

$$\begin{aligned}
S &= \sum_{n=1}^{\infty} \frac{1}{n(4n^2 - 1)} = \sum_{n=1}^{\infty} -\frac{1}{n} + \frac{1}{2n-1} + \frac{1}{2n+1} = -1 + \sum_{n=1}^{\infty} \frac{2}{2n-1} - \frac{1}{n} = \\
&= -1 + 2 \sum_{n=1}^{\infty} \frac{1}{2n-1} - \frac{1}{2n} = 1 + 2 \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} = -1 + 2 \ln 2
\end{aligned}$$