$$S = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sin n}{n} = \sum_{n=1}^{\infty} (-1)^{n+1} \int_0^1 \cos(xn)\, dx = -\frac{1}{2} \int_0^1 \sum_{n=1}^{\infty} (-e^{ix})^n + (-e^{-ix})^n \, dx$$

$$z = -e^{ix}$$

$$S = -\frac{1}{2} \int_0^1 \frac{z}{1-z} + \frac{\frac{1}{z}}{1-\frac{1}{z}}\, dx = -\frac{1}{2} \int_0^1 \frac{z}{1-z} - \frac{1}{1-z}\, dx = \frac{1}{2}$$