4.1

The formula for worst-case complexity of quicksort is derived by:

$[n + (n-1) + (n-2) + (n-3) + (n-4) + \ldots + 2] = [\frac{n(n+1)}{2} - 1] = O(n^2)$

4.2

```
for iteration 1, array is [1, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 16]
for iteration 2, array is [1, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 16]
for iteration 3, array is [1, 2, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 15, 16]
for iteration 4, array is [1, 2, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 15, 16]
for iteration 5, array is [1, 2, 3, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 14, 15, 16]
for iteration 6, array is [1, 2, 3, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 14, 15, 16]
for iteration 7, array is [1, 2, 3, 4, 12, 11, 10, 9, 8, 7, 6, 5, 13, 14, 15, 16]
for iteration 8, array is [1, 2, 3, 4, 12, 11, 10, 9, 8, 7, 6, 5, 13, 14, 15, 16]
for iteration 9, array is [1, 2, 3, 4, 5, 11, 10, 9, 8, 7, 6, 12, 13, 14, 15, 16]
for iteration 10, array is [1, 2, 3, 4, 5, 11, 10, 9, 8, 7, 6, 12, 13, 14, 15, 16]
for iteration 11, array is [1, 2, 3, 4, 5, 6, 10, 9, 8, 7, 11, 12, 13, 14, 15, 16]
for iteration 12, array is [1, 2, 3, 4, 5, 6, 10, 9, 8, 7, 11, 12, 13, 14, 15, 16]
for iteration 13, array is [1, 2, 3, 4, 5, 6, 7, 9, 8, 10, 11, 12, 13, 14, 15, 16]
for iteration 14, array is [1, 2, 3, 4, 5, 6, 7, 9, 8, 10, 11, 12, 13, 14, 15, 16]
for iteration 15, array is [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]
```

4.3

```
sizes = [10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 150, 200, 300, 350, 400, 500, 600, 750, 850, 975]
quicktimes = []

i = 0
for size in sizes:
    reversedlist = list(range(size, 0, -1)) # worst case where list is reversed
    start_time = time.time()
    quick_sort(reversedlist.copy(), 0, len(reversedlist)-1, 1)
    quicktimes.append(time.time() - start_time)
    print(f"Quick sort time for worst case for size {size} is {quicktimes[i]:.5f}")
    i += 1
```

```
Quick sort time for worst case for size 10 is 0.00000
Quick sort time for worst case for size 20 is 0.00000
Quick sort time for worst case for size 30 is 0.00000
Quick sort time for worst case for size 40 is 0.00000
Quick sort time for worst case for size 50 is 0.00000
Quick sort time for worst case for size 60 is 0.00000
Quick sort time for worst case for size 70 is 0.00000
Quick sort time for worst case for size 80 is 0.00000
Quick sort time for worst case for size 90 is 0.00000
Quick sort time for worst case for size 100 is 0.00000
Quick sort time for worst case for size 150 is 0.00000
Quick sort time for worst case for size 200 is 0.00100
Quick sort time for worst case for size 300 is 0.00100
Quick sort time for worst case for size 350 is 0.00200
Quick sort time for worst case for size 400 is 0.00200
Quick sort time for worst case for size 500 is 0.00451
Quick sort time for worst case for size 600 is 0.00600
Quick sort time for worst case for size 750 is 0.01002
Quick sort time for worst case for size 850 is 0.01101
Quick sort time for worst case for size 975 is 0.01551
```

4.4