CS375 Week 6

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Figure 1: Pumping Lemma Contradiction
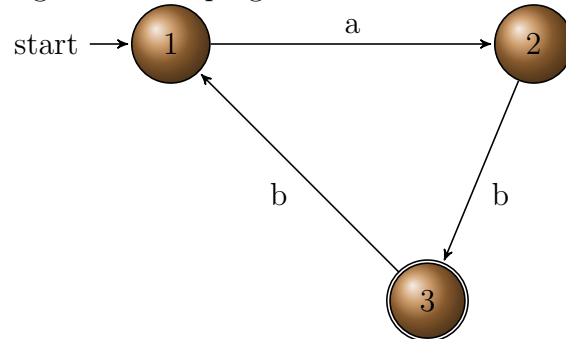
0.1

$$\mathbf{L} = \{\mathbf{a^n b^{2n}} \mid \mathbf{n \geq 1}\} = \{\mathbf{abb}, \mathbf{aabbbb}, \mathbf{aaabbbbbb}, ...\}$$

Proof. For any regular language L, there exists a number p such that for any string w in L of length at least p there are strings x,y,z such that

- $w = xyz$

- $\mid xy \mid \leq p$

- $\mid y \mid \geq 1$

- Then $x = a^n, y = a^n, z = b^{p+1}$

- $xy^2 z \ni L$

- This is a contradiction so the shown language is nonregular.

□

0.2

1. Palindromes

   Proof. For any regular language L, there exists a number p such that for any string w in L of length at least p there are strings x,y,z such that

   (a) If w = xyz.

1

(b) And if x = a, y = b, z = a.

(c) or if x = b, y = a, z = b.

(d) Then $w = a^{90+1}ba^{90}$

(e) and $w = b^{90+1}ab^{90}$

(f) Therefore Palindromes are nonregular

$\square$

2. Equal

   Proof. For any regular language L, there exists a number p such that for any string w in L of length at least p there are strings x,y,z such that

   (a) $EQUAL = \{\Lambda \quad ab \quad ba \quad aabb \quad abab \quad abba \quad baab \quad baba \quad bbaa \quad aaabbb...\}$
   (b) $\{a^n b^n\} = \mathbf{a^*b^*} \cap EQUAL$
   (c) If a^*b^* is regular then so is the result of $\mathbf{a^*b^*} \cap EQUAL$
   (d) $w = xyz$
   (e) $\mid xy \mid \le p$
   (f) $\mid y \mid \ge 1$
   (g) Then $x = a^n, y = a^2, z = b^n$
   (h) So then xyyz would allow aaab which is not in this language.
   (i) Therefore this language is nonregular

$\square$

0.3

Prove that the below generates the language defined by the regular expression: $\mathbf{a^*bb}$

$$\mathbf{Prod1} \quad \mathbf{S \to aS \mid bb}$$
$$S \Longrightarrow aS$$
$$\Longrightarrow aaS$$
$$\Longrightarrow aaaS$$
$$\Longrightarrow aaaaS \tag{1}$$
$$\Longrightarrow aaaaaS$$
$$\Longrightarrow aaaaabb$$

This derivation could continue infinitely until terminal bb is appended.

0.4

$$
\begin{array}{ll}
\textbf{Prod1} & \textbf{S} \rightarrow \textbf{XaXaX} \\
\textbf{Prod2} & \textbf{X} \rightarrow \textbf{aX} \mid \textbf{bX} \mid \boldsymbol{\Lambda}
\end{array}
\tag{2}
$$

0.5

0.6