CS375 Week 6

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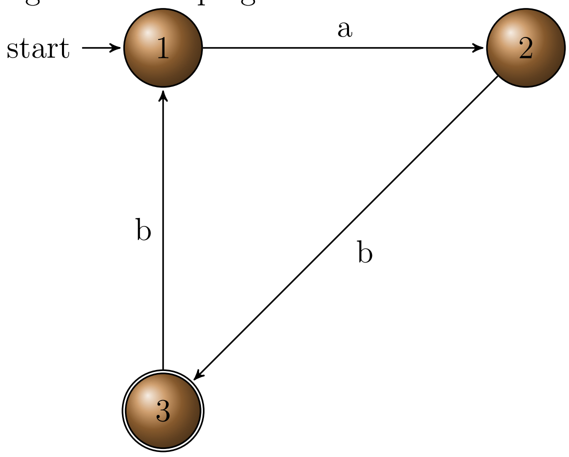
Figure 1: Pumping Lemma Contradiction



0.1

$$\mathbf{L = \{a^n b^{2n} \mid n \geq 1\} = \{abb, aabbbb, aaabbbbbb, ...\}}$$

Proof. For any regular language L, there exists a number p such that for any string w in L of length at least p there are strings x,y,z such that

- $w = xyz$
- $\mid xy \mid \leq p$
- $\mid y \mid \geq 1$
- Then $x = a^n, y = a^n, z = b^{p+1}$
- $xy^2z \ni L$
- This is a contradiction so the shown language is nonregular.

□

0.2

1. Palindromes

   Proof. For any regular language L, there exists a number p such that for any string w in L of length at least p there are strings x,y,z such that

(a) If w = xyz.
(b) And if x = a, y = b, z = a.
(c) or if x = b, y = a, z = b.
(d) Then $w = a^{90+1}ba^{90}$
(e) and $w = b^{90+1}ab^{90}$
(f) Therefore Palindromes are nonregular

□

2. Equal

Proof. For any regular language L, there exists a number p such that for any string w in L of length at least p there are strings x,y,z such that

(a) EQUAL $= \{\Lambda \quad ab \quad ba \quad aabb \quad abab \quad abba \quad baab \quad baba \quad bbaa \quad aaabbb...\}$
(b) $\{a^n b^n\} = \mathbf{a}^*\mathbf{b}^* \cap EQUAL$
(c) If a^*b^* is regular then so is the result of $\mathbf{a}^*\mathbf{b}^* \cap EQUAL$
(d) $w = xyz$
(e) $\mid xy \mid \leq p$
(f) $\mid y \mid \geq 1$
(g) Then $x = a^n, y = a^2, z = b^n$
(h) So then xyyz would allow aaab which is not in this language.
(i) Therefore this language is nonregular

□

0.3

Prove that the below generates the language defined by the regular expression: $\mathbf{a}^*\mathbf{bb}$

Prod1 S → aS | bb

$$\begin{aligned} S &\Longrightarrow aS \\ &\Longrightarrow aaS \\ &\Longrightarrow aaaS \\ &\Longrightarrow aaaaS \\ &\Longrightarrow aaaaaS \\ &\Longrightarrow aaaaabb \end{aligned} \tag{1}$$

This derivation could continue infinitely until terminal bb is appended.

0.4

To generate aabbab using: $a(a+b)*$

$$\begin{aligned}
&\textbf{Prod1} \quad \mathbf{S \rightarrow aX} \\
&\textbf{Prod2} \quad \mathbf{X \rightarrow aX \mid bX \mid \Lambda} \\
&S \implies aX \quad \text{(by Prod 1)} \\
&\implies aaX \quad \text{(by Prod 2)} \\
&\implies aabX \quad \text{(by Prod 2)} \\
&\implies aabbX \quad \text{(by Prod 2)} \\
&\implies aabbaX \quad \text{(by Prod 2)} \\
&\implies aabbabX \quad \text{(by Prod 2)} \\
&\implies aabbab\Lambda \quad \text{(by Prod 2)}
\end{aligned} \tag{2}$$

0.5

To generate bbabaaa using: $(a+b)^*a(a+b)^*a(a+b)^*$

$$\begin{aligned}
&\textbf{Prod1} \quad \mathbf{S \rightarrow XaXaX} \\
&\textbf{Prod2} \quad \mathbf{X \rightarrow aX \mid bX \mid \Lambda} \\
&S \implies XaXaX \quad \text{(by Prod 1)} \\
&\implies XaXaaX \quad \text{(by Prod 2)} \\
&\implies bXaXaaX \quad \text{(by Prod 2)} \\
&\implies bXaXaa \quad \text{(by Prod 2)} \\
&\implies bXabXaa \quad \text{(by Prod 2)} \\
&\implies bXabaXaa \quad \text{(by Prod 2)} \\
&\implies bbXabaXaa \quad \text{(by Prod 2)} \\
&\implies bbabaXaa \quad \text{(by Prod 2)} \\
&\implies bbabaaa \quad \text{(by Prod 2)}
\end{aligned} \tag{3}$$

0.6

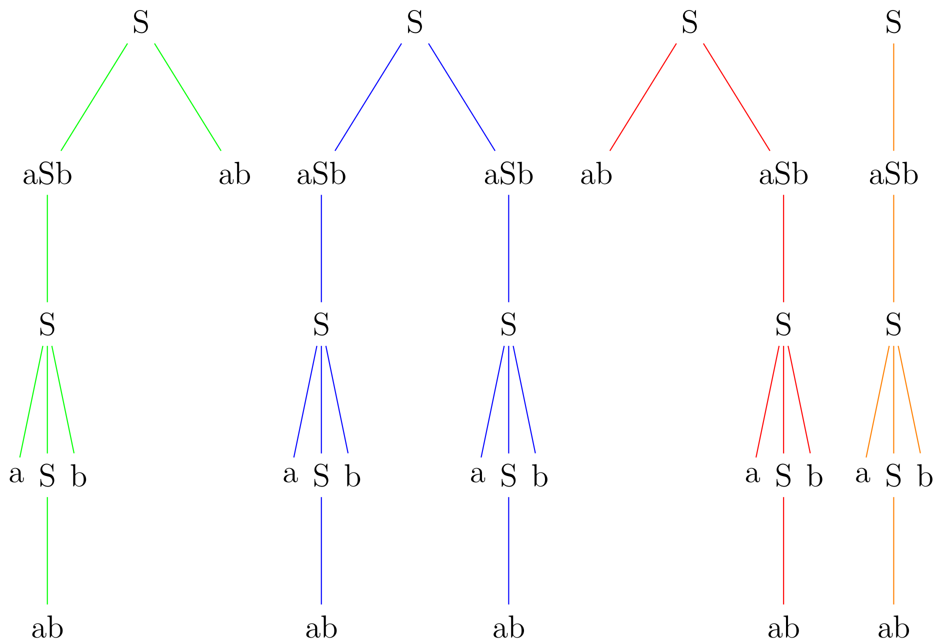
$$\begin{aligned}
&\textbf{Prod1} \quad \mathbf{S \rightarrow Xa} \\
&\textbf{Prod2} \quad \mathbf{X \rightarrow bbX \mid bbS \mid bb \mid \Lambda}
\end{aligned} \tag{4}$$

0.7

0.7.1

This CFG cannot match aaaa, abaa, or bbaa. The only string of length 4 is aabb:

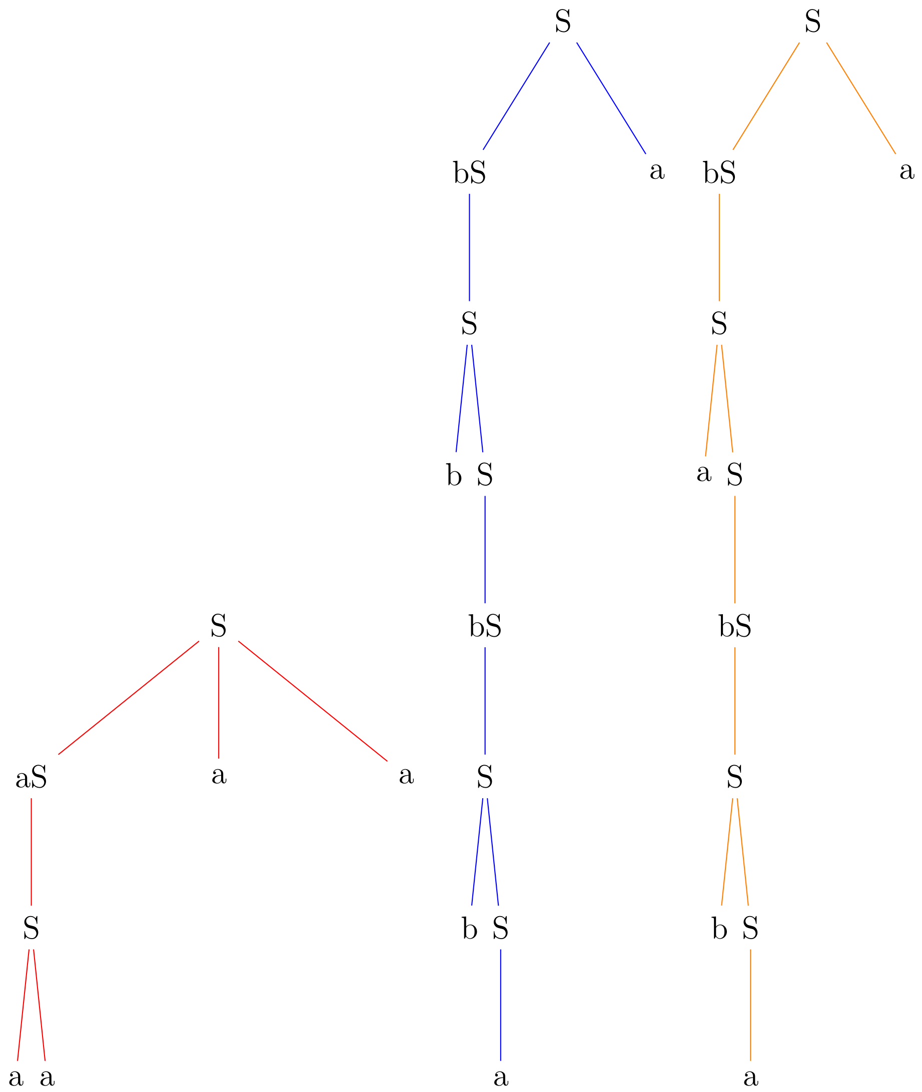
$$\textbf{Prod1} \quad \mathbf{S \rightarrow aSb \mid ab} \tag{5}$$



0.7.2

For strings aaaa, abaa, or bbaa. This CFG can create the string aaaa, bbaa:

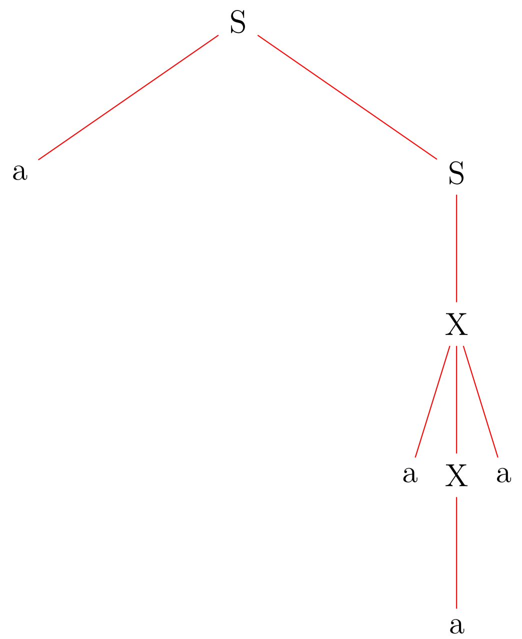
$$\textbf{Prod1} \quad \mathbf{S \rightarrow aS \mid bS \mid a} \tag{6}$$

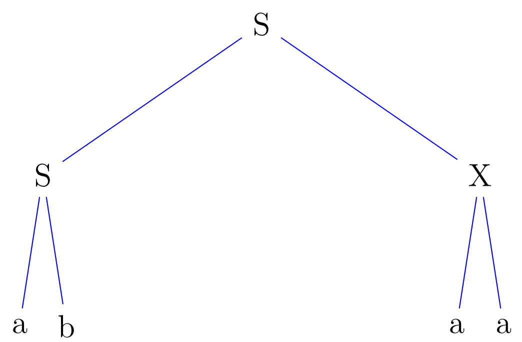


0.7.3

For strings aaaa, abaa, or bbaa. This CFG can create the string aaaa, abaa:

$$\begin{array}{ll} \textbf{Prod1} & \mathbf{S \rightarrow aS \mid aSb \mid X} \\ \textbf{Prod2} & \mathbf{X \rightarrow aXa \mid a} \end{array} \tag{7}$$

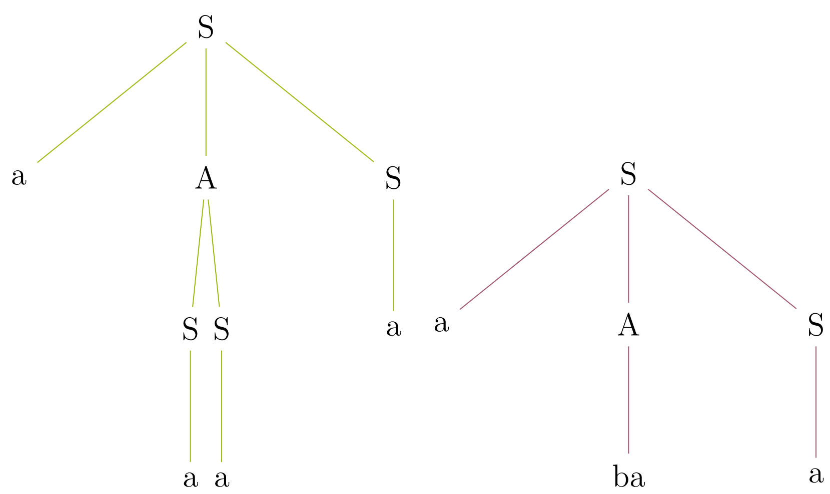




0.7.4

For strings aaaa, abaa, or bbaa. This CFG can create the string aaaa, abaa:

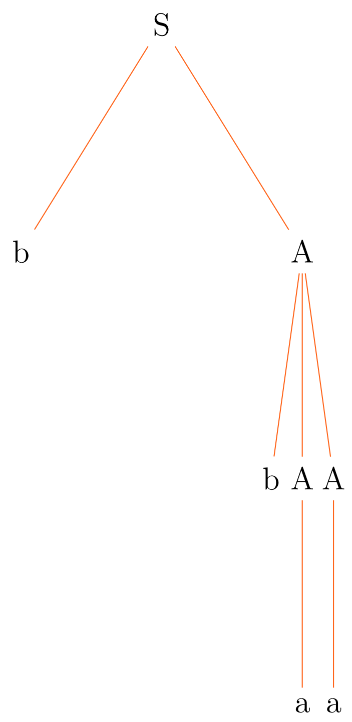
$$\begin{aligned} &\textbf{Prod1} \quad \mathbf{S \rightarrow aAS \mid a} \\ &\textbf{Prod2} \quad \mathbf{A \rightarrow SbA \mid SS \mid ba} \end{aligned} \tag{8}$$



0.7.5

For strings aaaa, abaa, or bbaa. This CFG can create the string bbaa:

$$\begin{aligned} &\textbf{Prod1} \quad \mathbf{S} \rightarrow \mathbf{aB} \mid \mathbf{bA} \\ &\textbf{Prod2} \quad \mathbf{A} \rightarrow \mathbf{a} \mid \mathbf{aS} \mid \mathbf{bAA} \\ &\textbf{Prod3} \quad \mathbf{B} \rightarrow \mathbf{b} \mid \mathbf{bS} \mid \mathbf{aBB} \end{aligned} \tag{9}$$

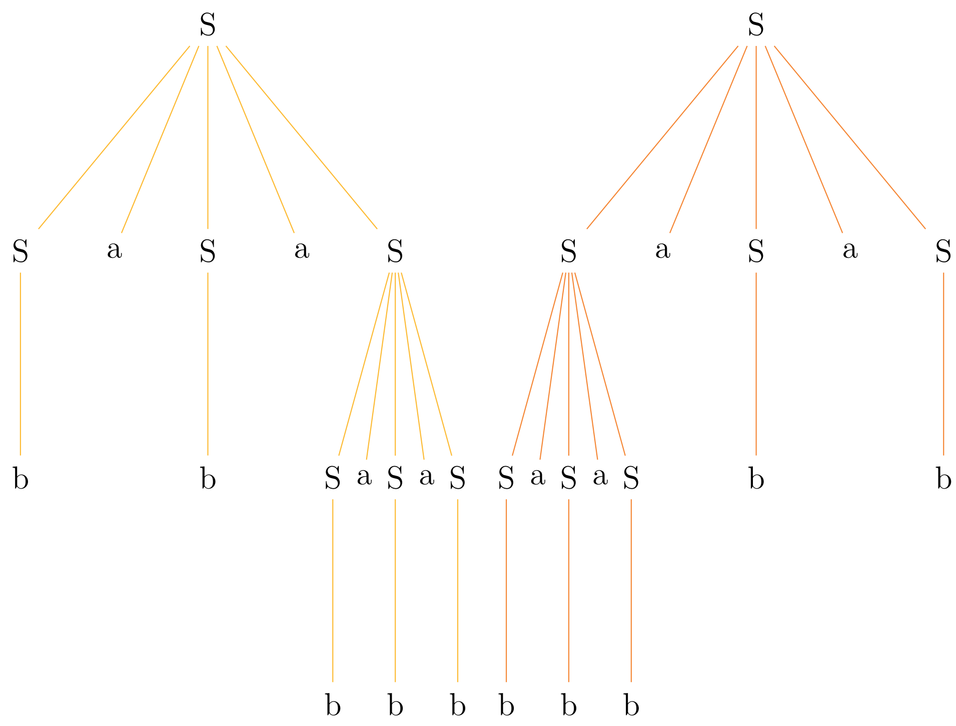


0.8

0.8.1

Ambiguous CFG with the string: babababab

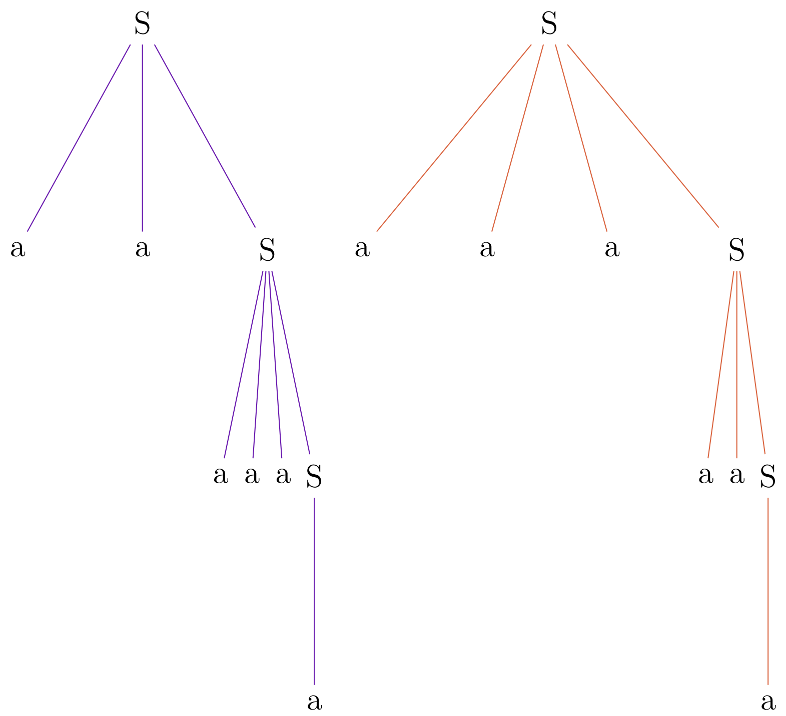
$$\textbf{Prod1} \quad \mathbf{S} \rightarrow \mathbf{SaSaS} \mid \mathbf{b} \tag{10}$$



0.8.2

Ambiguous CFG with the string: aaaaaa

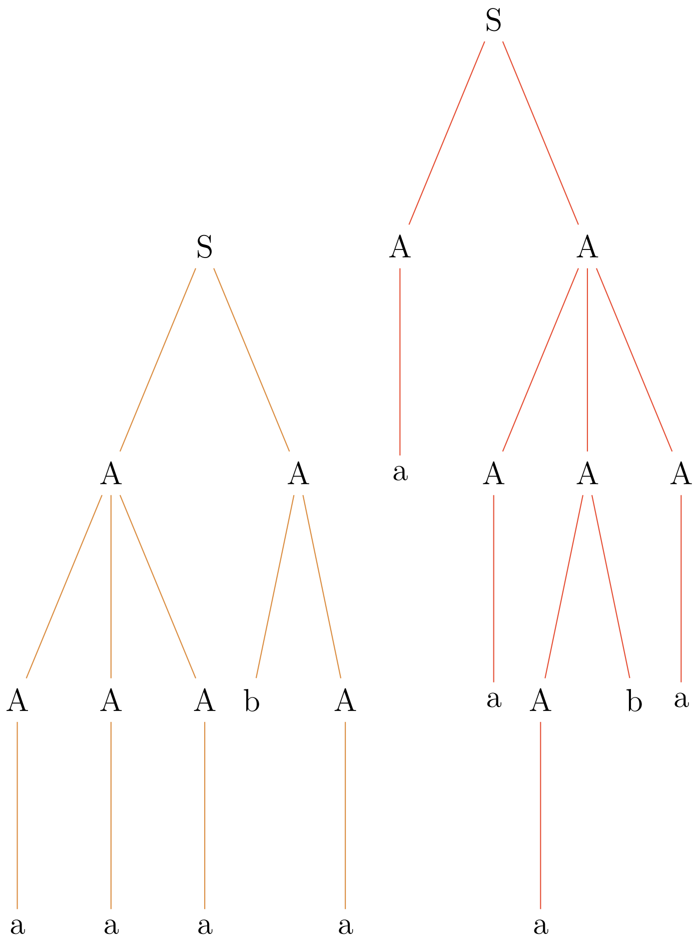
$$\textbf{Prod1} \quad \mathbf{S \rightarrow aaS \mid aaaS \mid a} \tag{11}$$



0.8.3

Ambiguous CFG with the string: aaaba

$$\begin{array}{ll} \textbf{Prod1} & \mathbf{S \rightarrow AA} \\ \textbf{Prod2} & \mathbf{A \rightarrow AAA \mid a \mid bA \mid Ab} \end{array} \tag{12}$$



0.9

0.9.1

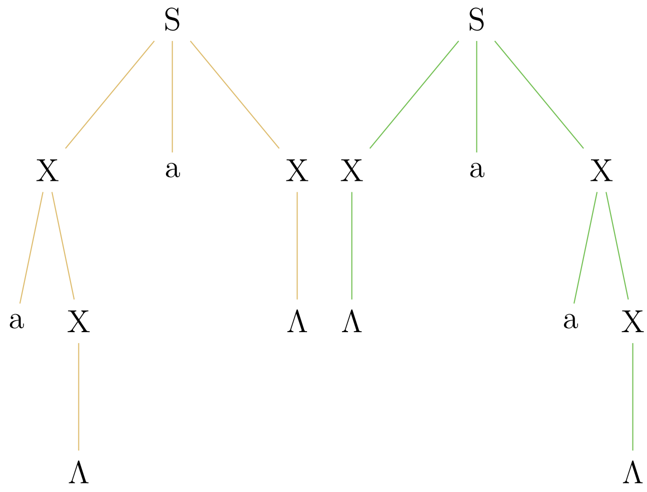
$$\begin{array}{ll} \textbf{Prod1} & \mathbf{S \rightarrow XaX} \\ \textbf{Prod2} & \mathbf{X \rightarrow aX \mid bX \mid \Lambda} \end{array} \tag{13}$$

Ambiguous

$$\begin{aligned} S &\Longrightarrow XaX \\ X &\Longrightarrow aXaX \\ &\Longrightarrow a\Lambda aX \\ &\Longrightarrow aaX \\ &\Longrightarrow aa\Lambda \\ &\Longrightarrow aa \end{aligned} \tag{14}$$

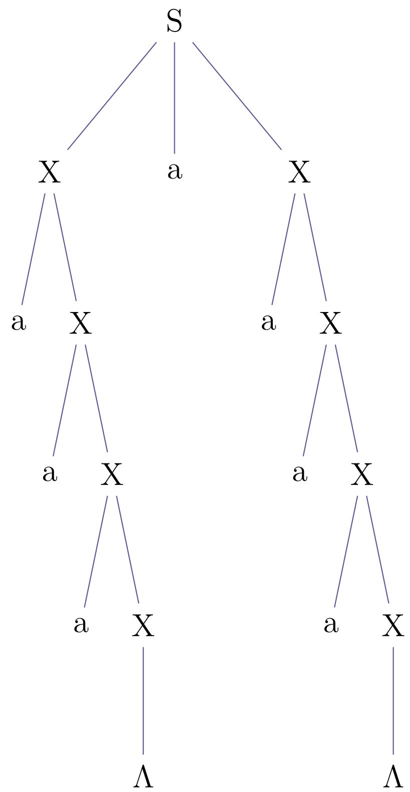
Second Example

$$\begin{aligned} S &\Longrightarrow XaX \\ X &\Longrightarrow \Lambda aX \\ &\Longrightarrow aaX \\ &\Longrightarrow aa\Lambda \\ &\Longrightarrow aa \end{aligned} \tag{15}$$



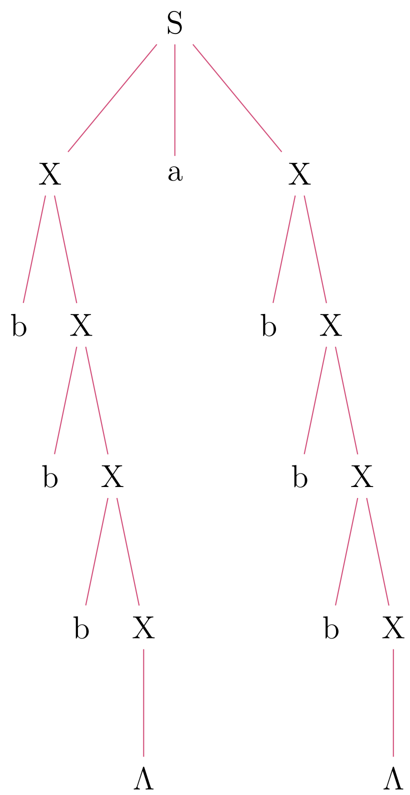
Non-Ambiguous

$$
\begin{aligned}
S &\Longrightarrow XaX \\
X &\Longrightarrow aXaX \\
&\Longrightarrow aaXaX \\
&\Longrightarrow aaaXaX \\
&\Longrightarrow aaa\Lambda aX \\
&\Longrightarrow aaaaaX \\
&\Longrightarrow aaaaaaX \\
&\Longrightarrow aaaaaaaX \\
&\Longrightarrow aaaaaaa\Lambda \\
&\Longrightarrow aaaaaaa
\end{aligned}
\tag{16}
$$



Second Example

$$
\begin{aligned}
S &\Longrightarrow XaX \\
X &\Longrightarrow bXaX \\
&\Longrightarrow bbXaX \\
&\Longrightarrow bbbXaX \\
&\Longrightarrow bbb\Lambda aX \\
&\Longrightarrow bbbbbX \\
&\Longrightarrow bbbbbbX \\
&\Longrightarrow bbbbbbbX \\
&\Longrightarrow bbbbbbb\Lambda \\
&\Longrightarrow bbbbbbb
\end{aligned} \tag{17}
$$



0.9.2

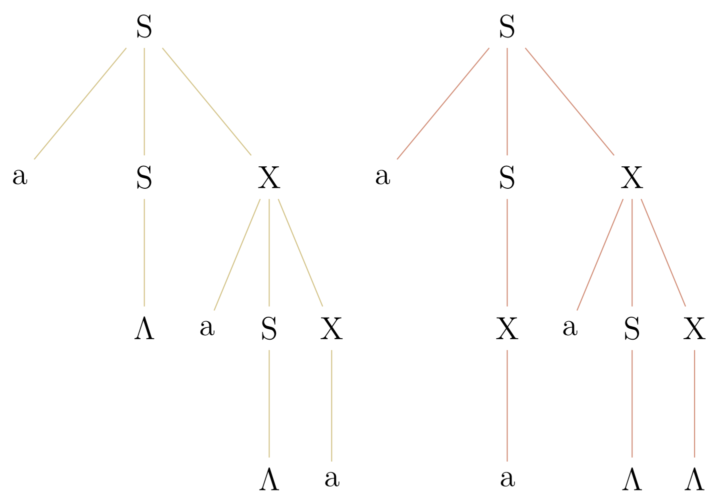
Ambiguous

$$\begin{aligned} &\textbf{Prod1} \quad \mathbf{S \rightarrow aSX \mid \Lambda} \\ &\textbf{Prod2} \quad \mathbf{X \rightarrow aX \mid a} \end{aligned} \tag{18}$$

$$\begin{aligned} S &\Longrightarrow aSX \\ &\Longrightarrow a\Lambda X \\ &\Longrightarrow aX \\ &\Longrightarrow aaSX \\ &\Longrightarrow aa\Lambda X \\ &\Longrightarrow aaa \end{aligned} \tag{19}$$

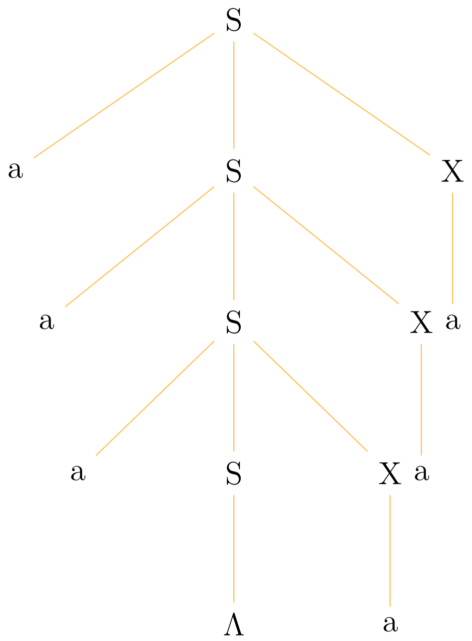
Second Example

$$\begin{aligned} S &\Longrightarrow aSX \\ &\Longrightarrow aXX \\ &\Longrightarrow aaX \\ &\Longrightarrow aaaSX \\ &\Longrightarrow aaa\Lambda X \\ &\Longrightarrow aaa\Lambda \end{aligned} \tag{20}$$



Non-Ambiguous

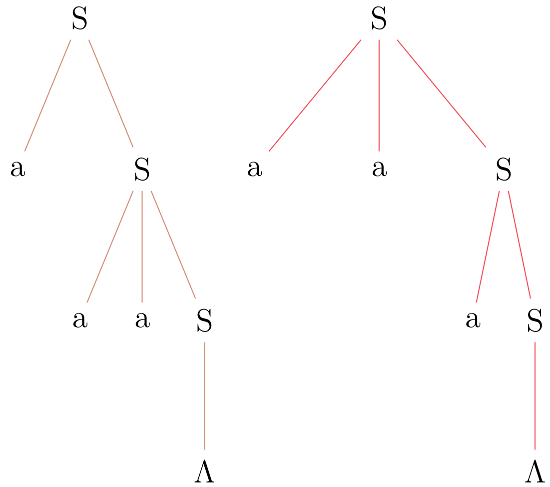
$$
\begin{aligned}
S &\Longrightarrow aSX \\
&\Longrightarrow aaSXX \\
&\Longrightarrow aaaSXXa \\
&\Longrightarrow aaa\Lambda Xaa \\
&\Longrightarrow aaaaaa
\end{aligned}
\tag{21}
$$



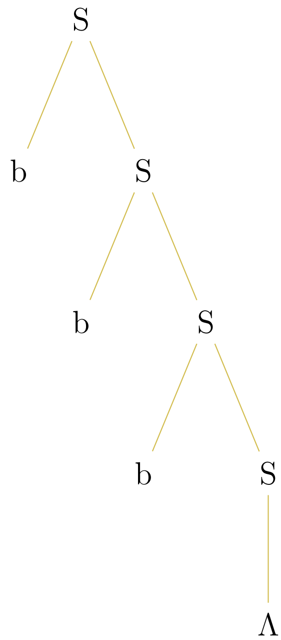
0.9.3

$$\textbf{Prod1} \quad \mathbf{S \rightarrow aS \mid bS \mid aaS \mid \Lambda} \tag{22}$$

Ambiguous



Non-Ambiguous



0.10

0.10.1

$$\textbf{Prod1} \quad \mathbf{S \rightarrow aS \mid bS \mid a} \tag{23}$$

0.10.2

$$\begin{array}{ll} \textbf{Prod1} & \mathbf{S \rightarrow aSb \mid bX} \\ \textbf{Prod2} & \mathbf{X \rightarrow bX \mid b} \end{array} \tag{24}$$

0.11

The CFG's generated by the Regular Languages over the alphabet $\Sigma = \{ab\}$:

0.11.1

The language defined by $(aaa + b)^*$:

0.11.2

The language defined by $(a + b)^*(bbb + aaa)(a + b)^*$:

0.11.3

All the strings that end in b and have an even number of b's in total:

0.11.4

The set of all strings of odd length:

0.11.5

All strings with exactly one a or exactly one b:

0.11.6

All strings with an odd number of a's or and even number of b's: