$$S=\sum_{n=0}^{\infty}\frac{1}{(2n+1)!!}=\sum_{n=0}^{\infty}\frac{1}{2^{n+1}\cdot\frac{1}{2}\cdot\ldots\cdot(\frac{1}{2}+n)}=\sum_{n=0}^{\infty}\frac{2^{-n}}{\left(\frac{3}{2}\right)_n}=$$

$$=\sum_{n=0}^{\infty}\frac{\left(1\right)_n}{\left(\frac{3}{2}\right)_n\left(1\right)_n}\left(\frac{1}{2}\right)^n={}_1F_1\left(1;\frac{3}{2};\frac{1}{2}\right)$$

$${}_1F_1(a;b;z)=\frac{\Gamma(b)}{\Gamma(a)\Gamma(b-a)}\int\limits_0^1 t^{a-1}(1-t)^{-a+b-1}e^{zt}\,dt$$

$${}_1F_1\left(1;\frac{3}{2};\frac{1}{2}\right)=\frac{\Gamma(\frac{3}{2})}{\Gamma(\frac{1}{2})}\int\limits_0^1(1-t)^{-1/2}e^{t/2}\,dt=$$

$$=\frac{1}{2}\sqrt{e}\int\limits_0^1(1-t)^{-1/2}e^{-\frac{1}{2}(1-t)}\,dt=\left[\begin{array}{c}(1-t)^{\frac{1}{2}}=u\sqrt{2}\\ t=1-2u^2\\ dt=-4u\,du\end{array}\right]=$$

$$=\frac{1}{2}\sqrt{e}\int\limits_{\frac{1}{\sqrt{2}}}^0 e^{-u^2}\frac{1}{u\sqrt{2}}(-4u\,du)=\sqrt{2e}\int\limits_{\frac{1}{\sqrt{2}}}^0 e^{-u^2}\,du=\sqrt{2e}\,\frac{\sqrt{\pi}}{2}\,erf\left(\frac{1}{\sqrt{2}}\right)$$

$$S=\sqrt{\frac{e\pi}{2}}\,erf\left(\frac{1}{\sqrt{2}}\right)$$