№1

$$\|\Lambda \circ M\| = \|(\lambda_0 + \vec{\lambda})(\mu_0 + \vec{\mu})\| =$$

$$= \|\lambda_0 \mu_0 - (\vec{\lambda}, \vec{\mu}) + \lambda_0 \vec{\mu} + \mu_0 \vec{\lambda} + [\vec{\lambda}, \vec{\mu}]\| =$$

$$= (\lambda_0 \mu_0 - (\vec{\lambda}, \vec{\mu}))^2 + (\lambda_0 \vec{\mu} + \mu_0 \vec{\lambda} + [\vec{\lambda}, \vec{\mu}]) \cdot$$

$$\cdot (-\lambda_0 \vec{\mu} + (-\mu_0 \vec{\lambda}) - [\vec{\lambda}, \vec{\mu}]) =$$

$$= (\lambda_0 \mu_0 - (\vec{\lambda}, \vec{\mu}))^2 + (+\lambda_0^2 |\vec{\mu}|^2 + (+\mu_0^2 |\vec{\lambda}|^2) -$$

$$+ |[\vec{\lambda}, \vec{\mu}]|^2 + \lambda_0 \mu_0 (+(\vec{\mu}, \vec{\lambda}) + \cancel{[\vec{\mu}, \vec{\lambda}]})$$

$$+ \mu_0 \lambda_0 (+(\vec{\lambda}, \vec{\mu}) + \cancel{[\vec{\lambda}, \vec{\mu}]})$$

$$+ \lambda_0 (-\cancel{(\vec{\mu}, [\vec{\lambda}, \vec{\mu}])}^{0} + \cancel{[\vec{\mu}, [\vec{\lambda}, \vec{\mu}]]})$$

$$+ \mu_0 (-\cancel{(\vec{\lambda}, [\vec{\lambda}, \vec{\mu}])}^{0} + \cancel{[\vec{\lambda}, [\vec{\lambda}, \vec{\mu}]]}) =$$

$$= (\lambda_0 \mu_0 - (\vec{\lambda}, \vec{\mu}))^2 + \lambda_0^2 |\vec{\mu}|^2 + \mu_0^2 |\vec{\lambda}|^2 +$$

$$+ |[\vec{\lambda}, \vec{\mu}]|^2 + 2\lambda_0 \mu_0 (\vec{\lambda}, \vec{\mu}) \neq$$

$$\cancel{+ \lambda_0 (\vec{\lambda}(\vec{\mu}, \vec{\mu}) - \vec{\mu}(\vec{\mu}, \vec{\lambda}))}$$

$$\cancel{+ \mu_0 (\vec{\lambda}(\vec{\lambda}, \vec{\mu}) - \vec{\mu}(\vec{\lambda}, \vec{\lambda}))}$$

$$\|\Lambda \circ M\| = \lambda_0^2 \mu_0^2 + (\vec{\lambda}, \vec{\mu})^2 + \lambda_0^2 |\vec{\mu}|^2 +$$

$$\mu_0^2 |\vec{\lambda}|^2 + |[\vec{\lambda}, \vec{\mu}]|^2 \overset{?}{=} (\lambda_0^2 + |\vec{\lambda}|^2)(\mu_0^2 + |\vec{\mu}|^2)$$

$$(\vec{\lambda}, \vec{\mu})^2 + |[\vec{\lambda}, \vec{\mu}]|^2 \overset{?}{=} |\vec{\lambda}|^2 |\vec{\mu}|^2$$

$$|\vec{\lambda}|^2 |\vec{\mu}|^2 (\cos^2\varphi + \sin^2\varphi) \overset{?}{=} |\vec{\lambda}|^2 |\vec{\mu}|^2$$

✓

чтд!

№2.

$$\Lambda \circ X^2 = X \circ \Lambda \Rightarrow \left[\begin{array}{l} \|X\| = 1 \\ \|X\| = 0 \to X = 0 \\ \|\Lambda\| = 0 \to X \in \mathbb{H} \end{array}\right.$$

$$\cancel{X = \cos\varphi + \sin\varphi \, |\vec{x}|}$$

$$X = \cancel{\cos\varphi + \sin\varphi \, \vec{x}} = x_0 + \vec{x}$$

$$|\vec{x}| = 1 \qquad x_0^2 + |\vec{x}|^2 = 1$$

$$\Lambda = \lambda_0 + \vec{\lambda}$$

$$X^2 = x_0^2 + 2x_0\vec{x} + \cancel{[\vec{x}, \vec{x}]}^{\,0} - (\vec{x}, \vec{x}) =$$

$$= x_0^2 - |\vec{x}|^2 + 2x_0\vec{x}$$

$$(\lambda_0 + \vec{\lambda})(x_0^2 - |\vec{x}|^2 + 2x_0\vec{x}) =$$

$$= \lambda_0(x_0^2 - |\vec{x}|^2) - 2x_0(\vec{\lambda}, \vec{x}) + \vec{\lambda}(x_0^2 - |\vec{x}|^2)$$

$$+ 2x_0\lambda_0\vec{x} + 2x_0[\vec{\lambda}, \vec{x}]$$

$$X \circ \Lambda = x_0\lambda_0 - (\vec{x}, \vec{\lambda}) + x_0\vec{\lambda} + \lambda_0\vec{x} + [\vec{x}, \vec{\lambda}]$$

$$\begin{cases} \lambda_0(x_0^2 - |\vec{x}|^2) - 2x_0(\vec{\lambda}, \vec{x}) = x_0\lambda_0 - (\vec{x}, \vec{\lambda}) \\ \vec{\lambda}(x_0^2 - |\vec{x}|^2) + 2x_0\lambda_0\vec{x} + 2x_0[\vec{\lambda}, \vec{x}] = \\ = x_0\vec{\lambda} + \lambda_0\vec{x} + [\vec{\lambda}, \vec{x}] \end{cases}$$

1) $\vec{x} = k\vec{\lambda}$

$$\begin{cases} x_0^2 - |k|^2 |\vec{\lambda}|^2 + 2x_0\lambda_0 k = x_0 + \lambda_0 k \\ \lambda_0 (x_0^2 - k^2 |\vec{\lambda}|^2) - (2x_0 k - k)|\vec{\lambda}|^2 = x_0\lambda_0 \\ x_0^2 + k^2 |\vec{\lambda}|^2 = 1 \end{cases}$$

$$2x_0^2 + 2x_0\lambda_0 k = x_0 + \lambda_0 k + 1$$

$$\lambda_0 x_0^2 + x_0^2 \cdot \frac{\lambda_0 k^2 + 2x_0 k - k}{k^2} =$$

$$= x_0\lambda_0 + \frac{\lambda_0 k^2 + 2x_0 k - k}{k^2}$$

$$k^2 x_0 \lambda_0 (x_0 - 1) + (x_0 - 1)(x_0 + 1)(\lambda_0 k^2 + 2x_0 k - k) = 0$$

$x_0 = 1$

$\vec{x} = \vec{0}$

$$x_0 \neq 1, \quad k \neq 0$$

$$2k^2 x_0 \lambda_0 + 2x_0^2 k - x_0 k + \lambda_0 k^2 + 2x_0 k - k = 0$$

$$2k x_0 \lambda_0 + 2x_0^2 - x_0 + \lambda_0 k + 2x_0 - 1 = 0$$

$$2k x_0 \lambda_0 + 2x_0^2 + \lambda_0 k + x_0 - 1 = 0$$

$$k = -\frac{2x_0^2 + x_0 - 1}{\lambda_0(2x_0 + 1)}$$

$$2x_0^2 \cancel{-} 2x_0 \frac{2x_0^2+x_0-1}{2x_0+1} = x_0+1 - \frac{2x_0^2+x_0-1}{2x_0+1}$$

$$\frac{2x_0}{2x_0+1} = x_0+1 - x_0 + \frac{1}{2x_0+1} = \frac{2x_0+2}{2x_0+1}$$

$$x_0 \neq -\frac{1}{2} \to \varnothing$$

$$x_0 = -\frac{1}{2}$$

$$\cancel{\frac{1}{2}} - \lambda_0 k = -\cancel{\frac{1}{2}} + \lambda_0 k + 1 \to \lambda_0 k = 0$$

2) $\vec{x} \parallel \vec{\lambda}$

$$\begin{cases} \lambda_0 (x_0^2 - |\vec{x}|^2) - 2x_0 (\vec{\lambda}, \vec{x}) = x_0 \lambda_0 - (\vec{x}, \vec{\lambda}) \\ x_0^2 - |\vec{x}|^2 = x_0, \quad |\vec{x}|^2 = \frac{1}{4} + \frac{1}{2} = \frac{3}{4} \\ 2x_0 \lambda_0 = \lambda_0 \quad \to \lambda_0 = 0 \\ 2x_0 = -1 \to x_0 = -\frac{1}{2} \\ x_0^2 + |\vec{x}|^2 = 1 \end{cases}$$

$$-\cancel{\frac{\lambda_0}{2}} + (\vec{\lambda}, \vec{x}) = -\cancel{\frac{\lambda_0}{2}} - (\vec{x}, \vec{\lambda})$$

$$(\vec{\lambda}, \vec{x}) = 0$$

Ответ: $X = 0$ или $X \in \mathbb{H}$ при $\Lambda = 0$ или $X = 1$ или $X = -\frac{1}{2} + \vec{x}$, где $|\vec{x}|^2 = \frac{3}{4}$ и $(\vec{\lambda}, \vec{x}) = 0$

№3.

$$\Lambda = \frac{\sqrt{2}}{2} + i_3 \frac{\sqrt{2}}{2}$$

$$\begin{pmatrix}-1\\0\\1\end{pmatrix} = \vec{R}' = \Lambda \circ R \circ \bar{\Lambda} =$$

$$= \left(\frac{\sqrt{2}}{2}\vec{r} - r_3\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\overset{[i_3,\, r]}{(\cancel{i_1 r_2 - i_2 r_1})}\right)\bar{\Lambda} =$$

$$= \frac{1}{2}\Big[\vec{r} - \cancel{r_3} + \overset{[i_3,\, r]}{\cancel{i_1 r_2 - i_2 r_1}} + \cancel{r_3} -$$

$$- \overset{[r,\, i_3]}{(\cancel{i_1 r_2} - i_2 r_1)} + \overset{r_3 i_3 - [[i_3,\, r],\, i_3]}{\cancel{r_3 i_3 - i_1 r_2 -}}$$

$$\cancel{- i_1 r_1}\Big] =$$

$$\cancel{= \frac{1}{2}\left(i_1 r_1 + i_2 r_2 + i_3 r_3 - i_3 r_3 - i_2 r_2 - i_1 r_1\right)}$$

$$= \frac{1}{2}\Big[\cancel{\vec{r}} + 2[i_3, r] + r_3 i_3 \quad \overset{\shortparallel}{0}$$

$$+ i_3 r_3 - \cancel{\vec{r}}\Big] = r_3 i_3 + [i_3, r] =$$

$$= i_1 r_2 - i_2 r_1 + i_3 r_3$$

$$\Rightarrow \boxed{\vec{r} = \begin{pmatrix}0\\-1\\1\end{pmatrix}}$$