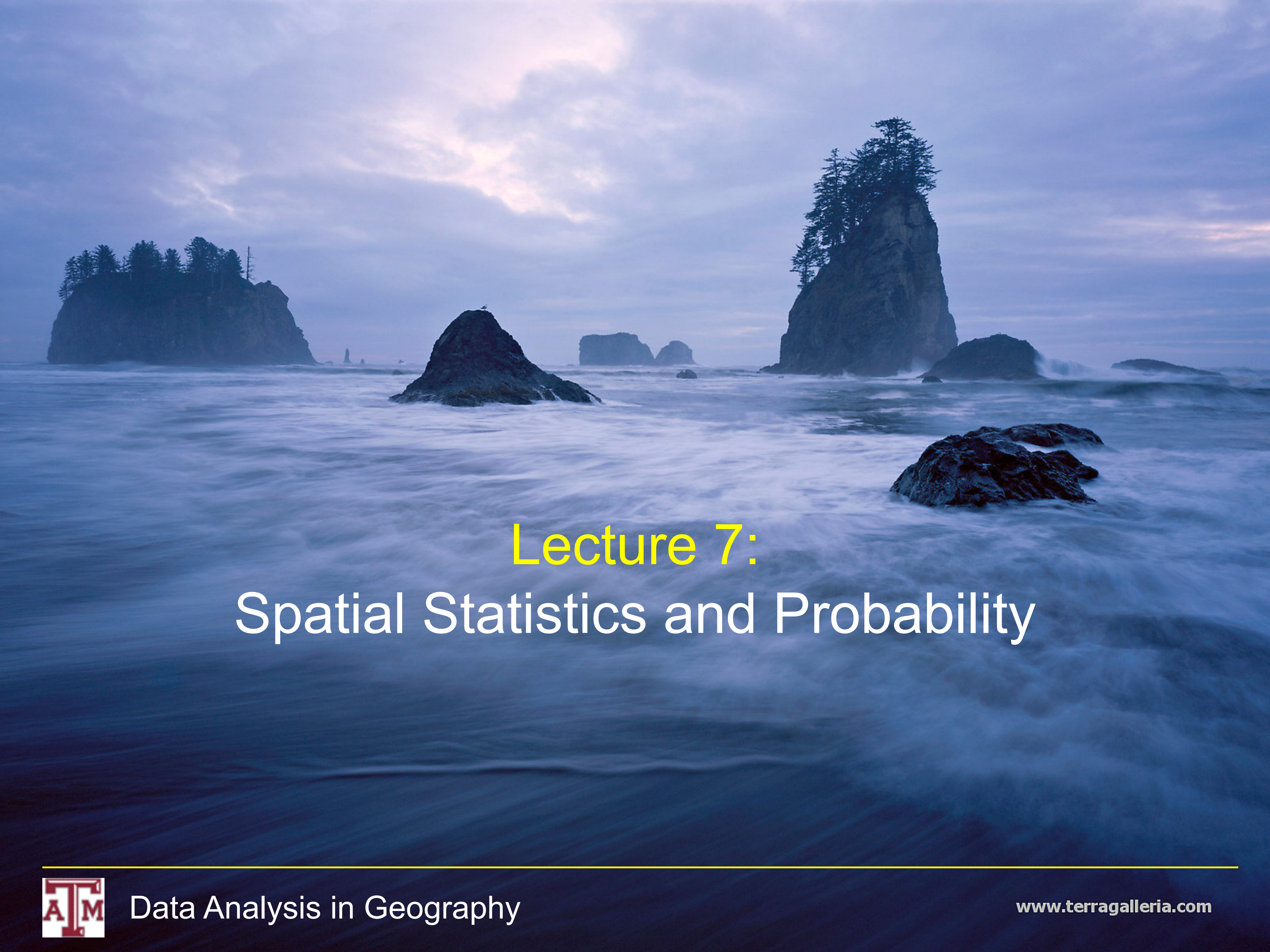


Lecture 7:

Spatial Statistics and Probability

www.terragalleria.com

Announcements

1. Please return the radiant thermometers to me now

Reminders:

1. HW#1 was due today via eCampus
2. HW#2 is due in one week (Feb. 14, 2019)

Penalties for Late Work

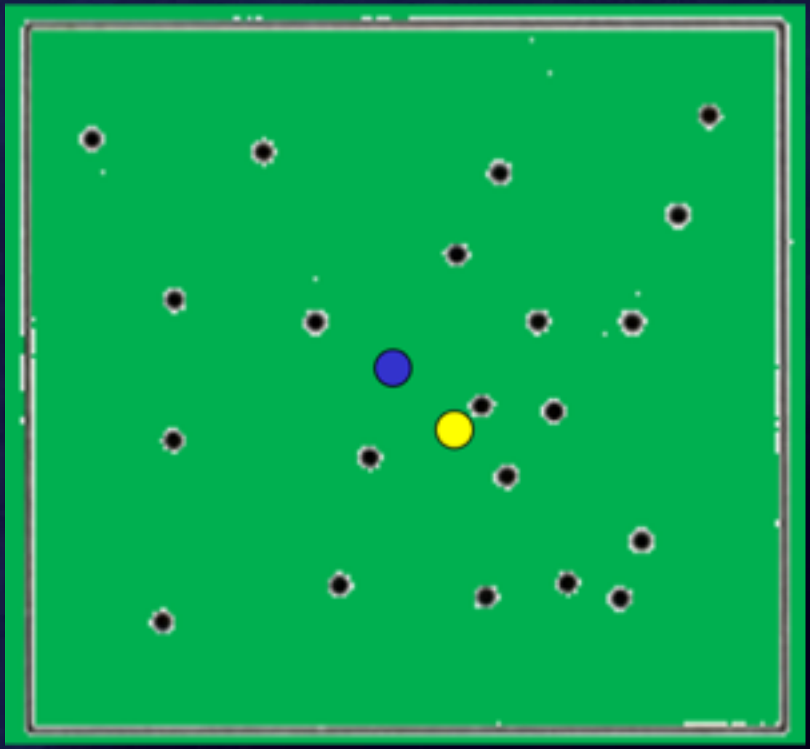
0-1 days late	Score * 90%
1-2 days late	Score * 70%
2-3 days late	Score * 50%
≥ 3 days late	No credit

Descriptive Spatial Statistics

- Spatial statistics (or spatial analysis) includes any of the formal techniques which study entities using their topological, geometric, or geographic properties.

John Snow's Map

Mean Center

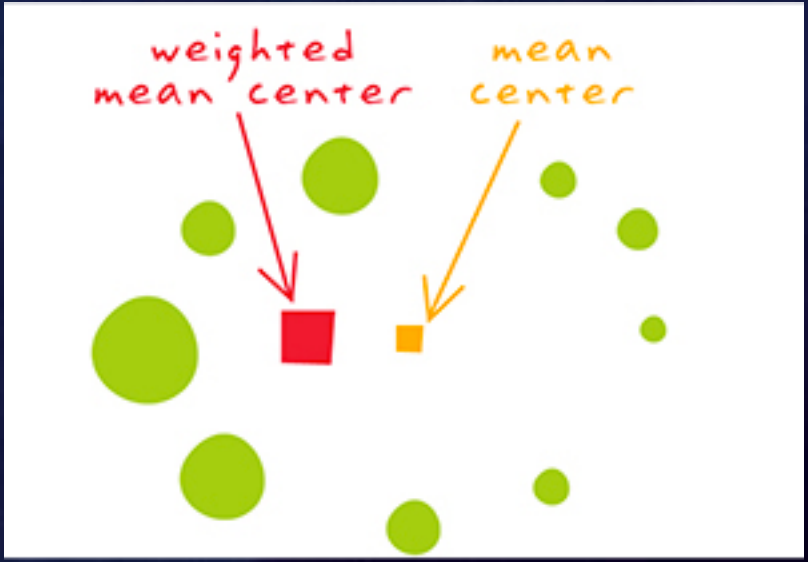


- Mean center (X_{bar}, Y_{bar}) is calculated as the arithmetic average of x and the average of y (mean center is not the same as the geometric center or centroid)

$$\bar{X} = \frac{x_1 + x_2 + \ldots\ldots + x_n}{n}$$

$$\bar{Y} = \frac{y_1 + y_2 + \ldots\ldots + y_n}{n}$$

Weighted Mean Center



Value dependent center

- **Weighted mean center** is weighted based on the sample values
- Previous method is "equal-weighted"

$$\bar{X} = \frac{v_1 x_1 + v_2 x_2 + ... + v_3 x_n}{v_1 + v_2 + ... + v_3}$$

$$\bar{Y} = \frac{v_1 y_1 + v_2 y_2 + ... + v_3 y_n}{v_1 + v_2 + ... + v_3}$$

Figure 4.3 Graph of Point Locations, Frequencies (in Parentheses) and Weighted Mean Center

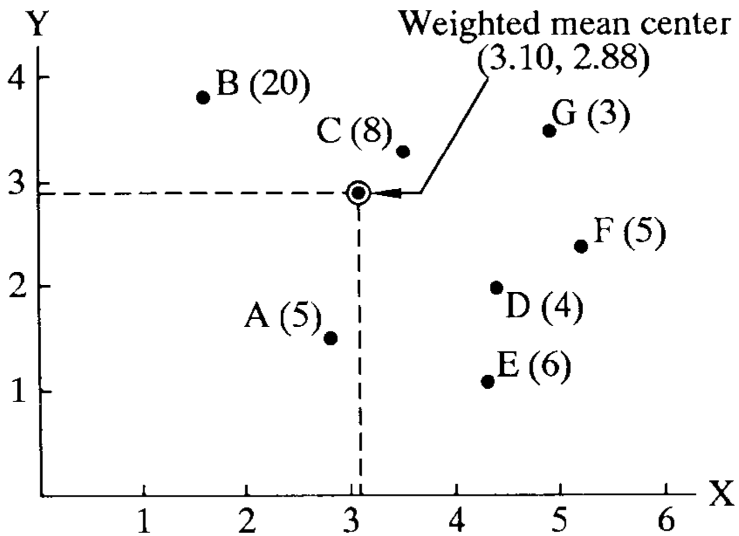


Table 4.3 Work Table for Calculating Weighted Mean Center

	Locational Coordinates*		**Weight**	**Weighted Coordinates**	
Point	X_i	Y_i	f_i	f_iX_i	f_iY_i
A	2.8	1.5	5	14.0	7.5
B	1.6	3.8	20	32.0	76.0
C	3.5	3.3	8	28.0	26.4
D	4.4	2.0	4	17.6	8.0
E	4.3	1.1	6	25.8	6.6
F	5.2	2.4	5	26.0	12.0
G	4.9	3.5	3	14.7	10.5
$n = 7$	$\Sigma f_i = 51$		$\Sigma f_i X_i = 158.1$		$\Sigma f_i Y_i = 147.0$

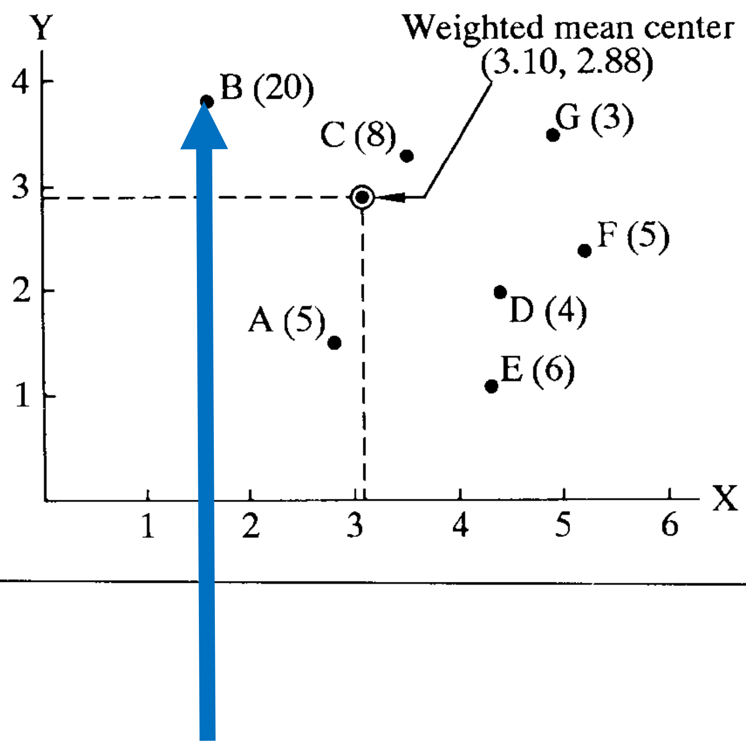
$$\overline{X}_{wc} = \frac{\Sigma f_i X_i}{\Sigma f_i} = \frac{158.1}{51} = 3.10$$

$$\overline{Y}_{wc} = \frac{\Sigma f_i Y_i}{\Sigma f_i} = \frac{147.0}{51} = 2.88$$

Weighted mean center coordinates: (3.10, 2.88)*

*See figure 4.3 for graph of point locations, frequencies, and weighted mean center.

Figure 4.3 Graph of Point Locations, Frequencies (in Parentheses) and Weighted Mean Center



Point B makes up nearly 40% of total weight

Table 4.3 Work Table for Calculating Weighted Mean Center

	Locational Coordinates*		Weight	Weighted Coordinates	
Point	X_i	Y_i	f_i	f_iX_i	f_iY_i
A	2.8	1.5	5	14.0	7.5
B	1.6	3.8	20	32.0	76.0
C	3.5	3.3	8	28.0	26.4
D	4.4	2.0	4	17.6	8.0
E	4.3	1.1	6	25.8	6.6
F	5.2	2.4	5	26.0	12.0
G	4.9	3.5	3	14.7	10.5
$n = 7$		$\Sigma f_i = 51$		$\Sigma f_iX_i = 158.1$	$\Sigma f_iY_i = 147.0$

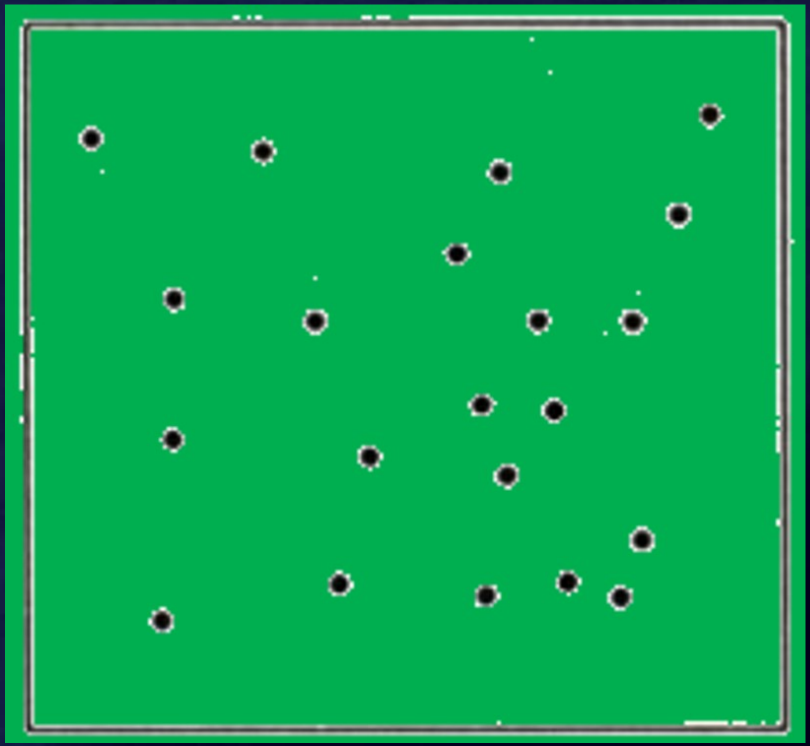
$$\overline{X}_{wc} = \frac{\Sigma f_i X_i}{\Sigma f_i} = \frac{158.1}{51} = 3.10$$

$$\overline{Y}_{wc} = \frac{\Sigma f_i Y_i}{\Sigma f_i} = \frac{147.0}{51} = 2.88$$

Weighted mean center coordinates: (3.10, 2.88)*

*See figure 4.3 for graph of point locations, frequencies, and weighted mean center.

Median Center



- **Median center** minimizes sum of distances from points to the location
- Iterative approach that guesses a location, *i*, and then minimizes distance from all points

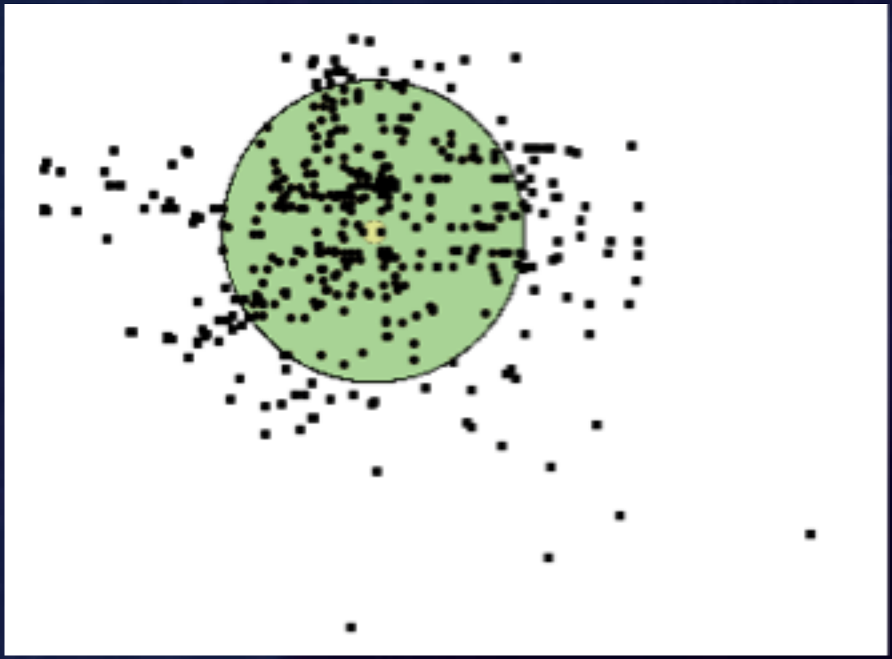
$$x' = \frac{\sum_{i=1}^{n} \frac{v_i x_i}{d_i}}{\sum_{i=1}^{n} \frac{v_i}{d_i}} \qquad y' = \frac{\sum_{i=1}^{n} \frac{v_i y_i}{d_i}}{\sum_{i=1}^{n} \frac{v_i}{d_i}}$$

Median Center of Population for the United States: 1880 to 2010

U.S. DEPARTMENT OF COMMERCE Economics and Statistics Administration U.S. Census Bureau

Prepared by the Geography Division

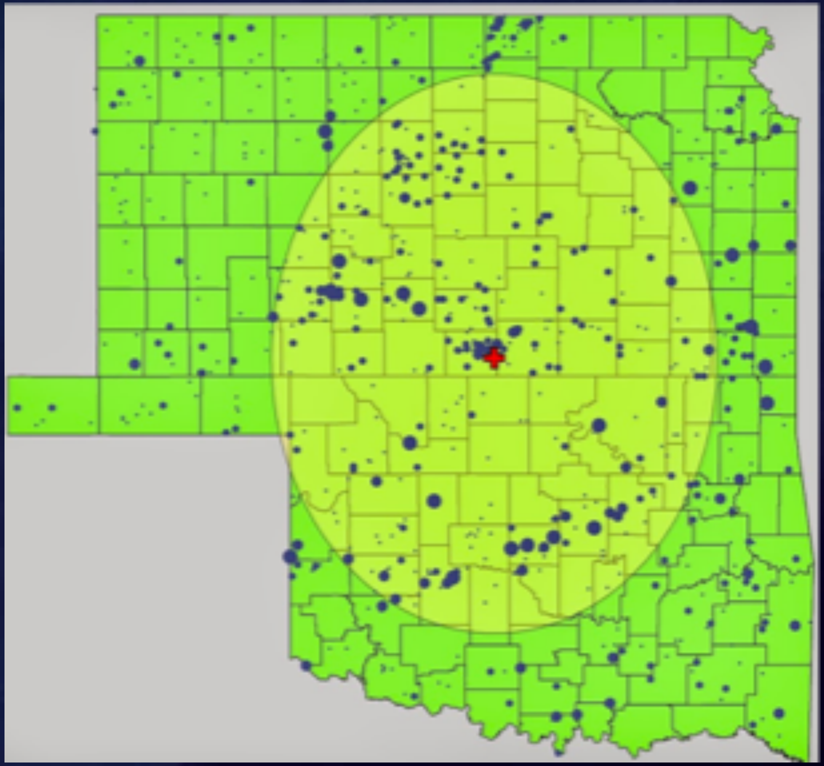
Standard Distance



- Average distance of each sample point to either the geometric or arithmetic center

$$S_D = \sqrt{\frac{\Sigma\left(X_i - \bar{X}_c\right)^2 + \Sigma\left(Y_i - \bar{Y}_c\right)^2}{n}}$$

Weighted Standard Distance

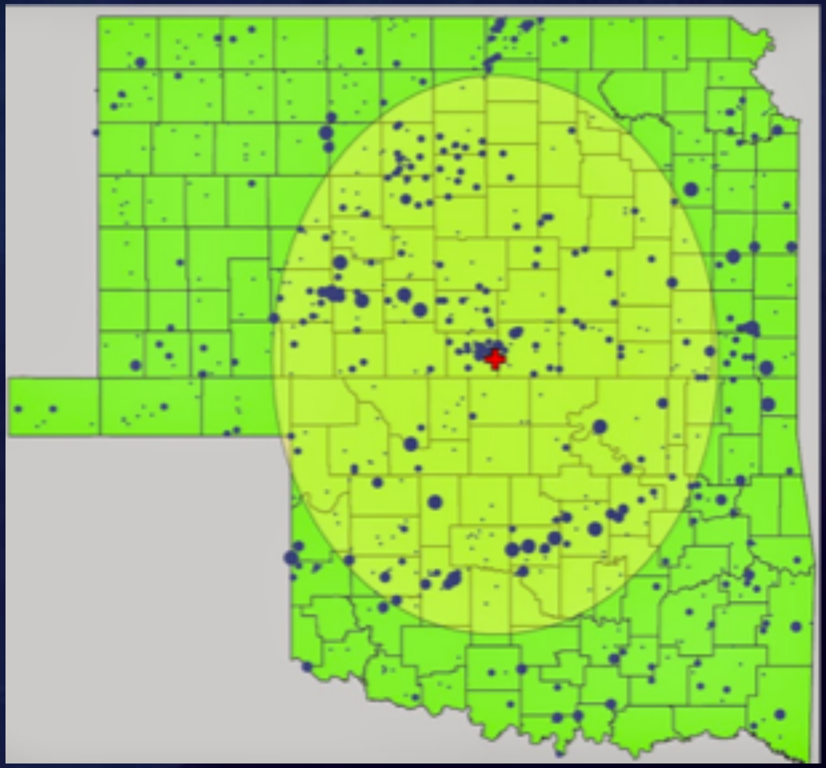


- Same as standard distance except the weights (or values) of the sampling points are considered:

$$SD_w = \sqrt{\frac{\sum_{i=1}^{n} w_i(x_i - \bar{X})^2}{\sum_{i=1}^{n} w_i} + \frac{\sum_{i=1}^{n} w_i(y_i - \bar{Y})^2}{\sum_{i=1}^{n} w_i}}$$

w_i - weight of sample i

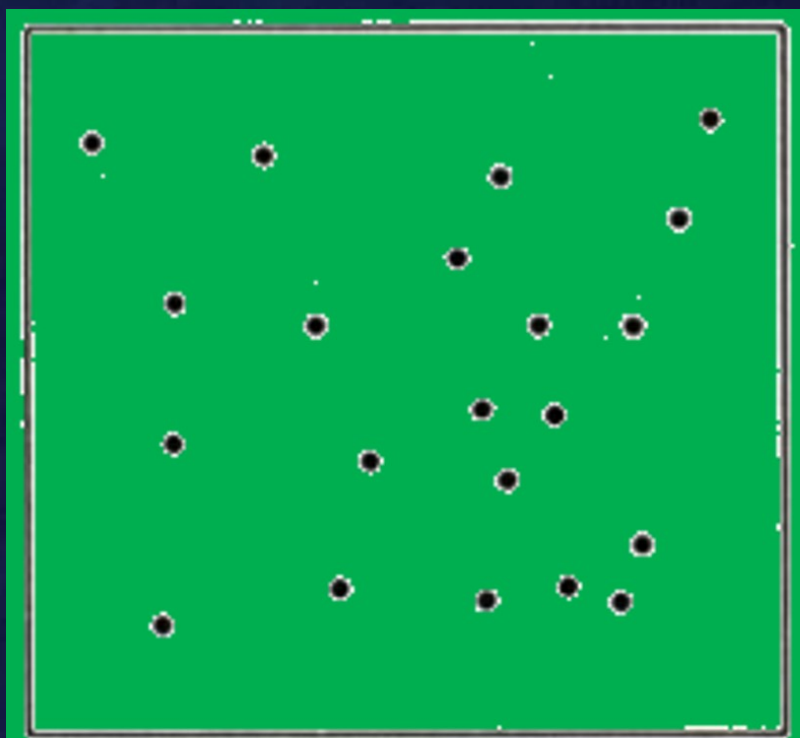
Relative Distance



- *Relative distance* is the standard distance divided by the radius of a circle, *r*, with area equal to the size of the study area
- Makes the measure of dispersion unitless and standardizes for the size of the study area
- Enables comparison of dispersion in study areas of different sizes

$$S_{D,\,rel} = S_D / r$$

Spatial Arrangement



- What spatial arrangement of a variable or your sampling in the study area?
- **Nearest Neighbor Distance**

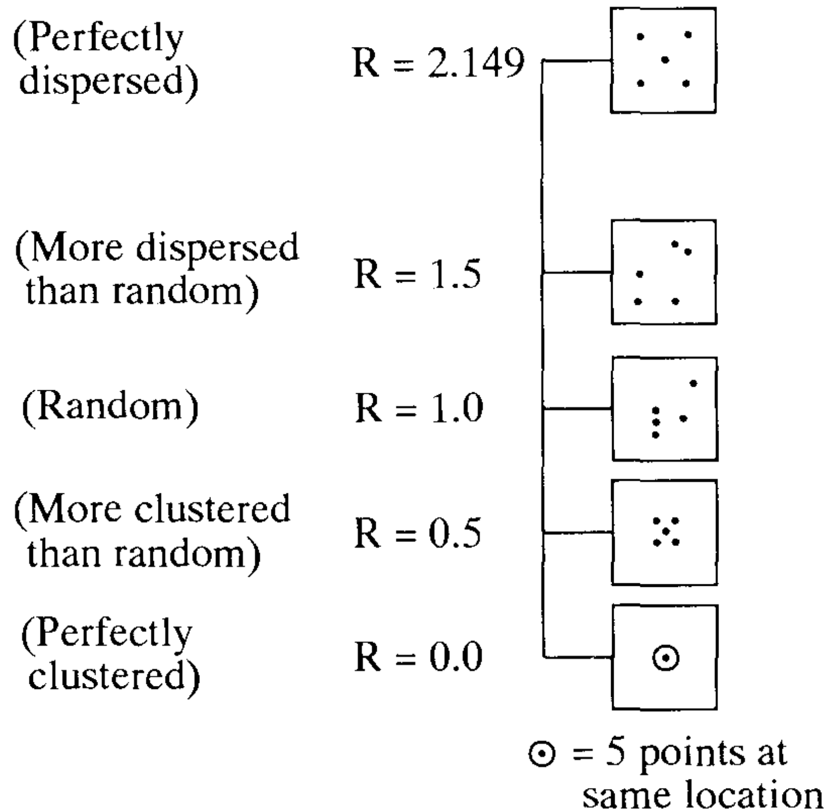
$$NND = \frac{\sum ND}{n}$$

$$NND_R = \frac{1}{2\sqrt{density}}$$

$$R = \frac{NND}{NND_R}$$

- The larger the R the more dispersed the samples, while a smaller R reflects clustering

Figure 11.3 Continuum of *R* Values in Nearest Neighbor Analysis



Source: Modified from Taylor, P. J. 1977. *Quantitative Methods in Geography*. Boston: Houghton Mifflin

Normal Distribution

- The shape of the normal curve is often illustrated as a **bell-shaped**
- The highest point is the **mean** of the distribution
- The normal curve is **symmetric**
- **Standard deviation** determines the width of the curve

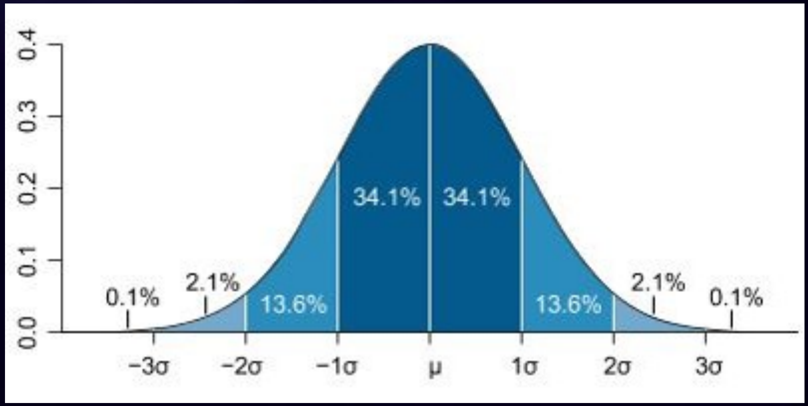
Normal Distribution

- The **total area** under the probability curve is **one**
- Probabilities for a random variable are given by areas under the curve, i.e., **intervals**

Normal Distribution

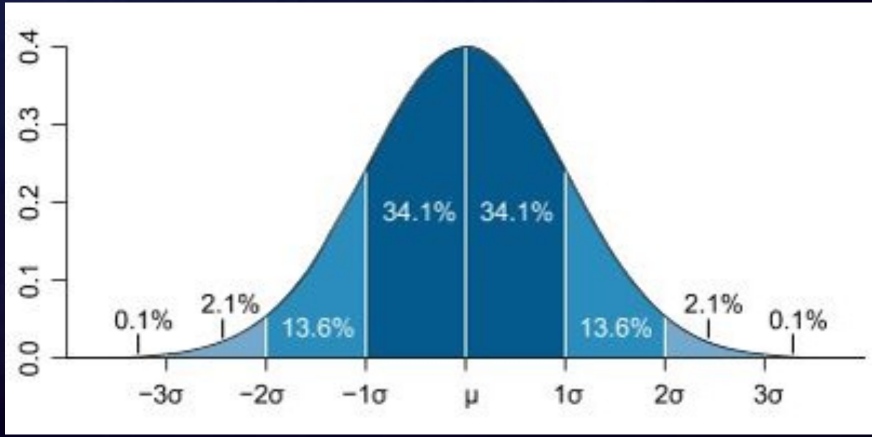
- Most common statistical tests are based on the assumption that the data you collect is normally distributed
- Allows for a simple description of how the data is distributed around the mean and between the maximum and minimum values
- **stdev: Standard deviation**

- 68% in -1 to +1 stdev from mean
- 95% in -2 to +2 stdev from mean
- 99.8% in -3 to +3 stdev from mean



Natural Systems

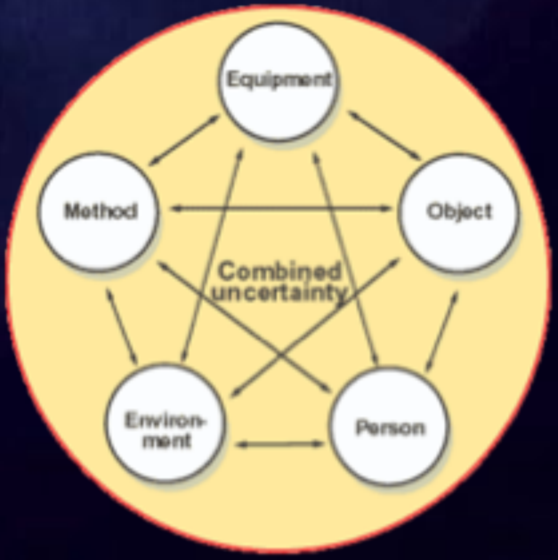
- Natural systems exhibit **variability**
- Sampling introduces **uncertainty**



Natural Systems

- We cannot account for everything in an experiment

- Sources of uncertainty:
  1. Equipment
  2. Object Measured
  3. Person
  4. Method
  5. Calibration
  6. Environment



Natural Systems

- We cannot account for everything in an experiment

- **Will these two hit it off?**

Natural Systems

- We cannot account for everything in an experiment
  - Can’t eliminate uncertainty
  - describe **probability** and **confidence** of our sample

Probability

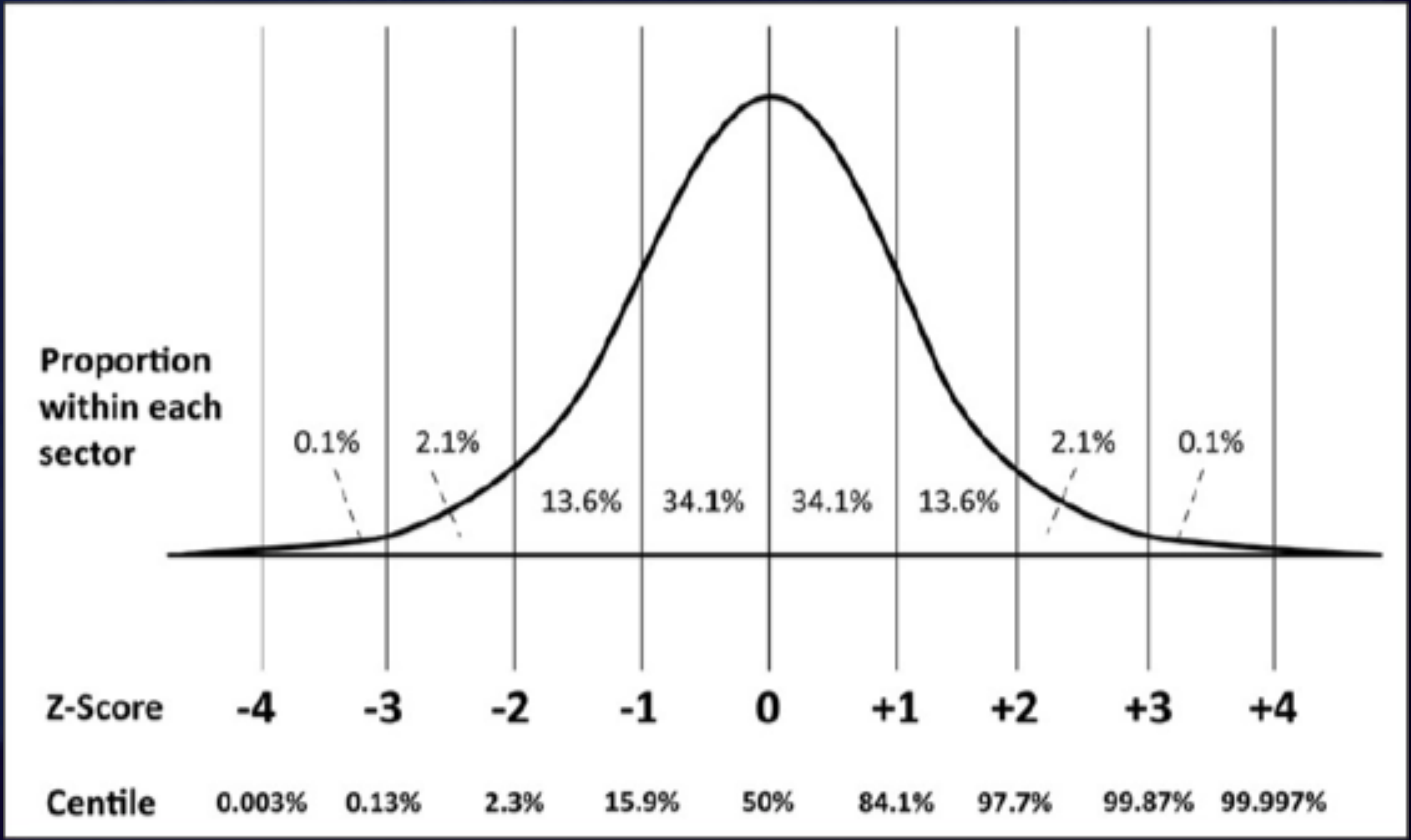
- Probability of a given data value is based on the ***Z* score**:

$$Z = \frac{X - \overline{X}}{s}$$

- Estimates how many standard deviations (s) a data value is away from the mean

- Probability of each *Z* score is listed in a ***Z* table**

Probability



Probability

- Average = 39.9 inches of annual rainfall in Washington DC
- Standard Deviation = 7.5 inches

- What is the probability of an annual rainfall > 48 inches?

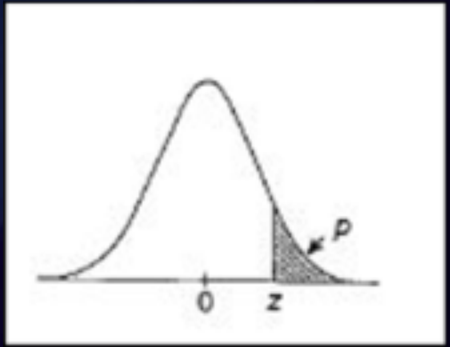
$$Z = \frac{X - \bar{X}}{s} = \frac{48 - 40}{7.5} = 1.07$$

- 48 inches is 1.07 stdevs away from the mean (the Z score)

Z Table

of standard deviations

Probability of getting that value



z	Second decimal place of z .00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010

Adapted with rounding from Table II of Fisher and Yates 1974.

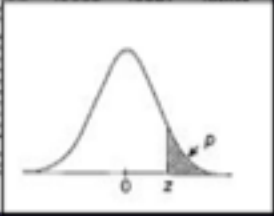
Annual Precipitation in Washington, DC

- What probability does a *Z* score of 1.07 correspond to?

- **Depends!**

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
	Second decimal place of z									
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	[illegible]			
2.5	.0062	.0060	.0059	.0057	.0055	.0054	[illegible]			
2.6	.0047	.0045	.0044	.0043	.0041	.0040	[illegible]			
2.7	.0035	.0034	.0033	.0032	.0031	.0030	[illegible]			
2.8	.0026	.0025	.0024	.0023	.0023	.0022	[illegible]			
2.9	.0019	.0018	.0018	.0017	.0016	.0016	[illegible]			
3.0	.0013	.0013	.0013	.0012	.0012	.0011	[illegible]			

Adapted with rounding from Table II of Fisher and Yates 1974.



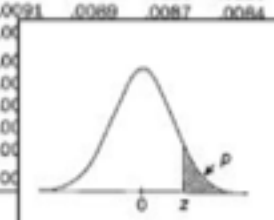
Annual Precipitation in Washington, DC

- What probability does a *Z* score of 1.07 correspond to?

- **Upper tail assessment:** Only 14.23% chance of greater than or equal to 48” of precipitation

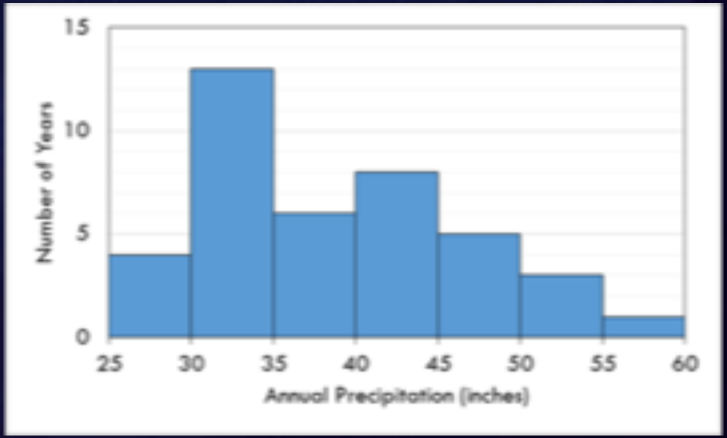
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
	Second decimal place of z									
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	$\boxed{.1423}$	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0376	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	[illegible]			
2.5	.0062	.0060	.0059	.0057	.0055	.0054	[illegible]			
2.6	.0047	.0045	.0044	.0043	.0041	.0040	[illegible]			
2.7	.0035	.0034	.0033	.0032	.0031	.0030	[illegible]			
2.8	.0026	.0025	.0024	.0023	.0023	.0022	[illegible]			
2.9	.0019	.0018	.0018	.0017	.0016	.0016	[illegible]			
3.0	.0013	.0013	.0013	.0012	.0012	.0011	[illegible]			

Adapted with rounding from Table II of Fisher and Yates 1974.



Example: Annual Precipitation in Washington, DC over 40 years

Ordered Sample			
26”	35”	39”	45”
26”	35”	40”	46”
28”	35”	40”	47”
29”	36”	41”	47”
32”	36”	41”	48”
32”	36”	41”	50”
33”	36”	41”	51”
33”	38”	41”	51”
34”	39”	43”	54”
35”	39”	43”	57”



- 6/ 40 = 15.0%
- 14.23% is good estimate

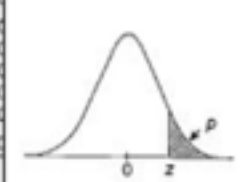
Annual Precipitation in Washington, DC

- What probability does a *Z* score of 1.07 correspond to?

- **Lower tail assessment:** 85.77% chance of less than or equal to 48” of precipitation

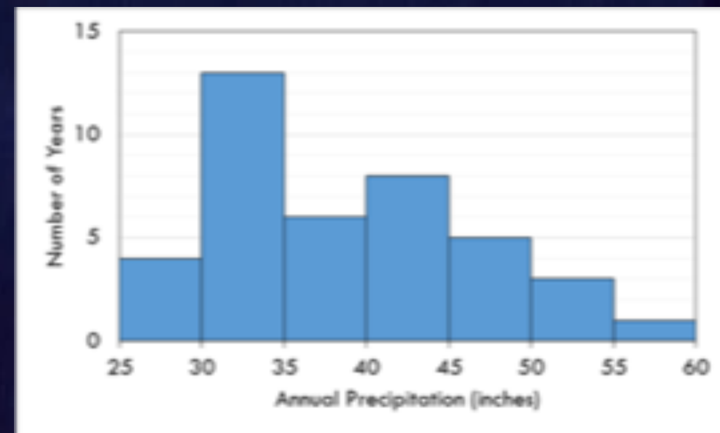
z	Second decimal place of *z* .00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	[illegible]	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0376	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	[illegible]	[illegible]	[illegible]	[illegible]
2.5	.0062	.0060	.0059	.0057	.0055	.0054	[illegible]	[illegible]	[illegible]	[illegible]
2.6	.0047	.0045	.0044	.0043	.0041	.0040	[illegible]	[illegible]	[illegible]	[illegible]
2.7	.0035	.0034	.0033	.0032	.0031	.0030	[illegible]	[illegible]	[illegible]	[illegible]
2.8	.0026	.0025	.0024	.0023	.0023	.0022	[illegible]	[illegible]	[illegible]	[illegible]
2.9	.0019	.0018	.0018	.0017	.0016	.0016	[illegible]	[illegible]	[illegible]	[illegible]
3.0	.0013	.0013	.0013	.0012	.0012	.0011	[illegible]	[illegible]	[illegible]	[illegible]

Adapted with rounding from Table II of Fisher and Yates 1974.



Example: Annual Precipitation in Washington, DC over 40 years

Ordered Sample			
26”	35”	39”	45”
26”	35”	40”	46”
28”	35”	40”	47”
29”	36”	41”	47”
32”	36”	41”	~~48”~~
32”	36”	41”	~~50”~~
33”	36”	41”	~~51”~~
33”	38”	41”	~~51”~~
34”	39”	43”	~~54”~~
35”	39”	43”	~~57”~~



- 34/ 40 = 85.0%
- 85.76% is good estimate

Annual Precipitation in Washington, DC

- What probability does a *Z* score of 1.07 correspond to?

- **Two-tail assessment:** A difference of more than 8” from the mean will occur 28.46% of the time.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
	Second decimal place of z									
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
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0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	[illegible]	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
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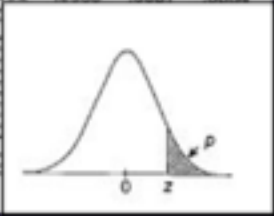
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	[illegible]	[illegible]	[illegible]	[illegible]
2.5	.0062	.0060	.0059	.0057	.0055	.0054	[illegible]	[illegible]	[illegible]	[illegible]
2.6	.0047	.0045	.0044	.0043	.0041	.0040	[illegible]	[illegible]	[illegible]	[illegible]
2.7	.0035	.0034	.0033	.0032	.0031	.0030	[illegible]	[illegible]	[illegible]	[illegible]
2.8	.0026	.0025	.0024	.0023	.0023	.0022	[illegible]	[illegible]	[illegible]	[illegible]
2.9	.0019	.0018	.0018	.0017	.0016	.0016	[illegible]	[illegible]	[illegible]	[illegible]
3.0	.0013	.0013	.0013	.0012	.0012	.0011	[illegible]	[illegible]	[illegible]	[illegible]

Adapted with rounding from Table II of Fisher and Yates 1974.



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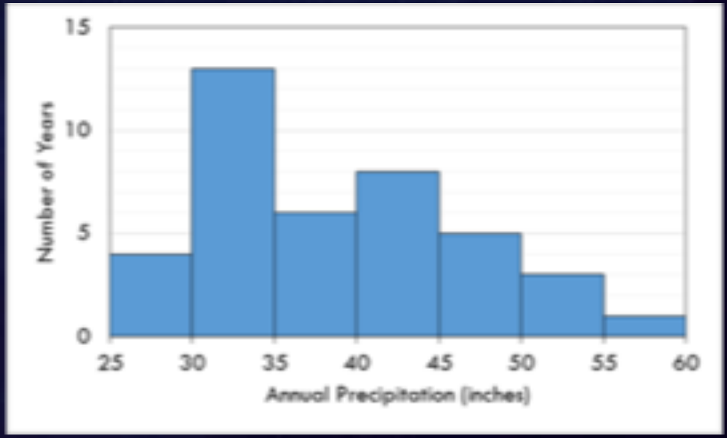

- 12/40 = 30.0%
- 28.46% is good estimate

Annual Precipitation in Washington, DC

- **Other questions:**
  1. What is the probability of drought conditions below 27 inches?
  2. What is the amount that has a probability of 1 in 100 years?
     - Lower and Upper
  3. What extreme events have a probability of only 5%?

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
	Second decimal place of z									
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
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1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
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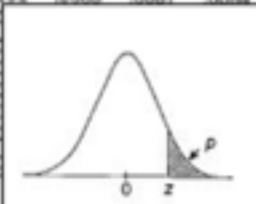
2.4	.0082	.0080	.0078	.0075	.0073	.0071	[illegible]			
2.5	.0062	.0060	.0059	.0057	.0055	.0054	[illegible]			
2.6	.0047	.0045	.0044	.0043	.0041	.0040	[illegible]			
2.7	.0035	.0034	.0033	.0032	.0031	.0030	[illegible]			
2.8	.0026	.0025	.0024	.0023	.0023	.0022	[illegible]			
2.9	.0019	.0018	.0018	.0017	.0016	.0016	[illegible]			
3.0	.0013	.0013	.0013	.0012	.0012	.0011	[illegible]			

Adapted with rounding from Table II of Fisher and Yates 1974.



Confidence Interval

- Express our confidence that the average of a sample is within a certain range of the population mean

$$\overline{X} \pm Z \frac{\sigma}{\sqrt{n}}$$

- To be 90% confident Z=1.65
- To be 95% confident Z=1.96
- You must know the standard deviation of the population (σ)

Confidence Interval

- Average = 39.9 inches
- Standard Deviation of population = 7.5 inches

$$\overline{X} \pm Z\frac{\sigma}{\sqrt{n}}$$

$$39.9 \pm 1.65\frac{7.5}{\sqrt{40}}$$

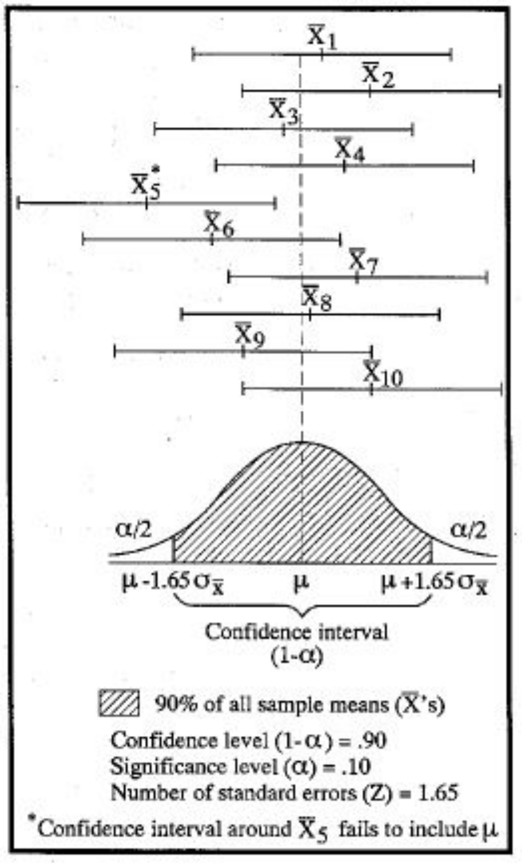
$$39.9 \pm 1.96\ inches$$

- What about 95% confidence interval?

$$39.9 \pm 2.32\ inches$$

Confidence Interval

- **Interpretation:**
  - Probability that the **sample mean** sample falls within a range of the population mean
- 90% chance that the mean of a sample will fall in interval within 1.65 standard deviations of population mean
- 95% chance that the mean of a sample will falls in interval within 1.96 standard deviations of population mean



Confidence Interval

- If you only have a standard deviation of your **sample** then:

$$\overline{X} \pm Z\sqrt{\frac{s^2}{n}\frac{(N-n)}{N}}$$

- Requires you to know the size of the population from which you are sampling
- Different equation if you use a stratified (non-random) sample

Confidence Interval

$$\bar{X} \pm Z\sqrt{\frac{s^2}{n}\frac{(N-n)}{N}}$$

- Average = 39.9 inches
- Standard Deviation = 7.5 inches
- Only 60 of 100 samples

$$39 \pm 1.65\sqrt{\frac{7.5^2}{60}\frac{(100-60)}{100}}$$

$$39 \pm 1.01\, inches$$

- What about 95% confidence interval?

$$39 \pm 1.20\, inches$$

Confidence Interval

- If you don’t know the size of the population then you can simply use the **standard equation:**

$$\bar{X} \pm Z \frac{s}{\sqrt{n}}$$

- Creates wider confidence interval under the assumption that the population is really large

Lecture 7:
Spatial Statistics and Probability