$$S=\sum_{n=2}^{\infty}\frac{n^4+n^3+n^2-n+1}{n^6-1}$$

$$\frac{n^4+n^3+n^2-n+1}{n^6-1}=\frac{1/2}{n-1}+\frac{-1/2}{n+1}+\frac{A+A'n}{n^2-n+1}+\frac{B+B'n}{n^2+n+1}=$$

$$=\frac{1}{n^2-1}+\frac{A+A'n}{n^2-n+1}+\frac{B+B'n}{n^2+n+1}$$

$$n^4+n^3+n^2-n+1=(n^2-n+1)(n^2+n+1)+(A+A'n)(n^2-1)(n^2+n+1)+$$

$$+(B+B'n)(n^2-1)(n^2-n+1)$$

Приравняем коэффициенты при степенях n:

$5:\ A'+B'=0$

$4:\ A+B+1=1$

$3:\ A-B=1$

$2:\ 1-A'+B'=1$

$1:\ -A+B-A'-B'=-1$

$0:\ 1-A-B=1$

$$\begin{cases}A'=B'=0\\ A=\dfrac{1}{2}\\ B=-\dfrac{1}{2}\end{cases}$$

$$S=\frac{1}{2}\sum_{n=2}^{\infty}\left(\frac{1}{n-1}-\frac{1}{n+1}\right)+\frac{1}{2}\sum_{n=2}^{\infty}\left(\frac{1}{n^2-n+1}-\frac{1}{n^2+n+1}\right)=$$

$$=\frac{1}{2}\sum_{n=2}^{\infty}\left(\frac{1}{n-1}-\frac{1}{n+1}\right)+\frac{1}{2}\sum_{n=2}^{\infty}\left(\frac{1}{(n-\frac{1}{2})^2+\frac{3}{4}}-\frac{1}{(n+\frac{1}{2})^2+\frac{3}{4}}\right)=$$

$$=\frac{1}{2}\left(\left(1+\frac{1}{2}\right)+\frac{1}{\frac{9}{4}+\frac{3}{4}}\right)=\frac{1}{2}\left(\frac{3}{2}+\frac{1}{3}\right)=\frac{11}{12}$$