Week 5 #

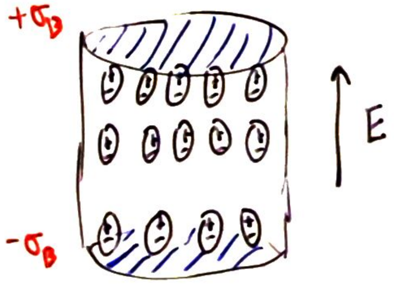
9.3 EM waves in matter

Maxwell's eq. in general

① $\vec{\nabla}\cdot\vec{E} = \frac{1}{\varepsilon_0}\rho$ $\qquad \vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\vec{\nabla}\cdot\vec{B} = 0$ $\qquad$ ④ $\vec{\nabla}\times\vec{B} = \mu_0 \vec{J} + \mu_0\varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

from Ch 4: $\rho = \rho_F + \rho_B$



$\rho_B = -\vec{\nabla}\cdot\vec{P}$

dipole moment per unit volume

① becomes $\vec{\nabla}\cdot(\varepsilon_0\vec{E} + \vec{P}) = \rho_F$

let $\vec{D} = \varepsilon_0\vec{E} + \vec{P}$ $\Rightarrow$ $\vec{\nabla}\cdot\vec{D} = \rho_F$

↳ we can control

Recall from Ch 6 $\quad \vec{J} = \vec{J}_F + \vec{J}_B$ ← bound current due to electron spins

↑ we can control

with $\vec{J}_B = \vec{\nabla}\times\vec{M}$ ← magnetization

use ④

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\vec{\nabla}\times\underbrace{\left(\frac{1}{\mu_0}\vec{B} - \vec{M}\right)}_{\equiv \vec{H}} = \vec{J}_f + \varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\vec{\nabla}\times\vec{H} = \vec{J}_F + \varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

but something is missing

↳ can control this one

when $E(t) \rightarrow P(t)$ (breathing dipoles)

$\frac{\partial\vec{P}}{\partial t} \neq 0$ when $\frac{\partial\vec{E}}{\partial t} \neq 0$

$\Rightarrow \frac{\partial\sigma_B}{\partial t} \neq 0$ "Breathing dipoles" $\Rightarrow$ effective current $I_p = \frac{\partial\sigma_B}{\partial t}\cdot da_\perp$

with $\sigma_B = \vec{P}\cdot\hat{n}$ $\qquad \frac{\partial I_p}{\partial a_\perp} = J_p = \frac{\partial(P\cdot\hat{n})}{\partial t}\frac{da_\perp}{da_\perp} \Rightarrow \vec{J}_p = \frac{\partial\vec{P}}{\partial t}$

so then $\vec{J} = \vec{J}_f + \vec{J}_b + \vec{J}_p$

and we get (in the end) $\vec{\nabla} \times \vec{H} = \vec{J}_f + \vec{J}_p + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

$$= \vec{J}_f + \frac{\partial}{\partial t}(\vec{P} + \varepsilon_0 \vec{E})$$

$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}}$$

Maxwells equations in terms of free charges and free currents

1. $\vec{\nabla} \cdot \vec{D} = \rho_f \qquad \vec{D} = \varepsilon_0 \vec{E} + \vec{P}$
2. $\vec{\nabla} \cdot \vec{B} = 0$
3. $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
4. $\vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t} \qquad \vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$

Special case: linear matter

$\vec{P} \propto \vec{E} \Rightarrow \vec{P} = \varepsilon_0 \chi_e \vec{E}$

$\vec{M} \propto \vec{H} \Rightarrow \vec{M} = \chi_m \vec{H}$

also, $\vec{D} = \varepsilon \vec{E} \qquad \vec{H} = \frac{1}{\mu}\vec{B} \qquad \varepsilon = \varepsilon_0(1+\chi_e)$

$\mu = \mu_0(1+\chi_m)$

Maxwell's equations in integral form

$$\int_V \vec{\nabla} \cdot \vec{D} = \int_V \rho_f$$

① $\oint_{surf} \vec{D} \cdot d\vec{a} = Q_{free}$

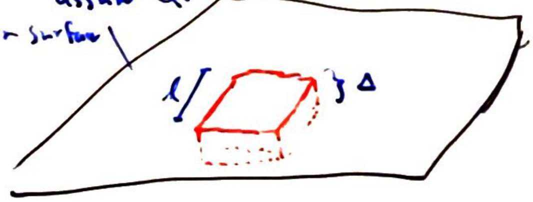
② $\oint_{surf} \vec{B} \cdot d\vec{a} = 0$

③ $\oint_{surf} \vec{\nabla} \times \vec{E} \cdot d\vec{a} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$

$\oint_{loop} \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$

④ $\oint \vec{H} \cdot d\vec{l} = I_{free,encl.} + \frac{d}{dt}\int_{surf} \vec{D} \cdot d\vec{a}$

1. assume $Q_f = 0$ on surface



we gonna take $\Delta \to 0$ at the end

2. Find the B.C.'s $D_{\perp,1}$ vs. $D_{\perp,2}$

Last time:

$$\vec{\nabla}\cdot\vec{D}=0$$
$$\vec{\nabla}\cdot\vec{B}=0$$
$$\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$
$$\vec{\nabla}\times\vec{H}=\frac{\partial\vec{D}}{\partial t}$$

Special case: linear media

$\vec{D}=\varepsilon\vec{E}$ $\qquad\qquad$ $\vec{H}=\frac{1}{\mu}\vec{B}$

$$\left.\begin{aligned}\vec{\nabla}\cdot\vec{E}&=0\\ \vec{\nabla}\cdot\vec{B}&=0\\ \vec{\nabla}\times\vec{B}&=\mu\varepsilon\frac{\partial\vec{E}}{\partial t}\\ \vec{\nabla}\times\vec{E}&=-\frac{\partial\vec{B}}{\partial t}\end{aligned}\right\}\rightarrow$$

note that these look exactly like maxwell's equations in a vacuum

but $\varepsilon_0\rightarrow\varepsilon$
$\mu_0\rightarrow\mu$

therefore, if an electromagnetic wave travels in <u>linear</u> media, the only thing that happens is a different wave speed

$v=\frac{1}{\sqrt{\varepsilon\mu}}$ $\qquad$ compare $\qquad$ $c=\frac{1}{\sqrt{\varepsilon_0\mu_0}}$

$v=\frac{c}{n}$ $\qquad$ n = refractive index

note: $\varepsilon=\varepsilon_0(1+\chi_e)$ $\qquad$ $\mu=\mu_0(1+\chi_m)$

$\Rightarrow n\geq 1$ $\qquad$ $n=\sqrt{(1+\chi_e)(1+\chi_m)}\geq 1$

all previous results for $\vec{E}(z,t)$ and $\vec{B}(z,t)$ still hold with

$\varepsilon_0\rightarrow\varepsilon$ $\qquad$ $\mu_0\rightarrow\mu$ $\qquad$ $c\rightarrow v=\frac{1}{\sqrt{\mu\varepsilon}}$

$u=\frac{1}{2}\left(\varepsilon E^2+\frac{1}{\mu}B^2\right)$ $\qquad$ $\vec{S}=\frac{1}{\mu}\left(\vec{E}\times\vec{B}\right)$

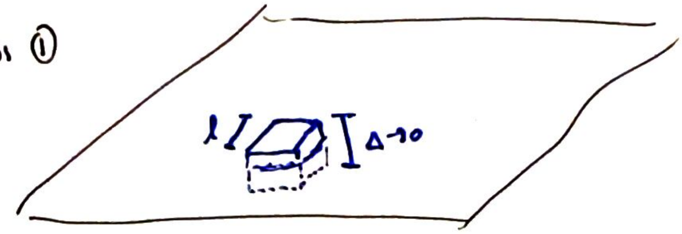
$\omega=kv$ $\qquad$ $I=\frac{1}{2}\varepsilon v E_0^2$

note usually $\chi_m\ll 1$ (so $\mu\approx\mu_0$)

$n\approx\sqrt{1+\chi_e}$

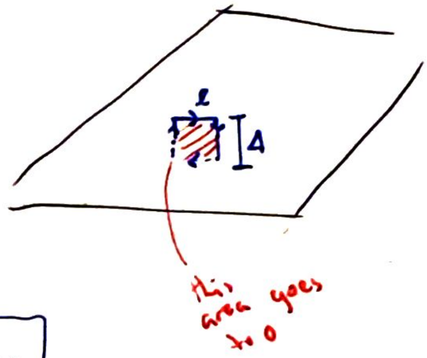
Boundary conditions, $\rho_F = 0$, $J_F = 0$

glass ①



Goal: find out if $\vec{D}, \vec{H}, \vec{E}, \vec{B}$ are continuous/discontinuous across boundary

plastic ②



1 $\oint_{surf} \vec{D} \cdot d\vec{a} = \cancel{Q_{free}}^{\,0}$ 2 $\oint_{surf} \vec{B} \cdot d\vec{a} = 0$

3 $\oint_{loop} \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int_{surf} \vec{B} \cdot d\vec{a}$ 4 $\oint \vec{H} \cdot d\vec{l} = \cancel{I_{free}}^{\,0} + \frac{\partial}{\partial t} \int_{surf} \vec{D} \cdot d\vec{a}$

from 1: the $D_\perp$ must be continuous $\boxed{D_{\perp,above} = D_{\perp,below}}$

and from 2: the $B_\perp$ must be continuous $\boxed{B_{\perp,above} = B_{\perp,below}}$

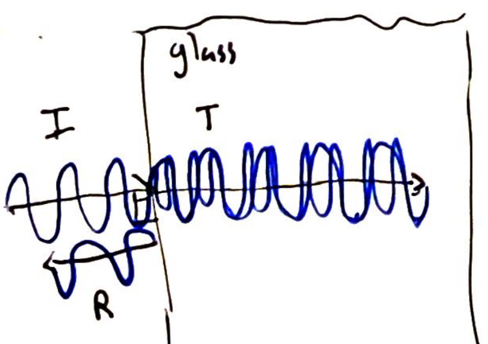
from 3:
$$l(E_{\parallel,above} - E_{\parallel,below}) = 0$$
(because $\int B \cdot da \to 0$)

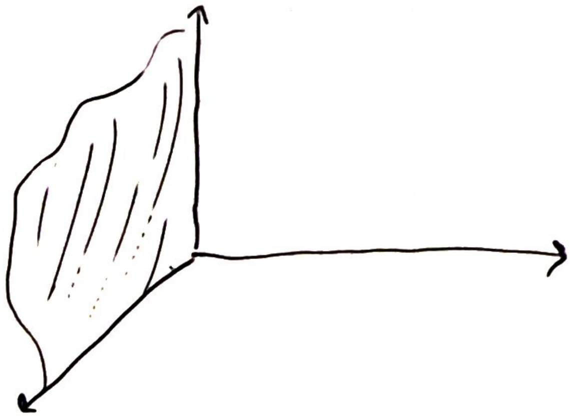
$$\Rightarrow E_{\parallel,above} - E_{\parallel,below} = 0 \Rightarrow \boxed{E_{\parallel,above} = E_{\parallel,below}}$$

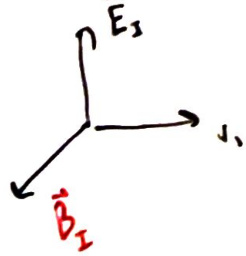
from 4: $\boxed{H_{\parallel,above} = H_{\parallel,below}}$

For linear media: $\boxed{\epsilon_{above} E_{\perp,above} = \epsilon_{below} E_{\perp,below}}$

$$\boxed{\frac{1}{\mu_{above}} B_{\parallel,above} = \frac{1}{\mu_{below}} B_{\parallel,below}}$$






$\vec{E}_I$
v_1
$\vec{B}_I$