Mesh $\mathcal{M}(K, V)$ $\qquad$ K - connectivity

$V = \{v_1, \cdots, v_n\}$ - vertices in $\mathbb{R}^3$

Neighbourhood ring $\quad N_i = \{j \mid (i,j) \in K\}$ $\qquad$ d_i - number of vertices in N_i.

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- 相对坐标 - relative coordinate : $\qquad$ set of differentials $\Delta = \{\delta_i\}$.

$$\delta_i = \mathcal{L}(V_i) = V_i - \frac{1}{d_i} \sum_{j \in N_i} V_j$$

解读：顶点 i 的相对坐标 δ_i 是该点绝对坐标 V_i 与 i 点邻域中平均坐标的差.

相对坐标与绝对坐标的转换的矩阵表示.

- A - mesh adjacency matrix $\qquad$ $D = \mathrm{diag}(d_1, \cdots, d_n)$ - degree matrix.

$*$ $\quad \Delta = LV, \quad L = I - D^{-1}A$ $\qquad$ L:具有连接性K的网格 laplacian operator

$$\Rightarrow \Delta = (I - D^{-1}A)V = V - D^{-1}AV$$

$$D^{-1}A = \begin{bmatrix} \frac{1}{d_1} & 0 & \cdots & 0 \\ 0 & \frac{1}{d_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{d_n} \end{bmatrix} \cdot \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & \cdots & \cdots & a_{nn} \end{bmatrix}, \quad a_{ij} = \begin{cases} 1 & , V_i \text{ and } V_j \text{ are connected} \\ 0 & , V_i \text{ and } V_j \text{ aren't connected.} \end{cases}$$

$I - D^{-1}A$

$$= \begin{bmatrix} \frac{a_{11}}{d_1} & \frac{a_{12}}{d_1} & \cdots & \frac{a_{1n}}{d_1} \\ \frac{a_{21}}{d_2} & \frac{a_{22}}{d_2} & \cdots & \frac{a_{2n}}{d_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{a_{n1}}{d_n} & \frac{a_{n2}}{d_n} & \cdots & \frac{a_{nn}}{d_n} \end{bmatrix} \qquad I - D^{-1}A = \begin{bmatrix} 1 - \frac{a_{11}}{d_1} & (-\frac{a_{12}}{d_1}) & \cdots & (-\frac{a_{1n}}{d_1}) \\ (-\frac{a_{21}}{d_2}) & 1 - \frac{a_{22}}{d_2} & \cdots & (-\frac{a_{2n}}{d_2}) \\ \vdots & \vdots & \ddots & \vdots \\ (-\frac{a_{n1}}{d_n}) & (-\frac{a_{n2}}{d_n}) & \cdots & 1 - \frac{a_{nn}}{d_n} \end{bmatrix}$$

$$V = \begin{bmatrix} V_1^x & V_1^y & V_1^z \\ V_2^x & V_2^y & V_2^z \\ \vdots & \vdots & \vdots \\ V_n^x & V_n^y & V_n^z \end{bmatrix} \qquad \Delta = \begin{bmatrix} \delta_1^x & \delta_2^y & \delta_3^z \\ \vdots & \vdots & \vdots \\ \delta_n^x & \delta_n^y & \delta_n^z \end{bmatrix} = \begin{bmatrix} 1 - \frac{a_{11}}{d_1} & (-\frac{a_{12}}{d_1}) & \cdots & (-\frac{a_{1n}}{d_1}) \\ (-\frac{a_{21}}{d_2}) & 1 - \frac{a_{22}}{d_2} & \cdots & (-\frac{a_{2n}}{d_2}) \\ \vdots & \vdots & \ddots & \vdots \\ (-\frac{a_{n1}}{d_n}) & (-\frac{a_{n2}}{d_n}) & \cdots & 1 - \frac{a_{nn}}{d_n} \end{bmatrix} \begin{bmatrix} V_1^x & V_1^y & V_1^z \\ V_2^x & V_2^y & V_2^z \\ \vdots & \vdots & \vdots \\ V_n^x & V_n^y & V_n^z \end{bmatrix}$$

$$\delta_1^x = (1 - \frac{a_{11}}{d_1})V_1^x - \frac{a_{12}}{d_1}V_2^x - \cdots - \frac{a_{1n}}{d_1}V_n^x = V_1^x - \frac{1}{d_1}(a_{11}V_1^x + a_{12}V_2^x + \cdots + a_{1n}V_n^x)$$

$$= V_1^x - \frac{1}{d_1}\sum_{j \in N_1} V_j^x$$

$$\therefore \Delta = LV \iff \delta_i = V_i - \frac{1}{d_i}\sum_{j \in N_i} V_i$$

- 确保控制点变型后在目标位置： $v_i' = u_i \quad i \in P_{controlPoints}$

  u_i：控制点目标位置