Modeling the Solar System

Trevor Wieland

April 24, 2018

1 Introduction

Perhaps one of the most interesting aspects of ODE modeling is its wide variety of applications, across countlessly many fields. One such application of ODE modeling is in the field of physics, and can be used to describe everything from subatomic interactions to longterm cosmic patterns. In this paper, the latter is discussed, specifically pertaining to several of the important bodies in our local solar system. ODE modeling is used to predict where these important bodies will be in the future, which is important to further our understanding of how celestial bodies interact and move over time. In addition to modeling how the solar system is and will be, several scenarios that greatly change the mechanics of our solar system will also be modeled, for the sake of comparing how these changes in the solar system would impact the system as a whole.

2 Gathering Data

Due to the sheer volume of objects in the solar system, not every small asteroid could be included, for obvious computational reasons. Instead, the model was limited to just 15 major bodies, being the 8 planets and Pluto, in addition to several other large asteroids, such as Ceres, Vesta, Pallas, Hygiea, and Interamnia. The information on these planets was gathered from the NASA Horizons public solar system database, and from this database the mass, initial position, and initial velocity of each of these bodies was taken. The day used as the starting point was April 1st of 2018, and the reference point used for the positions and velocities of the bodies is the Sun, also referred to as Sol. The information used by this model can be seen below in Tables 1 and 2. Then, by taking this data, the initial conditions for our system of ODE models could be formed.

Name	Mass (10^{23}kg)	X Position (AU)	Y Position (AU)	Z Position (AU)
Sol	19885440	0	0	0
Mercury	3.302	-0.112	-0.4522	-0.0267
Venus	48.685	-0.0518	0.7174	0.0128
Earth	59.7219	-0.8974	-0.5409	2.679e-05
Mars	6.4185	-0.5453	-1.402	-0.01599
Jupiter	18981.3	-3.699	-3.9497	0.09918
Saturn	5683.19	0.6368	-10.044	0.1492
Uranus	868.103	17.514	9.4259	-0.1918
Neptune	1024.1	28.778	-8.2589	-0.4932
Pluto	.1307	11.117	-31.652	0.1703
Ceres	.08958	-2.220	1.1898	0.4467
Vesta	.02108	-0.7517	-2.011	0.1516
Pallas	.02108	-0.1357	1.805	-1.236
Hygiea	.00867	2.8659	-1.325	0.1666
Interamnia	.0039	1.298	2.292	0.5255

Table 1: Horizons Data - Mass and Initial Positions

Name	X Velocity (AU/Day)	Y Velocity (AU/Day)	Z Velocity(AU/Day)
Sol	0	0	0
Mercury	0.02166	-0.005350	-0.002424
Venus	-0.0202	-0.001565	0.001148
Earth	0.008983	-0.01456	-6.1926e-08
Mars	0.01357	-0.00387	-0.0004141
Jupiter	0.005421	-0.00481	-0.0001013
Saturn	0.005267	0.0003318	-0.0002154
Uranus	-0.001887	0.003275	3.6598e-05
Neptune	0.0008513	0.003032	-8.255e-05
Pluto	0.003046	0.0003876	-0.0009171
Ceres	-0.005144	-0.004258	0.0006375
Vesta	0.0114	-0.004258	-0.00126
Pallas	-0.01259	-0.001184	0.001873
Hygiea	0.00498	0.00825	0.0004509
Interamnia	-0.008616	0.006648	-0.002272

Table 2: Horizons Data - Initial Velocities

3 Model

For this model, Newton's equations for gravity were used, due to their strong accuracy in large scale objects moving at relatively low speeds compared to light. The equations used for this was Newton's Gravitational Equation, shown in Equation 1. This can be transformed and related to our model by using Newton's force Equation, seen in 2. Since acceleration is equal to the second derivative of position, and net force is equal to the sum of the forces, we get 3. This equation tells us that the acceleration of any given body is equal to the sum of the accelerations caused by Newton's Gravitational Equation, which given the masses and locations of these other bodies, can be quickly computed.

$$F_{gravity} = G\frac{m_1 m_2}{r^2} \tag{1}$$

$$F = ma \tag{2}$$

$$\frac{d^2 r_i}{dr_i^2} = \sum_j^N G\frac{m_j}{r_i j^2} \tag{3}$$

For this model, computation and plotting of results was done through Matlab, and because of limits to Matlab's ODE solving capabilities, two key points of the model had to be accounted for. Firstly, only a numerical solution would be achievable. Due to the relatively complex nature of the model, finding a symbolic solution became impossible, and thus just stepping through and giving approximations became the only path forward. In addition to this, Matlab's ODE solver can not natively handle second order differential equations, meaning the system of 15 second order differential equations had to be turned into a system of 30 first order differential equations, of the form seen below, where N is the number of bodies in the model, and $r_i j$ is the distance between bodies i and j:

$$\begin{cases} r_i^{'} = r_{i+N} & i \leq N \\ r_i^{'} = \sum_j^N G\frac{m_j}{r_i j^2} & i > N \end{cases}$$

This allows us to use Matlab's native capabilities to solve this system of 15 second order differential equations instead as a more simple, yet longer system of 30 first order differential equations. This also gives us an easier way of implementing our initial conditions, as the first 15 variables will have their initial conditions being the initial positions, and the latter 15 variables will have their initial conditions be the initial velocities. From there, its just a matter of using Matlab's stiff ODE solver to get a numerical estimate of where the planets will be in the future.

4 Our Solar System

In this section, the model will be put to the test to determine how realistically it models the movements of the bodies in our solar system. First, the starting conditions of the model will be shown and discussed, detailing what we expect to happen in periods of a year, 10 years, 1,000 years and 10,000 years. Then, in the second subsection, each of the bodies will have their true solar year compared with the solar year that the model predicts, to be used as another measure of the model's accuracy. Finally, in the third subsection, the model's predictions of the previously mentions year milestones will be solved and compared both with the predictions lined out in the first subsection, and with predictions from NASA's Horizons database.

4.1 Initial Set up

Due to the nature of this model, having 15 different objects with vastly different distances from the sun, all being tracked on the same graph makes those closest to the sun look as if they have no meaningful distance from it. Because of this, each of these subsections will be split into 4 small subgroups, dealing with the inner planets, the outer planets, the extra bodies, and all at once. By doing this, the relationships between the planets and how they orbit should remain much clearer than if they were all shown overlaid on top of each other.

First, to look at what conditions the model will start in in a clearer light, each of the celestial bodies' initial positions will be graphed and shown, without any of the model's predicted movement included. This way, we have a good reference point for where these orbits start, and thus can compare distances between the initial points, and their predicted locations one orbit later, which will be done in more depth in the next subsection.

4.1.1 Inner Planets

As can be seen in Figure 4.1.1, these four innermost planets are within one Astronomical Unit of the sun, and thus in the future should remain very strictly in this bound. Due to the stability of the solar system, an accurate model would show this tight constraint for the planets, with only a small margin of error. These planets nearby are the planets most local to us, and thus are the most relevant to look at when it come to modeling Earth's behavior. Also, while this graph only depicts the initial positions of the inner planets, the outer planets will still have a slight, but important effect on these planets' orbits, especially later on, in the Extra Scenarios section of this paper.

4.1.2 Outer Planets

Highly different to the inner planets shown in Figure 4.1.1, the outer planets, shown in Figure 4.1.2, are much further from the Sun. This should make us expect to see much longer periods for orbits, due to the weakened effect of gravity. The orbits of these planets should also be more elliptical, due to the same reason. In later scenarios however, specifically when Jupiter's parameters are changed, these predictions will fall through, and only the model would be able to tell us what to expect. This is the reason why having an accurate model of the solar system in this first section is important, so that in more unrealistic scenarios where no data exists, we can put some faith that the model is correct.

4.1.3 Extra Bodies

In addition to the 8 planets, there are countlessly many asteroid and meteors in areas around our solar system, which will all have some minuscule, yet interesting effect on the rest of the system. In this model, we choose 6 of the more relevant of these extra bodies, namely Pluto, Ceres, Vesta, Pallas, Hygiea, and Interamnia, which are all large asteroids or dwarf planets. These extra bodies initial conditions can be seen in Figure 4.1.3 which shows just how far away or close by these random extra bodies cna be in the solar system. While these extra bodies likely won't have much of an effect on the 8 planets, we should expect to see a larger effect on them due to the planets, since the planets are much more massive than them, compared to each other. A strong model should support this prediction, and show the orbits as more erratic than the larger, and thus more stable, planets.

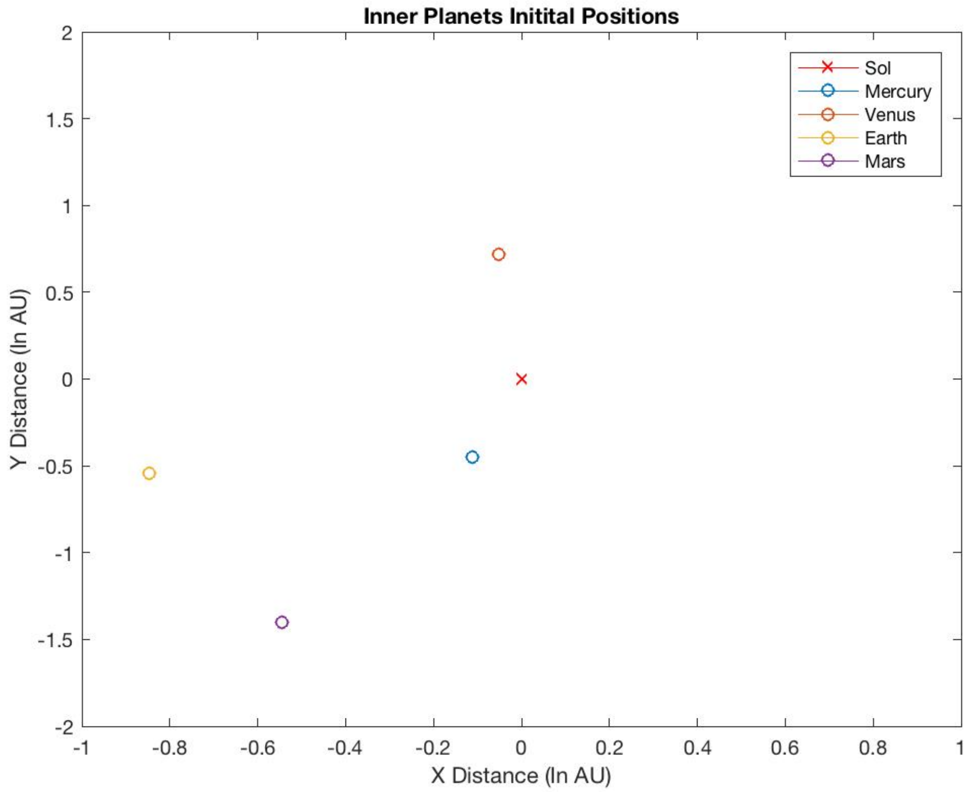

Figure 1: The four innermost planets, between the Sun and the Asteroid Belt.

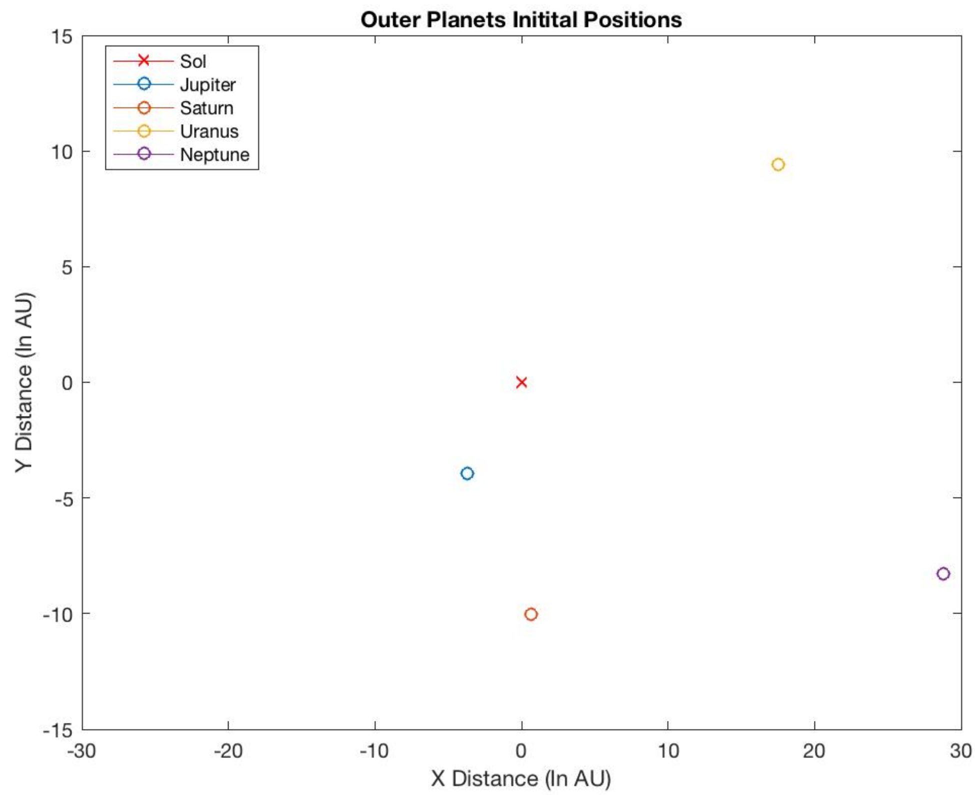

Figure 2: The four outermost planets, beyond the Asteroid Belt.

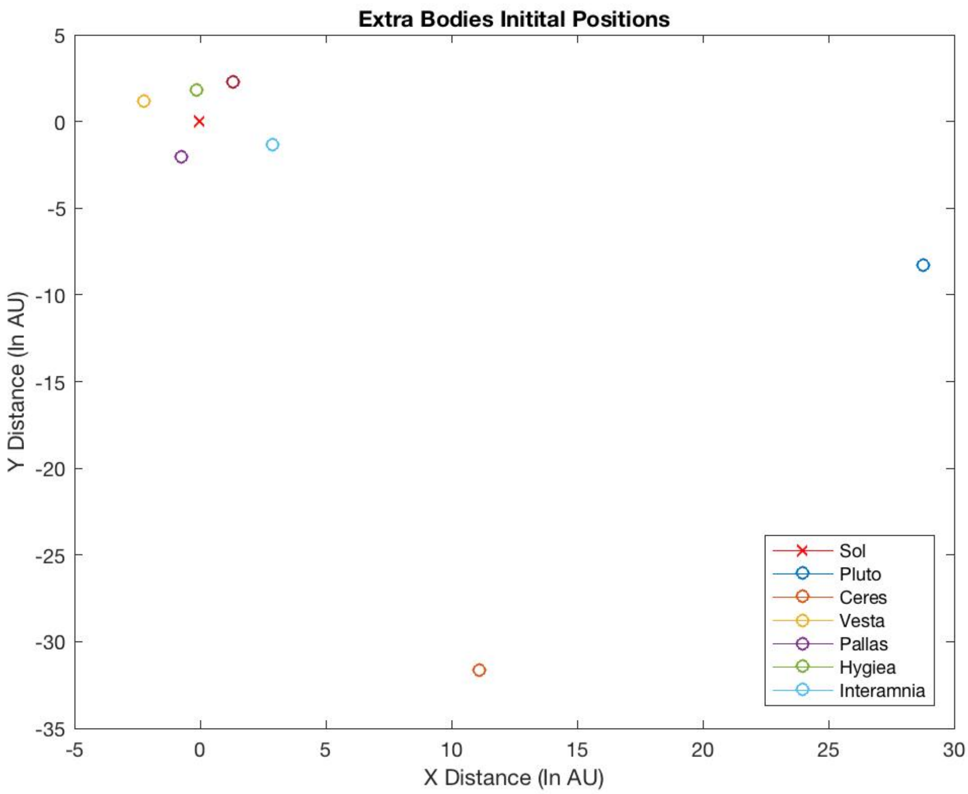

Figure 3: Other relevant celestial bodies to our solar system, including Pluto, Ceres, Vesta, Pallas, Hygiea, and Interamnia

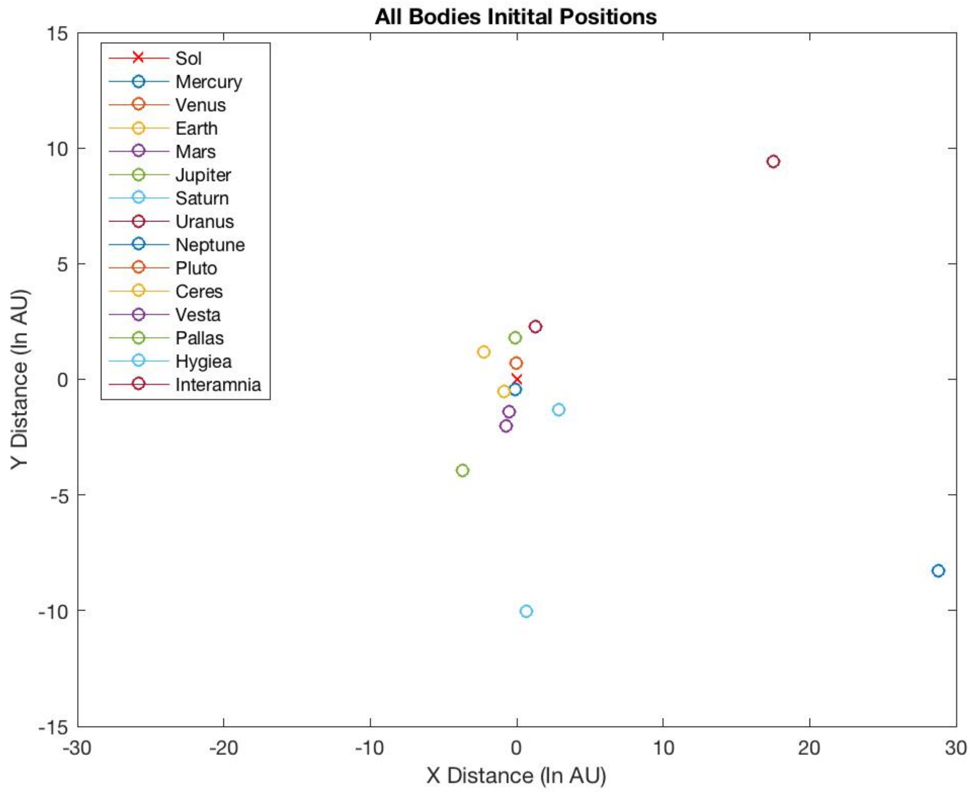

Figure 4: Each of the 15 relevant bodies are depicted in this graph, showing why it may not be a good idea to follow this form of depiction

4.1.4 All Bodies

Lastly, the reason we choose not to model every body in the solar system on one graph can be seen in Figure 4.1.4, where all of the bodies close to the sun are so close together relative to the outer planets that no meaningful information can be gained from looking at the close bodies. This will be especially true later on, when orbits are also included on the graphic, making the center near the sun close to impossible to understand. However, while meaningful information is hard to grasp from this last figure, it does serve a good purpose in illustrating the scale of the solar system, showing just how far apart everything is.

4.2 Verifying Years

4.3 Predictions

5 Alternate Scenarios

In the following subsections, a few different scenarios involving changes to our solar system will be described and their impacts modeled.

5.1 Slow Jupiter

5.2 Moved Mars

5.3 Jumbo Jupiter