the proposed resilient fault-tolerant guaranteed cost investment policy scheme are obtained by solving the derived LMI-based constraints that reduces the convergence time and also ensures the stability of the considered model.
- Simulation results reveal the robustness of the proposed control strategy with minimum guaranteed cost bound and also the applicability of the derived theoretical result.

1.4. Paper organization

The rest of this paper is organized as follows: In Section 2, the dynamical behavior of the finance system is formulated as a mathematical model and some preliminary lemmas and definitions are given. In Section 3, the sufficient stability conditions to ensure the finite-time boundedness of the finance system are established. In Section 4, the significance of the proposed theoretical result is examined by presenting the simulation results of two numerical examples. Atlast, the conclusion is provided in Section 5.

2. Problem formulation

In this section, we consider a nonlinear chaotic finance system with the interest rate x_1, investment demand x_2 and price index x_3, which is represented by the following differential equations [8]:

$$\begin{cases}\dot{x}_1(t) = x_3(t) + (x_2(t) - p)x_1(t),\\ \dot{x}_2(t) = 1 - qx_2(t) - x_1^2(t),\\ \dot{x}_3(t) = -x_1(t) - rx_3(t),\end{cases} \tag{1}$$

where the positive parameters p, q and r represent the saving amount, the cost per investment and the elasticity of demands of commercial market. Now by incorporating the control term (i.e, investment policy scheme) in the finance model, the system (1) can be rewritten as

$$\dot{x}(t) = Ax(t) + u(t) + Cw(t) + f(x(t)), \tag{2}$$

where $x(t) = (x_1(t), x_2(t), x_3(t))$ is the state vector; $w(t) \in \mathcal{R}^q$ represents the financial disturbance satisfies $\int_0^N w(s)^T w(s)ds \leq d,\ \forall d > 0$; $C \in \mathcal{R}^{3\times q}$ is the known real matrix; $u(t)$ represents the control input. Further, the co-efficient matrices are given by

$$A = \begin{bmatrix} -p & 0 & 1\\ 0 & -q & 0\\ -1 & 0 & -r\end{bmatrix} \text{ and } f(x(t)) = \begin{bmatrix} x_1x_2\\ 1 - x_1^2\\ 0\end{bmatrix}.$$

To restrain the chaos behavior, we design the resilient fault-tolerant controller with input-delay in the following form

$$u(t) = G(K_1 + \Delta K_1)(x(t) - x^*) + G(K_2 + \Delta K_2)(x(t - \tau(t)) - x^*), \tag{3}$$

where the fault effects are specified by the matrix $G = \text{diag}\{l_1, l_2, \ldots, l_m\},\ 0 \leq \underline{l}_\wp \leq l_\wp \leq \bar{l}_\wp \leq 1,\ \wp = 1, 2, \ldots, m$, wherein the kth actuator with complete failure is denoted by $l_k = 0$, without fault is represented by $l_k = 1$ and for partial case $l_k \in (0, 1)$; the differentiable time-varying delay function $\tau(t)$ satisfies $0 < \tau(t) < \tau$ and $\dot{\tau}(t) < \mu$; K_r represent the gain matrices and the gain fluctuations ΔK_r is described with the appropriate dimensioned constant matrices U_r and W_r as $\Delta K_r = U_r\Gamma_r(t)W_r$ and $\Gamma_r(t)$ satisfies $\Gamma_r^T(t)\Gamma_r(t) \leq I, r = (1, 2)$. Further, the equilibrium point $x^* = (x_1^*(t), x_2^*(t), x_3^*(t))$ of the model (1) is described by $Ax^* + f(x^*) = 0$.

Now by defining error $e(t) = x(t) - x^*$, the corresponding error system can be formulated as

$$\begin{aligned}\dot{e}(t) &= (A + G(K_1 + \Delta K_1))e(t) + G(K_2 + \Delta K_2)e(t - \tau(t)) \quad (4)\\ nonumber \quad &+ f(x(t)) - f(x^*) + Cw(t). \quad (4)\end{aligned}$$

For a given positive-definite matrices Q1 and Q2, the guaranteed cost function associated with the error system (4) is given by

$$J = \int_0^N [e^T(t)Q_1e(t) + u^T(t)Q_2u(t)]dt. \tag{5}$$

Now to obtain the sufficient constraints for the chaotic nonlinear finance system, the following definitions and lemmas are employed in the upcoming main result section.

Lemma 1. *[12] If $r - q - pqr \leq 0$, i.e.,$1 + pr - \frac{r}{q} \geq 0$ then the system (1) has a unique equilibrium point $P^*(0, \frac{1}{q}, 0)$.*

Lemma 2. *[12] If $r - q - pqr > 0$, i.e.,$1 + pr - \frac{r}{q} < 0$ then the system (1) has three equilibrium points such as $P^*(0, \frac{1}{q}, 0)$ and $P^*_\pm(\pm\sqrt{\frac{r-q-pqr}{r}}, \frac{1+pr}{r}, \mp\frac{1}{r}\sqrt{\frac{r-q-pqr}{r}})$.*

Definition 1 ([31]). System (4) is finite-time bounded with respect to (c_a, c_b, N, R, d), if $e^T(0)Re(0) \le c_a \Rightarrow e^T(t)Re(t) < c_b, \forall t \in \{1, \cdots, N\}$, where $c_a < c_b$, R is a positive-definite matrix and $N \in \mathbb{N}$.

Definition 2 ([20]). If there exists the control $u(t)$ and guaranteed cost bound $\hat{J} > 0$, the closed-loop system (4) is finite-time bounded with respect to (c_a, c_b, N, R, d) and the cost function (5) satisfies $J \le \hat{J}$, then $u(t)$ is said to be guaranteed cost control law.

Definition 3 ([32]). The closed-loop system (4) is finite-time bounded with a extended passivity performance index $\gamma > 0$ subject to (c_a, c_b, N, R, d), where R is a positive-definite matrix and $c_a < c_b$, if there exist $\theta \in [0, 1]$ and under zero initial condition, the error $e(t)$ satisfies

$$\int_0^N (-\gamma^{-1}\theta e^T(s)Se(s) + 2(1-\theta)e^T(s)w(s))ds \ge -\gamma \int_0^N w^T(s)w(s)ds$$

for any non-zero disturbance and symmetric matrix $S > 0$.

3. Main results

3.1. Finite-time boundedness analysis:

In this section, a resilient fault-tolerant investment policy scheme for the considered chaotic nonlinear finance system (1) is investigated. For this purpose, a new sufficient finite-time boundedness constraints is derived for the closed-loop system (4) with the known gain matrices in the investment policy scheme. Finally, the unknown gain matrices is taken in the investment policy scheme to establish finite-time boundedness conditions based on extended passivity performance index.

Theorem 1. *Let the control gain matrices without fluctuations to be known. For given positive scalar τ, μ and α, if there exist symmetric matrices $X_i > 0, i = (1, 2, .., 5)$ such that the following constraints hold*

$$\Pi = \begin{bmatrix} \Pi_{11} & X_1GK_2 & 0 & X_5 & X_1C & X_1 & L \\ & -(1-\mu)X_2 & 0 & -X_5 & 0 & 0 & 0 \\ & * & -X_3 & 0 & 0 & 0 & 0 \\ & * & * & -X_4 & 0 & 0 & 0 \\ & * & * & * & -\alpha I & 0 & 0 \\ & * & * & * & * & -I & 0 \\ & * & * & * & * & * & I \end{bmatrix} < 0, \tag{6}$$

$$c_a\Big[\lambda_2 + \tau(\lambda_3+\lambda_4) + \frac{\tau^3}{2}\lambda_5 + \tau^2\lambda_6\Big] + d(1-e^{\alpha N}) < \lambda_1 e^{\alpha N} c_b, \tag{7}$$

then the closed-loop system (4) without gain fluctuations is finite-time bounded subject to (c_a, c_b, N, R, d), where $\Pi_{11} = X_1A^T + AX_1 + X_1GK_1 + K_1^TG^TX_1 + X_2 + X_3 + \tau^2X_4 - \alpha X_1$, $\widetilde{X}_k = R^{\frac{-1}{2}}X_kR^{\frac{-1}{2}}, k = (1, ., 5)$, $\lambda_1 = \lambda_{\min}(\widetilde{X}_1)$, $\lambda_2 = \lambda_{\max}(\widetilde{X}_1)$ and $\lambda_i = \lambda_{\max}(\widetilde{X}_j), i = (3, 4, 5, 6), j = (2, ., 5)$.

Proof. In order to achieve a desired result, we consider the Lyapunov–Krasovskii functional candidate for the closed-loop system (4) without gain fluctuations in the following form

$$\begin{aligned} V(t) =& e^T(t)X_1e(t) + \int_{t-\tau(t)}^t e^T(s)X_2e(s)ds + \int_{t-\tau}^t e^T(s)X_3e(s)ds \\ &+ \tau\int_{t-\tau(t)}^t \int_s^t e^T(v)X_4e(v)dvds + \left[\int_{t-\tau(t)}^t e(s)ds\right]^T \\ &X_5\left[\int_{t-\tau(t)}^t e(s)ds\right]. \end{aligned} \tag{8}$$

By computing the time derivative of equation (8) along the trajectories of system (4) without gain fluctuations, we obtain

$$\begin{aligned} \dot{V}(t) \le& 2e^T(t)X_1[(A+GK_1)e(t) + GK_2e(t-\tau(t)) + f(x(t)) - f(x^*) + Cw(t)] + e^T(t)(X_2 + \\ &X_3)e(t) - (1-\mu)e^T(t-\tau(t))X_2e(t-\tau(t)) - e^T(t-\tau)X_3e(t-\tau) + \tau^2e^T(t)X_4e(t) \\ &- \tau\int_{t-\tau(t)}^t e^T(s)X_4e(s)ds + 2[e(t) - (1-\mu)e(t-\tau(t))]X_5\left[\int_{t-\tau(t)}^t e(s)ds\right]. \end{aligned} \tag{9}$$

The nonlinear term in above inequality can be rewritten by using the condition $S_1^TS_2 + S_2^TS_1 \le S_1^TS_1 + S_2^TS_2$, for any appropriate dimensioned S_1 and S_2 matrices.

$2e^T(t)X_1(f(x(t)) - f(x^*)) \le e^T(t)X_1X_1e(t) + (f(x(t)) - f(x^*))^T(f(x(t)) - f(x^*)) \le e^T(t)(X_1X_1 + L^2I)e(t)$, where L is a Lipschitz constant.

Further the integral term in (9) can be expressed in the following form via Jensen's inequality, we get

$$-\tau\int_{t-\tau(t)}^t e^T(s)X_4e(s)ds \le -\left[\int_{t-\tau(t)}^t e(s)ds\right]^T X_4\left[\int_{t-\tau(t)}^t e(s)ds\right], \tag{10}$$

From (9), it is easy to obtain that

$$\dot{V}(t) - \alpha V(t) - w^T(t)\alpha w(t) \le \eta^T(t)\bar{\Pi}\eta(t), \tag{11}$$

where

$$\begin{aligned} \bar{\Pi} &= \begin{bmatrix} \bar{\Pi}_{11} & X_1GK_2 & 0 & X_5 & X_1C \\ & -(1-\mu)X_2 & 0 & -X_5 & 0 \\ & * & -X_3 & 0 & 0 \\ & * & * & -X_4 & 0 \\ & * & * & * & -\alpha I \end{bmatrix}, \quad \eta^T(t) \\ &= \begin{bmatrix} e(t) & e(t-\tau(t)) & e(t-\tau) & \int_{t-\tau(t)}^t e(s)ds & w(t) \end{bmatrix}, \\ \bar{\Pi}_{11} &= X_1A^T + AX_1 + X_1^TX_1 + L^2I + X_1GK_1 + K_1^TG^TX_1 + X_2 \\ &\quad + X_3 + \tau^2X_4 - \alpha X_1. \end{aligned}$$

With the aid of Schur complement, we get the elements defined in (6) from (11). If the matrix inequality in (6) holds (i.e.,) $\bar{\Pi} < 0$, it is obvious that

$$\dot{V}(t) - \alpha V(t) - w^T(t)\alpha w(t) < 0 \quad \text{(or)} \quad \frac{d}{dt}(e^{-\alpha t}V(t)) < e^{-\alpha t}w^T(t)\alpha w(t). \tag{12}$$

Integrating the above inequality from 0 to N, it follows that

$$V(t) < e^{\alpha N}V(0) + e^{\alpha N}\int_0^N e^{-\alpha s}w^T(s)\alpha w(s)ds < e^{\alpha N}[V(0) + d(1-e^{-\alpha N})]. \tag{13}$$

On the other hand, we have

$$V(t) \ge e^T(t)X_1e(t) \ge \lambda_{min}(\widetilde{X}_1)e^T(t)Re(t) = \lambda_1 e^T(t)Re(t), \tag{14}$$

$$\begin{aligned} V(0) =& e^T(0)X_1e(0) + \int_{-\tau(0)}^0 e^T(s)X_2e(s)ds + \int_{-\tau}^0 e^T(s)X_3e(s)ds \\ &+ \tau\int_{0-\tau(0)}^0 \int_s^0 e^T(v)X_4e(v)dvds + \left[\int_{-\tau(0)}^0 e(s)ds\right]^T \\ &X_5\left[\int_{-\tau(0)}^0 e(s)ds\right], \\ &\le [\lambda_2 + \tau(\lambda_3+\lambda_4) + \frac{\tau^3}{2}\lambda_5 + \tau^2\lambda_6] \sup_{-\tau\le s\le 0}\{e^T(s)Re(s)\}, \\ &\le [\lambda_2 + \tau(\lambda_3+\lambda_4) + \frac{\tau^3}{2}\lambda_5 + \tau^2\lambda_6]c_a. \end{aligned} \tag{15}$$

By combining (13)-(15), we can obtain

$$e^T(t)Re(t) < \frac{e^{\alpha N}\{[\lambda_2 + \tau(\lambda_3+\lambda_4) + \frac{\tau^3}{2}\lambda_5 + \tau^2\lambda_6]c_a + d(1-e^{-\alpha N})\}}{\lambda_1}.$$

Thus we obtain $e^T(t)Re(t) < c_b$, $\forall t \in [0, N]$ only if the condition (7) holds which is defined in Theorem statement. This shows that the error system (4) with known gain matrices in investment policy scheme is finite-time bounded with respect to (c_a, c_b, N, R, d), which completes the proof. □

Now, by assuming the gain matrices in investment policy with fluctuations to be unknown, a finite-time resilient fault-tolerant controller with extended passivity index is developed in the sequel Theorem.

Theorem 2. *Let τ, μ, α, c_a, c_b, N and d be given positive constants and symmetric matrix R, the closed-loop system (4) is robustly finite-time bounded with a satisfactory disturbance attenuation level $\gamma > 0$, if there exist real-valued symmetric matrices $X_i > 0, i = (1, 2, ., 5), S > 0$ and scalars $\lambda > 0$, $\beta > 0$, $\theta \in [0, 1]$ satisfying the following constraints for $i, j \in \mathcal{R}$*

$$\left[\widehat{\Pi}_{i,j}\right] < 0, \tag{16}$$

$$\sigma R^{-1} < X < R^{-1}, \quad 0 < \bar{X}_2 < 2R^{-1}, \quad 0 < \bar{X}_3 < 2R^{-1}, 0 < \bar{X}_4 < 2R^{-1}, \quad 0 < \bar{X}_5 < 2R^{-1}, \tag{17}$$

$$\begin{bmatrix} c_a + [d - (d + c_b)e^{-\alpha N}]\sigma & \sqrt{(4\tau + \tau^3 + 2\tau^2)c_a} \\ & -\sigma \end{bmatrix} < 0, \tag{18}$$

where $\widehat{\Pi}_{1,1} = A^T X + XA + GZ_1 + Z_1^T G^T + \bar{X}_2 + \bar{X}_3 + \tau^2\bar{X}_3 - \alpha\bar{X}_1$, $\widehat{\Pi}_{1,2} = GZ_2$, $\widehat{\Pi}_{1,4} = \bar{X}_5$, $\widehat{\Pi}_{1,5} = C - X(1-\theta)$, $\widehat{\Pi}_{1,6} = I$, $\widehat{\Pi}_{1,7} = XL$, $\widehat{\Pi}_{1,8} = \lambda GU_1$, $\widehat{\Pi}_{1,9} = XW_1^T$, $\widehat{\Pi}_{1,10} = \beta GU_1$, $\widehat{\Pi}_{1,12} = X\sqrt{\theta}$, $\widehat{\Pi}_{2,2} = -(1-\mu)\bar{X}_2$, $\widehat{\Pi}_{2,4} = -(1-\mu)\bar{X}_5$, $\widehat{\Pi}_{2,11} = XW_2^T$, $\widehat{\Pi}_{3,3} = -\bar{X}_3$, $\widehat{\Pi}_{4,4} = -\bar{X}_4$, $\widehat{\Pi}_{5,5} = -\gamma I$, $\widehat{\Pi}_{6,6} = -I$, $\widehat{\Pi}_{7,7} = -I$, $\widehat{\Pi}_{8,8} = -\lambda I$, $\widehat{\Pi}_{9,9} = -\lambda I$, $\widehat{\Pi}_{10,10} = -\beta I$, $\widehat{\Pi}_{11,11} = -\beta I$ and $\widehat{\Pi}_{12,12} = -\gamma S$. Moreover, the relation $K_1 = Z_1X^{-1}$ and $K_2 = Z_2X^{-1}$ are used to compute resilient fault-tolerant gain matrices.

Proof. By following the similar approach along with the same Lyapunov–Krasovskii functional candidate as in Theorem 1 to prove the Theorem 2. It is noted that the desired conditions are obtained by taking gain fluctuation and for any non-zero external disturbance $w(t)$ with extended passivity performance index into account. Further, we have

$$\dot{V}(t) - \alpha V(t) + \gamma^{-1}\theta e(t)^T e(t) - 2(1-\theta)e^T(t)w(t) - \gamma w^T(t)w(t) \le \eta^T(t)\tilde{\Pi}\eta(t), \tag{19}$$

where $\tilde{\Pi} = \begin{bmatrix} \tilde{\Pi}_{11}(t) & X_1GK_2(t) & 0 & X_5 & X_1C - (1-\theta) \\ & -(1-\mu)X_2 & 0 & -X_5 & 0 \\ & * & -X_3 & 0 & 0 \\ & * & * & -X_4 & 0 \\ & * & * & * & -\gamma I \end{bmatrix}$,
$\tilde{\Pi}_{11}(t) = X_1A^T + AX_1 + X_1^T X_1 + L^2 I + X_1GK_1(t) + K_1^T(t)G^T X_1 + X_2 + X_3 + \tau^2 X_4 + \gamma^{-1}\theta - \alpha X_1$, the gain matrices $K_i(t) = K_i + \Delta K_i$, $(i = \{1, 2\})$ and the fluctuations $\Delta K_i = U_iF(t)W_i$. With the aid of *Fact* 1 and *Fact* 2 in [20], there exist a constants $\lambda > 0$ and $\beta > 0$ such that

$$\tilde{\Pi} = \begin{bmatrix} \tilde{\Pi}_1 & \tilde{\Pi}_2^T & \tilde{\Pi}_3^T & \tilde{\Pi}_4^T & \tilde{\Pi}_5^T \\ & -\lambda I & 0 & 0 & 0 \\ & * & -\lambda I & 0 & 0 \\ & * & * & -\beta I & 0 \\ & * & * & * & -\beta I \end{bmatrix}, \tag{20}$$

where $\tilde{\Pi}_1 = \begin{bmatrix} \Pi_{11} & X_1GK_2 & 0 & X_5 & X_1C - (1-\theta) & X_1 & L & \sqrt{\theta}I \\ & -(1-\mu)X_2 & 0 & -X_5 & 0 & 0 & 0 & 0 \\ & * & -X_3 & 0 & 0 & 0 & 0 & 0 \\ & * & * & -X_4 & 0 & 0 & 0 & 0 \\ & * & * & * & -\gamma I & 0 & 0 & 0 \\ & * & * & * & * & -I & 0 & 0 \\ & * & * & * & * & * & -I & 0 \\ & * & * & * & * & * & * & -\gamma I \end{bmatrix}$

$\tilde{\Pi}_2 = \begin{bmatrix}(X_1\lambda GU_1)^T & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$, $\tilde{\Pi}_3 = \begin{bmatrix}W_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$,
$\tilde{\Pi}_4 = \begin{bmatrix}(X_1\beta GU_2)^T & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$, $\tilde{\Pi}_5 = \begin{bmatrix}0 & W_2^T & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$.

To obtain the stated matrix (16), denote $X = X_1^{-1}$, $Z_1 = K_1X$, $Z_2 = K_2X$, $\bar{X}_i = XX_iX$, $i = (2, \ldots, 5)$, pre and post-multiplying the aforestated matrix $\tilde{\Pi}$ with $diag\{X_1^{-1}, X_1^{-1}, X_1^{-1}, X_1^{-1}, \underbrace{I, \ldots, I}_{8}\}$ and its transpose. Finally, we can obtain the condition (16). On the other hand, we notate $\hat{X} = R^{\frac{1}{2}} X R^{\frac{1}{2}}$ and considering the relation $\lambda_{\max}(\hat{X}) = \frac{1}{\lambda_{\min}(\hat{X}_1)}$ it follows that $I < R^{\frac{-1}{2}} X_1 R^{\frac{-1}{2}} < \frac{1}{\sigma} I$, thus we have $\lambda_2 < \frac{1}{\sigma}$ and $\lambda_1 > 1$. Further, it follows from $0 < R^{\frac{-1}{2}} X_2 R^{\frac{-1}{2}} < 2(R^{\frac{-1}{2}} X_1 R^{\frac{-1}{2}})^2 < \frac{2}{\sigma^2} I$ which implies $\lambda_3 < \frac{2}{\sigma^2}$. Similarly, we can obtain $\lambda_k < \frac{2}{\sigma^2}, k = (4,5,6)$ from $0 < \bar{X}_i < 2R^{-1}$, $i = (2,3,4,5)$. From the aforesaid relation, the condition (18) can be guaranteed by $\Big(\frac{1}{\sigma} + \frac{\tau}{\sigma^2}(4 + \tau^2 + 2\tau)\Big)c_a + d(1 - e^{-\alpha N}) < c_b e^{-\alpha N}$. By applying Schur complement, the above expression can be equivalently written in the form of (18). Therefore, if the LMIs (16)–(18) hold, the chaotic nonlinear finance system (1) via the proposed resilient fault-tolerant investment policy scheme is finite-time bounded together with minimum extended passivity performance index, which completes the proof. □

3.2. Finite-time guaranteed cost investment policy scheme

Theorem 3. *For given positive constants τ, μ and α, if there exist real-valued symmetric matrices $X_i > 0, i = (1, 2, ., 5)$ and positive scalars λ, β, δ_1, δ_2 and $\theta \in [0,\ 1]$ such that the following constraint together with the inequality (17) and (18) defined in Theorem 2 hold:*

$$\breve{\Pi} = \left[\breve{\Pi}_{i,j}\right] < 0, \forall i, j \in \mathcal{R}, \tag{21}$$

where $\breve{\Pi}_{1,1} = A^T X + XA + GZ_1 + Z_1^T G^T + \bar{X}_2 + \bar{X}_3 + \tau^2 \bar{X}_3 - \alpha + \bar{X}_1 X Q_1 X$, $\breve{\Pi}_{1,2} = GZ_2$, $\breve{\Pi}_{1,4} = \bar{X}_5$, $\breve{\Pi}_{1,5} = C - X(1-\theta)$, $\breve{\Pi}_{1,6} = I$, $\breve{\Pi}_{1,7} = XL$, $\breve{\Pi}_{1,8} = \lambda G U_1$, $\breve{\Pi}_{1,9} = XW_1^T$, $\breve{\Pi}_{1,10} = \beta G U_1$, $\breve{\Pi}_{1,12} = X\sqrt{\theta}$, $\breve{\Pi}_{1,13} = X$, $\breve{\Pi}_{1,14} = Z_1^T G^T$, $\breve{\Pi}_{1,15} = XU_1 G$, $\breve{\Pi}_{2,2} = -(1-\mu)\bar{X}_2$, $\breve{\Pi}_{2,4} = -(1-\mu)\bar{X}_5$, $\breve{\Pi}_{2,11} = XW_2^T$, $\breve{\Pi}_{2,14} = Z_2^T G^T$, $\breve{\Pi}_{2,17} = XU_2 G$, $\breve{\Pi}_{3,3} = -\bar{X}_3$, $\breve{\Pi}_{4,4} = -\bar{X}_4$, $\breve{\Pi}_{5,5} = -\gamma I$, $\breve{\Pi}_{6,6} = -I$, $\breve{\Pi}_{7,7} = -I$, $\breve{\Pi}_{8,8} = -\lambda I$, $\breve{\Pi}_{9,9} = -\lambda I$, $\breve{\Pi}_{10,10} = -\beta I$, $\breve{\Pi}_{11,11} = -\beta I$, $\breve{\Pi}_{12,12} = -\gamma I$, $\breve{\Pi}_{13,13} = -Q_1^{-1}$, $\breve{\Pi}_{14,14} = -Q_2^{-1}$, $\breve{\Pi}_{14,16} = \delta_1 W_1$, $\breve{\Pi}_{14,18} = \delta_2 W_2$, $\breve{\Pi}_{15,15} = -\delta_1 I$, $\breve{\Pi}_{16,16} = -\delta_1 I$, $\breve{\Pi}_{17,17} = -\delta_2 I$ and $\breve{\Pi}_{18,18} = -\delta_2 I$, then the resilient fault-tolerant guaranteed cost investment policy scheme (3) can be computed by the gain matrices $K_1 = Z_1 X^{-1}$ and $K_2 = Z_2 X^{-1}$ which makes the system (4) finite-time bounded with respect to (c_a, c_b, N, R, d). Furthermore, the finite-time guaranteed cost bound of (4) can be obtained as $\hat{J} = e^{\alpha N}[\lambda_2 + \tau(\lambda_3 + \lambda_4) + \frac{\tau^3}{2}\lambda_5 + \tau^2 \lambda_6] c_a$.

Proof. From (3), we obtain $u^T(t) Q_2 u(t) = (G(K_1 + \Delta K_1)e(t) + G(K_2 + \Delta K_2)e(t - \tau(t)))^T Q_2 (G(K_1 + \Delta K_1)e(t) + G(K_2 + \Delta K_2)e(t - \tau(t)))$. The similar steps of Theorems 1 and 2 are followed to prove the Theorem 3. Now, it is clear from (19) that

$$\dot{V}(t) - \alpha V(t) + \gamma^{-1}\theta e(t)^T e(t) - 2(1-\theta)e^T(t)w(t) - \gamma w^T(t)w(t) + e^T(t)Q_1 e(t) + u^T(t)Q_2 u(t) \le \eta^T(t)\breve{\Pi}\eta(t), \tag{22}$$

where $\breve{\Pi} = \begin{bmatrix} \breve{\Pi}_{11}(t) & X_1 G K_2(t) + (G(K_1 + \Delta K_1))^T Q_2 (G(K_2 + \Delta K_2)) & 0 & X_5 & X_1 C - (1-\theta) \\ & -(1-\mu)X_2 + (G(K_2 + \Delta K_2))^T Q_2 (G(K_2 + \Delta K_2)) & 0 & -X_5 & 0 \\ & * & -X_3 & 0 & 0 \\ & * & * & -X_4 & 0 \\ & * & * & * & -\gamma I \end{bmatrix}$,

$\breve{\Pi}_{11}(t) = X_1 A^T + AX_1 + X_1^T X_1 + L^2 I + X_1 G K_1(t) + K_1^T(t) G^T X_1 + X_2 + X_3 + \tau^2 X_4 + \gamma^{-1}\theta + Q_1 + (G(K_1 + \Delta K_1))^T Q_2 (G(K_1 + \Delta K_1)) - \alpha X_1$. In order to design the controller gains, first applying the *Fact* 1 and *Fact* 2 in [20] to the above matrix inequality. Secondly, pre and post multiplying with $diag\{X_1^{-1}, X_1^{-1}, X_1^{-1}, X_1^{-1}, \underbrace{I, \ldots, I}_{14}\}$ and its transpose, we obtain the LMI (21) which is stated in Theorem statement. If the condition 21 holds (i.e.,) $\breve{\Pi} < 0$, it is obvious that

$$\dot{V}(t) - \alpha V(t) + e^T(t) Q_1 e(t) + u^T(t) Q_2 u(t) < 0. \tag{23}$$

Now, integrate the above inequality from 0 to N with respect to time t, we get

$$J = \int_0^N [e^T(t) Q_1 e(t) + u^T(t) Q_2 u(t)]dt \le -e^{\alpha N} \int_0^N \frac{d}{dt}(e^{-\alpha t} V(t))dt \le e^{\alpha N}[V(0) - e^{-\alpha N} V(N)]. \tag{24}$$

Since $V(t) \ge 0$ and $\dot{V}(t) < 0$, then $\lim_{N \to \infty} V(N) = n$, where n denotes non-negative constant. Therefore from (15) we obtain

$$J \le e^{\alpha N}[\lambda_2 + \tau(\lambda_3 + \lambda_4) + \frac{\tau^3}{2}\lambda_5 + \tau^2 \lambda_6] c_a = \hat{J}. \tag{25}$$

It can be concluded from (25) that the designed control in (3) is a guaranteed cost controller with cost bound $\hat{J}$, which completes the proof. □

Remark 1. It is noted that in the chaotic finance system (1), the economic factors such as wrong economic policy, natural disasters and international share loss which may drastically affect the economy and also complicate the investment policy scheme. Further, the stabilization of the finance system in an infinite period of time may cause unpredictable effects in economy. Thus it is essential to predict and ensure the stability of the finance system within a finite period of time to regulate the economic crisis by overcoming the above stated issues. To overcome these facts, it is more important to find the degradation range incurred by unpredictable and undesirable changes in economy which motivates the study on designing finite-time resilient fault-tolerant guaranteed cost investment policy scheme.

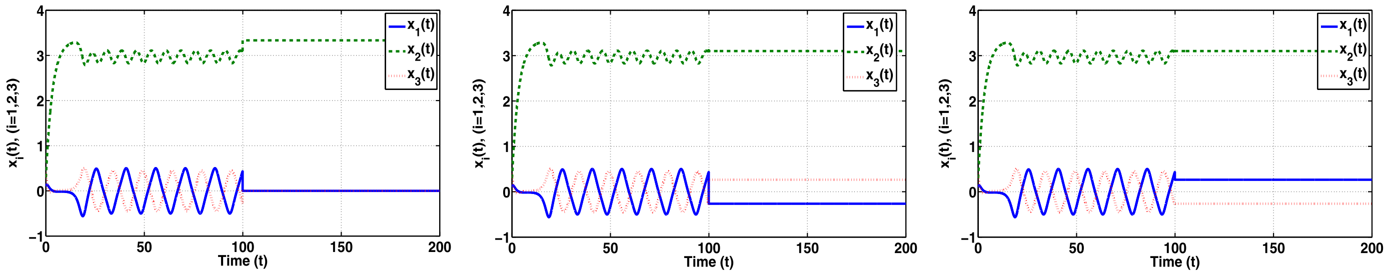

Fig. 1. The state trajectories of system (1) for different fixed points.

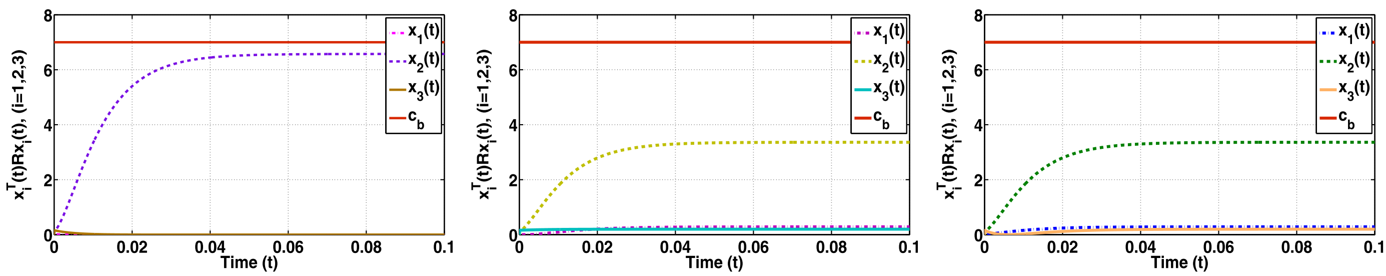

Fig. 2. Evaluation of $x_i^T(t)Rx_i(t)$ for different fixed points, (i = 1,2,3).

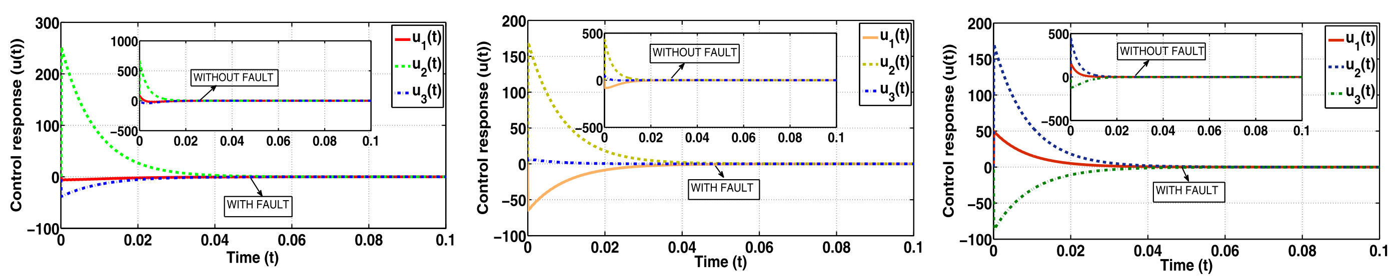

Fig. 3. Reliable control responses for different fixed points.

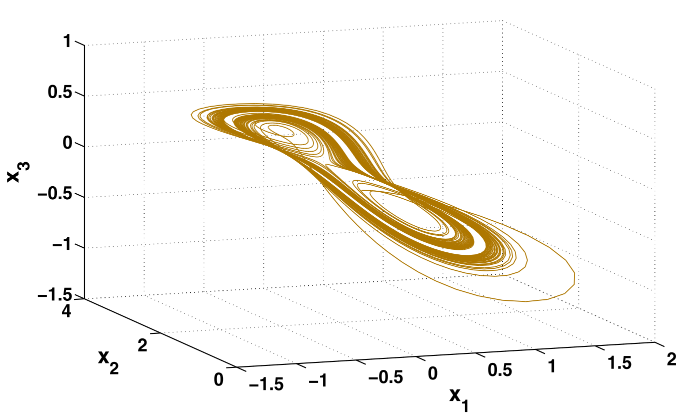

Fig. 4. Chaotic behaviour of the system (1) .

4. Numerical simulations

In this section, two numerical examples with simulation results for chaotic nonlinear finance system are discussed to validate the applicability of the proposed investment policy scheme designed in Theorem 3.

Example 1: The parameters of the financial system (1) are taken as $p = 2.1, q = 0.3, r = 1$ and the capital inflow risk matrix G is considered as $0.6I$. The Lemma 1 and 2 are used to obtain the equilibrium points of the considered system (1) with respect to the parameters p, q and r. For simulation purposes, the bounded disturbance function with the bound $d = 0.34$ is selected as $w(t) = 0.1sin(t)$. Also, the initial values of the state trajectories are taken as 0.1, 0.3 and 0.4, respectively. Moreover the uncertain matrices associated with proposed investment policy are chosen as $U_1^T = \begin{bmatrix}0.2 & 0.6 & 0.4\end{bmatrix}^T, W_1 = \begin{bmatrix}0.4 & 0.8 & 1.2\end{bmatrix}, U_2^T = \begin{bmatrix}0.4 & 0.3 & 0.5\end{bmatrix}^T$ and $W_2 = \begin{bmatrix}0.18 & 0.2 & 0.7\end{bmatrix}$. The remaining parameters involved in simulation are considered as $\alpha = 0.4, \mu = 0.3, L = 3, \theta = 0.5$ and $C = diag\{1, 2, 3\}$.

From the above considered parameters, the resilient fault-tolerant guaranteed cost investment policy scheme for the addressed nonlinear finance system is designed to achieve a robust performance with minimum cost bound. Now, by solving the derived LMI-based constraints in Theorem 3 through the LMI toolbox in MATLAB software, the optimal bound value of the considered delay $\tau(t)$, corresponding guaranteed cost bound and the controller gain matrices with the minimum disturbance attenuation index $\gamma = 0.1$ are obtained as

$$\tau = 1.3; \hat{J} = 6.6260; K_1 = \begin{bmatrix} -174.1836 & -25.4263 & -37.8756 \\ -25.4263 & -217.2244 & -76.8075 \\ -37.8756 & -76.8075 & -281.6521 \end{bmatrix};$$

$$K_2 = \begin{bmatrix} 0.5383 & 0.0035 & -0.0259 \\ 0.0035 & 0.5621 & 0.0025 \\ -0.0259 & 0.0025 & 0.3914 \end{bmatrix}.$$

Moreover to reveal the applicability and importance of the designed investment policy scheme, the responses of state trajectories of the addressed system (1) is shown in Fig. 1 wherein the developed investment policy design is activated only at $t = 100$. It is evident that the addressed system (1) cannot achieve stabil-

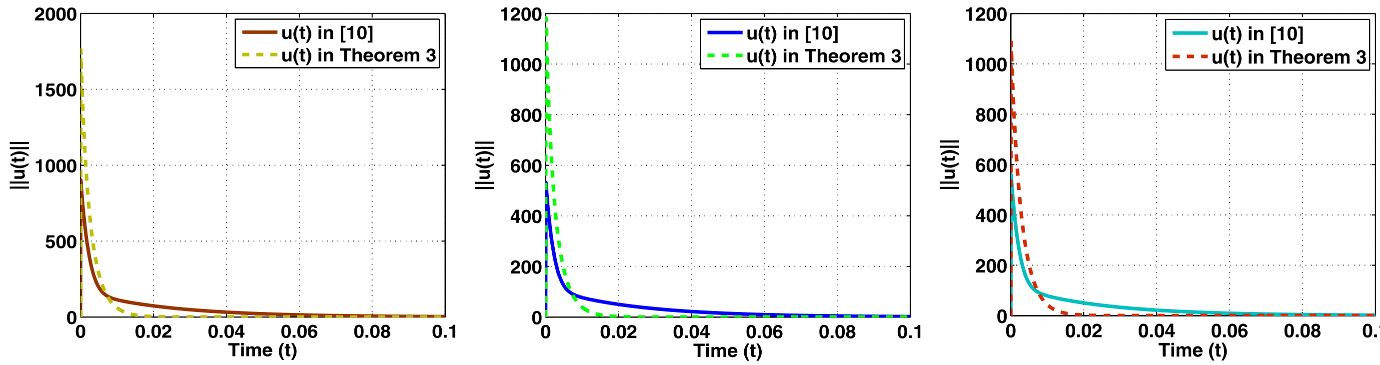

Fig. 5. Control response for different fixed points.

ity without the implementation of the developed controller that reveals the importance and robustness of the investment policy scheme designed in Theorem 3 for the nonlinear finance model (1). From Fig. 2, it can be seen that the designed investment policy scheme effectively stabilizes the considered system trajectories to its unstable equilibrium points within a finite period of time. The control responses in the absence and presence of actuator fault is shown in Fig. 3. To be precise, the impact of chaotic behaviour in nonlinear financial model (1) is illustrated by presenting the Fig. 4.

From these simulation results it is clear that the designed resilient fault-tolerant guaranteed cost investment policy scheme ensures the convergence to their desired equilibrium points within finite-time bound subject to (0.1, 7, 0.1, I, 0.34) for the nonlinear chaotic finance system (1) and also tackles unpredictable changes and irregular fluctuations in investment policy design. Hence, the efficiency and inherent potential of the developed theoretical result to achieve required performance with minimum cost bound by employing the developed investment policy scheme is clearly validated through this numerical example.

Example 1. In order to illustrate the effectiveness and potential of the proposed controller over the existing controller in [10], a comparative study is provided in this numerical example. The parameters of the financial system (1) are considered as same as in [10], such as $p = 2.5, q = 0.2, r = 1.2$ and the capital inflow risk matrix G is considered as $0.5I$. In view of the Lemmas 1 and 2, the addressed system has three different equilibrium points. Also the initial condition is given by $\begin{bmatrix}0.5 & 3 & -0.4\end{bmatrix}^T$. For simulation purposes, the other parameters are considered as $\tau = 1.9, L = 5\sqrt{(6)}, C = \begin{bmatrix}1 & 1 & 1\end{bmatrix}^T, c_a = 0.1, c_b = 26, N = 0.1, R = I, d = 0.34$ and $w(t) = 1/(1+2t)$.

To perform the comparison analysis, the performance is reduced to the H_∞ case by considering the value of θ to be 1 with respect to the above designed parameters and by solving the linear matrix inequalities in Theorem 3 with the aid of MATLAB LMI toolbox, we obtain guaranteed cost bound $\hat{J} = 2.8713$. Consequently, the gain matrices based on the proposed approach is determined as $K_1 = \begin{bmatrix}-316.9424 & -37.0517 & -54.9966\\ -37.0517 & -377.5121 & -111.1249\\ -54.9966 & -111.1249 & -465.5427\end{bmatrix}$ and $K_2 = \begin{bmatrix}0.4698 & -0.0599 & -0.1318\\ -0.0599 & 0.4736 & -0.1995\\ -0.1318 & -0.1995 & 0.0860\end{bmatrix}$.

Based on the gain matrices in [10] and the gain parameters determined above, the simulation results are given in Fig. (5). In particular, the control response based on developed approach and H_∞ controller in [10] for three different equilibrium points is shown in Fig. 5. Specifically, the convergence timing of the controller is reasonably reduced compared to the approach developed in [10].

Table 1
Comparison results.

Method	Upper bound τ	Minimum γ
[10]	1.2	0.16
Theorem 3	1.9	0.05

Moreover, to show the conservativeness of the proposed approach among the method in [10], the upper bound and the performance index values are computed and provided in Table. 1. Therefore, it is concluded that the proposed investment policy scheme effectively tolerates the faults, reduces the disturbance attenuation index and also ensures the better convergence over a finite-period of time than the method followed in [10].

5. Conclusion

In this paper, the finite-time resilient fault–tolerant guaranteed cost investment policy is designed for the nonlinear finance system to handle unpredictable changes and irregular fluctuation in policy design. The consistent gain matrices are calculated in an adequate manner via solving developed LMI-based criterion that is derived by using suitable Lyapunov–Krasovskii functional. Further the nonlinear chaotic finance system finite-time bounded with least possible disturbance attenuation index and guaranteed cost bound is achieved. Finally, the developed investment policy design has been verified by providing numerical example and its simulation results. The simulation results reveal the efficiency of the proposed investment policy scheme to achieve the robust performance within a finite period of time. Furthermore, in our future work, we will design a finite-time resilient fault-tolerant investment policy scheme for the stochastic fractional-order financial nonlinear system with randomly occurring fluctuations and undesirable changes in economic factors.

Declaration of Competing Interest

It is confirmed that the authors have no potential conflict of interest regarding the publication of this article.

CRediT authorship contribution statement

S. Harshavarthini: Conceptualization, Methodology, Software, Validation, Writing - original draft, Writing - review & editing. **R. Sakthivel:** Conceptualization, Validation, Writing - original draft, Writing - review & editing, Methodology. **Yong-Ki Ma:** Writing - review & editing. **M. Muslim:** Writing - review & editing.

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