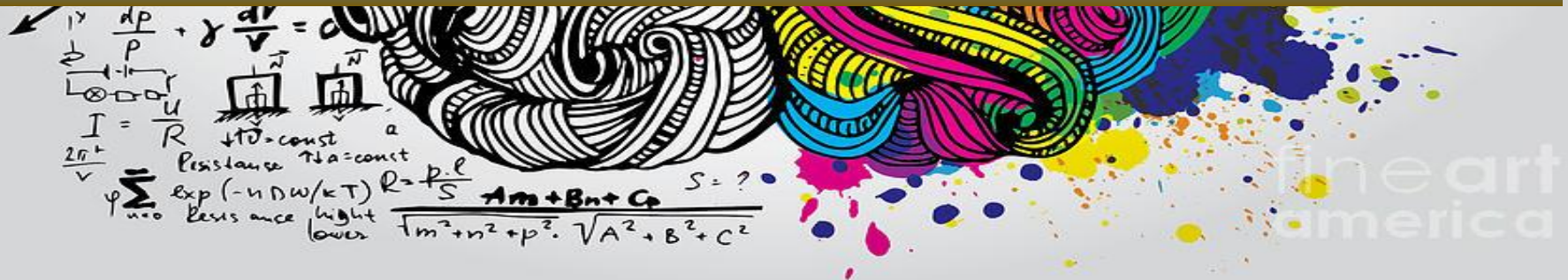


CorticalFlow: A Diffeomorphic Mesh Transformer for Cortical Surface Reconstruction

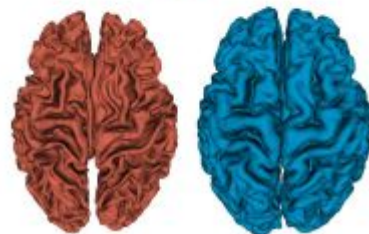
Authors: Leo Lebrat, Rodrigo Santa Cruz, Frederic de Gournay, Darren Fu, Pierrick Bourgeat, Jurgen Fripp, Clinton Fookes, and Olivier Salvado
 Presentation by: William Ashbee



Cortical Surface Reconstruction From MRI (CSR)

Given: Structural Image (mri)

Find: Surface model (mesh, spline, etc.)



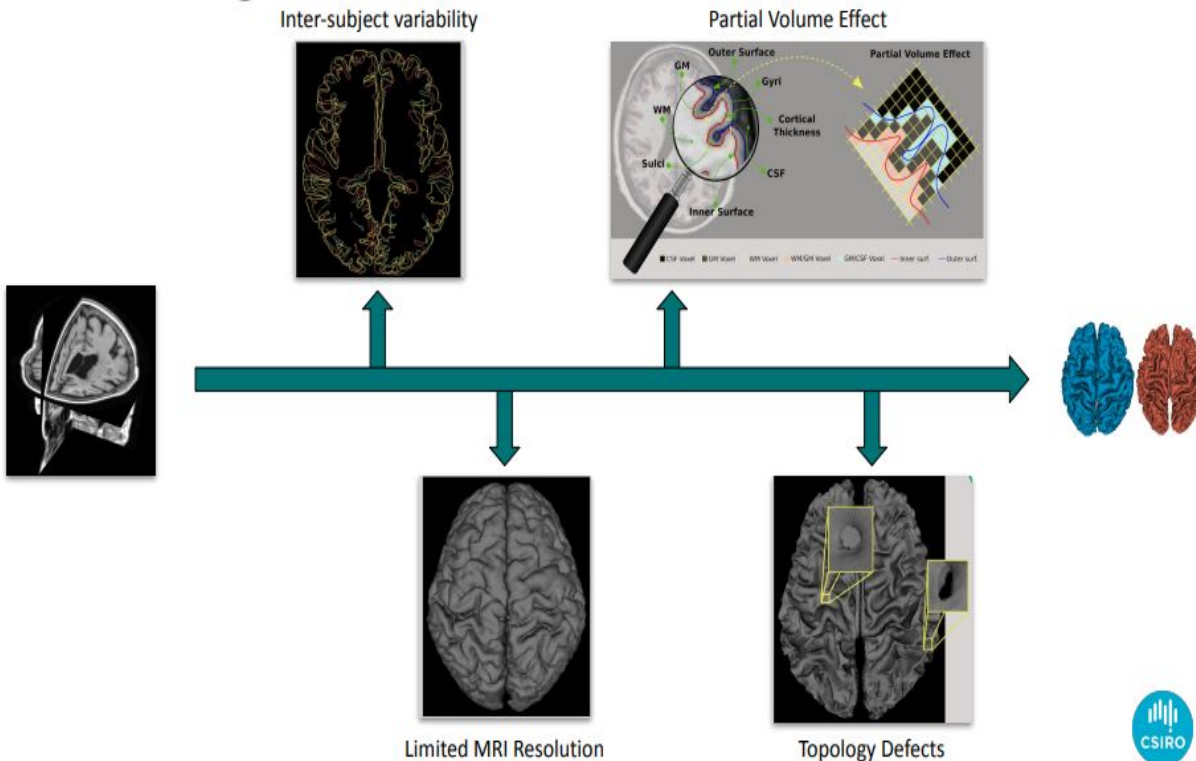
Cortical Surface Reconstruction Challenges

Inter Subject Variability: brain folds as unique as a **fingerprint**

Partial Volume effects: **tight corners** can easily be missed by surface algorithms

Resolution: MRIs are never high enough resolution or free of artifacts

Topological defects: e.g. self intersections



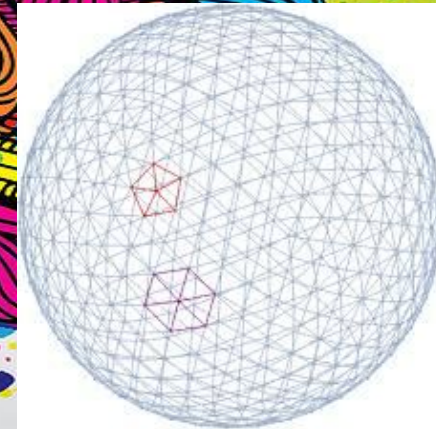
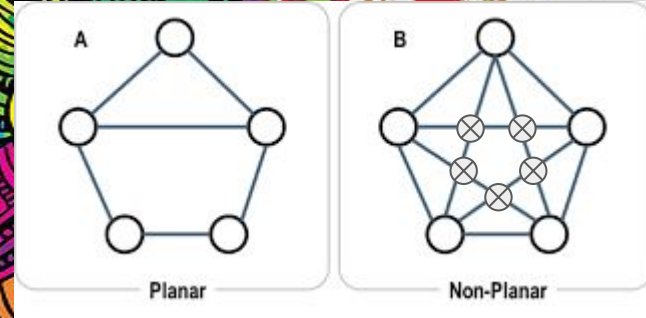
Challenges

Self intersections:

Meshes can't be said to be **planar graphs**, but they do strive to avoid **self intersections**

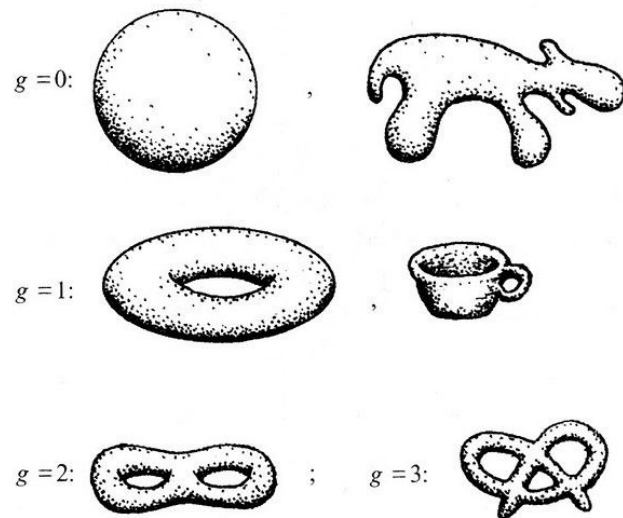
Topology correction algorithms in post processing are **expensive**

Cheaper to have **theoretical guarantees** during building of mesh



Related work

DeepCSR first predicts **implicit surface functions** and then employs an **iso-surface extraction** method along with a topology correction algorithm to obtain **genus zero** surfaces without handles or holes



$$I = \frac{U}{R} \quad \downarrow \text{TD} = \text{const} \quad a$$

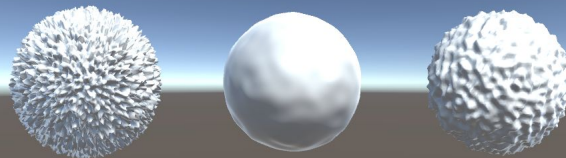
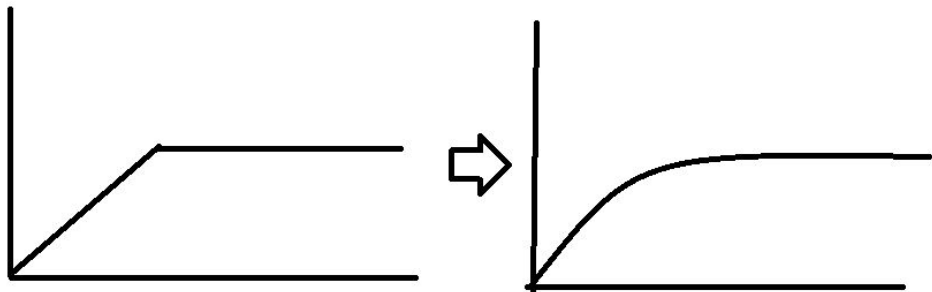
Resistance $\pi a = \text{const}$

$$\sum_{n=0}^{\infty} \exp(-n \pi D W / \kappa T) R = \frac{p \ell}{S} \frac{A m + B n + C p}{\sqrt{m^2 + n^2 + p^2} \sqrt{A^2 + B^2 + C^2}} \quad S = ?$$

Series $\sum_{n=0}^{\infty} \exp(-n \pi D W / \kappa T)$ Basis $\frac{p \ell}{S}$ $\frac{A m + B n + C p}{\sqrt{m^2 + n^2 + p^2} \sqrt{A^2 + B^2 + C^2}}$

Related work

Voxel2Mesh extends the vertex-wise template deformation approach of Wang et al. by optimizing several **mesh-smoothing penalty functions**



Related work

NMF builds an invertible mapping that enforces topology conservation upon the resolution of an Ordinary Differential Equation (ODE) through a sequence of residual blocks called **Neural Ordinary Differential Equation (NODE)**

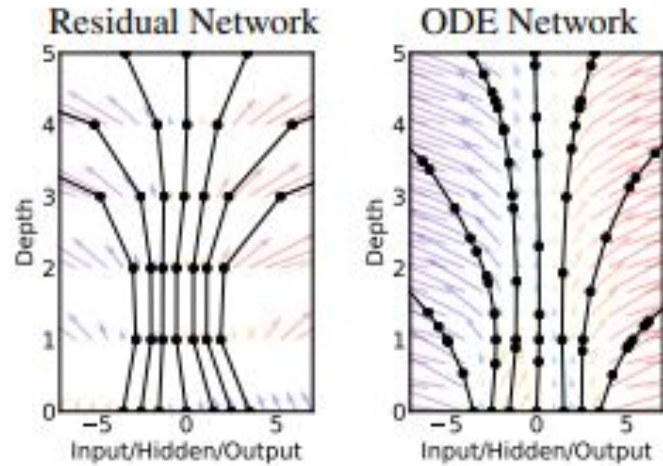
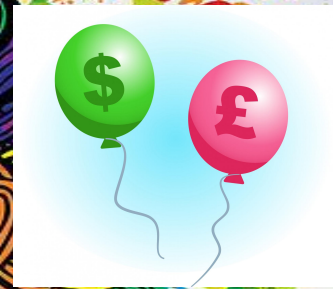


Figure 1: *Left:* A Residual network defines a discrete sequence of finite transformations. *Right:* A ODE network defines a vector field, which continuously transforms the state. *Both:* Circles represent evaluation locations.

Related work (Limitations)

The topology correction algorithm employed by DeepCSR is computationally **expensive** and is **blind to anatomy**.

Voxel2Mesh and NMF rely on time-demanding and vertex dependent building blocks such as graph convolution that **do not scale up well** as the number of **vertices** in the template mesh increases to accommodate complex shapes



Method

CorticalFlow (CF) is a multi-level deep learning architecture composed of several **Diffeomorphic Mesh Deformation** (DMD) modules.

It takes as input a 3-dimensional **Magnetic Resonance Image** (MRI) of a patient brain and a template.

CorticalFlow outputs the surface representation of an anatomical substructure by **composing stackable diffeomorphic deformations** generated by DMD modules



$$\begin{aligned} CF_{\theta_1}^1(\mathbf{I}, \mathcal{T}_1) &= \text{DMD}(\text{UNet}_{\theta_1}^1(\mathbf{I}), \mathcal{T}_1)) \\ CF_{\theta_{i+1}}^{i+1}(\mathbf{I}, \mathcal{T}_{i+1}) &= \text{DMD}(\text{UNet}_{\theta_{i+1}}^{i+1}(\mathbf{U}_1 \circ \dots \circ \mathbf{U}_i \circ \mathbf{I}), CF_i(\mathbf{I}, \mathcal{T}_{i+1})) \quad \text{for } i \geq 1, \end{aligned} \quad (2)$$

Method (objective)

CorticalFlow is trained in a supervised fashion, given a dataset D composed of pairs of MR-image I and triangle mesh S representing a cortical structure and for $i \in \{1, 2, 3\}$ we optimize the following objective

$$\arg \min_{\theta_i} \sum_{(I, S) \in D} \mathcal{L}(\text{CF}_{\theta_i}^i(I, T_i), S).$$

Method (Diffeomorphic deformation module--DMD)

Theorems guarantee surface created will not have self intersections.

The DMD solves a flow ordinary differential equation for each vertex V_i and assigns the result to V_i

$$\frac{d\Phi(s; \mathbf{x})}{ds} = v(\Phi(s; \mathbf{x})), \text{ with } \Phi(0; \mathbf{x}) = \mathbf{x}.$$

Algorithm 1 DMD module pseudo-code

Input: $\mathbf{U} \in \mathbb{R}^{H \times W \times D \times 3}$ ▷ Discrete flow
Input: \mathcal{T} with vertices $(V_i)_{i \in 1..m}$ ▷ Triangle mesh
Input: $n \in \mathbb{N}^*$ ▷ Number of integration steps
Output: Updated $(V_i)_{i \in 1..m}$
 $h \leftarrow \frac{1}{n}$
Ensure: $h < \frac{1}{L}$
 for $i \in [1, m]$ **do**
 for $j \in [1, n]$ **do**
 $V_i \leftarrow \Psi(h, V_i)$
 end for
 end for

Diffeomorphic Mesh Deformation (DMD)

Tractable framework for computing a diffeomorphic mapping Φ for each surface mesh vertex by solving the **flow ODE**,

$$\frac{d\Phi(s; \mathbf{x})}{ds} = v(\Phi(s; \mathbf{x})), \text{ with } \Phi(0; \mathbf{x}) = x$$

using the iterative approximation method,

$$V_{k+1}^i = V_k^i + hv(V_k^i), \text{ with } h = \frac{1}{N}$$

provided by the forward Euler method.

- ❖ Retains the initial mesh topology without producing self-intersections.
- ❖ We also provide sufficient and comprehensible conditions for meeting the diffeomorphic properties of these transformations.

Related work:

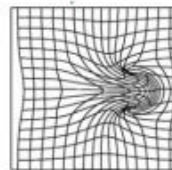
→ Scaling & Squaring in DL Registration [1]:

$$\Phi^{(1/8)} = x + v/8$$

$$\Phi^{(1/4)} = \Phi^{(1/8)} \circ \Phi^{(1/8)}$$

$$\Phi^{(1/2)} = \Phi^{(1/4)} \circ \Phi^{(1/4)}$$

$$\Phi^{(1)} = \Phi^{(1/2)} \circ \Phi^{(1/2)}$$



Voxel-wise integration

$$|I| \gg |V|$$

→ Neural ODEs [2]:

$$\Phi(x) = x + \int_0^1 f_{\theta}(x, I) dt$$

Neural Network with per vertex image feature extractor

1] - Dalca, Adrian V., et al. "Unsupervised learning of probabilistic diffeomorphic registration for images and surfaces." *Medical image analysis* 57 (2019): 226-236.

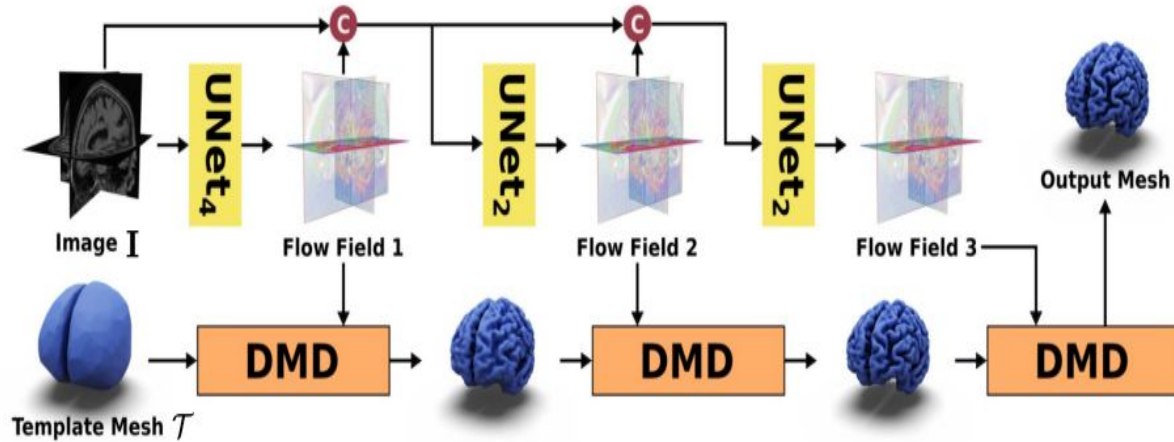
2] - Gupta and Chandraker. "Neural mesh flow: 3d manifold mesh generation via diffeomorphic flows." *In Advances in Neural Information Processing Systems*, 2020.



Cortical Flow architecture

Theorems guarantee surface created will not have self intersections.

The DMD solves a flow ordinary differential equation for each vertex V_i and assigns the result to V_i



$$CF_{\theta_1}^1(I, \mathcal{T}_1) = \text{DMD}\left(\text{UNet}_{\theta_1}^1(I), \mathcal{T}_1\right)$$

$$CF_{\theta_{i+1}}^{i+1}(I, \mathcal{T}_{i+1}) = \text{DMD}\left(\text{UNet}_{\theta_{i+1}}^{i+1}(U_1 \cap \dots \cap U_i I), CF_{\theta_i}^i(I, \mathcal{T}_{i+1})\right)$$

Cortical Flow architecture

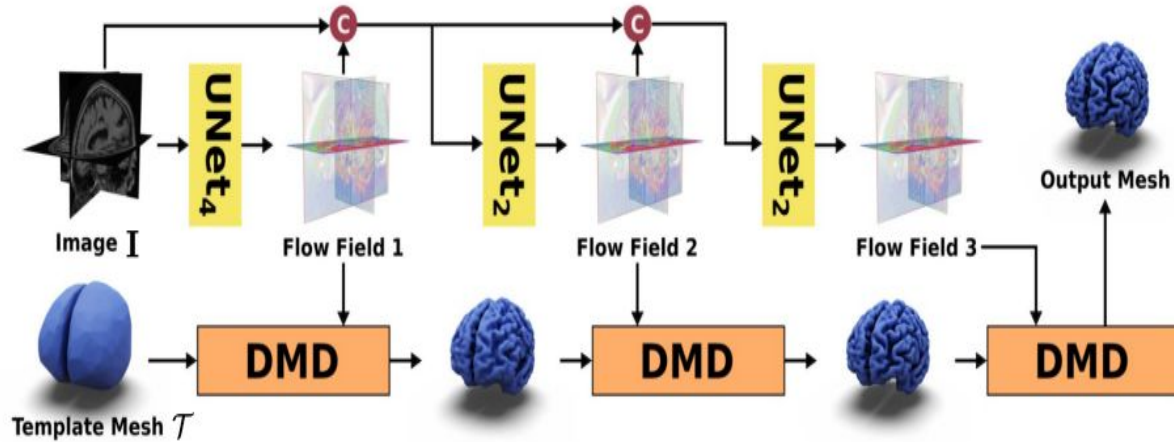
CorticalFlow consists of a chain of three deformations

The first deformation module receives as input a volumetric image and outputs a flow vector field with the same dimensions using UNet-3D

This discrete flow vector field is integrated by the DMD module to compute smooth deformations

The subsequent UNet-3D receives as input the image and the flow vector fields predicted by the previous deformation modules

The set of resulting mappings are composed to produce the final mesh

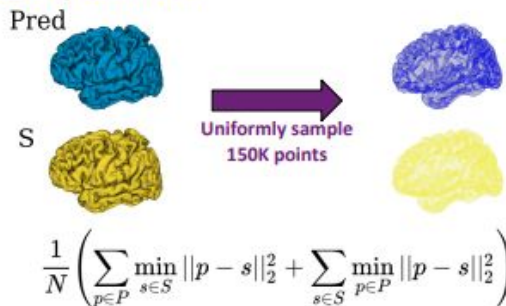


$$CF_{\theta_1}^1(I, \mathcal{T}_1) = \text{DMD}\left(\text{UNet}_{\theta_1}^1(I), \mathcal{T}_1\right)$$

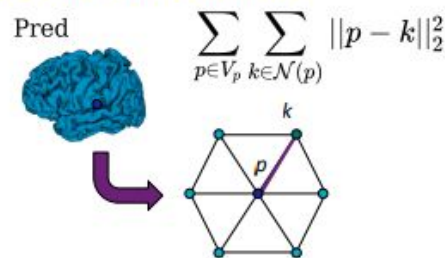
$$CF_{\theta_{i+1}}^{i+1}(I, \mathcal{T}_{i+1}) = \text{DMD}\left(\text{UNet}_{\theta_{i+1}}^{i+1}(U_1 \cap \dots \cap U_i I), CF_{\theta_i}^i(I, \mathcal{T}_{i+1})\right)$$

Multiscale Training/Loss

Chamfer distance:



Edge length regularizer:



$$\operatorname{argmin}_{\theta_i} \sum_{(I, S) \in \mathcal{D}} \mathcal{L} \left(CF_{\theta_i}^i(I, \mathcal{T}_i), S \right)$$

$$= h \frac{C}{\dots} \quad \begin{matrix} r \cdot U = EB \\ R = 13.50m \\ w = y \end{matrix}$$

Example results

$$w = \frac{mgL}{f}; \quad T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{L}{g}}$$

$$q = \frac{h}{\lambda} = \frac{h}{\frac{2\pi}{k}} = \frac{hk}{2\pi}$$

$$\begin{cases} x = x_0 + vt \\ y = y_0 + vt \\ z = z_0 + pt \end{cases}$$

Formula for's

$$1) T = \frac{t}{v} \quad 5) v = \frac{2\pi r}{T}$$

$$2) v = \frac{t}{t} \quad 6) v = \frac{v}{2\pi r}$$

$$3) T = \frac{v}{v} \quad \text{Physics - Resistance}$$

$$4) T = \frac{2\pi}{v}$$

$$w = BG \quad w = 0 \quad w = A$$

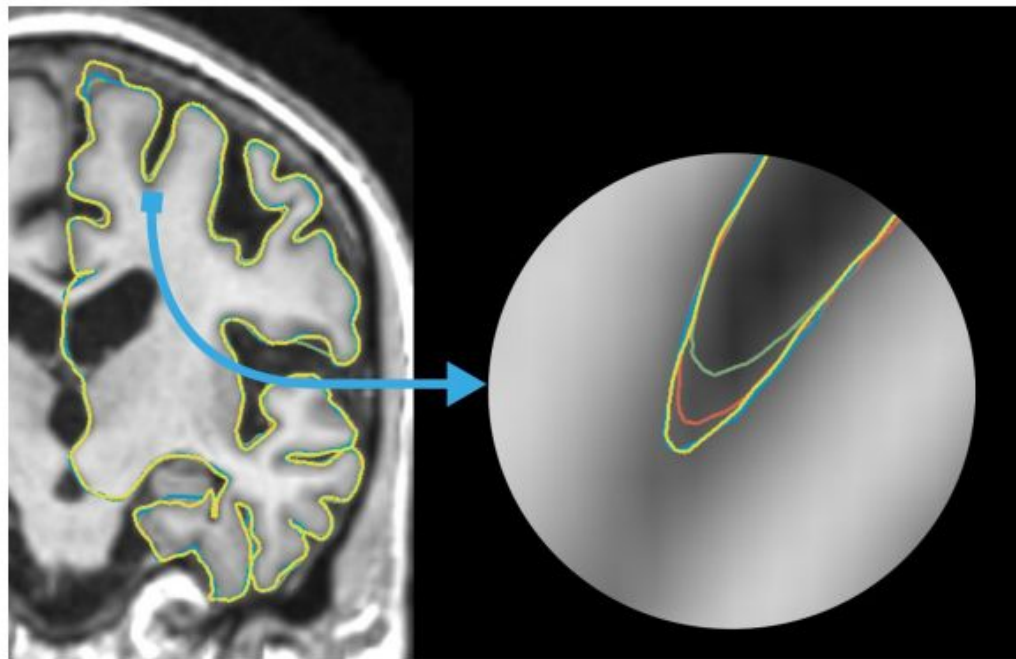
$$\frac{dp}{p} + \gamma \frac{dv}{v}$$

$$I = \frac{U}{R}$$

$$\frac{2\pi}{v}$$

$$\sum_{n=0}^{\infty} \exp(-nD)$$

Resistance



- 1st Deformation
- 2nd Deformation
- 3rd Deformation
- pseudo-ground-truth

fineart
america

Experiments

- **Dataset:**

- MRIs, Pseudo ground truth surfaces, and data splits proposed in [1].
- 3876 MRI images from **ADNI study**
- Pseudo ground truth surfaces generated with the **FreeSurfer V6.0 cross-sectional** pipeline.

- **Baselines:**

- **QuickNAT [2]:** Voxel-wise segmentation + surface extraction
- **Voxel2Mesh [3]:** Deformable model with regularity penalties
- **NMF* [4]:** Deformable model with diffeomorphic transformations
- **DeepCSR [1]:** Implicit surface prediction + surface extraction + Topology Correction

- **Metrics:**

- Geometric accuracy: Chamfer distance, Hausdorff distance, and Chamfer normals.
- Surface regularity: Percentage of self-intersecting faces using **PyMeshLab**.
- Time and space complexity: Average inference time (in seconds) and inference GPU memory footprint (in GB) to reconstruct the **four cortical surfaces**.

[1] - Santa Cruz et al. DeepCSR: A 3d deep learning approach for cortical surface reconstruction. In Proceedings of the IEEE/CVF Winter Conference on Applications of Computer Vision, 2021.

[2] - Roy et al. Quicknat: A fully convolutional network for quick and accurate segmentation of neuroanatomy. NeuroImage, 186:713–727, 2019.

[3] - Wickramasinghe et al. Voxel2mesh: 3d mesh model generation from volumetric data. In International Conference on Medical Image Computing and Computer-Assisted Intervention, 2020.

[4] - Gupta and Chandraker. Neural mesh flow: 3d manifold mesh generation via diffeomorphic flows. In Advances in Neural Information Processing Systems, 2020.

$$= h \frac{C}{\dots} \quad \begin{matrix} r, U = EB \\ R = 13.5D \\ w = \frac{y}{B} \end{matrix}$$

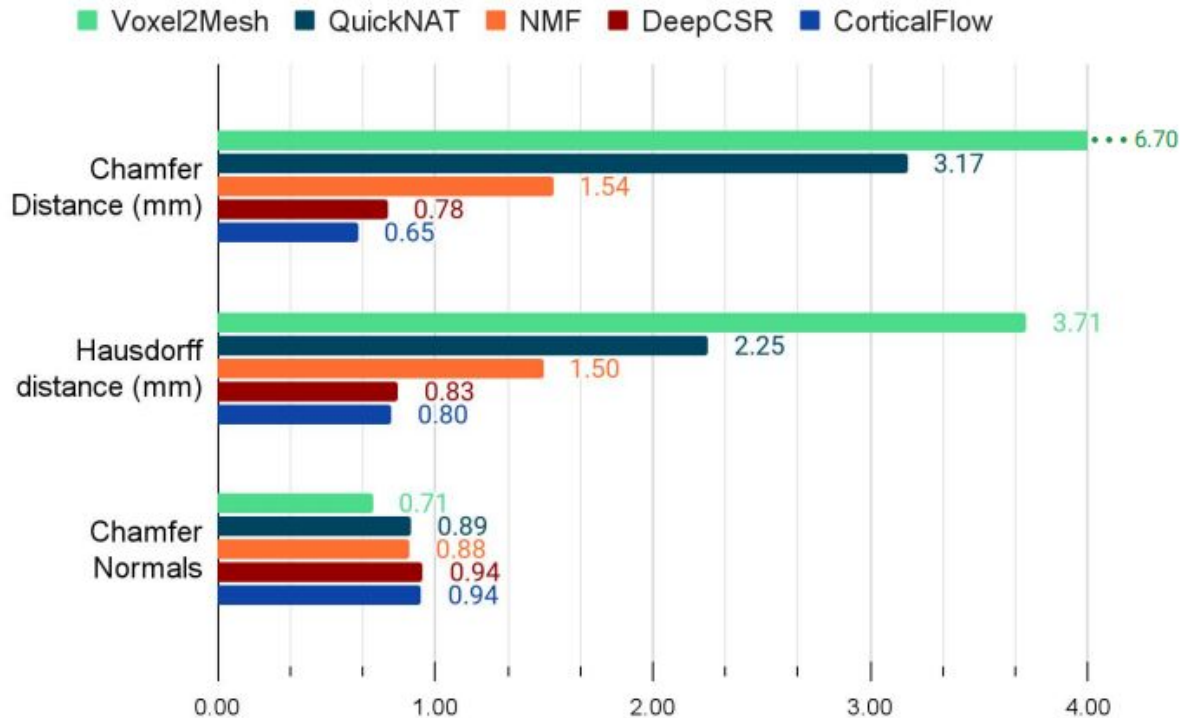
Average Geometric Accuracy (losses used)

$$\begin{aligned} 1) T &= \frac{t}{r} \\ 2) v &= \frac{r}{t} \\ 3) T &= \frac{r}{v} \\ 4) T &= \frac{2}{v} \\ w &= B \\ \omega &= D \end{aligned}$$

Smaller
Is better

Smaller
Is better

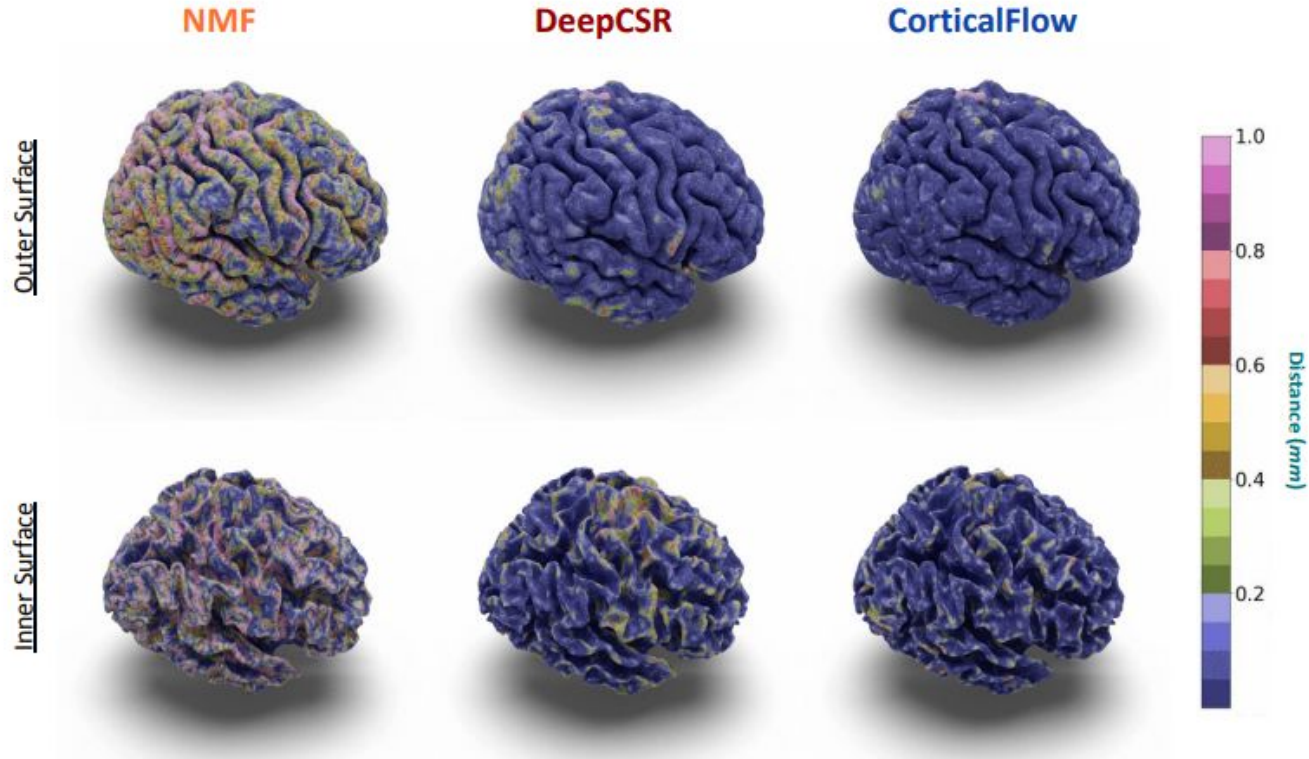
Greater
Is better



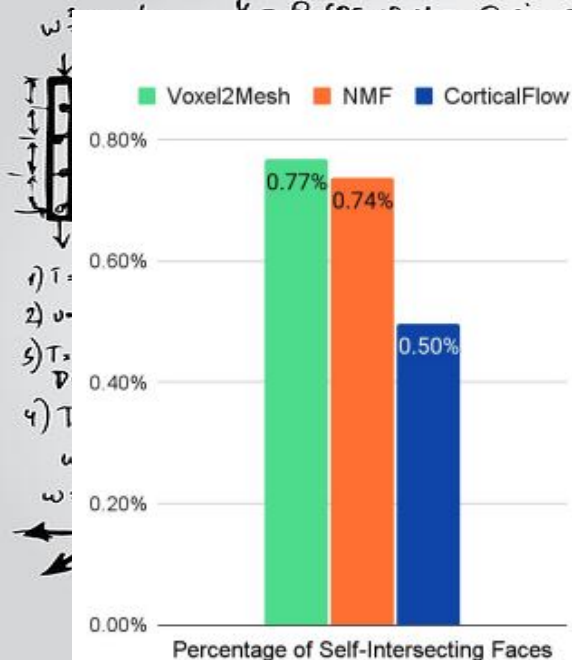
$$= h \frac{C}{B} \quad \text{with } U = \epsilon B, \quad R = 13.5 \text{ cm}, \quad w = \frac{Y}{B \epsilon}$$

Error color coded surfaces (loss)

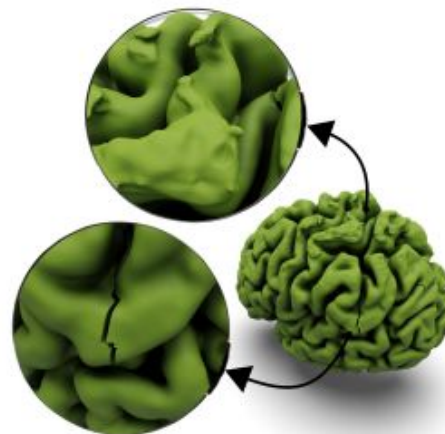
$$\begin{aligned} & \omega = \frac{m}{L} \\ & \text{1) } T = \frac{t}{r} \\ & \text{2) } v = \frac{t}{r} \\ & \text{3) } T = \frac{t}{r} \\ & \text{4) } T = \frac{2}{r} \\ & \omega = B \\ & \omega = D \\ & L = \frac{2}{r} \end{aligned}$$



Surface Regularity



QuickNAT
Multiple connected components, handles and holes.

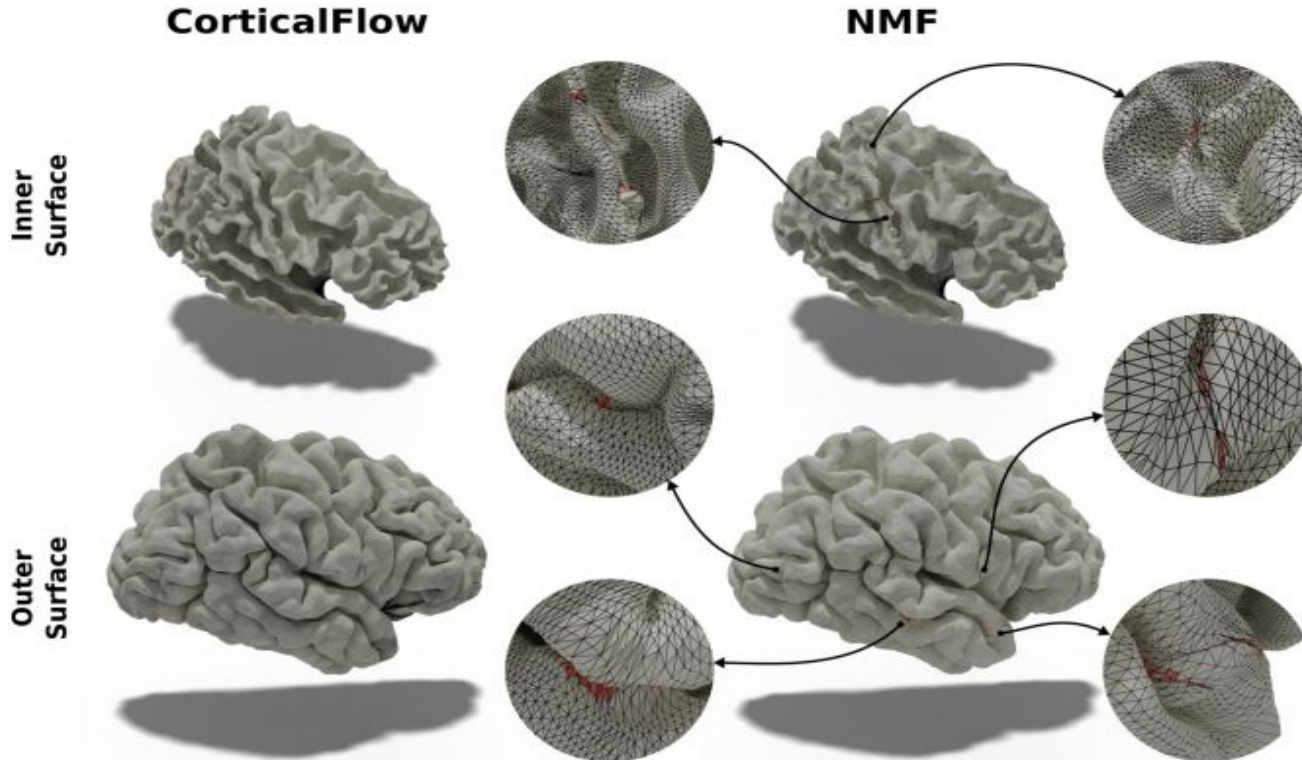


DeepCSR
Anatomical mistakes due to topology correction

$$= h \frac{C}{\dots} \quad \begin{matrix} r, U = EB \\ R = 13.5D \\ B = 2.0 \end{matrix} \quad w = \frac{Y}{BC}$$

Surface Regularity

$$\begin{aligned} & \omega = \frac{m}{g} \\ & 1) T = \frac{t}{r} \\ & 2) v = \frac{t}{r} \\ & 3) T = \frac{v}{t} \\ & 4) T = \frac{2\pi}{\sqrt{\dots}} \\ & \omega = B \\ & \omega = D \\ & \dots \end{aligned}$$



$$= h \frac{C}{\dots} \quad \begin{matrix} r \cdot U = EB \\ R = 13.50m \\ B = 2.0 \\ w = BG \end{matrix}$$

Inference time and gpu time

$$w = \frac{mg}{f}$$

$$x = \rho \cos \varphi, y = \rho \sin \varphi$$

$$1) T = \frac{t}{r}$$

$$2) v = \frac{r}{t}$$

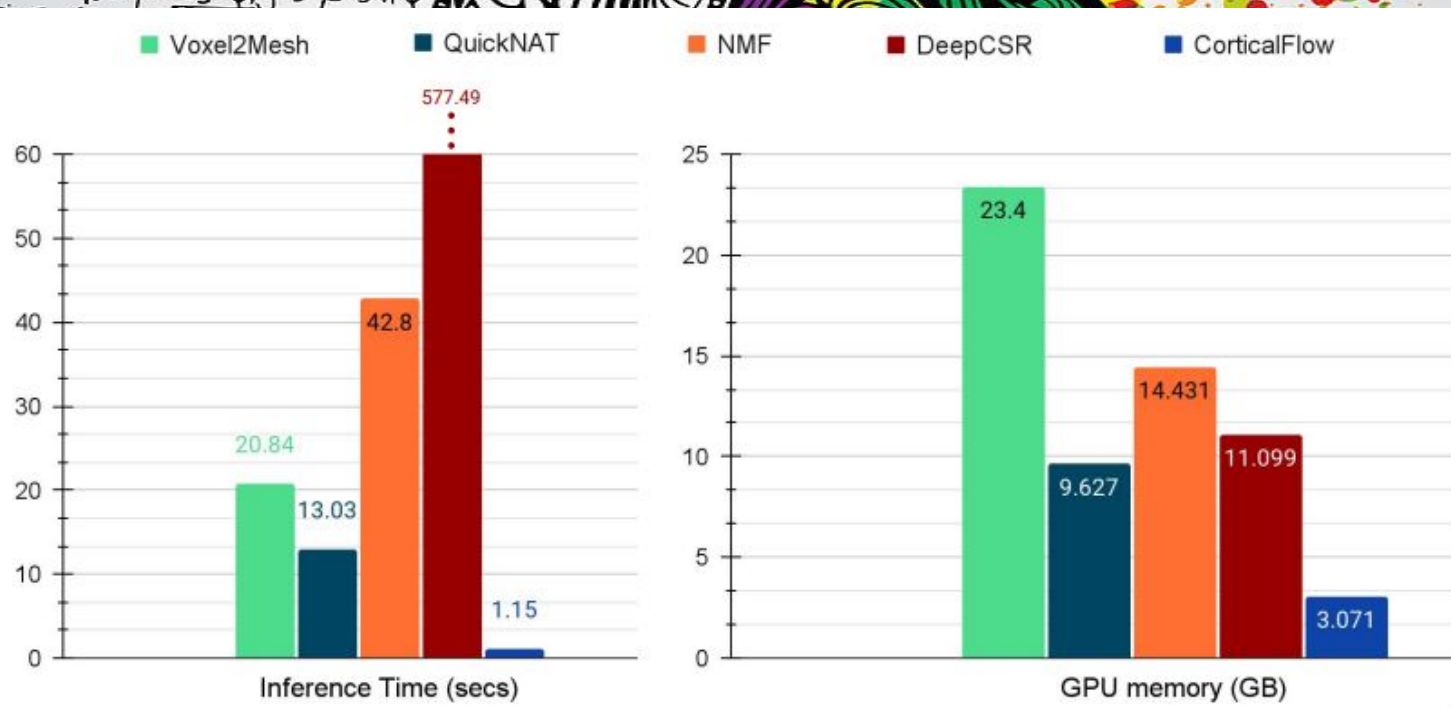
$$3) T = \frac{v}{D}$$

$$4) T = \frac{2\pi r}{v}$$

$$w = BG$$

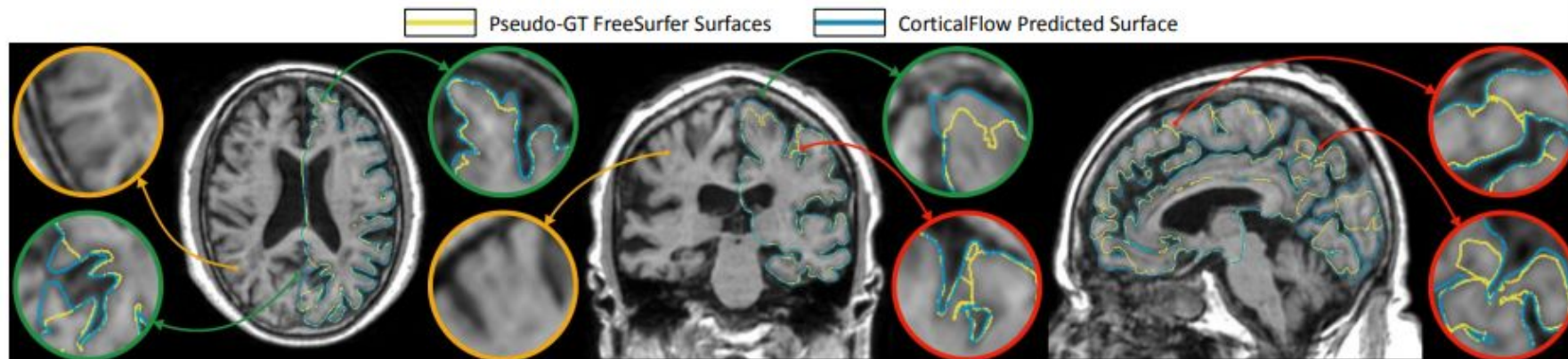
$$\omega = D$$

$$\frac{2\pi r}{v}$$



$$= b \frac{C}{\dots} \quad \begin{matrix} r.U = EB \\ R = 13.5D \\ B = 2.0 \end{matrix} \quad w = \frac{Y}{B \cdot C}$$

Comparison to Pseudo-Ground-Truth



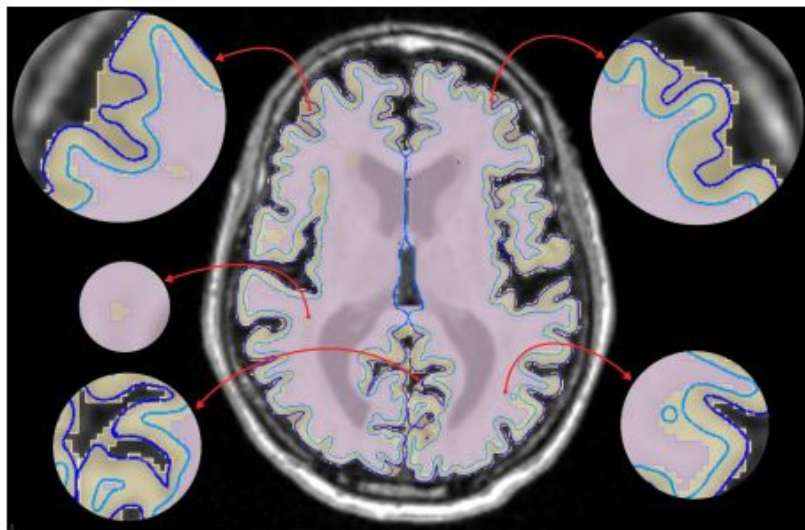
- Orange circles highlight blurry MRI regions.
- Green circles highlight FreeSurfer's underestimated area.
- Red circles highlight non-plausible predictions avoided by CorticalFlow thanks to the diffeomorphism of its predicted deformations.

$$= h \frac{C}{R} \quad \text{with } U = \epsilon B, \quad R = 13.5 \text{ cm}, \quad w = \frac{Y}{B \epsilon}$$

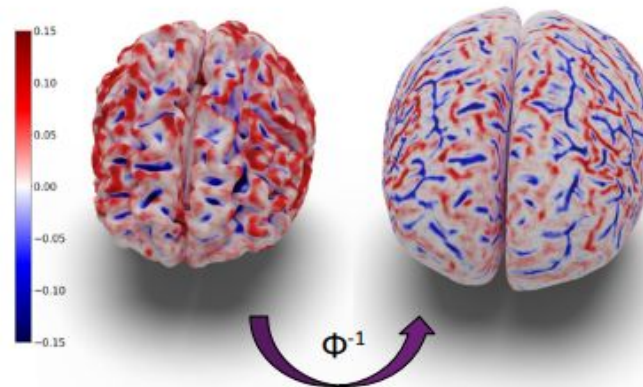
Future Applications

$$w = \frac{mgL}{\dots} \quad x = \rho \cos \varphi, y = \rho \sin \varphi$$

Segmentation at subvoxel resolution



Analysis of surface descriptors on a common reference surface



Conclusion

This paper ...

- introduces **CorticalFlow** - a novel geometric deep learning model for efficiently reconstructing high-resolution, accurate, and regular triangular meshes from volumetric images.
- derives a **diffeomorphic mesh deformable (DMD) module** that efficiently produces diffeomorphic mappings from stationary velocity field.
- shows that CorticalFlow is more accurate, robust, faster and memory efficient than state-of-the-art models in the **cortical surface reconstruction problem** which can facilitate large-scale medical studies and support new healthcare applications.

References

[CorticalFlow_NeurIPS21 \(lebrat.github.io\)](https://lebrat.github.io/CorticalFlow_NeurIPS21/)

https://lebrat.github.io/CorticalFlow/assets/pdf/CorticalFlow_NeurIPS21.pdf

<https://lebrat.github.io/CorticalFlow/>