Innovative Root Finding and Tracing Algorithms in Complex Domain for Treatment of Lossy Transmission Lines

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***Abstract* – Innovative complex root finding and tracing algorithms are applied to the analysis of losses in transmission line containing a thin graphene layer deposited on a silicone substrate. An equivalent transmission-line model of TM modes, which involves spatial dispersion of the graphene is considered. The root tracing scheme is modified to work effectively in complex space.**

***Index Terms* — Complex root tracing, complex root finding, propagation, graphene losses, spatial dispersion.**

1. Introduction

All real-world guiding structures are composed of lossy media which implies their lossy characteristics of propagation. In many practical applications estimation of the losses is crucial for proper design and performance. In mathematical model the losses are represented by real part of propagation coefficient $\gamma = \alpha + j\beta$ (α - the attenuation coefficient, β - the phase coefficient). The analysis of losses can be carried out with the use of discrete numerical techniques as well as with the analytical methods. The former ones are obviously more versatile and flexible, however such approach has also some disadvantages. For small or thin elements a very dense mesh is required which results in time-consuming analysis. Moreover modeling of unshielded guides in discrete methods is still an open problem (ambiguous choice of absorber parameters, spurious modes etc.). In many cases an application of dedicated (often continuous) methods is significantly more efficient. In this approach the problem boils down to finding complex root of function $F(\gamma)=0$ which represents complex propagation coefficient. Usually, the analysis must be performed for each frequency point f to obtain total dispersion characteristics ($F(\gamma, f)=0$). Hence, the efficiency of such procedure strongly depends on the efficiency of a root finding algorithm. In this paper we present the results of numerical tests carried out with the use of novel root finding techniques. The classic schemes for root finding like Newton's [1], Muller's [2] or simplex [3] methods require initial point to start the routine. These procedures are also very sensitive to singularities and branch cuts which commonly occur in this type of problems. The most general global algorithm (free of the mentioned drawbacks) is presented in [4], however its efficiency is low when each frequency point is considered separately. The algorithm tracking the root in a function with an extra parameter (commonly it is frequency) [5] seems to be much more efficient in the considered problem. Hence, the analysis should start from the global algorithm [4], which provides propagation coefficients of all modes guided in the structure for a fixed frequency. Next, such results become the initial points of the dispersion characteristics, which can be evaluated using scheme [5].

2. Root Tracking Algorithm

The algorithm [5] is based on creating a chain of simplexes (tetrahedrons in 3D) tracing the root in a function of frequency on complex plane. We start with a triangle (on a complex plane γ) containing a previously found root [4]. Then a tetrahedron is created by adding an extra point in 3D domain (see - Fig. 1). Next, three remaining faces (triangles) of the tetrahedron are examined. If the face with root is determined the process is repeated from that triangle. The main idea of the algorithm is very simple - if there is a root at the considered triangle, then the real and imaginary parts of the function change their signs at the triangle vertices.

Due to low number of function evaluations (only one in each step) the algorithm seems to be promising for considered issue. However, the main idea of the scheme is ineffective for complex domain. It must be emphasized that

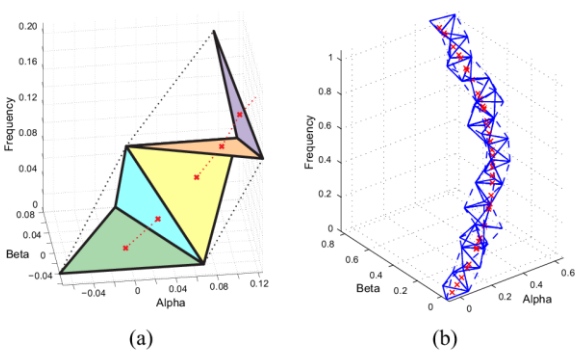

Fig. 1. Chain of the tetrahedrons tracing the propagation coefficient in a function of frequency; (a) 4th iteration, (b) 40th iteration.

the main condition of the scheme is insufficient and there is no guarantee that different signs of the real and imaginary parts of the function imply a root inside the triangle (see Fig. 2).

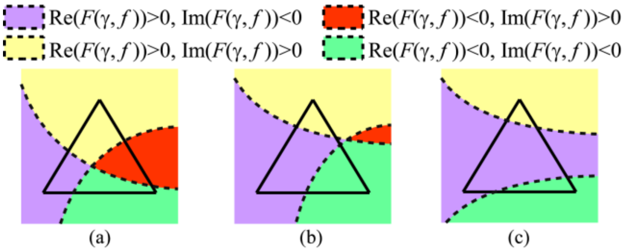

Fig. 2. Different cases of the real and imaginary part of the function signs at the vertices of the triangle; (a) root inside the triangle, (b) root outside the triangle, (c) no root in the neighborhood of the triangle.

There is only one method of unambiguous verification of the root validity - Cauchy's Argument Principle. However, such approach requires integration over the considered region (with the use of many points) and becomes inefficient for each step. In this paper we propose a modification of the tracing algorithm by separate approximation of the real and imaginary part of the function at the tetrahedron faces. Such approach causes that the model of the function is not holomorphic but can be used to estimate a complex root with a quite high accuracy. Then it must be checked if the estimated root is placed inside the considered triangle. Such treatment still does not guarantee the existence of the root but with the assumption of continuity of the function it is sufficient to trace the root for following frequencies.

3. Numerical Tests

In order to demonstrate the validity of the proposed modification, a graphene transmission line is considered. The structure contains a thin graphene layer deposited on a silicone substrate. An equivalent transmission-line model of TM modes, which involves spatial dispersion [6] of the graphene, is presented in [7] (equation 12). The parameters of the graphene sheet and the silicone substrate are the same as in Fig. 5 in [7]. The analysis is performed for the frequency range from 1THz to 7THz. The initial point is obtained from global root finding algorithm [4] then the modified root tracing scheme is used.

The global algorithm requires about 50000 function evaluations to estimate initial frequency points, which takes 85s (on Intel(R) Core(TM) i5 CPU 2.67GHz, 8GB RAM). To obtain the characteristics in the required range at least 30 points of frequency must be considered, which results in more than 40 minutes of calculations. The root tracing algorithm requires about 5000 of function evaluations for each of two modes. The total calculation time with the use of combined global and tracing algorithms takes about 91s (global: 85s, tracing: 6s). The final dispersion characteristics are presented in Fig. 3, where $\gamma = jk_\rho$. A very good agreement was achieved between obtained results and those presented in Fig. 5 in [7].

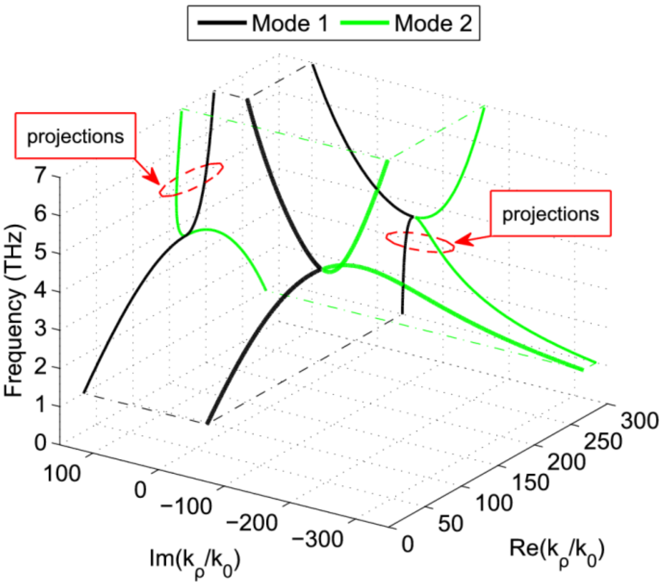

Fig. 3. Characteristics of TM surface waves propagating along a spatially dispersive graphene sheet deposited on a silicon substrate.

4. Conclusion

The obtained results suggest that the proposed algorithm can be effectively and efficiently applied in the analysis of propagation phenomena. In such issues complex analysis is necessary to include losses or/and radiation effects. The scheme presented in the paper can be also applied in any other technique where complex root finding is required (resonators, antennas etc.).

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