Rappels sur le spin : spin ½

• Représentation des états

Opérateurs de spin : $\hat{S}_x = \frac{\hbar}{2}\hat{\sigma}_x \qquad \hat{S}_y = \frac{\hbar}{2}\hat{\sigma}_y \qquad \hat{S}_z = \frac{\hbar}{2}\hat{\sigma}_z$

Etats propres associés : $\begin{cases}\hat{\sigma}_x|x+\rangle = |x+\rangle \\ \hat{\sigma}_x|x-\rangle = -|x-\rangle\end{cases} \qquad \begin{cases}\hat{\sigma}_y|y+\rangle = |y+\rangle \\ \hat{\sigma}_y|y-\rangle = -|y-\rangle\end{cases} \qquad \begin{cases}\hat{\sigma}_z|z+\rangle = |z+\rangle \\ \hat{\sigma}_z|z-\rangle = -|z-\rangle\end{cases}$

• Dans la base $\{|z+\rangle, |z-\rangle\}$ (états propres de $\hat{S}_z$)

$$|z+\rangle = \begin{pmatrix}1\\0\end{pmatrix} \quad \text{et} \quad |z-\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

$$\begin{cases}|x+\rangle = a|z+\rangle + b|z-\rangle = \begin{pmatrix}a\\b\end{pmatrix} \\ |x-\rangle = a'|z+\rangle + b'|z-\rangle = \begin{pmatrix}a'\\b'\end{pmatrix}\end{cases}$$

Matrices de Pauli : $\hat{\sigma}_x = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} \qquad \hat{\sigma}_y = \begin{pmatrix}0 & -i\\i & 0\end{pmatrix} \qquad \hat{\sigma}_z = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$

Par conséquent,

$$\begin{cases}\hat{\sigma}_x|x+\rangle = |x+\rangle \Leftrightarrow \hat{\sigma}_x\begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}a\\b\end{pmatrix} \Leftrightarrow \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}a\\b\end{pmatrix} \\ \langle x+|x+\rangle = |a|^2\langle z+|z+\rangle + |b|^2\langle z-|z-\rangle = |a|^2 + |b|^2 = 1\end{cases}$$

$$\Rightarrow \begin{cases}a = b \\ |a|^2 + |b|^2 = 1\end{cases} \Rightarrow a = b = \frac{1}{\sqrt{2}}$$

$$\begin{cases}\hat{\sigma}_x|x-\rangle = -|x-\rangle \Leftrightarrow \hat{\sigma}_x\begin{pmatrix}a'\\b'\end{pmatrix} = -\begin{pmatrix}a'\\b'\end{pmatrix} \Leftrightarrow \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}a'\\b'\end{pmatrix} = -\begin{pmatrix}a'\\b'\end{pmatrix} \\ \langle x-|x-\rangle = |a'|^2\langle z+|z+\rangle + |b'|^2\langle z-|z-\rangle = |a'|^2 + |b'|^2 = 1\end{cases}$$

$$\Rightarrow \begin{cases}a' = -b' \\ |a'|^2 + |b'|^2 = 1\end{cases} \Rightarrow a' = -b' = \frac{1}{\sqrt{2}}$$

d'où $\begin{cases}|x+\rangle = \frac{1}{\sqrt{2}}(|z+\rangle + |z-\rangle) \\ |x-\rangle = \frac{1}{\sqrt{2}}(|z+\rangle - |z-\rangle)\end{cases}$