20.3

1) $v + v = 2v \notin V$ ✗

2) $(0,\dots)+(0,\dots)=(0,\dots)\in V$

$\lambda(0,\dots)=(0,\dots)\in V$ ✓

3) $\sum(v_1^i+v_2^i)=\sum v_1^i+\sum v_2^i=0$

$\sum \lambda v_1^i=\lambda\sum v_1^i=0$ ✓

4) $\sum(v_1^i+v_2^i)=2\neq 1$ ✗

20.4 2)

$v_1+v_2 \perp l$

$\lambda v \perp l$ ✓

20.20

$1,\ t-\alpha,\ (t-\alpha)^2,\ \dots,\ (t-\alpha)^n$

Пусть $\lambda_n(t-\alpha)^n + \ldots + \lambda_0 = 0$ — нетр. лин. комб.

$$\parallel$$

$$a_n t^n + \ldots + a_1 t + a_0 = 0$$

$$a_n = 0 \Leftrightarrow \lambda_n = 0$$

$$\ldots$$

$$a_0 = 0 \Leftrightarrow \lambda_0 = 0$$

$\Rightarrow$ тривиальн. комб.

$$\lambda_0 = P(\alpha)$$

$$\lambda_1 = P'(\alpha)$$

$$\lambda_k = \frac{P^{(k)}(\alpha)}{k!}$$

$$(e_1', e_2', \ldots, e_n') = (e_1, \ldots, e_n)C$$

1) $e_i \Leftrightarrow e_j \Rightarrow c_i \Leftrightarrow c_j$

2) $e_i' \Leftrightarrow e_j' \Rightarrow c^i \Leftrightarrow c^j$

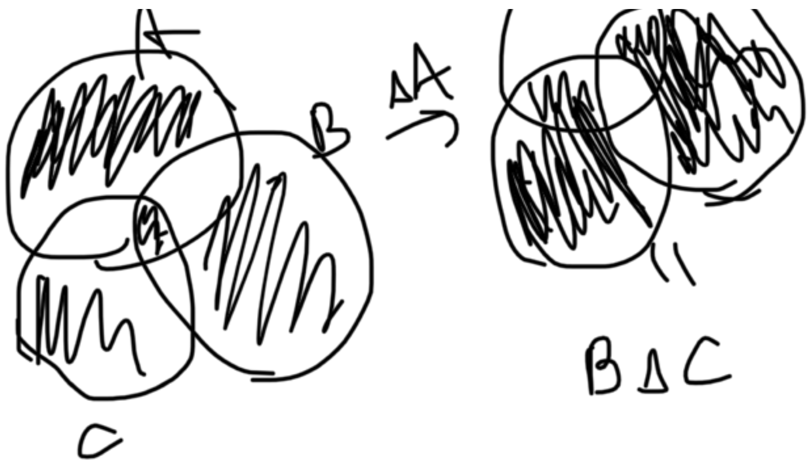
3) $(e_n', \ldots, e_1') = (e_n, \ldots, e_1) C_{sym}$

$$c_i^j \rightarrow c_{n-i}^{n-j}$$

$T.1^*$

$$A_1, \ldots, A_n$$

$$\sum_{0 \le k \le n} \binom{n}{k} = 2^n$$



$\boxed{\text{T. } 2^*}$ $\quad \mathbb{Z} \overset{?}{\not\cong} (V,+)$

От противного:

$$\varphi : \mathbb{Z} \to V$$

$$\varphi(0)=0$$

$$\varphi(1)=x$$

$$\varphi(2)=\varphi(1+1)=x+x=(1+1)x$$

$$\mathbb{K} \supset 7.1$$

$\ldots \; \overline{}\; 1+1$

Тогда $\exists\, k: \quad \varphi(k) = \frac{1}{1+1} x$

$$\varphi(k+k) = x = \varphi(1)$$

$$k+k = 1 \quad (?!)$$

II. <u>21.2</u> $\quad P_n(\mathbb{K}) = P_n^+(\mathbb{K}) \oplus P_n^-(\mathbb{K})$

$$\sum_{k=0}^{n} a_k x^k = \underbrace{\sum_{k=0}^{\lfloor n/2 \rfloor} a_{2k} x^{2k}}_{\in P_n^+(\mathbb{K})} + \underbrace{\sum_{k=0}^{\lfloor n/2 \rfloor} a_{2k+1} x^{2k+1}}_{\in P_n^-(\mathbb{K})}$$

$21.2)^*$ $\quad A \in \mathrm{Mat}_{n\times m}(\mathbb{K}),\ A = (v_1, \ldots, v_m)$

τ

$$A'x = 0\ ,\ \operatorname{rk} A = k$$

$$\searrow$$

$$\Phi\ ,\ \operatorname{rk}\Phi = n - k = \dim U$$

пр-во реш. $A^T x = 0$

$$\langle v_1, \ldots, v_m \rangle = \langle v_{\alpha_1}, \ldots, v_{\alpha_k} \rangle = V$$

$$U \cap V \neq \emptyset\ ?$$

$$A^T v_i = \begin{pmatrix} v_1^T \\ \vdots \\ v_m^T \end{pmatrix} v_i = 0$$

$$\Downarrow$$

$$v_i^T v_i = 0$$

$$\Downarrow$$

$$v_i = 0$$

$$\Downarrow$$

$v_i \notin \{v_{\alpha_1}, \dots, v_{\alpha_k}\}$

Значит $U \cap V = \emptyset$

$\dim(U+V) = \dim U + \dim V$

$\mathbb{K}^n = U \oplus V$

21.12. $P, Q \subset V$, $\dim V = n$

1) $\underbrace{\dim Q + \dim P}_{\geq n} - \underbrace{\dim(Q \cap P)}_{\geq 0} = \dim \overbrace{(P+Q)}^{\leq n}$

$Q \cap P \subset V \Rightarrow 0 \in Q \cap P$

2) $\dim Q + \dim P - \dim(P+Q) =$

$\dim(Q \cap P) \leq \min\{\dim Q, \dim P\}$

НУО $q \le p$

$$\dim(P+Q) \ge \max\{p, q\} = p$$

$$\dim(P \cap Q) \le q$$

$p=q$	$p=q$	$p=q+1$
$\dim(P+Q)=p+1$	$\dim(P+Q)=p$	$\dim(P+Q)=p$
$\dim(P\cap Q)=p$	$\dim(P\cap Q)=p-1$	$\dim(P\cap Q)=p-1$
$\downarrow$	$\downarrow$	$\downarrow$
$\times$	$Q\subset P$	$Q \subset P$

<u>35.10</u> $\dim V = n, \quad |F| = q = p^k$

а) q^n

б) $\frac{1}{n!}(q^n - 1)(q^n - q)\ldots(q^n - q^{n-1})$

n

35.13 a) $U\cap(V+W)=(U\cap V)+(U\cap W)$?

$U=\langle a\rangle$ $V=\langle b\rangle$, $W=\langle a+b\rangle$

$$\langle a\rangle\cap(\langle b\rangle+\langle a+b\rangle)=0+0$$
$$\parallel$$
$$\langle a\rangle$$

нет

б) $V\subseteq U$

$U\cap V=V$

$$U\cap(V+W)=V+(U\cap W)$$

$$\xi\in U\cap(V+W)\Leftrightarrow\begin{cases}\xi\in U\\ \exists v\in V, w\in W: \xi=v+w\end{cases}$$

$$\Leftrightarrow\begin{cases}\xi\in U\\ \exists w\in U\cap W: \xi=v+w\end{cases}\Leftrightarrow$$

$$\Leftrightarrow g \in V + (U \cap W)$$

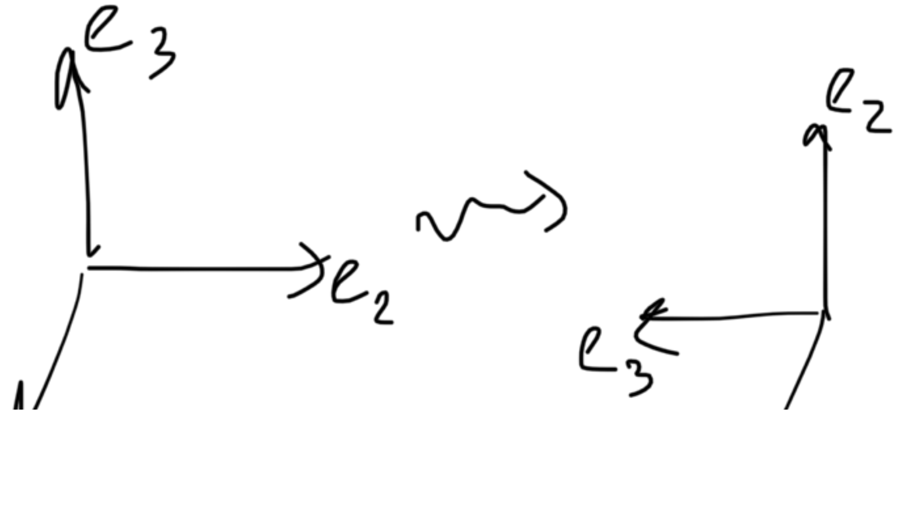
III. 23.6(5) $\varphi(x) = x - 2(x,n)\frac{n}{|n|^2}$

$$\varphi(\lambda x) = \lambda x - 2\lambda(x,n)\frac{n}{|n|^2} = \lambda\varphi(x)$$

$$\varphi(x+y) = x+y - 2(x,n)\frac{n}{|n|^2} - 2(y,n)\frac{n}{|n|^2} =$$

$$= \varphi(x) + \varphi(y)$$

23.14(2) $\pi/2$ вокруг e_1



e_1

e_1

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

23.70 (1) $\{e_1, \ldots, e_n\} \xrightarrow{e_i \cdot f_j} \{e_1, \ldots, e_j, \ldots, e_i, \ldots, e_n\}$

$A \qquad A'$

$e' = eA \qquad e' = \tilde{e} A'$

$A' =$ $i \to j$ строк в A

$\boxed{T.5}$ $\varphi_v : \mathbb{R}^3 \to \mathbb{R}^3$

$$\varphi_v(x) = [v, x]$$

$$x_1 \to v_2 x_3 - v_3 x_2$$

$$x_2 \to v_3 x_1 - v_1 x_3$$

$$x_3 \to -v_2 x_1 + v_1 x_2$$

$$A = \begin{pmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & +v_1 & 0 \end{pmatrix}$$

$$v \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \xrightarrow{\varphi} \varphi_v \begin{pmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & +v_1 & 0 \end{pmatrix}$$

$\cap$ $\cap$

$$\mathbb{R}^3 \cong \mathrm{Mat}_{3\times 3}(\mathbb{R})$$

I.6*

a) $V = \operatorname{Ker}\varphi \oplus \operatorname{Im}\varphi \Leftrightarrow$

$\Leftrightarrow \operatorname{Ker}(\varphi^2) = \operatorname{Ker}\varphi$

$\Rightarrow$ $v = \underset{\operatorname{Ker}(\varphi)}{\underset{\in}{u}} + \underset{\operatorname{Im}\varphi}{\underset{\in}{w}}$

$\varphi(u) = 0$, $\varphi(v) = \varphi(w) = \varphi(\varphi(\tilde{w}))$

$$\varphi(v) = \varphi^2(\tilde{w})$$

$$\operatorname{Ker}(\varphi) = \operatorname{Ker}\varphi^2$$

$\Leftarrow$ $v : \varphi(v) = 0$, $\exists w$ $\varphi(\varphi(\tilde{w})) = 0$

$$v = \underset{\ker\varphi}{\underset{\cap}{u}} + \underset{\operatorname{Im}\varphi}{\underset{\cap}{\overset{w}{\overset{\parallel}{\varphi(\tilde{w})}}}}$$

б) $\varphi^2 = \varphi$. Д-ть, что φ – проектор на $\operatorname{Im}\varphi$ вдоль $\ker\varphi$

$$\varphi(\underset{\operatorname{Im}\varphi}{\underset{\cap}{x}} + \underset{\ker\varphi}{\underset{\cap}{y}}) = x$$

$$\varphi^2(v) = \varphi(x) = \varphi(x+y) = \varphi(v)$$

23.82. 1)*

$$\varphi(a_i) = b_i$$

$$\psi(b_i) = c_i$$

$$\varphi(e) = eA, \quad \psi(e) = eB$$

$$\psi\varphi(e) = \psi(eA) = e\underbrace{BA}$$

$$(b_1, b_2, b_3) = (a_1, a_2, a_3)A$$

$$(c_1, c_2, c_3) = (b_1, b_2, b_3)B$$

$$A_{289}, \quad A_{229}, \quad A_{285}$$

$$A_{289} = \begin{pmatrix} -2 & 1 & 2 \\ -1 & 0 & 2 \\ -2 & 0 & 3 \end{pmatrix}, \quad A_{229} = \begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ -1 & -1 & 1 \end{pmatrix}, \quad A_{285} = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

$$A = \tilde{A}^{-1}\tilde{B}, \quad B = \tilde{B}^{-1}\tilde{C}$$

$$BA = \tilde{B}^{-1}\tilde{C}\tilde{A}^{-1}\tilde{B}$$

$$\tilde{B}^{-1} = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 2 & 4 \end{pmatrix}^T = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{pmatrix}$$

$$\tilde{A}^{-1} = -\begin{pmatrix} 0 & -1 & 0 \\ -3 & -2 & -2 \\ 2 & 2 & 1 \end{pmatrix}^T = \begin{pmatrix} 0 & 3 & -2 \\ 1 & 2 & -2 \\ 0 & 2 & -1 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & 3 & -2 \\ 1 & 2 & -2 \\ 0 & 2 & -1 \end{pmatrix} \begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ -1 & -1 & 1 \end{pmatrix} = \begin{pmatrix} -3 & 8 & -5 \\ 4 & 5 & -4 \\ 1 & 5 & -3 \end{pmatrix}$$

$$BA = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} -3 & 8 & -5 \\ 4 & 5 & -4 \\ 1 & 5 & -3 \end{pmatrix} =$$

$$= \begin{pmatrix} -3 & 8 & -5 \\ 1 & 13 & -9 \\ -4 & 13 & -8 \end{pmatrix}$$

23.95.1)*

$$\varphi : L \to M, \ \psi : M \to N$$

$$\dim M = m$$

$$\mathrm{rk}\,\varphi + \mathrm{rk}\,\psi - m \le \mathrm{rk}(\psi\varphi) \le \min(\mathrm{rk}\,\varphi, \mathrm{rk}\,\psi)$$

$$\begin{tikzcd} & M \arrow[dr, "\psi"] & \\ L \arrow[ur, "\varphi"] \arrow[rr, "\psi\varphi"'] & & N \end{tikzcd}$$

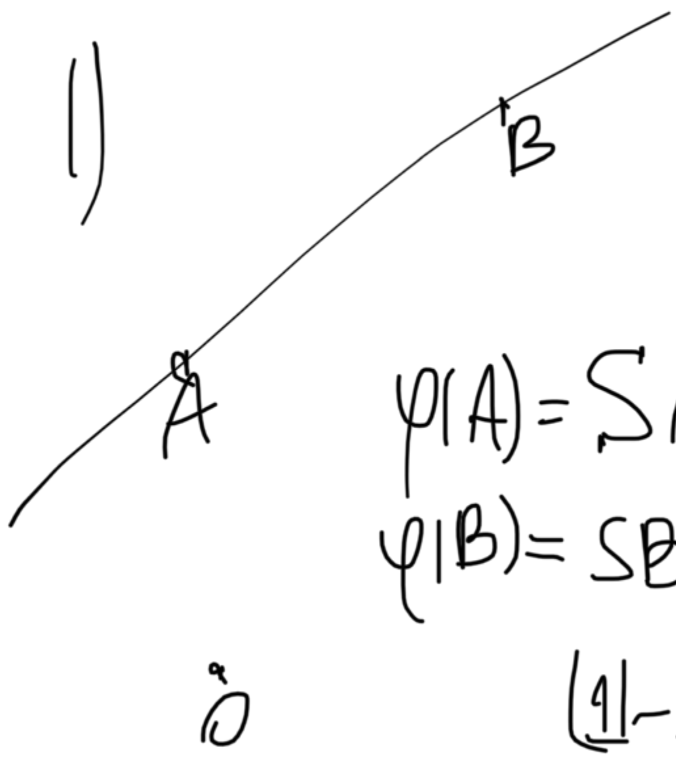
$$\dim M \geq \mathrm{rk}\,\varphi + \mathrm{rk}\,\psi - \mathrm{rk}\,\psi\varphi \qquad ?$$

$m \qquad m$

$$\mathrm{rk}(\psi\varphi) \le \min\{\mathrm{rk}\,\psi, \mathrm{rk}\,\varphi\} \quad \checkmark$$

12.28.

1)



$$\varphi(A) = SA + b = A$$

$$\varphi(B) = SB + b = B$$

$$(\mathbb{1} - S)A = b$$

$$(\mathbb{1} - S)B = b$$

$$\varphi(\alpha A + \beta B) =$$

$$= \alpha SA + b + \beta SB =$$

$$= \alpha A + \beta B + b - (\alpha + \beta) b = \alpha A + \beta B$$

$$\alpha+\beta=1$$

2)* $f(a) = Sa + b = a$

$$f(\delta_1 d + \gamma_1 c) = \delta_2 d + \gamma_2 c$$

$$\delta_1 S d + \gamma_1 S c + b = \delta_2 d + \gamma_2 c$$

$$\delta_i + \gamma_i = 1$$

Хочу: $\exists\, \delta_3, \gamma_3$:

$$\delta_3 d + \gamma_3 c = a$$

$$\delta_3 S d + \gamma_3 S c + b = \delta_3 d + \gamma_3 c$$

$\exists\, \alpha$: $\quad \alpha\delta_3 d + \alpha\gamma_3 c = \alpha a$

$\delta_1 \qquad \gamma_1$

$$\delta_2 d+\gamma_2 c=\alpha Sa+b=$$

$$=\alpha a+b(1-\alpha)$$

$$\delta_2 d+\gamma_2 c=\delta_1 d+\gamma_1 c+b(1-\alpha)$$

$$\Rightarrow \alpha=1$$

$\Rightarrow$ пр. через a.

<u>12.51</u> $\quad \frac{x^2}{5}-\frac{y^2}{4}=1$

$$\begin{pmatrix}x\\y\end{pmatrix}\to\begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}+\begin{pmatrix}b_1\\b_2\end{pmatrix}$$

$$\begin{cases}5=\alpha\sqrt{5}+b_1\\ 4=q\sqrt{5}+b_2\end{cases}$$

$$\frac{(\alpha x+\beta y+b_1)^2}{5}-\frac{(\gamma x+\delta y+b_2)^2}{4}=1$$

$$\frac{\frac{\alpha^2}{5}-\frac{\gamma^2}{4}}{1+\frac{b_2^2}{4}-\frac{b_1^2}{5}}=\frac{1}{5}, \quad \underline{20\alpha^2-25\gamma^2=20+5b_2^2-4b_1^2}$$

$$\begin{cases} 4\alpha\beta-5\gamma\delta=0 \\ \underline{4\alpha b_1-5\gamma b_2=0} \\ \underline{4\beta b_1-5\delta b_2=0} \\ \underline{5=\alpha\sqrt{5}+b_1} \\ \underline{4=\gamma\sqrt{5}+b_2} \end{cases}$$

$\alpha=$ $\quad\beta=\alpha k$

$\gamma=$ $\quad\delta=\gamma k$

$b_2=$ $\quad b_1$

$$4\alpha^2=5\gamma^2$$

$$\gamma=\sqrt{\frac{4}{5}}\alpha=\frac{2}{\sqrt{5}}\alpha$$

$$4=2\alpha+b_2$$

$$4b_1^2=5b_2^2, \quad b_2=-\frac{2}{\sqrt{5}}b_1$$

$$\begin{cases} 5=\alpha\sqrt{5}+b_1 \\ 4=2\alpha-\frac{2b_1}{\sqrt{5}} \end{cases} \qquad \begin{matrix} b_1=5-\alpha\sqrt{5} \\ 4=2\alpha-2\sqrt{5}+2\alpha \end{matrix}$$

$$\alpha = 1 + \frac{\sqrt{5}}{2}$$

$$x = \frac{2}{\text{[illegible]}} + 1$$

24.13 $\quad \dim V = 2n+1$

$$|A-\lambda E| = \chi_\varphi(\lambda) = \mathbb{R}[\lambda]_{2n+1}$$

$\Rightarrow \exists \geq$ вещ. корень $\chi_\varphi(\lambda) = 0$

24.22* $\quad$ 1) $A = (a_1, \dots, a_n)^T (b_1, \dots, b_n) \neq 0$

$$A_{ij} = a_i b_j$$

$$|A-\lambda E| = \varepsilon_{\alpha_1 \dots \alpha_n} C_{1\alpha_1} \dots C_{n\alpha_n}$$

$\parallel$

C

$$C_{ij} = A_{ij} - \lambda\delta_{ij}$$

$$|A-\lambda E| = \epsilon_{\alpha_1\ldots\alpha_n}\prod_{k=1}^{n}(A_{k\alpha_k} - \lambda\delta_{k\alpha_k})$$

$$\epsilon_{\alpha_1\ldots\alpha_{n-1}n} A_{1\alpha_1}\ldots A_{n-1\,\alpha_{n-1}} = 0,$$

т.к. это $|A_{n-1\times n-1}|$

тогда $|A-\lambda E| = (-\lambda)^n = 0$

$$\Rightarrow \lambda = 0$$

$$Ax = 0$$

$$\begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} \mid b_1 \dots b_n \mid \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = 0$$

$b_1x_1 + \dots + b_nx_n = 0$ — соотв. гиперпл-ть

24.37.3)*

$$A_1 = \begin{pmatrix} 2 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \qquad A_2 = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 3 & -1 & -2 \end{pmatrix}$$

$$D_1 = \begin{pmatrix} 1 & 0 & \\ 0 & -1 & 0 \end{pmatrix} \qquad D_2 = \begin{pmatrix} 1 & 0 & \\ 0 & 1 & 0 \end{pmatrix}$$

т.к. $\operatorname{tr} A = \mathrm{inv}$.

$23.98.2)^*$ $\varphi : V \to V$ $\dim V = n$

$$\varphi^2 = e$$

<u>Д-во:</u> $\dim \operatorname{Ker}(\varphi + e) + \dim \operatorname{Ker}(\varphi - e) = n$

$$v = \underset{\operatorname{Ker}(\varphi+\varphi^2)}{\underset{\cap}{u}} + \underset{\operatorname{Ker}(\varphi-\varphi^2)}{\underset{\cap}{w}}$$

$$\varphi(v) = \varphi(u) + \varphi^2(u) + \varphi(w) - \varphi^2(w) - \varphi^2(u) + \varphi^2(w)$$

$$= \varphi^2(w-u) = \varphi(\varphi(w-u))$$

$$v = \varphi(w-u) + g \in \operatorname{Ker}\varphi$$

$$u + w = u' + w'$$

$$\underset{\ker(\varphi+\varphi^2)}{\underset{\uparrow}{u - u'}} = \underset{\ker(\varphi-\varphi^2)}{\underset{\uparrow}{w - w'}}$$

$$\Rightarrow \varphi + \varphi^2 = \lambda(\varphi - \varphi^2)$$

$$\varphi\big((1+\lambda)\varphi + 1 - \lambda\big) = 0$$

$$(1+\lambda)\varphi^2(v) + (1-\lambda)\varphi(v) = 0$$

$$\varphi(v)\big[(1+\lambda)\varphi(v) + (1-\lambda)\big] = 0 \quad (?!)$$

$\Rightarrow$ разложение единственно.

Но почему сущ.?!

40.11* $P_n(t) = \chi_\varphi(t)$, $P_n(t) = (-1)^n t^n \dots$

для некоторого φ

$$P_n(t) = (-1)^n t^n + a_{n-1} t^{n-1} + \dots + a_0$$

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & \cdot & & \vdots \\ \vdots & & \ddots & \vdots \\ a_{n1} & \dots & & a_{nn} \end{pmatrix} \text{ — мат. } \varphi$$

$$\chi_\varphi(t) = (-1)^n t^n + (-1)^{n-1} \operatorname{tr} A \, t^{n-1} + \sum_{k=1}^{n-2} f_k(a_{ij}) t^k + |A|$$

$$\begin{cases} (-1)^{n-1} \operatorname{tr} A = b_{n-1} \\ f_k(a_{ij}) = b_k \end{cases}$$

fix a_{ij} $\forall (i,j): i \neq 1$

Тогда имеем СЛУ от a_{1j}

Она всегда решится при правильном подборе остальных a_{ij}

40.12 а)* $|AB - \lambda \mathbb{1}| =$

$= |B|\,|AB - \lambda \mathbb{1}|\,|B^{-1}| =$

$= |BA - \lambda \mathbb{1}|$ ✓

I.7

a) $\mathbb{F}_3$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 2 & 0 \end{pmatrix}$$

$$\begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 1 & 2 & -\lambda \end{vmatrix} = 0$$

$$-\lambda^3 + 2\lambda = 0$$

$$\lambda(2-\lambda^2) = 0$$

$$\lambda = 0$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 2 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

b) F_5 to me cause

T.9 A, B

$$\chi_A(t) = \chi_B(t)$$

$$Av = \lambda v \qquad (A - \lambda \mathbb{1})v_A = 0$$

$$(B - \lambda \mathbb{1})v_B = 0$$

$$v_B =$$

$$A = \mathcal{J}_A^{-1} \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix} \mathcal{J}_A$$

$$B = \gamma_B^{-1} \left(\qquad \right) \gamma_B$$

$$S^{-1} \gamma_A^{-1} = \gamma_B^{-1}$$

$$\gamma_B = \gamma_A S$$

$$(v_{B_1} \dots v_{B_n}) = (v_{A_1} \dots v_{A_n}) S$$

$$v_{B_i} = \alpha_i v_{A_i}$$

$$(B - \lambda_i I) v_{B_i} = 0$$

$$(A - \lambda_i \mathbb{1})\, v_{A_i} = 0$$

$$S^{-1}(A - \lambda_1 \mathbb{1}) S\, v_{B_1} = 0$$

Т. 10* $A = (a_{ij})$ $B = (b_{ij})$

$\exists^?$ диаг. $C = (c_{ij})$: $a_{ij} \le c_{ij} \le b_{ij}$

От против.: Если в (A, B) всегда есть крат. корни

$\rightarrow$ против. с непрерывн. многочл.

24.71 φ — невыр.

φ и φ^{-1} им. одинаковые инв. подпр-ва

$$\varphi(U) \subseteq U$$

$$\varphi^{-1}(\varphi(U)) \subseteq \varphi^{-1}(U)$$

$$U \subseteq \varphi^{-1}(U)$$

$$U \supseteq \varphi^{-1}(U)$$

$\Rightarrow$ ...

24.79.2) $\varphi(U) \subseteq U$

$$\chi_\varphi(t) = |A - t\mathbb{1}|$$

$$A = D^{-1}\tilde{A}D$$

$$\begin{array}{c} \| \\ \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix} \end{array}$$

$$\varphi(U) = AU \subseteq U = \langle v_1, \dots, v_k \rangle$$

$$\varphi(u_\sigma) = Au_\sigma = \lambda u_\sigma.$$

Если нет ни одного соб. век,

то $\langle u_1, \dots, u_n \rangle \not\supseteq U$ (?!)

<u>24.78</u>

$$n + \binom{n}{2} + \dots + \binom{n}{n} = 2^n - 1$$

<u>T.11</u>

$$\begin{pmatrix} \lambda - \alpha & & 0 \\ & \ddots & \\ 0 & & \lambda - \alpha \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = 0$$

$\alpha = \lambda$

$$\begin{pmatrix} 0 & & 0 \\ & \ddots & \\ 0 & & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = 0$$

$$V = \left\langle \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \right\rangle$$

T.12*

a) $\varphi : V \to V$ диаг. $\Rightarrow$

$\varphi|_U : U \to U$ — инв. $\Rightarrow$ диаг.

$$\varphi(U) \subseteq U$$

24.12 (5.5) $A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 2 & 4 \\ -1 & -2 & -3 \end{pmatrix}$

$A^2 = \begin{pmatrix} 2 & 4 & 6 \\ -1 & -2 & -3 \\ 0 & 0 & 0 \end{pmatrix}$ $A^4 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$$\begin{vmatrix} 1-\lambda & 2 & 1 \\ 1 & 2-\lambda & 4 \\ -1 & -2 & -3-\lambda \end{vmatrix} = 0$$

$$-(\lambda-1)(\lambda-2)(\lambda+3) \cancel{-8} \cancel{-2} - \lambda \cancel{+2}$$

$$-8\lambda \cancel{+8} + 2\lambda + 6 =$$

$$= -\lambda^3 + \cancel{7}\lambda - \cancel{6} - \cancel{7\lambda} + \cancel{6} = 0$$

$$\lambda = 0$$

$$\begin{pmatrix} 1 & 2 & 1 \\ 1 & 2 & 4 \\ -1 & -2 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 1 \\ 1 & 2 & 4 \\ -1 & -2 & -3 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \in \emptyset$$

$$\begin{cases} y_1 + 2y_2 = 1 \\ y_1 + 2y_2 = -5 \end{cases}$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} - \text{ЖНФ}$$

24.26.4)*

$$\varphi : \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix}$$

$$p(\varphi) : \begin{pmatrix} p(\lambda_1) & & 0 \\ & \ddots & \\ 0 & & p(\lambda_n) \end{pmatrix}$$

<u>41.8</u> A – нильпот., $A^k = 0$

Если $\exists \lambda \neq 0 :$ $A \sim \begin{pmatrix} \lambda & & \\ & 0 & \\ & & \ddots & \\ * & & & 0 \end{pmatrix}$

То $A^k \sim \begin{pmatrix} \lambda^k & & \\ & 0 & \\ & & \ddots & \\ & & & 0 \end{pmatrix} \neq 0$

4.17*. $A = \frac{\partial}{\partial x} + \frac{\partial}{\partial y}$

$$p(x,y)_{\leq n} \to (p_{ij})$$

$$p_{ij} \xrightarrow{A} i p_{i-1,j} + j p_{i,j-1}$$

$$\mathbb{K}_n[x,y] = a_n x^n + a_{n-1} x^{n-1} y + \dots + a_0 y^n$$

$$\begin{pmatrix} a_n \\ \vdots \\ a_0 \end{pmatrix} \xrightarrow{A} \begin{pmatrix} 0 \\ n a_n + a_{n-1} \\ (n-1) a_{n-1} + 2 a_{n-2} \\ \vdots \\ a_1 + n a_0 \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & 0 & \dots & & 0 \\ n & 1 & 0 & & 0 \\ 0 & n-1 & 2 & & 0 \\ & & \ddots & 3 & \\ & 0 & & \ddots & \\ & & & 1 & n \end{pmatrix}$$

$$\chi_A(t) = -\lambda(1-\lambda)(2-\lambda)\cdots(n-\lambda)$$

ЖНФ:

$$\begin{pmatrix} 0 & & & & 0 \\ & 1 & & & \\ & & 2 & & \\ & & & \ddots & \\ 0 & & & & n \end{pmatrix}$$

<u>41.30*.</u>

$$A^3 = A^2$$

$$J_\lambda^2 = \begin{pmatrix} \lambda & 1 & & 0 \\ & \lambda & \ddots & \\ & & \ddots & 1 \\ 0 & & & \lambda \end{pmatrix}^2 = \begin{pmatrix} \lambda^2 & 2\lambda & & 0 \\ & \ddots & \ddots & \\ & & \ddots & 2\lambda \\ 0 & & & \lambda^2 \end{pmatrix}$$

$$J_\lambda^3 = \begin{pmatrix} \lambda^3 & 3\lambda^2 & & 0 \\ & \ddots & \ddots & \\ & & \ddots & 3\lambda^2 \\ 0 & & & \lambda^3 \end{pmatrix}$$

$$\begin{cases} \lambda^3 = \lambda^2 \\ 3\lambda^2 = 2\lambda \end{cases} \to \lambda = 0$$

$\to \boxed{1}$ и $\boxed{\begin{matrix} 0 & \\ & 0 \end{matrix}}$ и $\boxed{0}$

41.15*

$AB - BA = B \overset{?}{\Rightarrow} B$ нильпот.

$$\begin{pmatrix} \lambda & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix} \quad \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix} \quad \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{pmatrix}$$

нильпот.

T. 14*

$\varphi: V \to V \quad \dim V = n$

$\chi_{\varphi}(\varphi): A^{-1} = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}^{-1} = \frac{1}{\lambda^2} \begin{pmatrix} \cancel{\lambda} & -1 \\ 0 & \lambda \end{pmatrix} =$

$= \begin{pmatrix} \frac{1}{\lambda} & -\frac{1}{\lambda^2} \\ 0 & \frac{1}{\lambda} \end{pmatrix} =$

$= \frac{1}{\lambda^2}\left[-A + 2\lambda \mathbb{1}\right]$

$\begin{pmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{pmatrix} = \frac{1}{\lambda^3} \begin{pmatrix} \lambda^2 & 0 & 0 \\ -\lambda & \lambda^2 & 0 \\ 1 & -\lambda & \lambda^2 \end{pmatrix}^T =$

$= \frac{1}{\lambda^3} \begin{pmatrix} \lambda^2 & -\lambda & 0 \\ 0 & \lambda^2 & -\lambda \\ 0 & & \lambda^2 \end{pmatrix}$

$$J^{-1} = \frac{1}{\lambda^k}\begin{pmatrix} \lambda^{k-1} & -\lambda^{k-2} & & 0 \\ & \ddots & \ddots & \\ & & & -\lambda^{k-2} \\ & & & \lambda^{k-1} \end{pmatrix}$$

$$J^{-1} = \frac{1}{\lambda^k}\left(-\lambda^{k-2} J + 2\lambda^{k-1} \mathbb{1}\right)$$

T.17*

$A \in M_{n\times n}(\mathbb{K})$

$L(A) = \{ f(A) \mid f(t) \in \mathbb{K}[t] \} \subset M_n(\mathbb{K})$

a) ord.

b) $f(I) = a\mathbb{1} + bI \Rightarrow L(I) \cong \mathbb{C}$

I.18

$$\chi_\varphi(t) = t^4(t-1)^3$$

$$m_\varphi(t) = t^2(t-1)^2$$

$$\begin{pmatrix} \boxed{\begin{matrix} 1 & 1 \\ 0 & 1 \end{matrix}} & & & 0 \\ & \boxed{1} & & \\ & & \boxed{\begin{matrix} 0 & 1 \\ 0 & 0 \end{matrix}} & \\ 0 & & & \boxed{\begin{matrix} 0 & 1 \\ 0 & 0 \end{matrix}} \end{pmatrix} \quad \text{um} \quad \begin{pmatrix} \ddots & & & \\ & \boxed{\begin{matrix} 0 & 1 \\ 0 & 0 \end{matrix}} & & \\ & & \boxed{0} & \\ & & & \boxed{0} \end{pmatrix}$$