 whether we choose the normal vector to point out of the cylinder or into the cylinder . Of course , in the inward pointing case , the curvature has the opposite

$$sign, k = -1/r.$$

For inward pointing normals , the formula breaks down when $h > r$, since we get multiple coverage of points in space by points of the form $y + t\nu(y)$. 3.Y is a sphere of radius R with outward normal , so Y_h is a spherical shell , and

$$\begin{aligned} V_3(Y_h) &= \frac{4}{3}\pi[(R+h)^3 - R^3] \\ &= h4\pi R^2 + h^2 4\pi R + h^3 \frac{4}{3}\pi \\ &= hA + h^2 \frac{1}{R} A + h^3 \frac{1}{3R^2} A \\ &= \frac{1}{3} \cdot A \cdot [3h + 3\frac{1}{R} \cdot h^2 + \frac{1}{R^2} h^3], \end{aligned}$$

where $A = 4\pi R^2$ is the area of the sphere . Once again , for inward pointing normals we must change the sign of the coefficient of h^2 and the formula thus obtained is only correct for $h \leq \frac{1}{R.}$ So in general , we wish to make the assumption that h is such that the map

$$Y \times [0, h] \to R^n, \quad (y, t) \mapsto y + t\nu(y)$$

is inj ective . For Y compact , there always exists an $h_0 > 0$ such that this condition holds for all $h < h_0$. This can be seen to be a consequence of the implicit function theorem . But so not to interrupt the discussion , we will take the injectivity of the map as an hypothesis , for the moment . In a moment we will define the notion of the various averaged curvatures , $H_1, ..., H_{n-1}$, of a hypersurface , and find for the case of the sphere with outward pointing normal , that

$$H_1 = \frac{1}{R,} \quad H_2 = \frac{1}{R^2,}$$

while for the case of the cylinder with outward pointing normal that

$$H_1 = \frac{1}{2r,} \quad H_2 = 0,$$

and for the case of the planar region that

$$H_1 = H_2 = 0.$$

We can thus write all three of the above the above formulas as

$$V_3(Y_h) = \frac{1}{3} A[3h + 3H_1 h^2 + H_2 h^3].$$