y_k := prob. that a random node of degree k is <u>not</u> part of the GCC

= prob. that <u>none</u> of the k edges leads to the GCC

u := prob. that a random edge <u>does not</u> lead to the GCC

$$\Rightarrow y_k = u^k$$

Therefore, the prob. y that a random node (of unspecified degree) is <u>not</u> part of the GCC: $y = \sum_k p_k y_k = \sum_k p_k u^k \equiv G_0(u)$

Once we know y, the fraction S of nodes in the GCC is:

$$S = 1 - y = 1 - G_0(u)$$

We need to compute u.

$$u = \sum_k \frac{k N_k}{\langle k \rangle N} u^{k-1} = \sum_k \frac{k p_k}{\langle k \rangle} u^{k-1} \overset{k'=k-1}{=} \sum_{k'} \overbrace{\frac{(k'+1) p_{k'+1}}{\langle k \rangle}}^{q_{k'}} u^{k'}$$

$$= \sum_k q_k u^k \equiv G_1(u)$$

In summary:

$$\begin{cases} S = 1 - G_0(u) \\ u = G_1(u) \end{cases}$$

random network $\mathcal{G}(N,p)$

$$p_k = \binom{N-1}{k} p^k (1-p)^{N-1-k} \quad ; \quad \langle k \rangle = p(N-1)$$

binomial theorem

$$G_0(x) = \sum_k p_k x^k = \sum_k \binom{N-1}{k} (px)^k (1-p)^{N-1-k} = (1-p+px)^{N-1}$$

$$= \left(1-p(1-x)\right)^{N-1} \xrightarrow{N\to\infty} e^{-\langle k \rangle (1-x)} \quad , \quad \langle k \rangle \to pN$$

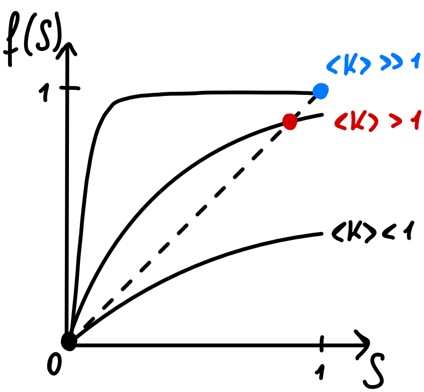
$$G_1(x) = \sum_k \frac{k p_k}{\langle k \rangle} x^{k-1} = \frac{1}{\langle k \rangle} \frac{d}{dx} G_0(x)$$

$$= \cancel{\frac{(N-1)p}{\langle k \rangle}} \left(1-p(1-x)\right)^{N-2} = \left(1-p(1-x)\right)^{N-2} \to e^{-\langle k \rangle (1-x)}$$

$$\Longrightarrow G_0(x) = G_1(x) \quad \text{for } N\to\infty$$

$S = 1 - e^{-\langle k \rangle (1-u)}$, where u is solution of $u = e^{-\langle k \rangle (1-u)}$

$$\Longrightarrow S = 1-u = 1 - e^{-\langle k \rangle S} \equiv f(S)$$



A non-zero solution emerges when:

$$\left.\frac{d f(S)}{dS}\right|_{S=0} = \left.\frac{dS}{dS}\right|_{S=0} \Longrightarrow \boxed{\langle k \rangle = 1}$$