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ORIGINAL PAPER

Analysis of labor strike based on evolutionary game and catastrophe theory

Ahmad Makui[1] · **Seyed Mohammad Seyedhosseini**[1] · **Seyed Jafar Sadjadi**[1] · **Parinaz Esmaeili**[1]

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Abstract
This paper analyzes the labor–employer relations during conditions that lead to strike using an evolutionary game and catastrophe theory. During a threat to strike, the employers may accept the whole or only a part of the demands of labors and improve the work conditions or decline the demands, and each selected strategies has its respective costs and benefits. The threat to strike action causes the formation of a game between the strikers and employers that in which, as time goes on, different strategies are evaluated by the players and the effective variables of strike faced gradual and continuous changes, which can lead to a sudden jump of the variables and push the system to very different conditions such as dramatic increase or decrease in the probability of selecting strategies. So the alliance between labors could suffer or reinforce. This discrete sudden change is called catastrophe. In this study after finding evolutionary stable strategies for each player, the catastrophe threshold analyzed by nonlinear evolutionary game and the managerial insight is proposed to employers to prevent the parameters from crossing the border of the catastrophe set that leads to a general strike.

Keywords Strike · Labor-management relations · Evolutionary game · Catastrophe theory · Evolutionary stable strategy

Mathematics Subject Classification 91A22 · 58K35

✉ Ahmad Makui
amakui@iust.ac.ir

Seyed Mohammad Seyedhosseini
seyedhosseini@iust.ac.ir

Seyed Jafar Sadjadi
sjsadjadi@iust.ac.ir

Parinaz Esmaeili
parinaz_esmaeili@ind.iust.ac.ir

1 Industrial, Engineering Department, Iran University of Science and Technology, Tehran, Iran

1 Introduction

Strike is a temporary work stoppage caused by workers, to achieve their goals and demands. The strike has different reasons such as asking for a pay raise, reducing working hours, and so on. As a result, employers can penalize employees, take no action, or compromise with labors (Lia 2016). Until the early nineteenth century in all judicial systems, the strike was an illegal act. So, repression and punishment were the first response to it (Monkam 2010).

There are many studies about the evolutionary relationship between employees and employers in the literature. Kim and Cho (2009) developed a multi-agent-based evolutionary labor market for modeling relationship between employees and employers using co-evolutionary game. He analyzed the effect of balanced power between employees and employers in maintenance and extension of labor market. Zhang et al. (2018) studied the effect of the labor division in the colony efficiency. Using the evolutionary game approach, they investigated the evolution of the division of labor with different strategies such as free riding.

There are few studies in which the relation of employers and employees during the strike is modeled by game models. Clemhout et al. (1975) studied the strike bargaining model in n-person player game. Lia (2016) modeled the role of labor laws in the promotion of strikes in negative-sum games.

During the strike, some of the workers prefer not to strike and gain the benefit of other ones strike. This is called the free-rider problem in economics. It occurs when those who benefit from resources, public goods, or services do not pay for them (Baumol 1952); so it is possible that the employee's strike alliance collapses. Xu et al. (2014) modeled such a condition in which discrete–continuous changes cause substantial cooperation in the beginning that led to complete betrayal at last in their evolutionary processes. The nonlinear approach to solve these models is catastrophe theory introduced by Thom (1972) and defined completely in Sect. 4. There are many application of catastrophe theory in simulating the labors' behavior. For example, Hu and Xia (2015) used a qualitative–quantitative hybrid catastrophe model for studying the person's sudden behavioral change such as that of an employee abruptly resigning from his job and to explore the trajectory and mechanism of the change. Xu et al. (2014) developed the catastrophe model based on qualitative simulation and fuzzy math in a manufacturing enterprise. They showed warning, critical and mutation area of employee turnover that helps the manufacturer to keep the enterprise human resource stability. Dimas et al. (2018) used a stochastic catastrophe Model to define the role of labors' satisfaction in the conflicts occurs during team works. Using this model, they proposed approaches to conflict management in teams.

By focusing on the discrete transition mechanism in the evolutionary games, this model will investigate the evolutionary game models of labor- employer relations during the strike in which the labors' alliance to strike experiences a catastrophic change.

We assume that labors have two strategies of strike (defection) or not strike (cooperation) and the employers have two strategies; improve labors' job condition (cooperation) or not-improve (defection). This study concentrates on the factors of continuous change in labors' strategies to find the threshold values on which the alliance to strike fails or strongly reinforces. The rest of this article is organized as follows. In Sects. 2 and 3, problem modeling and equilibrium analysis are discussed respectively. Section 4 presents a short background of catastrophe theory and the probability of catastrophe occurrence in the labor unions. Conclusions and suggestions are presented in Sect. 5.

2 Model and assumptions

Each strategy chosen by labors or employers during strike causes costs and benefits. If a labor strikes (defection), he will lose his monthly salary during the strike. In this case, if the employers agree with labors and improve the work conditions (cooperation), all labors will benefit from this; but if the condition does not improve, the labors that strike will bear the additional cost of losing credit. The payoff of the labors that do not strike depends on the employer's strategy. By improvement of working condition, the labors gain benefit without any costs and if the employer choose not to improve strategy (defection), there will be no changes in the players' payoff. The strategies of the players are shown in Fig. 1. Also, the Table 1 of relevant variables and form of payoff matrix are shown below (Fig. 2).

		employers	
		Improve (cooperation)	not improve (defection)
labors	Strike (defection)		
	not strike (cooperation)		

Fig. 1 The strategies of the players

Table 1 Relevant variables

Variables	Definitions
C_l	Cost of "strike" for labors
B_l	Benefit of "improve" for labors
A_l	Cost of "losing credit" for labors
C_e	Cost of "production stoppage" for the employers
B_e	Cost of "improve" for the employers

		employers	
		improve	not improve
labors	strike	$(B_l - C_l, -C_e - B_e)$	$(-C_l - A_l, -C_e)$
	not strike	$(B_l, -B_e)$	(0,0)

Fig. 2 Income matrix

3 Equilibrium analyses

At the primary stage, we assume that the probability that the labors choose the "strike" strategy (defection) is x, and the probability that the employers choose the "improve" strategy (cooperation) is y. The respective expectation values of "strike" and "not strike" for labors are U_{1Y} and U_{1N} and labors' average value is $\overline{U}_1$, then,

$$U_{1Y} = y(B_l - C_l) + (1-y)(-C_l - A_l) = yB_l - C_l - A_l + yA_l \tag{1}$$

$$U_{1N} = yB_l \tag{2}$$

$$\overline{U}_1 = x(yB_l - C_l - A_l + yA_l) + (1-x)(yB_l) = -x(C_l + A_l) + y(xA_l + B_l) \tag{3}$$

As the same way, the respective expectation values of "improve", "not improve" and the employer's average value are U_{2Y}, U_{2N} and $\overline{U}_2$.

$$U_{2Y} = x(-C_e - B_e) + (1-x)(-B_e) = -xC_e - B_e \tag{4}$$

$$U_{2N} = -xC_e \tag{5}$$

$$\overline{U}_2 = y(-xC_e - B_e) + (1-y)(-xC_e) = -yB_e - xC_e \tag{6}$$

3.1 Replicator dynamics equation of "strike" labor population

Replicator dynamic equation is a nonlinear approach to describe the evolutionary process in the infinite and well-mixed population. This equation is the multiplication of current probability of strategy and its fitness relative to the average. It shows the strategy reproducing chance and how fast it can grow.

Based on the evolutionary stable strategy of game, the games can be divided into four classes: the trivial game with no dilemma; the prisoner's dilemma (sometimes abbreviated PD); Chicken (also known as the Snow Drift or Hawk–Dove Game); and Stag Hunt (sometimes abbreviated SH). PD games are D-dominated and in the process of evolution, the defection dominates to cooperation. Trivial games are

C-dominated, and cooperation dominates to defection. The chicken games are polymorphic. It means the ESS for the game is a mixed strategy. The SH games are bistable in that there are two pure strategies when both players cooperate or both players defect (Tanimoto 2019). Finding the equilibrium of a 2 by 2 evolutionary game is explained below. According to the above equation,

$$F(x) = \frac{dx}{dt} = x\left(U_{1Y} - \overline{U}_1\right) = x(x-1)\left(A_l - yA_l + C_l\right) \tag{7}$$

Since y can not be equal to $\frac{A_l+C_l}{A_l}$ $(0 \le y \le 1)$, let $F(x) = 0$, then games are stable when $x = 0$ and $x = 1$.

Evolutionary stable strategies are attained when $\frac{dF(x)}{dx} < 0$. Because $A_l < A_l + C_l$ then $\frac{A_l+C_l}{A_l} > 1$ and $y < \frac{A_l+C_l}{A_l}$. Thus, evolutionary stable strategies are attained when $x=1$. It means that the strike strategy is the final choice for labors and the game is D-dominated.

3.2 Replicator dynamics equation of "improve" employer population

The replicator dynamics equation of "employer population" is:

$$F(y) = \frac{dy}{dt} = y\left(U_{2Y} - \overline{U}_2\right) = y(y-1)B_e \tag{8}$$

According to the above equation,

Let $F(y) = 0$, then games are stable when $y = 0$ and $y = 1$, because $B_e \ne 0$.

Evolutionary stable strategies are attained when $\frac{dF(y)}{dy} < 0$. Because $B_e > 0$, evolutionary stable strategies are attained when $y=0$. It means that the not improve strategy is the final choice for employer and the game is D-dominated for employers. So defection of both players (as the prisoner dilemma games) is ESS.

Supplementary explanation about finding evolutionary stable strategy mechanisms in more complicated models are defined in the studies of Tanimoto et al. Tanimoto in his book (The Evolutionary Games with Sociophysic), introduces the fundamentals of evolutionary game theory and how to calculate evolutionary stable strategies. He discussed comprehensively to explore how integrated reciprocity mechanisms can solve social dilemmas (Tanimoto 2019). In 2007, Tanimoto and Sagara surveyed the relationship between static and dynamic factors and the existence of a weakly dominated strategy games (Tanimoto and Sagara 2007). Tanimoto is a joint article with Wang et al. (2015) used evolutionary game theory to assess the conditions under which cooperation behavior evolves. They proposed new universal scaling parameters for the dilemma strength by five reciprocity mechanisms: direct reciprocity, indirect reciprocity, kin selection, group selection, and network reciprocity. Ito and Tanimoto (2018) proposed universal scaling parameters to measure two different types of dilemmas, gamble- intending and risk-averting.

In the rest of the paper, the existence of a discrete sudden change in the probability of strike strategy is diagnosed. This jump is called catastrophe. After a brief review of this theory, proves of its occurrence are presented.

4 Background of catastrophe theory

Catastrophe theory was proposed by René Thom in 1960s. It defines that small changes in certain parameters of a nonlinear system can cause equilibrium to appear or disappear leading to large and sudden changes in the behavior of the system. Thom (1972) called such instability "catastrophes".

This theory can be applied with particular effectiveness to situations in which gradually changing forces or motivations lead to abrupt changes in behavior (Zeeman 1976). The equation of the classic catastrophe theory which is used for deterministic systems is as below:

$$\mathrm{dx(t)} = \frac{-\partial \mathrm{V}\left(\mathrm{x(t)}, \vec{\mathrm{c}}\right)}{\partial \mathrm{x(t)}} \tag{9}$$

The Potential function, state variable and control variable vector are $\mathrm{V}\left(\mathrm{x(t)}, \vec{\mathrm{c}}\right)$, $\mathrm{x(t)}$ and $\vec{\mathrm{c}}$, respectively. By continuous changes in control variable, the discrete sudden jump will occur in the state variable.

Cusp catastrophe is one of the seven elementary catastrophes (Thom 1972), and it can be described by a smooth surface of equilibrium, as shown in Fig. 3. According to Zeeman (1976), a dynamic process can be considered a cusp catastrophe if it has five characteristics:

(1) Bimodality: If the control point enters the cusp, then the system state has two possible situations.
(2) Sudden transitions: Sudden transition means that the value of the potential function of the system changes abruptly from one minimum to another minimum.

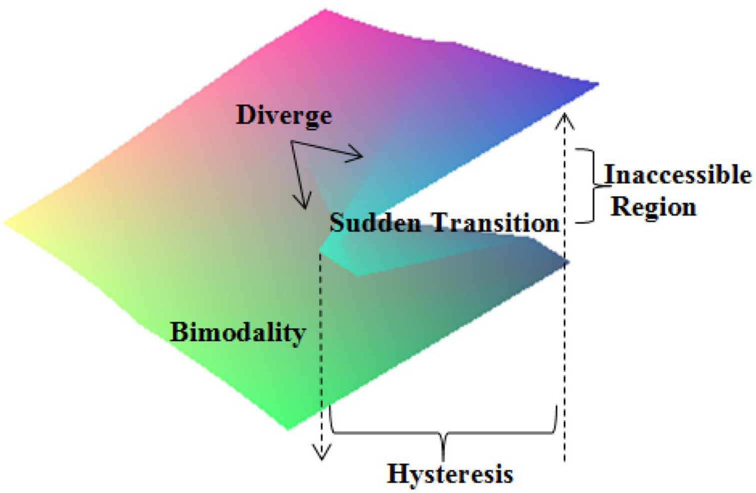

Fig. 3 The cusp catastrophe

(3) Hysteresis: The movement coming from one direction precipitates a catastrophe in a different place than the movement from a different direction.
(4) Inaccessibility: The only two situations of the state exist although the control surface has three lobes.
(5) Divergence: A small perturbation in the initial state of the system can result in a significant difference in its final state.

The cusp catastrophe model consists of one behavior variable (Z) and two control variables (α, β); the potential function is represented below:

$$F(Z, \alpha, \beta) = \frac{1}{4}Z^4 + \frac{1}{2}\alpha Z^2 + \beta Z \tag{10}$$

The equilibrium surface and the catastrophe set expressed below, respectively. In this set, as long as the control variables do not meet each other, the state variable does not change suddenly.

$$\frac{\partial F}{\partial x} = Z^3 + \alpha Z + \beta = 0 \tag{11}$$

$$27\alpha^2 = 4\beta^3 \tag{12}$$

The catastrophe equilibrium is observed in the replicator dynamics of the labor population. In the following after proving the existence of a catastrophe, the catastrophe set is obtained and the managerial insights are presented.

4.1 The existence of the catastrophe for the labor population

The replicator dynamics equation for the labor population is presented below. It is assumed that P(improve) = z · P(strike).

The replicator dynamics equation:

$$dx(t) = x(t)\left(U_x - \overline{U}\right)dt = x(x-1)\left(A_l - zxA_l + C_l\right)dt \tag{13}$$

By expanding the above equation:

$$\begin{aligned} dx(t) &= \left(x^3\left(-zA_l\right) + x^2\left(zA_l + A_l + C_l\right) + x\left(-A_l - C_l\right)\right)\, dt \\ &= \left(Ax^3 + Bx^2 + Cx\right)dt \end{aligned} \tag{14}$$

$$A = -zA_l; \quad B = zA_l + A_l + C_l; \quad C = -A_l - C_l \tag{15}$$

We assume α, β and Z as below and rewrite the replicator dynamics equation:

$$x = Z - \frac{B}{3A}, \quad \alpha = \left(C - \frac{B^2}{3A^2}\right), \quad \beta = \left(\frac{2}{27}\frac{B^3}{A^3} - \frac{1}{3}\frac{CB}{A^2}\right) \tag{16}$$

$$dx(t) = A\left(Z^3 + \alpha Z + \beta\right)dt \rightarrow dx(t) = 0 \rightarrow Z^3 + \alpha Z + \beta = 0 \tag{17}$$

The last equation is the equation of cusp catastrophe. The variables are replaced in the equation of catastrophe set. This set is a threshold in which the strategy of strike experiences a discrete increase or decrease abruptly.

$$\begin{aligned} 27\alpha^2 = 4\beta^3 \rightarrow 27\left(A_l + C_l + \frac{\left(zA_l + A_l + C_l\right)^2}{3\left(zA_l\right)^2}\right)^2 \\ = 4\left(\frac{2\left(zA_l + A_l + C_l\right)^3}{27\left(-zA_l\right)^3} + \frac{\left(A_l + C_l\right)\left(zA_l + A_l + C_l\right)}{3\left(zA_l\right)^2}\right)^3 \end{aligned} \tag{18}$$

- Example (1): In Fig. 4, catastroe set is plotted in Maple software by numbering the min and max ranges of each axes (the parameters of A_l, C_l and z). Each point in the surface shows a proportion of 3 variables in which the probability of strike experiences sudden and dramatic increases; this means all the labors unite to strike (defection). In the neighborhood of the points, small changes of parameters won't lead to major change of P(x), but as soon as reaching to the points on the surface, P(x), will experience sudden and discrete change.

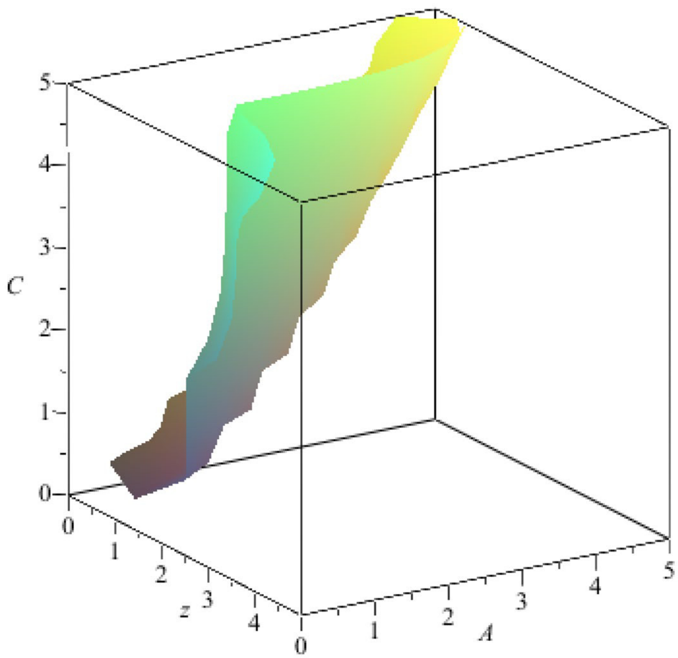

Fig. 4 The catastrophe surface of the example (1)

5 Conclusion

In this paper, we studied the evolutionary game model of labors strike due to inappropriate work conditions. The equilibrium analysis is done through the replicator dynamics equations. The ESS of the players shows that final choice for labors and employers are strike and not improve, respectively. So similar to prisoner dilemma game, the game is D-dominated. In the replicator dynamics of labor population, the possibility of catastrophe occurrence is proved. Using catastrophe theory, the threshold value at which the cooperation in an alliance faces a catastrophe is found. With the continuous changes in control variables, the sudden jump happens in the alliance strike. The parameters of A_l and C_l are the costs of strike for labors and z the proportion of P(improve) to P(strike). High values of P(improve) encourage the labors to strike and high costs of A_l and C_l discourage them. The employers can control labors intention to strike by setting up the strike penalties; but the better way is reducing the probability of strike by creating an appropriate work conditions.

For future studies, the role of labors interactions to the ones who are trying to free- riding and the imposed costs could be investigated. Different types of labors and their coalition in the threat to strike condition could be an interesting point of view. The benefit of work condition improvement for employers through job efficiency and its effect on preventing labor strike is another perspective to study.

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