Systematic solution of nonlinear equations with close or double roots in complex domain

Application of SCVS algorithm for crosswise complex dispersion characteristics

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***Abstract*—This paper shows the application of Simplex Search Vertices Searching (SCVS) algorithm in solving nonlinear equations of the form $F(f,\gamma)=0$, where $f\in\mathbb{R},\gamma\in\mathbb{C}$ and F is complex function, for the case where we face the problem of close or double complex roots. This problem is solved with the use of standard SCVS procedure, recently reported. In this case 2- and 3-Simplexes are used for root tracking routine. Analyzing the sign of function F at the vertices of considered simplexes we can infer whether there is one or two roots in its volume. Numerical results of calculating dispersion characteristics, for selected ferrite line, are shown to prove the performance of the proposed method.**

Keywords-component, root tracking, dispersion characteristic, nonlinear equation

I. INTRODUCTION AND MOTIVATION

A lot of electromagnetic problems can, in general, be described by non-linear equations. Solving them can be very difficult and time-consuming, especially if we work with many variables and in a complex domain. Solution of nonlinear equations can be considered as the problem of roots searching. Different methods can be used to search for roots of nonlinear equations. For equation in one real variable, e.g., Newton method or Muller method [1] in a complex domain can be used. Some of the methods can be applied to multiple variables. Recently, we have proposed a novel method, Simplex Chain Vertices Search (SCVS), which can be successfully used in solving nonlinear equations for many variables in a real and complex domain [3]. In general SCVS concept is a generalization of the concept presented many years ago in [2].

Numerical examples presented by us in [3] demonstrated the cases where we considered only one curve defining roots in the considered space. Further study of SCVS algorithm showed that it can be used in cases when curves defining roots lie very close to each other or even if they cross one another. The main goal of this paper is to demonstrate SCVS algorithm in such a case. Here we present the possibility of using SCVS algorithm for the purpose of solving dispersion equation in complex domain, which is a typical problem considered with regard to lossy transmission lines or, in general, to microwave transmission lines propagating complex modes.

To avoid replication of the basis of SCVS algorithm we encourage the reader to familiarize themselves with paper [3], where SCVS algorithm has been introduced for the first time. Here we present the explanation why root tracing in this case is possible, as well as new results for SCVS method applied in calculating the complex dispersion characteristics, which run very close or cross each other.

II. SCVS ALGORITHM FOR CLOSE OR DOUBLE ROOTS

In general, SCVS idea is realized as a root tracing process of a real or complex function of N variables in constrained space. In this algorithm it is assumed that the roots of the continuous function of N variables lie on the continuous (N-1)-dimensional hyperplane. The method uses regular N and (N-1)-Simplexes, at whose vertices the considered function changes its sign. Based on (N-1)-Simplex, the function is evaluated at two new points, which are vertices of new regular N-Simplexes, for which (N-1)-Simplex is one of their (N-1)-faces. The SCVS method uses the stacks and runs in an iterative mode, tracing the roots inside the volume of the considered simplexes. As a result, the algorithm creates a chain of simplexes in the constrained region, where the considered function evaluates the roots [3].

According to the procedure presented in [3], we consider the complex root between two points v^i and v^j, which are the vertices of the simplex, if the following condition is satisfied

$$\begin{cases} sign\left(\mathfrak{Re}\left(F(v^i)\right)\right) \neq sign\left(\mathfrak{Re}\left(F(v^j)\right)\right) \\ sign\left(\mathfrak{Im}\left(F(v^i)\right)\right) \neq sign\left(\mathfrak{Im}\left(F(v^j)\right)\right) \end{cases} \tag{1}$$

and

$$\lim\,(e^{ij}) \to 0 \tag{2}$$

where

$$e^{ij} = \left\|v^i - v^j\right\| \tag{3}$$

is the length of simplex edge, described as an Euclidian distance between two points (vertices of the considered simplex) v^i and v^j defined as

$$\left\|v^i - v^j\right\| = \sqrt{\sum_{n=1}^{N}\left(x_n^i - x_n^j\right)^2}. \tag{4}$$

where $\mathfrak{Re}$ and $\mathfrak{Im}$ indicate the real and imaginary part of the function value, respectively.

Considering complex dispersion characteristics in the function of frequency $\gamma(f)$, we work with N=3 variables: f, α, β. Variables α and β represent real and imaginary part of the complex propagation coefficient γ and, in the SCVS algorithm, we treat them independently. In practice we use initial 2-Simplex (triangle) in 3D space. Tracing the roots based on 2-Simplex we build new two 3-Simplexes and thus obtain six additional 2-Simplexes as candidates containing the roots inside their volume, at each new iteration step [3].

In numerical experiments presented in [3] we considered only the situation where the roots lay on one curve during the whole run of the algorithm. It gave us 3-Simplexes which have only two 2-Simplexes, at whose two vertices is the considered function satisfies condition (1). This situation is presented in Fig. 2a, where only simplexes (v^1,v^2,v^3) and (v^2,v^3,v^4) have the vertices which fulfill condition (1). As we can observe, curve A defining the roots goes through these two 2-Simplexes A.

If, in volume of 3-Simplex, two curves defining roots will run close to each other or cross, then all four 2-Simplexes have vertices which fulfill condition (1). It means that one curve defining roots will go through two 2-Simplexes, and the second one – through the next two 2-Simplexes (Fig. 1b), i.e., curves B and C respectively.

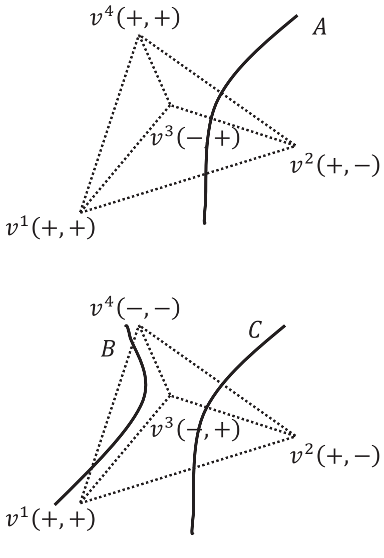

Figure 1. The 2- and 3-Simplexes considered in root tracking algorithm. a) one curve A defining roots goes through 3-Simplex (v^1,v^2,v^3,v^4). It goes through 2-Simplexes (v^1,v^2,v^3) and (v^2,v^3,v^4). b) Two curves defining roots, B and C, go through 3-Simplex (v^1,v^2,v^3,v^4) . The first one goes through 2-Simplexes (v^1,v^2,v^3) and (v^2,v^3,v^4), the second one goes through (v^1,v^3,v^4), and (v^1,v^2,v^4). v(+-) means that the real part of function value is positive and the imaginary part is negative.

This observation allows us to use SCVS algorithm in the case of close-running or even crosswise roots, for the purpose of root tracing of equation $F(f,\gamma)=0$ where $f\in\mathbb{R},\gamma\in\mathbb{C}$ and F is a function which evaluates complex values. In practice, the situation from Fig. 1b can be observed only if the distance between two curves defining roots will be smaller than the length of simplex edge defined by (3).

III. Numerical Results

To check the correctness of the proposed approach we performed numerical experiments, calculating the complex dispersion characteristics for ferrite coupled transmission line, whose mathematical model is presented in [5]. The cross-section of this line is presented below in Fig. 2. Real dispersion characteristics above the cut-off frequency for lossless line have been calculated and are presented in [3] (Fig. 9). In this case function F evaluated real values. Here we will investigate loosy FCL line. The loosy condition will be simulated by introducing ΔH in μ_a in Polder tensor, i.e.

$$\mu_a=\frac{\vartheta(M_S-i\Delta H)}{f} \tag{5}$$

where ϑ defines gyromagnetic coefficient.

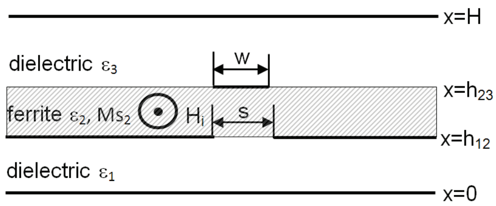

Fig. 2. A cross section of a slab dielectric waveguide of microstrip-sloted ferrite coupled line. Line dimensions [mm]: h_{12}=1, h_{23}=1.5, H=3, w=0.5, s=1 [mm]. Material parameters: ε_1=1, Ms_2=340 kA/m, H_i=0, ε_2=13.5, ε_3=1,

The dispersion equation we considered is of the form $F(f,\gamma)=0$, [1] where $\gamma=\alpha+j\beta$ is a propagation coefficient and f defines frequency. In this case function F evaluates complex values.

We performed the first calculation to find complex dispersion characteristics above the cut-off frequency, for lossy FCL line, with $\Delta H=50$kA/m. According to SCVS procedure, before we start a root tracing algorithm, we have to specify the vertices of one initial simplex, whose volume contains a root. In this case we have to specify the initial 2-Simplex, in whose volume the dispersion equation changes the sign fulfilling the condition (1). The following two points, defining the edge of the initial simplex, were chosen:$(\alpha=0.202,\beta=0.002,f=23)$and $(\alpha=0.207,\beta=0.004,f=23)$. At these two points dispersion function evaluates the values of the following sign for real and imaginary parts: $(+,-)$ and $(-,+)$ respectively. In

[1] Full analytical form of dispersion equation for this FCL line can be found in work [5].

calculations we defined the following limits for variables: frequency $f \epsilon (22, 25)$ [GHz], $\beta \epsilon (0, 3.5)$ [rad/m] and $\alpha \epsilon (0, 3.5)$ [rad/m]. The roots found in the algorithm run are presented below in Fig. 3. According to SCVS method, all roots are found with accuracy described by the edge length of used simplexes defined by (3). To obtain better root accuracy, the edge of the initial simplex must be decreased.

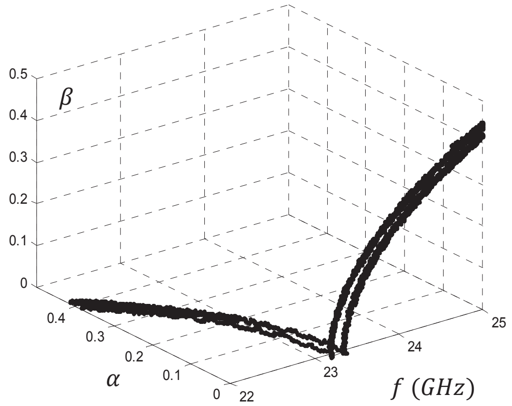

Figure 3. Complex roots of dispersion equation for FCL line found in one run of SCVS algorithm. Coordinates of points creating the curves are the solution of the dispersion equation $F(\alpha, \beta, f) = 0$.

While analyzing the curves in Fig. 3 we can observe that, although we started the algorithm run from one initial simplex defining the one root of one dispersion curve, we have obtained two dispersion curves. The conclusion is that the distance between the curves (modes of considered FCL transmission line), defining the roots of the dispersion equation $F(\alpha, \beta, f) = 0$ is, at some place of the space, smaller than the length of the considered simplexes used in the algorithm run.

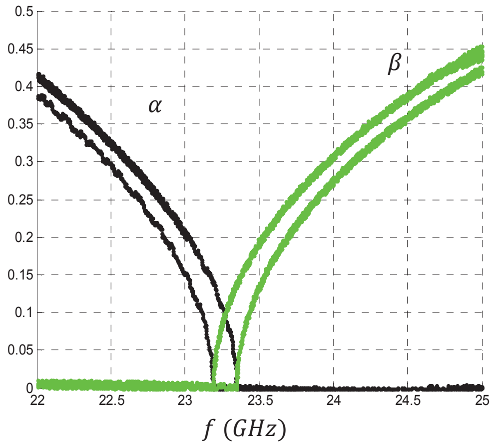

Figure 4. Roots of dispersion equation $F(\alpha, \beta, f) = 0$ of FCL [5]. Curves are presented as $\alpha(f)$ and $\beta(f)$.

The same roots, but presented independently for α and β in function of frequency, are shown below in Fig. 4.

Next we have calculated, for the same FCL line, dispersion characteristics at lower frequencies. The following two points, defining the first edge of initial simplex, were chosen: $(\alpha = 0.875, \beta = 1.23, f = 10)$and $(\alpha = 0.875, \beta = 1.25, f = 10)$. For these two points dispersion function evaluates values of the following sign for real and imaginary part: $(+, -)$ and $(-, +)$ respectively. In the algorithm run we have defined the following limits for variables: frequency $f \epsilon (7.5, 11.5)$ [GHz], $\beta \epsilon (0, 3.5)$ [rad/m] and $\alpha \epsilon (0, 3.5)$. The roots found in the algorithm run are presented below in Fig. 5.

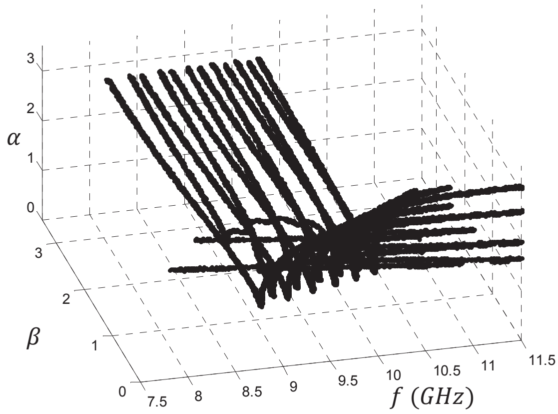

Figure 5. Complex roots of dispersion equation for FCL line found in one run of SCVS algorithm. Coordinates of points creating the curves are the solution of the dispersion equation $F(\alpha, \beta, f) = 0$. Curves defining roots cross each other in many points.

The same roots, but presented independently for α and β in function of frequency, are shown below in Fig. 6.

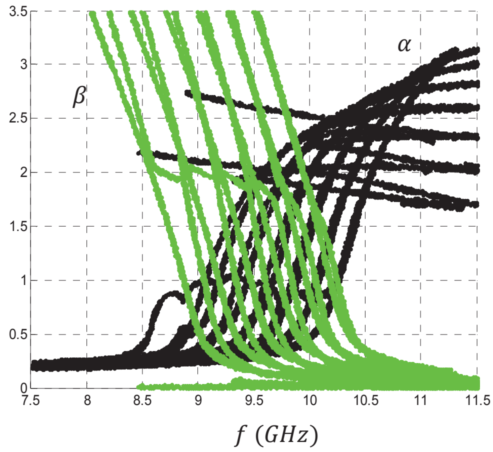

Figure 6. Roots of dispersion equation $F(\alpha, \beta, f) = 0$ of FCL [5]. Curves are presented as $\alpha(f)$ and $\beta(f)$.

In this case we have obtained a lot of dispersion curves (complex modes), although we started from one root only. Analyzing the curves from Fig. 5 we can observe that they cross each other many times.

We can see that SCVS algorithm in this case works well and according to our expectations. We do not analyze the obtained results from the physical point of view, as this is not the aim of this paper.

IV. Conclusion

Solution of nonlinear equations can be, in practice, transformed into a problem of finding the roots. In general, it is very difficult to solve this problem, especially when the roots lie very close to each other. In this paper we have demonstrated that SCVS algorithm can be successfully used in this case. Numerical calculations performed with the purpose of finding complex propagation constant in a frequency domain for ferrite coupled transmission line, demonstrated that SCVS algorithm can find roots laying on the curves which run very close to each other or even cross one another.

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References

[1] D. E. Muller, "A Method for Solving Algebraic Equations Using an Automatic Computer," MTAC, 10 (1956), 208-215.

[2] M. Mrozowski, “An Efficient Algorithm for Finding Zeros of a Real Function of Two Variables,” *IEEE Trans. Microwave Theory & Tech*., Volume 36, no. 3, pp. 601-604, March 1988.

[3] J. J. Michalski, P. Kowalczyk, “Efficient and Systematic Solution of Real and Complex Eigenvalue Problems Employing Simplex Chain Vertices Searching Procedure”, *IEEE Transactions on Microwave Theory and Techniques*, Volume: 59, Issue: 9, 2011.

[4] H. S. M. Coxeter, “Regular Polytopes”, Third edition, (1973), Dover edition, ISBN 0-486-61480-8.

[5] J. Michalski, M. Mazur, J. Mazur, “Scattering in a section of ferrite-coupled microstrip lines: theory and application in nonreciprocal devices,” *IEE -Proceedings. Microwaves, Antennas and Propagation.*,Volume 149, Issue 56, pp. 286-290, Oct.-Nov. 2002.