

0.1 Computaton of μ_t

$$\begin{aligned}
\mu_{t+1}(L_l, d, \mu_t) &= \mathbf{E}_{Z \sim \mathbf{N}(\mathbf{0}, \mathbf{1})} [p(\hat{\mu}_t + Z, t)] \\
&= \mathbf{E}_{Z \sim \mathbf{N}(\mathbf{0}, \mathbf{1})} \left[\frac{1}{\hat{L}_l} \sum_{k=0}^d \frac{\hat{\mu}_t^k (\hat{\mu}_t + Z)^k}{k!} \right] \\
&= \frac{1}{\hat{L}_l} \sum_{k=0}^d \frac{\hat{\mu}_t^k}{k!} \mathbf{E}_{Z \sim \mathbf{N}(\mathbf{0}, \mathbf{1})} [(\hat{\mu}_t + Z)^k]
\end{aligned}$$

0.2 Computation of $\mathbf{E}_{Z \sim \mathbf{N}(\mathbf{0}, \mathbf{1})} [(\hat{\mu}_t + Z)^k]$

$$\begin{aligned}
\mathbf{E}_{Z \sim \mathbf{N}(\mathbf{0}, \mathbf{1})} [(\hat{\mu}_t + Z)^k] &= \mathbf{E}_{Z \sim \mathbf{N}(\mathbf{0}, \mathbf{1})} \left[\sum_{i=0}^k \binom{k}{i} \hat{\mu}_t^{k-i} Z^i \right] \\
&= \sum_{i=0}^k \binom{k}{i} \hat{\mu}_t^{k-i} \mathbf{E}_{Z \sim \mathbf{N}(\mathbf{0}, \mathbf{1})} [Z^i] \\
&= \sum_{i=0}^k \binom{k}{i} \hat{\mu}_t^{k-i} (i \bmod 2) * (i-1)!!
\end{aligned}$$