CHAPTER 5: DISCRETE PROBABILITY DISTRIBUTIONS

Modeling Random Variables with Tables and Functions

I. Random Variables

A **Random Variable (RV)** is a numerical description of the outcome of an experiment.

- **Discrete RV:** Can take on a countable sequence of values (e.g., number of customers: 0, 1, 2).
- **Continuous RV:** Can take any value in an interval (e.g., time, weight).

II. Discrete Probability Distributions

A distribution is defined by a probability function, $f(x)$, which provides the probability for each value of the random variable. Two conditions must hold: 1. $f(x) \geq 0$ for all x. 2. $\sum f(x) = 1$.

Expected Value and Variance

The **Expected Value ($E(x)$ or μ)** is the weighted average of the random variable:

$$E(x) = \mu = \sum xf(x)$$

The **Variance (σ^2)** measures the "spread" or risk:

$$\sigma^2 = \sum (x-\mu)^2 f(x)$$

The **Standard Deviation (σ)** is simply $\sqrt{\sigma^2}$.

III. Special Discrete Distributions

Often, business problems follow specific mathematical patterns. We use predefined functions for these:

1. Binomial Distribution: Used when there are n independent trials, each with only two outcomes: **Success** or **Failure**. *Excel: `=BINOM.DIST(x, n, p, cumulative)`*

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

2. Poisson Distribution: Used to estimate the number of occurrences over a specified **interval of time or space** (e.g., arrivals per hour). *Excel: `=POISSON.DIST(x, mean, cumulative)`*

$$f(x) = \frac{\mu^x e^{-\mu}}{x!}$$

3. Hypergeometric Distribution: Similar to Binomial, but used when trials are **not independent** (sampling without replacement from a small population). *Excel: `=HYPGEOM.DIST(x, n, M, N, cumulative)`*

IV. Teacher's Strategy: Choosing the Model

Choosing the wrong distribution is a common student error. Use this guide:

- **Binomial:** Are there a fixed number of trials (n)? Are there only two outcomes? (e.g., 10 people enter a store, how many buy?)
- **Poisson:** Is there a "rate" over time/space? (e.g., Average of 5 calls per hour, what is the chance of 8?)
- **Hypergeometric:** Am I picking from a small group and *not* putting them back? (e.g., Picking 3 spoiled apples from a crate of 10).

V. Step-by-Step Example

Problem: A salesperson has a 20% ($p = 0.20$) chance of closing a deal. If they talk to 5 ($n = 5$) clients, what is the probability they close exactly 2 deals?

Logic: This is **Binomial** because we have fixed trials and two outcomes (Sale/No Sale). 1. $n = 5, x = 2, p = 0.20, (1-p) = 0.80$. 2. $f(2) = \binom{5}{2}(0.20)^2(0.80)^3$ 3. $\binom{5}{2} = 10$. 4. $f(2) = 10 \times 0.04 \times 0.512 = \mathbf{0.2048}$.

VI. Practice Set

1. A help desk receives an average of 3 calls per hour ($\mu = 3$). What is the probability of receiving exactly 0 calls in the next hour?

2. Given the following distribution, find $E(x)$: $x = 1, f(x) = 0.2$; $x = 2, f(x) = 0.5$; $x = 3, f(x) = 0.3$.

3. In Excel, what formula would you use to find the probability of **3 or fewer** successes in 10 trials with a 0.5 probability?

4. True/False: If the variance of a distribution is 16, the standard deviation is 4.

VII. Answer Key

1. **Poisson.** $f(0) = \frac{3^0 e^{-3}}{0!} = e^{-3} \approx \mathbf{0.0498}$. 2. $E(x) = (1\times0.2)+(2\times0.5)+(3\times0.3) = 0.2+1.0+0.9 = \mathbf{2.1}$. 3. `=BINOM.DIST(3, 10, 0.5, TRUE)` (Note: TRUE = cumulative). 4. **True.** $\sqrt{16} = 4$.