T4 a) $$G_{X_n}(s) = G(G_{X_{n-1}}(s))$$

$$G(z) = 1 - p(1-z)^{\alpha}$$

$$G_{X_n}(s) = \underbrace{G(G(\dots\ G}_{n+1 \text{ raz}}(s)\dots))$$

T5 a) $$G(s) = s$$

$$1 - p(1-s)^{\alpha} = s$$

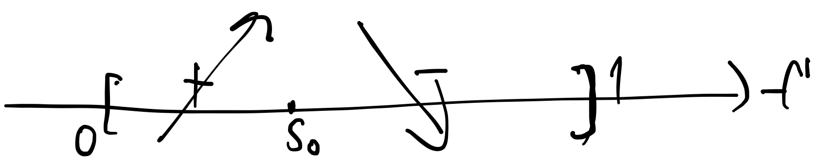
$$f(s) = s + p(1-s)^{\alpha} - 1 = 0$$

$$f'(s) = 1 - p\alpha(1-s)^{\alpha-1} = 0 \Rightarrow (1-s)^{1-\alpha} = p\alpha$$

$$s_0 = 1 - (p\alpha)^{\frac{1}{1-\alpha}} \in [0,1)$$

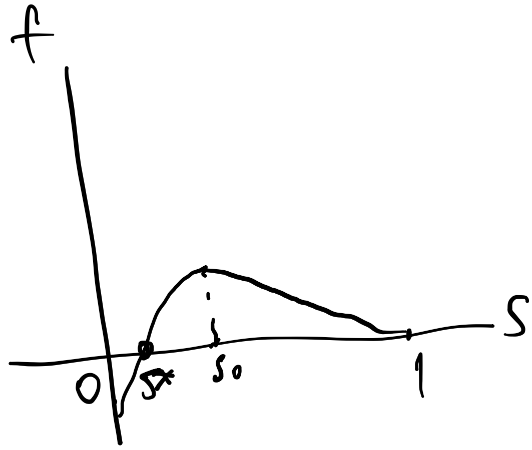
$$f(0) = p - 1 < 0 \quad , \quad f(1) = 0$$

$$f'(0) = 1 - p\alpha > 0 \quad , \quad f'(1-0) < 0$$



$\exists!\quad G(s^*) = s^*$

$0 < s^* < s_0$



T.8

$$P(\xi = k) = p(1-p)^k, \quad k \in \overline{0, 1, \ldots}$$

$$G_\xi(s) = E s^\xi = \sum_{k=0}^{\infty} s^k P(\xi = k) =$$

$$= \sum_{k=0}^{\infty} p\,(s(1-p))^k = \frac{p}{1 - s(1-p)} = s$$

$$\Rightarrow \quad s^2(1-p) - s + p = 0$$

По т. Виета $\quad s_1 s_2 = \frac{p}{1-p}$

$$s_1 = 1 \quad \Rightarrow \quad \boxed{s_2 = \frac{p}{1-p}}$$

$$G_Y(s) = \frac{s p}{1 - (1-p) G_Y(s)}$$

$$(1-p)\, G_{Y}^{2}(s) - G_Y(s) + sp = 0$$

$$G_Y(s) = \frac{1 \pm \sqrt{1-4(1-p)ps}}{2(1-p)}$$

$$G_Y(0) = P(Y=0) = 0$$

$$\Rightarrow \boxed{G_Y(s) = \frac{1 - \sqrt{1-4(1-p)ps}}{2(1-p)}}$$

$$P(Y=k) = \frac{G_Y^{(k)}(0)}{k!}$$

$$(1+x)^{1/2} = \sum_{k=0}^{\infty} C_{\frac{1}{2}}^{k} x^k \quad , \quad C_{\frac{1}{2}}^{k} = \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\cdots\left(\frac{1}{2}-k+1\right)}{k!}$$

$$\sqrt{1-4p(1-p)s} = \sum_{k=0}^{\infty} C_{\frac{1}{2}}^{k} (-1)^k (4p(1-p))^k s^k$$

$$\boxed{P(Y=k) = \frac{C_{\frac{1}{2}}^{k} (-1)^{k+1} (4p(1-p))^k}{2(1-p)\, k!}}$$

I.9 a) $G(s) = 1 - \frac{1}{2}\sqrt{1-s} = s$

$$\Rightarrow \quad 4(1-s)^2 = 1-s$$

$$\left[\begin{array}{l} 1-s=0 \\ \boxed{1-s=\frac{1}{4}} \end{array}\right.$$

$$s = \frac{3}{4}$$

δ) $G_y(s) = s\left(1 - \frac{1}{2}\sqrt{1-G_y(s)}\right)$

$$4\left(G_y(s) - s\right)^2 = s^2\left(1 - G_y(s)\right)$$

$$4G_y^2(s) - (8s - s^2)G_y(s) + 3s^2 = 0$$

$$G_y(s) = \frac{1}{8}\left[8s - s^2 \pm \sqrt{(8s-s^2)^2 - 48s^2}\right]$$

$$= \frac{1}{8}\left[s(8-s) \pm s\sqrt{16 - 16s + s^2}\right]$$

$G_y(1) = 1 \Rightarrow$

$$\Rightarrow \boxed{G_y(s) = \frac{1}{8}\left[s(8-s) + s\sqrt{16-16s+s^2}\right]}$$

b) $$P(Y=10) = \frac{G_Y^{(10)}(0)}{10!} = \frac{10\left(\sqrt{16-16s+s^2}\right)^{(9)}}{8\cdot 10!}\Bigg|_0 =$$

$$= \frac{\left(\sqrt{16-16s+s^2}\right)^{(9)}}{9!\cdot 8}\Bigg|_0 = \frac{\left(\sqrt{1-s\left(1-\frac{s}{16}\right)}\right)^{(9)}}{9!\cdot 2}\Bigg|_0$$

$$\sqrt{1-s\left(1-\frac{s}{16}\right)} = \sum_{k=0}^{\infty} C_{1/2}^{k}(-1)^k s^k\left(1-\frac{s}{16}\right)^k =$$

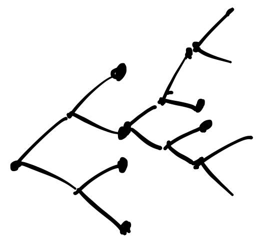
$$= \sum_{k=0}^{\infty} C_{1/2}^{k}(-1)^k s^k \sum_{n=0}^{k} C_k^n \frac{(-1)^n}{16^n} s^n =$$

$$\frac{\left(\sqrt{1-s\left(1-\frac{s}{16}\right)}\right)^{(9)}}{9!}\Bigg|_0 = -C_{1/2}^{5}C_5^4\frac{1}{16^4} - C_{1/2}^{6}C_6^3\frac{1}{16^3}$$

$$-C_{1/2}^{7}C_7^2\frac{1}{16^2} - C_{1/2}^{8}C_8^1\frac{1}{16} - C_{1/2}^{9} < 0$$

$$\Rightarrow P(Y=10) = 0$$

T.10



Это GW-процесс с законом $\xi = \begin{cases} 2, & p = 1/2 \\ 0, & p = 1/2 \end{cases}$

$$G_\xi(s) = \sum_{k=0}^{\infty} s^k P(\xi = k) = \frac{1}{2} + \frac{1}{2}s^2$$

$$= \frac{1+s^2}{2}$$

$$\frac{1+s^2}{2} = s \Rightarrow (s-1)^2 = 0 \Rightarrow s = 1$$

$\Rightarrow P(\text{пройти уровень}) = 1$