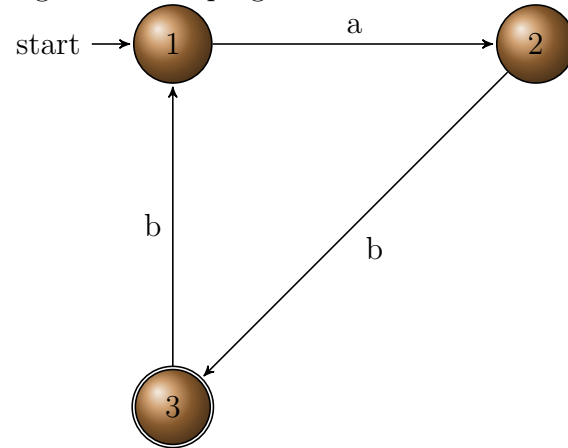


CS375 Week 6

Jason N Mansfield

August 14, 2011

Figure 1: Pumping Lemma Contradiction



0.1

$$L = \{a^n b^{2n} \mid n \geq 1\} = \{abb, aabbbb, aaabbbbbbb, \dots\}$$

Proof. For any regular language L , there exists a number p such that for any string w in L of length at least p there are strings x, y, z such that

- $w = xyz$
- $|xy| \leq p$
- $|y| \geq 1$
- Then $x = a^n, y = a^n, z = b^{p+1}$
- $xy^2z \notin L$
- This is a contradiction so the shown language is nonregular.

□

0.2

1. Palindromes

Proof. For any regular language L , there exists a number p such that for any string w in L of length at least p there are strings x, y, z such that

- (a) If $w = xyz$.
- (b) And if $x = a, y = b, z = a$.
- (c) or if $x = b, y = a, z = b$.
- (d) Then $w = a^{90+1}ba^{90}$
- (e) and $w = b^{90+1}ab^{90}$
- (f) Therefore Palindromes are nonregular

□

2. Equal

Proof. For any regular language L , there exists a number p such that for any string w in L of length at least p there are strings x,y,z such that

- (a) $EQUAL = \{\Lambda \quad ab \quad ba \quad aabb \quad abab \quad abba \quad baab \quad baba \quad bbaa \quad aaabbb...\}$
- (b) $\{a^n b^n\} = \mathbf{a^*b^*} \cap EQUAL$
- (c) If $\mathbf{a^*b^*}$ is regular then so is the result of $\mathbf{a^*b^*} \cap EQUAL$
- (d) $w = xyz$
- (e) $|xy| \leq p$
- (f) $|y| \geq 1$
- (g) Then $x = a^n, y = a^2, z = b^n$
- (h) So then $xxyz$ would allow $aaab$ which is not in this language.
- (i) Therefore this language is nonregular

□

0.3

Prove that the below generates the language defined by the regular expression: $\mathbf{a^*bb}$

Prod1 $S \rightarrow \mathbf{aS} \mid \mathbf{bb}$

$$\begin{aligned}
 S &\Rightarrow aS \\
 &\Rightarrow aaS \\
 &\Rightarrow aaaS \\
 &\Rightarrow aaaaS \\
 &\Rightarrow aaaaaS \\
 &\Rightarrow aaaaaabb
 \end{aligned}
 \tag{1}$$

This derivation could continue infinitely until terminal bb is appended.

0.4

To generate aabbab using: $a(a + b)^*$

$$\begin{aligned}
&\textbf{Prod1} \quad S \rightarrow \mathbf{aX} \\
&\textbf{Prod2} \quad X \rightarrow \mathbf{aX} \mid \mathbf{bX} \mid \Lambda \\
S &\Longrightarrow aX \quad (\text{by Prod 1}) \\
&\Longrightarrow aaX \quad (\text{by Prod 2}) \\
&\Longrightarrow aabX \quad (\text{by Prod 2}) \\
&\Longrightarrow aabbX \quad (\text{by Prod 2}) \\
&\Longrightarrow aabbaX \quad (\text{by Prod 2}) \\
&\Longrightarrow aabbabX \quad (\text{by Prod 2}) \\
&\Longrightarrow aabbab\Lambda \quad (\text{by Prod 2})
\end{aligned} \tag{2}$$

0.5

To generate bbabaaa using: $(a + b)^*a(a + b)^*a(a + b)^*$

$$\begin{aligned}
&\textbf{Prod1} \quad S \rightarrow \mathbf{XaXaX} \\
&\textbf{Prod2} \quad X \rightarrow \mathbf{aX} \mid \mathbf{bX} \mid \Lambda \\
S &\Longrightarrow XaXaX \quad (\text{by Prod 1}) \\
&\Longrightarrow XaXaaX \quad (\text{by Prod 2}) \\
&\Longrightarrow bXaXaaX \quad (\text{by Prod 2}) \\
&\Longrightarrow bXaXaa \quad (\text{by Prod 2}) \\
&\Longrightarrow bXabXaa \quad (\text{by Prod 2}) \\
&\Longrightarrow bXabaXaa \quad (\text{by Prod 2}) \\
&\Longrightarrow bbXabaXaa \quad (\text{by Prod 2}) \\
&\Longrightarrow bbabaXaa \quad (\text{by Prod 2}) \\
&\Longrightarrow bbabaaa \quad (\text{by Prod 2})
\end{aligned} \tag{3}$$

0.6

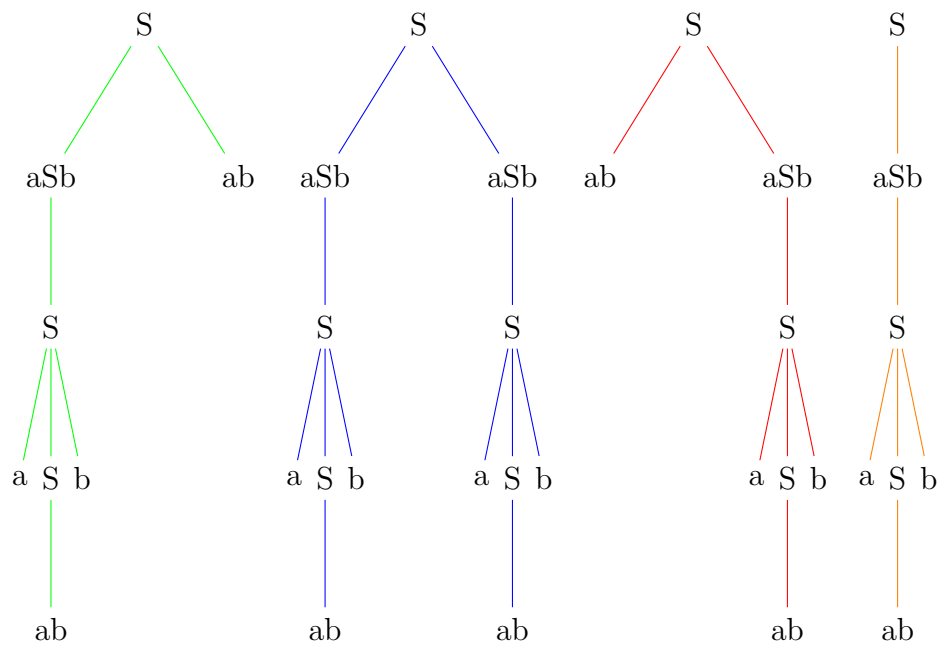
$$\begin{aligned}
&\textbf{Prod1} \quad S \rightarrow \mathbf{Xa} \\
&\textbf{Prod2} \quad X \rightarrow \mathbf{bbX} \mid \mathbf{bbS} \mid \mathbf{bb} \mid \Lambda
\end{aligned} \tag{4}$$

0.7

0.7.1

This CFG cannot match aaaa, abaa, or bbaa. The only string of length 4 is aabb:

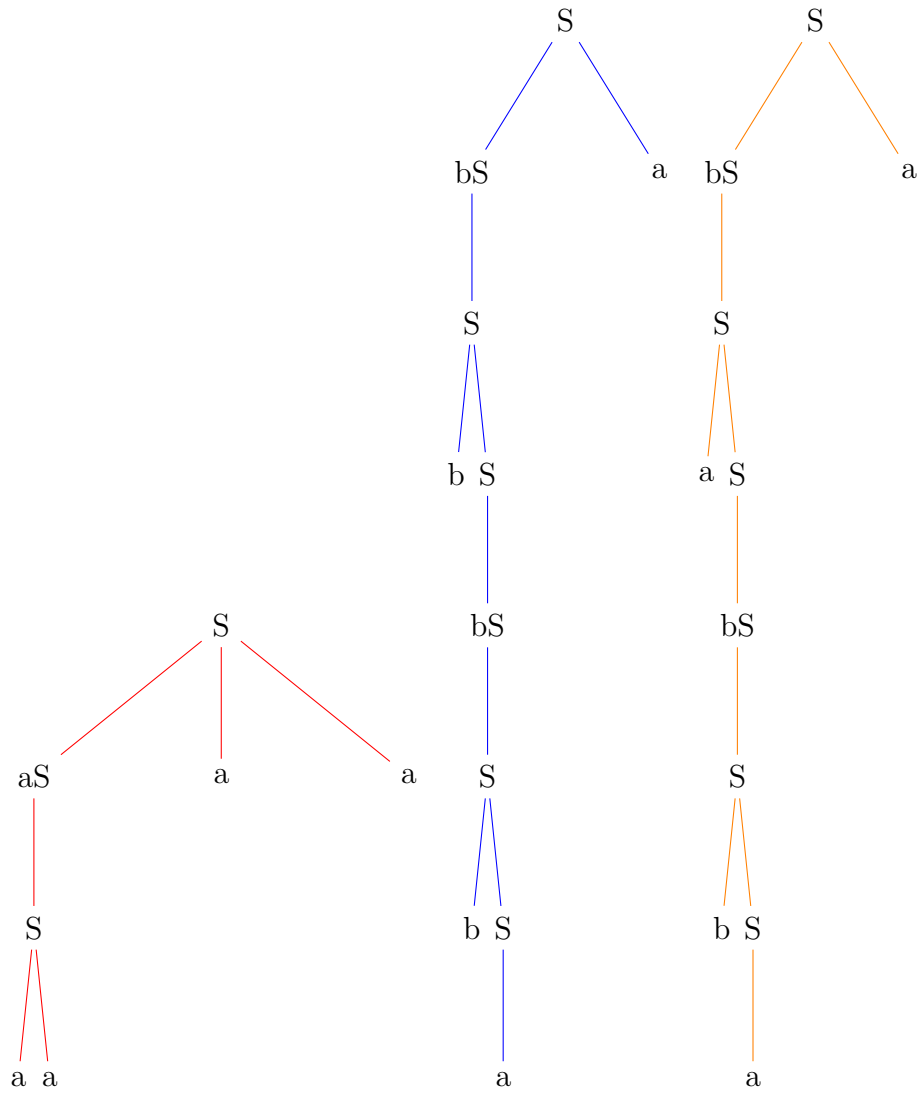
$$\text{Prod1} \quad S \rightarrow aSb \mid ab \quad (5)$$



0.7.2

For strings aaaa, abaa, or bbaa. This CFG can create the string aaaa, bbaa:

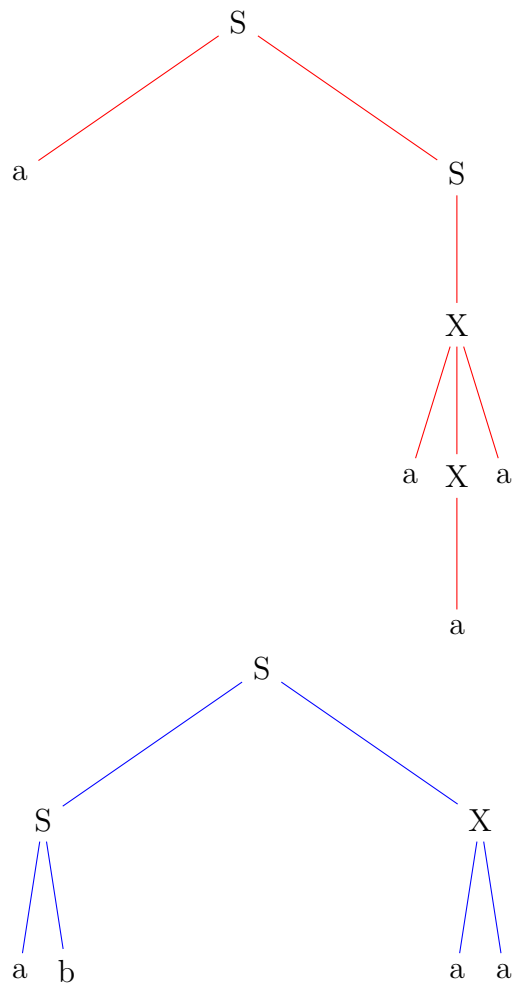
$$\text{Prod1} \quad S \rightarrow aS \mid bS \mid a \quad (6)$$



0.7.3

For strings aaaa, abaa, or bbaa. This CFG can create the string aaaa, abaa:

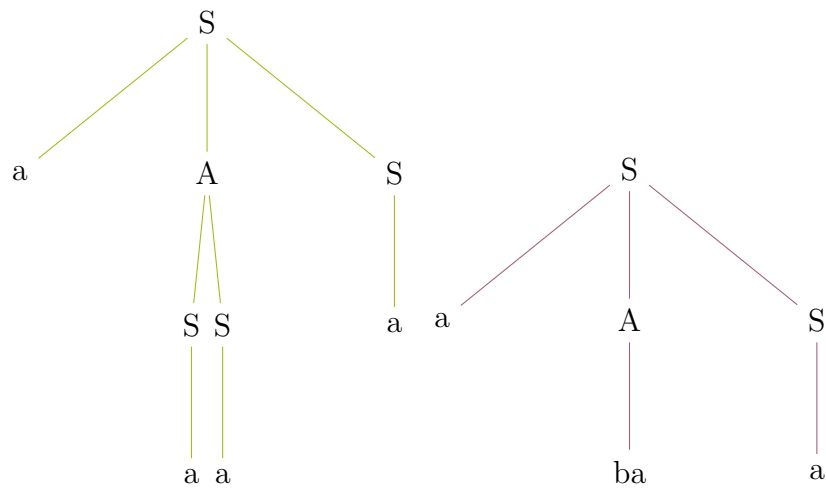
$$\begin{array}{ll} \text{Prod1} & S \rightarrow aS \mid aSb \mid X \\ \text{Prod2} & X \rightarrow aXa \mid a \end{array} \quad (7)$$



0.7.4

For strings aaaa, abaa, or bbaa. This CFG can create the string aaaa, abaa:

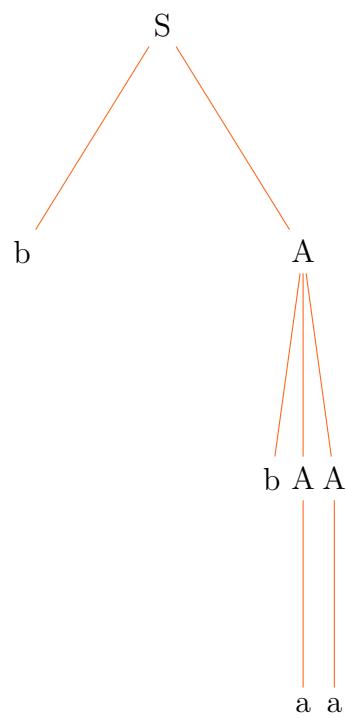
$$\begin{array}{ll} \text{Prod1} & S \rightarrow aAS \mid a \\ \text{Prod2} & A \rightarrow SbA \mid SS \mid ba \end{array} \quad (8)$$



0.7.5

For strings aaaa, abaa, or bbaa. This CFG can create the string bbaa:

$$\begin{array}{ll} \text{Prod1} & S \rightarrow aB \mid bA \\ \text{Prod2} & A \rightarrow a \mid aS \mid bAA \\ \text{Prod3} & B \rightarrow b \mid bS \mid aBB \end{array} \quad (9)$$

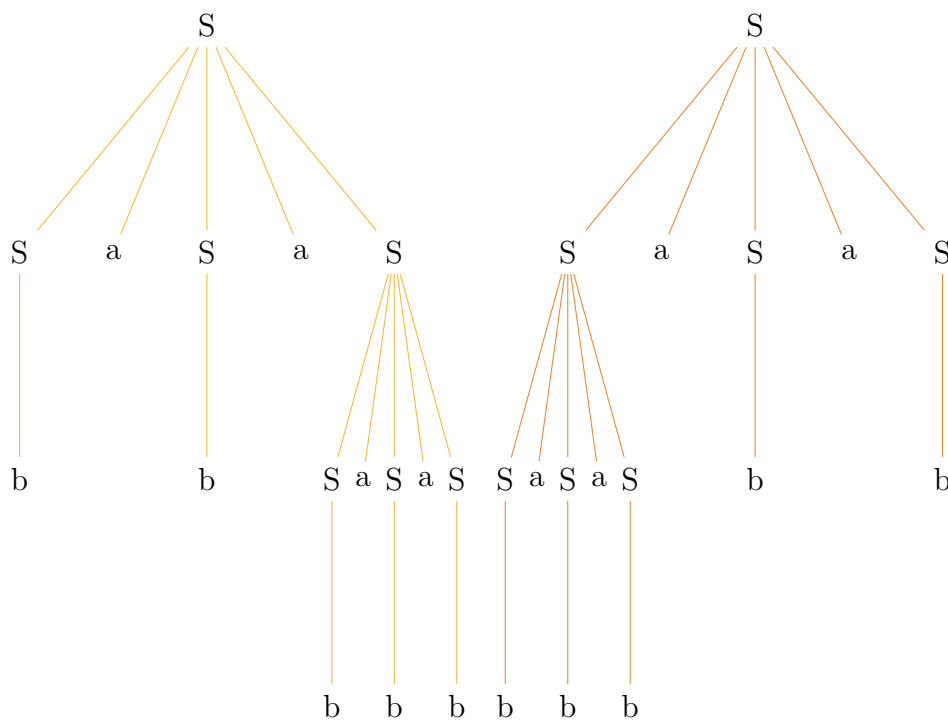


0.8

0.8.1

Ambiguous CFG with the string: babababab

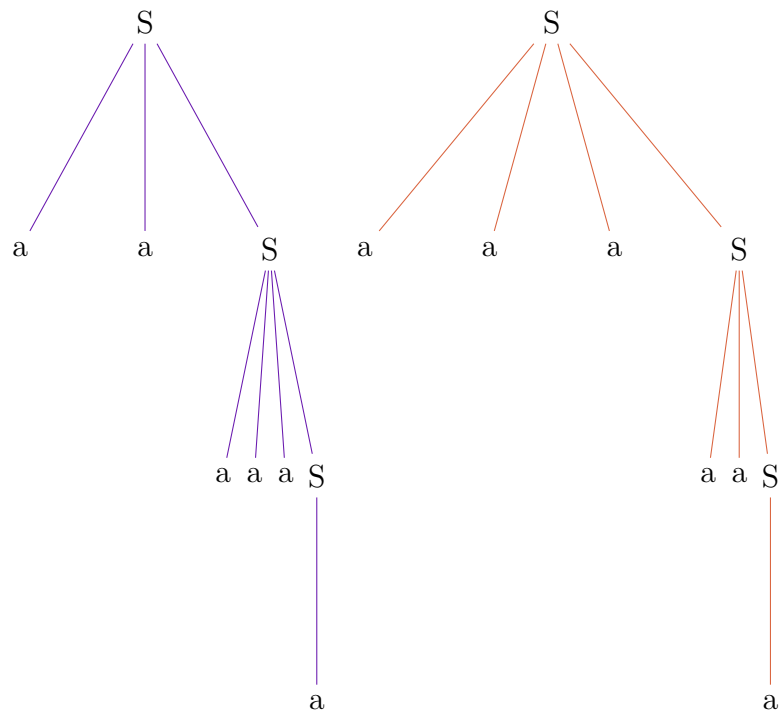
$$\text{Prod1} \quad S \rightarrow SaSaS \mid b \quad (10)$$



0.8.2

Ambiguous CFG with the string: aaaaaa

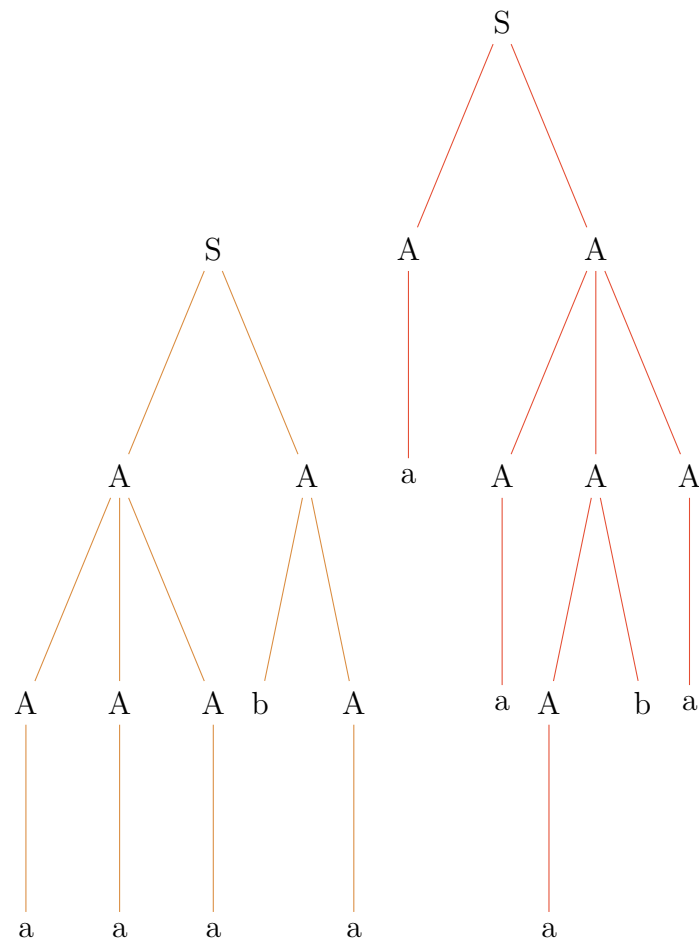
$$\text{Prod1} \quad S \rightarrow aaS \mid aaaS \mid a \quad (11)$$



0.8.3

Ambiguous CFG with the string: aaaba

$$\begin{array}{ll} \text{Prod1} & S \rightarrow AA \\ \text{Prod2} & A \rightarrow AAA \mid a \mid bA \mid Ab \end{array} \quad (12)$$



0.9

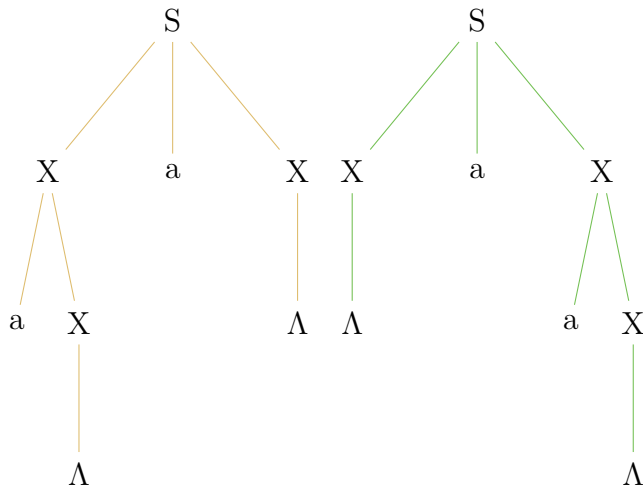
0.9.1

$$\begin{array}{ll} \text{Prod1} & S \rightarrow XaX \\ \text{Prod2} & X \rightarrow aX \mid bX \mid \Lambda \end{array} \quad (13)$$

Ambiguous

$$\begin{array}{l} S \Rightarrow XaX \\ X \Rightarrow aXaX \\ \quad \Rightarrow a\Lambda aX \\ \quad \Rightarrow aaX \\ \quad \Rightarrow aa\Lambda \\ \quad \Rightarrow aa \end{array} \quad (14)$$

$$\begin{array}{l} S \Rightarrow XaX \\ X \Rightarrow \Lambda aX \\ \quad \Rightarrow aaX \\ \quad \Rightarrow aa\Lambda \\ \quad \Rightarrow aa \end{array} \quad (15)$$



Non-Ambiguous

$$S \Rightarrow XaX$$

$$X \Rightarrow aXaX$$

$$\Rightarrow aaXaX$$

$$\Rightarrow aaaXaX$$

$$\Rightarrow aaaa\Lambda X$$

$$\Rightarrow aaaaaX$$

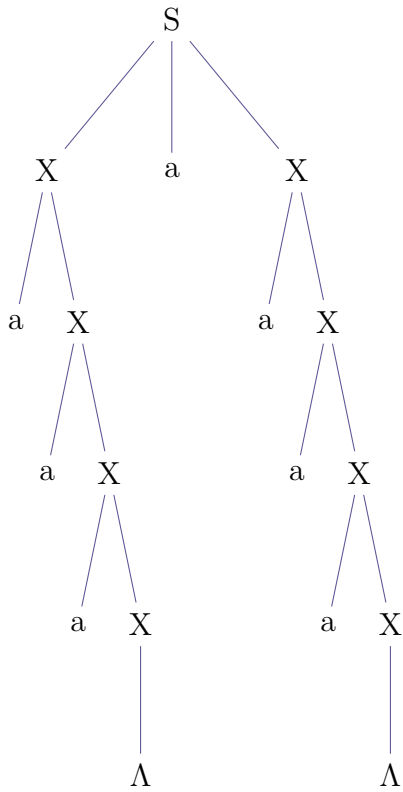
$$\Rightarrow aaaaaaX$$

$$\Rightarrow aaaaaaaX$$

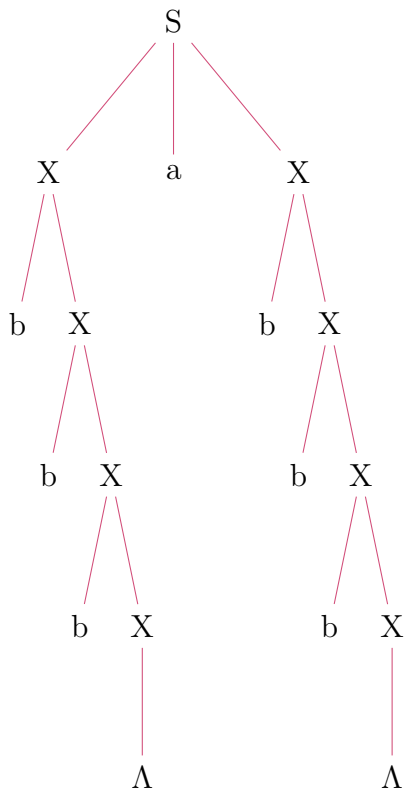
$$\Rightarrow aaaaaaa\Lambda$$

$$\Rightarrow aaaaaaa$$

(16)



$$\begin{aligned}
S &\Rightarrow XaX \\
X &\Rightarrow bXaX \\
&\Rightarrow bbXaX \\
&\Rightarrow bbbXaX \\
&\Rightarrow bbb\Lambda aX \\
&\Rightarrow bbbbbX \\
&\Rightarrow bbbbbbX \\
&\Rightarrow bbbbbb\Lambda \\
&\Rightarrow bbbbbb
\end{aligned}
\tag{17}$$



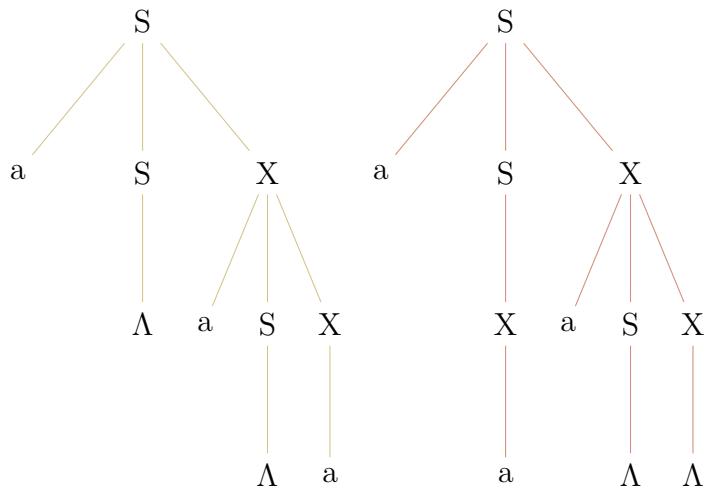
0.9.2

Ambiguous

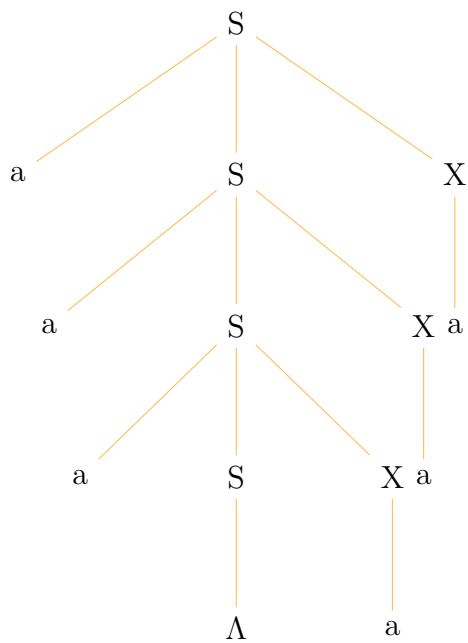
Prod1 $S \rightarrow aSX \mid \Lambda$

Prod2 $X \rightarrow aX \mid a$

(18)



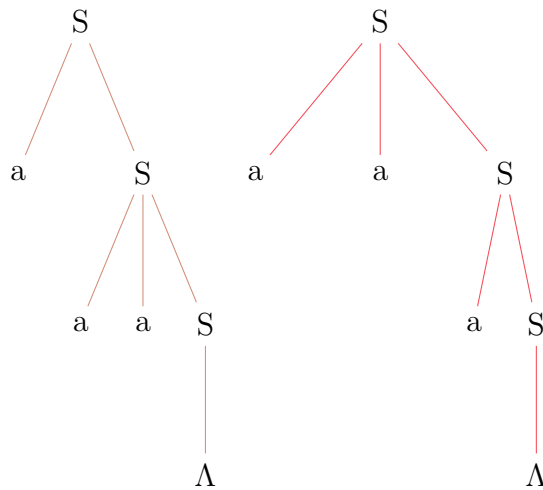
Non-Ambiguous



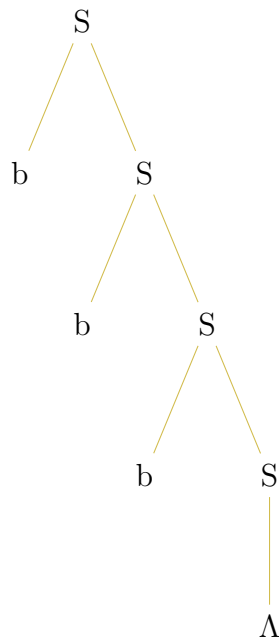
0.9.3

$$\text{Prod1} \quad S \rightarrow aS \mid bS \mid aaS \mid \Lambda \quad (19)$$

Ambiguous



Non-Ambiguous



0.10

0.10.1

$$\text{Prod1} \quad S \rightarrow aS \mid bS \mid a \quad (20)$$

0.10.2

$$\begin{array}{ll} \text{Prod1} & S \rightarrow aSb \mid bX \\ \text{Prod2} & X \rightarrow bX \mid b \end{array} \quad (21)$$

0.11

The CFG's generated by the Regular Languages over the alphabet $\Sigma = \{ab\}$:

0.11.1

The language defined by $(aaa + b)^*$:

0.11.2

The language defined by $(a + b)^*(bbb + aaa)(a + b)^*$:

0.11.3

All the strings that end in b and have an even number of b's in total:

0.11.4

The set of all strings of odd length:

0.11.5

All strings with exactly one a or exactly one b:

0.11.6

All strings with an odd number of a's or and even number of b's: