Bèzier Approximating an Image

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Definitions

Fat Bèzier Curve

We define a Bèzier Curve of degree n to be the image of the function $\phi : [0, 1] \to \mathbb{R}^2$ defined by

$$\phi(t) = (\sum_{i=0}^{n} a_i \binom{n}{i} t^i (1-t)^{n-i}, \sum_{i=0}^{n} b_i \binom{n}{i} t^i (1-t)^{n-i})$$

where the a_i and b_i are parameters of the curve. A Fat Bèzier Curve is the set of points in $\mathbb{R}^2$ which distance from Im ϕ is less then a defined quantity l.

Descriptions of Structures

Fat Bèzier Curve

We represent a Fat Bèzier Curve in a row matrix with the following structure:

$$\begin{pmatrix} l & a_0 & \dots & a_n & b_0 & \dots & b_n \end{pmatrix}$$

with notation as above.

Description of Methods

De Casteljau algorithm and curve splitting

Given a Bèzier Curve of degree n, it is possible to evaluate the curve at the time t_0 and split it in two curves with an algorithm taking only n steps. We set the recurrence relation:

$$\beta_i^{(0)} := \beta_i \qquad i = 0, \dots, n$$

$$\beta_i^{(j)} := \beta_i^{(j-1)}(1 - t_0) + \beta_{i+1}^{(j-1)} t_0 \qquad i = 0, \dots, n-j, \quad j = 1, \dots, n$$

where the β_i are multi-coordinate points. The evaluation of the Bèzier curve at time t_0 is $B(t_0) = \beta_0^{(n)}$ and the curve can be split into two curves with control points respectively: $\beta_0^{(0)}, \beta_0^{(1)}, \dots, \beta_0^{(n)}$ and $\beta_0^{(n)}, \beta_1^{(n-1)}, \dots, \beta_n^{(0)}$.

Description of Procedures

DistanceFromPoint

We use the De Casteljau algorithm for splitting a Bèzier curve in halfes recursively, and taking the minimum of the distances on the two halves. When the piece of curve is almost

a line, we calculate the minimum point distance and return.
We then base on paper *bez.pdf* to measure the flatness of a curve and to do the appropriate math for a line segment.