$$S=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{m+n+mn}{2^m(2^m+2^n)}=\frac{1}{2}\left(\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{m+n+mn}{2^m(2^m+2^n)}+\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{m+n+mn}{2^n(2^m+2^n)}\right)=$$

$$=\frac{1}{2}\left(\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{m+n+mn}{2^m+2^n}(\frac{1}{2^n}+\frac{1}{2^m}\right)=\frac{1}{2}\left(\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{m+n+mn}{2^{m+n}}\right)=$$

$$=\frac{1}{2}\left(\sum_{n=0}^{\infty}\frac{1}{2^n}\sum_{m=0}^{\infty}\frac{m}{2^m}+\sum_{n=0}^{\infty}\frac{n}{2^n}\sum_{m=0}^{\infty}\frac{1}{2^m}+\sum_{n=0}^{\infty}\frac{n}{2^n}\sum_{m=0}^{\infty}\frac{m}{2^m}\right)$$

$$\sum_{n=0}^{\infty}\frac{1}{2^n}=2$$

$$\sum_{n=0}^{\infty}\frac{n}{2^n}=\sum_{n=0}^{\infty}n\Delta(-2\cdot 2^{-n})=n\cdot(-2\cdot 2^{-n})\Big|_0^{\infty}-\sum_{n=1}^{\infty}(-2\cdot 2^{-n})\Delta n=$$

$$=2\sum_{n=1}^{\infty}(2^{-n})=2$$

$$S=\frac{1}{2}(2\cdot 2+2\cdot 2+2\cdot 2)=6$$