$$S=\sum_{n=1}^{\infty}\frac{1}{2^n(1+2^{2^{-n}})}=\left[\begin{array}{c} u=2^{2^{-n}} \\ \frac{1}{1+u}=\frac{2}{1-u^2}-\frac{1}{1-u}\end{array}\right]=$$

$$=\frac{1}{2(1+\sqrt{2})}+\sum_{n=2}^{\infty}\frac{2}{2^n(1+(2^{2^{-n}})^2)}-\frac{1}{2^n(1+2^{2^{-n}})}=$$

$$=\frac{1}{2(1+\sqrt{2})}+\sum_{n=2}^{\infty}\frac{1}{2^{n-1}(1+2^{2^{-(n-1)}})}-\frac{1}{2^n(1+2^{2^{-n}})}$$

$$f(n)=\frac{1}{2^n(1+2^{2^{-n}})}$$

$$S=\frac{1}{2(1+\sqrt{2})}+f(1)-\lim_{n\to\infty}f(n)=$$

$$=\frac{1}{2(1+\sqrt{2})}+\frac{1}{2(1-\sqrt{2})}-\lim_{n\to\infty}\frac{2^{-n}}{1+2^{2^{-n}}}=-1-L$$

$$L=\lim_{n\to\infty}\frac{2^{-n}}{1+2^{2^{-n}}}=\lim_{n\to\infty}\frac{-\ln 2\cdot 2^{-n}}{\ln^2 2\cdot 2^{-n}\cdot 2^{2^{-n}}}=-\frac{1}{\ln 2}$$

$$S=\frac{1}{\ln 2}-1$$