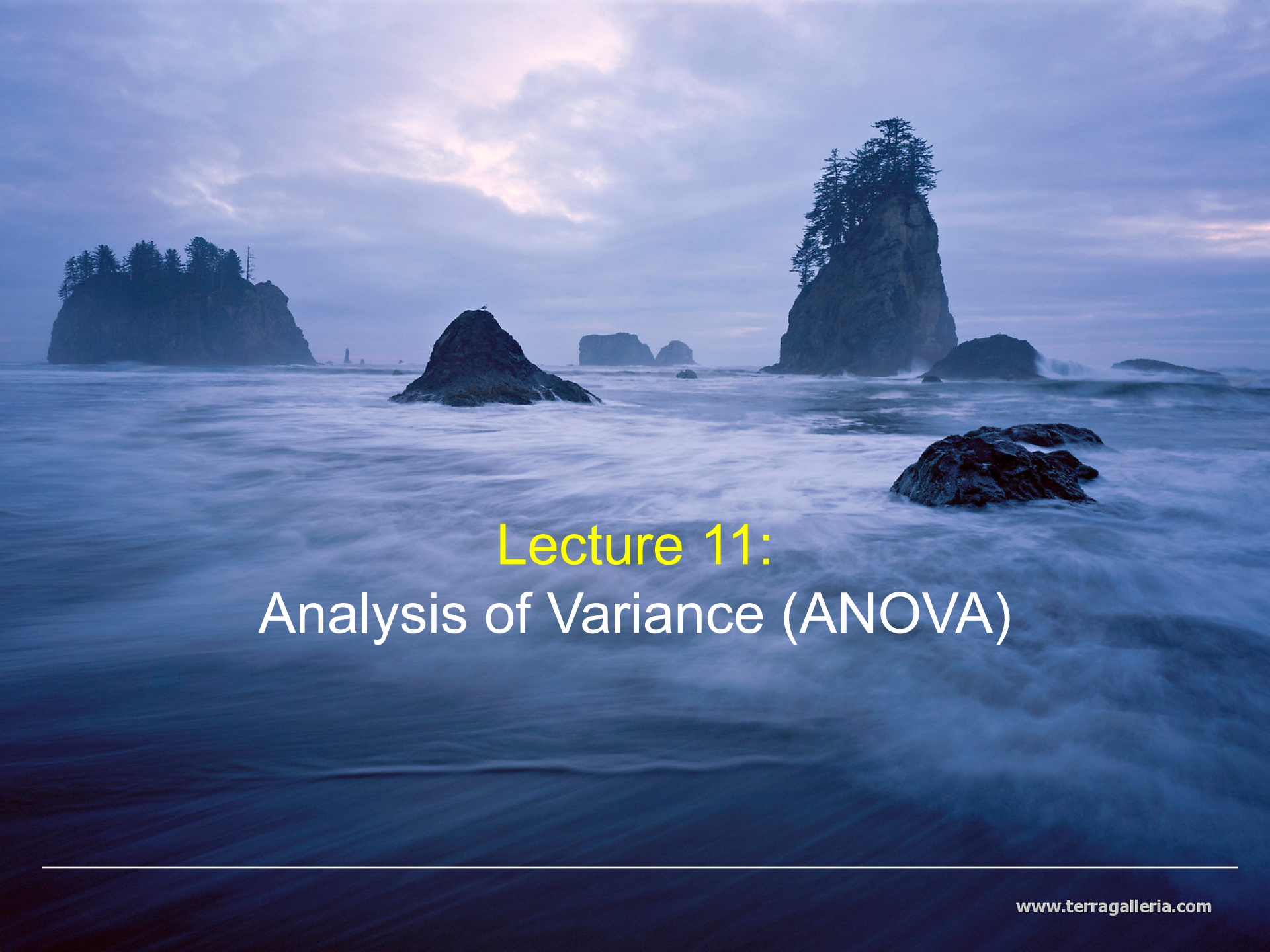
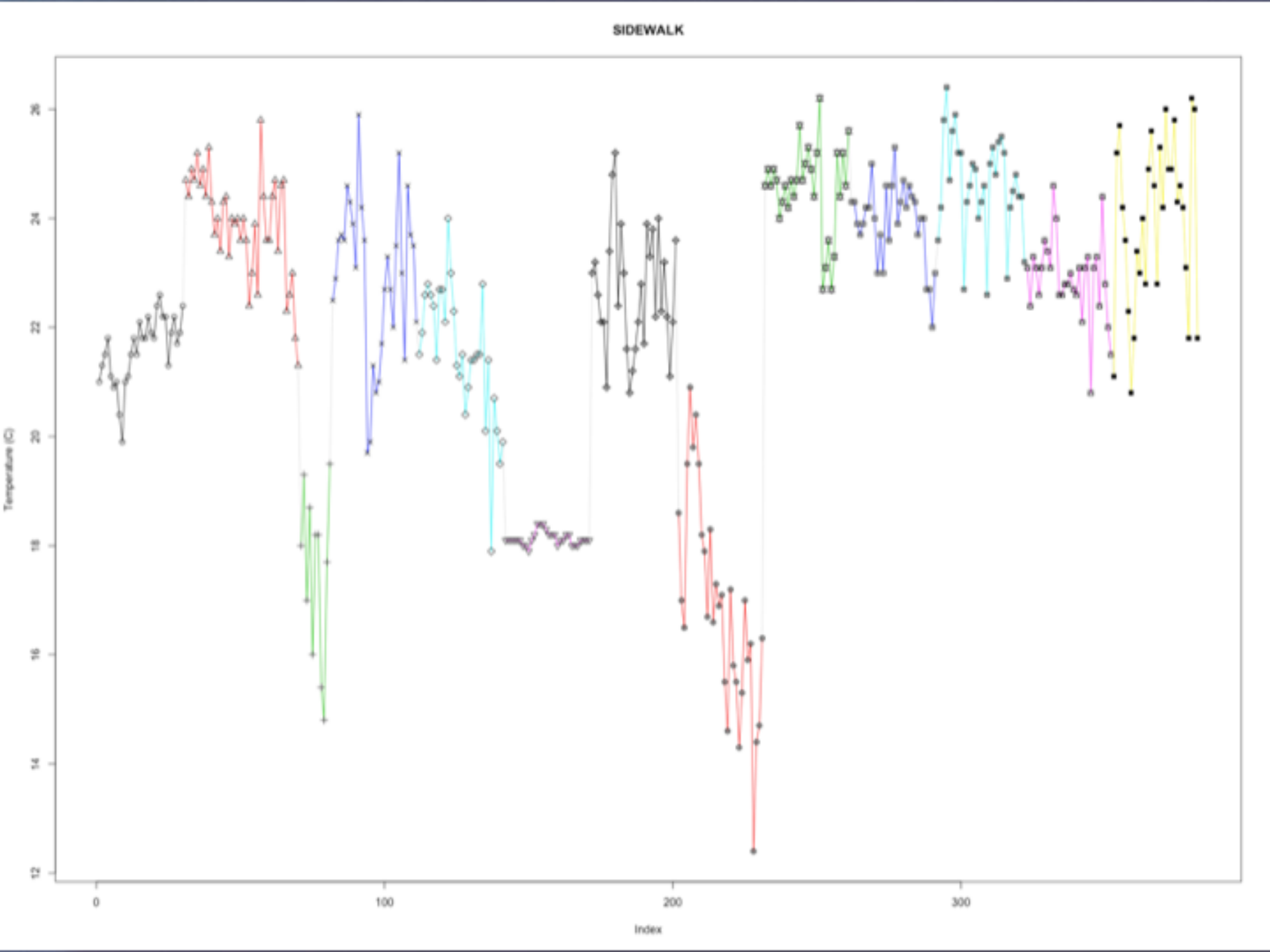


# Lecture 11:
# Analysis of Variance (ANOVA)

# Announcements

- If you have changed groups (let me know via email ASAP) - what was your old group (6-digit ID and group member names) and what is your new group (6-digit ID and group member names).

# Temperature Data from Assignment #2

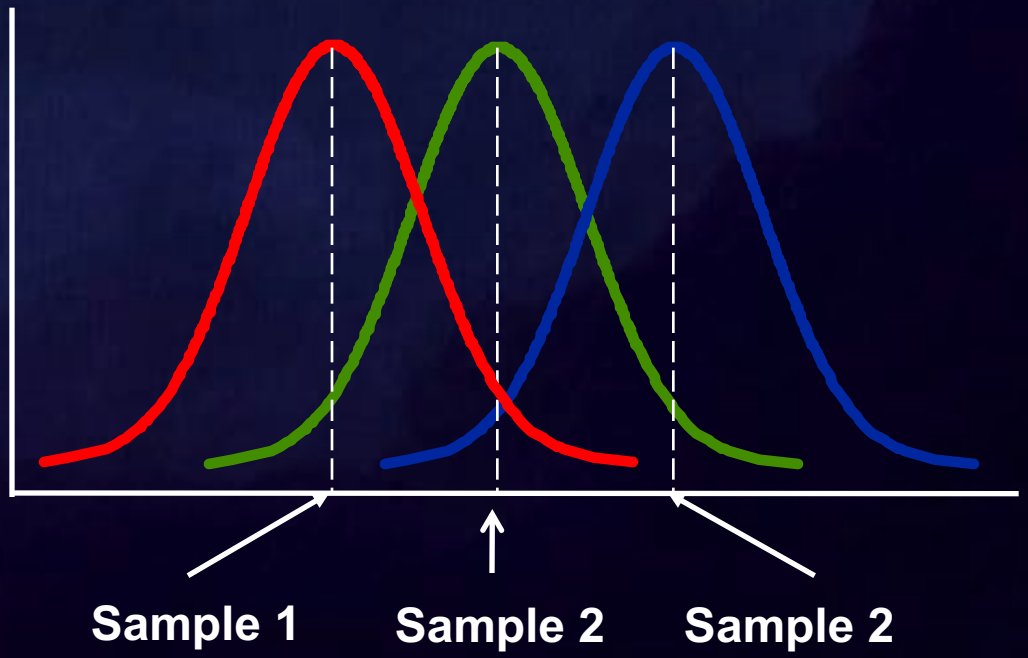

SIDEWALK
Temperature (C)
12
14
16
18
20
22
24
26
0
100
200
300
Index

# Let’s open up R…

# Differences

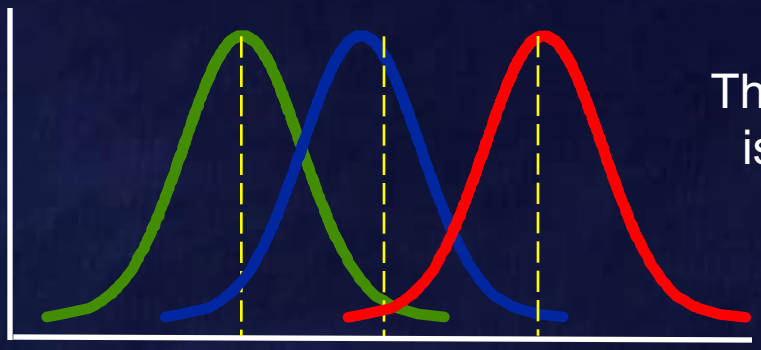
- t-test and z-test are used to determine if there is a statistically significant difference between 2 distributions

- Analysis of Variance (ANOVA) is a test to determine if there is a statistically significant difference between 3 or more distributions

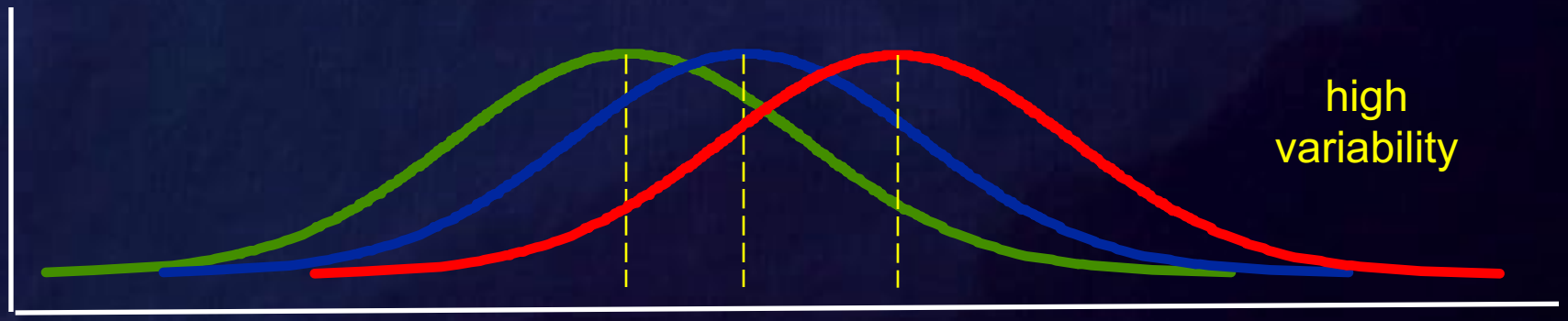



**Is there a *difference*?**

medium variability

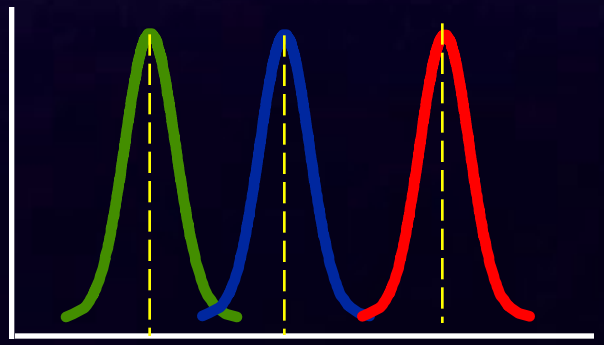


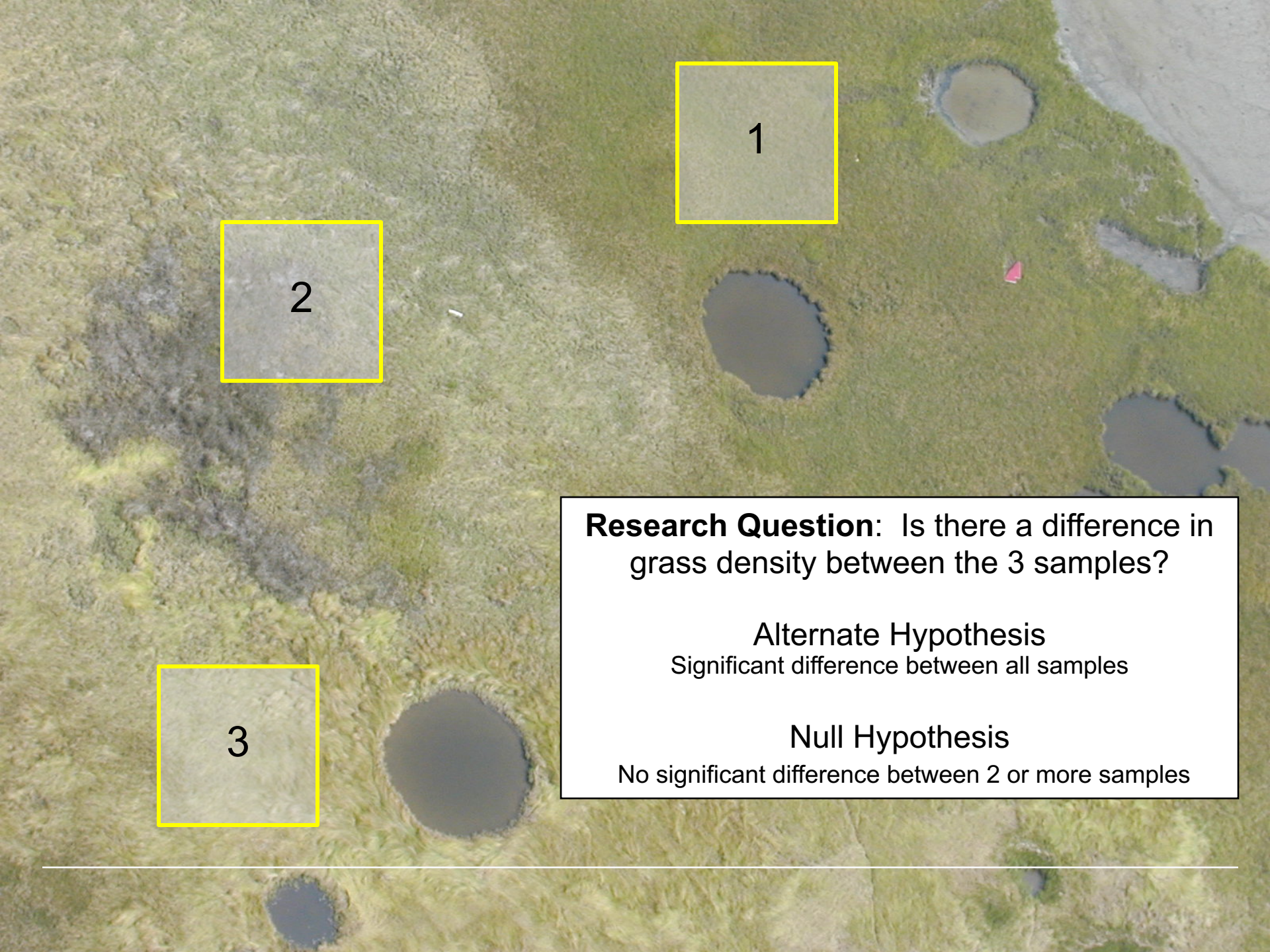
The mean difference is the *same* for all three cases



high variability

low variability






**Research Question**: Is there a difference in grass density between the 3 samples?

Alternate Hypothesis

Significant difference between all samples

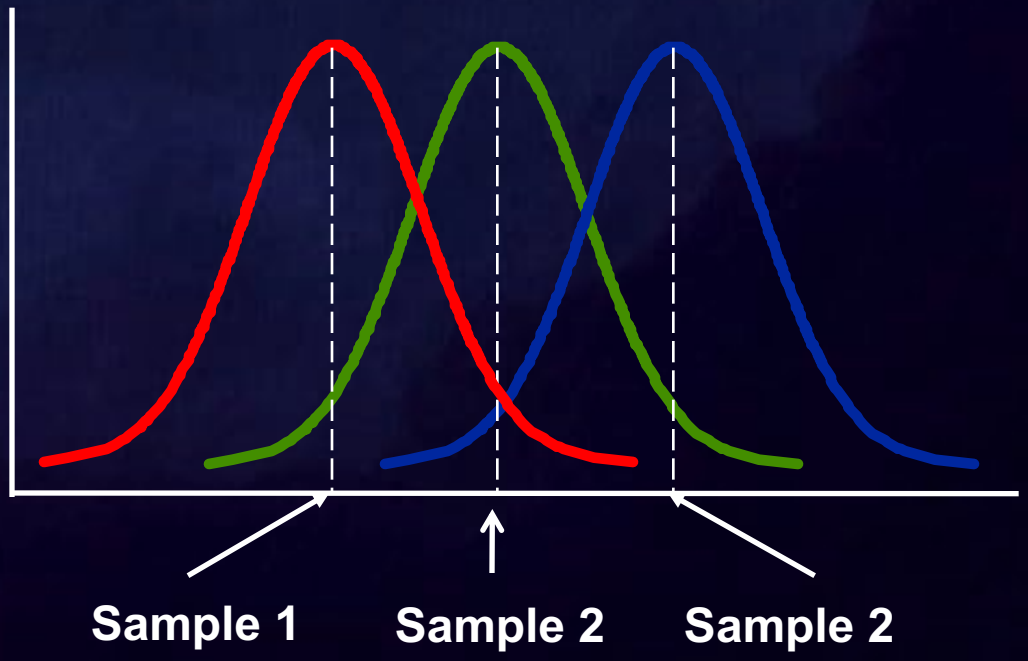
Null Hypothesis

No significant difference between 2 or more samples

# ANOVA

$$\frac{\text{Variability between groups}}{\text{Variability within groups}}$$

$$F = \frac{MS_B}{MS_W}$$




**Is there a *difference*?**

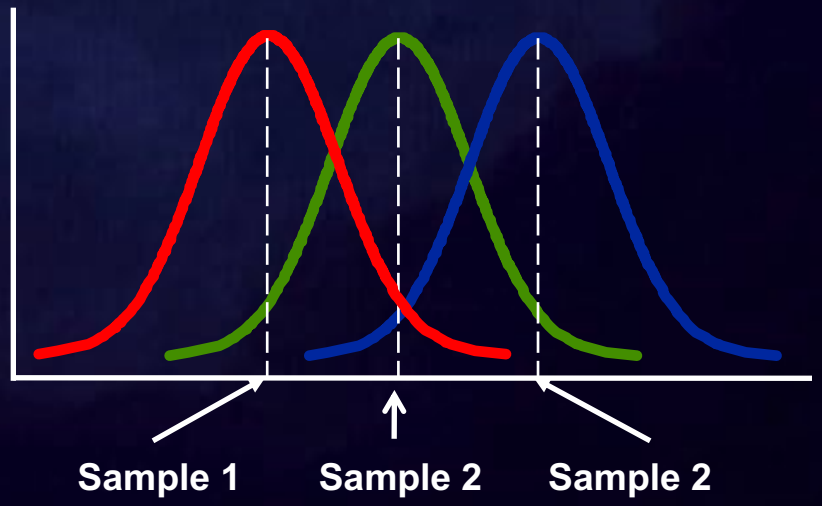
# ANOVA

- Step 1: Calculate the average of all samples weighted by # of observations in a sample (n) and the total number of samples (N)

$$\bar{X}_T = \frac{\sum n_i \bar{X}_i}{N}$$

- Step 2: Calculate between-group sum of squares ($SS_B$)

$$SS_B = \sum n_i \left( \bar{X}_i - \bar{X}_T \right)^2$$

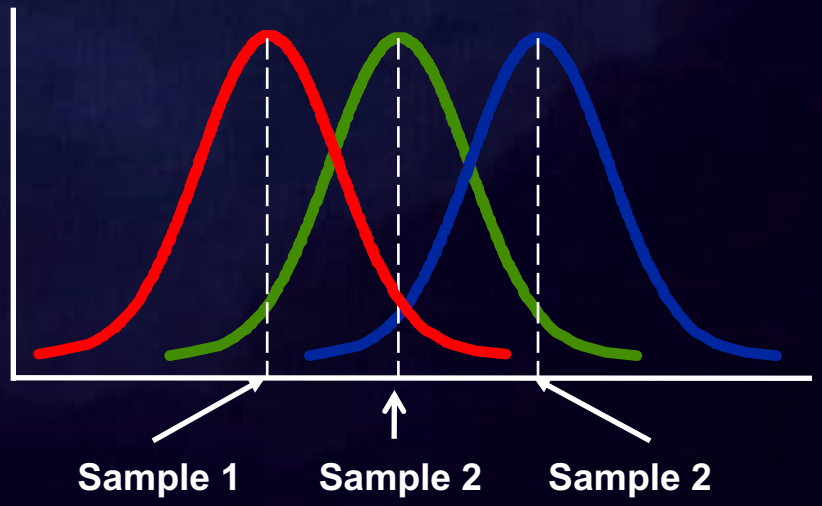



$$F = \frac{MS_B}{MS_W}$$

# ANOVA

- Step 3: Calculate the between-group mean squares ($MS_B$) based on the number of samples (k)

$$MS_B = \frac{SS_B}{df} = \frac{SS_B}{k-1}$$




- Step 4: Calculate the within-group sum of squares based on number of observations in each sample and the standard deviation

$$SS_W = \sum (n-1)s^2$$

$$F = \frac{MS_B}{MS_W}$$

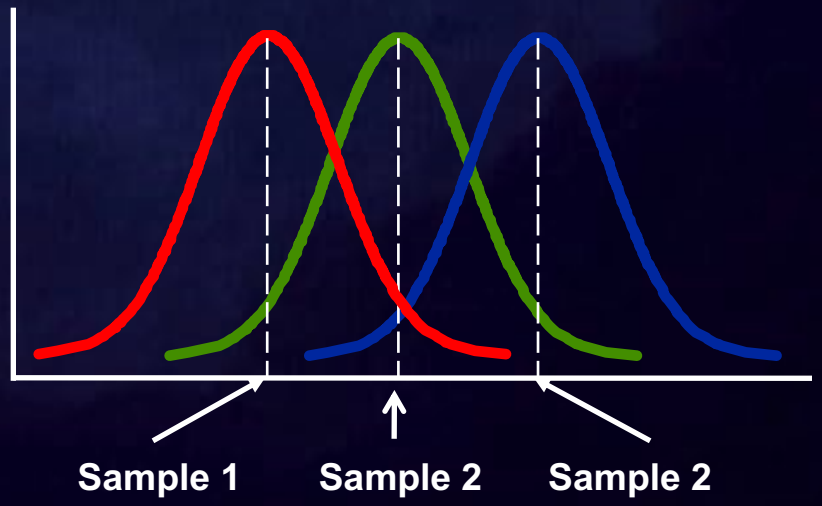
# ANOVA

- Step 5: Calculate the within group mean squares ($MS_w$) based on total number of samples (N) and the number of groups (k)

$$MS_W = \frac{SS_W}{df} = \frac{SS_W}{N-k}$$

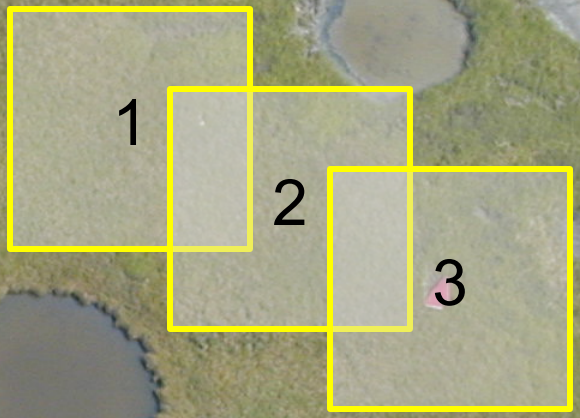
- Step 6: Calculate the test statistic (F-Statistic):

$$F = \frac{MS_B}{MS_W}$$



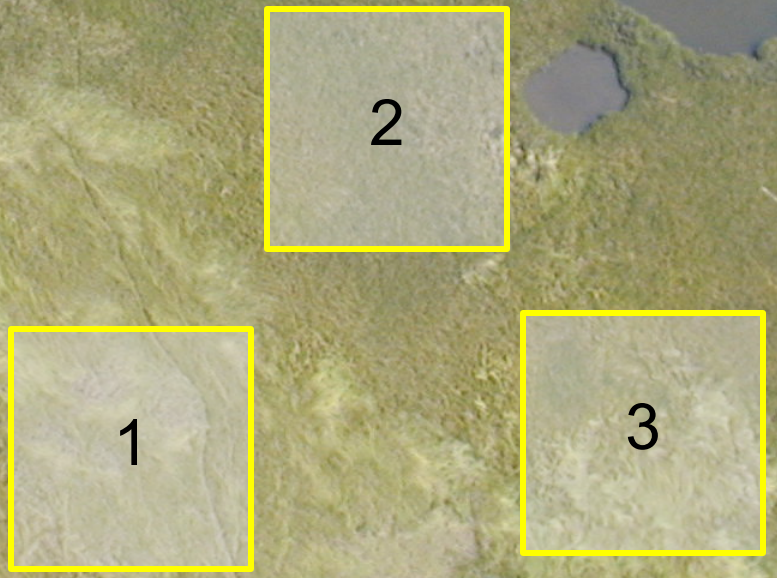

$$F = \frac{MS_B}{MS_W}$$

**Case A**: If the samples are from the same population then the between-group and within-group variance will be approximately the same and F=1




$$F = \frac{MS_B}{MS_W}$$

**Case B**: If the samples are not from the same population then the between-group variance will be greater than the within-group variance and F>1



Between group degrees of freedom *(k-1)*

Within group degrees of freedom *N-k*

Critical value of F for result to be significant at 95% confidence level

Significant if $F > F_{critical}$

Not Significant if $F \leq F_{critical}$

| $df_W$ | $df_B$ | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 15 | 20 | 24 | 30 | 40 | 60 | 120 | ∞ |
| 1 | 161.4 | 199.5 | 215.7 | 224.6 | 230.2 | 234.0 | 236.8 | 238.9 | 240.5 | 241.9 | 243.9 | 245.9 | 248.0 | 249.1 | 250.1 | 251.1 | 252.2 | 253.3 | 254.3 |
| 2 | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 | 19.40 | 19.41 | 19.43 | 19.45 | 19.45 | 19.46 | 19.47 | 19.48 | 19.49 | 19.50 |
| 3 | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.74 | 8.70 | 8.66 | 8.64 | 8.62 | 8.59 | 8.57 | 8.55 | 8.53 |
| 4 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.91 | 5.86 | 5.80 | 5.77 | 5.75 | 5.72 | 5.69 | 5.66 | 5.63 |
| 5 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.68 | 4.62 | 4.56 | 4.53 | 4.50 | 4.46 | 4.43 | 4.40 | 4.36 |
| 6 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 4.00 | 3.94 | 3.87 | 3.84 | 3.81 | 3.77 | 3.74 | 3.70 | 3.67 |
| 7 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.57 | 3.51 | 3.44 | 3.41 | 3.38 | 3.34 | 3.30 | 3.27 | 3.23 |
| 8 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.28 | 3.22 | 3.15 | 3.12 | 3.08 | 3.04 | 3.01 | 2.97 | 2.93 |
| 9 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.07 | 3.01 | 2.94 | 2.90 | 2.86 | 2.83 | 2.79 | 2.75 | 2.71 |
| 10 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.91 | 2.85 | 2.77 | 2.74 | 2.70 | 2.66 | 2.62 | 2.58 | 2.54 |
| 11 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.79 | 2.72 | 2.65 | 2.61 | 2.57 | 2.53 | 2.49 | 2.45 | 2.40 |
| 12 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.69 | 2.62 | 2.54 | 2.51 | 2.47 | 2.43 | 2.38 | 2.34 | 2.30 |
| 13 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.60 | 2.53 | 2.46 | 2.42 | 2.38 | 2.34 | 2.30 | 2.25 | 2.21 |
| 14 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.53 | 2.46 | 2.39 | 2.35 | 2.31 | 2.27 | 2.22 | 2.18 | 2.13 |
| 15 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.48 | 2.40 | 2.33 | 2.29 | 2.25 | 2.20 | 2.16 | 2.11 | 2.07 |
| 16 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 | 2.49 | 2.42 | 2.35 | 2.28 | 2.24 | 2.19 | 2.15 | 2.11 | 2.06 | 2.01 |
| 17 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 | 2.38 | 2.31 | 2.23 | 2.19 | 2.15 | 2.10 | 2.06 | 2.01 | 1.96 |
| 18 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 | 2.41 | 2.34 | 2.27 | 2.19 | 2.15 | 2.11 | 2.06 | 2.02 | 1.97 | 1.92 |
| 19 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 | 2.31 | 2.23 | 2.16 | 2.11 | 2.07 | 2.03 | 1.98 | 1.93 | 1.88 |
| 20 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 | 2.28 | 2.20 | 2.12 | 2.08 | 2.04 | 1.99 | 1.95 | 1.90 | 1.84 |
| 21 | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 | 2.32 | 2.25 | 2.18 | 2.10 | 2.05 | 2.01 | 1.96 | 1.92 | 1.87 | 1.81 |
| 22 | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 | 2.30 | 2.23 | 2.15 | 2.07 | 2.03 | 1.98 | 1.94 | 1.89 | 1.84 | 1.78 |
| 23 | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 | 2.27 | 2.20 | 2.13 | 2.05 | 2.01 | 1.96 | 1.91 | 1.86 | 1.81 | 1.76 |
| 24 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 | 2.25 | 2.18 | 2.11 | 2.03 | 1.98 | 1.94 | 1.89 | 1.84 | 1.79 | 1.73 |
| 25 | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 | 2.24 | 2.16 | 2.09 | 2.01 | 1.96 | 1.92 | 1.87 | 1.82 | 1.77 | 1.71 |
| 26 | 4.23 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 | 2.22 | 2.15 | 2.07 | 1.99 | 1.95 | 1.90 | 1.85 | 1.80 | 1.75 | 1.69 |
| 27 | 4.21 | 3.35 | 2.96 | 2.73 | 2.57 | 2.46 | 2.37 | 2.31 | 2.25 | 2.20 | 2.13 | 2.06 | 1.97 | 1.93 | 1.88 | 1.84 | 1.79 | 1.73 | 1.67 |
| 28 | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.24 | 2.19 | 2.12 | 2.04 | 1.96 | 1.91 | 1.87 | 1.82 | 1.77 | 1.71 | 1.65 |

# Temperature Data

| | Sample A | Sample B | Sample C |
|---|---|---|---|
| 1 | 30.7 | 25.7 | 27.0 |
| 2 | 29.6 | 27.6 | 28.3 |
| 3 | 26.8 | 26.6 | 28.9 |
| 4 | 33.4 | 28.0 | 28.3 |
| 5 | 31.4 | 27.3 | 27.9 |
| 6 | 31.1 | 26.7 | 27.8 |
| 7 | 30.6 | 27.3 | 28.2 |
| 8 | 28.2 | 26.7 | 27.0 |
| 9 | 30.4 | 26.8 | 26.4 |
| 10 | 28.6 | 27.4 | 26.6 |
| 11 | 28.2 | 30.2 | 26.3 |
| 12 | 30.6 | 30.3 | 26.2 |
| 13 | 29.8 | 31.8 | 27.4 |
| 14 | 28.9 | 28.1 | 28.0 |
| 15 | 29.9 | 30.1 | 28.5 |
| 16 | 29.8 | 34.1 | 27.3 |
| 17 | 30.3 | 33.9 | 27.4 |
| 18 | 32.3 | 30.8 | 26.6 |
| 19 | 29.6 | 30.7 | 26.7 |
| 20 | 31.4 | 30.4 | 26.1 |
| **Mean** | 30.1 | 29.0 | 27.3 |
| **Stdev** | 1.5 | 2.5 | 0.9 |

**Research Question**: Is there is a significant difference between surface temperatures on pavement, cement and grass

Alternate Hypothesis

Significant Difference

Null Hypothesis

No Significant Difference

# Temperature Data

| | Sample A | Sample B | Sample C |
|---|---|---|---|
| 1 | 30.7 | 25.7 | 27.0 |
| 2 | 29.6 | 27.6 | 28.3 |
| 3 | 26.8 | 26.6 | 28.9 |
| 4 | 33.4 | 28.0 | 28.3 |
| 5 | 31.4 | 27.3 | 27.9 |
| 6 | 31.1 | 26.7 | 27.8 |
| 7 | 30.6 | 27.3 | 28.2 |
| 8 | 28.2 | 26.7 | 27.0 |
| 9 | 30.4 | 26.8 | 26.4 |
| 10 | 28.6 | 27.4 | 26.6 |
| 11 | 28.2 | 30.2 | 26.3 |
| 12 | 30.6 | 30.3 | 26.2 |
| 13 | 29.8 | 31.8 | 27.4 |
| 14 | 28.9 | 28.1 | 28.0 |
| 15 | 29.9 | 30.1 | 28.5 |
| 16 | 29.8 | 34.1 | 27.3 |
| 17 | 30.3 | 33.9 | 27.4 |
| 18 | 32.3 | 30.8 | 26.6 |
| 19 | 29.6 | 30.7 | 26.7 |
| 20 | 31.4 | 30.4 | 26.1 |
| **Mean** | 30.1 | 29.0 | 27.3 |
| **Stdev** | 1.5 | 2.5 | 0.9 |

$$\bar{X}_T = \frac{\sum n_i \bar{X}_i}{N}$$

28.8

$$SS_B = \sum n_i \left(\bar{X}_i - \bar{X}_T\right)^2$$

76.1

$$MS_B = \frac{SS_B}{df} = \frac{SS_B}{k-1}$$

38.1

$$SS_W = \sum (n-1)s^2$$

171.9

$$MS_W = \frac{SS_W}{df} = \frac{SS_W}{N-k}$$

3.01

$$F = \frac{MS_B}{MS_W}$$

12.6

3-1=2 Between group degrees of freedom *(k-1)*

60-3=57

Within group degrees of freedom *N-k*

Critical value of F for result to be significant at 95% confidence level

Significant if $F>F_{critical}$

Not Significant if $F \leq F_{critical}$

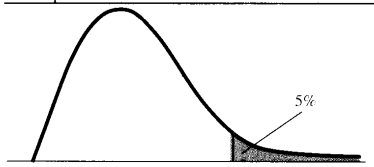


**Table E** Critical Values of the *F* Distribution

$\alpha = .05$

$df_1$

| $df_2$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 161 | 200 | 216 | 225 | 230 | 234 | 237 | 239 | 241 | 242 | 243 | 244 |
| 2 | 18.5 | 19.0 | 19.2 | 19.2 | 19.3 | 19.3 | 19.4 | 19.4 | 19.4 | 19.4 | 19.4 | 19.4 |
| 3 | 10.1 | 9.6 | 9.3 | 9.1 | 9.0 | 8.9 | 8.9 | 8.8 | 8.8 | 8.8 | 8.8 | 8.7 |
| 4 | 7.7 | 6.9 | 6.6 | 6.4 | 6.3 | 6.2 | 6.1 | 6.0 | 6.0 | 6.0 | 5.9 | 5.9 |
| 5 | 6.6 | 5.8 | 5.4 | 5.2 | 5.1 | 5.0 | 4.9 | 4.8 | 4.8 | 4.7 | 4.7 | 4.7 |
| 6 | 6.0 | 5.1 | 4.8 | 4.5 | 4.4 | 4.3 | 4.2 | 4.2 | 4.1 | 4.1 | 4.0 | 4.0 |
| 7 | 5.6 | 4.7 | 4.4 | 4.1 | 4.0 | 3.9 | 3.8 | 3.7 | 3.7 | 3.6 | 3.6 | 3.6 |
| 8 | 5.3 | 4.5 | 4.1 | 3.8 | 3.7 | 3.6 | 3.5 | 3.4 | 3.4 | 3.4 | 3.3 | 3.3 |
| 9 | 5.1 | 4.3 | 3.9 | 3.6 | 3.5 | 3.4 | 3.3 | 3.2 | 3.2 | 3.1 | 3.1 | 3.1 |
| 10 | 5.0 | 4.1 | 3.7 | 3.5 | 3.3 | 3.2 | 3.1 | 3.1 | 3.0 | 3.0 | 2.9 | 2.9 |
| 11 | 4.8 | 4.0 | 3.6 | 3.4 | 3.2 | 3.1 | 3.0 | 3.0 | 2.9 | 2.8 | 2.8 | 2.8 |
| 12 | 4.8 | 3.9 | 3.5 | 3.3 | 3.1 | 3.0 | 2.9 | 2.8 | 2.8 | 2.8 | 2.7 | 2.7 |
| 13 | 4.7 | 3.8 | 3.4 | 3.2 | 3.0 | 2.9 | 2.8 | 2.8 | 2.7 | 2.7 | 2.6 | 2.6 |
| 14 | 4.6 | 3.7 | 3.3 | 3.1 | 3.0 | 2.8 | 2.8 | 2.7 | 2.6 | 2.6 | 2.6 | 2.5 |
| 15 | 4.5 | 3.7 | 3.3 | 3.1 | 2.9 | 2.8 | 2.7 | 2.6 | 2.6 | 2.6 | 2.5 | 2.5 |
| 16 | 4.5 | 3.6 | 3.2 | 3.0 | 2.8 | 2.7 | 2.7 | 2.6 | 2.5 | 2.5 | 2.4 | 2.4 |
| 17 | 4.4 | 3.6 | 3.2 | 3.0 | 2.8 | 2.7 | 2.6 | 2.6 | 2.5 | 2.4 | 2.4 | 2.4 |
| 18 | 4.4 | 3.6 | 3.2 | 2.9 | 2.8 | 2.7 | 2.6 | 2.5 | 2.5 | 2.4 | 2.4 | 2.3 |
| 19 | 4.4 | 3.5 | 3.1 | 2.9 | 2.7 | 2.6 | 2.5 | 2.5 | 2.4 | 2.4 | 2.3 | 2.3 |
| 20 | 4.4 | 3.5 | 3.1 | 2.9 | 2.7 | 2.6 | 2.5 | 2.4 | 2.4 | 2.4 | 2.3 | 2.3 |
| 21 | 4.3 | 3.5 | 3.1 | 2.8 | 2.7 | 2.6 | 2.5 | 2.4 | 2.4 | 2.3 | 2.3 | 2.2 |
| 22 | 4.3 | 3.4 | 3.0 | 2.8 | 2.7 | 2.6 | 2.5 | 2.4 | 2.3 | 2.3 | 2.3 | 2.2 |
| 23 | 4.3 | 3.4 | 3.0 | 2.8 | 2.6 | 2.5 | 2.4 | 2.4 | 2.3 | 2.3 | 2.2 | 2.2 |
| 24 | 4.3 | 3.4 | 3.0 | 2.8 | 2.6 | 2.5 | 2.4 | 2.4 | 2.3 | 2.3 | 2.2 | 2.2 |
| 25 | 4.2 | 3.4 | 3.0 | 2.8 | 2.6 | 2.5 | 2.4 | 2.3 | 2.3 | 2.2 | 2.2 | 2.2 |
| 26 | 4.2 | 3.4 | 3.0 | 2.7 | 2.6 | 2.5 | 2.4 | 2.3 | 2.3 | 2.2 | 2.2 | 2.2 |
| 27 | 4.2 | 3.4 | 3.0 | 2.7 | 2.6 | 2.5 | 2.4 | 2.3 | 2.2 | 2.2 | 2.2 | 2.1 |
| 28 | 4.2 | 3.3 | 3.0 | 2.7 | 2.6 | 2.4 | 2.4 | 2.3 | 2.2 | 2.2 | 2.2 | 2.1 |
| 29 | 4.2 | 3.3 | 2.9 | 2.7 | 2.6 | 2.4 | 2.4 | 2.3 | 2.2 | 2.2 | 2.1 | 2.1 |
| 30 | 4.2 | 3.3 | 2.9 | 2.7 | 2.5 | 2.4 | 2.3 | 2.3 | 2.2 | 2.2 | 2.1 | 2.1 |
| 40 | 4.1 | 3.3 | 2.9 | 2.7 | 2.5 | 2.4 | 2.3 | 2.3 | 2.2 | 2.1 | 2.0 | 2.0 |
| 50 | 4.0 | 3.2 | 2.8 | 2.6 | 2.4 | 2.3 | 2.2 | 2.1 | 2.1 | 2.0 | 2.0 | 2.0 |
| 60 | 4.0 | 3.2 | 2.8 | 2.5 | 2.4 | 2.2 | 2.2 | 2.1 | 2.0 | 2.0 | 2.0 | 1.9 |
| 70 | 4.0 | 3.1 | 2.7 | 2.5 | 2.4 | 2.2 | 2.1 | 2.1 | 2.0 | 2.0 | 1.9 | 1.9 |
| 80 | 4.0 | 3.1 | 2.7 | 2.5 | 2.3 | 2.2 | 2.1 | 2.0 | 2.0 | 2.0 | 1.9 | 1.9 |
| 100 | 3.9 | 3.1 | 2.7 | 2.5 | 2.3 | 2.2 | 2.1 | 2.0 | 2.0 | 1.9 | 1.9 | 1.8 |
| 120 | 3.9 | 3.1 | 2.7 | 2.4 | 2.3 | 2.2 | 2.1 | 2.0 | 2.0 | 1.9 | 1.9 | 1.8 |
| ∞ | 3.8 | 3.0 | 2.6 | 2.4 | 2.2 | 2.1 | 2.0 | 1.9 | 1.9 | 1.8 | 1.8 | 1.8 |




From Leon F. Marzillier, *Elementary Statistics* ©1990 by Wm. C. Brown Publishers.

# Let’s watch a video…

https://www.youtube.com/watch?v=-yQb_ZJnFXw

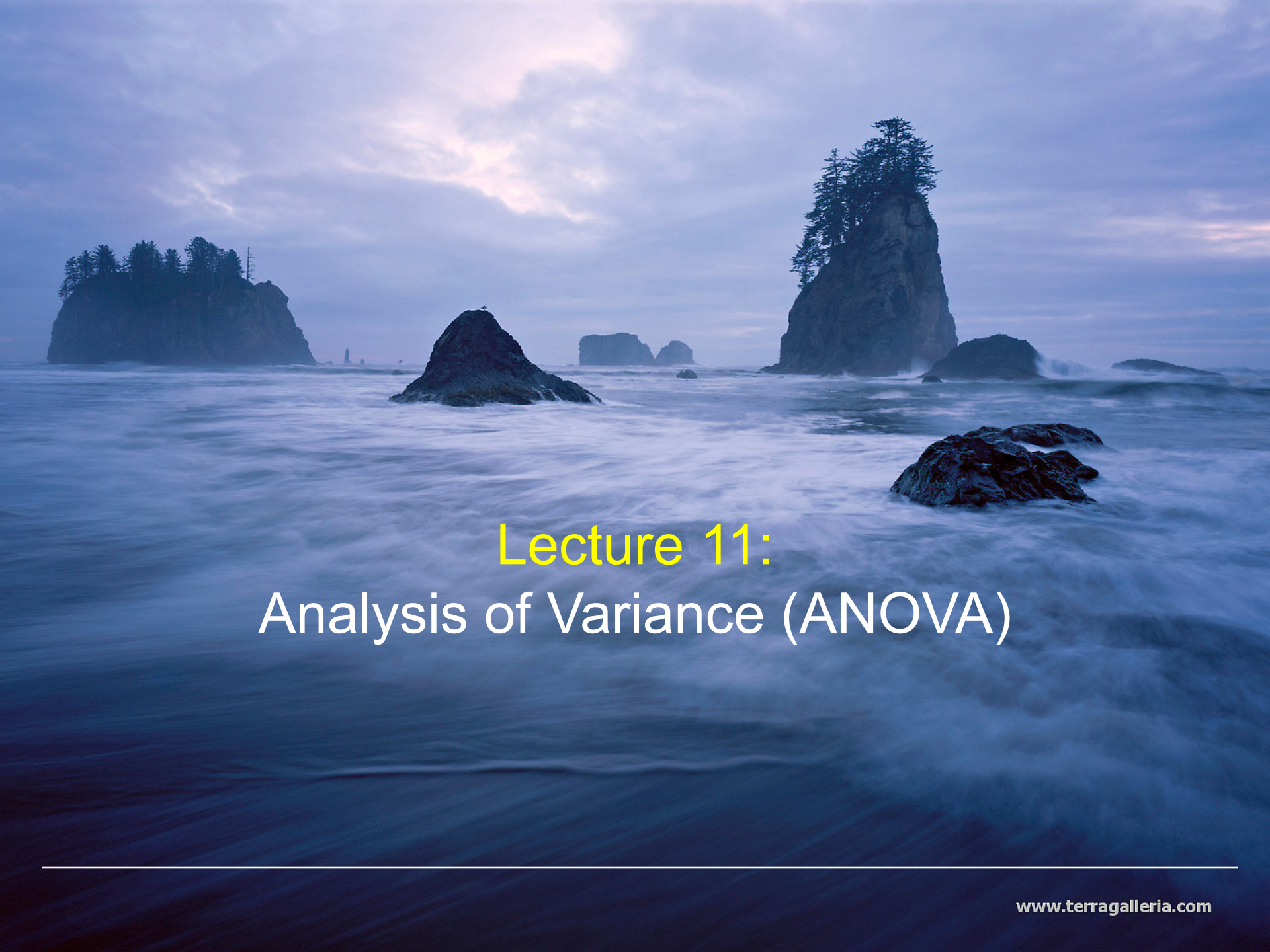
# Assumptions and Limitations

- Normality Assumption: The two populations are assumed to both follow a normal curve
- Independence Assumption: Two independent random samples
- Data Type Assumption: Interval or ratio data
- Variance Assumption: All samples have approximately the same variance

$$\frac{\text{Variability between groups}}{\text{Variability within groups}}$$

$$F = \frac{MS_B}{MS_W}$$

# Let’s open up R… again!



# Lecture 11:
# Analysis of Variance (ANOVA)