CS375 Week 6

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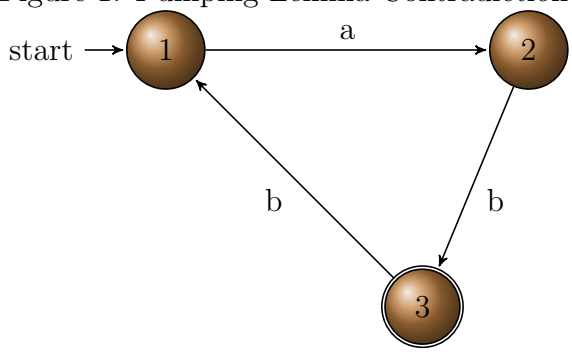
Figure 1: Pumping Lemma Contradiction



0.1

$$\mathbf{L = \{a^n b^{2n} \mid n \geq 1\} = \{abb, aabbbb, aaabbbbbb, ...\}}$$

Proof. For any regular language L, there exists a number p such that for any string w in L of length at least p there are strings x,y,z such that

- $w = xyz$
- $\mid xy \mid \leq p$
- $\mid y \mid \geq 1$
- Then $x = a^n, y = a^n, z = b^{p+1}$
- $xy^2z \ni L$
- This is a contradiction so the shown language is nonregular.

□

0.2

1. Palindromes

   Proof. For any regular language L, there exists a number p such that for any string w in L of length at least p there are strings x,y,z such that

   (a) If w = xyz.

(b) And if x = a, y = b, z = a.

(c) or if x = b, y = a, z = b.

(d) Then $w = a^{90+1}ba^{90}$

(e) and $w = b^{90+1}ab^{90}$

(f) Therefore Palindromes are nonregular

□

2. Equal

Proof. For any regular language L, there exists a number p such that for any string w in L of length at least p there are strings x,y,z such that

(a) EQUAL = $\{\Lambda \quad ab \quad ba \quad aabb \quad abab \quad abba \quad baab \quad baba \quad bbaa \quad aaabbb...\}$

(b) $\{a^n b^n\} = \mathbf{a}^*\mathbf{b}^* \cap EQUAL$

(c) If a^*b^* is regular then so is the result of $\mathbf{a}^*\mathbf{b}^* \cap EQUAL$

(d) $w = xyz$

(e) $\mid xy \mid \leq p$

(f) $\mid y \mid \geq 1$

(g) Then $x = a^n, y = a^2, z = b^n$

(h) So then xyyz would allow aaab which is not in this language.

(i) Therefore this language is nonregular

□

0.3

Prove that the below generates the language defined by the regular expression: $\mathbf{a}^*\mathbf{bb}$

$$\begin{aligned} \textbf{Prod1} \quad & \mathbf{S \rightarrow aS \mid bb} \\ & S \Longrightarrow aS \\ & \Longrightarrow aaS \\ & \Longrightarrow aaaS \\ & \Longrightarrow aaaaS \\ & \Longrightarrow aaaaaS \\ & \Longrightarrow aaaaabb \end{aligned} \tag{1}$$

This derivation could continue infinitely until terminal bb is appended.

0.4

To generate bbabaaa using: $(a+b)^*a(a+b)^*a(a+b)^*$

$$
\begin{aligned}
&\textbf{Prod1} \quad \mathbf{S \to XaXaX} \\
&\textbf{Prod2} \quad \mathbf{X \to aX \mid bX \mid \Lambda} \\
&S \Longrightarrow XaXaX \quad \text{(by Prod 1)} \\
&\Longrightarrow XaXaaX \quad \text{(by Prod 2)} \\
&\Longrightarrow bXaXaaX \quad \text{(by Prod 2)} \\
&\Longrightarrow bXaXaa \quad \text{(by Prod 2)} \\
&\Longrightarrow bXabXaa \quad \text{(by Prod 2)} \\
&\Longrightarrow bXabaXaa \quad \text{(by Prod 2)} \\
&\Longrightarrow bbXabaXaa \quad \text{(by Prod 2)} \\
&\Longrightarrow bbabaXaa \quad \text{(by Prod 2)} \\
&\Longrightarrow bbabaaa \quad \text{(by Prod 2)}
\end{aligned}
\tag{2}
$$

0.5

$$
\begin{aligned}
&\textbf{Prod1} \quad \mathbf{S \to Xa} \\
&\textbf{Prod2} \quad \mathbf{X \to bbX \mid bbS \mid bb \mid \Lambda}
\end{aligned}
\tag{3}
$$

0.6

0.7

(4)

0.8

0.9

(5)

0.10