Binary Neutron Star Simulations: New Tools and Insights

by

Trevor Vincent

A thesis submitted in conformity with the requirements
for the degree of Doctor of Philosophy
Graduate Department of Physics
University of Toronto

Abstract

Binary Neutron Star Simulations: New Tools and Insights

Trevor Vincent

Doctor of Philosophy

Graduate Department of Physics

University of Toronto

2018

On September 14th, 2015 LIGO detected gravitational waves from a merging binary black hole. Just under two years later, LIGO detected gravitational waves from merging neutron stars in coincidence with detections of a gamma ray burst and kilonova. Many more detections of this kind are expected in the future. There are several problems that are facing the numerical relativity community now that binary neutron stars (BNS) have been detected by LIGO and this thesis will tackle two of them. After an introductory chapter to provide the reader with needed background, we present a new numerical scheme and code in Chapter 2 that aims to lay the groundwork for more realistic BNS simulations in the future. This code uses discontinuous Galerkin numerical methods to solve elliptic problems on curvilinear and non-conforming meshes in parallel on large supercomputers. We test this code on several problems in numerical relativity, including one that mimicks the discontinuities in the phase transitions of a Neutron star, as well as the two and three black-hole initial gravitational data problem. This code will be used in the future to obtain highly accurate initial gravitational data for BNS simulations. In Chapter 3 of this thesis, we present a set of twelve new BNS simulations using a current state-of-the-art numerical relativity code. We study the neutrino and matter emission from this set of twelve simulations to establish trends between the source parameters and the emission from the binary. Large sets of simulations of BNS mergers will be crucial for the upcoming LIGO observation runs to maximize the extractions of astrophysical information from the instrument data. Finally in the last chapter we conclude the thesis by overviewing work that is currently being done by others to

progress the tools and insights presented in this thesis.

Acknowledgements

When I first began my PhD journey, I never would have dreamed that I would be attending the press conference for the first detection of gravitional waves at the Albert Einstein Institute in Hannover, Germany, or that I would later become a member of the Nobel prize-winning LIGO collaboration. For this and many other things, I have to give a big thank you to my supervisor Professor Harald Pfeiffer. Harald not only gave me the opportunity to research within one of the most exciting fields currently in astrophysics, but provided me with multiple opportunities to present my research abroad. He also gave me the opportunity to attend several summer schools and work overseas at the Albert Einstein Institute in Golm, Potsdam. On top of all of this, Harald has helped me get through multiple tough problems that arose in my research and provided the initial ideas for the projects in this thesis.

Secondly, thank you to Professor Francois Foucart who taught me how to run and analyze binary neutron stars simulations with the neutrino transport scheme in the Spectral Einstein Code. Chapter 3 of this thesis would not be possible without his knowledge, support and constant help with debugging the (seemingly) hundreds of problems that cropped up while running these simulations.

Thirdly, thank you to my committee members Professor Charles Dyer and Professor Amar Vutha who provided me excellent advice both during the committee meeting and outside of our formal meetings. I still miss the General Relativity class parties at Prof. Dyer's house.

Fourthly, thank you to Krystyna Biel for helping me with all the logistical duties required throughout the PhD, her understanding, thoughtfulness and efficiency was greatly appreciated.

Lastly, a big thank you to my family, the constant support of my dad, mother, sister and grandad, and the great friends I made in and outside the physics department at the U of T during my PhD: Gunjan Lakhlani, Nazim Boudjada, Amir Abdul-Reda, Daniel Dick, Heather Fong, Adam Lewis, Alex Kostenko, Helen Baxter, Daniel Baker, Fang Xi Lin, Phil Berger, George Stein, Dana Simard, Claire Yu, Mubdi Rahman, Julien Dyer, Alex Bercik, Hannah Fronenberg, Luca Talamo and Peter Pangarov and out of town friends Jonah Miller, Nils Deppe, Paul Sarte, Nils Fischer, Stephen Green, Peter Zimmerman, Kyohei Kawaguchi, Sergei Ossokine, Roland Haas, Tim Dietrich. These people made the PhD journey unforgettable and I thank them all for the great times.

Contents

List of Tables

List of Figures

Chapter 1

Introduction

The initial proposals for gravitational wave interferometers were constructed in the late 1980s with the scientific goal to detect the inspiral and merger of compact-object binaries. About two decades later, the LIGO (LIGO et al. (2018)), Virgo (Acernese et al. (2015)), and GEO600 (Affeldt et al. (2014)) detectors have been developed and operated as a network from the period of 2005 to 2010. No detections were made during this initial stage of sensitivity. Alongside the early-2000 interferometer development, numerical simulations of the Einstein equations were beginning to gather ground. In 2000, the first binary neutron star merger was simulated (Shibata & Uryū (2000)), five years later the first binary black hole merger was computed after years of trial and error (Pretorius (2005a)) and finally in 2006 the first binary neutron star - black hole simulations were performed (Shibata & Uryū (2006)). A variety of numerical relativity groups started forming at this time all around the globe, the Caltech-Cornell-CITA group (SXS; black-holes.org), the Kyoto/Tokyo group (Nagakura et al. (2014)), the University of Illinois at Urbana-Champaign group (UIUC), to name but a few. These groups started building simulation sets to aid in the parameter estimation studies that were expected to follow from the first detections. On September 9th, 2015, in a astonishing event during a engineering run, LIGO detected the gravitational wave from two coalescing black holes (Abbott et al. (2016b)). Since this day, there have been multiple binary black hole waveform detections (LIGO et al. (2018)). More recently however, LIGO made a landmark detection, GW170817 (Abbott et al. (2017b)).

GW170817 coincided with the detection of a gamma ray burst, GRB 170817A and a series of observations that followed across the electromagnetic spectrum codenamed atf2017gw (Villar et al. (2017)). The inferred masses of the bodies and the variety of electromagnetic observations imply that the source was a neutron star binary.

With these detections, and the awarding of the 2017 Nobel prize to three LIGO members, the gravitational era of astronomy was born. More detectors are expected in the future. With the upcoming earth-based detectors KAGRA (Somiya (2012)) and LIGO-India (gw-indigo.org), there will be a worldwide detector network capable of precise source localization. On top of this, there are already plans for third generation earth-based detectors such as the Einstein Telescope (Punturo et al. (2010)) and space based detectors such as LISA (elisascience.org) which will cover an entirely different frequency band. With more and more detectors, both space-based and earth-based, we will be able to gather data from the full frequency spectrum of gravitational waves directed towards Earth.

The LIGO detection search and parameter estimator pipeline relies heavily on waveforms computed from numerical relativity. LIGO detects signals using a technique called matched-filtering, which compares instrument data against large catalogs of theoretically modelled waveform templates. These templates are created using a mix of post-Newtonian expansions, semi-analytic models tuned to numerical relativity and full numerical relativity waveforms (Sachdev et al. (2019)). Furthermore, to extract the best astrophysical information from the instrument data, it is crucial to have numerical relativity waveforms. Parameter estimation in the first LIGO detection GW150914, of two coalescing black holes, used multiple semi-analytic models, tuned to numerical relativity waveforms, to determine source characteristics (Abbott et al. (2016a)). For the LIGO detection GW170817, there was both gravitational and electromagnetic emission. Post-Newtonian (PN) waveforms were used to estimate source parameters from the instrument data because in the frequency range detected the binary was well modelled by the PN expansion (Abbott et al. (2017b)). Even though the gravitational waveform did not capture details about the post-merger remnant, the electromagnetic emission from GW170817 provides information about this stage. Using the EM observations, fully relativistic simulations of binary neutron star mergers were used to help extract information about the remnant and constrain different aspects of the equation of state of nuclear matter. Radice et al. (2017); Shibata et al. (2017).

Even though numerical relativity has matured greatly since the major developments of the early 2000's, there is still a lot more that needs to be achieved. The biggest problems today in numerical relativity are the simulation of compact object binaries with matter and the simulation of supernovae, both requiring a great amount of microphysics, multi-scale grids, large sets of complex non-linear PDES, fast supercomputers and modern numerical techniques. With the coming of the exascale age of supercomputing there is an increasingly realistic chance of simulating these systems with all the known microphysics. This thesis aims to make progress in two areas of numerical relativity, both aiming at laying the groundwork for future BNS codes and analysis of LIGO BNS detections. First, we seek to improve the computational techniques used to solve the Einstein field equations so that more realistic microphysics may be introduced into the BNS simulations as computing power reaches exascale and beyond. To do this, we develop a new numerical scheme and code, which we test on several problems in numerical relativity, including a problem that mimicks the phase transitions discontinuities of a neutron star. This is outlined in Chapter 2. Secondly, we seek to further probe the parameter space of binary neutron star simulations with one of the most state of the art numerical relativity codes to help understand the emission properties of these LIGO sources for the next generation of detections. This is outlined in Chapter 3. Thus, this thesis is dedicated to the improving not only the current understanding of BNS mergers, but paving the way for more realistic future BNS merger simulations. The remaining portion of this introductory chapter will briefly describe the physics and computational techniques needed to understand the context of Chapters 2 and 3. We begin Chapter 1 with a high-level overview of BNS merger simulations and then in the following sections, we narrow down our focus to the numerical solving of the Einstein field equations for BNS simulations and the problems that current solvers have. We end Chapter 1 by introducing a new numerical method called discontinuous Galerkin which we plan on using to improve future BNS simulations.

1.1 Binary Neutron Stars (BNS): Einstein's Richest Laboratory

In the last section we outlined the grand scheme of this thesis and the chapters to follow. Now, we will start delving deeper into the binary neutron stars. In this section, we give a high-level overview of binary neutron star systems, and the computational work that has been performed to model them.

The first binary neutron star system discovered was PSR B1913+16 by Russell Hulse and Joseph Taylor in 1974 (Hulse & Taylor (1975)). From observations of radio-wave pulses of the stars, it was determined that the orbit of PSR B1913+16 was shrinking at a rate of 3.5 meters per year, precisely the amount predicted by the loss of energy in gravitational wave emission Weisberg et al. (1981). For their work, Hulse and Taylor were awarded the 1993 Nobel Prize in Physics. Since then, radio telescopes have discovered roughly a dozen binary neutron star systems, all within our Galaxy Baiotti & Rezzolla (2016). However, the presence of extragalactic short gamma-ray bursts, believed to be powered by binary neutron star mergers, gives indirect evidence for the existence of even more neutron star binaries. Furthermore, with the first BNS detection by LIGO, codenamed GW170817, we can expect many more gravitational wave detections of BNS systems in the future. Using updated models which include the GW170817 observation, LIGO expects a detection rate of $110 - 3840 Gpc^{-}3y^{-}1$ LIGO et al. (2018). This detection rate coupled with a detection radius of $\sim 107 Mpc$ for the LIGO-Hanford observatory (LIGO-Livingston has a $\sim 218 Mpc$ radius) Abbott et al. (2017b) means a lower bound of ~ 10 BNS mergers could be seen by LIGO per year. With the upcoming LIGO A+ upgrades it is expected that by the 2026, we may be detecting one per week (LIGO Document G1601435-v3).

There are three main stages of a neutron star binary system. These stages can be distinguished using the radius R of the neutron stars and their separation a. For $a >> R$, the binaries orbit each other with a separation that slowly decreases in a stage known as the **inspiral** Faber & Rasio (2012); Baumgarte & Shapiro (2010); Shibata (2015); Rezzolla & Zanotti (2013). In this stage, the stars are well approximated as point particles and orbit each other with speeds not exceeding $\sim 0.2c$ Shibata (2015). During the late inspiral, when the stars are orbitting each other multiple times per second ($O(10)Hz$), they start being visible by the LIGO detector. As the neutron stars approach and $a \sim 5R$, they become tidally deformed. The amount of deformation is governed by the equation of state (EOS) of the nuclear matter. At this stage the stars are orbitting at an extreme rate ($O(500)Hz$). As $a \sim 30km$ the stars plunge toward each other and merge into a single hypermassive neutron star, ejecting a large amount of material in the process Shibata (2015). This ejecta fuses together new nuclei whose decay releases optical radiation in an event called a "kilonova" Metzger (2016). This stage is known as the **merger** Faber & Rasio (2012); Baumgarte & Shapiro (2010); Shibata (2015); Rezzolla & Zanotti (2013). Whether this newly formed massive neutron star collapses down to a black-hole depends on the total mass of the system and the EOS. For most EOS, a binary of total mass close to $3M_{\odot}$ will collapse within a few milliseconds after merger Bauswein et al. (2013). Once the remant has settled into either a black-hole or a massive neutron star, we have reached the **ringdown** stage Faber & Rasio (2012); Baumgarte & Shapiro (2010); Shibata (2015); Rezzolla & Zanotti (2013). The dynamics and observational signatures of the ringdown stage are dominated by the left-over accretion disk, which drives magnetic, viscous and neutrino winds and potentially provides the energy source for a collimated polar jet creating one of the most luminous events in the universe, the short gamma ray burst. Both the merger and the ringdown stages are outside the LIGO band, so these stages cannot be probed via gravitational waves until the next-generation of detectors is here Abbott et al. (2017d).

Each stage of the binary coalescence is marked by differences in the gravitational wave. The early inspiral is represented by a sinusoidal pattern as the neutron stars slowly orbit each other like point particles Bauswein & Stergioulas (2019). As the binary first enters the LIGO band in the late inspiral, the stars become tidally deformed. This tidal deformation induces a quadrupole moment which changes the shape of the waveform Hinderer et al. (2010). The magnitude of the tidal deformation is governed by the EOS, so measurement of the tidal deformability can put constraints on the EOS and such constraints were computed for GW170817 Raithel (2019). As the stars plunge toward each other, the frequency grows rapidly and reaches its peak at merger, this rapid growth of the frequency is often called the “chirp” because it resembles the chirp sound of a sliding whistle if the frequencies were converted to sound waves. As the cores-bounce together and rotate during merger, the proto-remnant emits waves with a multiltude of frequencies with most of the emission in three major peaks (the largest being from the rotation of the proto-remnant and the other two from the fundamental frequencies of the bouncing spring-like cores) Takami et al. (2015). These peaks are EOS dependent and thus can lead to constraints on the EOS, but unfortunately these peaks were outside of the LIGO band for GW170817 Abbott et al. (2017d). Lastly as the remnant settles, there is a rapid decay of the amplitude as energy is lost from the remnant and it “rings down”. For GW170817, neither the ringdown nor the merger stage could provide us with constraints the EOS. The late inspiral however provided the a measure of the tidal deformability. Using this measurement, the radius of the primary neutron star if the binary had a mass-ratio $q = 1$ was constrained to be $9.8 < R < 13.2$ with a maximum mass of $2.3M_{\odot}$ if the remnant collapsed to a black hole Raithel (2019). However, some studies have argued that the late-time electromagnetic emission from GW170817 can be better explained by a long-lived neutron star remnant Yu et al. (2018). Therefore, depending on the assumed fate of the binary, implications for M_{max} may vary significantly. In future LIGO observation runs, post-merger gravitational waves from the remnant may provide a clear indication of the remnant’s fate and thus provide us with better constraints on R and M_{max}.

Until GW170817 there were two big mysteries surrounding BNS. The first mystery involved the so-called rapid neutron capture (r-process) elements. It was unclear for decades how approximately half of the elements in the Milky Way galaxy heavier than iron were formed.

The formation of these elements required environments where the density of free neutrons would be so high that neutron captures on nuclei proceed much faster than β decays. This series of neutron captures was called the r-process and such elements were therefore labelled r-process elements. In 1974, Lattimer and Schramm proposed that r-process elements could be formed in the neutron-rich matter ejected from neutron star - black-hole mergers (Lattimer & Schramm (1974)). Some years later, Symbalisty and Schramm proposed that that a similar mechanism of mass ejection could occur from binary neutron star systems and power the r-processSymbalisty & Schramm (1982). It was shown in the decade following that Newtonian BNS simulations could produce ejecta from tidal disruption with the desired neutron-richness, speed and mass to potentially produce the r-process elements Davies et al. (1994); Ruffert et al. (1996); Rosswog et al. (1998); Freiburghaus et al. (1999). In the years that followed fully-general relativistic simulations were performed and showed that there was another type of ejecta, matter ejected out the poles of the binary from the shocks of the cores contracting and recoiling upon impact Oechslin & Janka (2006); Hotokezaka et al. (2013); Sekiguchi et al. (2015); Foucart et al. (2015a). This type of ejecta was neutron-poor, much faster and in general less massive Foucart et al. (2015a). Further simulations have shown that the resulting accretion disk after a remnant settles can also ejecta matter via neutrino, magnetic and viscous winds with a variety of neutron-richness, masses and speeds Fernández & Metzger (2013). Alongside these simulations, work was being achieved on the theoretical side of astrophysical r-process emission models. In 1998, Li and Paczynski first proposed that the radioactive ejecta of a NS merger could power a supernova-like thermal transient, but they did not have a realistic model of the radioactive heating Li & Paczynski (1998). In 2010, Metzger et al. first calculated the emission luminosities following using a realistic modelling of the radioactive heating of decaying r-process nuclei Metzger et al. (2010). They predicted peak luminosities of $3x10^{41} ergs^{-1}$ for $10^{-2} M_\odot$ of ejecta expanding at $v \approx 0.1c$ with a spectral peak at visual wavelengths. As this was roughly 1000 times more luminous than classical novae, they named these events “kilonovae”. The color of the kilonova distinguishes the type of ejecta involved. In particular, ejecta with $Y_e \lessapprox 0.25$, such as the shocked polar ejecta, lacks enough neutrons to create r-process elements past $A \approx 140$ and a blue-colored fast moving kilonova is produced (Metzger & Fernández (2014)). At least all mergers should have $Y_e \lessapprox 0.2$ ejecta from tidal-tail ejecta or disk winds, this produces a red slower-moving kilonova which should be a universal feature of all mergers. Disk wind ejecta tends to be isotropic and produces kilonovae that are both blue and red in nature. For the LIGO GW170817 detection, the electromagnetic counterpart was codenamed AT2017fgo. Over the first few days the transient colors were blue and rapidly-evolving with a spectral peak at visual wavelengths (e.g. Soares-Santos et al. (2017;?); Nicholl et al. (2017;?); McCully et al. (2017); Cowperthwaite et al. (2017);). At later times, the colors became substantially redder and slowly evolved on timescales of several days. The total mass of the red ejecta was estimated to be $410^{-2}M$ with a somewhat lower velocity $v \approx 0.1c$ than the blue ejecta (e.g. Cowperthwaite et al. (2017); Chornock et al. (2017); Nicholl et al. (2017)). The quantity of blue ejecta from AT2017fgo was estimated to be $\sim 1 - 210^{-2}M_{\odot}$ with a faster velocity of $v \approx 0.2c$ (Cowperthwaite et al. (2017); Nicholl et al. (2017)), based on fitting the observed light curves to kilonova models (Metzger (2017)).

The second mystery involves the most luminous events in the universe, the Gamma ray burst (GRB). GRBs were first detected in the late 1960s by U.S military satellites sent into orbit to spy on the Soviet Union nuclear testing (Klebesadel et al. (1973)). The satellites were trained to detect short bursts of gamma-rays from expected nuclear explosions. The satellites started measuring gamma rays very frequently, but this work remained classified until it was determined the bursts were of cosmological origin (Klebesadel et al. (1973)). Very little was known about these gamma-ray bursts until the launch of the Compton Gamma Ray Observatory (CGRO) which operated between 1991 and 2000 (Fishman & Meegan (1995)). The CGRO satellite recorded over 2700 gamma ray bursts with the Burst And Transient Source Experiment (BATSE) (Fishman & Johnson (1989)). This experiment showed that the GRBs were isotropically distributed across the sky and originating outside of our galaxy. The BATSE results also showed the existence of two distinct populations of GRBs referred to as short and long GRBs (Kouveliotou et al.

(1993)). The former category typically last for less than 2 seconds and the latter for more than 2 seconds with distribution peaks at 0.2 seconds and 20 seconds respectively. The two types of bursts clearly had different origins. The main progenitors considered were supernovae and compact-object binaries with at least one neutron star (Levan et al. (2016)). It was found that the long bursts happened in star-forming galaxies and could usually be associated with a type Ic core-collapse supernova, whereas the short bursts could not (Hjorth (2013)). This meant that while supernovae were the likely progenitor for long bursts, the progenitor for short bursts must be different. The prevailing model for the short bursts was either a BNS or NSBH merger, although the latter has not yet been detected. It was only until GW170817 and the associated GRB 170817A that the BNS progenitor model for the short bursts was confirmed. About 1.7 seconds after the gravitational wave hit earth, FERMI and INTEGRAL registered a gamma ray burst which was short in duration (2.0s) but sub-luminous ($3x10^{46} ergs/s$) compared to other charted short GRBs (Abbott et al. (2017a;c)). The 1.7s delay was lucky because it allowed LIGO to place an upper bound on the speed of gravity, with calculations showing a relative difference between the speed of light and speed of gravity of about $\sim 7x10^{-16}$ (Abbott et al. (2017a)). While the sub-luminosity was mysterious, later analysis showed that it could be readily explained by an off-axis jet (20-40 degrees) and thus, if we had viewed GRB 170817A head-on it would have been as luminous as the other charted sGRBs.

On top of solving two mysteries, constraining the EOS and the speed of gravity, GW170817 also put constraints on the expansion rate of the universe, H_0 (Collaboration et al. (2017)). With so much science from one detection, the future appears very bright for gravitational wave astronomy and it is clear why BNS are called Einstein's Richest Laboratory (not just because of the 100 octillion U.S. dollars worth of gold produced in the ejecta!) (Baiotti & Rezzolla (2016)). However, on the numerical side, a lot still needs to be done. Despite continuous developments, current numerical relativity codes have not yet reached the accuracy required to model the gravitational wave signal and the electronmagnetic counterparts at the level required to extract as much information as possible from future detections (see (Baiotti & Rezzolla (2016)). High accuracy is required because small numerical errors in the initial data solves or the evolution lead to larger errors in the waveform (see e.g. Tsokaros et al. (2016)). High-accuracy is very difficult to obtain currently because of the multi-scale nature of the problem, we have to have high enough resolution to resolve the Neutron star and the MRI instability ($\sim 100 - 200m$ and $\sim 10cm$ respectively), but a large enough grid to extract the waveforms far away from the remnant ($1600km$). On top of this, most codes do not take into account all of the microphysics relevant to the evolution of the post-merger remnant, including a hot nuclear-theory based equation of state, a neutrino transport scheme accounting for both neutrino-matter and neutrino-neutrino interactions, and the evolution of the magnetic fields with enough resolution to resolve the growth of magneto-hydrodynamics instabilities (Foucart et al. (2015a)). That being said, slowly but surely, different levels of micro-physics are being added to the codes. The first papers that studied fully general relativistic BNS simulations including the effects of neutrinos were (Neilsen et al. (2014); Palenzuela et al. (2015)) with a simple leakage scheme and (Sekiguchi et al. (2015)) with a more complex M1 neutrino transport scheme. Both the leakage scheme and M1 transport scheme are approximate methods to deal with neutrinos which are preferred because solving the 7-dimensional Boltzmann transport equation for the neutrino distribution during a BNS simulation is currently numerically intractable. For a review of leakage and the M1 transport scheme see (Foucart et al. (2015b)). These BNS simulations with neutrino cooling and neutrino transport have focused mainly on equal mass systems with $M_{ns} = 1.35M_{\odot}$. Collectively these papers find that the ejected mass is only substantial enough to explain the total mass of r-process heavy elements in our galaxy for r-process nucleo-synthesis in the case of a softer equation of state (e.g., more compact stars). The first paper on BNS mergers with neutrino interactions using the SpEC code looked at $1.2M_{\odot}$ equal mass systems and compared a simple leakage cooling neutrino scheme with the more complicated gray M1 neutrino transport scheme, finding that the more realistic transport scheme had a significant affect on the disk composition and the outflows, producing more neutron rich material that could possibly seed r-process element creation. (Radice et al. (2016)) examined the effects of eccentricity and neutrino cooling on the matter outflows and remnant disk of a LS220 equal mass binary, finding that both had significant effects, with the absence of a neutrino scheme leading to matter outflows a factor of 2 off. Finally, only very recently have there been studies examining the effects on matter outflows due to mass asymmetry in the initial binaries, with both (Lehner et al. (2016)) and (Sekiguchi et al. (2016)) finding that mass asymmetry can affect the neutron-richness and total ejecta for both soft and stiff equations of state.

In Chapter 3 of this thesis, we will be contributing to the above growing set of BNS studies, by looking at the effect of mass asymmetry and different EOSs on matter and neutrino emissions. We will be using the SpEC code by the SXS collaboration (black-holes.org) which now has a state-of-the-art neutrino transport scheme.

1.2 Simulating BNS: General relativistic hydrodynamics

In the previous section we gave a broad overview of the physics of binary neutron star systems and the computational work to simulate their merger. In this section we take a closer look at the equations involved to simulate BNS systems, namely, the Einstein field equations and the equations of relativistic hydrodynamics.

In their usual form, space and time are treated on equal footing in the Einstein equations. From the perspective of performing a numerical evolution however, we need to reformulate the problem as an initial value problem where we have a set of initial gravitational and matter data at some time t, and a set of evolution equations which we can use to get the updated data at a later time. To do this, we make the ansatz that space-time can be treated as a time sequence of spatial hypersurfaces. With this ansatz, the space-time metric $g_{\mu\nu}$ can be decomposed into

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt), \tag{1.1}$$

where the spatial metric γ_{ij} is a function of the spatial coordinates x^i and t, α is the lapse function that measures proper time between neighboring hypersurfaces along their timelike unit normals n^μ and β^i is called the shift, which determines how coordinate labels move between each hypersurface. This is known as the 3+1 decomposition of the metric (Arnowitt et al. (1959)).

Now we need to decompose the Einstein field equations into a set of evolution and constraint equations involving the quantities β^i, α and γ_{ij}. To do this, we introduce the projection operator $\perp^\alpha_\beta = \delta^\alpha_\beta + n^\alpha n_\beta$, which can be easily proven to project the space-time components orthogonal to n^μ out of any space-time vector.

The three possible projections of the Einstein field equations: $n^\mu n^\nu G_{\mu\nu} = 8\pi n^\mu n^\nu T_{\mu\nu}$, $n^\mu \perp^\nu_\delta G_{\mu\nu} = 8\pi n^\mu \perp^\nu_\delta T_{\mu\nu}$, $\perp^\mu_\rho \perp^\nu_\delta G_{\mu\nu} = 8\pi \perp^\mu_\rho \perp^\nu_\delta T_{\mu\nu}$ lead to the three York/ADM 3+1 equations respectively:

$$0 = R + K^2 - K^{mn}K_{mn} - 16\pi\rho, \tag{1.2}$$

$$0 = D_i K - D_m K^m_i + 8\pi j_i, \tag{1.3}$$

$$\begin{aligned} \partial_t K_{ij} = {} & \beta^m \partial_m K_{ij} + K_{mj}\partial_i \beta^m + K_{im}\partial_j \beta^m - D_i D_j \alpha \\ & + \alpha(R_{ij} + KK_{ij} - 2K_{im}K^m_j) + 4\pi\alpha \left[(S-\rho)\gamma_{ij} - 2S_{ij}\right], \end{aligned} \tag{1.4}$$

where $K_{ij} = \beta^m \partial_m \gamma_{ij} + \gamma_{mj}\partial_i \beta^m + \gamma_{im}\partial_j \beta^m - \partial_t \gamma_{ij}$, is called the extrinsic curvature and it measures the rate at which the hypersurface deforms as it is carried forward along a normal (Baumgarte & Shapiro (2010)). We also introduced the projections of the stress tensor: $\rho = T_{\mu\nu}n^\mu n^\nu$, $j_\alpha = -\perp^\nu_\alpha T_{\mu\nu}n^\nu$, $S_{\alpha\beta} = \perp^\mu_\alpha \perp^\nu_\beta T_{\mu\nu}$ and $S = \gamma^{\mu\nu}S_{\mu\nu}$, while R_{ij} and R denote the Ricci tensor and scalar associated with γ_{ij}.

The 3+1 equations are a set of 10 equations. Equation 1.2 is known as the Hamiltonian constraint equation. Equation 1.3 is called the momentum constraint equation and is composed of three equations. In total, these four elliptic constraint equations play a similar role as the equations $\nabla \cdot \vec{E} = 4\pi\rho$ and $\nabla \cdot \vec{B} = 0$ which constrain the initial $\vec{E}$ and $\vec{B}$ fields in Maxwell's equations. The constraint equations must be solved prior to evolving the initial data and are usually called the **initial data equations**. The last set of six equations, Eqn. 1.4, are the evolution equations. As they stand, the 3+1 equations are ill-posed and still need to be manipulated a bit in order to discretize and solve them on a computer. For the set of evolution equations, the most used schemes are BSSN (Baumgarte & Shapiro (1998); Shibata & Nakamura (1995)), Z4 (Bona et al. (2003)) and the Generalized Harmonic Decomposition (Pretorius (2005b); technically, the Generalized Harmonic decomposition doesn't start from the ADM equations, see Chapter 3 for more details) which each manipulate the evolution equations into a slightly different well-posed hyperbolic system. For the initial data equations, the most used schemes are conformal tranverse traceless (CTT; Bowen & York Jr (1980)), conformal-thin sandwich (CTS; York Jr (1999)) and the extended-conformal thin-sandwich (XCTS; Pfeiffer & York Jr. (2005)) frameworks.

Alongside the field-equations, the matter equations must be solved. These come from the local conservation equations $\nabla_\mu T^{\mu\nu} = 0$ (this changes for the case of radiation-hydrodynamics, see Chapter 3). For Neutron-star matter, a perfect-fluid tensor and an irrotational velocity distribution are usually assumed and are coupled with some choice of equation of state (EOS) to complete the systemFaber & Rasio (2012). For black-holes, we set the source terms $\rho = j_\alpha = S_{\alpha\beta} = 0$ because there is only vacuum space. On top of the matter-equations, the equations of neutrino transport must be solved, we will look at this in detail in Chapter 3. Furthermore, if magnetic fields are included, we must also solve the equations of magnetohydronamics (Baumgarte & Shapiro (2003); Shibata & Sekiguchi (2012)).

To extract the gravitational waveform, the Weyl scalar ψ_4, which represents the outgoing transverse radiation, is extracted at a large radius away from the simulated binary system (the "wave-zone"). The energy, linear momentum, and angular momentum of the gravitational wave are computed by integrating the ψ_4 scalar in time (Kyutoku et al. (2015)).

For a more in-depth review, see (Sperhake (2014)), (Faber & Rasio (2012)) and (Shibata & Taniguchi (2011)) for BBH, NSNS and NSBH systems respectively. In the next section we discuss problems that are currently plaguing solvers for the initial data equations and then in the subsequent section we discuss new numerical methods that we plan to use to fix these problems.

1.3 Simulating BNS: Current problems

To solve the hydrodynamics equations $\nabla_\mu T_{\mu\nu} = 0$ for a binary neutron star simulation, we require an equation of state (EOS). In its most general form the term equation of state is used for any relation between thermodynamic state variables. For the case of a hydrodynamics description of a neutron star fluid, a equation of state relates the pressure of the fluid P, to it's density ρ, electron fraction Y_e, and temperature T. In simulations there are two current approaches to modeling neutron stars with temperature-dependent realistic EOSs in binaries. The first is to use EOSs in tabulated form (a set of (P, ρ, Y_e, T) points) and to interpolate between tabular points as required during the simulation. Since this can be computationally expensive, more approximate approaches have been tried. The most common approximant is to use n piece-wise polytropes with a polytropic thermal correction (Deaton et al. (2013)), (Kyutoku (2013)), (Bauswein et al. (2014)), (Kyutoku et al. (2015)). Following this approach, the cold ($T \approx 0$) piece-wise polytropic part of the EOS is defined as follows

$$P_{cold} = K_i \rho^{\Gamma_i}, \quad \epsilon_{cold} = \epsilon_i + \frac{K_i \rho^{\Gamma_i - 1}}{\Gamma_i - 1}, \tag{1.5}$$

where P, ρ, ϵ, K and Γ are the pressure, rest-mass density, specific internal energy, polytropic constant and polytropic index respectively, with i denoting the $i-th$ polytropic piece. It was found that four polytropic pieces ($n = 4$) will approximate most EOSs of cold neutron matter to good accuracy (Read (2008)). However, a piecewise-polytopic EOS is only valid for neutron stars with a temperature $T \approx 0$. In neutron star mergers, temperatures can reach up to $T \sim 50MeV$, so a polytropic EOS would not be realistic. To include thermal effects when using piecewise polytropes, one splits up the pressure $P = P_{cold} + P_{th}$ and internal energy $\epsilon = \epsilon_{cold} + \epsilon_{th}$ and assumes an ideal gas relationship $P_{th} = \rho\epsilon_{th}(\Gamma_{th} - 1)$ where the ideal gas index Γ_{th} is constant for all ρ and ϵ and usually set to 2. (Bauswein et al. (2010)),(Takami et al. (2014)).

The main issue with solving the Einstein equations and matter equations with realistic EOSs in tabulated or with an approximate piecewise-analytic form is that the EOS are not smooth due to the multiple phase transitions and the non-analytic behavior or very steep slopes at the surface of the NS. For example, let's consider the LS220 EOS (a commonly used phenomenological nuclear matter EOS for astrophysical application, see Lattimer & Swesty (1991)) at low temperature and electron fraction. Near the NS center the EOS can be approximated by a stiff $\Gamma \approx 7/2$ polytrope and then at slightly lower densities ($\approx 10^{14} g/cm^3$) the EOS begins to soften with $\Gamma \approx 1/2$ and then at even lower densities, the EOS asymptotically approaches the adiabatic index of a relativistic Fermi gas, $\Gamma \approx 4/3$. In certain coordinates, these changes can happen very close to the neutron star surface and are difficult to resolve (Deaton et al. (2013)). Furthermore, initial data solves for BNS and NSBH binaries have traditionally used multi-domain spectral finite element methods such as the SpEC elliptic solver Spells (Pfeiffer et al. (2003)) and LORENE (Gourgoulhon et al. (2001)). Spectral methods have the nice feature of exponential convergence when the underlying problem is smooth. However, when the problem is non-smooth such as is the case when we consider tabulated EOSs or piecewise-analytic EOS fits (e.g. piecewise-polytropes), spectral methods show poor convergence. In order to retain convergence, subdomains are often inserted by hand around the non-analytic parts (Deaton et al. (2013)).

Finally, for currently unknown reasons, the elliptic solver Spells (Pfeiffer et al. (2003),Foucart et al. (2008),Henriksson et al. (2014)) used by the SXS collaboration (black-holes.org) does not converge for high compactness NS initial-data without multiple extra corrective iteration schemes and even then, it cannot converge to high accuracy data when the binary objects are very close in separation (Henriksson et al. (2014)). This lack of convergence could be due to mathematical non-uniqueness in solutions of the XCTS formulation of the Einstein constraint equations (Cordero-Carrión et al. (2009)), but (Henriksson et al. (2014)) hints at the possibility that the compactness can be pushed far beyond previous simulations by using refined iteration schemes. In the next section we discuss a new powerful numerical technique that might provide us with a way around these problems.

1.4 Simulating BNS: New numerical methods

In the last section we outlined problems with current numerical solution techniques when handling binary neutron star initial data. In this section, we look at the current numerical techniques in detail and then describe a new method called discontinuous Galerkin that combines the best aspects of these current techniques. Discontinuous Galerkin (DG) methods will allow us to go beyond known problems with current binary neutron star initial data. To introduce DG methods, we must first look at the standard numerical techniques used today in numerical relativity. The most common methods used in Numerical Relativity to solve the evolution or initial data equations are finite difference, finite volume methods and spectral methods (Baumgarte & Shapiro (2010)). We introduce all three methods by examining the following PDE:

$$R(\bar{u}) = 0. \tag{1.6}$$

Here R is some partial differential operator, usually referred to as the residual, that operates on the solution u which could be composed of several fields we are solving for (e.g. the four fields in the initial data equations or the 10 components of the metric). Equation 1.6 represents a continuous equation which must be discretized in order to be solved on a computer. Finite difference, finite volume and spectral methods are different ways of taking continuous equations and making them discrete and therefore they are called discretization methods. Each of the different discretization methods used in numerical relativity demand in a different way, that the discrete version of Eqn. 1.6 be zero. In the finite difference method we set $R(u)$ to be zero at each of the grid points and then we Taylor expand the differential operators and represent the solution as a single number at each grid point. In finite volume methods we set the integral of the residual over an element (usually called a “cell”) of a mesh to be zero in each of the elements and we represent the solution as a single number per cell (the average of the solution over that cell). In the spectral finite element method we break up the domain into a set of elements with simple topologies and expand the solution over a set of basis functions. We demand that the

L_2-inner product between the residual and each of these basis functions is zero. That is, on each subdomain we have

$$\int R(\bar{u})\psi_j \mathrm{d}x = 0 \quad \forall \psi_j. \tag{1.7}$$

Recently, a new discretization method has become popular. Discontinuous Galerkin (DG) methods (Reed & Hill (1973); Hesthaven & Warburton (2008); Cockburn (2001); Cockburn & Shu (1998); Cockburn (1998); Cockburn et al. (2000)) combine the power of spectral methods in smooth regions with the shock-handling features of finite volume methods in discontinuous regions. To accomplish this, DG methods represent the solution on each element as an expansion over a set of basis functions with the residual satisfying Eqn. 1.7. To couple the solution between elements, DG methods use flux terms between neighboring elements which penalize deviation of the solution at the interface. These flux terms allow DG methods to borrow the discontinuity handling techniques of the finite volume method. With these features, DG methods can potentially obtain exponential convergence even when the solution is not smooth over the entire grid (see Chapter 2 for more details). On top of this, DG methods allow us to implement several nice features:

1. hp-adaptivity: In DG methods there are two types of refinement, you can increase the number of basis functions (p-refinement), or you can split an element of the mesh into smaller elements (h-refinement). In smooth regions, p-refinement is preferred and in non-smooth regions h-refinement is preferred. The combination of these two types of refinement (hp-refinement) can lead to rapid convergence of the solution.

2. Minimal communication: Since each element only needs the nearest-neighbour face data to compute the penalty flux terms, the amount of communication between processors is minimal.

3. Easy Boundary handling: Boundary elements can be treated just like interior elements with the boundary conditions being applied through the penalty terms in the flux. This

makes algorithms like Multigrid much easier to implement, since no special treatment is required for boundary elements.

4. Easy geometry handling: Unlike finite-difference methods which require special finite difference stencils for the boundaries of curved domains, DG methods can be applied as is to any type of domain, without any significant changes.

In the next Chapter we will outline a novel discontinuous Galerkin code we developed. We will showcase all of the features listed above by solving several problems in numerical relativity with our code. The future goal of this code will be to solve for BNS initial data with realistic EOS, spins and mass-ratios to high accuracy.

Chapter 2

A hp-adaptive discontinuous Galerkin solver for elliptic equations in numerical relativity

2.1 Chapter Overview

The material in this chapter is based on "A hp-adaptive discontinuous Galerkin solver for elliptic equations in numerical relativity" by Trevor Vincent, Harald Pfeiffer, Nils Fischer being prepared for Phys. Rev. D.

Chapter 3

Unequal Mass Binary Neutron Star Simulations with M1 Neutrino Transport: Ejecta and Neutrino Emission

3.1 Chapter Overview

The material in this chapter is based on ”Unequal Mass Binary Neutron Star Simulations with M1 Neutrino Transport:
Ejecta and Neutrino Emission" by Trevor Vincent, Francois Foucart, Roland Haas, Matthew Duez, Lawrence Kidder, Harald Pfeiffer, Mark Scheel being prepared for Phys. Rev. D.

Chapter 4

Conclusions & Future Work

4.1 Conclusions

It is an exciting time in gravitational-wave astrophysics and numerical relativity. With the first detection of gravitational waves from a a binary neutron star and the coincident detection of electromagnetic counterparts across the EM spectrum, a lot can be discovered about these extreme systems. With many more detections on the horizon, we will need to be prepared on the theoretical side. To help with the ongoing theoretical effort of properly modelling binary compact object systems and their emissions, this thesis has attempted to make progress along two different directions.

Firstly, we have sought to improve the numerical techniques used to solve the partial differential equations that arise in binary neutron star simulations. To this effect, Chapter 2 presented a completely new scheme based on discontinuous Galerkin methods, and we tested it to solve for constant density star initial data, which contains phase transitions similar to that in neutron stars. We also tested the code on multi-black hole puncture initial data, which contains multiple non-smooth points. All tests showed promising results. Currently, work is being done to extend this code by Nils Fischer and Prof. Harald Pfeiffer at the Albert Einstein Institute (AEI) in Potsdam, Germany by porting it into the task based parallel code SpECTRE (Kidder et al. (2016)), which is currently being developed at Cornell, Caltech and AEI to solve hyperbolic problems in numerical relativity on future exascale supercomputers using a task-based parallelism framework. The final goal will be to run our implementation on binary neutron star initial data with realistic microphysics and solve down to accuracies previously unobtainable.

Secondly, we wanted to increase the knowledge-base surrounding emissions from BNS mergers and their connection to binary parameters. Chapter 3 presented 12 state-of-art general relativistic radiation-hydrodynamics simulations of binary neutron star mergers with varying realistic EOS and mass-ratios. With these simulations we established that previous results using different codes were qualitatively correct, even though our neutrino schemes (amongst other things) were not identical. Talk is under way to use these simulations for new studies. Firstly, the subset of simulations which collapsed to a black hole were not evolved past this point and it would be interesting to see what emissions arose afterwards and to study the properties and evolution of the resulting accretion disks, which have been shown to produce a significant amount of ejecta when MHD is introduced (Fernandez et al. (2019)). Secondly, we initially ran each simulation with tracer particles that followed the fluid flow, but stopped this because it was slowing down the code too much to get a report out on time for this thesis. Tracer particles would allow us to better study the properties of the ejecta, by not only tracing out the fluid worldlines of the ejecta, but also by using the tracer data to seed light-curve simulations and nuclear reaction networks. All of this was done for neutron star - black-hole binaries in (Fernández et al. (2016)) by some of our collaborators. In terms of other source parameters, We also neglected in our study the eccentricity of the binary and the spins of the neutron stars, which can produce significantly more ejecta. In the future, the addition of magneto-hydrodynamics, eccentricity, spins and more realistic neutrino transport schemes will definitely be on the agenda.

In conclusion, while this thesis has helped push us closer to the goal of simulating and understanding binary neutron star mergers, a great deal of work still needs to be done. The next few decades should be a very interesting period for both numerical relativity and gravitational wave astrophysics, so stay tuned.

Bibliography

Abbott, B. P., et al. 2016a, Physical Review X, 6, 041014

—. 2016b, Phys. Rev. Lett., 116, 241102

—. 2017a, Astrophys. J., 848, L13

—. 2017b, Physical Review Letters, 119, 161101

—. 2017c, Astrophys. J., 848, L12

—. 2017d, Astrophys. J., 851, L16

Acernese, F., et al. 2015, Classical and Quantum Gravity, 32, 024001

Affeldt, C., et al. 2014, Classical and quantum gravity, 31, 224002

Arnowitt, R., Deser, S., & Misner, C. W. 1959, Physical Review, 116, 1322

Baiotti, L., & Rezzolla, L. 2016, ArXiv e-prints

Baiotti, L., & Rezzolla, L. 2016, arXiv preprint arXiv:1607.03540

Baumgarte, T. W., & Shapiro, S. L. 1998, Physical Review D, 59, 024007

—. 2003, The Astrophysical Journal, 585, 921

—. 2010, Numerical relativity: solving Einstein's equations on the computer (Cambridge University Press)

Bauswein, A., Baumgarte, T., & Janka, H.-T. 2013, Physical review letters, 111, 131101

Bauswein, A., Janka, H.-T., & Oechslin, R. 2010, Physical Review D, 82, 084043

Bauswein, A., & Stergioulas, N. 2019, arXiv preprint arXiv:1901.06969

Bauswein, A., Stergioulas, N., & Janka, H.-T. 2014, Physical Review D, 90, 023002

Bona, C., Ledvinka, T., Palenzuela, C., & Žáček, M. 2003, Physical Review D, 67, 104005

Bowen, J. M., & York Jr, J. W. 1980, Physical Review D, 21, 2047

Chornock, R., et al. 2017, "Astrophys. J. Lett.", 848, L19

Cockburn, B. 1998, in An introduction to the Discontinuous Galerkin method for convection-dominated problems; in Advanced Numerical Approximation of Nonlinear Hyperbolic Equations: Lectures given at the 2nd Session of the Centro Internazionale Matematico Estivo (C.I.M.E.) held in Cetraro, Italy, June 23–28, 1997, ed. A. Quarteroni, Lecture Notes in Physics (Berlin, New York: Springer), 150–268

Cockburn, B. 2001, J. Comput. Appl. Math., 128, 187

Cockburn, B., Karniadakis, G. E., & Shu, C.-W. 2000, in Discontinuous Galerkin Methods: Theory, Computation and Applications, ed. B. Cockburn, G. E. Karniadakis, & C.-W. Shu, Lecture Notes in Computational Science and Engineering (Berlin, New York: Springer), 3–50

Cockburn, B., & Shu, C.-W. 1998, J. Comput. Phys., 141, 199

Collaboration, L. S., et al. 2017, Nature, 551, 85

Cordero-Carrión, I., Cerdá-Durán, P., Dimmelmeier, H., Jaramillo, J. L., Novak, J., & Gourgoulhon, E. 2009, Physical Review D, 79, 024017

Cowperthwaite, P. S., et al. 2017, Astrophys. J., 848, L17

Davies, M., Benz, W., Piran, T., & Thielemann, F. 1994, arXiv preprint astro-ph/9401032

Deaton, M. B., et al. 2013, The Astrophysical Journal, 776, 47

Faber, J. A., & Rasio, F. A. 2012, Living Rev. Relativity, 15

Fernández, R., Foucart, F., Kasen, D., Lippuner, J., Desai, D., & Roberts, L. F. 2016, arXiv preprint arXiv:1612.04829

Fernández, R., & Metzger, B. D. 2013, Mon. Not. Roy. Astr. Soc., 435, 502

Fernandez, R., Tchekhovskoy, A., Quataert, E., Foucart, F., & Kasen, D. 2019, Bulletin of the American Physical Society

Fishman, G., & Johnson, W. 1989, Proc. GRO Science Workshop

Fishman, G. J., & Meegan, C. A. 1995, Annual Review of Astronomy and Astrophysics, 33, 415

Foucart, F., Kidder, L. E., Pfeiffer, H. P., & Teukolsky, S. A. 2008, Physical Review D, 77, 124051

Foucart, F., et al. 2015a, arXiv preprint arXiv:1510.06398

—. 2015b, arXiv preprint arXiv:1502.04146

Freiburghaus, C., Rosswog, S., & Thielemann, F.-K. 1999, The Astrophysical Journal Letters, 525, L121

Gourgoulhon, E., Grandclement, P., Taniguchi, K., Marck, J.-A., & Bonazzola, S. 2001, Physical Review D, 63, 064029

Henriksson, K., Foucart, F., Kidder, L. E., & Teukolsky, S. A. 2014, arXiv preprint arXiv:1409.7159

Hesthaven, J., & Warburton, T. 2008, Nodal Discontinuous Galerkin Methods: Algorithms, Analysis, and Applications, Texts in Applied Mathematics (Springer)

Hinderer, T., Lackey, B. D., Lang, R. N., & Read, J. S. 2010, Physical Review D, 81, 123016

Hjorth, J. 2013, Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, 371, 20120275

Hotokezaka, K., Kiuchi, K., Kyutoku, K., Okawa, H., Sekiguchi, Y., Shibata, M., & Taniguchi, K. 2013, Phys. Rev. D, 87, 024001

Hulse, R. A., & Taylor, J. H. 1975, Astrophys. J., 195, L51

Kidder, L. E., et al. 2016, arXiv preprint arXiv:1609.00098

Klebesadel, R. W., Strong, I. B., & Olson, R. A. 1973, The Astrophysical Journal, 182, L85

Kouveliotou, C., Meegan, C. A., Fishman, G. J., Bhat, N. P., Briggs, M. S., Koshut, T. M., Paciesas, W. S., & Pendleton, G. N. 1993, The Astrophysical Journal, 413, L101

Kyutoku, K. 2013, The Black Hole-Neutron Star Binary Merger in Full General Relativity: Dependence on Neutron Star Equations of State (Springer Science & Business Media)

Kyutoku, K., Ioka, K., Okawa, H., Shibata, M., & Taniguchi, K. 2015, arXiv preprint arXiv:1502.05402

Lattimer, J. M., & Schramm, D. N. 1974, The Astrophysical Journal, 192, L145

Lattimer, J. M., & Swesty, F. D. 1991, Nuclear Physics A, 535, 331

Lehner, L., Liebling, S. L., Palenzuela, C., Caballero, O., O'Connor, E., Anderson, M., & Neilsen, D. 2016, arXiv preprint arXiv:1603.00501

Levan, A., Crowther, P., de Grijs, R., Langer, N., Xu, D., & Yoon, S.-C. 2016, Space Science Reviews, 202, 33

Li, L.-X., & Paczynski, B. 1998, Astrophys. J., 507, L59

LIGO, Virgo, et al. 2018, arXiv preprint arXiv:1811.12907

McCully, C., et al. 2017, "Astrophys. J. Lett.", 848, L32

Metzger, B. D. 2016, arXiv:1610.09381

Metzger, B. D. 2017, Living Reviews in Relativity, 20, 3

Metzger, B. D., & Fernández, R. 2014, Mon. Not. Roy. Astr. Soc., 441, 3444

Metzger, B. D., et al. 2010, Mon. Not. Roy. Astr. Soc., 406, 2650

Nagakura, H., Hotokezaka, K., Sekiguchi, Y., Shibata, M., & Ioka, K. 2014, Astrophys.J., 784, L28

Neilsen, D., Liebling, S. L., Anderson, M., Lehner, L., O'Connor, E., & Palenzuela, C. 2014, Physical Review D, 89, 104029

Nicholl, M., et al. 2017, "Astrophys. J. Lett.", 848, L18

Oechslin, R., & Janka, H.-T. 2006, Monthly Notices of the Royal Astronomical Society, 368, 1489

Palenzuela, C., Liebling, S. L., Neilsen, D., Lehner, L., Caballero, O., O'Connor, E., & Anderson, M. 2015, arXiv preprint arXiv:1505.01607

Pfeiffer, H. P., Kidder, L. E., Scheel, M. A., & Teukolsky, S. A. 2003, Computer physics communications, 152, 253

Pfeiffer, H. P., & York Jr., J. W. 2005, Phys. Rev. Lett., 95, 091101

Pretorius, F. 2005a, Phys. Rev. Lett., 95, 121101

—. 2005b, Classical and Quantum Gravity, 22, 425

Punturo, M., et al. 2010, Class. Quantum Grav., 27, 084007

Radice, D., Galeazzi, F., Lippuner, J., Roberts, L. F., Ott, C. D., & Rezzolla, L. 2016, arXiv preprint arXiv:1601.02426

Radice, D., Perego, A., & Zappa, F. 2017, arXiv preprint arXiv:1711.03647

Raithel, C. A. 2019, arXiv preprint arXiv:1904.10002

Read, J. S. 2008, Neutron stars in compact binary systems: From the equation of state to gravitational radiation (ProQuest)

Reed, W., & Hill, T. 1973, Triangular mesh methods for the neutron transport equation, Conference: National topical meeting on mathematical models and computational techniques for analysis of nuclear systems, Ann Arbor, Michigan, USA, 8 Apr 1973 LA-UR–73-479, CONF-730414–2

Rezzolla, L., & Zanotti, O. 2013, Relativistic hydrodynamics (Oxford University Press)

Rosswog, S., Liebendörfer, M., Thielemann, F.-K., Davies, M., Benz, W., & Piran, T. 1998, Arxiv preprint astro-ph/9811367

Ruffert, M., Janka, H.-T., Takahashi, K., & Schaefer, G. 1996, arXiv preprint astro-ph/9606181

Sachdev, S., et al. 2019, arXiv preprint arXiv:1901.08580

Sekiguchi, Y., Kiuchi, K., Kyutoku, K., & Shibata, M. 2015, Physical Review D, 91, 064059

Sekiguchi, Y., Kiuchi, K., Kyutoku, K., Shibata, M., & Taniguchi, K. 2016, arXiv preprint arXiv:1603.01918

Shibata, M. 2015, Numerical Relativity, 100 Years of General Relativity (World Scientific Publishing Company)

Shibata, M., Kyutoku, K., Fujibayashi, S., Tanaka, M., Hotokezaka, K., Sekiguchi, Y., & Kiuchi, K. 2017, GW170817: Modeling based on numerical relativity and its implications, Tech. rep.

Shibata, M., & Nakamura, T. 1995, Physical Review D, 52, 5428

Shibata, M., & Sekiguchi, Y. 2012, Progress of theoretical physics, 127, 535

Shibata, M., & Taniguchi, K. 2011, Living Rev. Relativity, 14

Shibata, M., & Uryū, K. 2000, Physical Review D, 61, 064001

—. 2006, Physical Review D, 74, 121503

Soares-Santos, M., et al. 2017, "Astrophys. J. Lett.", 848, L16

Somiya, K. 2012, Classical and Quantum Gravity, 29, 124007

Sperhake, U. 2014, arXiv preprint arXiv:1411.3997

Symbalisty, E., & Schramm, D. N. 1982, Astrophysical Letters, 22, 143

Takami, K., Rezzolla, L., & Baiotti, L. 2014, Physical review letters, 113, 091104

Takami, K., Rezzolla, L., & Baiotti, L. 2015, Phys. Rev. D, 91, 064001

Tsokaros, A., Mundim, B. C., Galeazzi, F., Rezzolla, L., & Uryū, K. 2016, Physical Review D, 94, 044049

Villar, V. A., et al. 2017, Astrophys. J., 851, L21

Weisberg, J. M., Taylor, J. H., & Fowler, L. A. 1981, Scientific American, 245, 74

York Jr, J. W. 1999, Physical review letters, 82, 1350

Yu, Y.-W., Liu, L.-D., & Dai, Z.-G. 2018, The Astrophysical Journal, 861, 114