

Problem 1: parts 2 and 3

$$T(n) = T(n-1) + n$$

$$T(n-1) = T(n-2) + n-1 \quad T(n-2) = T(n-3) + 2n-2$$

$$T(n) = T(n-3) + 3n-3$$

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$$= T(n-n) + n * n - n$$

$$= T(0) + n * n - n$$

$$= c + n^2 - n$$

$$O(n^2)$$

Problem 2

$$\log_a n = \log_b n$$

$$\log_b n = \log_a n / \log_a b \rightarrow \log_a b \text{ is a constant}$$

$$\log_a n / \log_a b = \log_a n * (1 / \log_a b) = c * \log_a n$$

$$\log_b n = c * \log_a n$$

$$c * \log_a n \leq \log_b n \leq c * \log_a n$$

Problem 4:

best case

$$T(n) = 2T(n/2) + cn \quad T(n/2) = 2T(n/4) + cn$$

$$T(n) = 4T(n/4) + 2cn$$

$$= 8T(n/8) + 3cn$$

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$$= nT(n/n) + cn(\log_2 n)$$

$$= n + cn(\log_2 n)$$

$$O(n \log_2 n)$$

worst case

$$T(n) = T(n-1) + n$$

$$T(n-1) = T(n-2) + n-1 \quad T(n-2) = T(n-3) + 2n-2$$

$$T(n) = T(n-3) + 3n-3$$

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$$= T(n-n) + n * n - n$$

$$= T(0) + n * n - n$$

$$= c + n^2 - n$$

$$O(n^2)$$