№1.

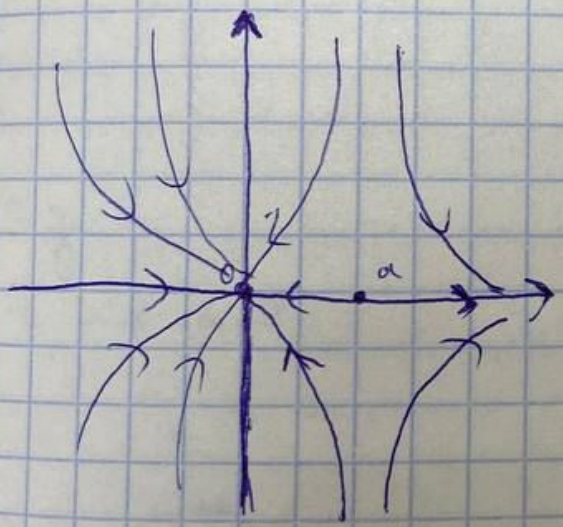
1) $\begin{cases} \dot{x} = x(a-x) \\ \dot{y} = -y \end{cases}$

$\dot{r} = \begin{pmatrix} a & 0 \\ 0 & -1 \end{pmatrix} r$ (в. т. $r^* = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$)

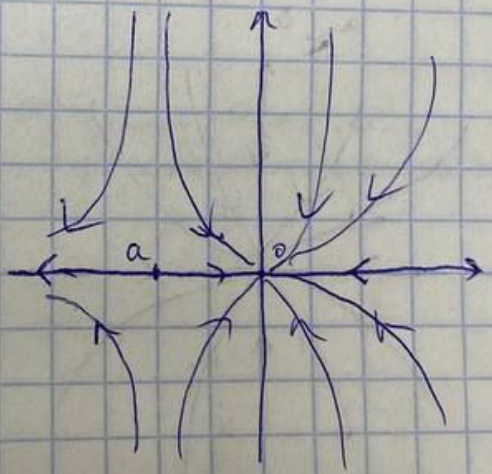
$\lambda_1 = a$

$\lambda_2 = -1$

$a > 0$



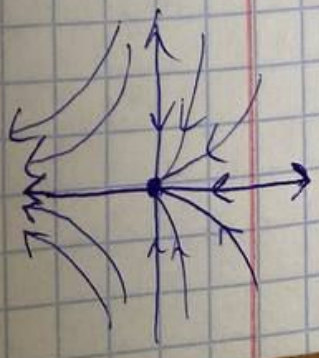
$a < 0$



$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ – уст. узел.

$\begin{pmatrix} a \\ 0 \end{pmatrix}$ – седло

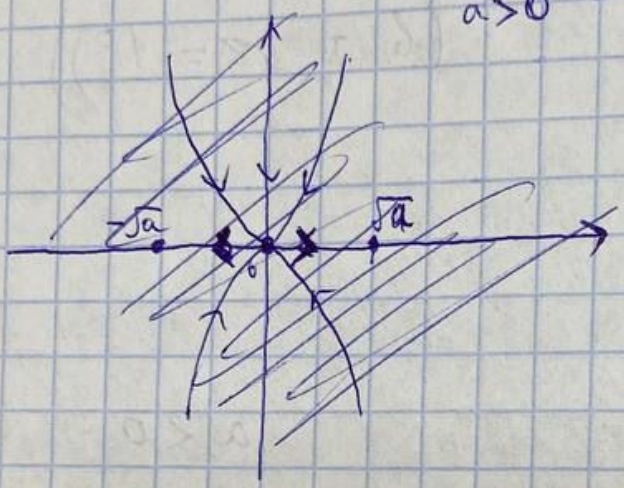
$a = 0$

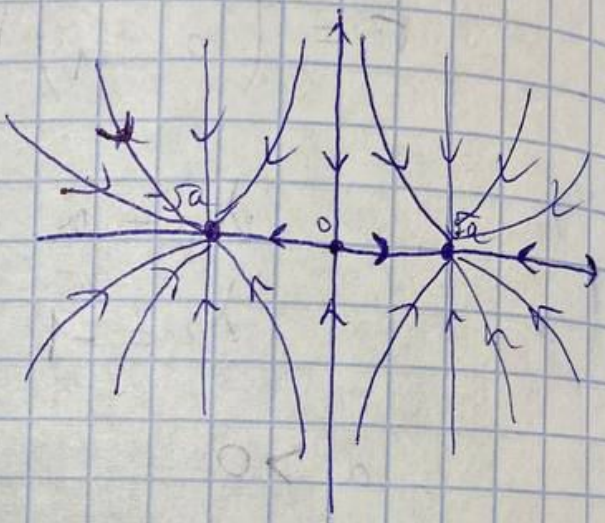


2) $\begin{cases} \dot{x} = x(a - x^2) \\ \dot{y} = -y \end{cases}$

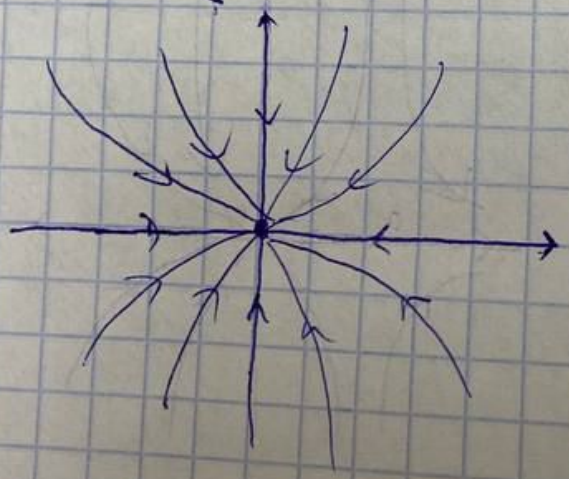
$\dot{x} = 0 \Rightarrow x^* = \{0, \sqrt{a}, -\sqrt{a}\}$

$a > 0$





$a \leq 0$



2.

1) $\ddot{x} + x + ax^3 = 0$

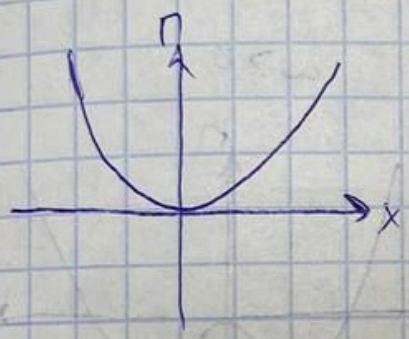
$x + ax^3 = \frac{d\Pi}{dx}$

$\Pi = \frac{1}{2}x^2 + \frac{a}{4}x^4 = \frac{ax^2}{4}\left(x^2 + \frac{2}{a}\right)$

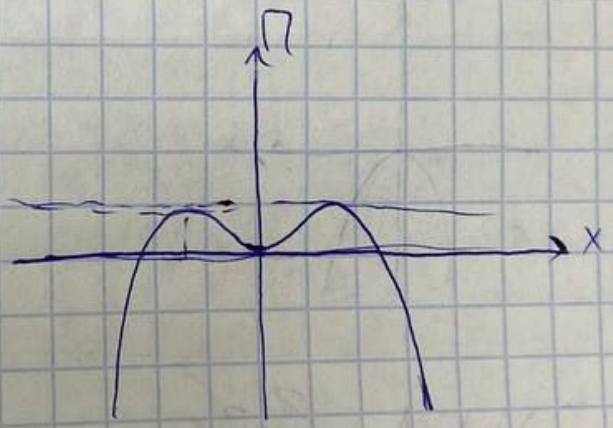
(с точностью до const)

$\Pi = \frac{a}{4}\left(x^2 + \frac{1}{a}\right)^2 - \frac{1}{4a}$

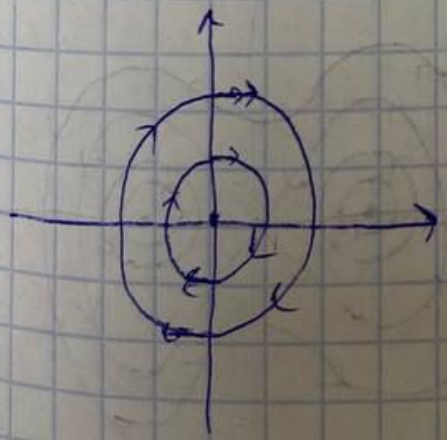
$a \geq 0$

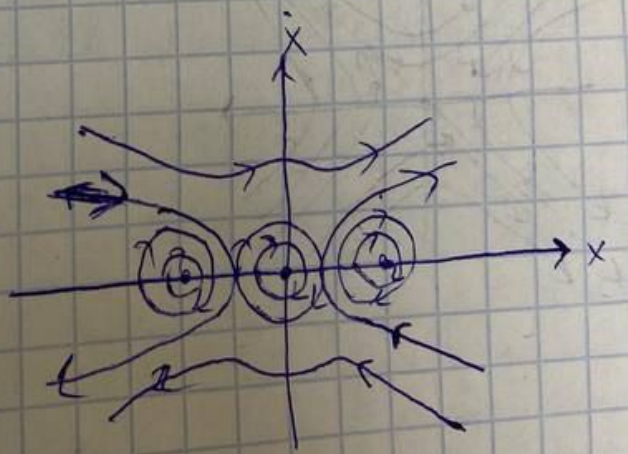


$a < 0$



$\begin{cases} \dot{x} = y \\ \dot{y} = -x - ax^3 \end{cases}$





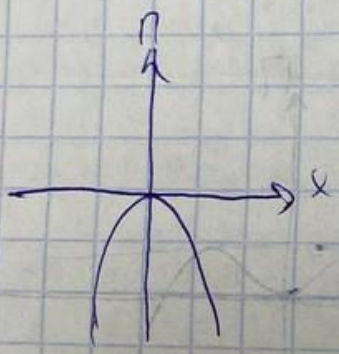
2) $\ddot{x} - x + ax^3 = 0$

- $\begin{cases} \dot{x} = y \\ \dot{y} = x - ax^3 = x\,a\left(\frac{1}{a} - x^2\right) \end{cases}$

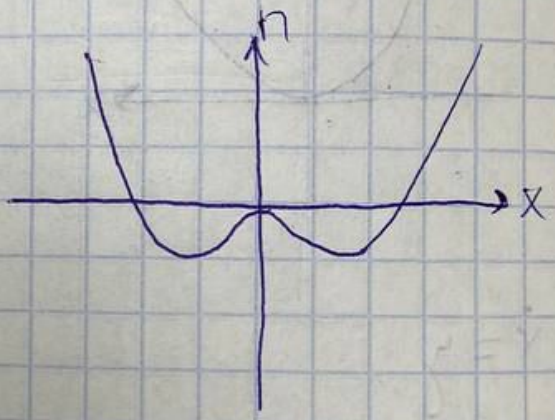
$$\Pi = \frac{a}{4}x^4 - \frac{1}{2}x^2 = \frac{a}{4}x^2\left(x^2 - \frac{2}{a}\right) =$$

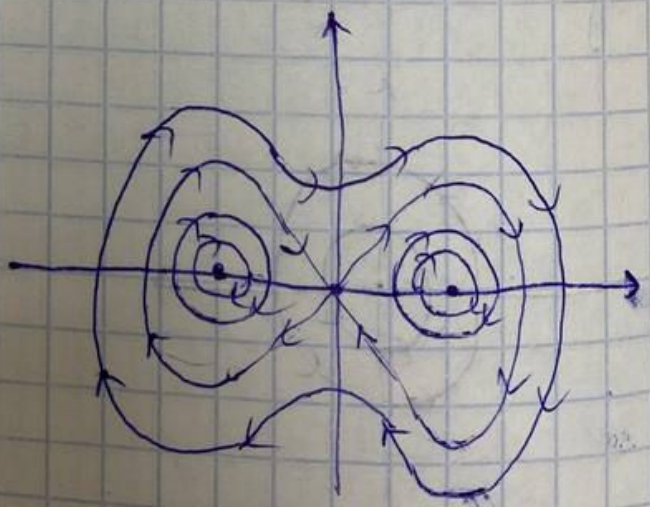
$$= \frac{a}{4}\left(x^2 - \frac{1}{a}\right)^2 - \frac{1}{4a}$$

$a \leq 0$



$a > 0$



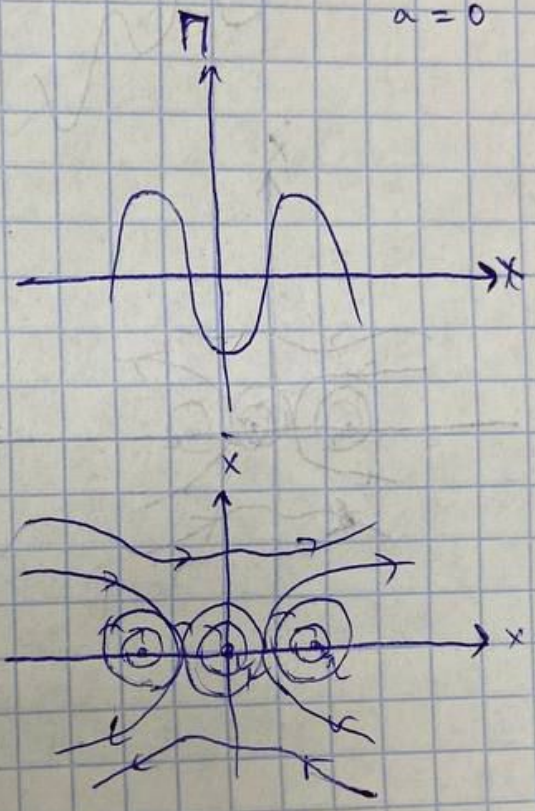


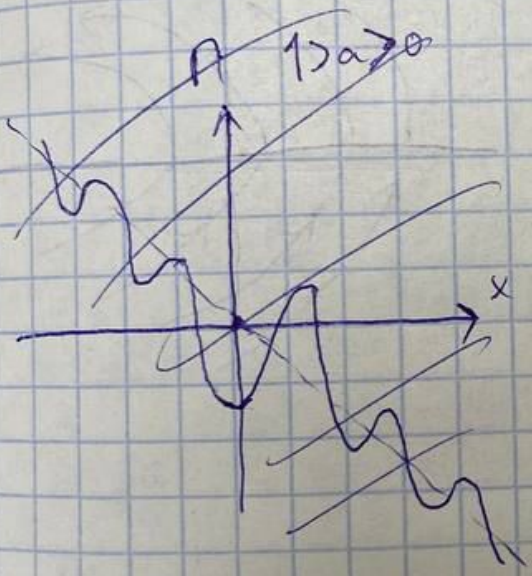
3) $\ddot{x} + \sin x = a$

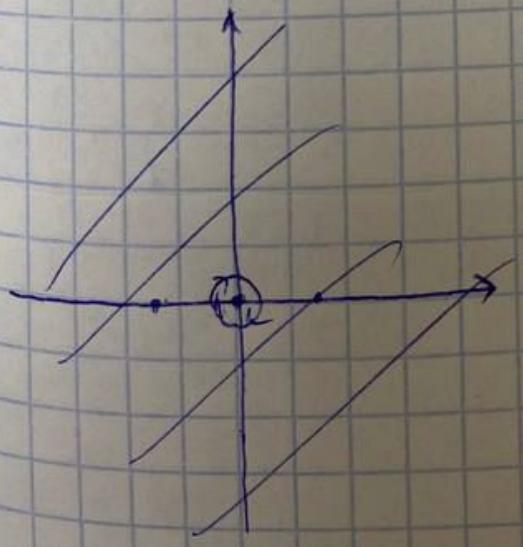
$\frac{d\Pi}{dx} = \sin x - a$

$\Pi(x) = -\cos x - ax$

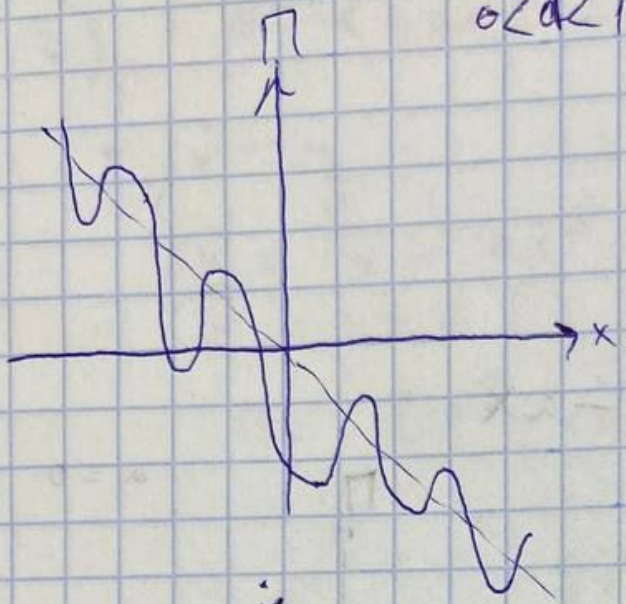
$\begin{cases} \dot{x} = y \\ \dot{y} = a - \sin x \end{cases}$

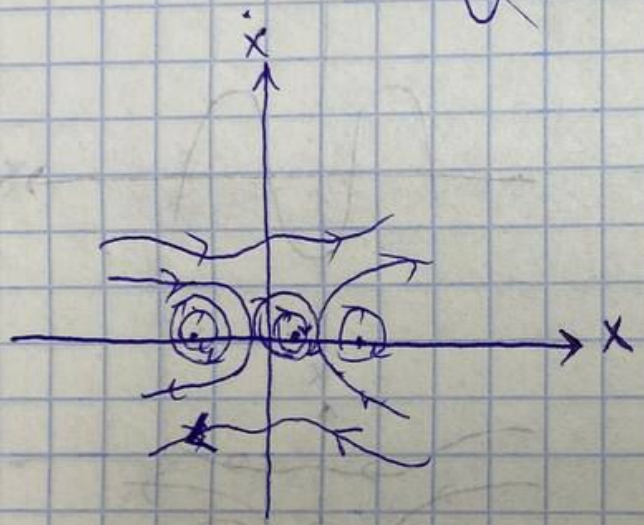




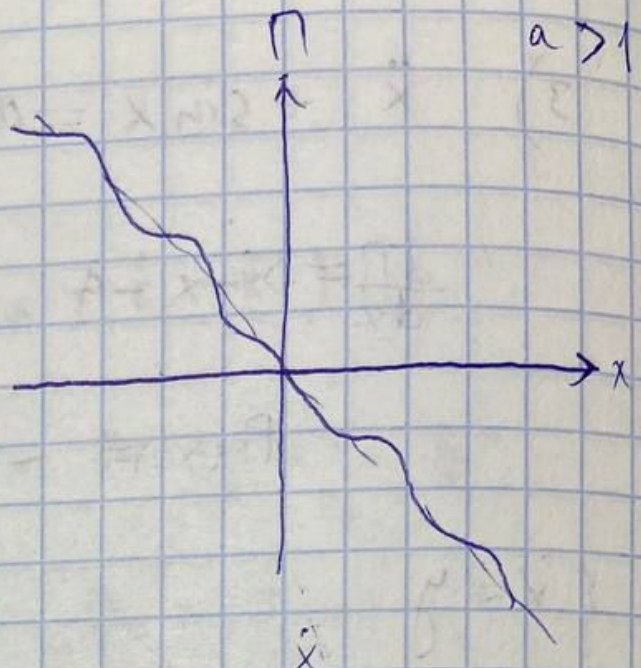


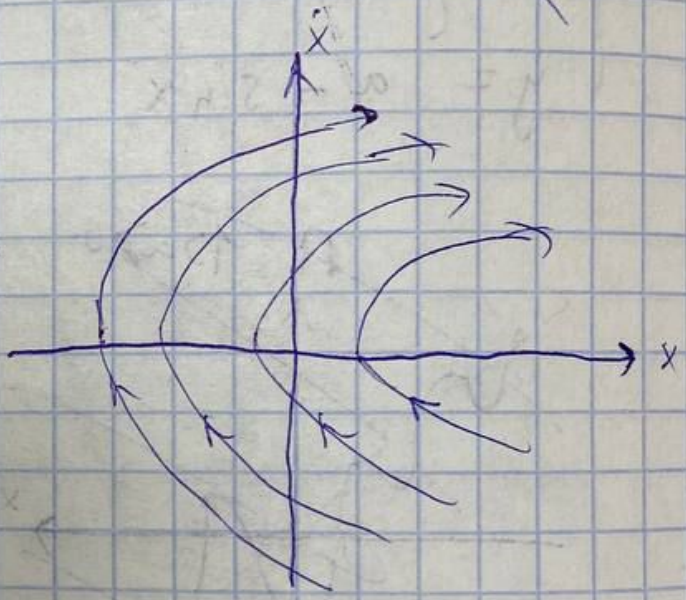
$0 < a < 1$





$a > 1$

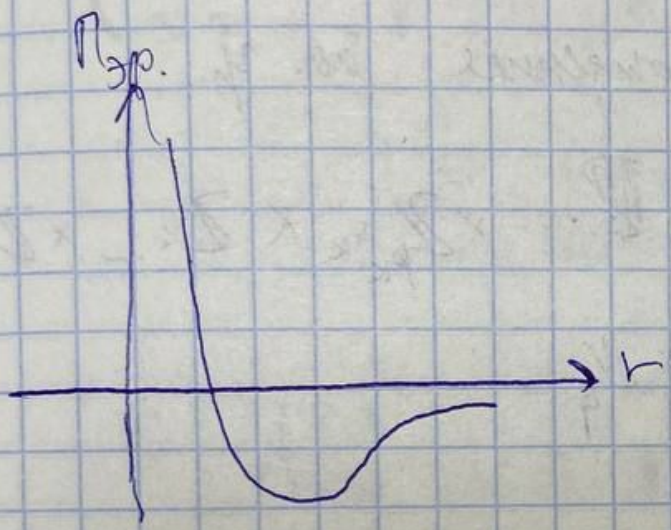




№ 3.

$$h = \frac{m(\dot{r}^2 + r^2\dot{\varphi}^2)}{2} - \frac{1}{r} = \text{const}$$

$$r^2\dot{\varphi} = C$$



№4.

$$\begin{cases} \dot{x} = \alpha x - y - x(x^2+y^2) \\ \dot{y} = x + \alpha y - y(x^2+y^2) \end{cases}$$

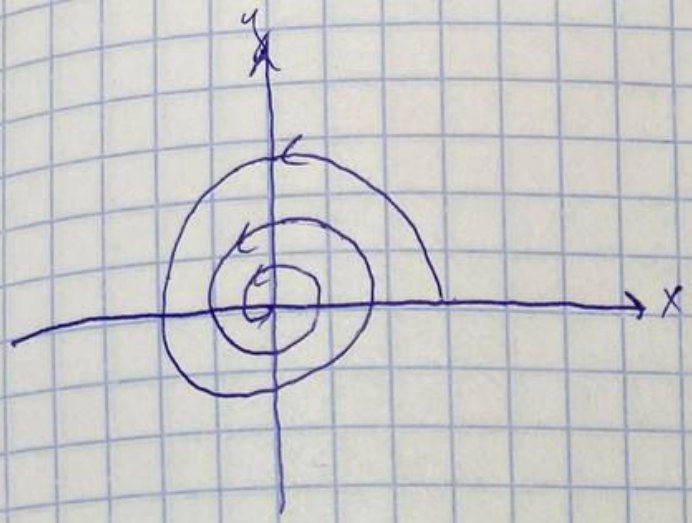
$\alpha \in \mathbb{R}$

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \\ \dot{x} = \dot{r}\cos\varphi - r\sin\varphi\,\dot{\varphi} \\ \dot{y} = \dot{r}\sin\varphi + r\cos\varphi\,\dot{\varphi} \end{cases}$$

$$\dot{r} = \dot{x}\cos\varphi + \dot{y}\sin\varphi = \alpha r\cos^2\varphi + \cancel{r\cos\varphi\sin\varphi} + \alpha r\sin^2\varphi - \cancel{r\sin\varphi\cos\varphi} - r^3 = \alpha r - r^3$$

$$r\dot{\varphi} = \dot{y}\cos\varphi - \dot{x}\sin\varphi = r\cos^2\varphi + \cancel{\alpha r\sin\varphi\cos\varphi} - \cancel{r^3\sin\varphi\cos\varphi} - \cancel{\alpha r\cos\varphi\sin\varphi} + r\sin^2\varphi + \cancel{r^3\cos\varphi\sin\varphi} = r$$

$$\begin{cases} \dot{\varphi} = 1 \\ \dot{r} = \alpha r - r^3 \end{cases} \qquad \begin{cases} \dot{\varphi} = 1 \\ \dot{r} = -r^3 \end{cases} \quad (\alpha = 0)$$



$$\frac{dr}{d\varphi} = -r^3 \qquad \frac{-dr}{r^3} = d\varphi$$

$$\frac{1}{2}\left(\frac{1}{r^2} - \frac{1}{r_0^2}\right) = \varphi - \varphi_0$$

$$\int_0^{2\pi} \frac{dr}{d\varphi} d\varphi = -r_0^3 \int_0^{2\pi} d\varphi = -2\pi r_0^3 = 0$$

$$\Downarrow$$

$$r_0 = 0 = \sqrt{\alpha} = \sqrt{0}.$$