Bèzier Approximating an Image

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Definitions

Fat Bèzier Curve

We define a Bèzier Curve of degree n to be the image of the function $\phi : [0, 1] \rightarrow \mathbb{R}^2$ defined by

$$\phi(t) = (\sum_{i=0}^{n} a_i \binom{n}{i} t^i (1-t)^{n-i}, \sum_{i=0}^{n} b_i \binom{n}{i} t^i (1-t)^{n-i})$$

where the a_i and b_i are parameters of the curve. A Fat Bèzier Curve is the set of points in $\mathbb{R}^2$ which distance from $\operatorname{Im} \phi$ is less then a defined quantity l.

Descriptions of Structures

Fat Bèzier Curve

We represent a Fat Bèzier Curve in a row matrix with the following structure:

$$\begin{pmatrix} l & a_0 & \ldots & a_n & b_0 & \ldots & b_n \end{pmatrix}$$

with notation as above.

Description of Methods

De Casteljau algorithm and curve splitting

Given a Bèzier Curve of degree n, it is possible to evaluate the curve at the time t_0 and split it in two curves with an algorithm taking only n steps. We set the recurrence relation:

$$\beta_i^{(0)} := \beta_i \qquad i = 0, \ldots, n$$

$$\beta_i^{(j)} := \beta_i^{(j-1)}(1 - t_0) + \beta_{i+1}^{(j-1)} t_0 \qquad i = 0, \ldots, n-j, \quad j = 1, \ldots, n$$

where the β_i are multi-coordinate points. The evaluation of the Bèzier curve at time t_0 is $B(t_0) = \beta_0^{(n)}$ and the curve can be split into two curves with control points respectively: $\beta_0^{(0)}, \beta_0^{(1)}, \ldots, \beta_0^{(n)}$ and $\beta_0^{(n)}, \beta_1^{(n-1)}, \ldots, \beta_n^{(0)}$.

Description of Procedures

DistanceFromPoint

We use the De Casteljau algorithm for splitting a Bèzier curve in halfes recursively, and taking the minimum of the distances on the two halves. When the piece of curve is almost

a line, we calculate the minimum point distance and return.
We then base on paper *bez.pdf* to measure the flatness of a curve and to do the appropriate math for a line segment.

DESCRIPTION OF .BEZIER FILES

We describe the structure of the .bezier files, that are produced after the regolarization of the input image.
On the first line of the file two numbers are given, that are the size of the canvas image, in this order: height width.
On every subsequent line the parameters of a bèzier curve are written, following the convention for the bèzier curve memorization (a row matrix).