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Measuring Top 4 Banks in Indonesia Portfolio Risks: Estimating *Value at Risk* (VaR) and *Expected Shortfall* (ES) by Copula-GARCH Method

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ABSTRACT

Investments have diverse levels of risk according to their form. The primary goal of investors is to create an investment portfolio that optimizes profits while minimizing risk. As we know, market conditions are not always stable, so measuring risk is necessary. Value at Risk (VaR) is a measuring standard used in financial analysis to measure the maximum loss of an asset or portfolio. Several classical methods of estimating VaR are used, i.e., historical data, Monte-Carlo simulations, and variance-covariance approach. VaR has the disadvantage that the loss value often exceeds the VaR estimation. Shortfall (ES) estimation is needed to explain the loss value that often exceeds the VaR estimation. In the VaR calculation, there is a dependency between the stocks in a portfolio. This paper resorts to copula theory to overcome this problem. In addition, high volatility returns lead to heteroskedasticity, so the GARCH model is required to deal with this problem. This paper presents a new method, copula-GARCH, for calculating the VaR and ES of portfolios by combining the copula and GARCH models. This paper describes how to use the copula-GARCH model to estimate the VaR and ES of a BBNI-BMRI portfolio and BBCA-BBRI portfolio. This research shows that the student-t Copula model best describes the dependence structure of both portfolios' return series. This study showed that the risk value of BBNI-BMRI portfolios was higher than BBCA-BBRI portfolios.

Keywords: *Value at Risk, Expected Shortfall, GARCH, Copula, Portfolio*

1. Introduction

Stocks are one of the investments that many people are interested in. The reason is that the return on the stock investment is more significant than other investments, so it can increase the amount of wealth on a large scale. Even though there is a substantial risk in large profits or commonly called high-risk high return, in such circumstances, one-way investors can reduce the level of risk is to make investments in the form of a portfolio. A portfolio is a form of investment in which investors invest in multiple stocks to obtain optimal returns with minimal risk. Before this, investors will invest in the most efficient portfolio among all portfolios. The market conditions which unstable are a problem in the portfolio. Therefore, it is necessary to estimate the value of risk to determine the value of portfolio losses.

Value at Risk (VaR) has become the standard measure used by financial analysts to estimate an asset's or a portfolio's market risk (Hotta et al., 2008). VaR measures how an asset portfolio's market risk is likely to diminish over a specified period under normal conditions. Securities firms or investment banks commonly use it to analyze the market risk of their asset portfolios. Then it became a general concept with broader applicability. Calculating VaR is simple when a portfolio consists of only a single asset. However, it becomes extremely complicated due to the complexity of the joint multivariate distribution. Furthermore, one of the most challenging aspects of calculating VaR is modelling the dependence structure, as VaR is concerned with the tail of the distribution (Hotta et al., 2008).

VaR can be calculated using three classical methods, i.e., historical data, Monte-Carlo simulation, and variance-covariance approach. The classical methods for calculating VaR presuppose that the joint distribution is known, such as the most widely applied normality of the joint distribution of asset returns in theoretical and empirical models. Linear correlation assumes the variance of the return on a risky asset portfolio is determined by the variances of the individual assets and the linear correlation between the assets in the portfolio. The tails of the financial asset return distribution are fatter than normal distributions. Hence, several empirical studies have confirmed that the multivariate normal distribution does not provide accurate results due to asymmetry and excess financial data. However, VaR has a disadvantage. The loss value often exceeds the estimated VaR value. Therefore, Expected Shortfall (ES) is proposed to describe loss values that often exceed the estimated VaR value. This Expected Shortfall (ES) represents the conditional expectation when the loss value exceeds VaR.

Longin and Solnik (2001) and Ang and Chen (2002) discovered that asset returns are more highly correlated during volatile markets and market downturns. There is a stronger dependence between significant losses than between significant gains. These asymmetries cannot be modelled by symmetric distribution. Consequently, the problem found by normality may result in an inaccurate VaR estimation.

This paper uses the copula theory to overcome these problems. Copulas have been commonly used in finance, including risk management, derivative asset pricing, option valuation, et cetera. Furthermore, copulas can describe more complex multivariate dependence structures, such as non-linear and tail dependence (Hürlimann, 2004). The volatility in the financial market known as heteroskedasticity would be explained and fitted by the GARCH model.

Sklar introduced copula in 1959, which is a function that combines multivariate distribution functions for the lower-dimensional marginal distribution function, at a one-dimensional function (Seth & Myers, 2007). The copula is widely used in joint distribution because it does not require the assumption of normality and decomposes the n-dimensional joint distribution into n-marginal distributions and combines them using a copula. Patton (2001) developed the constant copula into the conditional copula by allowing the first and second conditional moments to change over time. Following Patton's (2001) methodological development, the conditional copula began to be used in the estimate of VaR, e.g., Rockinger and Jondeau (2001) used the Plackett copula with the GARCH process to model innovations using the student-t asymmetrical generalized distribution of Hansen (1994). Palaro and Hotta (2006) estimated the VaR of a portfolio consisting of the NASDAQ and S&P500 indices using a mixed model with the conditional copula and multivariate GARCH. Huang et al. (2009) apply Copula-GARCH to estimate the VaR of a portfolio that consists of NASDAQ and TAIEX.

This paper combines GARCH and copula to estimate the VaR and ES of two banking stock portfolios. Portfolio one consists of BBNI and BMRI stocks, while the other portfolio consists of BBCA and BBRI stocks. First, we apply GARCH models that can represent experimentally reported time-varying mean values and variances of banking returns. Then, we use copulas to illustrate the dependency relationship of both portfolios. After that, we estimate the VaR using the Monte-Carlo simulation and continue to calculate the ES. Finally, we compare the result of the two portfolios.

There are several reasons for using data on Indonesia's top four bank stocks. First, the high returns and affordable stock prices. Second, it has the most outstanding market capitalization of all banks in Indonesia. Third, high financial conditions, growth prospects, and transaction value. Last, it has strong fundamentals to make it easier and faster to stabilize in a crisis.

The contribution of this paper is first to carry out risk management for investors to make decisions before investing. Second, provide an overview to investors regarding the possible risks faced when investing in a portfolio. The last is an alternative method to measure the value of losses in stock portfolios with high volatility.

The rest of the paper is organized as follows. Section 2 introduces the method such as GARCH and Copula families. Section 3 describes the empirical procedure and results, and section 5 summarizes.

2. Methodology

2.1. Stock Return

Return in economic terms is the return received by investors from shares traded in the capital market (company shares are public). Return or return on this investment can be expressed as a form of price difference or percentage. While shares are letters that prove that a person owns a share of the capital of a company. So, it can be concluded that stock return is the level of profit enjoyed by investors on an investment they make (Robert Ang, 2001). There are two kinds of return equations, namely rate of return arithmetic and rate of return Geometric. The rate of return arithmetic equation is as follows:

$$R(P_{i,j}) = \frac{P_{i,j} - P_{i,j-1}}{P_{i,j-1}}, \ i = 1, 2, 3, \ \ldots$$

In the time series for stock price data, the Geometric rate of return equation or so-called log return series is used. The equation is as follows:

$$R(P_{i,j}) = \ln\left(\frac{P_{i,j}}{P_{i,j-1}}\right) \times 100\%, \ i = 1, 2, 3, \ \ldots$$

where

$R(P_{i,j})$ = Return of the i-th stock at the j-th period

$P_{i,j}$ = The i-th stock price at the j-th period

$P_{i,j-1}$ = The i-th stock price at time (j-1)

2.2. Portfolio

A portfolio is a combination of various investment instruments, whether in the form of accidental or indeed decided through planning supported by calculations and rational considerations to maximize investment risk (Sulistiyowati, 2017). When forming a portfolio, investors seek to maximize the expected return from a certain level of risk. In other words, the formed portfolio can offer the lowest level of risk with a certain expected return. According to Choiriyah, efficient portfolio is a portfolio that provides maximum return for investors with a certain level of risk, while the portfolio chosen by investors from the many choices available in the efficient portfolio is the optimal portfolio.

Therefore, the optimal expected return portfolio value will be searched using the following equation:

$$E(R_i) = \sum_{j=1}^{n} \frac{R_{ij}}{t}$$

$$E(R_p) = \sum_{i=1}^{n} X_i E(R_i)$$

where:

$E(R_i)$ = the level of profit expected from the stock i

R_{ij} = profit rate of stock I in the j-th period

t = number of observation periods

$E(R_p)$ = the level of profit expected from the

X_i = weight of funds invested in the i-th stock

2.3. *Autoregressive Integrated Moving Average (ARIMA) Model*

ARIMA is a combination of autoregressive, differencing, and moving average characterized by three terms: p, d, q. ARIMA model is formed through several stages, plotting ACF and PACF, generating the ARIMA model, coefficient test, diagnostic checking, and selecting the best model.

Auto Regressive (AR) = AR (p)

$$\left(1 - \phi_1 B - \phi_1 B^2 - \cdots - \phi_p B^p\right) Y_t = \delta + \varepsilon_t$$

where:

δ = constant value

ϕ_p = auto regressive parameters

ε_t = error value at time t

Moving Average (MA) = MA (q)

$$Y_t = \mu + \left(1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q\right)\varepsilon_t$$

$$Y_t = \mu + \theta_q(B)\varepsilon_t$$

2.3.1 Autocorrelation Function (ACF)

According to Hanke, Wichern and Reitsch (2003), autocorrelation is a correlation between two values in a time series with a time difference (lag) of 0,1,2 periods or more.

2.3.2 Partial Autocorrelation Function (PACF)

If the lag $1,\ 2,\ \ldots,\ k-1$ is considered separately, the linear correlation level between Z_t and $Z_{(t+k)}$ can be measured using the partial autocorrelation function (Makridakis & McGee, 1988). Cryer (1986) described the PACF estimation formula as follows:

$$\phi_k k = \frac{\rho_k - \sum_{j=1}^{k-1} \phi_{k-1,j}\, \rho_{k-j}}{1 - \sum_{j=1}^{k-1} \phi_{k-1,j}\, \rho_j}$$

Note that $\phi_k k = \phi_{(k-1,j)} - \phi_k k \phi_{(k-1,j-k)}$ for $j =\ 1,\ 2,\ \ldots,\ k-1$, where:

$\phi_k k$ = coefficient of partial autocorrelation at lag k

ρ_k = coefficient of partial autocorrelation at lag k estimated by r_k

ρ_j = coefficient of partial autocorrelation at lag j estimated by r_j

$\rho_{(k-j)}$ = coefficient of partial autocorrelation at lag (k-j) estimated by $r_{(k-j)}$

2.3.3 ARIMA modelling

ARIMA model (p,d,q) is a nonstationary time series model, i.e., the mean and variance are not constant for any observation time. According to Box & Jenkins (1976), p is the order of autoregressive parameters, d is the number of nonseasonal differences needed for stationarity, and q is the order of moving average parameters. Based on AR(p), I(d), and MA(q), the general equation is constructed as follows:

$$\phi_p(B)(1-B)^d Y_t = \mu + +\theta_q(B)\varepsilon_t$$

2.3.4 Coefficient Test

A good ARIMA model is a model whose parameter estimates are not significantly different from zero. The coefficient hypothesis test can be performed as follows:

$H_0 : \emptyset = 0$
$H_1 : \emptyset \neq 0$
Significance level: $\alpha\ =\ 0.05$
Critical region: Reject when p-value < α

2.3.5 Diagnostic Checking

The model is considered good if it meets the assumption of residual normality and has no autocorrelations of residuals. Residual normality means the error/disturbance is normally distributed. Residual normality can be tested with the following hypothesis,

H_0: Residual is normally distributed
H_1: Residual is not normally distributed
Significance level: $\alpha\ =\ 0.05$
Critical region: Reject H_0 when p-value < α

Autocorrelation is a deviation where there is a relationship between a series of errors that cause the model to be biased. The resulting residual is white noise if the errors are not related. White noise can be tested with the following hypothesis.

$H_0: \rho_1 = \rho_2 = \cdots = \rho_k = 0$ (Residual is white noise)
H_1: at least one $\rho_k \neq 0$ for $k = 1, 2, \dots, n$ (Residual is not white noise)
Significance level: $\alpha = 0.05$
Critical region: Reject when p-value < α

2.3.6 Best model selection

According to Ramanathan (1995), the selection of the best model can be seen from the model that has the smallest AIC (Akaike's Information Criterion), smallest BIC (Bayesian Information Criterion), and the most significant Log Likelihood. Small AIC and BIC values along with significant log likelihood indicate that the error is small, so the model quality is good. The AIC method is based on the maximum likelihood estimation (MLE) method, maximizing the likelihood, and minimizing error. The AIC formula can be written as follows:

$$AIC = N + \ln\left(\frac{SS_e}{N}\right) + 2K$$

where
N = number of observations
SS_e = sum square of errors
K = number of parameters

2.4. GARCH Model

Generalized Autoregressive Residual Homoskedasticity (GARCH) model plays a significant role in analyzing and forecasting time series data with high volatility and unpredictable shock, especially stocks' return data. We would like to use this model whether the conditional expected value of ARIMA model's residual variance is not constant, in other words having ARCH effects or heteroskedastic.

Tsay (2005) mentioned Lagrange-Multiplier Test (ARCH-LM Test) as such one method to verify the existence of ARCH effects in the ARIMA model. The aim of the test is to show that variance of the model's residual not only the function of independent variable, but also depends on squared residual series of previous periods (Enders, 1995).

Let $a_t = r_t - \mu_t$ be the residual of the model, then it is equivalent with the linear regression:

$$a_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_m a_{t-m}^2 + e_t,\ t = m+1, \dots, T$$

where e_t denotes the error term, m is an integer, and T is a sample size.

Assume that given the return of stock $\{r_t\}$, where $t = 1,2, \dots, T$, then GARCH(p,q) model can be expressed as follows:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \beta_i \, \sigma_{t-i}^2 + \sum_{j=1}^{q} \alpha_j \, a_{t-j}^2$$

$$a_t = \sigma_t \varepsilon_t$$

$\{\varepsilon_t\}$ sequence of iid random variables = error term

$$\omega > 0,\ \alpha_j \geq 0,\ \beta_i \geq 0, \text{ and } \sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i) < 1$$

2.5.Dependence

Dependence or association between random variables can be measured through numerous ways. Practically, linear correlation coefficient or Pearson's correlation is commonly used as a measure of dependence. Nevertheless, Pearson's correlation is not copula-based and sometimes becomes misleading due to most random variables not being jointly elliptically distributed (Embrechts et al., 2001).

2.5.1 Concordance

Let (x_i, y_i) and (x_j, y_j) represent two observations from a continuous random variable vector (X, Y), then (x_i, y_i) and (x_j, y_j) are *concordant* if $(x_i - x_j)(y_i - y_j) > 0$ and *discordant* if $(x_i - x_j)(y_i - y_j) < 0$ (Nelsen, 2006).

2.5.2 Kendall's Tau

Let (X_1, Y_1) and (X_2, Y_2) independent random variables with copula C_1 and C_2, then the population version of Kendall's Tau is defined as follows:

$$\tau = \tau_{X,Y} = P[(X_1 - X_2)(Y_1 - Y_2) > 0] - P[(X_1 - X_2)(Y_1 - Y_2) < 0]$$

If Q denotes the probability of concordance minus discordance, then Kendall's Tau can be rewritten as follows:

$$\tau = Q(C_1, C_2) = 4 \iint_{I^2} C_2(u, v)\, dC_1(u, v) - 1$$

Kendall's Tau is one of two measures of dependence/concordance that provide more appropriate results for nonelliptical distributions. Kendall's Tau dependency test for random variables test can be expressed with the following hypothesis:

$H_0 : \tau = 0$ (Two random variables are independent)
$H_1 : \tau \neq 0$ (Two random variables are dependent)
Significance level: $\alpha = 0.05$
Critical Region: Reject H_0 when p-value < α

2.5.3 Spearman's Rho

Let (X_1, Y_1), (X_2, Y_2), and (X_3, Y_3) independent random variable vectors with a joint distribution function H , and marginal F and G as well as copula C . Therefore, the population version of Spearman's Rho is defined as follows:

$$\rho_{X,Y} = 3(P[(X_1 - X_2)(Y_2 - Y_3) > 0] - P[(X_1 - X_2)(Y_2 - Y_3) < 0])$$

which simply be the probability of concordance and discordance of two vectors, (X_1, Y_1) i.e., a pair of random variables with distribution function H and copula C , while (X_2, Y_3) is a pair of independent random variables (also applied equally for (X_3, Y_2)) with copula Π. Based on that definition, the formula could be rewritten as:

$$\rho_{X,Y} = 3Q(C, \Pi) = 12 \iint_{I^2} C(u, v)\ du\, dv\ - 3$$

Hence, Spearman's Rho dependency test for random variables test can be expressed with the following hypothesis:

$$H_0 : \rho = 0$$

$$H_1 : \rho \neq 0$$

Significance level: $\alpha = 0.05$
Critical Region: Reject H_0 when p-value < α

2.6. Copula

2.6.1 Copula Theory

Copula is a function that relates the univariate marginal distribution to the multivariate distribution (Bob, 2013). Copula serves as a tool to study nonlinear dependence between events in multivariate cases. Copula has several advantages, among others, that it does not require an assumption of normality at the marginal it is quite flexible in various data, especially for financial data that does not meet the characteristics of a normal distribution and can show the distribution pattern of the data on the tail of the distribution of each variable. There are basically three Copulas used in financial applications, namely the Elliptical Copula, the Archimedean Copula, and Plackett Copula.

2.6.2 Elliptical Copula

The Elliptical Copula is derived from a multivariate elliptical distribution. The Elliptical Copula consists of 2 family members, that is the Gaussian Copula and the t-Student Copula.

(1) Gaussian Copula

Gaussian Copula is a type of copula that uses a bivariate standard normal distribution. The formulation of the Gaussian copula can be written as follows:

$$C_p^{Ga}(u_1,u_2) = \int_{-\infty}^{\Phi^{-1}(u_1)} \int_{-\infty}^{\Phi^{-1}(u_2)} \frac{1}{2\pi(1-\rho_{12}^2)^{1/2}} exp\left\{-\frac{s^2-2\rho_{12}st+t^2}{2(1-\rho_{12}^2)}\right\} ds\, dt$$

where

$s = \Phi^{-1}(u_2)$

$t = \Phi^{-1}(u_1)$

$\rho_{12} =$ ordinary linear correlation coefficient according to bivariate normal distribution with $-1 < \rho_{12} < 1$

(2) t-Student Copula

t-Student Copula is a type of copula that uses the t-student distribution. This copula form uses a bivariate student distribution which can be written as follows:

$$C_{v,\rho}^{t}(u_1,u_2) = \int_{-\infty}^{t_{v^1}(u_1)} \int_{-\infty}^{t_{v^1}(u_2)} \frac{1}{2\pi(1-\rho_{12}^2)^{1/2}} \left\{1+\frac{s^2-2\rho_{12}st+t^2}{v(1-\rho_{12}^2)}\right\}^{-(v+2)/2} ds\, dt$$

where

$s = t_v^{-1}(u_2)$

$t = t_v^{-1}(u_1)$

$\rho_{12} =$ ordinary linear correlation coefficient according to bivariate normal distribution

2.6.3 Archimedan Copula

Meanwhile, Archimedean Copula is a continuous multivariate copula that has a simple shape but has a wide range of dependency structures and has a dependency approach that is easy to implement. In addition, Archimedean copula is also a simple bivariate copula in describing dependencies. This copula is widely used in various applications in finance, insurance, and the like because of its simple shape and nature. The flexibility of the Archimedean Copula can be expressed by the generator function $\varphi(u)$. In the bivariate case, the equation for the Archimedean Copula function is as follows:

$$C(u_1,u_2) = \varphi^{-1}(\varphi(u_1)+\varphi(u_2))$$

with:

$C(u_1,u_2) =$ bivariate copula distribution function

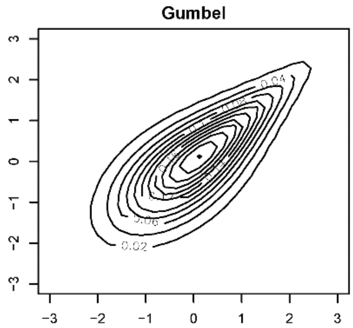
$\varphi(u) =$ copula generator function

$\varphi^{-1} =$ the inverse of the copula generator function

The Archimedean Copula has three important family members, that is Clayton copula, Gumbel copula, and Frank copula.

(1) Gumbel Copula

Gumbel copula is a copula that has the best sensitivity to high risk (Damasari, 2015). This copula is used to model asymmetric dependencies in the data. One of the advantages of Copula Gumbel is that it has the ability to capture strong upper tail dependencies and weak lower tail dependencies.



The function of the Gumbel Copula generator is:

$$\varphi(u) = (-\ln(u))^{\theta}$$

while the inverse of the generator function above can be written as follows:

$$\varphi^{-1}(y) = u = e^{-y^{1/\theta}}$$

Based on the generator function equation above, the cumulative distribution function of Clayton Copula can be expressed as follows:

$$C(u_1, u_2) = exp\left(-\left[(-\ln u_1)^{\theta} + (-\ln u_2)^{\theta}\right]^{1/\theta}\right)$$

with:

$C(u_1, u_2)$ = bivariate copula distribution function

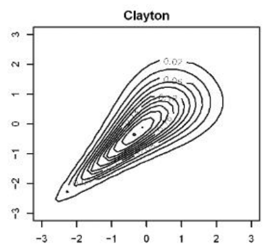
$\varphi(u)$ = copula generator function

φ^{-1} = the inverse of the copula generator function

θ =parameter copula

(2) *Clayton Copula*

Clayton Copula is a copula that can be used to study correlation risk because of its ability to capture lower tail dependencies.



The generator function of Clayton Copula is as follows:

$$\varphi(u) = u^{-\theta} - 1$$

while the inverse of the generator function above is as follows:

$$\varphi^{-1}(t) = (t+1)^{-\frac{1}{\theta}}$$

So, based on the generator function above, we get the following:

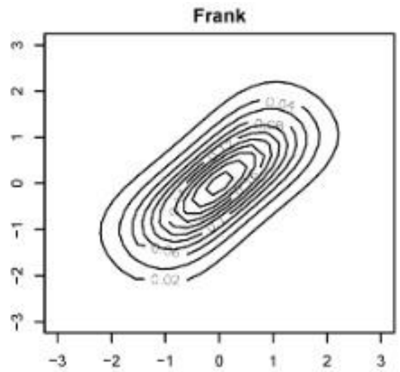
$$C(u_1, \dots, u_d) = \left[\sum_{i=1}^{d} u_i^{-\theta} - d + 1\right]^{-\frac{1}{\theta}}$$

while the bivariate form of Gumbel Copula can be written as follows:

$$C(u_1, u_2) = \left[u_1^{-\theta} + u_2^{-\theta} - 1\right]^{-\frac{1}{\theta}}$$

(3) *Frank Copula*

According to Cherubini et al. (2004), Frank copula becomes the most commonly used Archimedian copula in solving empirical and practical case. Frank copula itself has an advantage of having a wide dependency length, and also an ability to explain postive and negative dependence between random variables.



The generator function of Frank copula is defined below:

$$\varphi(t) = -\ln\left(\frac{e^{-\theta t} - 1}{e^{-\theta} - 1}\right),\ \theta > 0$$

Based on the generator function above, the Frank copula cumulative distribution function would be:

$$C(u_1, u_2) = -\frac{1}{\theta}\ln\left(1 + \frac{\left(e^{-\theta u_1} - 1\right)\left(e^{-\theta u_2} - 1\right)}{e^{-\theta} - 1}\right)$$

One property of Frank copula is having radial symmetry, which means there will be no upper or lower tail dependencies, or could be expressed as:

$$C(u_1, u_2) = \hat{C}(u_1, u_2)$$

2.6.6 Plackett Copula

In 1965, the Plackett Copula was developed from a bivariate distribution with continuous marginals. One of the weaknesses of this type of copula is that it cannot be easily expanded if the dimensions are more than 2. The Plackett Copula function can be defined as follows (Nelsen, 2006):

$$C(u, v) = \frac{[1 + (\theta - 1)(u + v)] \pm \sqrt{[1 + (\theta - 1)(u + v)]\ - 4uv\theta(\theta - 1)}}{2(\theta - 1)}$$

Plackett Copula was known as a comprehensive copula, for the following reasons:

$\theta \to 0 \;\Rightarrow$ the copula becomes $\mathrm{W}(u, v)$ => countermonotonicity
$\theta \to \infty \;\Rightarrow$ the copula becomes $\mathrm{M}(u, v)$ => comonotonicity
$\theta = 1 \;\Rightarrow$ the copula becomes $\Pi(u, v)$ => independence

2.7. Value at Risk (VaR)

Value at Risk is one of the measurements that can calculate the estimated maximum loss of shares invested over a certain period of time with a confidence interval (confidence level) (1-α). VaR in general can be written as follows:

$$P\left(r \leq \widehat{VaR}\right) = 1 - \alpha$$

where
r = the return for a certain period
α = the error rate (Jorion, 2006)

In the portfolio, the VaR value with a confidence level $(1 - \alpha)$ in a time period of t days can be calculated using the following equation (Maruddani and Purbowati, 2009):

$$VaR_{(1-\alpha)}(t) = W_0 R * \sqrt{t}$$

where:
W_0 = portfolio initial investment fund

$R*$ = value of the quantile α of the return distribution
t = time period

There are three methods for calculating VaR estimates, to be spesific:

a. *Variant-Covariance Method*
In this method, VaR will be calculated using a parametric approach based on the assumption that the return or percentage change in price is normally distributed. The Variance-Covariance method uses a matrix containing the elements of volatility, correlation, and asset weights. In this method, the variance is calculated using the covariance matrix of the return and then the variance will be used to calculate the VaR. The VaR value of a single asset can be calculated using the following equation:

$$VaR = Pz_{\alpha}\sigma\sqrt{t}$$

where:
Pz_{α} = standard normal distribution quantile on significance α
σ = single asset return volatility
t = stock determination period

b. *Historical Simulation Method*
The Historical Simulation method is in its calculation because it does not require assumptions about normality (non-parametric) and time series so that it can be directly used to calculate the VaR. The return data in this method is sorted in a certain order which is divided into percentiles. In the Historical Simulation method, the VaR for the portfolio is estimated at the return value by creating a hypothetical time. The formula that can be used in calculating VaR is as follows:

$$VaR_{(1-\alpha)} = \mu(R) - R_{\alpha}$$

where:
$VaR_{(1-\alpha)}$ = maximum potential loss
$\mu(R)$ = the average value of the return
R_{α} = maximum loss α

c. *Monte Carlo Simulation Method*
In 1997, Boyle introduced the use of the Monte Carlo Simulation method. This method is also non-parametric because it does not use the assumption of a normal distribution. This simulation method has several types of algorithms for estimating the VaR value on both single assets and portfolios. Perform simulations by generating random numbers based on the characteristics of the data which are then used to estimate the VaR value. The VaR value using the Monte Carlo method can be calculated using the following equation:

$$VaR = \mu - (Z \times \sigma)$$

where:
VaR = maximum potential loss
μ = the average value of the return
Z = average return
σ = standard deviation

In this paper, we will use the estimation of VaR parameters using the Monte Carlo simulation method because the shape of the simulation method is simpler than the Variant-Covariance method with great accuracy. In addition, in this method, the return data does not need to be normally distributed.

2.8. Expected Shortfall (ES)

Expected Shortfall or Conditional Value at Risk is a measure of the risk expectation value if the return exceeds the maximum return limit (VaR). The advantages of ES are, among others, that the expected shortfall can calculate risk on data with normal or abnormal distribution so that the expected shortfall can properly reflect the effect of diversification to minimize risk. ES can overcome the weakness of VaR because VaR does not pay attention to any loss that exceeds the VaR level. ES takes into account losses above the VaR value that may occur, so ES always has a value less than VaR. The ES equation can be expressed as follows:

$$\begin{aligned} ES_{(1-\alpha)}(R) &= E\left(R \middle| R \leq VaR_{(1-\alpha)}(R)\right) \\ &= \int_{-\infty}^{VaR_{(1-\alpha)}} \frac{f(x)}{P(R \leq VaR_{(1-\alpha)}} \, dx \\ &= \int_{-\infty}^{VaR_{(1-\alpha)}} x \frac{f(x)}{\alpha} \, dx \end{aligned}$$

where:
$f(x) = \frac{1}{\sigma\sqrt{2\pi}} exp^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$
x = average count
μ = standard deviation

3. Empirical Results

3.1. Data Description

This study uses data on the closing price of BBNI, BMRI, BBRI, and BBCA shares sourced from finance.yahoo.com from January 2, 2017, to December 30, 2021, with approximately 1261 daily data. The market returns of BBNI, BMRI, BBRI, and BBCA are shown in **Figure 1** below.

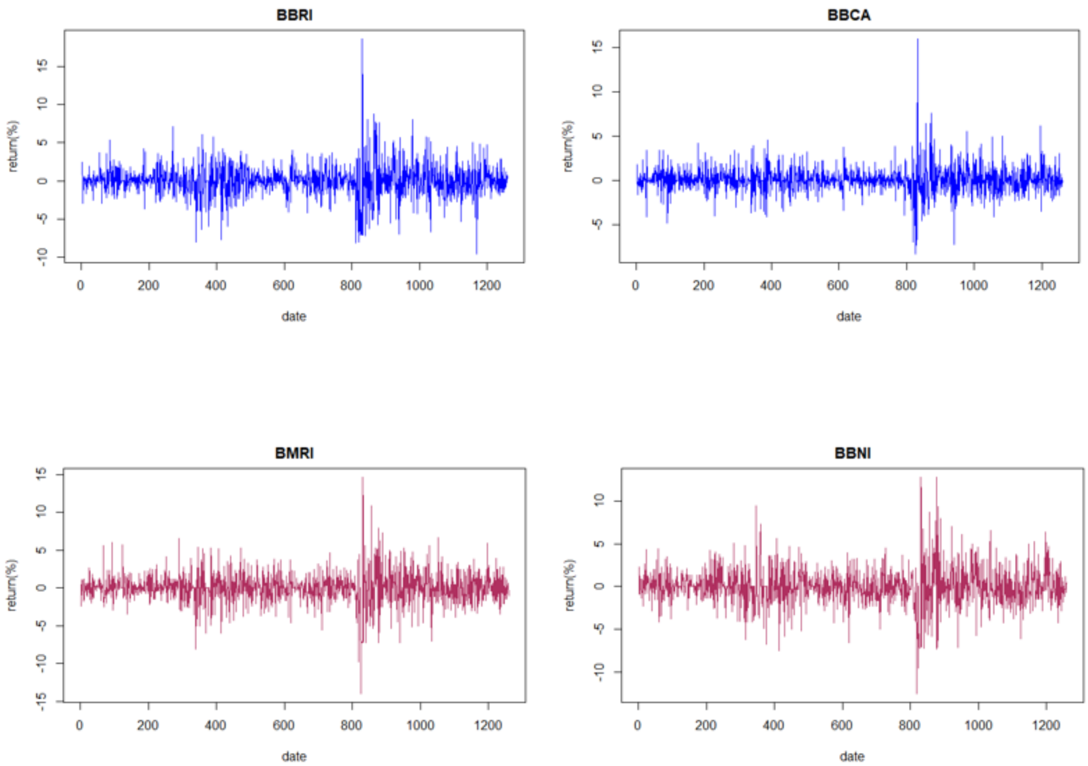


Figure 1 Market stock returns of top four Indonesian banks

3.1.1 Normality Test

A normality assumption test was carried out to the four stocks with a hypothesis
H_0: Data is normally distributed

H_1: Data is not normally distributed

With critical region reject when p-value < $\alpha = 0.05$, the results and conclusions are as follows:

Normality Test		
Stock	**P-value**	**Conclusion**
BBNI	<2.2e-16	Data is not normally distributed
BMRI	<2.2e-16	Data is not normally distributed
BBRI	<2.2e-16	Data is not normally distributed
BBCA	<2.2e-16	Data is not normally distributed

3.1.2 Descriptive analysis

We calculate the mean, variance, and skewness of the four stocks and obtain the following results:

Stock	Mean	Variance	Skewness
BBNI	0.01589359	5.009742	0.1602712
BMRI	0.01537895	4.526171	0.01710295
BBRI	0.04490954	4.537506	0.4153271
BBCA	0.06792008	2.353353	0.797866

From the mean, variance, and skewness values above, it can be concluded that the average return on BBNI, BMRI, BBRI, and BBCA is positive. Stocks tend to provide benefits. The highest variance return is owned by BBNI, so BBNI tends to provide the highest losses. The skewness value from BBNI, BMRI, BBRI and BBCA returns more than zero, which means that the four stocks shift from the mean value so that the return is asymmetrical.

3.2. *Marginal Distributions Modelling*

3.2.1 ARIMA Modelling

The stationary test is performed using the Augmented Dickey-Fuller Test for the four stocks returns. The result obtained that the return of the four stocks had a p-value $<\alpha = 0.05$. Thus, it could be concluded that the return of the four stocks was stationary to mean and variance. Furthermore, we continue to get the ARIMA model using R software. We obtained the ARIMA model for BBNI return that is ARIMA (4,0,4) with zero mean, BMRI return that is ARIMA (2,0,1) with zero mean, BBCA return that is ARIMA (0,0,3) with zero mean, and BBRI return that is ARIMA (2,0,1) with zero mean. The four models were declared to have passed the significance test of parameters.

We apply Ljung-box test, augmented dickey-fuller test, and Jarque-Bera test on residuals. The results are shown in the following tables.

Ljung-Box Test			
Stock	**Model**	**P-value**	**Conclusion**
BBNI	ARIMA (4,0,4) with zero mean	0.1707	No autocorrelation
BMRI	ARIMA (2,0,1) with zero mean	0.192	No autocorrelation
BBCA	ARIMA (0,0,3) with zero mean	0.2065	No autocorrelation
BBRI	ARIMA (2,0,1) with zero mean	0.6827	No autocorrelation

Augmented Dickey-Fuller Test			
Stock	**Model**	**P-value**	**Conclusion**
BBNI	ARIMA (4,0,4) with zero mean	0.01	Residual stationary is fulfilled
BMRI	ARIMA (2,0,1) with zero mean	0.01	Residual stationary is fulfilled
BBCA	ARIMA (0,0,3) with zero mean	0.01	Residual stationary is fulfilled
BBRI	ARIMA (2,0,1) with zero mean	0.01	Residual stationary is fulfilled

Jarque-Bera test for normality			
Stock	**Model**	**P-value**	**Conclusion**
BBNI	ARIMA (4,0,4) with zero mean	< 2.2e-16	Residual normality is fulfilled
BMRI	ARIMA (2,0,1) with zero mean	< 2.2e-16	Residual normality is fulfilled
BBCA	ARIMA (0,0,3) with zero mean	< 2.2e-16	Residual normality is fulfilled
BBRI	ARIMA (2,0,1) with zero mean	< 2.2e-16	Residual normality is fulfilled

Based on the best ARIMA model above, we derive the model equation for each stock return as follows:

- **BBNI**

$$X_t = 0.396180X_{(t-1)} + 0.979032X_{(t-2)} + 0.388230X_{(t-3)} - 0.940901X_{(t-4)} + \epsilon_t - 0.390262\epsilon_{(t-1)} - 0.988418\epsilon_{(t-2)} - 0.364570\epsilon_{(t-3)} + 0.943131\epsilon_{(t-4)}$$

- **BMRI**

$$X_t = -0.649690X_{(t-1)} - 0.066927X_{(t-2)} + \epsilon_t + 0.665532\epsilon_{(t-1)}$$

- **BBCA**

$$X_t = \epsilon_t - 0.631\epsilon_{(t-1)} - 0.0403\epsilon_{(t-2)} + 0.0665\epsilon_{(t-3)}$$

- **BBRI**

$$X_t = -0.525403X_{(t-1)} - 0.092658X_{(t-2)} + \epsilon_t + 0.566376\epsilon_{(t-1)}$$

3.2.2 ARMA-GARCH Modelling

Lagrange Multiplier test is performed on the four ARIMA models. The test results concluded that the four data were heteroscedastic, so the GARCH model was formed by adding the ARMA effect of the best model (ARMA-GARCH). We obtained the ARMA-GARCH model for BBNI return that is ARMA(4,4)-GARCH(1,1) with non-zero mean, BMRI return that is ARMA(2,1)-GARCH (1,1) with non-zero mean, BBCA return that is ARMA(0,3)-GARCH(1,1) with non-zero mean, and BBRI return that is ARMA(2,1)-GARCH(1,1) with non-zero mean. All model parameters are declared significant to be used for further analysis.

Based on the ARMA-GARCH model, we can derive the equation as follows:

- **BBNI**

$$X_t = 0.0148171 + 0.5231988X_{(t-1)} + 0.9216182X_{(t-2)} + 0.0571051X_{(t-3)} - 0.6648074X_{(t-4)} + \epsilon_t - 0.5343346\epsilon_{(t-1)} - 0.9414196\epsilon_{(t-2)} + 0.0090355\epsilon_{(t-3)} + 0.6330995\epsilon_{(t-4)}$$

$$\sigma_t^2 = 0.0718439 + 0.1011234\epsilon_{(t-1)}^2 + 0.8881150\sigma_{(t-1)}^2$$

- **BMRI**

$$X_t = 0.13367 - 0.70755X_{(t-1)} - 0.09496X_{(t-2)} + \epsilon_t + 0.65429\epsilon_{(t-1)}$$

$$\sigma_t^2 = 0.09247 + 0.06515\epsilon_{(t-1)}^2 + 0.91163\sigma_{(t-1)}^2$$

- **BBCA**

$$X_t = 0.0998870 + \epsilon_t - 0.1454479\epsilon_{(t-1)} - 0.0668100\epsilon_{(t-1)} - 0.0016012\epsilon_{(t-3)}$$

$$\sigma_t^2 = 0.1314215 + 0.1026322\epsilon_{(t-1)}^2 + 0.8335889\sigma_{(t-2)}^2$$

- **BBRI**

$$X_t = 0.147216 - 0.513636X_{(t-1)} - 0.085351X_{(t-2)} + \epsilon_t + 0.499028\epsilon_{(t-1)}$$

$$\sigma_t^2 = 0.120151 + 0.097798\epsilon_{(t-1)}^2 + 0.873912\sigma_{(t-1)}^2$$

3.3. Copulas Modelling

3.3.1. Normality Test

The Kolmogorov Smirnov test is used to measure the residual normality of ARMA-GARCH in each of the stocks of BBNI, BMRI, BBRI, and BBCA. Based on the results, it can be concluded that the distribution of four residual stocks is not normal.

3.3.2. Dependency Test

Kendall's Tau and Spearman's Rho correlation is used to measure dependencies between the returns of BBNI, BMRI, BBRI, and BBCA stocks. Based on both Kendall's Tau and Spearman's Rho dependency tests' results, it can be concluded that there is a correlation between the returns of BBNI and BMRI stock portfolios. Likewise, the return of BBCA and BBRI stock portfolios. Therefore, when the two pairs of stocks are dependent on each other, copula method will become the most adequate and manageable calculation to accommodate the dependence properties.

3.3.3. Copula Model Selection

After estimating the parameters of the marginal distribution, the residual distributions of four stocks are not normal, and each stock has mutual dependencies. We will move ahead to estimate the copula parameters. In our work, we use five copula functions: Gaussian, Gumbel, Frank, Student-t, and Plackett. According to the table, the student-t copula has the highest log likelihood value. This indicates that the student-t copula best describes the dependence structure in both stock portfolios.

Copula Estimation and Selection for BBNI and BMRI Stock Portfolios		
Copula Model	Parameter	Loglikelihood
Gaussian	0.6585	354.1
Clayton	1.275	320.7
Gumbel	1.745	332
Frank	4.626	284.6
Student-t	**0.6337**	**373**
Plackett	8.026	305.8

Copula Estimation and Selection for BBCA and BBRI Stock Portfolios		
Copula Model	Parameter	Loglikelihood
Gaussian	0.2089	27.62
Clayton	-	-
Gumbel	1.159	33.4
Frank	1.339	27.88
Student-t	**0.2159**	**66.4**
Plackett	2.139	32.58

Based on the table above presents the parameter estimation results for the four stocks with Copula Gaussian, Clayton, Gumbel, Frank, t-Student, and Plackett calculated using r studio. The highest maximum likelihood value is owned by copula student-t, which is 373 for BNI and MANDIRI shares and 66.4 for BCA and BRI shares. This shows that the best copula student-copula model is better able to catch heavy tails compared to other copula models.

Therefore, the student-t copula model for BBCA and BBRI stock portfolio is:

$$C^t_{3.3940,0.2159} = P(t_{3.3940}(X_1) \leq u_1, t_{3.3940}(X_2) \leq u_2) = t^{0.2159}_{3.3940}(t^{-1}_{3.3940}(u_1), t^{-1}_{3.3940}(u_2))$$

and the student-t copula model for BBNI and BMRI stock portfolio is:

$$C^t_{4.0235,0.6337} = P(t_{4.0235}(X_1) \leq u_1, t_{4.0235}(X_2) \leq u_2) = t^{0.6337}_{4.0235}(t^{-1}_{4.0235}(u_1), t^{-1}_{4.0235}(u_2))$$

3.4. Estimation of Value at Risk (VaR)

An optimal portfolio weight calculation will be carried out before estimating the Value at Risk (VaR). In this paper, each stock's weight will be defined, with BBNI receiving 50% of the weight and BMRI receiving 50%. Likewise, BBCA stock weight is 50%, and BBRI is also 50%. These compositions of weights are assumed as an optimal portfolio.

VaR calculations on the bivariate portfolio will be analyzed for 21 days (about 3 weeks) at a 95 percent confidence level using Monte-Carlo simulations that generate up to 1259 data and repeat five times. Because of the confidence level and the VaR estimation formula, we estimate VaR_{1260} at time t = 1260, and at each subsequent observation, we re-estimate VaR. We estimate VaR_{1261} using observations from t = 2 to t = 1260 and VaR_{1262} using observations from t = 3 to t = 1261, and so on until the sample-out observations we have updated are depleted.

By taking the average value of those five times repetition results, the summary of the VaR estimation results is shown as follows:

Estimation of Value at Risk based on Student-t Copula-GARCH model using Monte-Carlo Simulation		
Stock Portfolios	α	VaR (%)
BBRI dan BBCA	0.05	-4.18139
BMRI dan BBNI	0.05	-6.10168

Based on the results above, the estimated VaR value for BBRI and BBCA portfolio is –4.18139% and the estimated VaR value for BMRI and BBNI portfolio is –6.10168%, while the minus sign indicates the portfolio return's loss. In other words, on the 5% significance level, BMRI and BBNI portfolio will own a higher risk of loss rate than the other portfolio.

Particularly, assumed that an investor would like to put some of their money e.g., Rp10,000,000.00 into both portfolios. With the following estimation of VaR, about the next 21 days, the investor will be having a maximum loss at around Rp418,139.00 from BBRI and BBCA stocks portfolio and around Rp610,168.00 from BMRI and BBNI stocks portfolio. In 95% confidence level, there might be 5% possibilities of experiencing returns loss more than those amount for both portfolios.

3.5. Estimation of Expected Shortfall (ES/CVaR)

After estimating the Value at Risk value, we continue to estimate the Expected Shortfall (ES) or Conditional Value at Risk (CVaR) for a 95 percent confidence level. The ES calculation results for both bivariate portfolios are as follows:

Estimation of Conditional Value at Risk		
Stock Portfolios	α	CVaR/ES (%)
BBRI dan BBCA	0.05	-5.6481
BMRI dan BBNI	0.05	-7.7254

On the previous section, Expected Shortfall or Condition Value at Risk was already defined as an expected value of stocks' returns whether return values exceed the VaR value. Regarding the estimation results above, it could be seen that BBRI and BBCA portfolio's CVaR/ES estimate value is –5.6481%, while BMRI and BBNI portfolio's CVaR/ES estimate value is –7.7254%. Thus, the interpretation remains the same that BMRI and BBNI portfolio will own a higher risk of loss rate than the other portfolio on 5% significance level.

Once more assuming an investor would like to put Rp10,000,000.00 into both portfolios, with the following estimation results of CVaR/ES, the investor will be having the bigger loss than VaR at Rp564,810.00 maximum from BBRI and BBCA stocks portfolio and Rp772,540.00 maximum from BMRI and BBNI stocks portfolio if the worst case happened about the next 21 days (about 3 weeks). In other words, the investor might need to spare some extra money for the difference between CVaR and VaR. For BBRI and BBCA portfolio, if something extremely worse happened, the investor should spare about Rp146,671.00, while for BMRI and BBNI portfolio is about Rp162,372.00.

4. Conclusion

Every stock portfolio has a risk of facing loss. Therefore, the portfolio's total losses must be estimated as accurately as possible so investors can diversify their funding well. Because stock portfolio consists of two or more random variables that correlate with each other, joint distribution would most likely be applied to estimate portfolio's risk measures (VaR and CVaR). The joint distribution should be rid of any restrictions that can make the estimation results inaccurate, such as normality assumptions because stock has such high volatility and non-linear returns. Because of that, Monte-Carlo simulation will be the most suitable method to do the VaR and CVaR estimation.

This paper makes use of copula-GARCH method to estimate the portfolio's risk measures for the top 4 banks in Indonesia. This method offers a flexible way to estimate the value, by making marginal ARMA-GARCH distribution and accounting the dependence of portfolio with copula distribution that fitted the data. Through the study, it is proven that student-t copula describes the dependence of both stock portfolios the best, by looking at the biggest log-likelihood value. At the end, using Monte-Carlo simulation and averaging the 21 days estimation, it can be concluded that BBRI and BBCA portfolio is preferable for the investor due to the lower risk rates for the next 21 days. Looking at the estimation results, we can also conclude that the more we simulate, the better and more accurate the estimation will be. This shows that copula-GARCH method can account for the high volatility effect variable and distribution, which is appropriate for investment method such as stocks.

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