

Rapid and simultaneous estimation of fault slip and heterogeneous lithospheric viscosity from postseismic deformation

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SUMMARY

Key words:

1 INTRODUCTION

Geodetic observations of surface deformation in the months to years following an earthquake are often attributed to afterslip (e.g. Marone et al. 1991), viscoelastic relaxation in the lithosphere (e.g. Nur & Mavko 1974), and/or poroelastic-relaxation (e.g. Peltzer et al. 1998; Jónsson et al. 2003). If post-seismic deformation can be entirely described by afterslip, then one could easily constrain the spatial distribution of slip with a linear least squares inversion (e.g. Harris & Segall 1987; Bürgmann et al. 2002; Freed 2007), which could then provide insight into the frictional properties of faults (e.g. Hsu et al. 2006; Barbot et al. 2009). However, postseismic deformation following large ($M_w \geq 7$) earthquakes is often attributed to viscoelastic relaxation in the lithosphere (e.g. Hetland & Hager 2003; Pollitz 2003, 2005) or a combination of both afterslip and viscoelastic relaxation (e.g. Freed et al. 2006a; Hearn et al. 2009; Johnson et al. 2009; Rollins et al. 2015). In such cases, postseismic deformation can be used to constrain the viscous properties of the lithosphere, although this is a more difficult task than constraining a slip distribution. Not only are there potentially competing deformation mechanism which must be discerned, finding the viscosity distribution of the lithosphere from postseismic deformation is a computationally expensive nonlinear inverse problem. Typically, the estimation of viscosities is approached with a forward modeling, grid search method. These forward

modeling techniques require the number of unknown parameters being estimated to be small, meaning that significant and potentially inappropriate modeling assumptions must be made (Riva & Govers 2008; Hines & Hetland 2013).

In this paper we propose a relatively fast method to kinematically invert coseismic and postseismic deformation to simultaneously estimate a time-dependent distribution of fault slip and an arbitrarily discretized viscosity structure of the lithosphere. Our method is based on an approximation which linearizes the rate of early postseismic deformation with respect to the viscosity of the lithosphere. We demonstrate the efficacy and limitations of our method through a synthetic test.

2 LINEARIZING EARLY POSTSEISMIC DEFORMATION

We assume that the lithosphere can be approximated as a Maxwell viscoelastic material on the timescales of postseismic deformation, where shear stress, σ , and strain, ε , are related by

$$\frac{\partial \varepsilon}{\partial t} = \frac{\sigma}{2\eta} + \frac{1}{2\mu} \frac{\partial \sigma}{\partial t}. \quad (1)$$

We use η and μ to represent viscosity and shear modulus, respectively. This constitutive relationship implies that a sudden strain in the lithosphere from an earthquake will instantaneously propagate stresses through the lithosphere elastically (assuming the lithosphere is undergoing quasi-static deformation). Viscoelastic creep will also initiate immediately after the earthquake, where the initial viscous strain rate in each parcel of the lithosphere will be proportional to the fluidity ($1/\eta$) in that parcel, and independent of the fluidity elsewhere because the initial stresses are only controlled by the elastic properties of the lithosphere. Stresses from the earthquake will dissipate over time through viscoelastic relaxation and while the stress changes from viscoelastic relaxation are small compared to the initial elastic stresses, each parcel will continue to creep at a rate that is approximately proportional to its fluidity. In this early postseismic period, the surface deformation from creep in each parcel will have an amplitude that is also proportional to the fluidity in that parcel and independent of the fluidity elsewhere. As we will show, the early surface expression of creep in the entire lithosphere is therefore a sum of the surface deformation from each parcel and is linear with respect to lithospheric fluidity. We demonstrate this property of early postseismic surface deformation in this section using simple infinite length, strike-slip earthquake models, where the lithosphere is approximated as a layered halfspace. In section 2.2 we consider a finite fault model with an arbitrarily discretized lithospheric viscosity structure.

2.1 Two-dimensional earthquake models

The easiest way to demonstrate how postseismic deformation can be linearized with respect to lithospheric viscosity is with a two-dimensional earthquake model consisting of a long, vertical, surface rupturing, strike-slip fault that is embedded in a viscoelastic horizontal layer overlying a viscoelastic halfspace. We make use of the correspondence principle of viscoelasticity (e.g. Flügge 1975), which states that the Laplace transform of deformation in a viscoelastic body has the same form as the Laplace transform of deformation in an elastic body with the same geometry and subjected to the same boundary conditions. The solution for displacements following an earthquake in a viscoelastic lithosphere can then be readily found provided that the corresponding elastic solution is known (e.g. Nur & Mavko 1974; Savage & Prescott 1978; Hetland & Hager 2005). One only needs to replace the shear modulus in the Laplace transform of the elastic solution with the effective viscoelastic shear modulus and then compute the inverse Laplace transform.

2.1.1 Two layered model

From the solution of Rybicki (1971), surface displacements, $u_e(x, t)$, resulting from slip on a fault in an elastic surface layer overlying a semi-infinite elastic substrate are

$$u_e(x, t) = b(t) \left(\frac{1}{2} W(0) + \sum_{n=1}^{\infty} \Gamma^n W(n) \right), \quad (2)$$

where

$$W(n) = \frac{1}{\pi} \left(\tan^{-1} \left(\frac{2nH + D}{x} \right) - \tan^{-1} \left(\frac{2nH - D}{x} \right) \right) \quad (3)$$

and

$$\Gamma = \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}. \quad (4)$$

In the above equation, $b(t)$ describes cumulative slip on the fault through time and can describe coseismic slip and/or afterslip. D is the locking depth of the fault, H is the thickness of the upper layer, and μ_1 and μ_2 are the shear moduli in the upper layer and lower substrate, respectively. The Laplace transform of eq. (2) is

$$\hat{u}_e(x, s) = \hat{b}(s) \left(\frac{1}{2} W(0) + \sum_{n=1}^{\infty} \Gamma^n W(n) \right). \quad (5)$$

We replace μ_1 and μ_2 in eq. 5 with the equivalent shear moduli for Maxwell materials in the Laplace domain, $\hat{\mu}_1$ and $\hat{\mu}_2$, to get the Laplace transform of surface displacements in the two-layered, viscoelastic half-space,

$$\hat{u}_v(x, s) = \hat{b}(s) \left(\frac{1}{2} W(0) + \sum_{n=1}^{\infty} \hat{\Gamma}^n W(n) \right), \quad (6)$$

where

$$\hat{\Gamma} = \frac{\hat{\mu}_1 - \hat{\mu}_2}{\hat{\mu}_1 + \hat{\mu}_2} \quad (7)$$

and

$$\hat{\mu}_i = \frac{s}{\frac{s}{\mu_i} + \frac{1}{\eta_i}}. \quad (8)$$

To find the surface displacements in the time domain one must find the inverse Laplace transform of eq. (6), which is typically done using the method of residues. However, we are interested in characterizing the behavior of only the early postseismic deformation and it better serves us to instead perform the inverse Laplace transform with an extension of the initial value theorem (Appendix A). We assume for simplicity that the shear modulus for the viscoelastic lithosphere is homogenous (i.e. $\mu_1 = \mu_2$) and demonstrate in a supplementary IPython notebook that our conclusions still hold when $\mu_1 \neq \mu_2$. The surface displacements in the time domain are

$$u_v(x, t) = b(t) \frac{1}{2} W(0) + b(t) * \mathcal{L}^{-1} \left[\sum_{n=1}^{\infty} \hat{\Gamma}^n W(n) \right]. \quad (9)$$

Evaluating the above inverse Laplace transform using the method described in Appendix A, we find

$$\begin{aligned} u_v(x, t) = & b(t) \frac{1}{2} W(0) + \\ & b(t) * \left(\frac{\mu}{2\eta_2} W(1) - \frac{\mu}{2\eta_1} W(1) \right) + \\ & b(t) * \left(\left(\frac{\mu^2 t}{4\eta_2^2} - \frac{\mu^2 t}{4\eta_1 \eta_2} \right) (W(1) - W(2)) + \left(\frac{\mu^2 t}{4\eta_1 \eta_2} - \frac{\mu^2 t}{4\eta_1^2} \right) (W(1) + W(2)) \right) + \\ & \dots \end{aligned} \quad (10)$$

The first term in eq. (10) is the elastic response to slip on the fault. The remaining terms describe the surface displacement due to viscoelastic relaxation. We refer to the first of these remaining terms as the initial viscoelastic response, which describes surface deformation resulting from viscoelastic creep while the stress regime created by elastic dislocations is unaltered by viscoelastic relaxation. As suggested, the initial viscoelastic response is a linear expression with respect to the fluidity in each of the two layers.

If the time since the rupture is sufficiently small compared to the relaxation times of each layer, $\tau_i = \eta_i/\mu$, (i.e. the third and following terms in eq. (10) are small) and the timescale of slip described by $b(t)$ is also short compared to the relaxation times in the halfspace, then we can truncate the series and approximate early surface deformation using only the elastic response and the initial viscoelastic response,

$$u_v(x, t) \approx b(t) \frac{1}{2} W(0) + \int_0^t b(\theta) \left(\frac{\mu}{2\eta_2} W(1) - \frac{\mu}{2\eta_1} W(1) \right) d\theta. \quad (11)$$

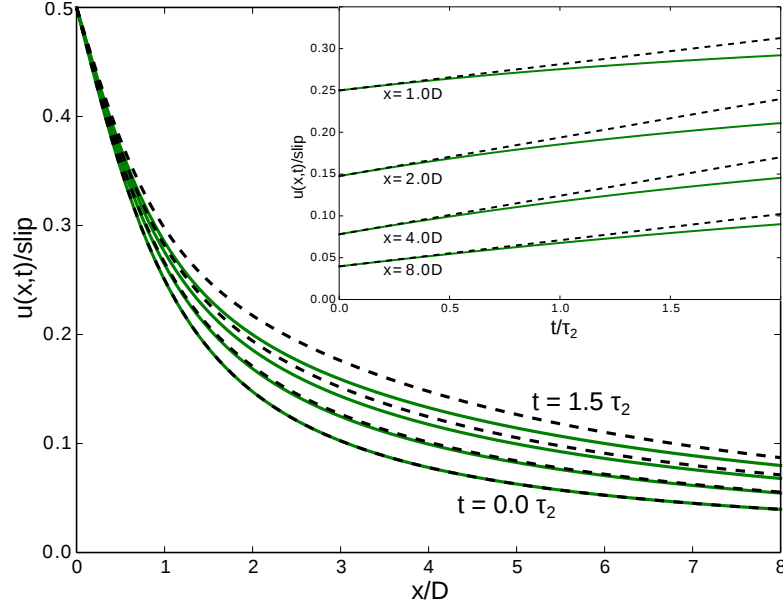


Figure 1. Surface displacements predicted by eq. (10) truncated after ten terms (green) and the approximation given by eq. (11) (dotted black). Times are normalized by the lowest relaxation time in the lithosphere, τ_2 , and distances are normalized by the fault locking depth, D . Displacements are shown as a function of distance from the fault at times $t/\tau_2 = 0.0, 0.5, 1.0$ and 1.5 . The inset figure shows displacement time series at locations $x/D = 1.0, 2.0, 4.0$ and 8.0

An approximation similar to eq. (11) was demonstrated by Segall (2010) for an elastic layer over a Maxwell viscoelastic substrate.

Figure 1 shows the series solution from eq. (10) truncated after sufficiently many terms along with the approximation given by eq. (11). In this comparison, we use $H = 15$ km, $D = 10$ km and a shear modulus of 32.0 GPa throughout the lithosphere. The upper layer is given a viscosity of 10^{20} Pa s ($\tau \approx 100$ years) and the substrate is given a viscosity of 10^{19} Pa s ($\tau \approx 10$ years). We let $b(t)$ describe a unit of instantaneous slip at $t = 0.0$. In the series solution, the rate of surface deformation decreases over time as stresses in the halfspace decay through viscoelastic relaxation. Because $b(t)$ is a constant after $t = 0$, the initial viscoelastic response in eq. (11) describes a constant rate of surface deformation and so eq. (11) is a good approximation for as long as the rate of deformation predicted by (10) is also approximately constant. We find that the approximation is indistinguishable from the series solution for at least as long as half the lowest of the two relaxation times, regardless of our choice of model parameters. The approximation breaks down faster than what is shown in Figure 1 when the upper layer is weaker than the substrate or when we decrease the depth of the material interface (i.e. when the weaker material is closer to the fault). We also note that the approximation has

more longevity for locations further away from the fault, where it starts to break down at about the minimum relaxation time in the lithosphere.

2.1.2 Three layered and continuous depth dependent model

We follow the same procedure from above to find the surface deformation resulting from slip on a strike-slip fault in a three layered viscoelastic half-space. Starting from the layered elastic solution from Chinnery & Jovanovich (1972), we evaluate the solution for the viscoelastic problem in our supplementary IPython notebook. We find the initial viscoelastic response to a unit of slip to be

$$\frac{\partial}{\partial t} u(x, t)|_{t=0} = \frac{\mu}{2\eta_3} W(1, 1) + \frac{\mu}{2\eta_2} (W(0, 1) - W(1, 1)) - \frac{\mu}{2\eta_1} W(0, 1), \quad (12)$$

where

$$W(n, m) = \frac{1}{\pi} \left(\tan^{-1} \left(\frac{2nH_2 + 2mH_1 + D}{x} \right) - \tan^{-1} \left(\frac{2nH_2 + 2mH_1 - D}{x} \right) \right). \quad (13)$$

We see that eq. (12) is once again linear with respect to the fluidity in each of the three layers. We can approximate early postseismic deformation resulting from slip described by $b(t)$ as

$$u(x, t) \approx b(t) \frac{1}{2} W(0, 0) + \int_0^t b(\theta) \left(\frac{\mu}{2\eta_3} W(1, 1) + \frac{\mu}{2\eta_2} (W(0, 1) - W(1, 1)) - \frac{\mu}{2\eta_1} W(0, 1) \right) d\theta, \quad (14)$$

where η_1 , η_2 , and η_3 are the viscosities of the top, middle, and bottom layers, respectively, and H_1 and H_2 are the thicknesses of the top and middle layer, respectively. We can see that eq. (14) recovers eq. (11) when $\eta_3 = \eta_2$.

At this point we posit that a similar approximation can be made for an arbitrarily layered lithosphere. In Appendix B we use eq. (12) to find an initial viscoelastic response kernel. We then integrate that kernel over the depth of the lithosphere to find the initial viscoelastic response for an arbitrary depth dependent viscosity structure. If the lithosphere is elastic above the fault depth, D , and described by $\eta(z)$ below D then early postseismic deformation can be approximated as

$$u(x, t) \approx \frac{b(t)}{\pi} \tan^{-1} \left(\frac{D}{x} \right) + \int_0^t \int_D^\infty \frac{\mu b(\theta)}{2\pi \eta(z)} \left(\frac{2x}{x^2 + (D + 2z)^2} - \frac{2x}{x^2 + (2z - D)^2} \right) dz d\theta. \quad (15)$$

Although the above equation is capable of describing surface deformation for an arbitrary depth dependent viscosity structure, it falls short of being useful as the forward solution in an inverse problem aimed at estimating lithospheric viscosity. This shortcoming is because the above equation makes the unphysical assumption that the fault is infinitely long, in addition to the restriction of only being applicable to a vertical strike-slip fault. The assumption of infinite length would introduce first order errors, which would likely wash out the second order effect of viscosity. However, eq. (15) is useful for making estimates of the depth sensitivity of postseismic deformation.

2.2 arbitrarily discretized earthquake models

Motivated by our above results, we make the assertion that the initial viscoelastic response to an instantaneous dislocation in a three-dimensional Maxwell viscoelastic medium, which has been arbitrarily discretized into N regions, will have the form

$$\frac{\partial}{\partial t} \vec{u}(\vec{x}, t) \Big|_{t=0} = \sum_j^N \frac{1}{\eta_j} G_j(\vec{x}). \quad (16)$$

We denote \vec{u} and \vec{x} as vectors to emphasize that eq. (16) is generalized to three-dimensional problems. We use $G_j(\vec{x})$ to represent the initial rate of surface deformation at position \vec{x} resulting from viscoelastic creep in region j with unit fluidity, where fluidity is zero (i.e. elastic) in all other regions. In this sense, $G_j(x)$ can be thought of as a Green's function for the initial viscoelastic response, and thus we refer to $G_j(x)$ as the initial viscoelastic Green's function. We verify eq. (16) numerically in section 4.3.1 and save a theoretical justification for a later paper.

Using eq. (16), we can approximation early surface deformation in a form that is consistent with eqs. (11) and (14):

$$\vec{u}(\vec{x}, t) \approx b(t)F(\vec{x}) + \sum_j^N \int_0^t \frac{b(\theta)}{\eta_j} G_j(\vec{x}) d\theta, \quad (17)$$

where $F(x)$ is the elastic Green's function, which describes the elastic deformation resulting from a dislocation. We further generalize the approximation of surface deformation in eq. (17) to allow for an arbitrary spatial distribution of slip by using linear superposition. If the elastic deformation in a viscoelastic lithosphere can be described in terms of M elastic dislocation sources, then early surface deformation resulting from both elastic dislocations and viscous creep can be approximated as

$$\vec{u}(\vec{x}, t) \approx \sum_i^M b_i(t)F_i(\vec{x}) + \sum_i^M \sum_j^N \int_0^t \frac{b_i(\theta)}{\eta_j} G_{ij}(\vec{x}) d\theta. \quad (18)$$

The initial viscoelastic Green's function is dependent upon both the region it represents as well as the dislocation source inducing the viscoelastic creep in that region, hence the two indices. It is worth restating that the approximation given above does not account for the viscoelastic coupling between the regions, since in eq. (18) each region's contribution to surface deformation is independent of the viscosity elsewhere. This approximation is therefore appropriate for as long as the regions do not significantly transfer stresses between eachother through viscoelastic deformation. Alternatively, since our initial viscoelastic Green's functions do not have time dependence, one could view eq. (18) as being appropriate up until surface velocities resulting from viscoelastic creep have decayed appreciably.

3 INVERSION METHOD

The approximation of postseismic deformation given by eq. (18) can be cast as an inverse problem aimed at finding the distribution of slip on a fault and an arbitrarily complicated lithosphere viscosity structure from postseismic deformation. We assume that the slip history in any one direction on each fault patch, $b_i(t)$, can be expressed as P linear terms such that

$$b_i(t) = \sum_k^P \alpha_{ik} A_k(t), \quad (19)$$

where $A_k(t)$ consists of either unit step functions describing coseismic slip on a fault patch, or ramp functions, which increase from 0 to 1 over some time interval, that are intended to represent afterslip. The coefficient α_{ik} then represents either the amount of coseismic slip or the cumulative slip over a time interval. The approximation given by eq. (18) now becomes

$$\vec{u}(\vec{x}, t) \approx \sum_i^M \sum_k^P \alpha_{ik} F_i(\vec{x}) A_k(t) + \sum_i^M \sum_j^N \sum_k^P \int_0^t \frac{\alpha_{ik}}{\eta_j} G_{ij}(\vec{x}) A_k(\theta) d\theta. \quad (20)$$

If we assume a fault geometry and the elastic properties of the lithosphere, $F_i(\vec{x})$ can be computed with finite element software or with an analytical solution, for instance using Okada (1992) or Meade (2007). Likewise, $G_{ij}(\vec{x})$ can be computed using finite element software. If the assumed geometry of the viscoelastic regions is sufficiently simple, $G_{ij}(\vec{x})$ may also be computed with semi-analytic techniques (e.g. Pollitz 1997; Fukahata & Matsu'ura 2006; Barbot & Fialko 2010).

We estimate the unknown slip parameters, α_{ik} , and unknown viscosities in each region of the lithosphere, η_j , from observations of surface deformation in a least squares sense. Let \mathbf{u}_{obs} be a vector of observed coseismic and postseismic surface displacements at various locations and points in time. Let \mathbf{m} be a vector of all the unknown parameters α_{ik} and η_j with length $Q = M + N + P$, and let $\mathbf{u}(\mathbf{m})$ be a vector of postseismic surface displacements predicted by eq. (20). We seek to solve

$$\min \|\mathbf{f}(\mathbf{m})\|_2^2 \quad (21)$$

subject to the constraint that

$$\mathbf{m} \geq 0, \quad (22)$$

where

$$\mathbf{f}(\mathbf{m}) = \begin{bmatrix} \mathbf{W}(\mathbf{u}(\mathbf{m}) - \mathbf{u}_{\text{obs}}) \\ \lambda_s \mathbf{L}_s \mathbf{m} \\ \lambda_v \mathbf{L}_v \mathbf{m} \end{bmatrix}. \quad (23)$$

In the above equation, \mathbf{W} is a diagonal matrix containing the reciprocal of the data uncertainties (i.e. $\mathbf{W}^T \mathbf{W} = \mathbf{C}_d^{-1}$ where \mathbf{C}_d is the data covariance matrix), and \mathbf{L}_s and \mathbf{L}_v are regularization matrices.

We impose a nonnegativity constraint on \mathbf{m} which ensures that inferred slip is in one predominant direction and that viscosities are positive. Specifically, the rake of the inferred slip on each fault patch is to be within a 90° window defined by the rakes of chosen orthogonal basis slip directions. For instance, the basis slip directions could be chosen such that only slip rakes within 45° of pure strike-slip, normal, or thrust are permissible.

Because this inverse problem inevitably has nonunique solutions for \mathbf{m} , we put additional constraints on the inferred slip and inferred viscosity with the matrices \mathbf{L}_s and \mathbf{L}_v , respectively. In our following synthetic test, we constrain the solution by minimizing the Laplacian of the spatial distribution of fault slip and lithospheric viscosity by letting \mathbf{L}_s and \mathbf{L}_v be umbrella operators (Desbrun et al. 1999). The parameters λ_v and λ_s in eq. (23) control how much we enforce the smoothness constraint. In our synthetic test, we choose these parameters using L-curves, which describe the trade off between the model smoothness and data misfit (Figure 2). We first set $\lambda_v = 0$ and then use an L-curve to pick λ_s . We then fix λ_s at our chosen value and use another L-curve to pick λ_v . We have attempted to choose our model parameters through cross-validation but we found that the optimal pair of penalty parameters picked through cross-validation tended to significantly degrade our fit to the most near-field stations.

We find \mathbf{m} that satisfies the above conditions using the Gauss-Newton method (e.g. Aster et al. 2013). The best fit model parameters are found by making an initial guess for the solution and then iteratively solving

$$\mathbf{J}(\mathbf{m}^k)\mathbf{m}^{k+1} = -\mathbf{f}(\mathbf{m}^k) + \mathbf{J}(\mathbf{m}^k)\mathbf{m}^k \quad (24)$$

for \mathbf{m}^{k+1} , where $\mathbf{J}(\mathbf{m}^k)$ is the Jacobian of $\mathbf{f}(\mathbf{m})$ with respect to \mathbf{m} evaluated at \mathbf{m}^k . We impose the nonnegativity constraint on \mathbf{m} by solving eq. (24) with a nonnegative least squares algorithm (Lawson & Hanson 1995). In a nonlinear least squares algorithm, computing the Jacobian can often be the largest computational burden when an analytic solution for the Jacobian is not available. By linearizing the viscoelastic response in early postseismic deformation with respect to η_j , we have made our forward problem, eq. (20), sufficiently simple that evaluating its Jacobian for a given \mathbf{m} only requires a few computationally inexpensive matrix operations. Consequently, our nonlinear least squares algorithm converges to a solution for \mathbf{m} in a matter of seconds on a desktop computer. The main computational burden is in computing $F_i(x)$ and $G_{ij}(x)$ which is done with finite element software and only needs to be done once for a given fault and lithosphere geometry.

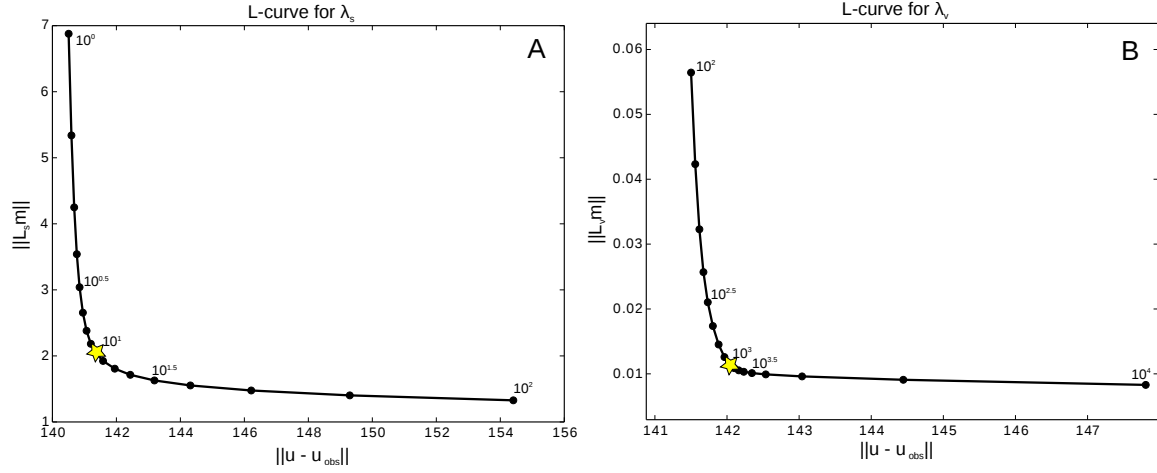


Figure 2. L-curves used to select the penalty parameters. Panel A shows the trade off between slip smoothness and data misfit while varying λ_s and keeping λ_v fixed at zero. Panel B shows trade off between smoothness of inferred viscosity and misfit while varying λ_v and keeping λ_s fixed at the value chosen from Panel A. Stars indicate our chosen penalty parameters.

4 SYNTHETIC TEST

4.1 Synthetic postseismic deformation

We demonstrate with a synthetic test that our inverse method is capable of recovering fault slip and lithospheric viscosity from postseismic deformation. We use the finite element software, Pylith (Aagard et al. 2013), to compute the surface deformation resulting from a specified amount of slip on a fault in a lithosphere with a specified viscosity. We invert this synthetic surface deformation using the method described above to recover the imposed model parameters. The synthetic test also serves to demonstrate that eqs. (16) and (18) are indeed valid for three dimensional earthquake models.

Our synthetic model consists of a 50 km long by 20 km wide strike-slip fault, striking to the north and dipping 60° to the east (figure 4). At $t = 0$ we impose 6.54×10^{19} N m of surface rupturing, right-lateral coseismic slip with a distribution shown in Figure 3. After the coseismic slip, we impose a constant rate of afterslip from $t = 0$ to 0.5 years. The cumulative moment over this interval is about 1.07×10^{19} N m. The spatial distribution of afterslip is shown in Figure 3. During the interval $t = 0.5$ to 1.0 years the rate of afterslip is decreased by a factor of 2. We do not impose any fault slip beyond $t = 1$ year.

The lithosphere in our synthetic model is Maxwell viscoelastic with homogenous Lamé parameters $\lambda = 32$ GPa and $\mu = 32$ GPa. The viscosity in the lithosphere decays from 10^{21} Pa s ($\tau \approx 1,000$ years) at the surface to 10^{19} Pa s ($\tau \approx 10$ years) at 75 km depth (Figure 4). We compute displacements at 0.1 year intervals up until $t = 10$ years, which makes the uppermost lithosphere effectively elastic

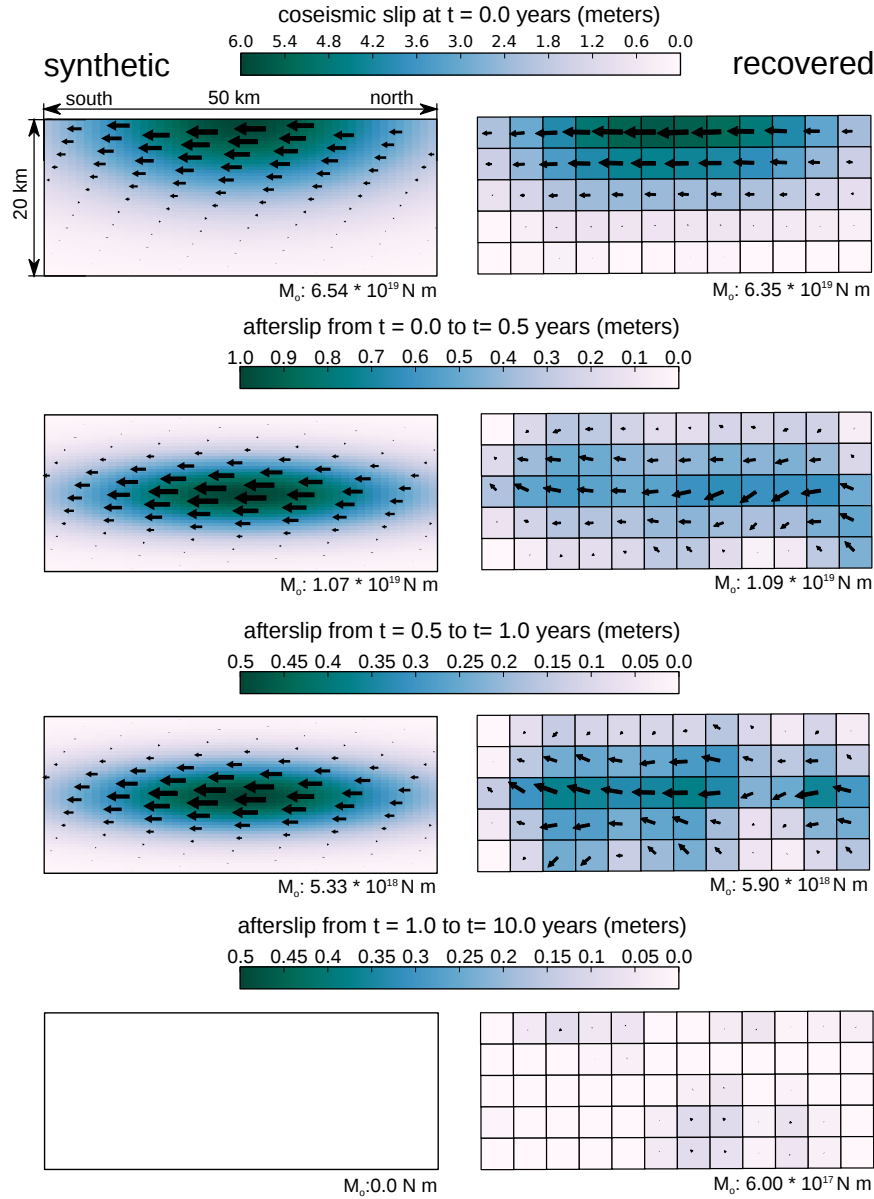


Figure 3. Slip distribution imposed in the synthetic models and slip recovered from the inversion. Colors indicate magnitude of slip and arrows indicate direction of slip (arrows pointing right indicate left-lateral and up is thrust). The panels showing afterslip display cumulative slip over the specified time interval.

on these timescales. We compute surface displacements at 50 randomly chosen locations within a 400 km square centered about the fault (Figure 5), which is intended to roughly correspond with the density of GPS station at a well instrumented plate boundary. We add temporally correlated noise to the computed displacements through time, consistent with what one would expect from GPS observations. The standard deviation of northing and easting displacements is 1 mm, and the standard deviation of the vertical displacements is 2.5 mm. We add temporal covariance with an exponential noise model

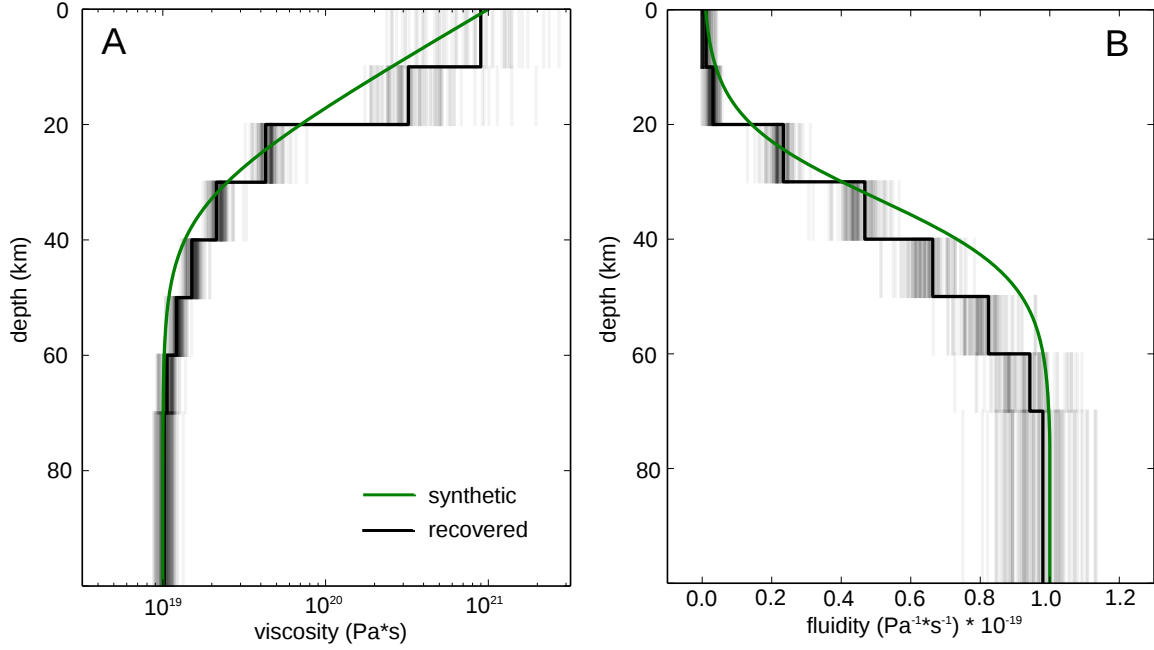


Figure 4. Synthetic and recovered lithospheric viscosities (right) and associated fluidities (left). Semi-transparent lines are recovered models found through bootstrapping and indicate the degree of uncertainty on the inferred viscosity structure.

that has a characteristic timescale of 0.25 years, which is intended to simulate seasonal processes that are typically present in GPS time series.

4.2 Green’s functions

We invert the synthetic surface deformation for fault slip on a 4 km by 4 km discretization of the fault segment. We estimate a constant viscosity in 10 km thick horizontal layers from the surface down to 70 km depth and for a lower substrate. We compute the elastic Green’s functions, $F_i(\vec{x})$, and initial viscoelastic Green’s functions, $G_{ij}(\vec{x})$, numerically using Pylith. The elastic Green’s functions are the initial surface displacements resulting from 1 m of imposed slip on fault patch i . For each fault patch, we use basis slip directions with rake 45° updip and 45° downdip of pure right-lateral slip. These slip basis directions restrict all inferred slip to be within 45° of right-lateral. We find the initial viscoelastic Green’s functions, $G_{ij}(\vec{x})$, by computing the initial rate of surface deformation due to 1 m of slip on fault patch i in a model that is elastic everywhere except in region j , which is assigned a viscosity of 10¹⁸ Pa s.

We define the basis slip functions, $A_k(t)$, as a heaviside function centered at $t = 0$ and three ramp functions which increase from 0 to 1 m of slip over the time intervals $t = 0$ to 0.5 years, $t = 0.5$ to 1 year, and $t = 1$ to 10 years. Although our synthetic model does not have any fault slip from $t = 1$ to

10 years, we include the last ramp function to test if postseismic deformation over that interval, which is resulting purely from viscoelastic creep in the synthetic model, can be describe with continued fault slip.

4.3 Recovered model

Our best fitting model of slip on the fault is shown in Figure 3. The spatial distribution and direction of inferred coseismic slip are a good match to the synthetic coseismic slip. The distribution of afterslip was decently recovered but not as well as the coseismic slip was recovered. There are a few artifacts in the distribution of afterslip which are not present in the synthetic model, such as slip on the north-most section of the fault. The distribution of afterslip also has been considerably smoothed out by the regularization. Our inability to recover the details of the imposed afterslip as well as the coseismic slip could be because the data noise is obscuring some of the postseismic signal (figure 5b and 5c compared to figure 5a) or possibly because the imposed afterslip is deeper than the coseismic slip and thus more difficult to accurately recover. Nevertheless, the inferred moment of both coseismic slip and afterslip, which is proportional to slip integrated over the fault plane, is in good agreement with the moment in the synthetic model. Although the spatial distribution of inferred slip may be more difficult to recover, the cumulative slip seems to be consistently recovered with ease.

The inferred slip over the last time interval, $t = 1$ to 10 years, is also consistent with the synthetic model. The moment of slip over this interval is 5×10^{17} N m, which is two orders of magnitude smaller than the moment for the coseismic slip or cummulative afterslip in the synthetic model. This means that the inferred slip is accounting for, at most, a few mm's of displacement from $t = 1.0$ to $t = 10.0$ years. This is on order of the data uncertainty and so the inferred slip is negligably small. The majority of surface deformation during this time interval (figure 5d) is therefore being properly attributed to viscoelastic relaxation.

The inferred viscosities in each of the eight layers are shown in Figure 4a. The recovered viscosities correspond well with the synthetic model. We used bootstrapping to estimate the uncertainties of the recovered viscosities and we found that the strongest layers near the surface, despite being proximal to the earthquake source, have the highest uncertainties. However, viscosities greater than 10^{20} Pa s are effectively elastic on the timescales of this synthetic test and so a wide range of high viscosities for the upper layers would just as adequately be able to describe the synthetic surface displacements. When looking at inferred values of fluidity (Figure 4b), we see that the uncertainties are lowest at the surface and increase with depth, as is perhaps more intuitive.

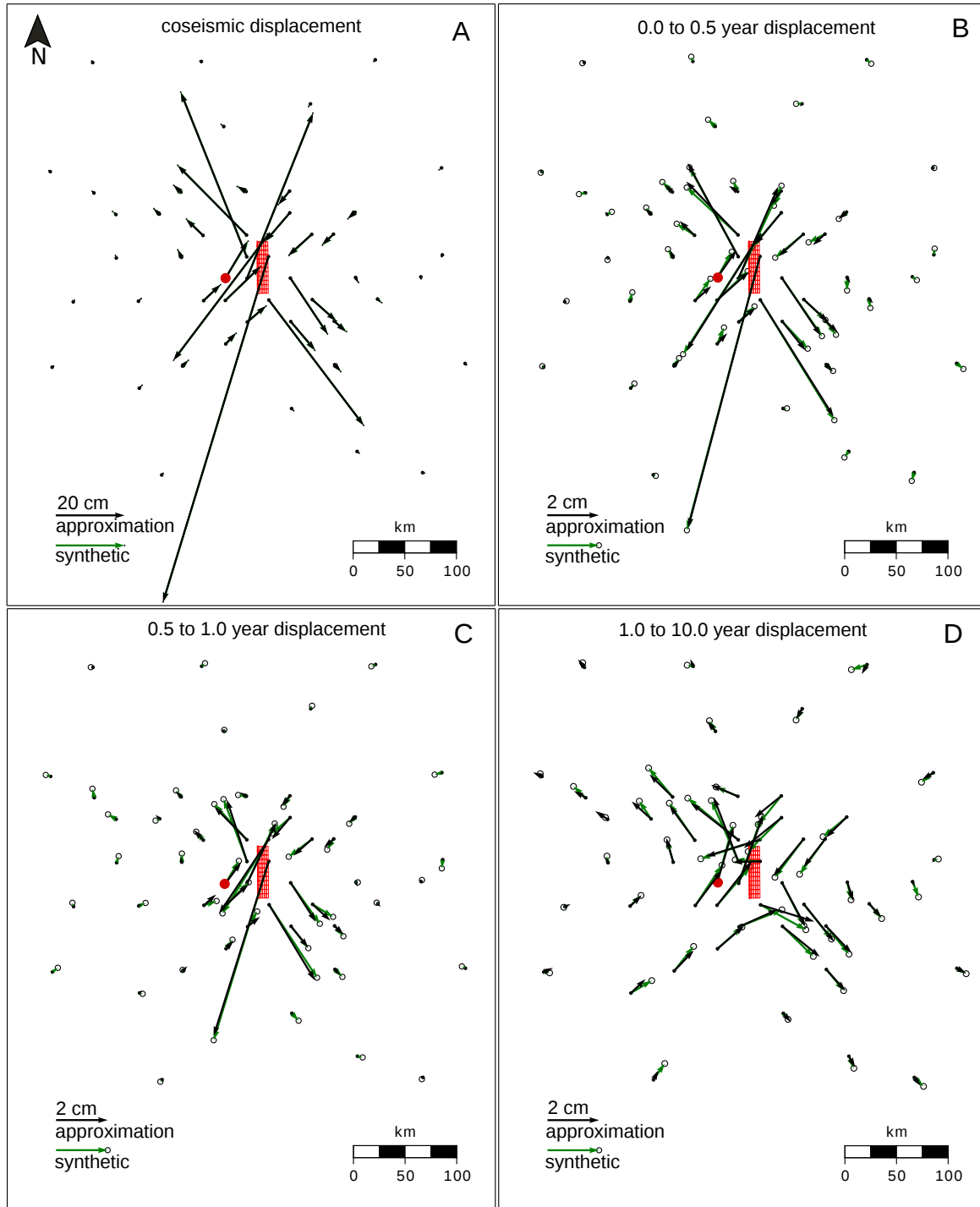


Figure 5. Synthetic surface displacements (green) and best fitting surface displacements (black). Vertical displacements are used in the inversion but are not shown here. The top left panel shows coseismic displacements and the remaining panels show the cumulative displacements over the indicated time intervals. Ellipses indicate 1 standard deviation uncertainty in the synthetic data. Red dot indicates the position of the time series shown in Figure 6. The surface projection of the discretized synthetic fault is depicted in red.

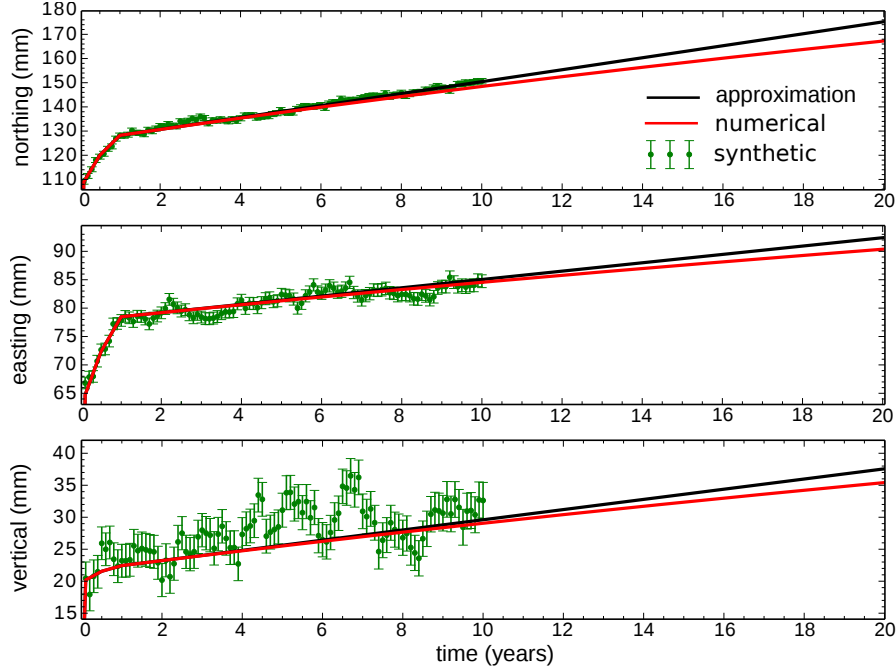


Figure 6. Displacement time series for the position shown in Figure 5 (green), best fitting surface displacements using the approximation from eq. (18) (black) and surface displacements computed with Pylith using the inferred slip distribution and viscosity structure. Coseismic displacements at $t = 0$ are not shown.

4.3.1 Validation

The fact that our recovered fault slip and lithospheric viscosity are in good agreement with the synthetic model suggests that the approximation given by eq. (18) is accurate over the 10 years of synthetic data. We further assess the accuracy of eq. (18) by running a forward model with Pylith where the imposed fault slip and lithospheric viscosity are those estimated from the synthetic data. We then compare the displacements from the numerically computed forward model with the displacements predicted by eq. (18). We refer to the numerically computed displacements as $\vec{u}_{\text{true}}(\vec{x}, t)$ and the displacements predicted by the approximation as $u(x, t)$ (Figure 6). We refer to the difference between $\vec{u}_{\text{true}}(\vec{x}, t)$ and $\vec{u}(\vec{x}, t)$ as the approximation error (Figure 7). At $t = 10$ years the approximation error is on order of a few mm's in length for each location, which is the magnitude of the data uncertainty. Additionally, the approximation error is small compared to the cm's of deformation resulting from viscoelastic relaxation, indicating that eq. (18) is indeed a fair approximation up to $t = 10$ years.

At $t = 20$ years the approximation error is about one cm in magnitude for near field sites, indicating that the approximation has broken down in the near field by this time, while the error is still on order of a few mm's for the far field sites. The faster divergence for the near field sites is consistent with the comparison we made between the approximate and true displacements for a two-dimensional, two layered earthquake model in section 2.1.1 (Figure 1).

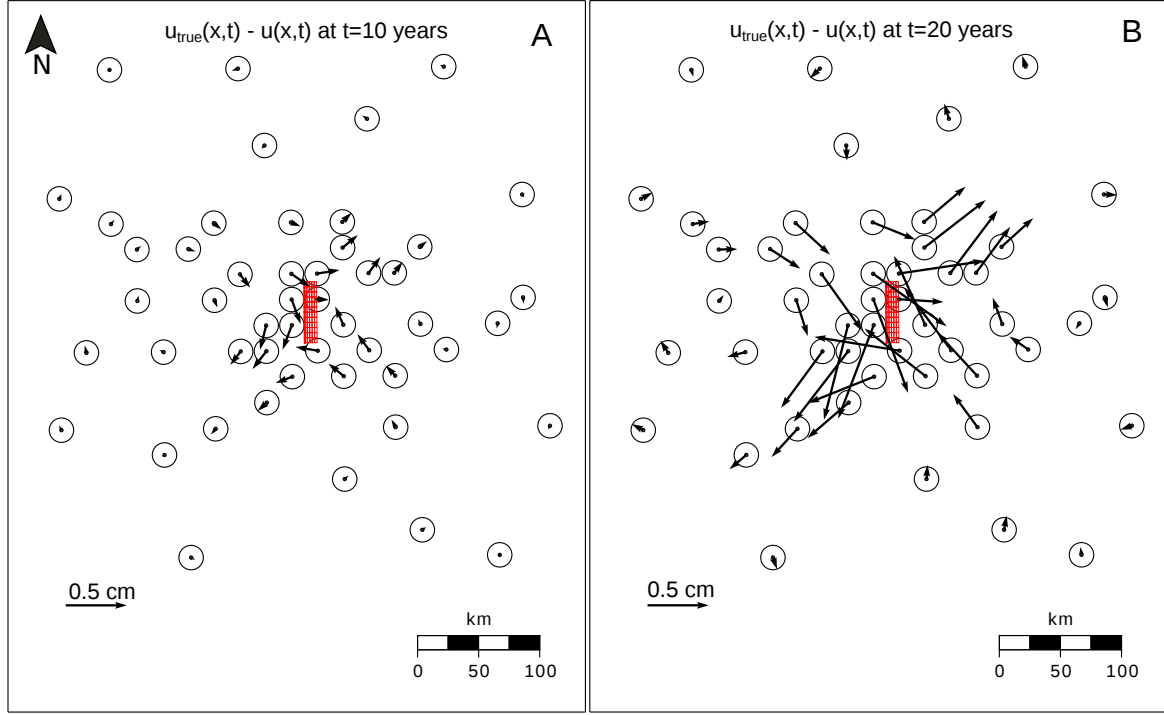


Figure 7. Difference between $\vec{u}(\vec{x}, t)$ and $\vec{u}_{\text{true}}(\vec{x}, t)$ at $t = 10$ years and $t = 20$ years. Circles with 1 mm radius are centered at each station to compare the accuracy of $\vec{u}(x, t)$ to the noise in the synthetic data.

The accuracy of eq. (18) is also demonstrated in Figure 6, which shows $\vec{u}(\vec{x}, t)$ and $\vec{u}_{\text{true}}(\vec{x}, t)$ at a sample site near the fault. The numerical solution asymptotically approaches the rate of deformation predicted by eq. (18) as time goes to zero, demonstrating that eq. (16) accurately describes the initial viscoelastic response. Additionally, the magnitude of the difference between $\vec{u}(\vec{x}, t)$ and $\vec{u}_{\text{true}}(\vec{x}, t)$ is smaller than the uncertainty on our synthetic data throughout the time series, indicating that the approximation given by eq. (18) is appropriate for this synthetic test. For this site and other near field sites, the approximation starts to break down at about $t = 10$ years. The lowest relaxation time in our synthetic lithosphere is also about 10 years and so the duration over which eq. (18) is accurate is longer than what we found in our analysis for a two-dimensional, two layered earthquake model in section 2.1.1.

5 DISCUSSION

A fundamental assumption in our method for estimating slip and viscosity from postseismic deformation is that the timescale of relaxation in the weakest part of the lithosphere is at least as long as the timescales over which postseismic deformation is observed. This assumption allows us to approximate the surface expression of viscous creep as a linear system with respect to lithospheric fluidity,

which greatly facilitates and expedites the inverse problem. However, the lithosphere's relaxation time is generally not well known. There is then an added complication of deciding how much of the post-seismic time series to use in the inverse problem. In our synthetic example, we conveniently picked the length of our time series to correspond with the weakest relaxation time in the lithosphere. In general, the length of the postseismic time series should be determined iteratively in an inversion using our method. For example, if the approximation given by eq. (18) is incapable of adequately describing observed postseismic deformation or if inferred relaxation times are significantly less than the length of the time series, then the postseismic time series used in the inversion should be shortened.

Our synthetic data is characterized by transient and rapid postseismic deformation in the first year after the earthquake followed by steady deformation in the later years. Surface deformation following large ($\geq M_w = 7$) earthquakes often is characterized by a similar temporal evolution (e.g. Savage & Svarc 1997; Savage et al. 2005; Ergintav et al. 2009). Several studies have attributed rapid early transience to afterlip and described the later steady deformation with viscous relaxation in a Maxwell viscoelastic lower crust or upper mantle (e.g. Perfettini et al. 2005; Johnson et al. 2009; Hearn et al. 2009; Freed et al. 2006a; Rollins et al. 2015). These studies have found that lithospheric relaxation times no shorter than years or decades are needed to describe postseismic deformation. Indeed, Perfettini et al. (2005) described the trend in two years of postseismic deformation following the 2001 $M_w = 8.4$ Peru earthquake by assuming that the lithospheric viscosity was sufficiently high that the rate of deformation from viscoelastic creep could be considered constant, which is the assumption that we make in formulating eq. (18). If transient postseismic deformation can be attributed to fault slip followed by steady viscoelastic creep, then eq. (18) should be appropriate on the timescale of years or decades after an earthquake.

Postseismic transient deformation could also be the result of creep in a weak ductile shear zone which is embedded in a stronger viscoelastic lithosphere (e.g. Hetland & Zhang 2014). Sufficiently localized creep in a shear zone can be modeled as slip on a downdip extension of the ruptured fault (e.g. Hearn et al. 2002; Kenner & Segall 2003; Johnson & Segall 2004) because the two processes are kinematically indistinguishable. Likewise, Freed et al. (2006a) noted that deep fault slip could serve as a proxy for distributed viscous relaxation in a weak lower crust. The applicability of our method should therefore be unaffected by localized viscous deformation with the understanding that inferences of fault slip could be absorbing that deformation.

Several studies have explored other lithospheric rheologies to explain observed transient post-seismic deformation. For example, Pollitz (2003, 2005) invoked a Burgers rheology upper mantle to explain surface displacements following the 2002 $M_w = 7.9$ Denali earthquake and the 1999 $M_w 7.1$ Hector Mine earthquake. In both cases the best fitting transient relaxation time was on the order of

a month and the best fitting steady-state relaxation time was on the order of years. Postseismic deformation following the Denali earthquake was also successfully modeled by Freed et al. (2006b) with a power-law rheology in the upper mantle, consistent with laboratory studies (e.g. Kirby and Kronenberg 1987). The power-law rheology was able to reproduce the observed transient surface deformation because the high stresses in the earthquake decreased the effective viscosity of the upper mantle to $\sim 10^{17}$ Pa s resulting in fast surface deformation. As stresses from the earthquake relaxed, the effective viscosity increased and the predicted surface deformation became steadier.

Based on the success of Pollitz (2003, 2005) and Freed et al. (2006b), one may dismiss our method as being unrealistic because we assume that the lithosphere is Maxwell viscoelastic. However, our method does not necessarily preclude the possibility of a Burgers rheology or a stress-nonlinear viscosity. As long as stresses in the lithosphere remain roughly equal to the stresses transferred elastically through fault slip, then a viscosity structure inferred using our method could be interpreted as the effective viscosity in a power-law rheology or the initial transient viscosity in a Burgers rheology. If the commonly observed early transient postseismic deformation truly is the result viscous relaxation in the lithosphere rather than afterslip, then the results from Pollitz (2003, 2005) and Freed et al. (2006b) suggest that the time interval over which eq. (18) is appropriate is on order of a month after an earthquake. Although one month of postseismic data is a short time window and potentially valuable data beyond that window would be unavailable for our inverse method, such a low effective viscosity in the upper mantle would yield a strong signal over this time. We postulate that the method described in this paper would then be able to identify the location of the weak regions in the lithosphere with much higher resolution than what could be obtained through the typical forward modeling, grid search method. Any excised portion of the timeseries could then be used to further constrain estimates of fault slip and lithospheric viscosity by including it in a gradient based nonlinear inverse method where the forward problem is computed numerically rather than with eq. (18). In such case, the initial guess for the model parameters would be the estimates of slip and viscosity obtained using our method with the truncated time series.

6 CONCLUSION

We present a method to invert coseismic and postseismic deformation to simultaneously estimate a time dependent distribution of fault slip and an arbitrarily discretized viscosity structure of the lithosphere. We take advantage of the fact that early postseismic deformation resulting from viscoelastic relaxation can be approximated as a linear function with respect to the lithospheric viscosity. Our method is then only applicable for as long as this approximation is appropriate, that is, for as long as stresses resulting from coseismic slip and afterslip have not significantly decayed due to viscoelas-

tic relaxation. Based on estimates of lithospheric viscosity from other studies, we estimate that our method could be used for postseismic deformation ranging from months to years after an earthquake. Despite this limitation, Our method is much faster than forward modeling techniques that are commonly used in studies seeking to estimate the rheologic properties of the lithosphere and our method allows for unprecedented resolving power of a lithospheric viscosity structure.

APPENDIX A: INVERSE LAPLACE TRANSFORM THROUGH SERIES EXPANSION

let $f(t)$ be analytic at $t = 0$ and let there be a real valued M , and C such that

$$\left| f^{(n)}(t) \right| < C e^{Mt} \quad \forall t \geq 0 \text{ and } \forall n \in \{0, 1, 2, \dots\}, \quad (\text{A.1})$$

where $f^{(n)}(t)$ denotes the n^{th} derivative of $f(t)$. We define the Laplace transform of $f(t)$ as

$$\mathcal{L}[f(t)] := \hat{f}(s) := \int_0^\infty f(t) e^{-st} dt \quad (\text{A.2})$$

and we restrict our attention to $s \in \mathbb{R}$. The constraints on $f^{(n)}(t)$ from eq. (A.1) ensure that

$$\lim_{s \rightarrow \infty} \mathcal{L}[f^{(n)}(t)] = 0. \quad (\text{A.3})$$

It can be shown using integration by parts that

$$\mathcal{L}[f^{(n)}(t)] = s^n \hat{f}(s) - \sum_{m=1}^n s^{m-1} f^{(n-m)}(0) \quad \forall s > M. \quad (\text{A.4})$$

Substituting eq. (A.4) into eq. (A.3) and then rearranging the terms gives us a recursive formula for $f^{(n)}(0)$ in terms of $\hat{f}(s)$:

$$f^{(n)}(0) = \lim_{s \rightarrow \infty} s^{n+1} \hat{f}(s) - \sum_{m=1}^n s^m f^{(n-m)}(0), \quad (\text{A.5})$$

where the base case, $n = 0$, is the initial value theorem:

$$f(0) = \lim_{s \rightarrow \infty} s \hat{f}(s). \quad (\text{A.6})$$

Since we request $f(t)$ to be analytic at $t = 0$, we can construct a Taylor series expansion of $f(t)$ such that

$$f(t) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} t^n \quad (\text{A.7})$$

for all values of t within some neighborhood of $t = 0$. We find the inverse Laplace transform of $\hat{f}(s)$ by combining eq. (A.7) with eqs. (A.5) and (A.6) so that $f(t)$ is expressed in terms of $\hat{f}(s)$.

APPENDIX B: POSTSEISMIC APPROXIMATION FOR A TWO-DIMENSIONAL EARTHQUAKE MODEL WITH AN ARBITRARY DEPTH DEPENDENT VISCOSITY

We seek to find an approximation for early postseismic deformation in a two-dimensional, strike-slip earthquake model with an arbitrary depth-dependent viscosity below the fault locking depth, D . We first find the initial rate of surface deformation following a unit of slip in a lithosphere that is elastic except for a viscoelastic layer which is at depth z and with thickness Δz . This is found by making the substitutions $H_1 \rightarrow z$, $H_2 \rightarrow \Delta z$, $\eta_1 \rightarrow \infty$, $\eta_3 \rightarrow \infty$, and $\eta_2 \rightarrow \eta$ in eq. (12), which gives us

$$\frac{\partial}{\partial t} u_1(x, t)|_{t=0} = \frac{1}{\eta} (W(z + \Delta z) - W(z)), \quad (\text{B.1})$$

where

$$W(z) = \frac{\mu}{2\pi} \left(\tan^{-1} \left(\frac{2z - D}{x} \right) - \tan^{-1} \left(\frac{2z + D}{x} \right) \right). \quad (\text{B.2})$$

From eq. (16) we know that the initial rate of surface deformation for a lithosphere composed of N discrete layers, each with viscosity η_i , at depth z_i , and having thickness Δz , is then

$$\frac{\partial}{\partial t} u_N(x, t)|_{t=0} = \sum_i^N \frac{1}{\eta_i} (W(z_i + \Delta z) - W(z_i)). \quad (\text{B.3})$$

The initial rate of surface deformation for a viscosity structure given by $\eta(z)$ is found by taking the limit as $\Delta z \rightarrow 0$ and $N \rightarrow \infty$:

$$\frac{\partial}{\partial t} u(x, t)|_{t=0} = \int_D^\infty \frac{1}{\eta(z)} \frac{\partial}{\partial z} W(z) dz \quad (\text{B.4})$$

$$= \int_D^\infty \frac{\mu}{2\pi\eta(z)} \left(\frac{2x}{x^2 + (D + 2z)^2} - \frac{2x}{x^2 + (2z - D)^2} \right) dz. \quad (\text{B.5})$$

Finally, we add the elastic component of deformation and integrate eq. (B.5) with the fault slip history to obtain an approximation for early postseismic deformation:

$$u(x, t) \approx \frac{b(t)}{\pi} \tan^{-1} \left(\frac{D}{x} \right) + \int_0^t \int_D^\infty \frac{\mu b(\theta)}{2\pi\eta(z)} \left(\frac{2x}{x^2 + (D + 2z)^2} - \frac{2x}{x^2 + (2z - D)^2} \right) dz d\theta. \quad (\text{B.6})$$

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