Problem solving project

Lucian Trestioreanu & Nina Çaushi

* Brief Introduction of the problem “ Minimum Makespan Minimization”

Job scheduling is an optimization problem in which N jobs with certain processing times are assigned to M machines. The goal is to find the best assignment of the jobs to the machines such that the completion time of the last job (all jobs have finished processing) is minimized.

Two constraints on this problem are that a job can be scheduled only on a single machine and the makespan is the maximum completion time for all machines. This is an NP hard problem.

* Description of your algorithms

o Parameters Tuning

o Operators (Crossover, Mutation, etc.)

o **Important! Please justify your implementation**

To solve the minimum makespan problem, the genetic algorithm was used. There are three main parameters in GA: crossover probability, mutation probability and population size. Crossover randomly selects one or two points in the parents’ chromosomes and interchanges the two parents chromosomes at this point(s) to produce two new offsprings in hopes of producing better chromosomes. A crossover probability is between 0.6 and 1. After crossover, a mutation might happen which means that part of the offsprings chromosome might change in order to introduce diversity within the population. The probability of mutating a particular bit is typically between 0.001 and 0.1. Population size says how many individuals are in the population. For the instance: **u\_s\_hihi\_512\_16.txt** we only tuned the parameters Pm and Pc and ran 9 possible combinations. For the crossover probability we chose the values 0.6; 0.8 and 1 while for the mutation probability we chose the values 0.001953125; 0.00390625 and 9.765625E-4. For the instance: **u\_s\_lohi\_512\_16.txt** the population size was also changed this the values 512; 1024; 2048. For the mutation probability we used the values 0.001953125; 0.009765625 and 0.01953125 and for the crossover probability we still kept the values 0.6; 0.8 and 1. This means that 27 possible algorithms were tested.

* Use ANOVA or Non-parametric analysis to study the effect of tuned parameters in your

algorithms (regarding the performance) on two specific instances; **u\_s\_hihi\_512\_16.txt** and **u\_s\_lohi\_512\_16.txt** (You only have to perform statistical analysis on these two instances)

* Explain the outcome from statistical analysis

For the instance: **u\_s\_hihi\_512\_16.txt:**

We have tuned the parameters crossover and mutation probabilities, for three values each, which results in 9 possible algorithms.

Algorithm 1: pc=0.8, pm=0.001953125 (1/512)

Algorithm 2: pc=0.6, pm=0.001953125 (1/512)

Algorithm 3: pc=1.0, pm=0.001953125 (1/512)

Algorithm 4: pc=1.0, pm=0.00390625 (1/512)

Algorithm 5: pc=0.6, pm=0.00390625 (2/512)

Algorithm 6: pc=0.8, pm=0.00390625 (2/512)

Algorithm 7: pc=0.8, pm=9.765625E-4 (0.5/512)

Algorithm 8: pc=0.6, pm=9.765625E-4 (0.5/512)

Algorithm 9: pc=1.0, pm=9.765625E-4 (0.5/512)

We ran each algorithm 50 times and collected the following results.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Alg1 | Alg2 | Alg3 | Alg4 | Alg5 | Alg6 | Alg7 | Alg8 | Alg9 |
| 1 | 26594395 | 25827243 | 26342091 | 26401053 | 26692129 | 27252061 | 27386220 | 25268109 | 26730117 |
| 2 | 27033346 | 26691659 | 25133489 | 25686431 | 24325386 | 26317239 | 27072528 | 26551773 | 26188261 |
| 3 | 27364718 | 27041751 | 25504343 | 27218389 | 25069822 | 27342290 | 27192246 | 26904594 | 25939730 |
| 4 | 25889296 | 25813770 | 25871440 | 26833533 | 26814483 | 26833017 | 25328776 | 25415389 | 25989096 |
| 5 | 26474980 | 26745896 | 26672764 | 26693418 | 27364920 | 27010498 | 26768679 | 26046592 | 24866368 |
| 6 | 27095611 | 26664714 | 26476074 | 27280296 | 25848873 | 25112606 | 26004897 | 27230748 | 27062763 |
| 7 | 26493422 | 27119310 | 26506886 | 27842227 | 26594802 | 26438128 | 26890947 | 27509745 | 26993575 |
| 8 | 26897736 | 27100301 | 27281816 | 25904689 | 26744623 | 26326812 | 25926142 | 27514051 | 27000559 |
| 9 | 25965540 | 27171977 | 26202433 | 26462369 | 26752628 | 27086764 | 26801473 | 27332045 | 26901126 |
| 10 | 26066978 | 27451665 | 25371683 | 26829451 | 26472045 | 27415848 | 26818930 | 27098449 | 25678367 |
| 11 | 26454109 | 26927610 | 27137803 | 26110625 | 27480172 | 26538533 | 26292431 | 26937334 | 26024443 |
| 12 | 26438318 | 26730249 | 27151746 | 26183676 | 26306641 | 26685167 | 25939838 | 26403717 | 26983695 |
| 13 | 26451776 | 26682255 | 27006431 | 26327865 | 25578294 | 25860777 | 26231173 | 26851887 | 26790041 |
| 14 | 25926304 | 26807010 | 27533559 | 26759405 | 26796393 | 27443000 | 27066550 | 25378799 | 26563600 |
| 15 | 25631190 | 27240909 | 25978881 | 25182339 | 27910700 | 25908068 | 26723136 | 26346249 | 26294140 |
| 16 | 26273749 | 27467896 | 26704479 | 26747855 | 26908168 | 26486038 | 25844965 | 26807232 | 26132867 |
| 17 | 26742412 | 27127287 | 25157040 | 26273839 | 26800916 | 26707223 | 27239953 | 26488256 | 25763326 |
| 18 | 25589119 | 27008843 | 26709634 | 26571189 | 27257358 | 26419142 | 27227354 | 26073602 | 24538806 |
| 19 | 26711424 | 27317566 | 26562414 | 26937609 | 27307220 | 26606427 | 26511442 | 26531662 | 26532324 |
| 20 | 26872270 | 27318356 | 26157490 | 26589493 | 26568873 | 26697749 | 25896742 | 26335463 | 25090882 |
| 21 | 26466776 | 26458811 | 26190578 | 25723812 | 25525759 | 27150187 | 26310043 | 26743261 | 25556788 |
| 22 | 26177981 | 26472676 | 26157050 | 26706725 | 27214120 | 25633757 | 25971090 | 27022907 | 27023630 |
| 23 | 26399450 | 26080163 | 26731043 | 26763806 | 26236101 | 26309618 | 27293115 | 27106528 | 26589459 |
| 24 | 27107922 | 27215332 | 26995136 | 26481351 | 26945347 | 26063753 | 26075752 | 26543842 | 26887636 |
| 25 | 26266433 | 26440883 | 26200569 | 27016693 | 26834188 | 26957698 | 26107975 | 26737263 | 25484291 |
| 26 | 26780305 | 27197957 | 26787494 | 25964798 | 26470691 | 26830400 | 26681135 | 26872360 | 26273723 |
| 27 | 26029602 | 26452449 | 25936509 | 25985569 | 27004891 | 26200262 | 26322213 | 26346221 | 26795384 |
| 28 | 25539606 | 26646493 | 25883356 | 25440559 | 26495150 | 25885208 | 25742487 | 26992738 | 26259833 |
| 29 | 26488870 | 25652329 | 26253236 | 25561647 | 26410444 | 27068430 | 26930466 | 26799551 | 27035400 |
| 30 | 25907675 | 26865274 | 26813952 | 26398826 | 27632668 | 25981114 | 26867838 | 27471605 | 26187471 |
| 31 | 26622681 | 26220706 | 26127958 | 26092832 | 25634536 | 27317033 | 27548241 | 26783840 | 26319888 |
| 32 | 26898313 | 26476630 | 26450020 | 27180858 | 26635996 | 26886612 | 26307371 | 26592434 | 26449263 |
| 33 | 26424556 | 26842881 | 26142145 | 24935552 | 26568479 | 27167194 | 26639652 | 26796633 | 27089516 |
| 34 | 25870696 | 27060088 | 25714339 | 26111659 | 26647930 | 26403835 | 26821342 | 26980971 | 26637698 |
| 35 | 25735548 | 26473513 | 25794663 | 26864925 | 26341157 | 26998252 | 26962944 | 25350776 | 26625280 |
| 36 | 27326426 | 26203892 | 26130670 | 26760895 | 25858973 | 26969864 | 26075493 | 26427182 | 26376142 |
| 37 | 26808832 | 26739519 | 25962969 | 24978156 | 27785568 | 26059265 | 26164432 | 25586427 | 26353374 |
| 38 | 25789128 | 26421911 | 27049981 | 26399742 | 26908697 | 26815110 | 26399459 | 25871406 | 26566075 |
| 39 | 25522315 | 26507885 | 26639062 | 26234438 | 26792049 | 26220877 | 26497170 | 27523824 | 26253296 |
| 40 | 26740842 | 26765608 | 26444165 | 26741555 | 26228519 | 27146296 | 26727608 | 27555155 | 26877820 |
| 41 | 26096419 | 26981057 | 26074336 | 25819799 | 26548532 | 27247522 | 26860832 | 26902932 | 26545780 |
| 42 | 24686216 | 26369027 | 25921481 | 26161220 | 26672685 | 26125063 | 27422401 | 27747923 | 25134235 |
| 43 | 27090203 | 27199209 | 26208430 | 26018638 | 26434046 | 27023816 | 26454344 | 25253503 | 26254867 |
| 44 | 26669205 | 27114238 | 26472156 | 25964060 | 25072897 | 25840173 | 26288524 | 27289602 | 26470600 |
| 45 | 25302155 | 25741969 | 26860805 | 26561732 | 26475937 | 27387669 | 25693190 | 27479792 | 26822716 |
| 46 | 26080895 | 25854347 | 26587446 | 26610997 | 26228505 | 26315194 | 27058614 | 26837980 | 25997518 |
| 47 | 26477969 | 27102455 | 26266558 | 26164946 | 26843446 | 26715690 | 26800452 | 27065513 | 25722062 |
| 48 | 27601653 | 26573529 | 24923826 | 25697051 | 26949602 | 26479819 | 26688299 | 26890964 | 26095462 |
| 49 | 26554091 | 26436355 | 26935687 | 25168551 | 26216639 | 26804592 | 27698261 | 25174993 | 27320377 |
| 50 | 25644571 | 26434335 | 26256067 | 26034195 | 27080384 | 26512473 | 26500259 | 27274382 | 26577638 |

We processed the results in Matlab, with the Kruskal-Wallis test which tests the hypothesis that all samples have the same mean or not (meaning the sample data from each column come from the same distribution or not) .

[pval, kwtab, stats] = kruskalwallis([Alg1, Alg2, Alg3, Alg4, Alg5, Alg6, Alg7, Alg8, Alg9])



The Kruskal-Wallis test returns a stats structure which we further used to perform a follow-up multiple comparison test.

Multcompare(stats)

The p values per pairs of algorithms:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Alg A | Alg B |  |  |  | P Value |
| 1 | 2 | -158.8948 | -78.2200 | 2.4548 | 0.0658 |
| 1 | 3 | -71.6348 | 9.0400 | 89.7148 | 1.0000 |
| 1 | 4 | -69.6948 | 10.9800 | 91.6548 | 1.0000 |
| 1 | 5 | -131.8348 | -51.1600 | 29.5148 | 0.5666 |
| 1 | 6 | -139.0148 | -58.3400 | 22.3348 | 0.3776 |
| 1 | 7 | -126.5548 | -45.8800 | 34.7948 | 0.7061 |
| 1 | 8 | -156.5148 | -75.8400 | 4.8348 | 0.0849 |
| 1 | 9 | -81.5948 | -0.9200 | 79.7548 | 1.0000 |
| 2 | 3 | 6.5852 | 87.2600 | 167.9348 | 0.0226 |
| 2 | 4 | 8.5252 | 89.2000 | 169.8748 | 0.0176 |
| 2 | 5 | -53.6148 | 27.0600 | 107.7348 | 0.9820 |
| 2 | 6 | -60.7948 | 19.8800 | 100.5548 | 0.9978 |
| 2 | 7 | -48.3348 | 32.3400 | 113.0148 | 0.9469 |
| 2 | 8 | -78.2948 | 2.3800 | 83.0548 | 1.0000 |
| 2 | 9 | -3.3748 | 77.3000 | 157.9748 | 0.0727 |
| 3 | 4 | -78.7348 | 1.9400 | 82.6148 | 1.0000 |
| 3 | 5 | -140.8748 | -60.2000 | 20.4748 | 0.3331 |
| 3 | 6 | -148.0548 | -67.3800 | 13.2948 | 0.1904 |
| 3 | 7 | -135.5948 | -54.9200 | 25.7548 | 0.4652 |
| 3 | 8 | -165.5548 | -84.8800 | -4.2052 | 0.0304 |
| 3 | 9 | -90.6348 | -9.9600 | 70.7148 | 1.0000 |
| 4 | 5 | -142.8148 | -62.1400 | 18.5348 | 0.2898 |
| 4 | 6 | -149.9948 | -69.3200 | 11.3548 | 0.1604 |
| 4 | 7 | -137.5348 | -56.8600 | 23.8148 | 0.4147 |
| 4 | 8 | -167.4948 | -86.8200 | -6.1452 | 0.0239 |
| 4 | 9 | -92.5748 | -11.9000 | 68.7748 | 1.0000 |
| 5 | 6 | -87.8548 | -7.1800 | 73.4948 | 1.0000 |
| 5 | 7 | -75.3948 | 5.2800 | 85.9548 | 1.0000 |
| 5 | 8 | -105.3548 | -24.6800 | 55.9948 | 0.9901 |
| 5 | 9 | -30.4348 | 50.2400 | 130.9148 | 0.5916 |
| 6 | 7 | -68.2148 | 12.4600 | 93.1348 | 0.9999 |
| 6 | 8 | -98.1748 | -17.5000 | 63.1748 | 0.9991 |
| 6 | 9 | -23.2548 | 57.4200 | 138.0948 | 0.4005 |
| 7 | 8 | -110.6348 | -29.9600 | 50.7148 | 0.9661 |
| 7 | 9 | -35.7148 | 44.9600 | 125.6348 | 0.7289 |
| 8 | 9 | -5.7548 | 74.9200 | 155.5948 | 0.0934 |

P values < 0.05 mean that we have statistically significant differences with 95% confidence.



The results in the picture above indicate that there is a difference between algorithm 2 and 4 so the test rejects the hypothesis that the data in these two groups come from the same distribution. The same is true for algorithm 4 and 8. However, the test does not reject the hypothesis that algorithm 4 and the remaining six come from the same distribution. Therefore, these results suggest that the data from algorithms 1, 3, 4, 5, 6, 7, 9 come from the same distribution and the algorithms 2 and 8 come from the same distribution.

Based on the information gathered we have chosen to go ahead with algorithm 4 for this instance of the problem.

For the instance: **u\_s\_lohi\_512\_16.txt:**

We tuned the parameters crossover, mutation probability, and population size for three values each, which results in 27 possible algorithms. We ran each algorithm 50 times and collected the results. The results were further processed in Matlab, with the Kruskal-Wallis test.

The values tested were:

Alg1: pc=1.0, pm=0.001953125 (1/512), popsize=512

Alg2: pc=1.0, pm=0.001953125 (1/512), popsize=1024

Alg3: pc=1.0, pm=0.001953125 (1/512), popsize=2048

Alg4: pc=1.0, pm=0.009765625 (5/512), popsize=512

Alg5: pc=1.0, pm=0.009765625 (5/512), popsize=1024

Alg6: pc=1.0, pm=0.009765625 (5/512), popsize=2048

Alg7: pc=1.0, pm=0.01953125 (10/512), popsize=512

Alg8: pc=1.0, pm=0.01953125 (10/512), popsize=1024

Alg9: pc=1.0, pm=0.01953125 (10/512), popsize=2048

Alg10: pc=0.8, pm=0.001953125 (1/512), popsize=512

Alg11: pc=0.8, pm=0.001953125 (1/512), popsize=1024

Alg12: pc=0.8, pm=0.001953125 (1/512), popsize=2048

Alg13: pc=0.8, pm=0.009765625 (5/512), popsize=512

Alg14: pc=0.8, pm=0.009765625 (5/512), popsize=1024

Alg15: pc=0.8, pm=0.009765625 (5/512), popsize=2048

Alg16: pc=0.8, pm=0.01953125 (10/512), popsize=512

Alg17: pc=0.8, pm=0.01953125 (10/512), popsize=1024

Alg18: pc=0.8, pm=0.01953125 (10/512), popsize=2048

Alg19: pc=0.6, pm=0.001953125 (1/512), popsize=512

Alg20: pc=0.6, pm=0.001953125 (1/512), popsize=1024

Alg21: pc=0.6, pm=0.001953125 (1/512), popsize=2048

Alg22: pc=0.6, pm=0.009765625 (5/512), popsize=512

Alg23: pc=0.6, pm=0.009765625 (5/512), popsize=1024

Alg24: pc=0.6, pm=0.009765625 (5/512), popsize=2048

Alg25: pc=0.6, pm=0.01953125 (10/512), popsize=512

Alg26: pc=0.6, pm=0.01953125 (10/512), popsize=1024

Alg27: pc=0.6, pm=0.01953125 (10/512), popsize=2048



|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **B** | **lower** | **mean difference** | **upper** | **P Value** | **A** | **B** | **lower** | **mean difference** | **upper** | **P Value** |
| 1 | 2 | -509.69359 | -221.48000 | 66.73359 | 0.46062 | 8 | 23 | -449.81359 | -161.60000 | 126.61359 | 0.94995 |
| 1 | 3 | -799.61359 | -511.40000 | -223.18641 | 0.00000 | 8 | 24 | -912.92359 | -624.71000 | -336.49641 | 0.00000 |
| 1 | 4 | -83.33359 | 204.88000 | 493.09359 | 0.63437 | 8 | 25 | -115.82359 | 172.39000 | 460.60359 | 0.90279 |
| 1 | 5 | -380.93359 | -92.72000 | 195.49359 | 0.99999 | 8 | 26 | -517.08359 | -228.87000 | 59.34359 | 0.38697 |
| 1 | 6 | -970.20359 | -681.99000 | -393.77641 | 0.00000 | 8 | 27 | -961.20359 | -672.99000 | -384.77641 | 0.00000 |
| 1 | 7 | -110.65359 | 177.56000 | 465.77359 | 0.87225 | 9 | 10 | 282.00641 | 570.22000 | 858.43359 | 0.00000 |
| 1 | 8 | -508.86359 | -220.65000 | 67.56359 | 0.46916 | 9 | 11 | 12.76641 | 300.98000 | 589.19359 | 0.02821 |
| 1 | 9 | -1,087.95359 | -799.74000 | -511.52641 | 0.00000 | 9 | 12 | -226.73359 | 61.48000 | 349.69359 | 1.00000 |
| 1 | 10 | -517.73359 | -229.52000 | 58.69359 | 0.38073 | 9 | 13 | 642.98641 | 931.20000 | 1,219.41359 | 0.00000 |
| 1 | 11 | -786.97359 | -498.76000 | -210.54641 | 0.00000 | 9 | 14 | 221.06641 | 509.28000 | 797.49359 | 0.00000 |
| 1 | 12 | -1,026.47359 | -738.26000 | -450.04641 | 0.00000 | 9 | 15 | -228.58359 | 59.63000 | 347.84359 | 1.00000 |
| 1 | 13 | -156.75359 | 131.46000 | 419.67359 | 0.99662 | 9 | 16 | 567.46641 | 855.68000 | 1,143.89359 | 0.00000 |
| 1 | 14 | -578.67359 | -290.46000 | -2.24641 | 0.04535 | 9 | 17 | 166.35641 | 454.57000 | 742.78359 | 0.00000 |
| 1 | 15 | -1,028.32359 | -740.11000 | -451.89641 | 0.00000 | 9 | 18 | -357.93359 | -69.72000 | 218.49359 | 1.00000 |
| 1 | 16 | -232.27359 | 55.94000 | 344.15359 | 1.00000 | 9 | 19 | 143.10641 | 431.32000 | 719.53359 | 0.00001 |
| 1 | 17 | -633.38359 | -345.17000 | -56.95641 | 0.00288 | 9 | 20 | -177.22359 | 110.99000 | 399.20359 | 0.99979 |
| 1 | 18 | -1,157.67359 | -869.46000 | -581.24641 | 0.00000 | 9 | 21 | -372.75359 | -84.54000 | 203.67359 | 1.00000 |
| 1 | 19 | -656.63359 | -368.42000 | -80.20641 | 0.00073 | 9 | 22 | 515.36641 | 803.58000 | 1,091.79359 | 0.00000 |
| 1 | 20 | -976.96359 | -688.75000 | -400.53641 | 0.00000 | 9 | 23 | 129.27641 | 417.49000 | 705.70359 | 0.00003 |
| 1 | 21 | -1,172.49359 | -884.28000 | -596.06641 | 0.00000 | 9 | 24 | -333.83359 | -45.62000 | 242.59359 | 1.00000 |
| 1 | 22 | -284.37359 | 3.84000 | 292.05359 | 1.00000 | 9 | 25 | 463.26641 | 751.48000 | 1,039.69359 | 0.00000 |
| 1 | 23 | -670.46359 | -382.25000 | -94.03641 | 0.00031 | 9 | 26 | 62.00641 | 350.22000 | 638.43359 | 0.00216 |
| 1 | 24 | -1,133.57359 | -845.36000 | -557.14641 | 0.00000 | 9 | 27 | -382.11359 | -93.90000 | 194.31359 | 0.99999 |
| 1 | 25 | -336.47359 | -48.26000 | 239.95359 | 1.00000 | 10 | 11 | -557.45359 | -269.24000 | 18.97359 | 0.10802 |
| 1 | 26 | -737.73359 | -449.52000 | -161.30641 | 0.00000 | 10 | 12 | -796.95359 | -508.74000 | -220.52641 | 0.00000 |
| 1 | 27 | -1,181.85359 | -893.64000 | -605.42641 | 0.00000 | 10 | 13 | 72.76641 | 360.98000 | 649.19359 | 0.00115 |
| 2 | 3 | -578.13359 | -289.92000 | -1.70641 | 0.04643 | 10 | 14 | -349.15359 | -60.94000 | 227.27359 | 1.00000 |
| 2 | 4 | 138.14641 | 426.36000 | 714.57359 | 0.00002 | 10 | 15 | -798.80359 | -510.59000 | -222.37641 | 0.00000 |
| 2 | 5 | -159.45359 | 128.76000 | 416.97359 | 0.99754 | 10 | 16 | -2.75359 | 285.46000 | 573.67359 | 0.05626 |
| 2 | 6 | -748.72359 | -460.51000 | -172.29641 | 0.00000 | 10 | 17 | -403.86359 | -115.65000 | 172.56359 | 0.99957 |
| 2 | 7 | 110.82641 | 399.04000 | 687.25359 | 0.00010 | 10 | 18 | -928.15359 | -639.94000 | -351.72641 | 0.00000 |
| 2 | 8 | -287.38359 | 0.83000 | 289.04359 | 1.00000 | 10 | 19 | -427.11359 | -138.90000 | 149.31359 | 0.99248 |
| 2 | 9 | -866.47359 | -578.26000 | -290.04641 | 0.00000 | 10 | 20 | -747.44359 | -459.23000 | -171.01641 | 0.00000 |
| 2 | 10 | -296.25359 | -8.04000 | 280.17359 | 1.00000 | 10 | 21 | -942.97359 | -654.76000 | -366.54641 | 0.00000 |
| 2 | 11 | -565.49359 | -277.28000 | 10.93359 | 0.07890 | 10 | 22 | -54.85359 | 233.36000 | 521.57359 | 0.34485 |
| 2 | 12 | -804.99359 | -516.78000 | -228.56641 | 0.00000 | 10 | 23 | -440.94359 | -152.73000 | 135.48359 | 0.97394 |
| 2 | 13 | 64.72641 | 352.94000 | 641.15359 | 0.00185 | 10 | 24 | -904.05359 | -615.84000 | -327.62641 | 0.00000 |
| 2 | 14 | -357.19359 | -68.98000 | 219.23359 | 1.00000 | 10 | 25 | -106.95359 | 181.26000 | 469.47359 | 0.84716 |
| 2 | 15 | -806.84359 | -518.63000 | -230.41641 | 0.00000 | 10 | 26 | -508.21359 | -220.00000 | 68.21359 | 0.47587 |
| 2 | 16 | -10.79359 | 277.42000 | 565.63359 | 0.07845 | 10 | 27 | -952.33359 | -664.12000 | -375.90641 | 0.00000 |
| 2 | 17 | -411.90359 | -123.69000 | 164.52359 | 0.99869 | 11 | 12 | -527.71359 | -239.50000 | 48.71359 | 0.29126 |
| 2 | 18 | -936.19359 | -647.98000 | -359.76641 | 0.00000 | 11 | 13 | 342.00641 | 630.22000 | 918.43359 | 0.00000 |
| 2 | 19 | -435.15359 | -146.94000 | 141.27359 | 0.98395 | 11 | 14 | -79.91359 | 208.30000 | 496.51359 | 0.59872 |
| 2 | 20 | -755.48359 | -467.27000 | -179.05641 | 0.00000 | 11 | 15 | -529.56359 | -241.35000 | 46.86359 | 0.27612 |
| 2 | 21 | -951.01359 | -662.80000 | -374.58641 | 0.00000 | 11 | 16 | 266.48641 | 554.70000 | 842.91359 | 0.00000 |
| 2 | 22 | -62.89359 | 225.32000 | 513.53359 | 0.42177 | 11 | 17 | -134.62359 | 153.59000 | 441.80359 | 0.97211 |
| 2 | 23 | -448.98359 | -160.77000 | 127.44359 | 0.95271 | 11 | 18 | -658.91359 | -370.70000 | -82.48641 | 0.00064 |
| 2 | 24 | -912.09359 | -623.88000 | -335.66641 | 0.00000 | 11 | 19 | -157.87359 | 130.34000 | 418.55359 | 0.99703 |
| 2 | 25 | -114.99359 | 173.22000 | 461.43359 | 0.89824 | 11 | 20 | -478.20359 | -189.99000 | 98.22359 | 0.77789 |
| 2 | 26 | -516.25359 | -228.04000 | 60.17359 | 0.39500 | 11 | 21 | -673.73359 | -385.52000 | -97.30641 | 0.00025 |
| 2 | 27 | -960.37359 | -672.16000 | -383.94641 | 0.00000 | 11 | 22 | 214.38641 | 502.60000 | 790.81359 | 0.00000 |
| 3 | 4 | 428.06641 | 716.28000 | 1,004.49359 | 0.00000 | 11 | 23 | -171.70359 | 116.51000 | 404.72359 | 0.99951 |
| 3 | 5 | 130.46641 | 418.68000 | 706.89359 | 0.00003 | 11 | 24 | -634.81359 | -346.60000 | -58.38641 | 0.00266 |
| 3 | 6 | -458.80359 | -170.59000 | 117.62359 | 0.91218 | 11 | 25 | 162.28641 | 450.50000 | 738.71359 | 0.00000 |
| 3 | 7 | 400.74641 | 688.96000 | 977.17359 | 0.00000 | 11 | 26 | -238.97359 | 49.24000 | 337.45359 | 1.00000 |
| 3 | 8 | 2.53641 | 290.75000 | 578.96359 | 0.04477 | 11 | 27 | -683.09359 | -394.88000 | -106.66641 | 0.00014 |
| 3 | 9 | -576.55359 | -288.34000 | -0.12641 | 0.04973 | 12 | 13 | 581.50641 | 869.72000 | 1,157.93359 | 0.00000 |
| 3 | 10 | -6.33359 | 281.88000 | 570.09359 | 0.06538 | 12 | 14 | 159.58641 | 447.80000 | 736.01359 | 0.00000 |
| 3 | 11 | -275.57359 | 12.64000 | 300.85359 | 1.00000 | 12 | 15 | -290.06359 | -1.85000 | 286.36359 | 1.00000 |
| 3 | 12 | -515.07359 | -226.86000 | 61.35359 | 0.40653 | 12 | 16 | 505.98641 | 794.20000 | 1,082.41359 | 0.00000 |
| 3 | 13 | 354.64641 | 642.86000 | 931.07359 | 0.00000 | 12 | 17 | 104.87641 | 393.09000 | 681.30359 | 0.00015 |
| 3 | 14 | -67.27359 | 220.94000 | 509.15359 | 0.46617 | 12 | 18 | -419.41359 | -131.20000 | 157.01359 | 0.99672 |
| 3 | 15 | -516.92359 | -228.71000 | 59.50359 | 0.38851 | 12 | 19 | 81.62641 | 369.84000 | 658.05359 | 0.00067 |
| 3 | 16 | 279.12641 | 567.34000 | 855.55359 | 0.00000 | 12 | 20 | -238.70359 | 49.51000 | 337.72359 | 1.00000 |
| 3 | 17 | -121.98359 | 166.23000 | 454.44359 | 0.93236 | 12 | 21 | -434.23359 | -146.02000 | 142.19359 | 0.98521 |
| 3 | 18 | -646.27359 | -358.06000 | -69.84641 | 0.00137 | 12 | 22 | 453.88641 | 742.10000 | 1,030.31359 | 0.00000 |
| 3 | 19 | -145.23359 | 142.98000 | 431.19359 | 0.98881 | 12 | 23 | 67.79641 | 356.01000 | 644.22359 | 0.00154 |
| 3 | 20 | -465.56359 | -177.35000 | 110.86359 | 0.87359 | 12 | 24 | -395.31359 | -107.10000 | 181.11359 | 0.99989 |
| 3 | 21 | -661.09359 | -372.88000 | -84.66641 | 0.00056 | 12 | 25 | 401.78641 | 690.00000 | 978.21359 | 0.00000 |
| 3 | 22 | 227.02641 | 515.24000 | 803.45359 | 0.00000 | 12 | 26 | 0.52641 | 288.74000 | 576.95359 | 0.04887 |
| 3 | 23 | -159.06359 | 129.15000 | 417.36359 | 0.99742 | 12 | 27 | -443.59359 | -155.38000 | 132.83359 | 0.96798 |
| 3 | 24 | -622.17359 | -333.96000 | -45.74641 | 0.00536 | 13 | 14 | -710.13359 | -421.92000 | -133.70641 | 0.00002 |
| 3 | 25 | 174.92641 | 463.14000 | 751.35359 | 0.00000 | 13 | 15 | -1,159.78359 | -871.57000 | -583.35641 | 0.00000 |
| 3 | 26 | -226.33359 | 61.88000 | 350.09359 | 1.00000 | 13 | 16 | -363.73359 | -75.52000 | 212.69359 | 1.00000 |
| 3 | 27 | -670.45359 | -382.24000 | -94.02641 | 0.00031 | 13 | 17 | -764.84359 | -476.63000 | -188.41641 | 0.00000 |
| 4 | 5 | -585.81359 | -297.60000 | -9.38641 | 0.03296 | 13 | 18 | -1,289.13359 | -1,000.92000 | -712.70641 | 0.00000 |
| 4 | 6 | -1,175.08359 | -886.87000 | -598.65641 | 0.00000 | 13 | 19 | -788.09359 | -499.88000 | -211.66641 | 0.00000 |
| 4 | 7 | -315.53359 | -27.32000 | 260.89359 | 1.00000 | 13 | 20 | -1,108.42359 | -820.21000 | -531.99641 | 0.00000 |
| 4 | 8 | -713.74359 | -425.53000 | -137.31641 | 0.00002 | 13 | 21 | -1,303.95359 | -1,015.74000 | -727.52641 | 0.00000 |
| 4 | 9 | -1,292.83359 | -1,004.62000 | -716.40641 | 0.00000 | 13 | 22 | -415.83359 | -127.62000 | 160.59359 | 0.99785 |
| 4 | 10 | -722.61359 | -434.40000 | -146.18641 | 0.00001 | 13 | 23 | -801.92359 | -513.71000 | -225.49641 | 0.00000 |
| 4 | 11 | -991.85359 | -703.64000 | -415.42641 | 0.00000 | 13 | 24 | -1,265.03359 | -976.82000 | -688.60641 | 0.00000 |
| 4 | 12 | -1,231.35359 | -943.14000 | -654.92641 | 0.00000 | 13 | 25 | -467.93359 | -179.72000 | 108.49359 | 0.85793 |
| 4 | 13 | -361.63359 | -73.42000 | 214.79359 | 1.00000 | 13 | 26 | -869.19359 | -580.98000 | -292.76641 | 0.00000 |
| 4 | 14 | -783.55359 | -495.34000 | -207.12641 | 0.00000 | 13 | 27 | -1,313.31359 | -1,025.10000 | -736.88641 | 0.00000 |
| 4 | 15 | -1,233.20359 | -944.99000 | -656.77641 | 0.00000 | 14 | 15 | -737.86359 | -449.65000 | -161.43641 | 0.00000 |
| 4 | 16 | -437.15359 | -148.94000 | 139.27359 | 0.98092 | 14 | 16 | 58.18641 | 346.40000 | 634.61359 | 0.00269 |
| 4 | 17 | -838.26359 | -550.05000 | -261.83641 | 0.00000 | 14 | 17 | -342.92359 | -54.71000 | 233.50359 | 1.00000 |
| 4 | 18 | -1,362.55359 | -1,074.34000 | -786.12641 | 0.00000 | 14 | 18 | -867.21359 | -579.00000 | -290.78641 | 0.00000 |
| 4 | 19 | -861.51359 | -573.30000 | -285.08641 | 0.00000 | 14 | 19 | -366.17359 | -77.96000 | 210.25359 | 1.00000 |
| 4 | 20 | -1,181.84359 | -893.63000 | -605.41641 | 0.00000 | 14 | 20 | -686.50359 | -398.29000 | -110.07641 | 0.00011 |
| 4 | 21 | -1,377.37359 | -1,089.16000 | -800.94641 | 0.00000 | 14 | 21 | -882.03359 | -593.82000 | -305.60641 | 0.00000 |
| 4 | 22 | -489.25359 | -201.04000 | 87.17359 | 0.67358 | 14 | 22 | 6.08641 | 294.30000 | 582.51359 | 0.03826 |
| 4 | 23 | -875.34359 | -587.13000 | -298.91641 | 0.00000 | 14 | 23 | -380.00359 | -91.79000 | 196.42359 | 0.99999 |
| 4 | 24 | -1,338.45359 | -1,050.24000 | -762.02641 | 0.00000 | 14 | 24 | -843.11359 | -554.90000 | -266.68641 | 0.00000 |
| 4 | 25 | -541.35359 | -253.14000 | 35.07359 | 0.19133 | 14 | 25 | -46.01359 | 242.20000 | 530.41359 | 0.26932 |
| 4 | 26 | -942.61359 | -654.40000 | -366.18641 | 0.00000 | 14 | 26 | -447.27359 | -159.06000 | 129.15359 | 0.95804 |
| 4 | 27 | -1,386.73359 | -1,098.52000 | -810.30641 | 0.00000 | 14 | 27 | -891.39359 | -603.18000 | -314.96641 | 0.00000 |
| 5 | 6 | -877.48359 | -589.27000 | -301.05641 | 0.00000 | 15 | 16 | 507.83641 | 796.05000 | 1,084.26359 | 0.00000 |
| 5 | 7 | -17.93359 | 270.28000 | 558.49359 | 0.10383 | 15 | 17 | 106.72641 | 394.94000 | 683.15359 | 0.00014 |
| 5 | 8 | -416.14359 | -127.93000 | 160.28359 | 0.99777 | 15 | 18 | -417.56359 | -129.35000 | 158.86359 | 0.99736 |
| 5 | 9 | -995.23359 | -707.02000 | -418.80641 | 0.00000 | 15 | 19 | 83.47641 | 371.69000 | 659.90359 | 0.00060 |
| 5 | 10 | -425.01359 | -136.80000 | 151.41359 | 0.99394 | 15 | 20 | -236.85359 | 51.36000 | 339.57359 | 1.00000 |
| 5 | 11 | -694.25359 | -406.04000 | -117.82641 | 0.00007 | 15 | 21 | -432.38359 | -144.17000 | 144.04359 | 0.98749 |
| 5 | 12 | -933.75359 | -645.54000 | -357.32641 | 0.00000 | 15 | 22 | 455.73641 | 743.95000 | 1,032.16359 | 0.00000 |
| 5 | 13 | -64.03359 | 224.18000 | 512.39359 | 0.43319 | 15 | 23 | 69.64641 | 357.86000 | 646.07359 | 0.00138 |
| 5 | 14 | -485.95359 | -197.74000 | 90.47359 | 0.70627 | 15 | 24 | -393.46359 | -105.25000 | 182.96359 | 0.99992 |
| 5 | 15 | -935.60359 | -647.39000 | -359.17641 | 0.00000 | 15 | 25 | 403.63641 | 691.85000 | 980.06359 | 0.00000 |
| 5 | 16 | -139.55359 | 148.66000 | 436.87359 | 0.98137 | 15 | 26 | 2.37641 | 290.59000 | 578.80359 | 0.04509 |
| 5 | 17 | -540.66359 | -252.45000 | 35.76359 | 0.19572 | 15 | 27 | -441.74359 | -153.53000 | 134.68359 | 0.97224 |
| 5 | 18 | -1,064.95359 | -776.74000 | -488.52641 | 0.00000 | 16 | 17 | -689.32359 | -401.11000 | -112.89641 | 0.00009 |
| 5 | 19 | -563.91359 | -275.70000 | 12.51359 | 0.08404 | 16 | 18 | -1,213.61359 | -925.40000 | -637.18641 | 0.00000 |
| 5 | 20 | -884.24359 | -596.03000 | -307.81641 | 0.00000 | 16 | 19 | -712.57359 | -424.36000 | -136.14641 | 0.00002 |
| 5 | 21 | -1,079.77359 | -791.56000 | -503.34641 | 0.00000 | 16 | 20 | -1,032.90359 | -744.69000 | -456.47641 | 0.00000 |
| 5 | 22 | -191.65359 | 96.56000 | 384.77359 | 0.99998 | 16 | 21 | -1,228.43359 | -940.22000 | -652.00641 | 0.00000 |
| 5 | 23 | -577.74359 | -289.53000 | -1.31641 | 0.04723 | 16 | 22 | -340.31359 | -52.10000 | 236.11359 | 1.00000 |
| 5 | 24 | -1,040.85359 | -752.64000 | -464.42641 | 0.00000 | 16 | 23 | -726.40359 | -438.19000 | -149.97641 | 0.00001 |
| 5 | 25 | -243.75359 | 44.46000 | 332.67359 | 1.00000 | 16 | 24 | -1,189.51359 | -901.30000 | -613.08641 | 0.00000 |
| 5 | 26 | -645.01359 | -356.80000 | -68.58641 | 0.00147 | 16 | 25 | -392.41359 | -104.20000 | 184.01359 | 0.99993 |
| 5 | 27 | -1,089.13359 | -800.92000 | -512.70641 | 0.00000 | 16 | 26 | -793.67359 | -505.46000 | -217.24641 | 0.00000 |
| 6 | 7 | 571.33641 | 859.55000 | 1,147.76359 | 0.00000 | 16 | 27 | -1,237.79359 | -949.58000 | -661.36641 | 0.00000 |
| 6 | 8 | 173.12641 | 461.34000 | 749.55359 | 0.00000 | 17 | 18 | -812.50359 | -524.29000 | -236.07641 | 0.00000 |
| 6 | 9 | -405.96359 | -117.75000 | 170.46359 | 0.99942 | 17 | 19 | -311.46359 | -23.25000 | 264.96359 | 1.00000 |
| 6 | 10 | 164.25641 | 452.47000 | 740.68359 | 0.00000 | 17 | 20 | -631.79359 | -343.58000 | -55.36641 | 0.00315 |
| 6 | 11 | -104.98359 | 183.23000 | 471.44359 | 0.83273 | 17 | 21 | -827.32359 | -539.11000 | -250.89641 | 0.00000 |
| 6 | 12 | -344.48359 | -56.27000 | 231.94359 | 1.00000 | 17 | 22 | 60.79641 | 349.01000 | 637.22359 | 0.00232 |
| 6 | 13 | 525.23641 | 813.45000 | 1,101.66359 | 0.00000 | 17 | 23 | -325.29359 | -37.08000 | 251.13359 | 1.00000 |
| 6 | 14 | 103.31641 | 391.53000 | 679.74359 | 0.00017 | 17 | 24 | -788.40359 | -500.19000 | -211.97641 | 0.00000 |
| 6 | 15 | -346.33359 | -58.12000 | 230.09359 | 1.00000 | 17 | 25 | 8.69641 | 296.91000 | 585.12359 | 0.03401 |
| 6 | 16 | 449.71641 | 737.93000 | 1,026.14359 | 0.00000 | 17 | 26 | -392.56359 | -104.35000 | 183.86359 | 0.99993 |
| 6 | 17 | 48.60641 | 336.82000 | 625.03359 | 0.00459 | 17 | 27 | -836.68359 | -548.47000 | -260.25641 | 0.00000 |
| 6 | 18 | -475.68359 | -187.47000 | 100.74359 | 0.79926 | 18 | 19 | 212.82641 | 501.04000 | 789.25359 | 0.00000 |
| 6 | 19 | 25.35641 | 313.57000 | 601.78359 | 0.01541 | 18 | 20 | -107.50359 | 180.71000 | 468.92359 | 0.85106 |
| 6 | 20 | -294.97359 | -6.76000 | 281.45359 | 1.00000 | 18 | 21 | -303.03359 | -14.82000 | 273.39359 | 1.00000 |
| 6 | 21 | -490.50359 | -202.29000 | 85.92359 | 0.66094 | 18 | 22 | 585.08641 | 873.30000 | 1,161.51359 | 0.00000 |
| 6 | 22 | 397.61641 | 685.83000 | 974.04359 | 0.00000 | 18 | 23 | 198.99641 | 487.21000 | 775.42359 | 0.00000 |
| 6 | 23 | 11.52641 | 299.74000 | 587.95359 | 0.02987 | 18 | 24 | -264.11359 | 24.10000 | 312.31359 | 1.00000 |
| 6 | 24 | -451.58359 | -163.37000 | 124.84359 | 0.94367 | 18 | 25 | 532.98641 | 821.20000 | 1,109.41359 | 0.00000 |
| 6 | 25 | 345.51641 | 633.73000 | 921.94359 | 0.00000 | 18 | 26 | 131.72641 | 419.94000 | 708.15359 | 0.00003 |
| 6 | 26 | -55.74359 | 232.47000 | 520.68359 | 0.35301 | 18 | 27 | -312.39359 | -24.18000 | 264.03359 | 1.00000 |
| 6 | 27 | -499.86359 | -211.65000 | 76.56359 | 0.56345 | 19 | 20 | -608.54359 | -320.33000 | -32.11641 | 0.01097 |
| 7 | 8 | -686.42359 | -398.21000 | -109.99641 | 0.00011 | 19 | 21 | -804.07359 | -515.86000 | -227.64641 | 0.00000 |
| 7 | 9 | -1,265.51359 | -977.30000 | -689.08641 | 0.00000 | 19 | 22 | 84.04641 | 372.26000 | 660.47359 | 0.00058 |
| 7 | 10 | -695.29359 | -407.08000 | -118.86641 | 0.00006 | 19 | 23 | -302.04359 | -13.83000 | 274.38359 | 1.00000 |
| 7 | 11 | -964.53359 | -676.32000 | -388.10641 | 0.00000 | 19 | 24 | -765.15359 | -476.94000 | -188.72641 | 0.00000 |
| 7 | 12 | -1,204.03359 | -915.82000 | -627.60641 | 0.00000 | 19 | 25 | 31.94641 | 320.16000 | 608.37359 | 0.01107 |
| 7 | 13 | -334.31359 | -46.10000 | 242.11359 | 1.00000 | 19 | 26 | -369.31359 | -81.10000 | 207.11359 | 1.00000 |
| 7 | 14 | -756.23359 | -468.02000 | -179.80641 | 0.00000 | 19 | 27 | -813.43359 | -525.22000 | -237.00641 | 0.00000 |
| 7 | 15 | -1,205.88359 | -917.67000 | -629.45641 | 0.00000 | 20 | 21 | -483.74359 | -195.53000 | 92.68359 | 0.72751 |
| 7 | 16 | -409.83359 | -121.62000 | 166.59359 | 0.99900 | 20 | 22 | 404.37641 | 692.59000 | 980.80359 | 0.00000 |
| 7 | 17 | -810.94359 | -522.73000 | -234.51641 | 0.00000 | 20 | 23 | 18.28641 | 306.50000 | 594.71359 | 0.02174 |
| 7 | 18 | -1,335.23359 | -1,047.02000 | -758.80641 | 0.00000 | 20 | 24 | -444.82359 | -156.61000 | 131.60359 | 0.96488 |
| 7 | 19 | -834.19359 | -545.98000 | -257.76641 | 0.00000 | 20 | 25 | 352.27641 | 640.49000 | 928.70359 | 0.00000 |
| 7 | 20 | -1,154.52359 | -866.31000 | -578.09641 | 0.00000 | 20 | 26 | -48.98359 | 239.23000 | 527.44359 | 0.29351 |
| 7 | 21 | -1,350.05359 | -1,061.84000 | -773.62641 | 0.00000 | 20 | 27 | -493.10359 | -204.89000 | 83.32359 | 0.63427 |
| 7 | 22 | -461.93359 | -173.72000 | 114.49359 | 0.89544 | 21 | 22 | 599.90641 | 888.12000 | 1,176.33359 | 0.00000 |
| 7 | 23 | -848.02359 | -559.81000 | -271.59641 | 0.00000 | 21 | 23 | 213.81641 | 502.03000 | 790.24359 | 0.00000 |
| 7 | 24 | -1,311.13359 | -1,022.92000 | -734.70641 | 0.00000 | 21 | 24 | -249.29359 | 38.92000 | 327.13359 | 1.00000 |
| 7 | 25 | -514.03359 | -225.82000 | 62.39359 | 0.41680 | 21 | 25 | 547.80641 | 836.02000 | 1,124.23359 | 0.00000 |
| 7 | 26 | -915.29359 | -627.08000 | -338.86641 | 0.00000 | 21 | 26 | 146.54641 | 434.76000 | 722.97359 | 0.00001 |
| 7 | 27 | -1,359.41359 | -1,071.20000 | -782.98641 | 0.00000 | 21 | 27 | -297.57359 | -9.36000 | 278.85359 | 1.00000 |
| 8 | 9 | -867.30359 | -579.09000 | -290.87641 | 0.00000 | 22 | 23 | -674.30359 | -386.09000 | -97.87641 | 0.00024 |
| 8 | 10 | -297.08359 | -8.87000 | 279.34359 | 1.00000 | 22 | 24 | -1,137.41359 | -849.20000 | -560.98641 | 0.00000 |
| 8 | 11 | -566.32359 | -278.11000 | 10.10359 | 0.07630 | 22 | 25 | -340.31359 | -52.10000 | 236.11359 | 1.00000 |
| 8 | 12 | -805.82359 | -517.61000 | -229.39641 | 0.00000 | 22 | 26 | -741.57359 | -453.36000 | -165.14641 | 0.00000 |
| 8 | 13 | 63.89641 | 352.11000 | 640.32359 | 0.00194 | 22 | 27 | -1,185.69359 | -897.48000 | -609.26641 | 0.00000 |
| 8 | 14 | -358.02359 | -69.81000 | 218.40359 | 1.00000 | 23 | 24 | -751.32359 | -463.11000 | -174.89641 | 0.00000 |
| 8 | 15 | -807.67359 | -519.46000 | -231.24641 | 0.00000 | 23 | 25 | 45.77641 | 333.99000 | 622.20359 | 0.00535 |
| 8 | 16 | -11.62359 | 276.59000 | 564.80359 | 0.08111 | 23 | 26 | -355.48359 | -67.27000 | 220.94359 | 1.00000 |
| 8 | 17 | -412.73359 | -124.52000 | 163.69359 | 0.99854 | 23 | 27 | -799.60359 | -511.39000 | -223.17641 | 0.00000 |
| 8 | 18 | -937.02359 | -648.81000 | -360.59641 | 0.00000 | 24 | 25 | 508.88641 | 797.10000 | 1,085.31359 | 0.00000 |
| 8 | 19 | -435.98359 | -147.77000 | 140.44359 | 0.98274 | 24 | 26 | 107.62641 | 395.84000 | 684.05359 | 0.00013 |
| 8 | 20 | -756.31359 | -468.10000 | -179.88641 | 0.00000 | 24 | 27 | -336.49359 | -48.28000 | 239.93359 | 1.00000 |
| 8 | 21 | -951.84359 | -663.63000 | -375.41641 | 0.00000 | 25 | 26 | -689.47359 | -401.26000 | -113.04641 | 0.00009 |
| 8 | 22 | -63.72359 | 224.49000 | 512.70359 | 0.43007 | 25 | 27 | -1,133.59359 | -845.38000 | -557.16641 | 0.00000 |
|  |  | means |  |  |  | 26 | 27 | -732.33359 | -444.12000 | -155.90641 | 0.00001 |

We have chosen algorithm 4 because it provides the lowest results and is statistically different from those algorithm that provide higher values.

For obtaining the lower bound though, we have used a larger population size (1024/2048) and a high number of steps - ran the code for a long time in order to better explore the solution space.

* Show average performance (from 50 independent runs) of your best algorithms on **all**

**instances** (If you have a lot of variations, state the parameter you use) e.g. Genetic Algorithm (Population: x, Crossover: x.xx%, Mutation: x.xx%, Local Search: Gradient, etc.)

Average performance for the algorithms was run on the following setup:

MakeSpans for 50 iterations, MAX\_ISTEPS 100000, pc=1.0, pm=0.009765625 (5/512) , popsize=512.

The results are as follows:

|  |  |
| --- | --- |
| Instance | Average |
| u\_c\_hihi\_512\_16 | 15,966,430 |
| u\_c\_hilo\_512\_16 | 204,159 |
| u\_c\_lohi\_512\_16 | 529,462 |
| u\_c\_lolo\_512\_16 | 6,908 |
| u\_i\_hihi\_512\_16 | 16,662,829 |
| u\_i\_hilo\_512\_16 | 210,435 |
| u\_i\_lohi\_512\_16 | 540,188 |
| u\_i\_lolo\_512\_16 | 7,043 |
| u\_s\_hihi\_512\_16 | 15,757,344 |
| u\_s\_hilo\_512\_16 | 203,290 |
| u\_s\_lohi\_512\_16 | 481,462 |
| u\_s\_lolo\_512\_16 | 7,374 |

The table with data from all the 50 runs is in the annex \Project\_GA\Data\Average\_for\_50\_runs\_all\_instances\average\_for\_50\_runs\_all\_instances\_Excel.xlsx

• Show the lower bound of your algorithm on **all instances** and present it in a similar

fashion to the file “LowerBound\_BestValue\_Instances”

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Instance** | **LB** | **Our Best** | **%** | **mean %** | **Comments: 1000000000 steps, pc/pm/popsize** |
| u\_c\_hihi\_512\_16.txt | 7346524 | 8478411 | 15.41% | 11.61% | pc=1.0, pm=5/512, popsize=512 |
| u\_c\_hilo\_512\_16.txt | 152700 | 167048 | 9.40% | pc=1.0, pm=5/512, popsize=2048 |
| u\_c\_lohi\_512\_16.txt | 238138 | 270916 | 13.76% | pc=1.0, pm=5/512, popsize=2048 |
| u\_c\_lolo\_512\_16.txt | 5132 | 5536 | 7.87% | pc=1.0, pm=5/512, popsize=1024 |
| u\_i\_hihi\_512\_16.txt | 2909326 | 3505029 | 20.48% | 24.07% | pc=1.0, pm=5/512, popsize=2048 |
| u\_i\_hilo\_512\_16.txt | 73057 | 90416 | 23.76% | pc=1.0, pm=5/512, popsize=2048 |
| u\_i\_lohi\_512\_16.txt | 101063 | 135997 | 34.57% | pc=1.0, pm=5/512, popsize=1024 |
| u\_i\_lolo\_512\_16.txt | 2529 | 2971 | 17.48% | pc=1.0, pm=5/512, popsize=1024 |
| u\_s\_hihi\_512\_16.txt | 4063563 | 5009057 | 23.27% | 20.11% | pc=1.0, pm=5/512, popsize=1024 |
| u\_s\_hilo\_512\_16.txt | 95419 | 110505 | 15.81% | pc=1.0, pm=5/512, popsize=1024 |
| u\_s\_lohi\_512\_16.txt | 120452 | 151330 | 25.64% | pc=1.0, pm=5/512, popsize=2048 |
| u\_s\_lolo\_512\_16.txt | 3414 | 3951 | 15.73% | pc=1.0, pm=5/512, popsize=2048 |

Future Work

• Explain the future work if you were to have more time to improve the work and justify

your answer

/////////////////todo //////////////////