For the instance: **u\_s\_hihi\_512\_16.txt:**

We have tuned the parameters crossover and mutation probability, for three values each, which results in 9 possible algorithms. We ran each algorithm 50 times and collected the results. The results were further processed in Matlab, with the Kruskal-Wallis test.

The values tested were:

Algorithm 1: pc=0.8, pm=0.001953125 (1/512)

Algorithm 2: pc=0.6, pm=0.001953125 (1/512)

Algorithm 3: pc=1.0, pm=0.001953125 (1/512)

Algorithm 4: pc=1.0, pm=0.00390625 (1/512)

Algorithm 5: pc=0.6, pm=0.00390625 (2/512)

Algorithm 6: pc=0.8, pm=0.00390625 (2/512)

Algorithm 7: pc=0.8, pm=9.765625E-4 (0.5/512)

Algorithm 8: pc=0.6, pm=9.765625E-4 (0.5/512)

Algorithm 9: pc=1.0, pm=9.765625E-4 (0.5/512)



Our data:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Alg1 | Alg2 | Alg3 | Alg4 | Alg5 | Alg6 | Alg7 | Alg8 | Alg9 |
| 1 | 26594395 | 25827243 | 26342091 | 26401053 | 26692129 | 27252061 | 27386220 | 25268109 | 26730117 |
| 2 | 27033346 | 26691659 | 25133489 | 25686431 | 24325386 | 26317239 | 27072528 | 26551773 | 26188261 |
| 3 | 27364718 | 27041751 | 25504343 | 27218389 | 25069822 | 27342290 | 27192246 | 26904594 | 25939730 |
| 4 | 25889296 | 25813770 | 25871440 | 26833533 | 26814483 | 26833017 | 25328776 | 25415389 | 25989096 |
| 5 | 26474980 | 26745896 | 26672764 | 26693418 | 27364920 | 27010498 | 26768679 | 26046592 | 24866368 |
| 6 | 27095611 | 26664714 | 26476074 | 27280296 | 25848873 | 25112606 | 26004897 | 27230748 | 27062763 |
| 7 | 26493422 | 27119310 | 26506886 | 27842227 | 26594802 | 26438128 | 26890947 | 27509745 | 26993575 |
| 8 | 26897736 | 27100301 | 27281816 | 25904689 | 26744623 | 26326812 | 25926142 | 27514051 | 27000559 |
| 9 | 25965540 | 27171977 | 26202433 | 26462369 | 26752628 | 27086764 | 26801473 | 27332045 | 26901126 |
| 10 | 26066978 | 27451665 | 25371683 | 26829451 | 26472045 | 27415848 | 26818930 | 27098449 | 25678367 |
| 11 | 26454109 | 26927610 | 27137803 | 26110625 | 27480172 | 26538533 | 26292431 | 26937334 | 26024443 |
| 12 | 26438318 | 26730249 | 27151746 | 26183676 | 26306641 | 26685167 | 25939838 | 26403717 | 26983695 |
| 13 | 26451776 | 26682255 | 27006431 | 26327865 | 25578294 | 25860777 | 26231173 | 26851887 | 26790041 |
| 14 | 25926304 | 26807010 | 27533559 | 26759405 | 26796393 | 27443000 | 27066550 | 25378799 | 26563600 |
| 15 | 25631190 | 27240909 | 25978881 | 25182339 | 27910700 | 25908068 | 26723136 | 26346249 | 26294140 |
| 16 | 26273749 | 27467896 | 26704479 | 26747855 | 26908168 | 26486038 | 25844965 | 26807232 | 26132867 |
| 17 | 26742412 | 27127287 | 25157040 | 26273839 | 26800916 | 26707223 | 27239953 | 26488256 | 25763326 |
| 18 | 25589119 | 27008843 | 26709634 | 26571189 | 27257358 | 26419142 | 27227354 | 26073602 | 24538806 |
| 19 | 26711424 | 27317566 | 26562414 | 26937609 | 27307220 | 26606427 | 26511442 | 26531662 | 26532324 |
| 20 | 26872270 | 27318356 | 26157490 | 26589493 | 26568873 | 26697749 | 25896742 | 26335463 | 25090882 |
| 21 | 26466776 | 26458811 | 26190578 | 25723812 | 25525759 | 27150187 | 26310043 | 26743261 | 25556788 |
| 22 | 26177981 | 26472676 | 26157050 | 26706725 | 27214120 | 25633757 | 25971090 | 27022907 | 27023630 |
| 23 | 26399450 | 26080163 | 26731043 | 26763806 | 26236101 | 26309618 | 27293115 | 27106528 | 26589459 |
| 24 | 27107922 | 27215332 | 26995136 | 26481351 | 26945347 | 26063753 | 26075752 | 26543842 | 26887636 |
| 25 | 26266433 | 26440883 | 26200569 | 27016693 | 26834188 | 26957698 | 26107975 | 26737263 | 25484291 |
| 26 | 26780305 | 27197957 | 26787494 | 25964798 | 26470691 | 26830400 | 26681135 | 26872360 | 26273723 |
| 27 | 26029602 | 26452449 | 25936509 | 25985569 | 27004891 | 26200262 | 26322213 | 26346221 | 26795384 |
| 28 | 25539606 | 26646493 | 25883356 | 25440559 | 26495150 | 25885208 | 25742487 | 26992738 | 26259833 |
| 29 | 26488870 | 25652329 | 26253236 | 25561647 | 26410444 | 27068430 | 26930466 | 26799551 | 27035400 |
| 30 | 25907675 | 26865274 | 26813952 | 26398826 | 27632668 | 25981114 | 26867838 | 27471605 | 26187471 |
| 31 | 26622681 | 26220706 | 26127958 | 26092832 | 25634536 | 27317033 | 27548241 | 26783840 | 26319888 |
| 32 | 26898313 | 26476630 | 26450020 | 27180858 | 26635996 | 26886612 | 26307371 | 26592434 | 26449263 |
| 33 | 26424556 | 26842881 | 26142145 | 24935552 | 26568479 | 27167194 | 26639652 | 26796633 | 27089516 |
| 34 | 25870696 | 27060088 | 25714339 | 26111659 | 26647930 | 26403835 | 26821342 | 26980971 | 26637698 |
| 35 | 25735548 | 26473513 | 25794663 | 26864925 | 26341157 | 26998252 | 26962944 | 25350776 | 26625280 |
| 36 | 27326426 | 26203892 | 26130670 | 26760895 | 25858973 | 26969864 | 26075493 | 26427182 | 26376142 |
| 37 | 26808832 | 26739519 | 25962969 | 24978156 | 27785568 | 26059265 | 26164432 | 25586427 | 26353374 |
| 38 | 25789128 | 26421911 | 27049981 | 26399742 | 26908697 | 26815110 | 26399459 | 25871406 | 26566075 |
| 39 | 25522315 | 26507885 | 26639062 | 26234438 | 26792049 | 26220877 | 26497170 | 27523824 | 26253296 |
| 40 | 26740842 | 26765608 | 26444165 | 26741555 | 26228519 | 27146296 | 26727608 | 27555155 | 26877820 |
| 41 | 26096419 | 26981057 | 26074336 | 25819799 | 26548532 | 27247522 | 26860832 | 26902932 | 26545780 |
| 42 | 24686216 | 26369027 | 25921481 | 26161220 | 26672685 | 26125063 | 27422401 | 27747923 | 25134235 |
| 43 | 27090203 | 27199209 | 26208430 | 26018638 | 26434046 | 27023816 | 26454344 | 25253503 | 26254867 |
| 44 | 26669205 | 27114238 | 26472156 | 25964060 | 25072897 | 25840173 | 26288524 | 27289602 | 26470600 |
| 45 | 25302155 | 25741969 | 26860805 | 26561732 | 26475937 | 27387669 | 25693190 | 27479792 | 26822716 |
| 46 | 26080895 | 25854347 | 26587446 | 26610997 | 26228505 | 26315194 | 27058614 | 26837980 | 25997518 |
| 47 | 26477969 | 27102455 | 26266558 | 26164946 | 26843446 | 26715690 | 26800452 | 27065513 | 25722062 |
| 48 | 27601653 | 26573529 | 24923826 | 25697051 | 26949602 | 26479819 | 26688299 | 26890964 | 26095462 |
| 49 | 26554091 | 26436355 | 26935687 | 25168551 | 26216639 | 26804592 | 27698261 | 25174993 | 27320377 |
| 50 | 25644571 | 26434335 | 26256067 | 26034195 | 27080384 | 26512473 | 26500259 | 27274382 | 26577638 |

The p values per pairs of algorithms:



|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Alg A | Alg B |  |  |  | P Value |
| 1 | 2 | -158.8948 | -78.2200 | 2.4548 | 0.0658 |
| 1 | 3 | -71.6348 | 9.0400 | 89.7148 | 1.0000 |
| 1 | 4 | -69.6948 | 10.9800 | 91.6548 | 1.0000 |
| 1 | 5 | -131.8348 | -51.1600 | 29.5148 | 0.5666 |
| 1 | 6 | -139.0148 | -58.3400 | 22.3348 | 0.3776 |
| 1 | 7 | -126.5548 | -45.8800 | 34.7948 | 0.7061 |
| 1 | 8 | -156.5148 | -75.8400 | 4.8348 | 0.0849 |
| 1 | 9 | -81.5948 | -0.9200 | 79.7548 | 1.0000 |
| 2 | 3 | 6.5852 | 87.2600 | 167.9348 | 0.0226 |
| 2 | 4 | 8.5252 | 89.2000 | 169.8748 | 0.0176 |
| 2 | 5 | -53.6148 | 27.0600 | 107.7348 | 0.9820 |
| 2 | 6 | -60.7948 | 19.8800 | 100.5548 | 0.9978 |
| 2 | 7 | -48.3348 | 32.3400 | 113.0148 | 0.9469 |
| 2 | 8 | -78.2948 | 2.3800 | 83.0548 | 1.0000 |
| 2 | 9 | -3.3748 | 77.3000 | 157.9748 | 0.0727 |
| 3 | 4 | -78.7348 | 1.9400 | 82.6148 | 1.0000 |
| 3 | 5 | -140.8748 | -60.2000 | 20.4748 | 0.3331 |
| 3 | 6 | -148.0548 | -67.3800 | 13.2948 | 0.1904 |
| 3 | 7 | -135.5948 | -54.9200 | 25.7548 | 0.4652 |
| 3 | 8 | -165.5548 | -84.8800 | -4.2052 | 0.0304 |
| 3 | 9 | -90.6348 | -9.9600 | 70.7148 | 1.0000 |
| 4 | 5 | -142.8148 | -62.1400 | 18.5348 | 0.2898 |
| 4 | 6 | -149.9948 | -69.3200 | 11.3548 | 0.1604 |
| 4 | 7 | -137.5348 | -56.8600 | 23.8148 | 0.4147 |
| 4 | 8 | -167.4948 | -86.8200 | -6.1452 | 0.0239 |
| 4 | 9 | -92.5748 | -11.9000 | 68.7748 | 1.0000 |
| 5 | 6 | -87.8548 | -7.1800 | 73.4948 | 1.0000 |
| 5 | 7 | -75.3948 | 5.2800 | 85.9548 | 1.0000 |
| 5 | 8 | -105.3548 | -24.6800 | 55.9948 | 0.9901 |
| 5 | 9 | -30.4348 | 50.2400 | 130.9148 | 0.5916 |
| 6 | 7 | -68.2148 | 12.4600 | 93.1348 | 0.9999 |
| 6 | 8 | -98.1748 | -17.5000 | 63.1748 | 0.9991 |
| 6 | 9 | -23.2548 | 57.4200 | 138.0948 | 0.4005 |
| 7 | 8 | -110.6348 | -29.9600 | 50.7148 | 0.9661 |
| 7 | 9 | -35.7148 | 44.9600 | 125.6348 | 0.7289 |
| 8 | 9 | -5.7548 | 74.9200 | 155.5948 | 0.0934 |

P values < 0.05 means that we have statistically significant differences with 95% confidence.

Based on the information gathered we have chosen to go ahead with algorithm 4 for this instance of the problem.

For the instance: **u\_s\_lohi\_512\_16.txt:**

We tuned the parameters crossover, mutation probability, and population size for three values each, which results in 27 possible algorithms. We ran each algorithm 50 times and collected the results. The results were further processed in Matlab, with the Kruskal-Wallis test.

The values tested were:

Alg1: pc=1.0, pm=0.001953125 (1/512), popsize=512

Alg2: pc=1.0, pm=0.001953125 (1/512), popsize=1024

Alg3: pc=1.0, pm=0.001953125 (1/512), popsize=2048

Alg4: pc=1.0, pm=0.009765625 (5/512), popsize=512

Alg5: pc=1.0, pm=0.009765625 (5/512), popsize=1024

Alg6: pc=1.0, pm=0.009765625 (5/512), popsize=2048

Alg7: pc=1.0, pm=0.01953125 (10/512), popsize=512

Alg8: pc=1.0, pm=0.01953125 (10/512), popsize=1024

Alg9: pc=1.0, pm=0.01953125 (10/512), popsize=2048

Alg10: pc=0.8, pm=0.001953125 (1/512), popsize=512

Alg11: pc=0.8, pm=0.001953125 (1/512), popsize=1024

Alg12: pc=0.8, pm=0.001953125 (1/512), popsize=2048

Alg13: pc=0.8, pm=0.009765625 (5/512), popsize=512

Alg14: pc=0.8, pm=0.009765625 (5/512), popsize=1024

Alg15: pc=0.8, pm=0.009765625 (5/512), popsize=2048

Alg16: pc=0.8, pm=0.01953125 (10/512), popsize=512

Alg17: pc=0.8, pm=0.01953125 (10/512), popsize=1024

Alg18: pc=0.8, pm=0.01953125 (10/512), popsize=2048

Alg19: pc=0.6, pm=0.001953125 (1/512), popsize=512

Alg20: pc=0.6, pm=0.001953125 (1/512), popsize=1024

Alg21: pc=0.6, pm=0.001953125 (1/512), popsize=2048

Alg22: pc=0.6, pm=0.009765625 (5/512), popsize=512

Alg23: pc=0.6, pm=0.009765625 (5/512), popsize=1024

Alg24: pc=0.6, pm=0.009765625 (5/512), popsize=2048

Alg25: pc=0.6, pm=0.01953125 (10/512), popsize=512

Alg26: pc=0.6, pm=0.01953125 (10/512), popsize=1024

Alg27: pc=0.6, pm=0.01953125 (10/512), popsize=2048



|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **B** | **lower** | **mean difference** | **upper** | **P Value** | **A** | **B** | **lower** | **mean difference** | **upper** | **P Value** |
| 1 | 2 | -509.69359 | -221.48000 | 66.73359 | 0.46062 | 8 | 23 | -449.81359 | -161.60000 | 126.61359 | 0.94995 |
| 1 | 3 | -799.61359 | -511.40000 | -223.18641 | 0.00000 | 8 | 24 | -912.92359 | -624.71000 | -336.49641 | 0.00000 |
| 1 | 4 | -83.33359 | 204.88000 | 493.09359 | 0.63437 | 8 | 25 | -115.82359 | 172.39000 | 460.60359 | 0.90279 |
| 1 | 5 | -380.93359 | -92.72000 | 195.49359 | 0.99999 | 8 | 26 | -517.08359 | -228.87000 | 59.34359 | 0.38697 |
| 1 | 6 | -970.20359 | -681.99000 | -393.77641 | 0.00000 | 8 | 27 | -961.20359 | -672.99000 | -384.77641 | 0.00000 |
| 1 | 7 | -110.65359 | 177.56000 | 465.77359 | 0.87225 | 9 | 10 | 282.00641 | 570.22000 | 858.43359 | 0.00000 |
| 1 | 8 | -508.86359 | -220.65000 | 67.56359 | 0.46916 | 9 | 11 | 12.76641 | 300.98000 | 589.19359 | 0.02821 |
| 1 | 9 | -1,087.95359 | -799.74000 | -511.52641 | 0.00000 | 9 | 12 | -226.73359 | 61.48000 | 349.69359 | 1.00000 |
| 1 | 10 | -517.73359 | -229.52000 | 58.69359 | 0.38073 | 9 | 13 | 642.98641 | 931.20000 | 1,219.41359 | 0.00000 |
| 1 | 11 | -786.97359 | -498.76000 | -210.54641 | 0.00000 | 9 | 14 | 221.06641 | 509.28000 | 797.49359 | 0.00000 |
| 1 | 12 | -1,026.47359 | -738.26000 | -450.04641 | 0.00000 | 9 | 15 | -228.58359 | 59.63000 | 347.84359 | 1.00000 |
| 1 | 13 | -156.75359 | 131.46000 | 419.67359 | 0.99662 | 9 | 16 | 567.46641 | 855.68000 | 1,143.89359 | 0.00000 |
| 1 | 14 | -578.67359 | -290.46000 | -2.24641 | 0.04535 | 9 | 17 | 166.35641 | 454.57000 | 742.78359 | 0.00000 |
| 1 | 15 | -1,028.32359 | -740.11000 | -451.89641 | 0.00000 | 9 | 18 | -357.93359 | -69.72000 | 218.49359 | 1.00000 |
| 1 | 16 | -232.27359 | 55.94000 | 344.15359 | 1.00000 | 9 | 19 | 143.10641 | 431.32000 | 719.53359 | 0.00001 |
| 1 | 17 | -633.38359 | -345.17000 | -56.95641 | 0.00288 | 9 | 20 | -177.22359 | 110.99000 | 399.20359 | 0.99979 |
| 1 | 18 | -1,157.67359 | -869.46000 | -581.24641 | 0.00000 | 9 | 21 | -372.75359 | -84.54000 | 203.67359 | 1.00000 |
| 1 | 19 | -656.63359 | -368.42000 | -80.20641 | 0.00073 | 9 | 22 | 515.36641 | 803.58000 | 1,091.79359 | 0.00000 |
| 1 | 20 | -976.96359 | -688.75000 | -400.53641 | 0.00000 | 9 | 23 | 129.27641 | 417.49000 | 705.70359 | 0.00003 |
| 1 | 21 | -1,172.49359 | -884.28000 | -596.06641 | 0.00000 | 9 | 24 | -333.83359 | -45.62000 | 242.59359 | 1.00000 |
| 1 | 22 | -284.37359 | 3.84000 | 292.05359 | 1.00000 | 9 | 25 | 463.26641 | 751.48000 | 1,039.69359 | 0.00000 |
| 1 | 23 | -670.46359 | -382.25000 | -94.03641 | 0.00031 | 9 | 26 | 62.00641 | 350.22000 | 638.43359 | 0.00216 |
| 1 | 24 | -1,133.57359 | -845.36000 | -557.14641 | 0.00000 | 9 | 27 | -382.11359 | -93.90000 | 194.31359 | 0.99999 |
| 1 | 25 | -336.47359 | -48.26000 | 239.95359 | 1.00000 | 10 | 11 | -557.45359 | -269.24000 | 18.97359 | 0.10802 |
| 1 | 26 | -737.73359 | -449.52000 | -161.30641 | 0.00000 | 10 | 12 | -796.95359 | -508.74000 | -220.52641 | 0.00000 |
| 1 | 27 | -1,181.85359 | -893.64000 | -605.42641 | 0.00000 | 10 | 13 | 72.76641 | 360.98000 | 649.19359 | 0.00115 |
| 2 | 3 | -578.13359 | -289.92000 | -1.70641 | 0.04643 | 10 | 14 | -349.15359 | -60.94000 | 227.27359 | 1.00000 |
| 2 | 4 | 138.14641 | 426.36000 | 714.57359 | 0.00002 | 10 | 15 | -798.80359 | -510.59000 | -222.37641 | 0.00000 |
| 2 | 5 | -159.45359 | 128.76000 | 416.97359 | 0.99754 | 10 | 16 | -2.75359 | 285.46000 | 573.67359 | 0.05626 |
| 2 | 6 | -748.72359 | -460.51000 | -172.29641 | 0.00000 | 10 | 17 | -403.86359 | -115.65000 | 172.56359 | 0.99957 |
| 2 | 7 | 110.82641 | 399.04000 | 687.25359 | 0.00010 | 10 | 18 | -928.15359 | -639.94000 | -351.72641 | 0.00000 |
| 2 | 8 | -287.38359 | 0.83000 | 289.04359 | 1.00000 | 10 | 19 | -427.11359 | -138.90000 | 149.31359 | 0.99248 |
| 2 | 9 | -866.47359 | -578.26000 | -290.04641 | 0.00000 | 10 | 20 | -747.44359 | -459.23000 | -171.01641 | 0.00000 |
| 2 | 10 | -296.25359 | -8.04000 | 280.17359 | 1.00000 | 10 | 21 | -942.97359 | -654.76000 | -366.54641 | 0.00000 |
| 2 | 11 | -565.49359 | -277.28000 | 10.93359 | 0.07890 | 10 | 22 | -54.85359 | 233.36000 | 521.57359 | 0.34485 |
| 2 | 12 | -804.99359 | -516.78000 | -228.56641 | 0.00000 | 10 | 23 | -440.94359 | -152.73000 | 135.48359 | 0.97394 |
| 2 | 13 | 64.72641 | 352.94000 | 641.15359 | 0.00185 | 10 | 24 | -904.05359 | -615.84000 | -327.62641 | 0.00000 |
| 2 | 14 | -357.19359 | -68.98000 | 219.23359 | 1.00000 | 10 | 25 | -106.95359 | 181.26000 | 469.47359 | 0.84716 |
| 2 | 15 | -806.84359 | -518.63000 | -230.41641 | 0.00000 | 10 | 26 | -508.21359 | -220.00000 | 68.21359 | 0.47587 |
| 2 | 16 | -10.79359 | 277.42000 | 565.63359 | 0.07845 | 10 | 27 | -952.33359 | -664.12000 | -375.90641 | 0.00000 |
| 2 | 17 | -411.90359 | -123.69000 | 164.52359 | 0.99869 | 11 | 12 | -527.71359 | -239.50000 | 48.71359 | 0.29126 |
| 2 | 18 | -936.19359 | -647.98000 | -359.76641 | 0.00000 | 11 | 13 | 342.00641 | 630.22000 | 918.43359 | 0.00000 |
| 2 | 19 | -435.15359 | -146.94000 | 141.27359 | 0.98395 | 11 | 14 | -79.91359 | 208.30000 | 496.51359 | 0.59872 |
| 2 | 20 | -755.48359 | -467.27000 | -179.05641 | 0.00000 | 11 | 15 | -529.56359 | -241.35000 | 46.86359 | 0.27612 |
| 2 | 21 | -951.01359 | -662.80000 | -374.58641 | 0.00000 | 11 | 16 | 266.48641 | 554.70000 | 842.91359 | 0.00000 |
| 2 | 22 | -62.89359 | 225.32000 | 513.53359 | 0.42177 | 11 | 17 | -134.62359 | 153.59000 | 441.80359 | 0.97211 |
| 2 | 23 | -448.98359 | -160.77000 | 127.44359 | 0.95271 | 11 | 18 | -658.91359 | -370.70000 | -82.48641 | 0.00064 |
| 2 | 24 | -912.09359 | -623.88000 | -335.66641 | 0.00000 | 11 | 19 | -157.87359 | 130.34000 | 418.55359 | 0.99703 |
| 2 | 25 | -114.99359 | 173.22000 | 461.43359 | 0.89824 | 11 | 20 | -478.20359 | -189.99000 | 98.22359 | 0.77789 |
| 2 | 26 | -516.25359 | -228.04000 | 60.17359 | 0.39500 | 11 | 21 | -673.73359 | -385.52000 | -97.30641 | 0.00025 |
| 2 | 27 | -960.37359 | -672.16000 | -383.94641 | 0.00000 | 11 | 22 | 214.38641 | 502.60000 | 790.81359 | 0.00000 |
| 3 | 4 | 428.06641 | 716.28000 | 1,004.49359 | 0.00000 | 11 | 23 | -171.70359 | 116.51000 | 404.72359 | 0.99951 |
| 3 | 5 | 130.46641 | 418.68000 | 706.89359 | 0.00003 | 11 | 24 | -634.81359 | -346.60000 | -58.38641 | 0.00266 |
| 3 | 6 | -458.80359 | -170.59000 | 117.62359 | 0.91218 | 11 | 25 | 162.28641 | 450.50000 | 738.71359 | 0.00000 |
| 3 | 7 | 400.74641 | 688.96000 | 977.17359 | 0.00000 | 11 | 26 | -238.97359 | 49.24000 | 337.45359 | 1.00000 |
| 3 | 8 | 2.53641 | 290.75000 | 578.96359 | 0.04477 | 11 | 27 | -683.09359 | -394.88000 | -106.66641 | 0.00014 |
| 3 | 9 | -576.55359 | -288.34000 | -0.12641 | 0.04973 | 12 | 13 | 581.50641 | 869.72000 | 1,157.93359 | 0.00000 |
| 3 | 10 | -6.33359 | 281.88000 | 570.09359 | 0.06538 | 12 | 14 | 159.58641 | 447.80000 | 736.01359 | 0.00000 |
| 3 | 11 | -275.57359 | 12.64000 | 300.85359 | 1.00000 | 12 | 15 | -290.06359 | -1.85000 | 286.36359 | 1.00000 |
| 3 | 12 | -515.07359 | -226.86000 | 61.35359 | 0.40653 | 12 | 16 | 505.98641 | 794.20000 | 1,082.41359 | 0.00000 |
| 3 | 13 | 354.64641 | 642.86000 | 931.07359 | 0.00000 | 12 | 17 | 104.87641 | 393.09000 | 681.30359 | 0.00015 |
| 3 | 14 | -67.27359 | 220.94000 | 509.15359 | 0.46617 | 12 | 18 | -419.41359 | -131.20000 | 157.01359 | 0.99672 |
| 3 | 15 | -516.92359 | -228.71000 | 59.50359 | 0.38851 | 12 | 19 | 81.62641 | 369.84000 | 658.05359 | 0.00067 |
| 3 | 16 | 279.12641 | 567.34000 | 855.55359 | 0.00000 | 12 | 20 | -238.70359 | 49.51000 | 337.72359 | 1.00000 |
| 3 | 17 | -121.98359 | 166.23000 | 454.44359 | 0.93236 | 12 | 21 | -434.23359 | -146.02000 | 142.19359 | 0.98521 |
| 3 | 18 | -646.27359 | -358.06000 | -69.84641 | 0.00137 | 12 | 22 | 453.88641 | 742.10000 | 1,030.31359 | 0.00000 |
| 3 | 19 | -145.23359 | 142.98000 | 431.19359 | 0.98881 | 12 | 23 | 67.79641 | 356.01000 | 644.22359 | 0.00154 |
| 3 | 20 | -465.56359 | -177.35000 | 110.86359 | 0.87359 | 12 | 24 | -395.31359 | -107.10000 | 181.11359 | 0.99989 |
| 3 | 21 | -661.09359 | -372.88000 | -84.66641 | 0.00056 | 12 | 25 | 401.78641 | 690.00000 | 978.21359 | 0.00000 |
| 3 | 22 | 227.02641 | 515.24000 | 803.45359 | 0.00000 | 12 | 26 | 0.52641 | 288.74000 | 576.95359 | 0.04887 |
| 3 | 23 | -159.06359 | 129.15000 | 417.36359 | 0.99742 | 12 | 27 | -443.59359 | -155.38000 | 132.83359 | 0.96798 |
| 3 | 24 | -622.17359 | -333.96000 | -45.74641 | 0.00536 | 13 | 14 | -710.13359 | -421.92000 | -133.70641 | 0.00002 |
| 3 | 25 | 174.92641 | 463.14000 | 751.35359 | 0.00000 | 13 | 15 | -1,159.78359 | -871.57000 | -583.35641 | 0.00000 |
| 3 | 26 | -226.33359 | 61.88000 | 350.09359 | 1.00000 | 13 | 16 | -363.73359 | -75.52000 | 212.69359 | 1.00000 |
| 3 | 27 | -670.45359 | -382.24000 | -94.02641 | 0.00031 | 13 | 17 | -764.84359 | -476.63000 | -188.41641 | 0.00000 |
| 4 | 5 | -585.81359 | -297.60000 | -9.38641 | 0.03296 | 13 | 18 | -1,289.13359 | -1,000.92000 | -712.70641 | 0.00000 |
| 4 | 6 | -1,175.08359 | -886.87000 | -598.65641 | 0.00000 | 13 | 19 | -788.09359 | -499.88000 | -211.66641 | 0.00000 |
| 4 | 7 | -315.53359 | -27.32000 | 260.89359 | 1.00000 | 13 | 20 | -1,108.42359 | -820.21000 | -531.99641 | 0.00000 |
| 4 | 8 | -713.74359 | -425.53000 | -137.31641 | 0.00002 | 13 | 21 | -1,303.95359 | -1,015.74000 | -727.52641 | 0.00000 |
| 4 | 9 | -1,292.83359 | -1,004.62000 | -716.40641 | 0.00000 | 13 | 22 | -415.83359 | -127.62000 | 160.59359 | 0.99785 |
| 4 | 10 | -722.61359 | -434.40000 | -146.18641 | 0.00001 | 13 | 23 | -801.92359 | -513.71000 | -225.49641 | 0.00000 |
| 4 | 11 | -991.85359 | -703.64000 | -415.42641 | 0.00000 | 13 | 24 | -1,265.03359 | -976.82000 | -688.60641 | 0.00000 |
| 4 | 12 | -1,231.35359 | -943.14000 | -654.92641 | 0.00000 | 13 | 25 | -467.93359 | -179.72000 | 108.49359 | 0.85793 |
| 4 | 13 | -361.63359 | -73.42000 | 214.79359 | 1.00000 | 13 | 26 | -869.19359 | -580.98000 | -292.76641 | 0.00000 |
| 4 | 14 | -783.55359 | -495.34000 | -207.12641 | 0.00000 | 13 | 27 | -1,313.31359 | -1,025.10000 | -736.88641 | 0.00000 |
| 4 | 15 | -1,233.20359 | -944.99000 | -656.77641 | 0.00000 | 14 | 15 | -737.86359 | -449.65000 | -161.43641 | 0.00000 |
| 4 | 16 | -437.15359 | -148.94000 | 139.27359 | 0.98092 | 14 | 16 | 58.18641 | 346.40000 | 634.61359 | 0.00269 |
| 4 | 17 | -838.26359 | -550.05000 | -261.83641 | 0.00000 | 14 | 17 | -342.92359 | -54.71000 | 233.50359 | 1.00000 |
| 4 | 18 | -1,362.55359 | -1,074.34000 | -786.12641 | 0.00000 | 14 | 18 | -867.21359 | -579.00000 | -290.78641 | 0.00000 |
| 4 | 19 | -861.51359 | -573.30000 | -285.08641 | 0.00000 | 14 | 19 | -366.17359 | -77.96000 | 210.25359 | 1.00000 |
| 4 | 20 | -1,181.84359 | -893.63000 | -605.41641 | 0.00000 | 14 | 20 | -686.50359 | -398.29000 | -110.07641 | 0.00011 |
| 4 | 21 | -1,377.37359 | -1,089.16000 | -800.94641 | 0.00000 | 14 | 21 | -882.03359 | -593.82000 | -305.60641 | 0.00000 |
| 4 | 22 | -489.25359 | -201.04000 | 87.17359 | 0.67358 | 14 | 22 | 6.08641 | 294.30000 | 582.51359 | 0.03826 |
| 4 | 23 | -875.34359 | -587.13000 | -298.91641 | 0.00000 | 14 | 23 | -380.00359 | -91.79000 | 196.42359 | 0.99999 |
| 4 | 24 | -1,338.45359 | -1,050.24000 | -762.02641 | 0.00000 | 14 | 24 | -843.11359 | -554.90000 | -266.68641 | 0.00000 |
| 4 | 25 | -541.35359 | -253.14000 | 35.07359 | 0.19133 | 14 | 25 | -46.01359 | 242.20000 | 530.41359 | 0.26932 |
| 4 | 26 | -942.61359 | -654.40000 | -366.18641 | 0.00000 | 14 | 26 | -447.27359 | -159.06000 | 129.15359 | 0.95804 |
| 4 | 27 | -1,386.73359 | -1,098.52000 | -810.30641 | 0.00000 | 14 | 27 | -891.39359 | -603.18000 | -314.96641 | 0.00000 |
| 5 | 6 | -877.48359 | -589.27000 | -301.05641 | 0.00000 | 15 | 16 | 507.83641 | 796.05000 | 1,084.26359 | 0.00000 |
| 5 | 7 | -17.93359 | 270.28000 | 558.49359 | 0.10383 | 15 | 17 | 106.72641 | 394.94000 | 683.15359 | 0.00014 |
| 5 | 8 | -416.14359 | -127.93000 | 160.28359 | 0.99777 | 15 | 18 | -417.56359 | -129.35000 | 158.86359 | 0.99736 |
| 5 | 9 | -995.23359 | -707.02000 | -418.80641 | 0.00000 | 15 | 19 | 83.47641 | 371.69000 | 659.90359 | 0.00060 |
| 5 | 10 | -425.01359 | -136.80000 | 151.41359 | 0.99394 | 15 | 20 | -236.85359 | 51.36000 | 339.57359 | 1.00000 |
| 5 | 11 | -694.25359 | -406.04000 | -117.82641 | 0.00007 | 15 | 21 | -432.38359 | -144.17000 | 144.04359 | 0.98749 |
| 5 | 12 | -933.75359 | -645.54000 | -357.32641 | 0.00000 | 15 | 22 | 455.73641 | 743.95000 | 1,032.16359 | 0.00000 |
| 5 | 13 | -64.03359 | 224.18000 | 512.39359 | 0.43319 | 15 | 23 | 69.64641 | 357.86000 | 646.07359 | 0.00138 |
| 5 | 14 | -485.95359 | -197.74000 | 90.47359 | 0.70627 | 15 | 24 | -393.46359 | -105.25000 | 182.96359 | 0.99992 |
| 5 | 15 | -935.60359 | -647.39000 | -359.17641 | 0.00000 | 15 | 25 | 403.63641 | 691.85000 | 980.06359 | 0.00000 |
| 5 | 16 | -139.55359 | 148.66000 | 436.87359 | 0.98137 | 15 | 26 | 2.37641 | 290.59000 | 578.80359 | 0.04509 |
| 5 | 17 | -540.66359 | -252.45000 | 35.76359 | 0.19572 | 15 | 27 | -441.74359 | -153.53000 | 134.68359 | 0.97224 |
| 5 | 18 | -1,064.95359 | -776.74000 | -488.52641 | 0.00000 | 16 | 17 | -689.32359 | -401.11000 | -112.89641 | 0.00009 |
| 5 | 19 | -563.91359 | -275.70000 | 12.51359 | 0.08404 | 16 | 18 | -1,213.61359 | -925.40000 | -637.18641 | 0.00000 |
| 5 | 20 | -884.24359 | -596.03000 | -307.81641 | 0.00000 | 16 | 19 | -712.57359 | -424.36000 | -136.14641 | 0.00002 |
| 5 | 21 | -1,079.77359 | -791.56000 | -503.34641 | 0.00000 | 16 | 20 | -1,032.90359 | -744.69000 | -456.47641 | 0.00000 |
| 5 | 22 | -191.65359 | 96.56000 | 384.77359 | 0.99998 | 16 | 21 | -1,228.43359 | -940.22000 | -652.00641 | 0.00000 |
| 5 | 23 | -577.74359 | -289.53000 | -1.31641 | 0.04723 | 16 | 22 | -340.31359 | -52.10000 | 236.11359 | 1.00000 |
| 5 | 24 | -1,040.85359 | -752.64000 | -464.42641 | 0.00000 | 16 | 23 | -726.40359 | -438.19000 | -149.97641 | 0.00001 |
| 5 | 25 | -243.75359 | 44.46000 | 332.67359 | 1.00000 | 16 | 24 | -1,189.51359 | -901.30000 | -613.08641 | 0.00000 |
| 5 | 26 | -645.01359 | -356.80000 | -68.58641 | 0.00147 | 16 | 25 | -392.41359 | -104.20000 | 184.01359 | 0.99993 |
| 5 | 27 | -1,089.13359 | -800.92000 | -512.70641 | 0.00000 | 16 | 26 | -793.67359 | -505.46000 | -217.24641 | 0.00000 |
| 6 | 7 | 571.33641 | 859.55000 | 1,147.76359 | 0.00000 | 16 | 27 | -1,237.79359 | -949.58000 | -661.36641 | 0.00000 |
| 6 | 8 | 173.12641 | 461.34000 | 749.55359 | 0.00000 | 17 | 18 | -812.50359 | -524.29000 | -236.07641 | 0.00000 |
| 6 | 9 | -405.96359 | -117.75000 | 170.46359 | 0.99942 | 17 | 19 | -311.46359 | -23.25000 | 264.96359 | 1.00000 |
| 6 | 10 | 164.25641 | 452.47000 | 740.68359 | 0.00000 | 17 | 20 | -631.79359 | -343.58000 | -55.36641 | 0.00315 |
| 6 | 11 | -104.98359 | 183.23000 | 471.44359 | 0.83273 | 17 | 21 | -827.32359 | -539.11000 | -250.89641 | 0.00000 |
| 6 | 12 | -344.48359 | -56.27000 | 231.94359 | 1.00000 | 17 | 22 | 60.79641 | 349.01000 | 637.22359 | 0.00232 |
| 6 | 13 | 525.23641 | 813.45000 | 1,101.66359 | 0.00000 | 17 | 23 | -325.29359 | -37.08000 | 251.13359 | 1.00000 |
| 6 | 14 | 103.31641 | 391.53000 | 679.74359 | 0.00017 | 17 | 24 | -788.40359 | -500.19000 | -211.97641 | 0.00000 |
| 6 | 15 | -346.33359 | -58.12000 | 230.09359 | 1.00000 | 17 | 25 | 8.69641 | 296.91000 | 585.12359 | 0.03401 |
| 6 | 16 | 449.71641 | 737.93000 | 1,026.14359 | 0.00000 | 17 | 26 | -392.56359 | -104.35000 | 183.86359 | 0.99993 |
| 6 | 17 | 48.60641 | 336.82000 | 625.03359 | 0.00459 | 17 | 27 | -836.68359 | -548.47000 | -260.25641 | 0.00000 |
| 6 | 18 | -475.68359 | -187.47000 | 100.74359 | 0.79926 | 18 | 19 | 212.82641 | 501.04000 | 789.25359 | 0.00000 |
| 6 | 19 | 25.35641 | 313.57000 | 601.78359 | 0.01541 | 18 | 20 | -107.50359 | 180.71000 | 468.92359 | 0.85106 |
| 6 | 20 | -294.97359 | -6.76000 | 281.45359 | 1.00000 | 18 | 21 | -303.03359 | -14.82000 | 273.39359 | 1.00000 |
| 6 | 21 | -490.50359 | -202.29000 | 85.92359 | 0.66094 | 18 | 22 | 585.08641 | 873.30000 | 1,161.51359 | 0.00000 |
| 6 | 22 | 397.61641 | 685.83000 | 974.04359 | 0.00000 | 18 | 23 | 198.99641 | 487.21000 | 775.42359 | 0.00000 |
| 6 | 23 | 11.52641 | 299.74000 | 587.95359 | 0.02987 | 18 | 24 | -264.11359 | 24.10000 | 312.31359 | 1.00000 |
| 6 | 24 | -451.58359 | -163.37000 | 124.84359 | 0.94367 | 18 | 25 | 532.98641 | 821.20000 | 1,109.41359 | 0.00000 |
| 6 | 25 | 345.51641 | 633.73000 | 921.94359 | 0.00000 | 18 | 26 | 131.72641 | 419.94000 | 708.15359 | 0.00003 |
| 6 | 26 | -55.74359 | 232.47000 | 520.68359 | 0.35301 | 18 | 27 | -312.39359 | -24.18000 | 264.03359 | 1.00000 |
| 6 | 27 | -499.86359 | -211.65000 | 76.56359 | 0.56345 | 19 | 20 | -608.54359 | -320.33000 | -32.11641 | 0.01097 |
| 7 | 8 | -686.42359 | -398.21000 | -109.99641 | 0.00011 | 19 | 21 | -804.07359 | -515.86000 | -227.64641 | 0.00000 |
| 7 | 9 | -1,265.51359 | -977.30000 | -689.08641 | 0.00000 | 19 | 22 | 84.04641 | 372.26000 | 660.47359 | 0.00058 |
| 7 | 10 | -695.29359 | -407.08000 | -118.86641 | 0.00006 | 19 | 23 | -302.04359 | -13.83000 | 274.38359 | 1.00000 |
| 7 | 11 | -964.53359 | -676.32000 | -388.10641 | 0.00000 | 19 | 24 | -765.15359 | -476.94000 | -188.72641 | 0.00000 |
| 7 | 12 | -1,204.03359 | -915.82000 | -627.60641 | 0.00000 | 19 | 25 | 31.94641 | 320.16000 | 608.37359 | 0.01107 |
| 7 | 13 | -334.31359 | -46.10000 | 242.11359 | 1.00000 | 19 | 26 | -369.31359 | -81.10000 | 207.11359 | 1.00000 |
| 7 | 14 | -756.23359 | -468.02000 | -179.80641 | 0.00000 | 19 | 27 | -813.43359 | -525.22000 | -237.00641 | 0.00000 |
| 7 | 15 | -1,205.88359 | -917.67000 | -629.45641 | 0.00000 | 20 | 21 | -483.74359 | -195.53000 | 92.68359 | 0.72751 |
| 7 | 16 | -409.83359 | -121.62000 | 166.59359 | 0.99900 | 20 | 22 | 404.37641 | 692.59000 | 980.80359 | 0.00000 |
| 7 | 17 | -810.94359 | -522.73000 | -234.51641 | 0.00000 | 20 | 23 | 18.28641 | 306.50000 | 594.71359 | 0.02174 |
| 7 | 18 | -1,335.23359 | -1,047.02000 | -758.80641 | 0.00000 | 20 | 24 | -444.82359 | -156.61000 | 131.60359 | 0.96488 |
| 7 | 19 | -834.19359 | -545.98000 | -257.76641 | 0.00000 | 20 | 25 | 352.27641 | 640.49000 | 928.70359 | 0.00000 |
| 7 | 20 | -1,154.52359 | -866.31000 | -578.09641 | 0.00000 | 20 | 26 | -48.98359 | 239.23000 | 527.44359 | 0.29351 |
| 7 | 21 | -1,350.05359 | -1,061.84000 | -773.62641 | 0.00000 | 20 | 27 | -493.10359 | -204.89000 | 83.32359 | 0.63427 |
| 7 | 22 | -461.93359 | -173.72000 | 114.49359 | 0.89544 | 21 | 22 | 599.90641 | 888.12000 | 1,176.33359 | 0.00000 |
| 7 | 23 | -848.02359 | -559.81000 | -271.59641 | 0.00000 | 21 | 23 | 213.81641 | 502.03000 | 790.24359 | 0.00000 |
| 7 | 24 | -1,311.13359 | -1,022.92000 | -734.70641 | 0.00000 | 21 | 24 | -249.29359 | 38.92000 | 327.13359 | 1.00000 |
| 7 | 25 | -514.03359 | -225.82000 | 62.39359 | 0.41680 | 21 | 25 | 547.80641 | 836.02000 | 1,124.23359 | 0.00000 |
| 7 | 26 | -915.29359 | -627.08000 | -338.86641 | 0.00000 | 21 | 26 | 146.54641 | 434.76000 | 722.97359 | 0.00001 |
| 7 | 27 | -1,359.41359 | -1,071.20000 | -782.98641 | 0.00000 | 21 | 27 | -297.57359 | -9.36000 | 278.85359 | 1.00000 |
| 8 | 9 | -867.30359 | -579.09000 | -290.87641 | 0.00000 | 22 | 23 | -674.30359 | -386.09000 | -97.87641 | 0.00024 |
| 8 | 10 | -297.08359 | -8.87000 | 279.34359 | 1.00000 | 22 | 24 | -1,137.41359 | -849.20000 | -560.98641 | 0.00000 |
| 8 | 11 | -566.32359 | -278.11000 | 10.10359 | 0.07630 | 22 | 25 | -340.31359 | -52.10000 | 236.11359 | 1.00000 |
| 8 | 12 | -805.82359 | -517.61000 | -229.39641 | 0.00000 | 22 | 26 | -741.57359 | -453.36000 | -165.14641 | 0.00000 |
| 8 | 13 | 63.89641 | 352.11000 | 640.32359 | 0.00194 | 22 | 27 | -1,185.69359 | -897.48000 | -609.26641 | 0.00000 |
| 8 | 14 | -358.02359 | -69.81000 | 218.40359 | 1.00000 | 23 | 24 | -751.32359 | -463.11000 | -174.89641 | 0.00000 |
| 8 | 15 | -807.67359 | -519.46000 | -231.24641 | 0.00000 | 23 | 25 | 45.77641 | 333.99000 | 622.20359 | 0.00535 |
| 8 | 16 | -11.62359 | 276.59000 | 564.80359 | 0.08111 | 23 | 26 | -355.48359 | -67.27000 | 220.94359 | 1.00000 |
| 8 | 17 | -412.73359 | -124.52000 | 163.69359 | 0.99854 | 23 | 27 | -799.60359 | -511.39000 | -223.17641 | 0.00000 |
| 8 | 18 | -937.02359 | -648.81000 | -360.59641 | 0.00000 | 24 | 25 | 508.88641 | 797.10000 | 1,085.31359 | 0.00000 |
| 8 | 19 | -435.98359 | -147.77000 | 140.44359 | 0.98274 | 24 | 26 | 107.62641 | 395.84000 | 684.05359 | 0.00013 |
| 8 | 20 | -756.31359 | -468.10000 | -179.88641 | 0.00000 | 24 | 27 | -336.49359 | -48.28000 | 239.93359 | 1.00000 |
| 8 | 21 | -951.84359 | -663.63000 | -375.41641 | 0.00000 | 25 | 26 | -689.47359 | -401.26000 | -113.04641 | 0.00009 |
| 8 | 22 | -63.72359 | 224.49000 | 512.70359 | 0.43007 | 25 | 27 | -1,133.59359 | -845.38000 | -557.16641 | 0.00000 |
|  |  | means |  |  |  | 26 | 27 | -732.33359 | -444.12000 | -155.90641 | 0.00001 |

We have chosen algorithm 4.

For obtaining the lower bound though, we have used a larger population size (1024/2048) and a high number of steps - ran the code for a long time in order to better explore the solution space.