$$S = 5\left(\frac{1}{2}\right)^3\left(1+\frac{1}{2}\right) - 9\left(\frac{1\cdot 3}{2\cdot 4}\right)^3\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right) + ... = \sum_{n=1}^{\infty}(4n+1)\left[\frac{(2n-1)!!}{(2n)!!}\right]^3 H_{2n}(-1)^{n-1}$$

$$4n+1 = \frac{(5/4)_n}{(1/4)_n}$$

$$H_{2n} = \frac{1}{2}\frac{(1)_n}{(1/2)_n}\partial_c\Big|_{c=\frac{1}{2}}\frac{(c)_n}{(\frac{3}{2}-c)_n}$$

$$S = -\frac{1}{2}\partial_c\Big|_{c=\frac{1}{2}}\sum_{n=1}^{\infty}\frac{(5/4)_n}{(1/4)_n}\left[\frac{(1/2)_n}{(1)_n}\right]^3\frac{(1)_n(c)_n}{(1/2)_n(\frac{3}{2}-c)_n}(-1)^n$$

$$S = -\frac{1}{2}\partial_c\Big|_{c=\frac{1}{2}}{}_4F_3\left(\begin{array}{c}\frac{5}{4},\frac{1}{2},\frac{1}{2},c\\\frac{1}{4},1,\frac{3}{2}-c\end{array};-1\right)$$

$${}_4F_3\left(\begin{array}{c}a,1+\frac{a}{2},b,c\\\frac{a}{2},a-b+1,a-c+1\end{array};-1\right) = \frac{\Gamma(a-b+1)\Gamma(a-c+1)}{\Gamma(1+a)\Gamma(a-b-c+1)}$$

$$a = b = \frac{1}{2}$$

$$S = -\frac{1}{2}{}_4F_3\left(\begin{array}{c}\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{1}{2}\\\frac{1}{4},1,1\end{array};-1\right)\left(-\psi(1)+\psi(1/2)\right) = \frac{2}{\pi}\ln 2$$