I. 14.4. 2) $\begin{vmatrix} 3 & 4 \\ 5 & 7 \end{vmatrix} = 21 - 20 = 1$

5) $\begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix} = \cos^2\alpha + \sin^2\alpha = 1$

14.7. 1) $\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = -1$

3) $\begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{vmatrix} = 1 + (-8) - 8 - 4 - 4 - 4 =$
$= -27$

II. 15.2. 1) $3\begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} 3 & 2 \\ 3 & 2 \end{pmatrix} - 4\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} =$

$= \begin{pmatrix} 3 & 6 \\ 3 & 6 \end{pmatrix} - \begin{pmatrix} 3 & 2 \\ 3 & 2 \end{pmatrix} - \begin{pmatrix} 0 & 4 \\ 0 & 4 \end{pmatrix} =$

$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

15.5. 1) $\begin{pmatrix} 2 & -3 & 0 \end{pmatrix}\begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix} = 8 - 9 + 0 = -1$

2) $\begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix}\begin{pmatrix} 2 & -3 & 0 \end{pmatrix} = \begin{pmatrix} 8 & -12 & 0 \\ 6 & -9 & 0 \\ 2 & -3 & 0 \end{pmatrix}$

15.10. 1) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ не сущ.

2) $\begin{pmatrix} 2 \\ 4 \end{pmatrix}\begin{pmatrix} 1 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 4 & 8 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 8 \\ 16 \end{pmatrix}$

15.11. 1)

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^n = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^{n-2} =$$

$$= 2 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^{n-1} = \dots =$$

$$= 2^{n-1} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

III. 17.1.4)

$$\begin{cases} y + 3z = -1 \\ 2x + 3y + 5z = 3 \\ 3x + 5y + 7z = 6 \end{cases}$$

$$\left(\begin{array}{ccc|c} 0 & 1 & 3 & -1 \\ 2 & 3 & 5 & 3 \\ 3 & 5 & 7 & 6 \end{array}\right) \to \left(\begin{array}{ccc|c} 0 & 1 & 3 & -1 \\ 0 & -1 & 1 & -3 \\ 3 & 5 & 7 & 6 \end{array}\right) \to$$

$$\to \left(\begin{array}{ccc|c} 0 & 0 & 4 & -4 \\ 0 & -1 & 1 & -3 \\ 3 & 5 & 7 & 6 \end{array}\right) \to \left(\begin{array}{ccc|c} 0 & 0 & 1 & -1 \\ 0 & -1 & 0 & -2 \\ 3 & 5 & 7 & 6 \end{array}\right) \to$$

$$\to \left(\begin{array}{ccc|c} 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 2 \\ 3 & 0 & 0 & 3 \end{array}\right) \to \left(\begin{array}{ccc|c} 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 1 \end{array}\right)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

19.1.5)
$$\begin{cases} x+2y+3z=-4 \\ 2x+3y+4z=1 \\ 3x+4y+5z=6 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & -4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 5 & 6 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & 3 & -4 \\ 0 & -1 & -2 & 9 \\ 0 & -2 & -4 & 18 \end{array}\right) \to$$

$$\to \left(\begin{array}{ccc|c} 1 & 0 & -1 & 14 \\ 0 & 1 & 2 & -9 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{cases} x-z=14 \\ y+2z=-9 \end{cases} \qquad \begin{cases} x=14+z \\ y=-9-2z \end{cases}$$

IV. 1.6 $\quad a(-5,-1) \quad b(-1,3)$

$$\begin{vmatrix} -5 & -1 \\ -1 & 3 \end{vmatrix} = -15-1 \neq 0$$

$\Rightarrow$ ab - базис.

$$c(-1,2) = (-5k_1-k_2,\ -k_1+3k_2)$$

$$\begin{pmatrix} -5 & -1 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = \frac{1}{-16}\begin{pmatrix} 3 & 1 \\ 1 & -5 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix} = \frac{-1}{16}\begin{pmatrix} -1 \\ -11 \end{pmatrix} = \begin{pmatrix} \frac{1}{16} \\ \frac{11}{16} \end{pmatrix}$$

$$c = \frac{1}{16}a + \frac{11}{16}b$$

$d(2,-6)$. $\begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = -\frac{1}{16}\begin{pmatrix} 3 & 1 \\ 1 & -5 \end{pmatrix}\begin{pmatrix} 2 \\ -6 \end{pmatrix} =$

$= -\frac{1}{16}\begin{pmatrix} 0 \\ 32 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$

$d = -2b$

1.11. 2) $l = a+b+c$ $m = b+c$ $n = -a+c$

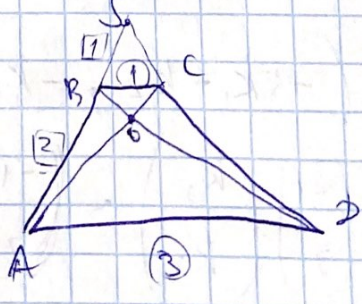
$\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ -1 & 0 & 1 \end{vmatrix} = 1-1+1 = 1 \neq 0 \Rightarrow$ не компл.

3) $l = c$ $m = a-b-c$ $n = a-b+c$

$\begin{vmatrix} 0 & 0 & 1 \\ 1 & -1 & -1 \\ 1 & -1 & 1 \end{vmatrix} = -1+1 = 0 \Rightarrow$ компл.

$m + 2l = n$

1.16.



$\overline{AC} = \overline{AB} + \frac{1}{3}\overline{AD}$

$AO = \frac{3}{4}\overline{AC} =$

$= \frac{3}{4}\overline{AB} + \frac{1}{4}\overline{AD}$

$\overline{AS} = \frac{3}{2}\overline{AB}$

1.17. Возьмём произв. т. O.

$$\overline{OE} = \frac{\overline{OA}+\overline{OB}}{2} \qquad \overline{OF} = \frac{\overline{OC}+\overline{OD}}{2}$$

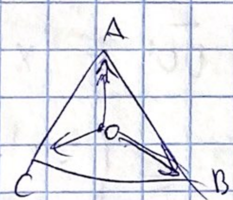
$$\overline{EF} = \overline{OF}-\overline{OE} = \frac{\overline{BC}+\overline{AD}}{2}$$

1.24. 1) O, A, B

$|AM| : |BM| = m : n$

$$\overline{OM} = \frac{n}{n+m}\overline{OA} + \frac{m}{n+m}\overline{OB}$$

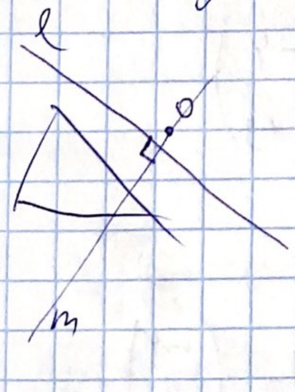
1.37.



$\overline{v} = \overline{OA}+\overline{OB}+\overline{OC} = \overline{0}$

т. O — ц.м. ΔABC

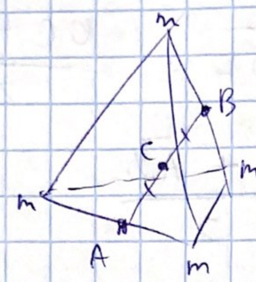
Вне пл-ти Δ не сущ. т.к. $\exists$ пр. l и пр. $m \perp l$: ΔABC и т. O в разных полуплоскостях. Тогда $\text{Пр}_m^{\perp} \overline{v} \neq 0$



1.38. $\overline{r}_1, \dots, \overline{r}_n \leftrightarrow m_1, \dots, m_n$

$$\overline{r}_c = \frac{m_1\overline{r}_1 + \dots + m_n\overline{r}_n}{\sum_{i=1}^{n} m_i}$$

1.51.



ц. AB – ц.м. тетр. для любых таких отр. AB.
=> такие отрезки пересек. и дел. в т. пересеч. пополам.

V. 4.3. $O' = (-1, 3)$

$e_1' = \begin{pmatrix}2\\3\end{pmatrix} \qquad e_2' = \begin{pmatrix}1\\1\end{pmatrix}$

1) $A = (e_1' \; e_2')\begin{pmatrix}x'\\y'\end{pmatrix} + \overline{OO'} = x'\begin{pmatrix}2\\3\end{pmatrix} + y'\begin{pmatrix}1\\1\end{pmatrix} + \begin{pmatrix}-1\\3\end{pmatrix} =$

$= \begin{pmatrix}2x'+y'-1\\3x'+y'+3\end{pmatrix}$

2) $A = \begin{pmatrix}x\\y\end{pmatrix} = \overline{OO'} + (e_1' \; e_2')\begin{pmatrix}k_1\\k_2\end{pmatrix} = \begin{pmatrix}-1\\3\end{pmatrix} + \begin{pmatrix}2&1\\3&1\end{pmatrix}\begin{pmatrix}k_1\\k_2\end{pmatrix}$

$\begin{pmatrix}k_1\\k_2\end{pmatrix} = \begin{pmatrix}2&1\\3&1\end{pmatrix}^{-1}\begin{pmatrix}x+1\\y-3\end{pmatrix} = \begin{pmatrix}-1&1\\3&-2\end{pmatrix}\begin{pmatrix}x+1\\y-3\end{pmatrix} =$

$= \begin{pmatrix}y-x-4\\3x-2y+9\end{pmatrix}$

3) O: $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$

$\begin{pmatrix}k_1\\k_2\end{pmatrix} = \begin{pmatrix}0-0-4\\0-0+9\end{pmatrix} = \begin{pmatrix}-4\\9\end{pmatrix}$

e_1: $\begin{pmatrix}k_1\\k_2\end{pmatrix} = \begin{pmatrix}e_1^2-e_1^1-4\\3e_1^1-2e_1^2+9\end{pmatrix} \xlongequal{e_1=\binom{1}{0}} \begin{pmatrix}-5\\12\end{pmatrix}$

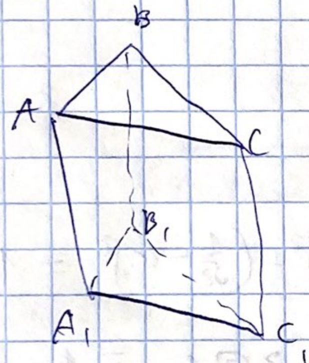
e_2 : $\begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = \begin{pmatrix} e_2^2 - e_2^1 - 4 \\ 3e_2^1 - 2e_2^2 + 9 \end{pmatrix} \overset{e_2 = \binom{0}{1}}{=\!=} \begin{pmatrix} -3 \\ 7 \end{pmatrix}$

4.26. 1) $\begin{pmatrix} x \\ y \end{pmatrix} = \overline{OO'} + (e_1' \; e_2') \begin{pmatrix} x' \\ y' \end{pmatrix}$

$(e_1' \; e_2') = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} (e_1 \; e_2)$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} \overset{\varphi = \frac{\pi}{3}}{=\!=} \begin{pmatrix} 1 + \frac{1}{2}x' + \frac{\sqrt{3}}{2}y' \\ 3 - \frac{\sqrt{3}}{2}x' + \frac{1}{2}y' \end{pmatrix}$

4.19.



$\overline{AA_1} = \overline{AB_1} - \overline{AB}$

$\overline{A_1M} = \frac{1}{3}(\overline{A_1B_1} + \overline{A_1C_1}) =$

$= \frac{1}{3}(\overline{AB} + \overline{AC})$

$x'\,\overline{A_1B} + y'\,\overline{A_1C} + z'\,\overline{A_1M} =$

$= x'(2\overline{AB} - \overline{AB_1}) + y'(\overline{AC} + \overline{AB} - \overline{AB_1}) + \frac{z'}{3}(\overline{AB} + \overline{AC}) =$

$= \overline{AB}\left(2x' + y' + \frac{z'}{3}\right) + \overline{AC}\left(y' + \frac{z'}{3}\right) + \overline{AB_1}(-x' - y')$

$\boxed{\text{VI}}$. 2.27(2) $\bar{a} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ $\bar{b} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$

$pr = \frac{\bar{a}\bar{b}}{|a|} = \frac{1 + (-1) + 4}{\sqrt{1 + 1 + 4}} = \frac{4}{\sqrt{6}}$; $|a| = \sqrt{6}$

$\overline{ort} =$ ~~...~~ $\bar{b} - pr \cdot \frac{\bar{a}}{|a|} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} - \frac{4/\sqrt{6}}{\sqrt{6}} \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} =$

$= \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} - \frac{2}{3}\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1/3 \\ 5/3 \\ 2/3 \end{pmatrix}$

2.35. $\bar{a} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ $\bar{b} = \begin{pmatrix} 5 \\ 1 \\ 1 \end{pmatrix}$ $|\bar{b}| = \sqrt{25+1+1} = \sqrt{27}$

$|\bar{c}| = 1$ $\cos(\widehat{b,c}) = \sqrt{\frac{2}{27}} = \frac{\bar{b}\bar{c}}{|\bar{b}||\bar{c}|} = \frac{b c}{|b|}$

$\bar{a}\bar{c} = 0$

$$\begin{cases} c_1^2 + c_2^2 + c_3^2 = 1 \\ c_1 - c_2 + c_3 = 0 \\ 5c_1 + c_2 + c_3 = \sqrt{2} \end{cases} \quad \begin{cases} c_2 = c_1 + c_3 \\ 3c_1 + c_3 = \frac{1}{\sqrt{2}} \\ c_1^2 + c_2^2 + c_3^2 = 1 \end{cases}$$

$$\begin{cases} c_3 = \frac{1}{\sqrt{2}} - 3c_1 \\ c_2 = \frac{1}{\sqrt{2}} - 2c_1 \\ c_1^2 + \left(\frac{1}{\sqrt{2}} - 2c_1\right)^2 + \left(\frac{1}{\sqrt{2}} - 3c_1\right)^2 = 1 \end{cases}$$

$14c_1^2 + \cancel{1} - 2\sqrt{2}c_1 - 3\sqrt{2}c_1 = \cancel{1}$

$c_1\left(c_1 - \frac{5\sqrt{2}}{14}\right) = 0$

$c = \begin{pmatrix} 0 \\ \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{pmatrix}$ или $c = \begin{pmatrix} \frac{5\sqrt{2}}{14} \\ -\frac{3\sqrt{2}}{14} \\ -\frac{4\sqrt{2}}{7} \end{pmatrix}$

2.21 $|e_1| = 3$ $|e_2| = \sqrt{2}$ $|e_3| = 4$

$\cos(\widehat{e_1, e_2}) = \frac{1}{\sqrt{2}} = \frac{e_1 e_2}{|e_1||e_2|} = \frac{e_1 e_2}{3\sqrt{2}} \Rightarrow e_1 e_2 = 3$

$\cos(\widehat{e_2, e_3}) = \frac{1}{\sqrt{2}} = \frac{e_2 e_3}{|e_2||e_3|} = \frac{e_2 e_3}{4\sqrt{2}}$, $e_2 e_3 = 4$

$\cos(e_1, e_3) = \frac{1}{2} = \frac{e_1 e_3}{|e_1||e_3|} = \frac{e_1 e_3}{14}$, $e_1 e_3 = 7$

	e_1	e_2	e_3
e_1	9	3	7
e_2	3	2	4
e_3	7	4	16

$$\bar{a}$$

$$l_1 = |\langle 1, -3, 0\rangle| = \sqrt{e_1^2 + 9e_2^2 + 0} = \sqrt{3 + 9\cdot 2} = \sqrt{21}$$

$$\bar{b}$$

$$l_2 = |\langle -1, 2, 1\rangle| = \sqrt{e_1^2 + 4e_2^2 + e_3^2} = \sqrt{9 + 4\cdot 2 + 16} = \sqrt{33}$$

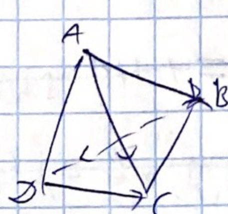
$$\cos(\bar{a}, \bar{b}) = \frac{\bar{a}\,\bar{b}}{|\bar{a}||\bar{b}|} = \frac{-e_1^2 + 2e_1e_2 + e_1e_3 +}{l_1 l_2}$$

$$\frac{3e_2e_1 - 6e_2^2 - 3e_2e_3}{l_1 l_2} = \frac{-9 + 2\cdot 3 + 7 + 3\cdot 3 - 6\cdot 2 - 3\cdot 4}{l_1 l_2} =$$

$$= \frac{22 - 33}{\sqrt{7}\sqrt{3}\sqrt{3}\sqrt{11}} = \frac{-11}{3\sqrt{7}\sqrt{11}} = \frac{-1}{3}\sqrt{\frac{11}{7}}$$

ъгл вер-ма $\in \left\{ \arccos\left(\pm\frac{1}{3}\sqrt{\frac{11}{7}}\right), \pi - \arccos\left(\frac{1}{3}\sqrt{\frac{11}{7}}\right) \right\}$

2.45.



$$\overline{BC}\cdot\overline{AD} = (\overline{BA} + \overline{AC})(\overline{AC} + \overline{CD}) =$$

$$= \overline{BA}\cdot\overline{AC} + \overline{BA}\cdot\overline{CD} + \overline{AC}^2 + \overline{AC}\cdot\overline{CD} =$$

$$= \overline{AC}(\overline{AC} + \overline{CD} + \overline{BA}) = \overline{AC}(\overline{AD} - \overline{AB}) = \overline{AC}\cdot\overline{BD} = 0$$

$\Rightarrow$ $BC \perp AD$

VII. 3.1(1) $\langle 3, -1, 2\rangle \times \langle 2, -3, -5\rangle =$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 2 & -3 & -5 \end{vmatrix} = 11\hat{i} + 19\hat{j} - 7\hat{k} = \langle 11, 19, -7\rangle$$

3.7(2) $\bar{a} = \langle 1, -1, 1\rangle$ $b = \langle 5, 1, 1\rangle$

$$\bar{v} = [\bar{a}, b] = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 5 & 1 & 1 \end{vmatrix} = -2\hat{i} + 4\hat{j} + 6\hat{k} = \langle -2, 4, 6\rangle$$

$\bar{v} \cdot \bar{c} = \langle -2, 4, 6\rangle \langle 0, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\rangle = 5\sqrt{2} > 0$ подходит ✓

$\bar{v} \cdot \bar{c} = \langle -2, 4, 6\rangle \langle \frac{5\sqrt{2}}{14}, -\frac{3\sqrt{2}}{14}, -\frac{8\sqrt{2}}{14}\rangle = \frac{2\sqrt{2}}{14} \langle -1, 2, 3\rangle \langle 5, -3, -8\rangle =$

$= \frac{\sqrt{2}}{7}(-5 - 6 - 24) < 0$ не подходит ✗

3.12. $[a, b] = [b, c] = [c, a]$

a, b одн. род. пл-ть α

Тогда $[a, b] \perp \alpha$
$\quad\quad\; \shortparallel$
$\quad\;\; [b, c] \perp \alpha$

$\Leftrightarrow$ a, b, c лежат в одной пл-ти α

$\Updownarrow$

$c = \lambda_1 a + \lambda_2 b$

~~$|a||b| \sin(a \hat{} b) = |b||c| \sin(b \hat{} c) = |a||c| \sin(a \hat{} c)$~~

~~$a^2 \sin^2(a \hat{} b) = (\lambda_1^2 a^2 + 2\lambda_1\lambda_2 ab + \lambda_2^2 b^2) \sin^2(b \hat{} c)$~~

~~$b^2 \sin^2(a \hat{} b) =$~~

$[a,b]+[c,b]=0$

$[a+c, b]=0$, $[a+c, b]=0$, $a+c=\lambda_1 b$

аналогично $a+b=\lambda_2 c$

$b+c=\lambda_3 a$

$2(a+b+c)=\lambda_3 a+\lambda_1 b+\lambda_2 c$

$\Leftrightarrow \quad \lambda_1=\lambda_2=\lambda_3=\lambda$

$$\begin{cases} a+c=\lambda b \\ a+b=\lambda c \\ b+c=\lambda a \end{cases} - \quad c-b=\lambda(b-c)=-\lambda(c-b)$$

$\Updownarrow$

$\lambda=-1$

$a+b+c=0$

чтд!

3.13. 1) $|[a,b]|^2=\begin{vmatrix} (a,a) & (a,b) \\ (a,b) & (b,b) \end{vmatrix}$

$$\cancel{\left\|\begin{matrix} e_1 & e_2 & e_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{matrix}\right\|^2 = (a_2b_3-a_3b_2)^2+(a_1b_3-a_3b_1)^2+ + (a_1b_2-a_2b_1)^2=}$$

$|[a,b]|^2=a^2b^2\sin^2\alpha$

$\Rightarrow \begin{vmatrix} (a,a) & (a,b) \\ (a,b) & (b,b) \end{vmatrix} = a^2b^2(1-\cos^2\alpha)=a^2b^2\sin^2\alpha$

✓

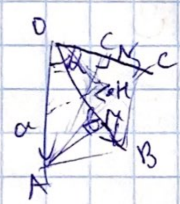
2) $[a,[b,c]]^2 = \begin{vmatrix} (a,a) & (a,[b,c]) \\ (a,[b,c]) & ([b,c],[b,c]) \end{vmatrix} =$

$$= b^2(a,c)^2 + c^2(a,b)^2 - 2(b,c)(a,b)(a,c)$$

$$= a^2\left(b^2c^2 - (b,c)^2\right) - (a,b,c)^2$$

$$(a,b,c)^2 + a^2(b,c)^2 + b^2(a,c)^2 + c^2(a,b)^2 = a^2b^2c^2 + 2(b,c)(a,b)(a,c)$$

$$\frac{(a,b,c)^2}{a^2b^2c^2} + \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 + 2\cos\alpha\cos\beta\cos\gamma$$



$$V^2 = a^2b^2c^2\left(1 + 2\cos\alpha\cos\beta\cos\gamma - \cos^2\alpha - \cos^2\beta - \cos^2\gamma\right)$$

$V = |AH|\, bc\sin\alpha$

M и N на сф. диам. OH $\Rightarrow OH = MN/\sin\alpha$

$$MN^2 = (a\cos\gamma)^2 + (a\cos\beta)^2 - 2a^2\cos\beta\cos\gamma\cos\alpha$$

$$AH^2 = a^2 - OH^2 = a^2 - \frac{a^2}{\sin^2\alpha}\left(\cos^2\gamma + \cos^2\beta - 2\cos\alpha\cos\beta\cos\gamma\right)$$

$$= \frac{a^2}{\sin^2\alpha}\left(1 + 2\cos\alpha\cos\beta\cos\gamma - \cos^2\alpha - \cos^2\beta - \cos^2\gamma\right)$$

$$V^2 = AH^2 b^2c^2\sin^2\alpha = a^2b^2c^2(\ldots)$$

ЧТД!

3.31. $\begin{cases} (x, a) = p \\ (x, b) = q \\ (x, c) = s \end{cases}$ (пересеч. 3 пл-тей)

x сущ. когда $(a, b, c) \neq 0 \Rightarrow x = \dfrac{X}{(a,b,c)}$

Заметим, что если $X = \sum_i [a, y_i] = [a, \sum y_i]$

то $(X, a) = 0$.

То же самое относится к b и c.

В силу сим. задачи $X = \alpha [a, b] + \beta [b, c] + \gamma [a, c]$

Тогда перепишем сист.

$$\begin{cases} \beta ([b, c], a) = p (a, b, c) \\ \gamma ([a, c], b) = q (a, b, c) \\ \alpha ([a, b], c) = s (a, b, c) \end{cases}$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} s \\ p \\ -q \end{pmatrix}$$

$$x = \frac{s [a, b] + p [b, c] + q [c, a]}{(a, b, c)}$$

VIII 5.4. $M_0 = r_0$ $(r, n) = D$

1) $r_{пр} = r_0 + \alpha n$

$(r_0, n) + \alpha |n|^2 = D$

$\alpha = \frac{1}{|n|^2}(D - (r_0, n))$

$r_{пр} = r_0 + \frac{n}{|n|^2}(D - (r_0, n))$

2) $M_1 = r_0 + 2\alpha n = r_0 + \frac{2n}{|n|^2}(D - (r_0, n))$

6.1. 1) $r = r_0 + au + bv$ $(r, n) = D$

$(r - r_0, [a, b]) = 0$

$(r, [a, b]) = (r_0, [a, b])$

3) $[r, a] = b$ $r = r_0 + at$

$[r - r_0, a] = 0$

$b = [r_0, a]$ $| \cdot a$

$(a, b) = 0$

$(r_0, b) = 0$

$[r_0 + at, a] = b$

$[r_0, a] = b$

$r_0 = \frac{[a, b]}{|a|^2}$

$r = \frac{[a, b]}{|a|^2} + at$

4) $(r, n_i) = D_i \quad i = \overline{0,1} \qquad [r, a] = b$

$r \perp n_i$

$r \parallel [n_1, n_2]$

$a = [n_1, n_2]$

$b = [r_0, a] = [r_0, [n_1, n_2]] = n_1(r_0, n_2) - n_2(r_0, n_1)$

$D_i = (r_0, n_i) \qquad r_0 = \frac{[a, b]}{|a|^2}$

$\parallel$

$n_1 D_2 - n_2 D_1$

$[r, [n_1, n_2]] = n_1 D_2 - n_2 D_1$

6.11. 1) $M_0(r_0) \qquad (r, n) = D$

из. зад. 5.4(1) $\quad d = |\alpha n| = \left| \frac{D - (r_0, n)}{|n|} \right|$

3) $(r, n) = D_1$

$(r, n) = D_2$

$$D_1 = (r_{0_1}, n) = \alpha_1 |n|^2$$

$$D_2 = (r_{0_2}, n) = \alpha_2 |n|^2$$

$$d = |\alpha_1 - \alpha_2| |n| = \left| \frac{D_1 - D_2}{|n|} \right|$$

4) $M(r_0) \qquad r = r_1 + at$

$\xrightarrow[\text{пар.-пер.}]{} \quad M(r_0 + r_1) \qquad r \to r + r_1 \Rightarrow r = at$

$$d = \left| \frac{[r_0 + r_1, a]}{|a|} \right|$$

8) $r = r_1 + a_1 t$ и $r = r_2 + a_2 t$

$$d = \min \left| \frac{[r_2 + r_1 + a_2 t, a_1]}{|a_1|} \right| =$$

$$= \min \left| \frac{[r_1 + r_2, a_1] + t[a_2, a_1]}{|a_1|} \right|$$

$$\xrightarrow{\min} A = [r_1 + r_2, a_1]^2 + 2t([a_2, a_1], [r_1 + r_2, a_1]) + t^2 [a_2, a_1]^2$$

min

$$t = \frac{-([a_2,a_1],[r_1+r_2,a_1])}{[r_1+r_2,a_1]^2}$$

$\cancel{d = r_1+r_2}$ $\quad at^2+bt+c = \frac{b^2}{4a} - \frac{b^2}{2a} + c = -\frac{b^2}{4a}+c = -\frac{b^2-4ac}{4a}$

$$A_{min} = \cancel{[r_1+r_2,a_1]^2 - \frac{([a_2,a_1],[r_1+r_2,a_1])^2 - [a_2,a_1]^2[r_1+r_2,a_1]^2}{[a_2,a_1]^2}} \quad \frac{\left(\frac{b}{2}\right)^2-ac}{a}$$

$$d = \frac{\sqrt{[a_2,a_1]^2[r_1+r_2,a_1]^2 - ([a_2,a_1],[r_1+r_2,a_1])^2}}{|a_1|\,|[a_1,a_2]|}$$

$$= \frac{|[[a_2,a_1],[r_1+r_2,a_1]]|}{|a_1|\cdot|[a_1,a_2]|}$$

IX. 5.15. $\begin{pmatrix}0\\2\end{pmatrix} = \frac{1}{3}\begin{pmatrix}3+1+x\\-1+4+y\end{pmatrix} = \frac{1}{3}\begin{pmatrix}4+x\\3+y\end{pmatrix}$

$$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-4\\3\end{pmatrix}$$

ур-е: $\quad r = \begin{pmatrix}3\\-1\end{pmatrix} + \begin{pmatrix}-7\\4\end{pmatrix}t, \quad t\in[0,1]$

6.15. $\cancel{A_1}, \cancel{n_i} = \begin{pmatrix} A_i \\ B_i \\ C_i \end{pmatrix}$

$\cancel{(r, n_i) = -D_i}$

$$\begin{cases} x = -\frac{D_1}{A_1} + \alpha_1 t \\ y = \beta_1 t \\ z = \gamma_1 t \end{cases}$$

$$A_1 \alpha_1 + B_1 \beta_1 + C_1 \gamma_1 = 0$$

$$n_i = \begin{pmatrix} A_i \\ B_i \\ C_i \end{pmatrix} = (A_i, B_i, C_i) \begin{pmatrix} i \\ j \\ k \end{pmatrix} = (\tilde{A}_i, \tilde{B}_i, \tilde{C}_i) \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}$$

$$(r, n_i) = -D_i$$

$$r = (x, y, z) \begin{pmatrix} i \\ j \\ k \end{pmatrix} = (\tilde{x}, \tilde{y}, \tilde{z}) \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}$$

$$a = [n_1, n_2] = [\tilde{n}_1, \tilde{n}_2] \begin{vmatrix} [e_2 \times e_3] & [e_3 \times e_1] & [e_1 \times e_2] \\ \tilde{n}_1 & & \\ \tilde{n}_2 & & \end{vmatrix}$$

=

6.15. $\begin{cases} x = x_0 + \alpha t \\ y = y_0 + \beta t \\ z = z_0 + \gamma t \end{cases}$

$$A_i \alpha + B_i \beta + C_i \gamma = 0$$

$$\gamma = -1$$

$$\begin{pmatrix} A_1 & B_1 \\ A_2 & B_2 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \frac{1}{A_1 B_2 - A_2 B_1} \begin{pmatrix} B_2 & -B_1 \\ -A_2 & A_1 \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} =$$

$$= \begin{pmatrix} C_1 B_2 - C_2 B_1 \\ C_2 A_1 - C_1 A_2 \end{pmatrix} \frac{1}{A_1 B_2 - A_2 B_1}$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \sim \begin{pmatrix} C_1 B_2 - C_2 B_1 \\ C_2 A_1 - C_1 A_2 \\ A_2 B_1 - A_1 B_2 \end{pmatrix} \sim \begin{pmatrix} C_2 B_1 - C_1 B_2 \\ C_1 A_2 - C_2 A_1 \\ A_2 B_1 - A_1 B_2 \end{pmatrix} = \begin{pmatrix} \begin{vmatrix} B_1 & B_2 \\ C_1 & C_2 \end{vmatrix} \\ \begin{vmatrix} C_1 & C_2 \\ A_1 & A_2 \end{vmatrix} \\ \begin{vmatrix} B_1 & B_2 \\ A_1 & A_2 \end{vmatrix} \end{pmatrix}$$

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} = (e_1\ e_2\ e_3) \begin{pmatrix} x \\ y \\ z \end{pmatrix} = (e_1\ e_2\ e_3) \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + (e_1\ e_2\ e_3) \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$

$$= \begin{pmatrix} \tilde{x}_0 \\ \tilde{y}_0 \\ \tilde{z}_0 \end{pmatrix} + e_1 \begin{vmatrix} B_1 & B_2 \\ C_1 & C_2 \end{vmatrix} + e_2 \begin{vmatrix} C_1 & C_2 \\ A_1 & A_2 \end{vmatrix} + e_3 \begin{vmatrix} B_1 & B_2 \\ A_1 & A_2 \end{vmatrix}$$

чтд!

X. 5.30. $2x-3y+4=0$

$\rho(A, l)=2$ $l: \ 3y=4x$

$A(x_0, y_0)$ - ?

$$\begin{cases} \frac{|4x_0-3y_0|}{5}=2 \\ 2x_0-3y_0+4=0 \end{cases}$$

$|2x_0-4|=10$

$|x_0-2|=5$

$x_0=7$ $x_0=-3$

$y_0=\frac{2x_0+4}{3}=6$ $y_0=\frac{2\cdot(-3)+4}{3}=-\frac{2}{3}$

Отв.: $A(7; 6)$

$A(-3, -\frac{2}{3})$

5.35. $3x - y + 5 = 0$ сим. от. $x + y - 1 = 0$

$$\frac{3}{\sqrt{10}}x - \frac{y}{\sqrt{10}} + \frac{5}{\sqrt{10}} = 0 \qquad \cancel{\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0}$$

$$\alpha x + \sqrt{1-\alpha^2}\, y + \beta = 0$$

$$\begin{pmatrix} \frac{3}{\sqrt{10}} + \alpha \\ \sqrt{1-\alpha^2} - \frac{1}{\sqrt{10}} \\ \beta + \frac{5}{\sqrt{10}} \end{pmatrix} \sim \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$\left(\lambda - \frac{3}{\sqrt{10}}\right)^2 + \left(\lambda + \frac{1}{\sqrt{10}}\right)^2 = 1$$

$$2\lambda^2 - \frac{4}{\sqrt{10}}\lambda = 0 \qquad \lambda = \frac{2}{\sqrt{10}}$$

$$\alpha = -\frac{1}{\sqrt{10}}$$

$$\beta = \frac{-7}{\sqrt{10}}$$

$$-x + 3y - 7 = 0$$

$$x - 3y + 7 = 0$$

6.49. 2) сост. ур-е пл-ти $\perp$ $x+3y-z+2=0$

и прох. через пр.: $\begin{cases} 2x-y+z=0 \\ x+2y+z-3=0 \end{cases}$

$$n = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$$

$$\begin{cases} 2\alpha - \beta + 1 = 0 \\ \alpha + 2\beta + 1 = 0 \end{cases}$$

$$5\alpha + 3 = 0 \qquad \alpha = -\frac{3}{5}$$

$$\beta = -\frac{1}{5}$$

$$a = \begin{pmatrix} 3 \\ 1 \\ -5 \end{pmatrix}$$

$$r_0 = \begin{pmatrix} 3/5 \\ 6/5 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} t + \begin{pmatrix} 3 \\ 1 \\ -5 \end{pmatrix} v$$

$$n' = \begin{vmatrix} i & j & k \\ 1 & 3 & -1 \\ 3 & 1 & -5 \end{vmatrix} = \begin{pmatrix} -14 \\ +2 \\ -8 \end{pmatrix} \to \begin{pmatrix} 7 \\ -1 \\ 4 \end{pmatrix}$$

$$D = -\left(7 \cdot \frac{3}{5} - 6/5\right) = -3$$

$$\boxed{7x - y + 4z - 3 = 0}$$

6.80. $x - z - 5 = 0 \longrightarrow \frac{x}{\sqrt{2}} - \frac{z}{\sqrt{2}} - \frac{5}{\sqrt{2}} = 0$

$3x + 5y + 4z = 0 \longrightarrow \frac{3x}{5\sqrt{2}} + \frac{y}{\sqrt{2}} + \frac{4z}{5\sqrt{2}} = 0$

$\frac{8}{5\sqrt{2}}x + \frac{y}{\sqrt{2}} - \frac{1}{5\sqrt{2}}z - \frac{5}{\sqrt{2}} = 0$

$\downarrow$

$\boxed{8x + 5y - z - 25 = 0}$