$$I = \int_0^{\frac{\pi}{2}} \frac{\tan^{-1}(\sin t)}{\sin t} dt = \int_0^{\frac{\pi}{2}} \int_0^1 \frac{dy\ dt}{1 + (y \sin t)^2} = \int_0^1 \int_0^{\frac{\pi}{2}} \frac{\csc^2 t\ dt}{1 + \cot^2 t + y^2}\, dy =$$

$$= \int_0^1 \int_0^{\infty} \frac{dx}{\left(\sqrt{1+y^2}\right)^2 + x^2}\, dy = \frac{\pi}{2} \int_0^1 \frac{dy}{\sqrt{1+y^2}} =$$

$$= \frac{\pi}{2} \sinh^{-1} y \Big|_0^1 = \frac{\pi}{2} \sinh^{-1} 1 = \frac{\pi}{2} \ln(1 + \sqrt{2})$$