CHAPTER 4: PROBABILITY CONCEPTS

Quantifying Uncertainty for Decision Making

I. The Basics of Probability

Probability is a numerical measure of the likelihood that an event will occur. It is always expressed on a scale from **0 to 1**.

- $P(E) = 0$: The event is impossible.
- $P(E) = 1$: The event is certain.
- **Sample Space (S):** The set of all possible experimental outcomes.

II. Counting Rules

Before calculating probability, we must know how many outcomes are possible. **1. Multi-Step Experiments:** If there are k steps with n_i outcomes at each step, the total outcomes $= n_1 \times n_2 \times ... \times n_k$.

2. Combinations: Used when we select n items from N, and **order does not matter**.

$$C_n^N = \binom{N}{n} = \frac{N!}{n!(N-n)!}$$

3. Permutations: Used when **order matters**.

$$P_n^N = \frac{N!}{(N-n)!}$$

III. Basic Relationships

1. Complement of an Event (A^c): The probability that event A does *not* occur.

$$P(A) + P(A^c) = 1$$

2. Addition Law: Used to find the probability that event A **OR** event B occurs ($A \cup B$).

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Note: If events are **Mutually Exclusive**, $P(A \cap B) = 0$.

IV. Conditional Probability

Conditional probability is the likelihood of an event occurring *given* that another event has already occurred.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Independence

Two events are **Independent** if the occurrence of one does not change the probability of the other. Mathematically, A and B are independent if:

$$P(A|B) = P(A) \quad \text{OR} \quad P(B|A) = P(B)$$

Multiplication Law

Used to find the probability of the **intersection** (A AND B):

$$P(A \cap B) = P(B)P(A|B)$$

If independent: $P(A \cap B) = P(A)P(B)$.

V. Inferential Statistics

This is the process of using data from a sample to make estimates or test hypotheses about a population. Probability is the "bridge" that allows us to state how confident we are in those inferences.

VI. Step-by-Step Example

Problem: A committee of 3 people is to be formed from a pool of 8 employees. How many different committees are possible? If one specific employee (John) must be on the committee, what is the probability he is chosen at random?

Logic: 1. **Counting:** Does order matter? No, a committee is a group. Use **Combinations**.

$$\binom{8}{3} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56 \text{ ways.}$$

2. **Specific Event:** If John is on the committee, we only need to choose 2 more people from the remaining 7.

$$\binom{7}{2} = \frac{7 \times 6}{2 \times 1} = 21 \text{ ways.}$$

3. **Probability:** $21/56 = 0.375$ or 37.5%.

VII. Practice Set

1. You have 10 stocks and want to pick the "Top 3" in specific order of performance. Is this a Combination or Permutation? How many ways?

2. $P(A) = 0.60, P(B) = 0.40$. If $P(A \cap B) = 0.24$, are the events independent?

3. A manager says there is a 70% chance of hitting sales targets (A) and a 40% chance of getting a bonus (B). If the probability of hitting targets AND getting a bonus is 30%, what is the probability of hitting targets OR getting a bonus?

4. What is the complement of rolling a "4" on a standard 6-sided die?

VIII. Answer Key

1. **Permutation** (Order matters). $P_3^{10} = 10 \times 9 \times 8 = \mathbf{720}$. 2. **Yes.** $0.60 \times 0.40 = 0.24$. Since $P(A)P(B) = P(A \cap B)$, they are independent. 3. $0.70 + 0.40 - 0.30 = \mathbf{0.80}$. 4. $1 - 1/6 = \mathbf{5/6}$.