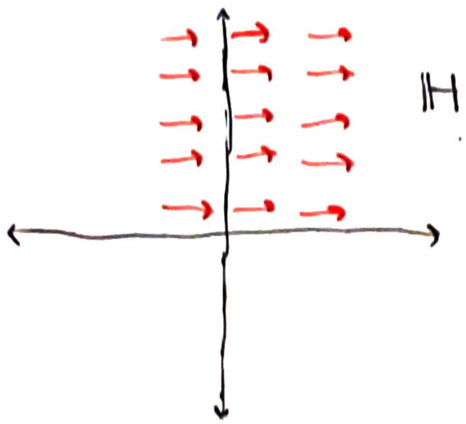
Week 4 pt 3



$V(x,y) = (\alpha, 0)$

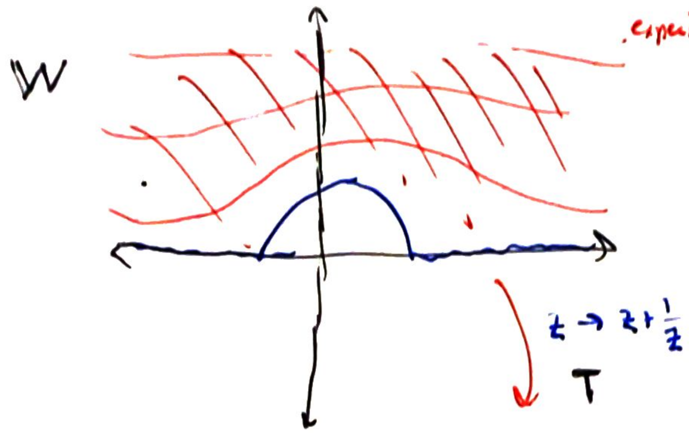
$u(x,y) = \alpha x$

$u = \mathrm{Re}[\alpha z]$

complex potential is

$f(z) = \alpha z = \alpha x + i(\alpha y)$

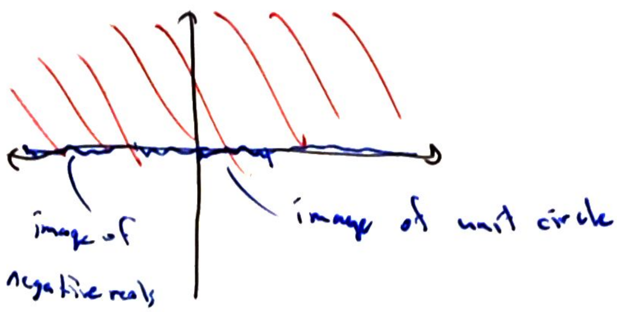
if u_1 is harmonic and u_2 is harmonic then $au_1 + bu_2$ is still harmonic



to get this conformally from the first case use the transform

$z \to z + \frac{1}{z}$

real number a goes to $a + \frac{1}{a}$



then if you're on the unit circle you get twice the real part

$u: \mathbb{H} \to \mathbb{R}$

$\mathbb{W} \quad \mathbb{H} \quad \mathbb{R}$

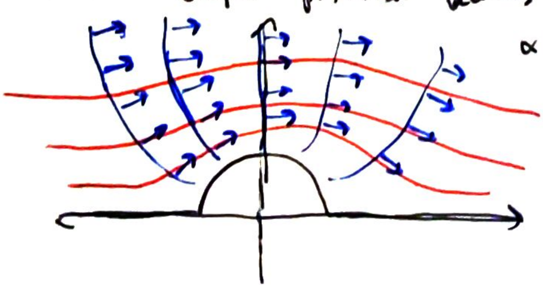
$w \to T(w) \to u(T(w))$

$\phi = u \circ T$

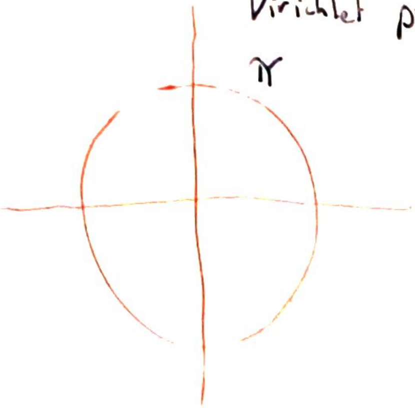
If u is harmonic and T is conformal then $u \circ T$ is harmonic

so the complex potential becomes $f(z) = \alpha\left(z + \frac{1}{z}\right)$

$$\alpha\left(x+iy + \frac{1}{x+iy}\right) = \underbrace{\alpha\left(x + \frac{x}{x^2+y^2}\right)}_{\boxed{\phi}} + \underbrace{i\alpha\left(y + \frac{y}{x^2+y^2}\right)}_{\boxed{\psi}}$$



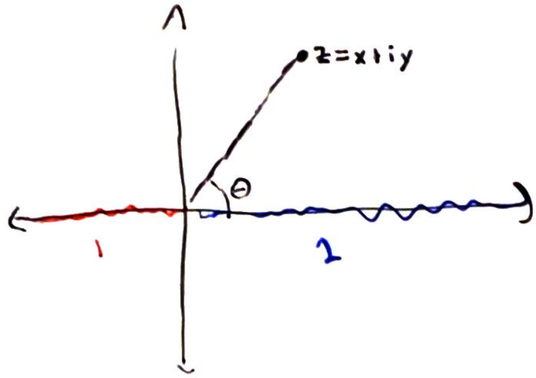
Dirichlet problem



prescribe u on $\mathbb{T}$, u cont.

u = poisson integral

half-plane standard solution



$u = \frac{1}{\pi}\theta$

recall $\operatorname{Log} z = \ln(r) + i\arg(z)$

$\frac{1}{\pi i}\operatorname{Log}(z)$

$\frac{1}{\pi}\theta = \operatorname{Re}\left[\frac{1}{i\pi}\operatorname{Log}(z)\right]$