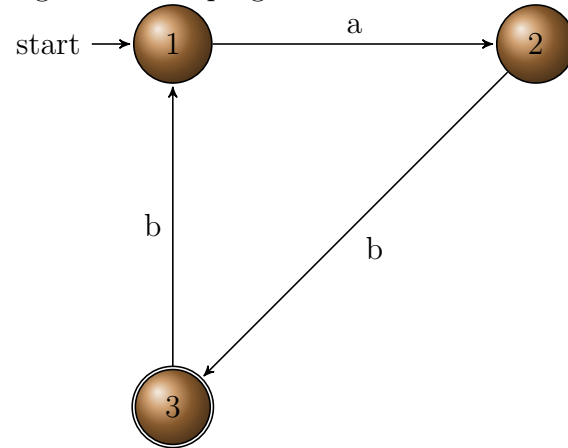# CS375 Week 6

Jason N Mansfield

August 13, 2011

Figure 1: Pumping Lemma Contradiction

0.1

$$\mathbf{L = \{a^n b^{2n} \mid n \geq 1\} = \{abb, aabbbb, aaabbbbbb, ...\}}$$

Proof. For any regular language L, there exists a number p such that for any string w in L of length at least p there are strings x,y,z such that

- $w = xyz$

- $\mid xy \mid \leq p$

- $\mid y \mid \geq 1$

- Then $x = a^n, y = a^n, z = b^{p+1}$

- $xy^2 z \ni L$

- This is a contradiction so the shown language is nonregular.

$\square$

0.2

1. Palindromes

   Proof. For any regular language L, there exists a number p such that for any string w in L of length at least p there are strings x,y,z such that

(a) If w = xyz.

(b) And if x = a, y = b, z = a.

(c) or if x = b, y = a, z = b.

(d) Then $w = a^{90+1}ba^{90}$

(e) and $w = b^{90+1}ab^{90}$

(f) Therefore Palindromes are nonregular

$\square$

2. Equal

Proof. For any regular language L, there exists a number p such that for any string w in L of length at least p there are strings x,y,z such that

(a) EQUAL = $\{\Lambda \quad ab \quad ba \quad aabb \quad abab \quad abba \quad baab \quad baba \quad bbaa \quad aaabbb...\}$

(b) $\{a^n b^n\} = \mathbf{a}^* \mathbf{b}^* \cap EQUAL$

(c) If $a^* b^*$ is regular then so is the result of $\mathbf{a}^* \mathbf{b}^* \cap EQUAL$

(d) $w = xyz$

(e) $|xy| \le p$

(f) $|y| \ge 1$

(g) Then $x = a^n, y = a^2, z = b^n$

(h) So then xyyz would allow aaab which is not in this language.

(i) Therefore this language is nonregular

$\square$

0.3

Prove that the below generates the language defined by the regular expression: $\mathbf{a}^* \mathbf{bb}$

$$\mathbf{Prod1} \quad \mathbf{S} \to \mathbf{aS} \mid \mathbf{bb}$$

$$\begin{aligned}
S &\Longrightarrow aS \\
&\Longrightarrow aaS \\
&\Longrightarrow aaaS \\
&\Longrightarrow aaaaS \\
&\Longrightarrow aaaaaS \\
&\Longrightarrow aaaaabb
\end{aligned} \tag{1}$$

This derivation could continue infinitely until terminal bb is appended.

0.4

To generate aabbab using: $a(a+b)*$

$$
\begin{aligned}
&\textbf{Prod1} \quad \textbf{S} \to \textbf{aX} \\
&\textbf{Prod2} \quad \textbf{X} \to \textbf{aX} \mid \textbf{bX} \mid \boldsymbol{\Lambda} \\
&S \implies aX \quad \text{(by Prod 1)} \\
&\implies aaX \quad \text{(by Prod 2)} \\
&\implies aabX \quad \text{(by Prod 2)} \\
&\implies aabbX \quad \text{(by Prod 2)} \\
&\implies aabbaX \quad \text{(by Prod 2)} \\
&\implies aabbabX \quad \text{(by Prod 2)} \\
&\implies aabbab\Lambda \quad \text{(by Prod 2)}
\end{aligned} \tag{2}
$$

0.5

To generate bbabaaa using: $(a+b)^*a(a+b)^*a(a+b)^*$

$$
\begin{aligned}
&\textbf{Prod1} \quad \textbf{S} \to \textbf{XaXaX} \\
&\textbf{Prod2} \quad \textbf{X} \to \textbf{aX} \mid \textbf{bX} \mid \boldsymbol{\Lambda} \\
&S \implies XaXaX \quad \text{(by Prod 1)} \\
&\implies XaXaaX \quad \text{(by Prod 2)} \\
&\implies bXaXaaX \quad \text{(by Prod 2)} \\
&\implies bXaXaa \quad \text{(by Prod 2)} \\
&\implies bXabXaa \quad \text{(by Prod 2)} \\
&\implies bXabaXaa \quad \text{(by Prod 2)} \\
&\implies bbXabaXaa \quad \text{(by Prod 2)} \\
&\implies bbabaXaa \quad \text{(by Prod 2)} \\
&\implies bbabaaa \quad \text{(by Prod 2)}
\end{aligned} \tag{3}
$$

0.6

$$
\begin{aligned}
&\textbf{Prod1} \quad \textbf{S} \to \textbf{Xa} \\
&\textbf{Prod2} \quad \textbf{X} \to \textbf{bbX} \mid \textbf{bbS} \mid \textbf{bb} \mid \boldsymbol{\Lambda}
\end{aligned} \tag{4}
$$

0.7

0.7.1

This CFG cannot match aaaa, abaa, or bbaa as it will always be abab:

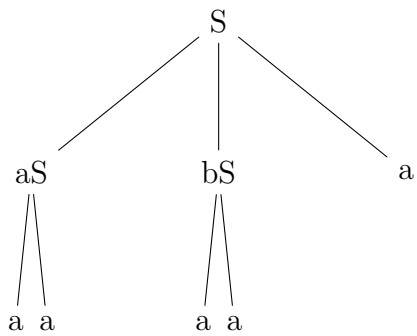$$\textbf{Prod1} \quad \textbf{S} \rightarrow \textbf{aSb} \,|\, \textbf{ab} \tag{5}$$

```
    S              S               S
   / \            / \             / \
 aSb   ab      aSb   aSb   ab   aSb
  |            |      |           |
  ab           ab     ab          ab
```
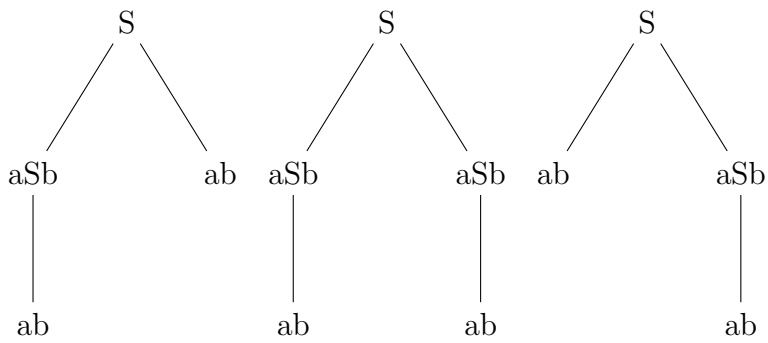
0.7.2

This CFG can create the string aaaa:

$$\textbf{Prod1} \quad \textbf{S} \rightarrow \textbf{aS} \,|\, \textbf{bS} \,|\, \textbf{a} \tag{6}$$

```
           S
        /  |  \
      aS   bS    a
      /\   /\
     a  a  a  a
```

0.7.3

$$\begin{aligned}\textbf{Prod1} \quad &\textbf{S} \rightarrow \textbf{aS} \mid \textbf{aSb} \mid \textbf{X} \\ \textbf{Prod2} \quad &\textbf{X} \rightarrow \textbf{aXa} \mid \textbf{a}\end{aligned} \qquad (7)$$

```
          S
        / | \
       /  |  \
     aS  aSb   X
```

0.7.4
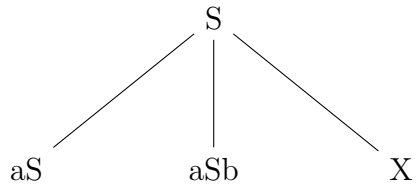
$$\begin{aligned}\textbf{Prod1} \quad &\textbf{S} \rightarrow \textbf{aAS} \mid \textbf{a} \\ \textbf{Prod2} \quad &\textbf{A} \rightarrow \textbf{SbA} \mid \textbf{SS} \mid \textbf{ba}\end{aligned} \qquad (8)$$

```
S
|
|
|
```

0.7.5

$$\begin{aligned}\textbf{Prod1} \quad &\textbf{S} \rightarrow \textbf{aB} \mid \textbf{bA} \\ \textbf{Prod2} \quad &\textbf{A} \rightarrow \textbf{a} \mid \textbf{aS} \mid \textbf{bAA} \\ \textbf{Prod3} \quad &\textbf{B} \rightarrow \textbf{b} \mid \textbf{bS} \mid \textbf{aBB}\end{aligned} \qquad (9)$$

```
S
|
|
|
```

0.8

0.9

0.10

0.11