Notes

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1 Problem

$$\int_{e_j} \psi \cdot t \; dx$$

where$\psi = \begin{bmatrix} B - Ay \\ C + Ax \end{bmatrix}$, and $t = \begin{bmatrix} x_2^{(j)} - x_1^{(j)} \\ y_2^{(j)} - y_1^{(j)} \end{bmatrix} \frac{1}{|e_j|}$

$$\frac{1}{|e_j|} \int_{e_j} \begin{bmatrix} B - Ay \\ C + Ax \end{bmatrix} \cdot \begin{bmatrix} x_2^{(j)} - x_1^{(j)} \\ y_2^{(j)} - y_1^{(j)} \end{bmatrix} \; dS$$

2 Parameterization

$$v(t) = \langle x_1^{(j)}, y_1^{(j)} \rangle + t \langle x_2^{(j)} - x_1^{(j)}, \; y_2^{(j)} - y_1^{(j)} \rangle$$

$$x = x_1^{(j)} + t[x_2^{(j)} - x_1^{(j)}]$$
$$y = y_1^{(j)} + t[y_2^{(j)} - y_1^{(j)}]$$

3 Integration

4 Results

$$A \begin{bmatrix} x_1^{(1)} y_2^{(1)} + y_1^{(1)} x_2^{(1)} \\ x_1^{(2)} y_2^{(2)} + y_1^{(2)} x_2^{(2)} \\ x_1^{(3)} y_2^{(3)} + y_1^{(3)} x_2^{(3)} \end{bmatrix} + B \begin{bmatrix} x_2^{(1)} - x_1^{(1)} \\ x_2^{(2)} - x_1^{(2)} \\ x_2^{(3)} - x_1^{(3)} \end{bmatrix} + C \begin{bmatrix} y_2^{(1)} - y_1^{(1)} \\ y_2^{(2)} - y_1^{(2)} \\ y_2^{(3)} - y_1^{(3)} \end{bmatrix}$$