1. $$\begin{cases} \dot{x} = \sigma(y-x) \\ \dot{y} = rx - y - xz \\ \dot{z} = -bz + xy \end{cases} \qquad \sigma, r, b > 0$$

ПР: $\dot{x} = 0 \Rightarrow x = y$

$\dot{y} = 0 \Rightarrow x = 0$ или $z = r-1$

$\dot{z} = 0 \Rightarrow z = \frac{x^2}{b}$

1) $r \leq 1 \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ 1 ПР.

2) $r > 1 \Rightarrow$ 3 ПР.: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

и $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\sqrt{b(r-1)} \\ -\sqrt{b(r-1)} \\ r-1 \end{pmatrix}$

и $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \sqrt{b(r-1)} \\ \sqrt{b(r-1)} \\ r-1 \end{pmatrix}$

$r = 1 \Rightarrow$ 1ПР: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$$\frac{d}{dt}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\sigma & \sigma & 0 \\ r & -1 & 0 \\ 0 & 0 & -b \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

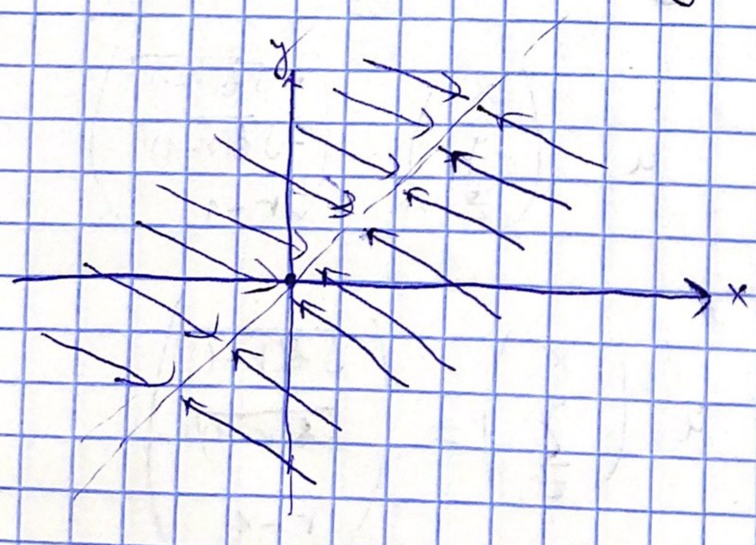
$\dot{\bar{r}}$ $\qquad$ J

$$\dot{\bar{r}} = J\,\bar{r}$$

$$\det J = -\sigma b + br\sigma = \sigma b(r-1) \underset{r=1}{=} 0$$

$$\dot{x} = \sigma(y - x) = -j\sigma + o(|\bar{r}|)$$

$$\dot{z} = -bz + o(|\bar{r}|)$$



$$\begin{cases} \bar f = \begin{pmatrix} \sigma(y-x) \\ rx - y - xz \\ -bz + xy \end{pmatrix} = 0 \\ \operatorname{div} \bar f = 0 \end{cases}$$

$$\begin{cases} \bar f = 0 \\ -\sigma - 1 - b = 0 \end{cases}$$

$$\begin{cases} x = y = z = 0 \\ \sigma + 1 + b = 0 \end{cases} \quad \text{или} \quad \begin{cases} x = y = \pm\sqrt{b(r-1)} \\ z = r - 1 \\ \sigma + 1 + b = 0 \end{cases}$$

$$r = \sigma \frac{\sigma+b+3}{\sigma-b-1} = \sigma \frac{\sigma+b+1+2}{-\sigma-1-b+2\sigma} = \sigma \frac{0+2}{0+2\sigma} = 1$$

$\Rightarrow$ при $r = 1$ и $r = \sigma \frac{\sigma+b+3}{\sigma-b-1}$ происх. бифур.

$$\det(J - \lambda \cdot \mathbb{1}) = 0$$

$$\det \begin{pmatrix} -\sigma-\lambda & \sigma & 0 \\ r & -1-\lambda & 0 \\ 0 & 0 & -b-\lambda \end{pmatrix} = 0$$

$$-(\lambda+1)(\lambda+\sigma)(\lambda+b) + (b+\lambda) r \sigma = 0$$

$$(b+\lambda)\left(\lambda^2 + (\sigma+1)\lambda + \sigma - r\sigma\right) = 0$$

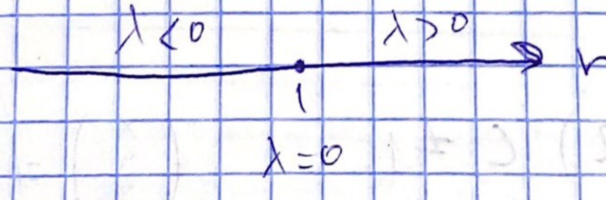
$$\lambda = -b$$

$$\lambda = -\frac{\sigma+1}{2} \pm \frac{1}{2}\sqrt{\sigma^2+2\sigma+1-4\sigma+4r\sigma} =$$

$$= -\frac{\sigma+1}{2} \pm \frac{1}{2}\sqrt{(\sigma-1)^2+4r\sigma}$$

$$\lambda_+ = 0 \Rightarrow \cancel{\sigma^2+2\sigma+1}\ 4r\sigma - 4\sigma = 0$$

$$r = 1$$



2. $\begin{cases} \dot{x} = -y - z \\ \dot{y} = x + a y \\ \dot{z} = b + z(x - c) \end{cases}$

$a = 1/4 \quad b = 1 \quad , \quad c > 0$

$\begin{cases} y + z = 0 \\ x + \frac{1}{4} y = 0 \\ 1 + z(x - c) = 0 \end{cases}$

$\begin{cases} y = -4x \\ z = 4x \\ 4x^2 - 4xc + 1 = 0 \end{cases}$

$$x^* = \frac{1}{4}\left(2c \pm \sqrt{4c^2 - 4}\right) =$$

$$= \frac{1}{2}c \pm \frac{1}{2}\sqrt{c^2 - 1}$$

ПР : 1) $c = 1$ $\quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1/2 \\ -2 \\ 2 \end{pmatrix}$

2) $c \neq 1$ $\quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x^*_- \\ -4x^*_- \\ 4x^*_- \end{pmatrix}$

и $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x^*_+ \\ -4x^*_+ \\ 4x^*_+ \end{pmatrix}$

$$\dot{r} = J r$$

$$J = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 1/4 & 0 \\ 2 & 0 & x-c \end{pmatrix}\Bigg|_{\text{в н.р. при } c=1} = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 1/4 & 0 \\ 2 & 0 & \frac{1}{2}-c \end{pmatrix}$$

$$\det(J - \lambda \cdot \mathbb{1}) = -\lambda\left(\frac{1}{4}-\lambda\right)\left(\frac{1}{2}-c-\lambda\right) + \frac{1}{2} + \frac{1}{2} - c = 0$$

$$1 - c - \lambda\left(\lambda - \frac{1}{4}\right)\left(\lambda - \frac{1}{2} + c\right) = 0$$

$$c = 1 \Rightarrow \lambda\left(\lambda - \frac{1}{4}\right)\left(\lambda + \frac{1}{2}\right) = 0$$

$$\lambda \in \left\{0, \frac{1}{4}, -\frac{1}{2}\right\}$$

$$\vee\ 0$$

$$\downarrow$$

бифуркация