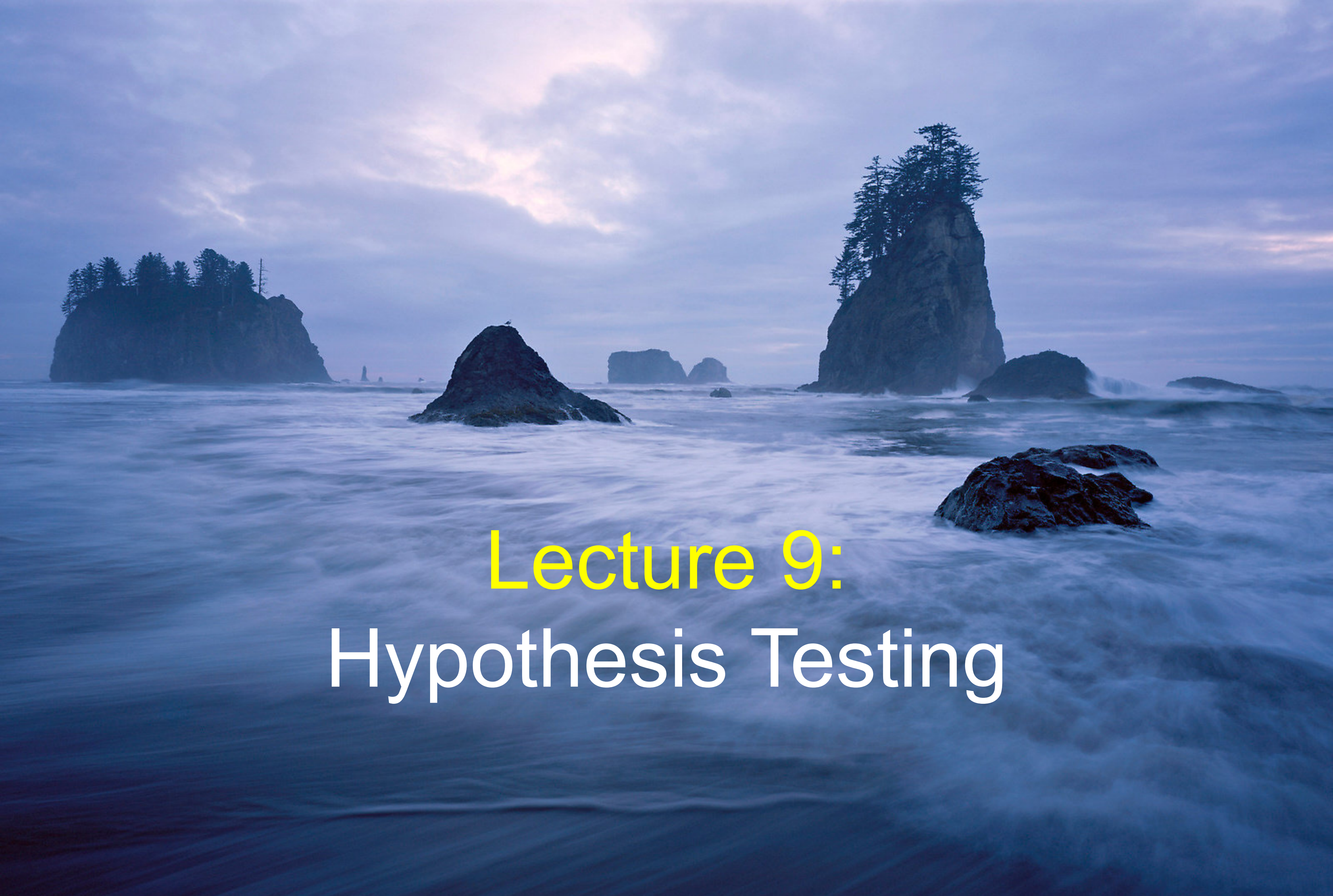

Lecture 9:
Hypothesis Testing

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Announcements

1. Please return the missing radiant thermometer, if you don’t, I will find you!

Reminders:

1. HW#3 will be assigned on Tuesday

Hypothesis Testing

- We want to make statements that are couched in likelihood of being true
- It is easier to show something is not true than to show it is true – falsifying is possible.
- Strategy: set up a specific (null) hypothesis which you intend to reject. If the null hypothesis is ACCEPTED there is still an unknown probability that it is wrong!
- But if it is rejected, we have determined *a priori* what the risk is of being wrong; we have preset the confidence limits

Kinds of Error

	Hypothesis Correct	**Hypothesis Incorrect**
Hypothesis Accepted	Correct Decision	Type II Error β
Hypothesis Rejected	Type I error α	Correct Decision

- α is a measure of percent risk we are willing to take or tolerate an incorrect rejection of the null hypothesis.
- For α = 0.05 (95% confidence) we are willing to accept an error 1 in 20 times doing a test.
- We do not specify β since it is unknown we cannot allow an “accepted” hypothesis. We say the test has failed.

Null Hypothesis

- We choose a null hypothesis that is specific
- We intend to reject the null hypothesis so we can ultimately accept the more general alternative
- If we CANNOT reject the null hypothesis we say: <u>Fail to Reject</u>! The test failed!
- We do <u>not</u> say that we accept the null hypothesis and reject the alternative

Research Question: Is there a difference in the color of the grass?

Alternate Hypothesis

$\mu_{intertidal} \neq \mu_{supratidal}$

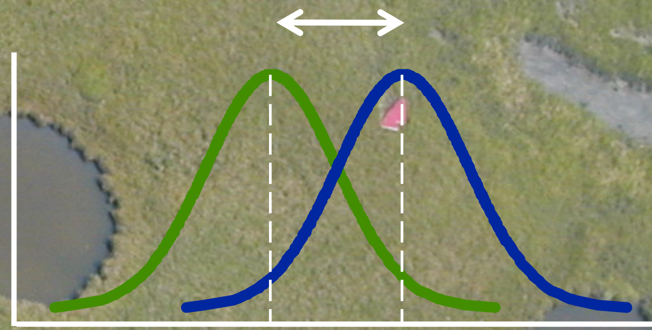
Significant Difference

Null Hypothesis

$\mu_{intertidal} = \mu_{supratidal}$

No Difference

Two-tailed Test



Research Question: Is the intertidal grass greener than the supratidal grass?

Alternate Hypothesis

$\mu_{intertidal} > \mu_{supratidal}$

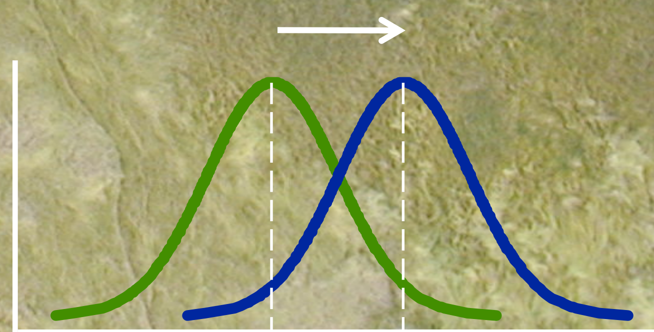
Significant Difference

Null Hypothesis

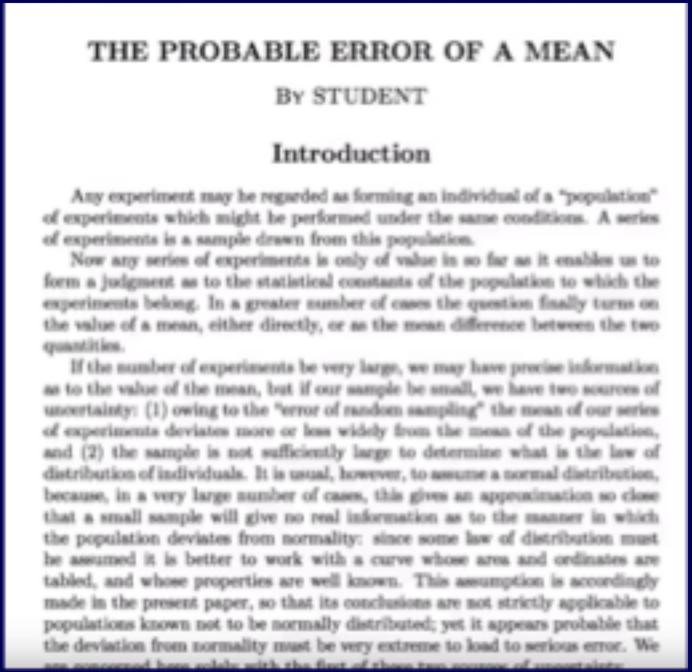
$\mu_{intertidal} \leq \mu_{supratidal}$

No Difference

One-tailed Test



Student's *t*-test



THE PROBABLE ERROR OF A MEAN

By STUDENT

Introduction

Any experiment may be regarded as forming an individual of a "population" of experiments which might be performed under the same conditions. A series of experiments is a sample drawn from this population.

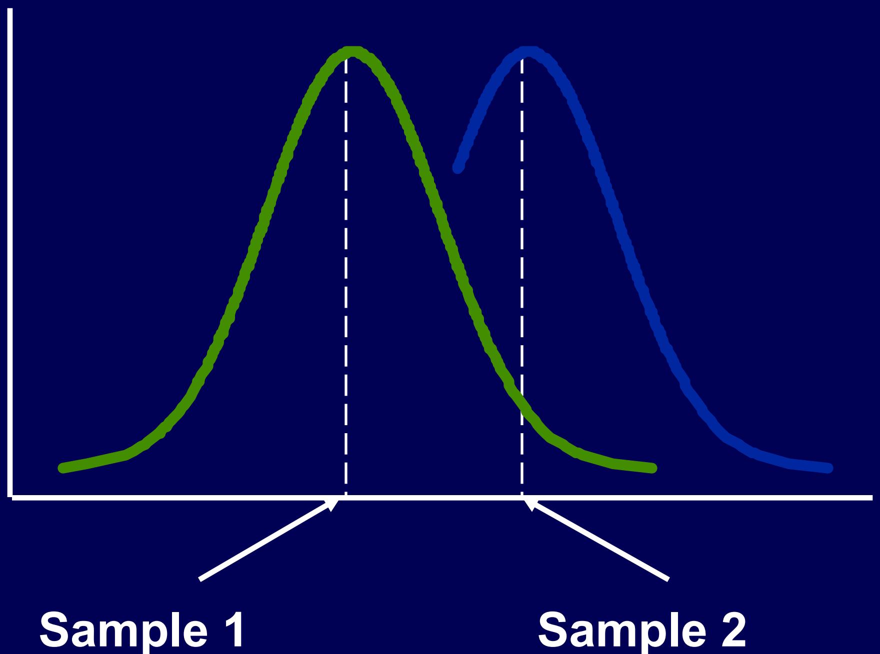
Now any series of experiments is only of value in so far as it enables us to form a judgment as to the statistical constants of the population to which the experiments belong. In a greater number of cases the question finally turns on the value of a mean, either directly, or as the mean difference between the two quantities.

If the number of experiments be very large, we may have precise information as to the value of the mean, but if our sample be small, we have two sources of uncertainty: (1) owing to the "error of random sampling" the mean of our series of experiments deviates more or less widely from the mean of the population, and (2) the sample is not sufficiently large to determine what is the law of distribution of individuals. It is usual, however, to assume a normal distribution, because, in a very large number of cases, this gives an approximation so close that a small sample will give no real information as to the manner in which the population deviates from normality: since some law of distribution must be assumed it is better to work with a curve whose area and ordinates are tabled, and whose properties are well known. This assumption is accordingly made in the present paper, so that its conclusions are not strictly applicable to populations known not to be normally distributed; yet it appears probable that the deviation from normality must be very extreme to load to serious error. We

William Sealy Gosset

Student's *t*-test

$$\frac{\text{Signal}}{\text{Noise}}$$

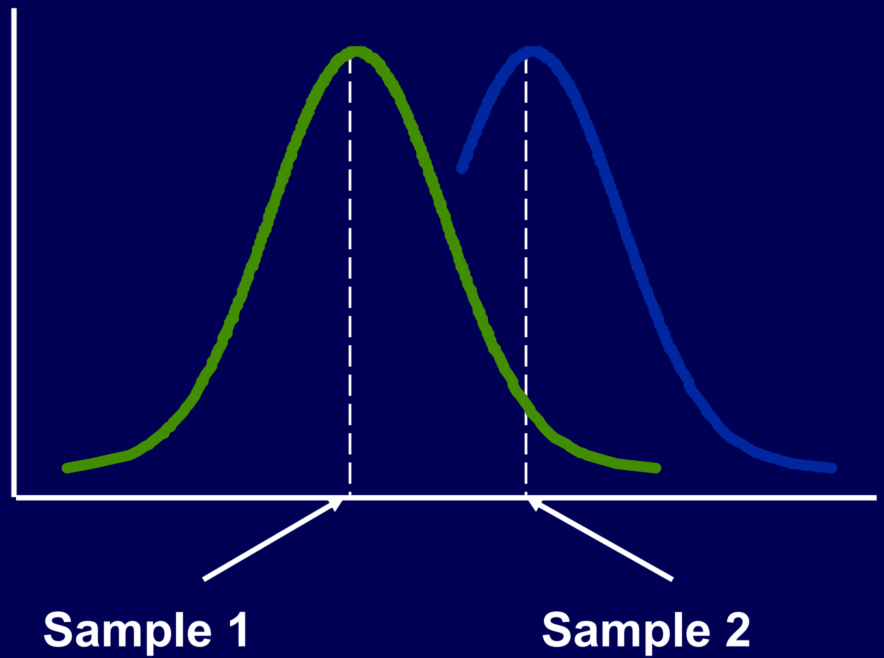


Is there a *difference*?

Student's *t*-test

$$\frac{\text{Signal}}{\text{Noise}}$$

$$\frac{\text{Difference between group means}}{\text{Variability of groups}}$$



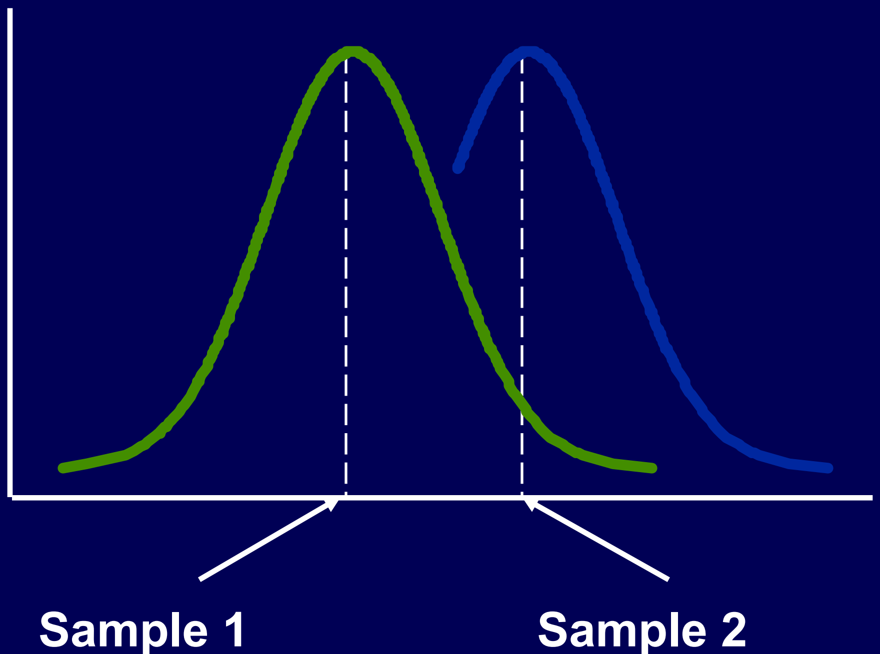
Is there a *difference*?

Student's *t*-test

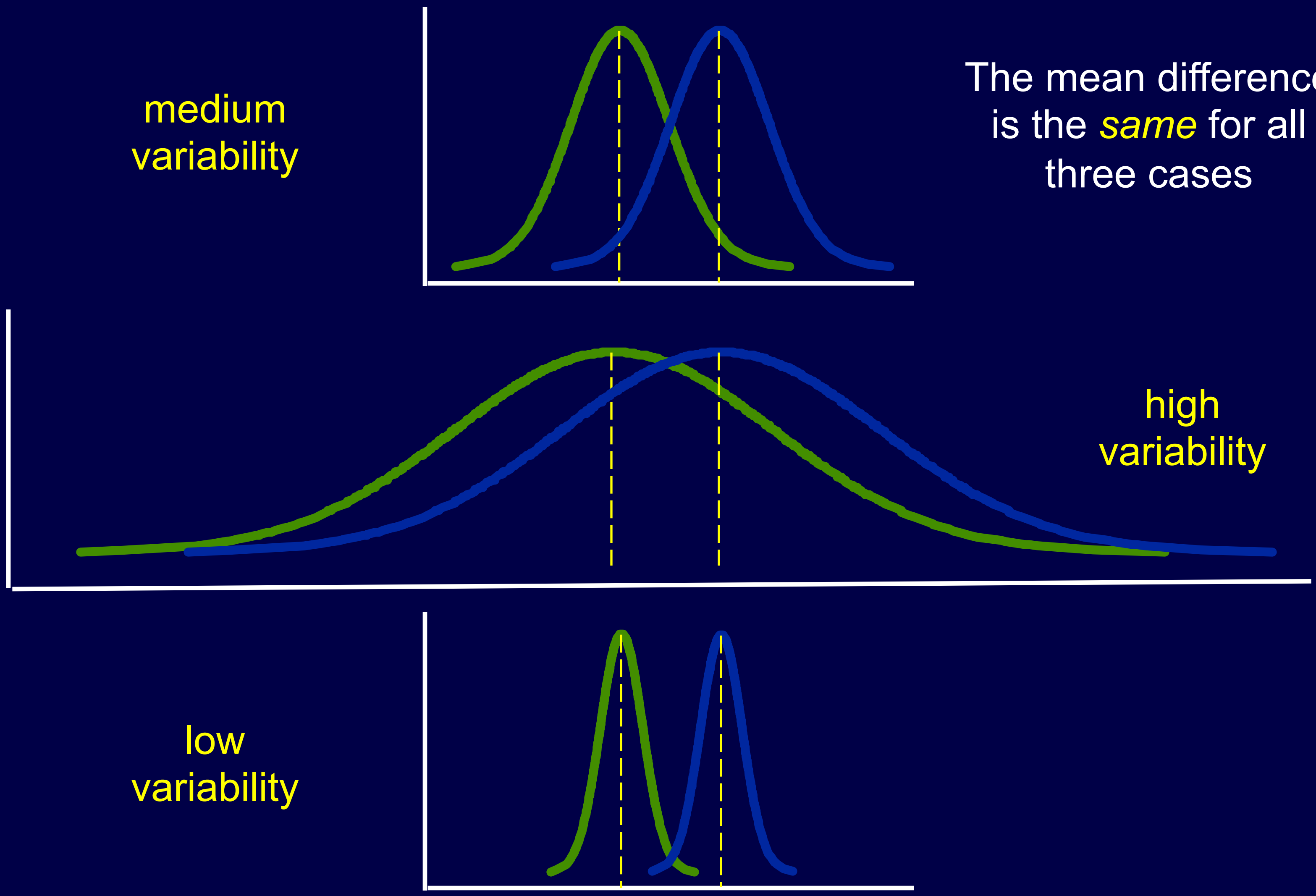
$$\frac{\text{Signal}}{\text{Noise}}$$

$$\frac{\text{Difference between group means}}{\text{Variability of groups}}$$

$$\frac{\text{Observed difference}}{\text{Expected difference based on chance}}$$



Is there a *difference*?



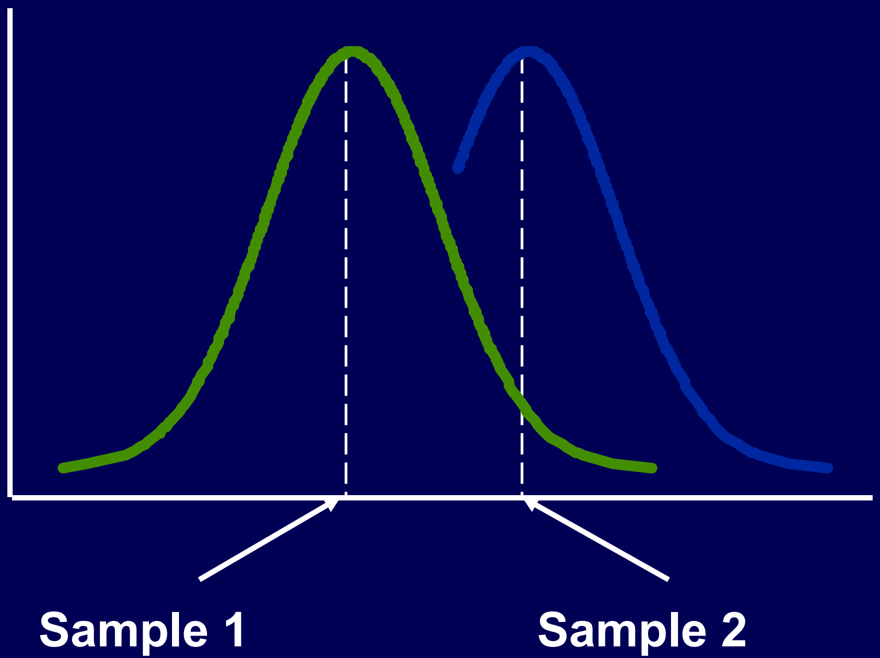
The mean difference is the *same* for all three cases

Student's *t*-test

$$\frac{\text{Signal}}{\text{Noise}}$$

$$\frac{\text{Difference between group means}}{\text{Variability of groups}}$$

$$t = \frac{\left(\bar{x}_1 - \bar{x}_2\right)}{SE_{\bar{x}_1 - \bar{x}_2}} = \frac{\left(\bar{x}_1 - \bar{x}_2\right)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$



Is there a *difference*?

Tree DBH Data

	Sample A	Sample B
1	56.8	21.7
2	60.2	51.8
3	47.4	41.3
4	48.4	27.3
5	50	50
6	18.6	32.3
7	64.6	59.6
8	99	62.2
9	63.3	25.5
10	32.1	46
11	79.3	17.5
12	63.7	22.9
13	50.6	50.7
14	25.5	63.8
15	42.7	47.5
16	36.9	58.6
17	25.5	80
18	49	53.4
19	58.9	39.3
20	55.7	35.7
Mean	**51.4**	**46.4**
Stdev	**18.9**	**18.6**

Research Question: Is there is a significant difference between tree diameters at the 95% confidence level

Alternate Hypothesis

$\mu_{\text{sample 1}} \neq \mu_{\text{sample 2}}$

Significant Difference

Null Hypothesis

$\mu_{\text{sample 1}} = \mu_{\text{sample 2}}$

No Difference

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{SE_{\bar{x}_1 - \bar{x}_2}} = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

t Table

Confidence levels
1-α

Degrees of freedom
$df = (n_1+n_2 - 2)$

Critical value of t for result to be different

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{SE_{\bar{x}_1 - \bar{x}_2}} = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

	PROPORTION IN ONE TAIL					
	0.25	0.10	0.05	0.025	0.01	0.005
	PROPORTION IN TWO TAILS COMBINED					
df	0.50	0.20	0.10	0.05	0.02	0.01
1	1.000	3.078	6.314	12.706	31.821	63.657
2	0.816	1.886	2.920	4.303	6.965	9.925
3	0.765	1.638	2.353	3.182	4.541	5.841
4	0.741	1.533	2.132	2.776	3.747	4.604
5	0.727	1.476	2.015	2.571	3.365	4.032
6	0.718	1.440	1.943	2.447	3.143	3.707
7	0.711	1.415	1.895	2.365	2.998	3.499
8	0.706	1.397	1.860	2.306	2.896	3.355
9	0.703	1.383	1.833	2.262	2.821	3.250
10	0.700	1.372	1.812	2.228	2.764	3.169
11	0.697	1.363	1.796	2.201	2.718	3.106
12	0.695	1.356	1.782	2.179	2.681	3.055
13	0.694	1.350	1.771	2.160	2.650	3.012
14	0.692	1.345	1.761	2.145	2.624	2.977
15	0.691	1.341	1.753	2.131	2.602	2.947
16	0.690	1.337	1.746	2.120	2.583	2.921
17	0.689	1.333	1.740	2.110	2.567	2.898
18	0.688	1.330	1.734	2.101	2.552	2.878
19	0.688	1.328	1.729	2.093	2.539	2.861
20	0.687	1.325	1.725	2.086	2.528	2.845
21	0.686	1.323	1.721	2.080	2.518	2.831
22	0.686	1.321	1.717	2.074	2.508	2.819
23	0.685	1.319	1.714	2.069	2.500	2.807
24	0.685	1.318	1.711	2.064	2.492	2.797
25	0.684	1.316	1.708	2.060	2.485	2.787
26	0.684	1.315	1.706	2.056	2.479	2.779
27	0.684	1.314	1.703	2.052	2.473	2.771
28	0.683	1.313	1.701	2.048	2.467	2.763
29	0.683	1.311	1.699	2.045	2.462	2.756
30	0.683	1.310	1.697	2.042	2.457	2.750
40	0.681	1.303	1.684	2.021	2.423	2.704
60	0.679	1.296	1.671	2.000	2.390	2.660
120	0.677	1.289	1.658	1.980	2.358	2.617
∞	0.674	1.282	1.645	1.960	2.326	2.576

Let's open up R…

t Table

Confidence levels
1-α

Degrees of freedom
df = (n_1+n_2 - 2)

Critical value of *t* for result to be different

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{SE_{\bar{x}_1 - \bar{x}_2}} = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

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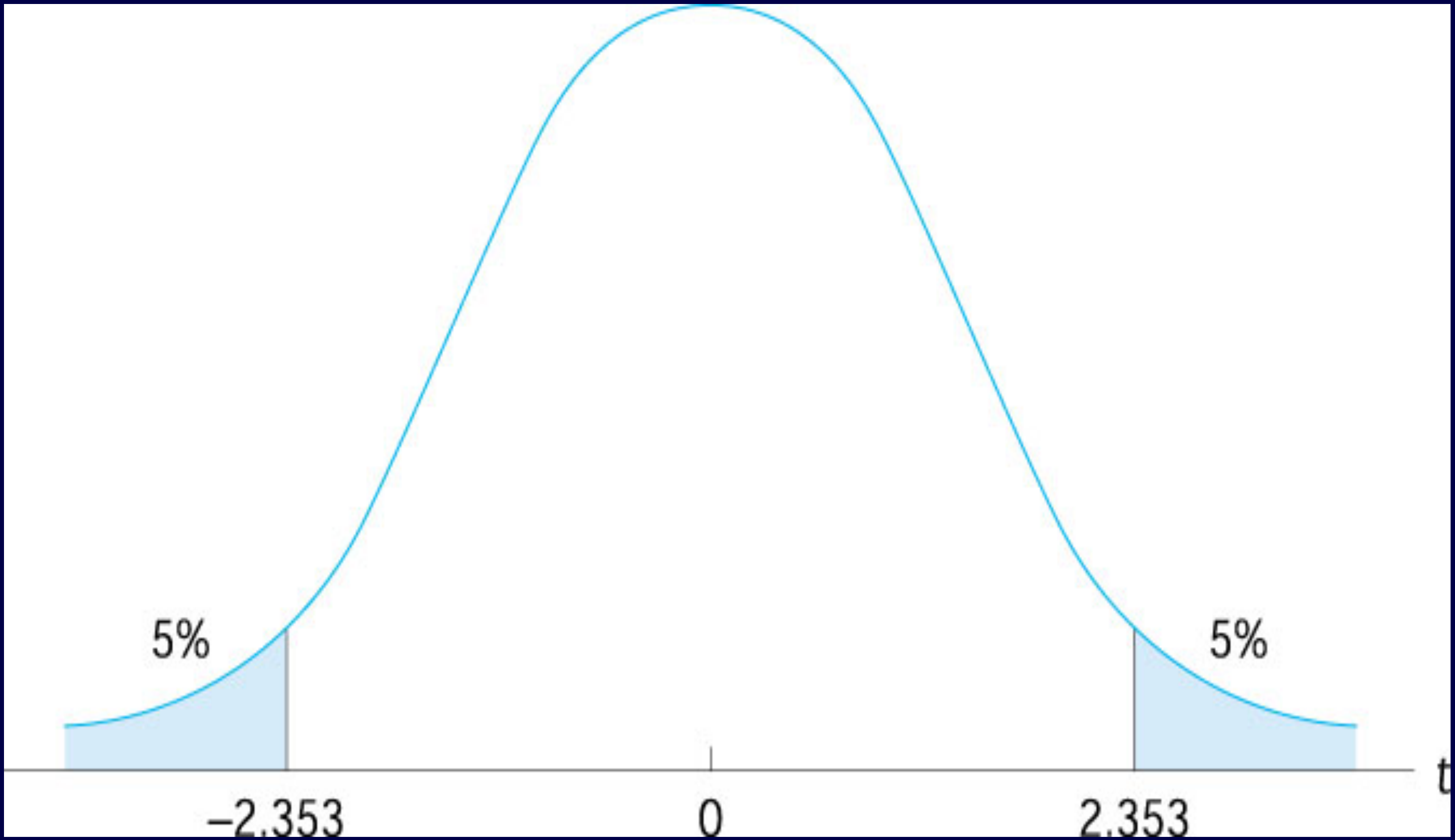
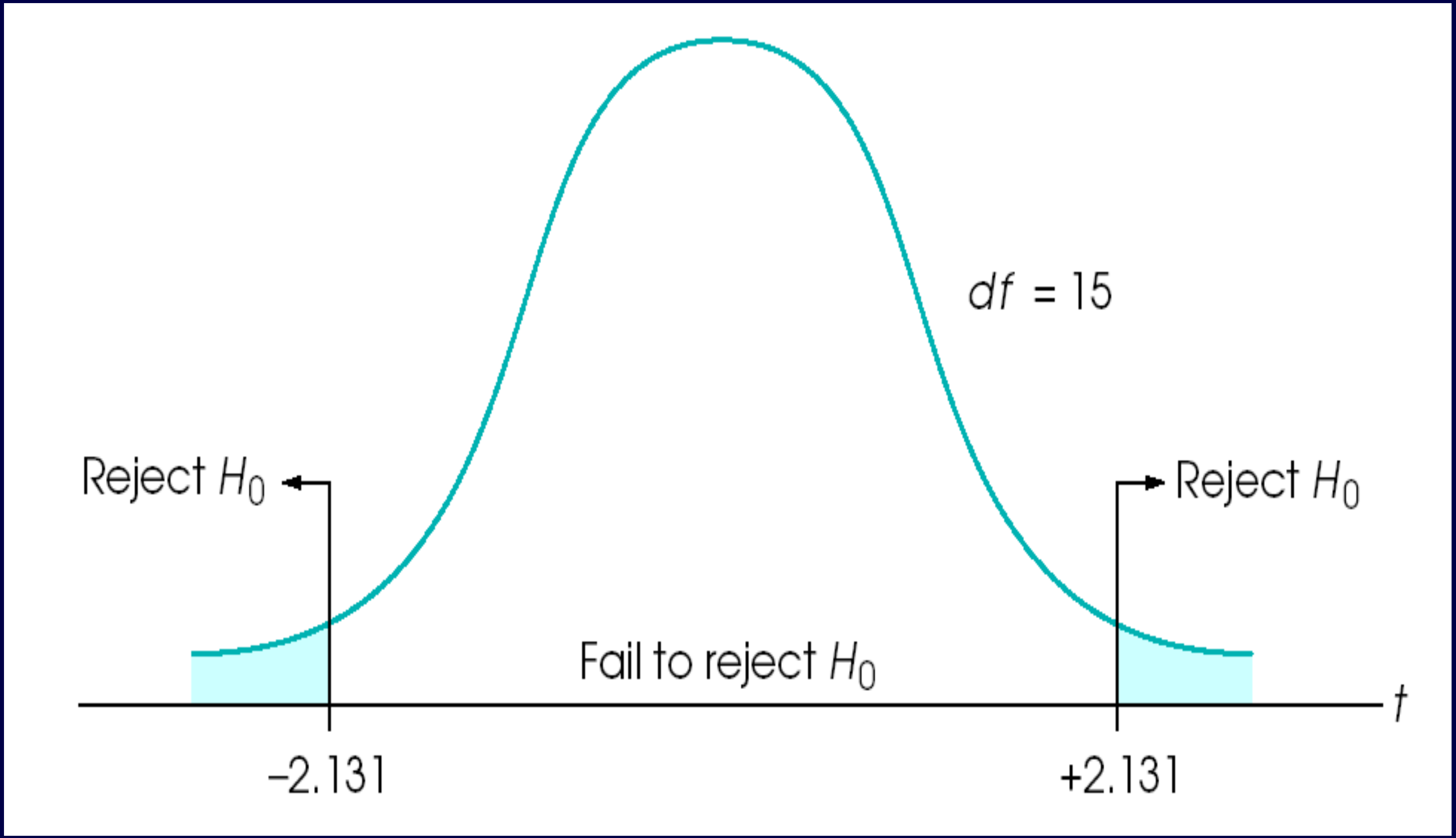
Finding Critical Values

The t-distribution for df = 3, 2-tailed α = 0.10



Finding Critical Values

The t-distribution for df =15, 2-tailed α = 0.05



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∞	0.674	1.282	1.645	1.960	2.326	2.576

Reporting Your Results

- If $t > t_{critcal}$ then the null hypothesis is rejected
  - "There is a statistically significant difference between the two sample groups at the 95% confidence level."
- If $t \leq t_{critcal}$ then the null hypothesis is accepted
  - "There is no statistically significant difference between the two sample groups at the 95% confidence level."

t-test Assumptions and Limitations

- Size Assumption: Sample < 30

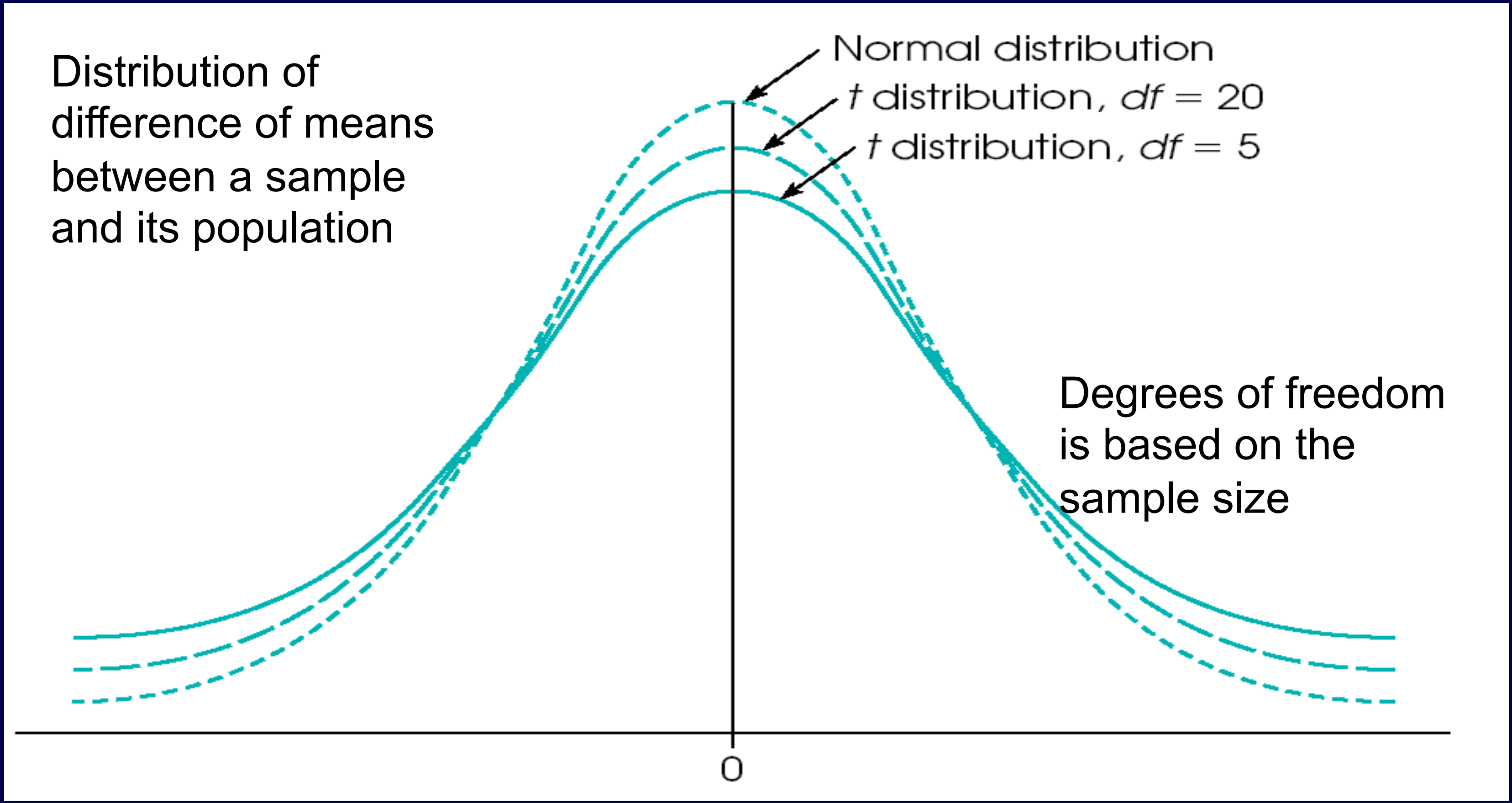
- Normality Assumption: The two populations are assumed to both follow a normal curve

- Independence Assumption: Two independent random samples

- Data Type Assumption: Interval or ratio data

$$t = \frac{\left(\overline{x}_1 - \overline{x}_2\right)}{SE_{\overline{x}_1 - \overline{x}_2}} = \frac{\left(\overline{x}_1 - \overline{x}_2\right)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Survey Results…



The larger the *df* (the sample size), the more closely the *t* distribution approximates a normal distribution- can use a z-test rather than a t-test

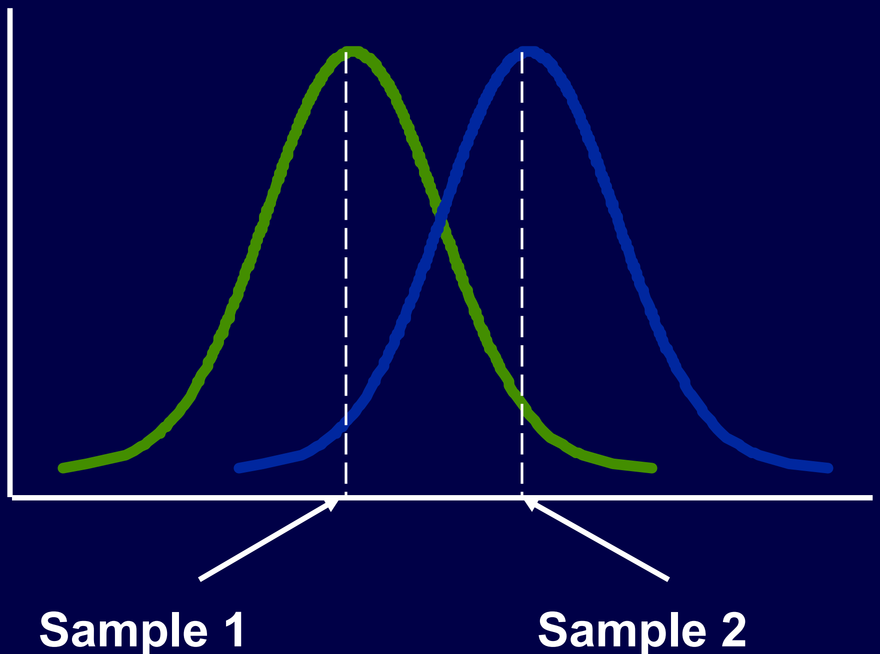
z-test

$$\frac{\text{signal}}{\text{noise}}$$

$$\frac{\text{difference between group means}}{\text{variability of groups}}$$

$$\frac{\text{Observed difference}}{\text{Expected difference based on chance}}$$

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{SE_{\bar{x}_1 - \bar{x}_2}} = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$



Is there a *difference*?

Z Table

Critical Value of Z →

Probability of getting that value →

$$Z = \frac{(\bar{x}_1 - \bar{x}_2)}{SE_{\bar{x}_1 - \bar{x}_2}} = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

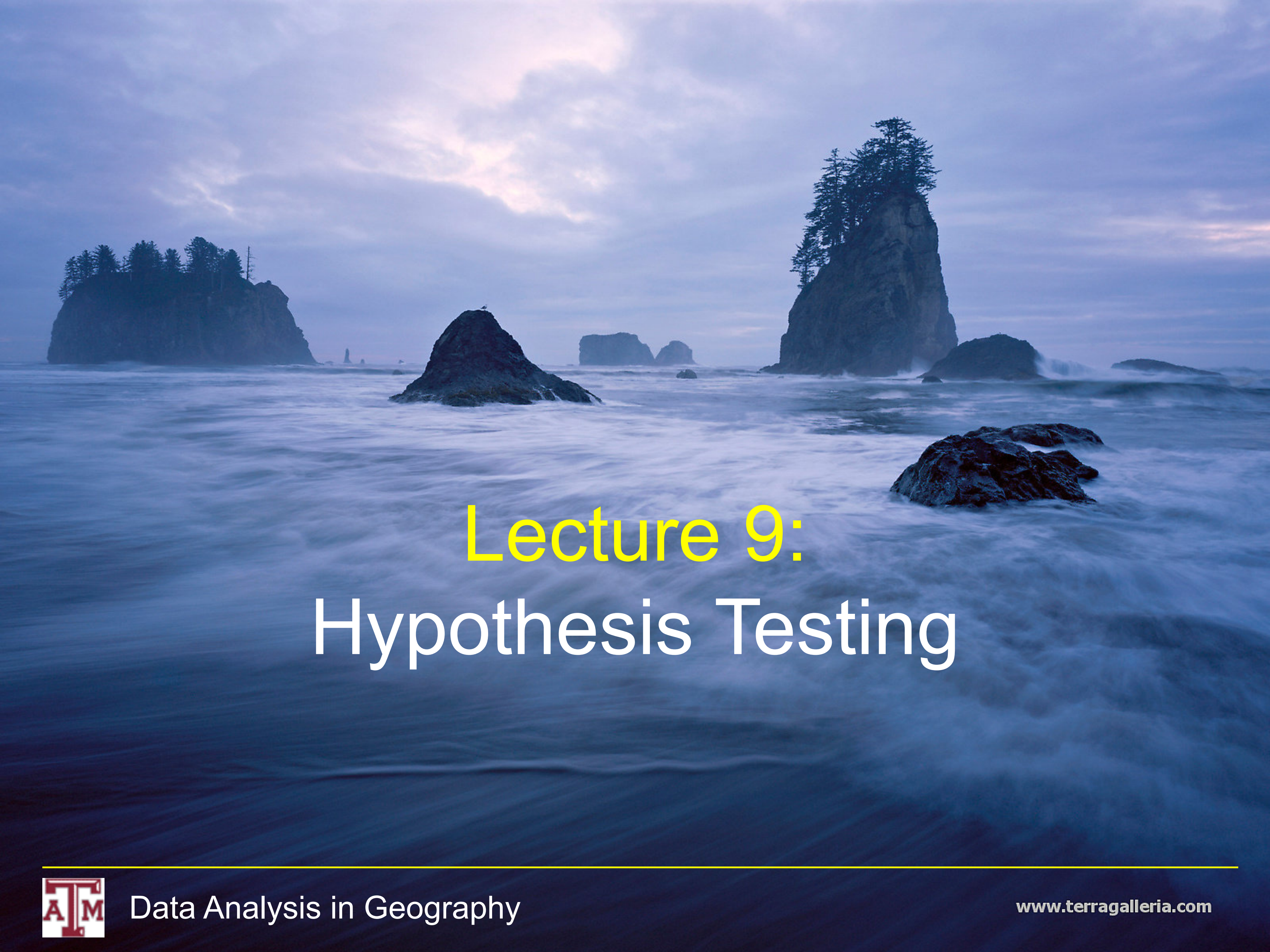
	Second decimal place of z									
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010

Adapted with rounding from Table II of Fisher and Yates 1974.

Assumptions and Limitations

- Size Assumption: Sample >30
- Normality Assumption: The two populations are assumed to both follow a normal curve
- Independence Assumption: Two independent random samples
- Data Type Assumption: Interval or ratio data

$$Z = \frac{\left(\bar{x}_1 - \bar{x}_2\right)}{SE_{\bar{x}_1 - \bar{x}_2}} = \frac{\left(\bar{x}_1 - \bar{x}_2\right)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$



Lecture 9:
Hypothesis Testing
Data Analysis in Geography
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