

SU - DÚ1 - TREŠTÍK

$$X = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 3 \\ 1 & 2 & 1 \end{pmatrix}, y = \begin{pmatrix} 2 \\ 1 \\ 7 \end{pmatrix}$$

$$X = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 9 \end{pmatrix}, \quad \begin{matrix} x^0 \\ x^1 \\ x^2 \end{matrix}$$

$$y = \theta_2 x^2 + \theta_1 x + \theta_0$$

$$y = X \vec{\theta} + \vec{\epsilon}$$

$$\vec{\theta} = (X^T X)^{-1} X^T \vec{y}$$

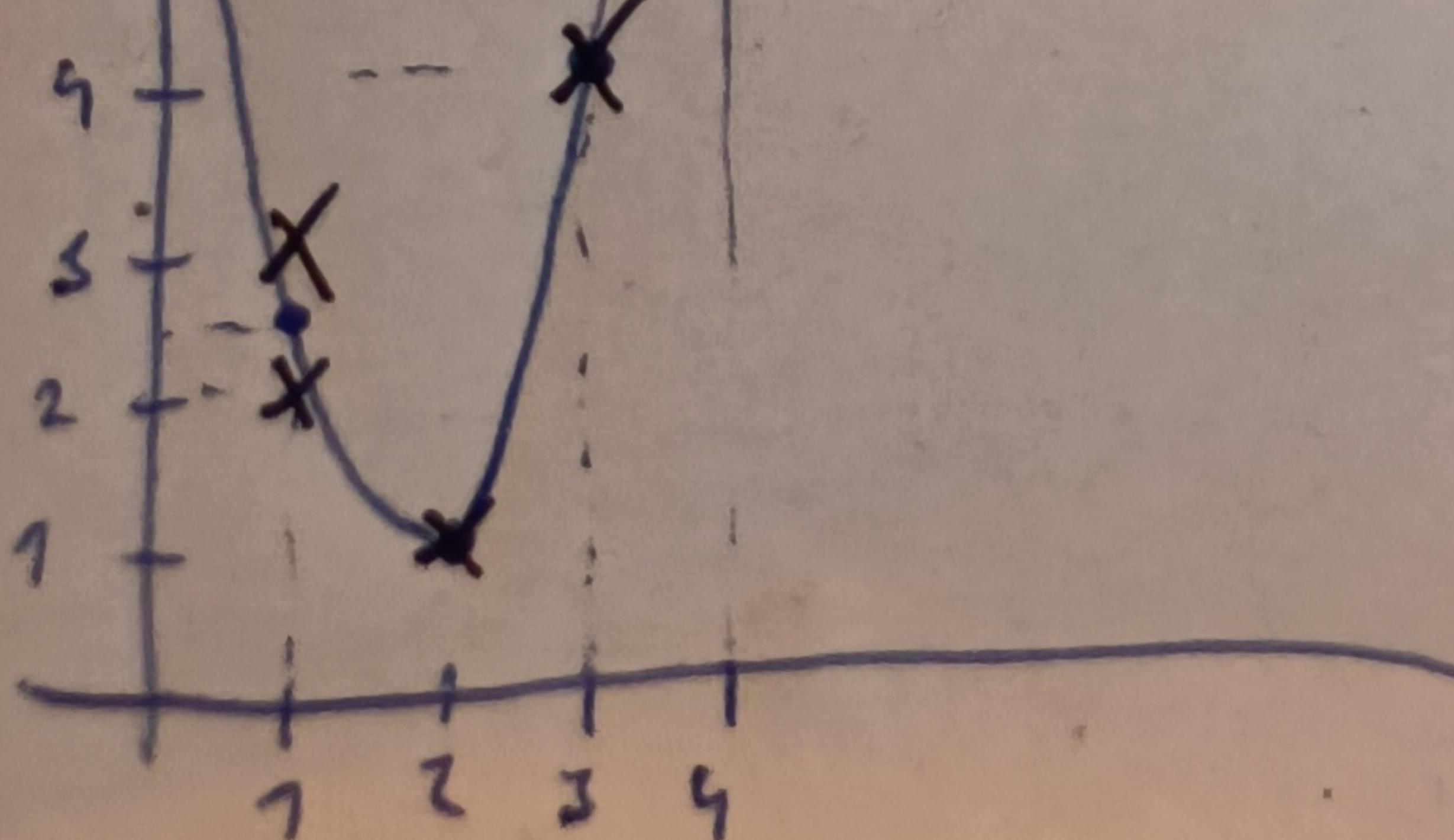
$$X^T = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 9 & 9 \end{pmatrix}$$

$$\vec{\theta} = \begin{pmatrix} 29 & -59 & 17 \\ 2 & 5 & 5 \\ 5 & 17 & -8 \\ 17 & -8 & 8 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 9 & 9 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 7 \end{pmatrix}$$

$$\vec{\theta} = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & -\frac{5}{2} & 1 \\ -\frac{5}{2} & -\frac{5}{2} & \frac{5}{2} & -\frac{5}{2} \\ 1 & 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 7 \end{pmatrix}$$

$$\vec{\theta} = \begin{pmatrix} \frac{17}{2} \\ -\frac{55}{4} \\ \frac{9}{4} \end{pmatrix} = \theta_0 \\ = \theta_1 \\ = \theta_2$$

$$! y = \frac{9}{4}x^2 - \frac{53}{4}x + \frac{17}{2} !$$



approx.

$$y_0(x=0) = \frac{17}{2} = 8,5$$

$$y_1(x=1) = \frac{10}{4} = 2,5$$

$$y_2(x=2) = \cancel{15,5} = 1$$

$$y_3(x=3) = 4$$

$$y_4(x=4) = 11,5$$

$$\vec{\theta} = \begin{pmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \end{pmatrix}$$

$$(X^T X) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 9 & 9 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 9 & 9 \end{pmatrix} = \begin{pmatrix} 4 & 7 & 15 \\ 7 & 15 & 37 \\ 15 & 37 & 99 \end{pmatrix}$$

$$(X^T X)^{-1} = \begin{pmatrix} 4 & 7 & 15 \\ 7 & 15 & 37 \\ 15 & 37 & 99 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}^{\frac{1}{2}} =$$

$$= \begin{pmatrix} 1 & \frac{7}{9} & \frac{15}{27} \\ \frac{7}{9} & 15 & 37 \\ 15 & 37 & 99 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & \frac{7}{9} & \frac{15}{27} \\ 0 & \frac{21}{27} & \frac{15}{27} \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{7}{27} & \frac{5}{9} \\ 0 & \frac{7}{9} & \frac{5}{9} \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -11 & & \\ 0 & \frac{43}{9} & \frac{171}{9} \\ & -\frac{15}{9} & 0 \end{pmatrix} \cdot \frac{1}{9} = \begin{pmatrix} \frac{7}{9} & \frac{15}{9} & \frac{1}{9} \\ 0 & 1 & \frac{45}{81} \\ 0 & \frac{43}{9} & \frac{171}{81} \end{pmatrix} = \begin{pmatrix} \frac{7}{9} & \frac{15}{9} & \frac{1}{9} \\ 0 & 1 & \frac{45}{81} \\ 0 & \frac{43}{9} & \frac{171}{81} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & \frac{7}{9} & \frac{15}{27} \\ 0 & \frac{7}{9} & \frac{15}{27} \\ 0 & 0 & \frac{8}{27} \end{pmatrix} \cdot \frac{1}{27} = \begin{pmatrix} 1 & \frac{7}{9} & \frac{15}{27} \\ 0 & 1 & \frac{45}{81} \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & \frac{7}{9} & \frac{15}{27} \\ 0 & 1 & \frac{45}{81} \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & \frac{7}{9} & \frac{15}{27} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{9} & 0 & 0 \\ -\frac{69}{8} & \frac{171}{8} & -\frac{43}{8} \\ \frac{7}{9} & -\frac{53}{8} & \frac{11}{8} \end{pmatrix} = \begin{pmatrix} 1 & \frac{7}{9} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{9} & 0 & 0 \\ -\frac{69}{8} & \frac{171}{8} & -\frac{43}{8} \\ \frac{7}{9} & -\frac{53}{8} & \frac{11}{8} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{29}{2} & -\frac{69}{5} & \frac{17}{4} \\ -\frac{69}{5} & \frac{171}{8} & -\frac{43}{8} \\ \frac{17}{8} & -\frac{53}{8} & \frac{11}{8} \end{pmatrix}$$