线性回归：$\hat{y} = x^T w + b$

$E = \frac{1}{2}(\hat{y} - y)^2$ 单样本

$\therefore dw = \frac{\partial E}{\partial w} = \frac{\partial E}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w} = (\hat{y} - y) \cdot x^T$

$db = \frac{\partial E}{\partial b} = (\hat{y} - y)$

$w = w - \alpha dw$

$b = b - \alpha db$

多样本，dw, db 分别累加求均值.

逻辑回归：$\hat{y} = \frac{1}{1+e^{-z}}$，$z = x^T w + b$

$\ln \frac{y}{1-y} = z$，$p(y|x) = \hat{y}^y \cdot (1-\hat{y})^{1-y}$

$E = -[y \log \hat{y} + (1-y) \log(1-\hat{y})]$ …… 单样本

$dw = \frac{\partial E}{\partial w} = \frac{\partial E}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial z}{\partial w}$

$= -\left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}}\right) \cdot \hat{y}(1-\hat{y}) \cdot x^T$

$= (\hat{y} - y) x^T$

$db = \hat{y} - y$

$w = w - \alpha dw$，$b = b - \alpha db$

LDA 核心最大化类间距离最小化类内距离.

$$\max_w J(w) = \frac{\|w^T \mu_1 - w^T \mu_2\|}{\sum_{i=1, x \in X_i}^{N} w^T (x^{(i)} - \mu_i)(x^{(i)} - \mu_i)^T w}$$

$$= \frac{w^T (\mu_1 - \mu_2)(\mu_1 - \mu_2)^T w}{w^T \sum_{i=1}^{N} (x_i - \mu_i)(x_i - \mu_i)^T w}$$

$S_B = (\mu_1 - \mu_2)(\mu_1 - \mu_2)^T$，$S_w = \sum_{i=1}^{N} (x^{(i)} - \mu_i)(x^{(i)} - \mu_i)^T$

$$\max_w J(w) = \frac{w^T S_B w}{w^T S_w w}$$

$\frac{\partial J(w)}{\partial w} = 0 \Rightarrow S_B w \cdot (w^T S_w w) = S_w w (w^T S_B w)$

$S_B w = S_w w \cdot J$

此时 J 为常数 . $S_B w = \lambda S_w w$，$S_w^{-1} S_B w = \lambda w$

J 即求 $S_w^{-1} S_B$ 最大特征值，w 特征向量.

$S_B = (\mu_1 - \mu_2)(\mu_1 - \mu_2)^T$　　　$w = S_w^{-1} (\mu_1 - \mu_2)$

对于高维 S_w 不变，

$S_B = S_{全局} - S_w = \sum_{i=1}^{N} (x^i - \mu_{全})(x^i - \mu_{全})^T - \sum_{i=1}^{N} (x^{(i)} - \mu_i)(x^{(i)} - \mu_i)^T$

$= \sum_{i=1}^{N} m_i (\mu_i - \mu_{全})(\mu_i - \mu_{全})^T$