CS375 Week 6

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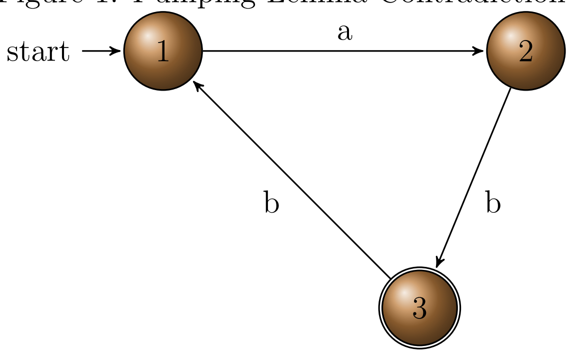
Figure 1: Pumping Lemma Contradiction



0.1

$$\mathbf{L = \{a^n b^{2n} \mid n \geq 1\} = \{abb, aabbbb, aaabbbbbb, ...\}}$$

Proof. For any regular language L, there exists a number p such that for any string w in L of length at least p there are strings x,y,z such that

- $w = xyz$
- $\mid xy \mid \leq p$
- $\mid y \mid \geq 1$
- Then $x = a^n, y = a^n, z = b^{p+1}$
- $xy^2z \ni L$
- This is a contradiction so the shown language is nonregular.

□

0.2

1. Palindromes

   Proof. For any regular language L, there exists a number p such that for any string w in L of length at least p there are strings x,y,z such that

   (a) If w = xyz.

(b) And if x = a, y = b, z = a.

(c) or if x = b, y = a, z = b.

(d) Then $w = a^{90+1}ba^{90}$

(e) and $w = b^{90+1}ab^{90}$

(f) Therefore Palindromes are nonregular

$\square$

2. Equal

Proof. For any regular language L, there exists a number p such that for any string w in L of length at least p there are strings x,y,z such that

(a) EQUAL = $\{\Lambda \quad ab \quad ba \quad aabb \quad abab \quad abba \quad baab \quad baba \quad bbaa \quad aaabbb...\}$

(b) $\{a^n b^n\} = \mathbf{a}^*\mathbf{b}^* \cap EQUAL$

(c) If a^*b^* is regular then so is the result of $\mathbf{a}^*\mathbf{b}^* \cap EQUAL$

(d) $w = xyz$

(e) $\mid xy \mid \leq p$

(f) $\mid y \mid \geq 1$

(g) Then $x = a^n, y = a^2, z = b^n$

(h) So then xyyz would allow aaab which is not in this language.

(i) Therefore this language is nonregular

$\square$

0.3

Prove that the below generates the language defined by the regular expression: $\mathbf{a^*bb}$

$$\textbf{Prod1} \quad \mathbf{S \rightarrow aS \mid bb}$$

$$S \Longrightarrow aS$$
$$\Longrightarrow aaS$$
$$\Longrightarrow aaaS$$
$$\Longrightarrow aaaaS$$
$$\Longrightarrow aaaaaS$$

This derivation could continue infinitely until terminal bb is appended.

$$\Longrightarrow aaaaabb$$

0.4