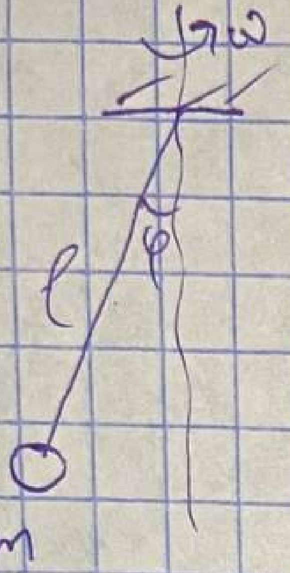
№1.



$$E = \frac{m\omega^2 l^2 \sin^2\varphi}{2} + mgl(1-\cos\varphi)$$

$$+ \frac{m l^2 \dot{\varphi}^2}{2}$$

1) $E(\varphi) > 0 \quad \forall\ 0 < |\varphi| < \varepsilon$

2) $E(0) = 0$

3) $\dot{E}(\varphi) = \dot{\varphi} m l^2 \left(\varphi \left(\omega^2 - \frac{g}{l} \right) + \ddot{\varphi} \right)$

$\dot{E}(\varphi) \le 0$ при $\omega < \sqrt{\frac{g}{l}}$ $\longrightarrow$ устойчивое п.-р.

Выберем обл. σ: $\begin{cases} \dot\varphi < 0 \\ \varphi < 0 \end{cases}$

$$\dot E(\varphi) = m l^2 \underset{\substack{\wedge \\ 0}}{\dot\varphi} \Big(\underset{\substack{\wedge \\ 0}}{\varphi} \underset{\substack{\vee \\ 0}}{\left(\omega^2 - \frac{g}{l} \right)} + \underset{\substack{\wedge \\ 0}}{\ddot\varphi} \Big) < 0$$

и $\dot E(\varphi) = 0 \iff \varphi = 0$

$\Rightarrow$ при $\omega \geq \sqrt{\frac{g}{l}}$ — неуст. п.р.

№2.

$$\begin{cases} J_x \dot{\omega}_x + (J_z - J_y)\omega_y \omega_z = 0 \\ J_y \dot{\omega}_y + (J_x - J_z)\omega_x \omega_z = 0 \\ J_z \dot{\omega}_z + (J_y - J_x)\omega_x \omega_y = 0 \end{cases}$$

$$\bar{\omega} = \begin{pmatrix} \omega \\ 0 \\ 0 \end{pmatrix}$$

1) $$\tilde{\bar{\omega}} = \begin{pmatrix} \omega + x \\ y \\ z \end{pmatrix}$$

$$\begin{cases} J_x \dot{x} + (J_z - J_z) y z = 0 \\ J_y \dot{y} + (J_x - J_z)(\omega + x) z = 0 \\ J_z \dot{z} + (J_y - J_x)(\omega + x) y = 0 \end{cases}$$

$$\begin{cases} \dot{x} = \dfrac{-J_z + J_y}{J_x} y z \\ \dot{y} = \dfrac{J_z - J_x}{J_y}(\omega + x)\cancel{z} \\ \dot{z} = \dfrac{J_x - J_y}{J_z}(\omega + x) y \end{cases}$$

2) $U_1 = J_x x^2 + J_y y^2 + J_z z^2 + 2J_x \omega x$

$U_2 = J_x^2 x^2 + J_y^2 y^2 + J_z^2 z^2 + 2J_x^2 \omega x$

$$\dot U_{1f} = 2J_x x\dot x + 2J_y y\dot y + 2J_z z\dot z + 2J_x \omega \dot x =$$

$$= 2xyz(-J_z + J_y + J_z - J_x + J_x - J_y) +$$

$$+ \cancel{2J_x \omega \dot x}\ 2yz(J_y - J_z) +$$

$$+ 2yz(J_z - J_x) + 2yz(J_x - J_y) = 0$$

$\dot U_{2f} = 0$ Аналогично.

3) $V = U_1^2 + U_2^2$

$V(\bar r) > 0 \quad \forall\, 0 < |\bar r| < \varepsilon$

$V(\bar 0) = 0$

$$\dot V_f(\bar r) = 2U_1 \cancelto{0}{\dot U_{1f}} + 2U_2 \cancelto{0}{\dot U_{2f}} = 0 \quad \forall\, |\bar r| < \varepsilon$$

$\Rightarrow$ н.р. $\bar r = \bar 0$ — <u>уст.</u>

№3.

$$\begin{cases} \dot{x} = -\frac{\partial G}{\partial x} \\ \dot{y} = -\frac{\partial G}{\partial y} \end{cases}$$

$$\dot{\bar{r}} = -\nabla G$$

$$G(\cancel{x}, y) = \text{const}$$

$$dG = 0$$

$$\frac{\partial G}{\partial x} dx + \frac{\partial G}{\partial y} dy = 0$$

$\Rightarrow$ кас. к $G = \text{const}$: $\left\langle \frac{\partial G}{\partial y}, -\frac{\partial G}{\partial x} \right\rangle = \bar{\tau}$

$$(\dot{\bar{r}}, \bar{\tau}) = \left\langle -\frac{\partial G}{\partial x}, -\frac{\partial G}{\partial y} \right\rangle \cdot \left\langle \frac{\partial G}{\partial y}, -\frac{\partial G}{\partial x} \right\rangle =$$

$$= -\frac{\partial G}{\partial x}\frac{\partial G}{\partial y} + \frac{\partial G}{\partial y}\frac{\partial G}{\partial x} = 0$$

$\Rightarrow \quad \dot{\bar{r}} \perp \bar{\tau}$

чтд!