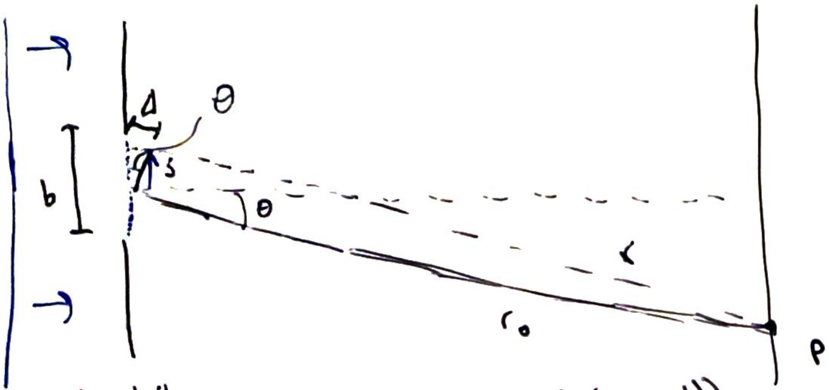
Week 8 #



Contribution at P goes as $\frac{\exp(i(kr-\omega t))}{r}$

$$dE_p = \frac{E_L\,ds\,\exp(i(kr-\omega t))}{r}$$

what is E_L? we have N oscillators each of "strength" E_0

so the total strength/length $= NE_0/b = E_L$

(check canvas page for Huygen's lab)

so $$dE_p = \frac{E_L\,ds\,\exp(i(k(r_0+\Delta)-\omega t))}{r_0+\Delta}$$

$$\approx \frac{E_L\,ds\,\exp(i(kr_0-\omega t))}{r_0}\exp(ik\Delta)$$

$$E_p = E_L\frac{\exp(i(kr_0-\omega t))}{r_0}\underbrace{\int_{-b/2}^{b/2}\exp(iks\sin\theta)\,ds}$$

recall $\sin x = \frac{e^{ix}-e^{-ix}}{2i}$

$$\frac{1}{ik\sin\theta}\exp(ik\sin\theta s)\Big|_{-b/2}^{b/2}$$

$$\to \frac{b\sin(k\frac{b}{2}\sin\theta)}{k\,b/2\,\sin\theta}$$

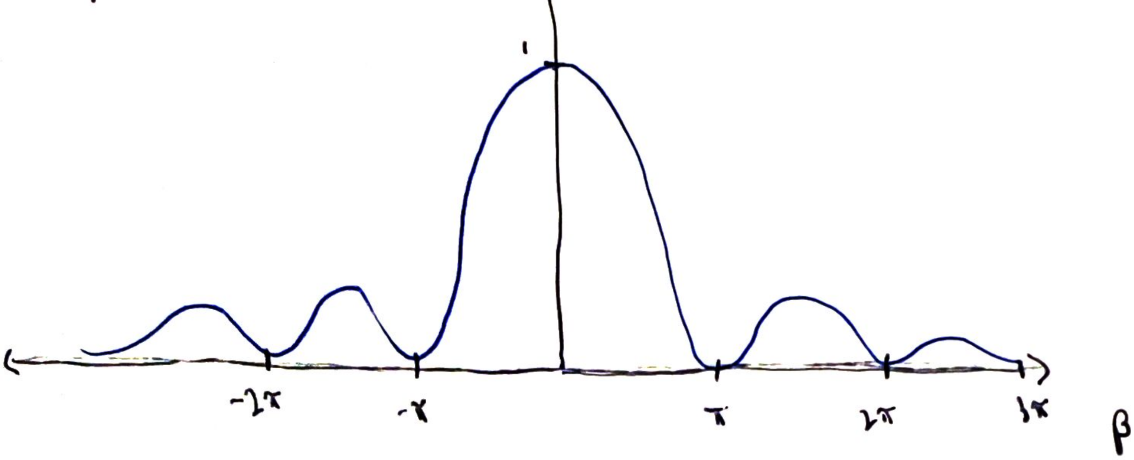
$\beta = k\frac{b}{2}\sin\theta$

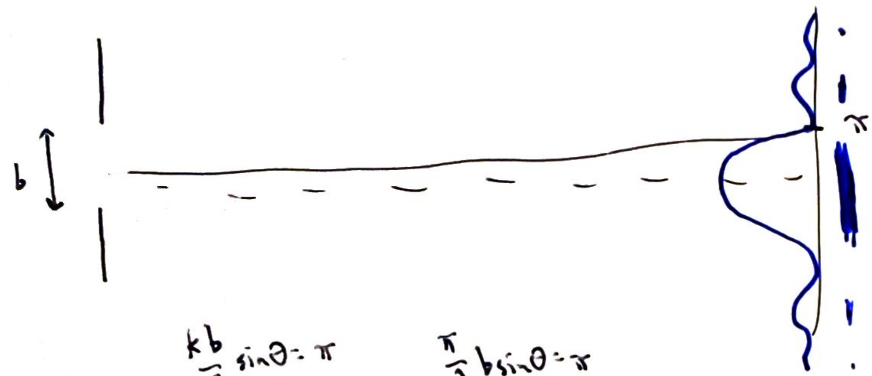
$$E_p = \frac{E_L b}{r_0}\exp(i(kr_0-\omega t))\frac{\sin(\beta)}{\beta}$$

$I_0 = \left(\frac{E_L b}{r_0}\right)^2$

$I_{at\,p} = I_0\left(\frac{\sin\beta}{\beta}\right)^2$

$\frac{\sin\beta}{\beta} = \text{sinc}\,\beta$

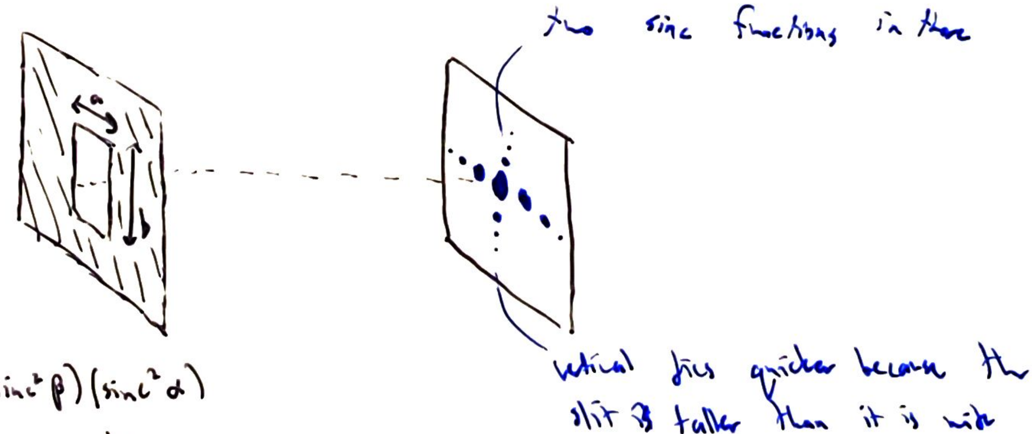




$\frac{kb}{2}\sin\theta = \pi \qquad \frac{\pi}{\lambda} b \sin\theta = \pi$

$\Rightarrow \sin\theta = \frac{\lambda}{b} \quad \text{or} \quad \theta \approx \frac{\lambda}{b}$

This can be extended to 2-D rectangular aperture

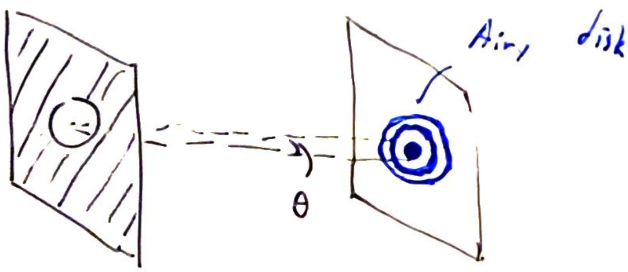


$I = I_0 (\text{sinc}^2\beta)(\text{sinc}^2\alpha)$

$\beta = \frac{kb}{2}\sin\theta \qquad \alpha = \frac{ka}{2}\sin\theta$

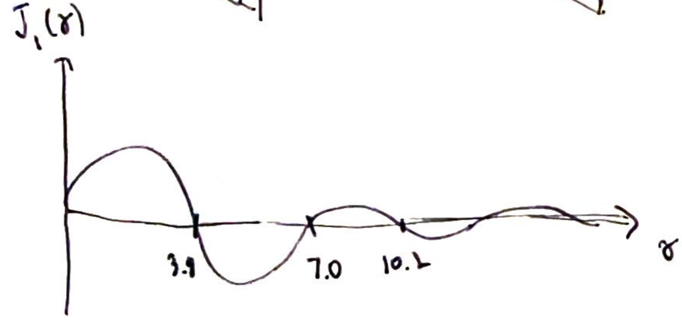
Circular aperture

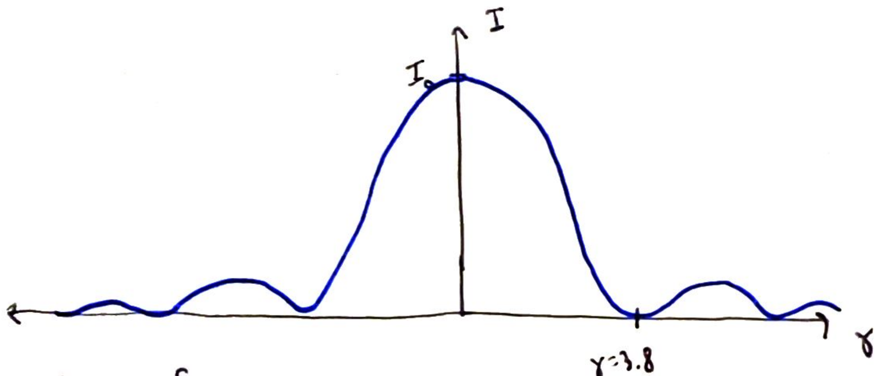
circular aperture



$$I = I_0 \left(\frac{2 J_1(\gamma)}{\gamma} \right)^2$$

$$\gamma = \frac{1}{2} k D \sin\theta$$





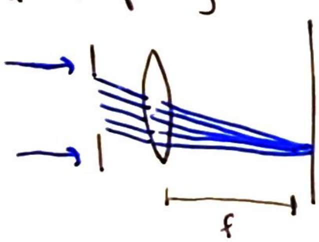
so the first 0 occurs when

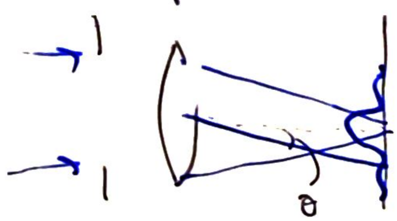
$$\sin\theta = \frac{1.22\lambda}{D}$$

since θ is usually small $\theta = \frac{1.22\lambda}{D}$

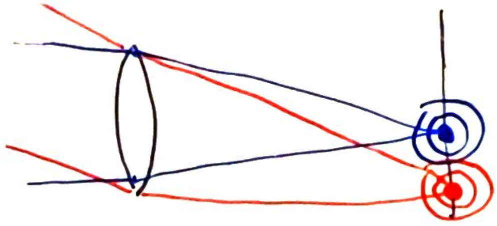
and $\Delta\theta = \frac{2.44\lambda}{D}$

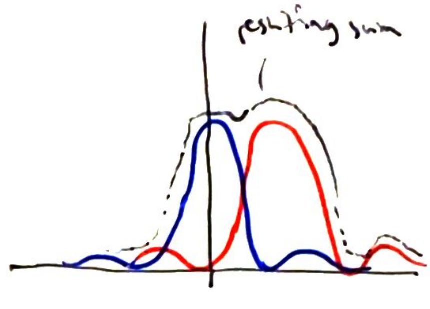
go back to putting a lens behind the aperture





$$\theta = \frac{1.22\lambda}{D}$$

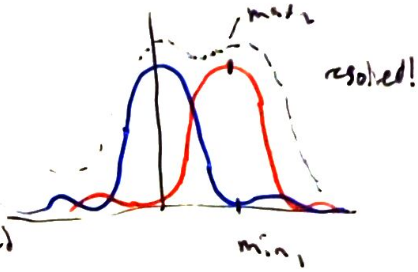




Objects resolved when the diffraction spots are not overlapped

Rayleigh's criterion

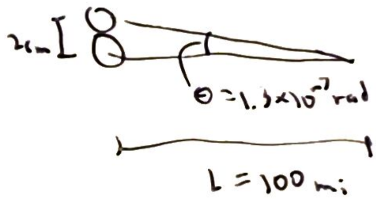
when max_2 and min_1 overlaps the objects are resolved



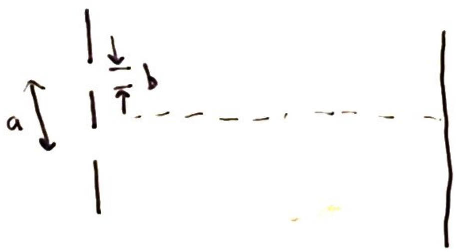
what is the resolution of say a 5_m telescope @ $\lambda = 550$ nm

$$\Delta\theta = \frac{1.22\lambda}{D} = 1.3\times10^{-7}\text{ rad} = 8\times10^{-6}\text{ deg.}$$

so for 2 pennies



<u>Double slit</u>



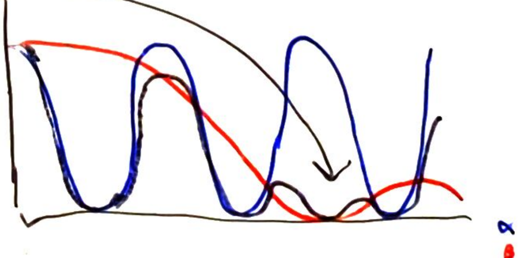
$$I = 4I_0\left(\frac{\sin\beta}{\beta}\right)^2\cos^2(\alpha)$$

$$\alpha = \tfrac{1}{2}ka\sin\theta$$

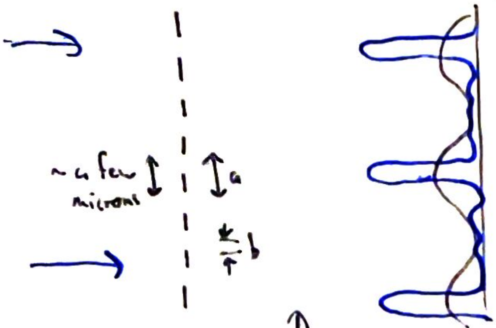
$$\beta = \tfrac{1}{2}kb\sin\theta$$

finite slit width changes the envolope on the $\cos^2$ fringes

"missing order" when the sinc cancells out the max of the $\cos^2$



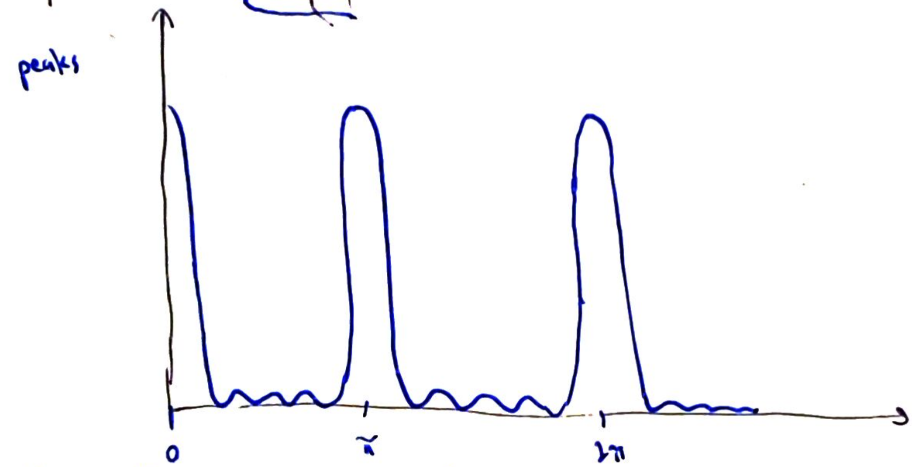
multiple slits - diffraction grating



$$I = I_0 \left(\frac{\sin\beta}{\beta}\right)^2 \left(\frac{\sin N\alpha}{\sin\alpha}\right)^2$$

$\left(\frac{\sin\beta}{\beta}\right)^2$: envelope; $\left(\frac{\sin N\alpha}{\sin\alpha}\right)^2$: peaks

N = total number of slits

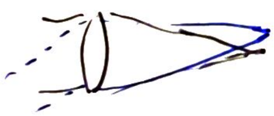


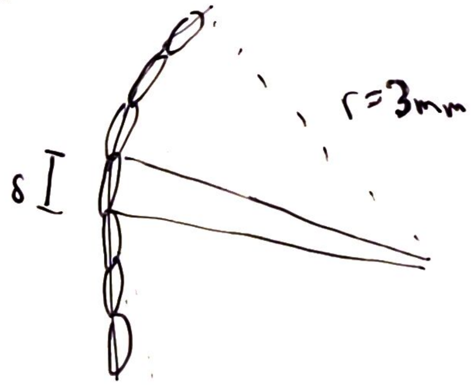
peaks at the same position as 2 slits but must narrower

Diffraction and resolution in a bee's eye
(small detour from multi-slit diffraction)
Bee's use UV light (~400 nm), eye curvature ~3 mm

the Bee would like to have small lens

$$\Delta\Theta_{geometrical} = \frac{\delta}{r}$$





$$\Delta\Theta_{diffraction} = \frac{\lambda}{\delta}$$

$$\Delta\Theta_{tot} = \Delta\Theta_{geometrical} + \Delta\Theta_{diffraction}$$

$$\Delta\Theta_{tot} = \frac{\delta}{r} + \frac{\lambda}{\delta}$$

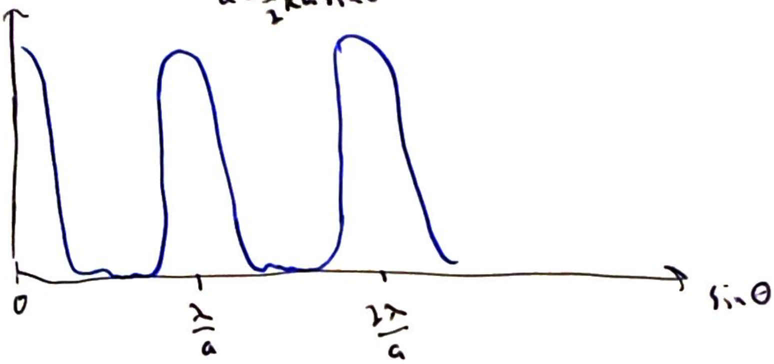
$$\frac{d\,\Delta\Theta_{tot}}{d\delta} = \frac{1}{r} - \frac{\lambda}{\delta^2} = 0 \Rightarrow \delta = \sqrt{\lambda r} \approx \sqrt{400\text{nm} \cdot 3\text{mm}} \approx 35\,\mu\text{m}$$

(turns out to be accurate)

fun!

peaks when $\alpha = m\pi \Rightarrow a\sin\theta = m\lambda$

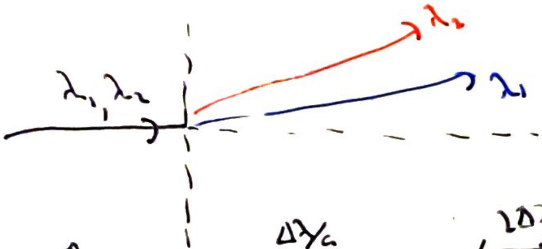
$\alpha = \frac{1}{2}ka\sin\theta$

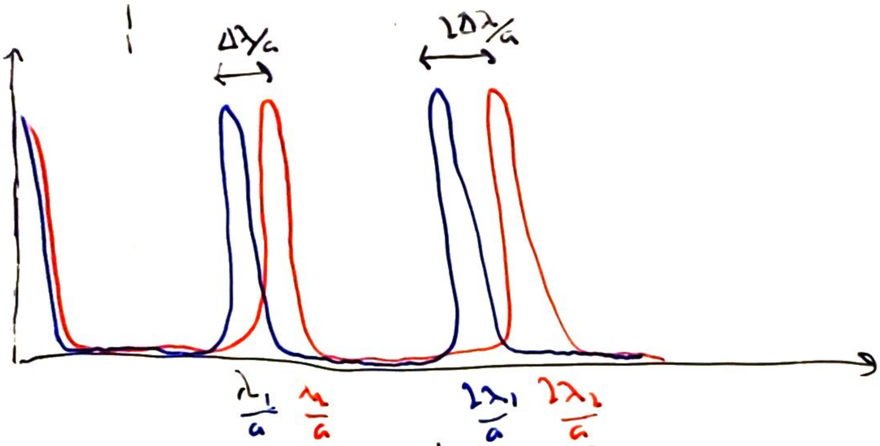


$m=1$ $\quad$ $m=2$ $\quad$ the m is the order of the peak

Use this as a spectrometer

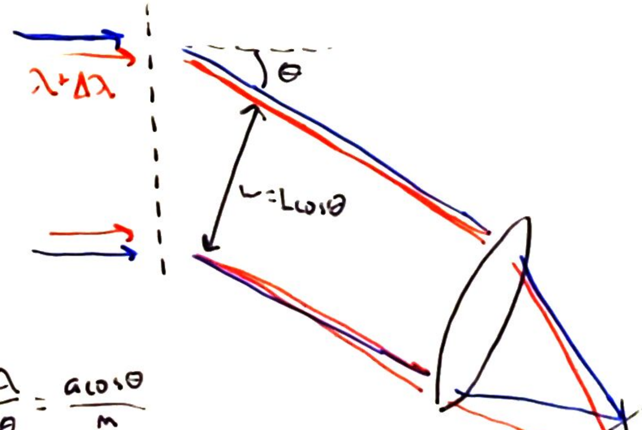
$\lambda_2 > \lambda_1$





better separation of λ at higher orders

Resolution $R = \frac{\lambda}{\Delta\lambda}$



$a\sin\theta = m\lambda$

$\lambda = \frac{a\sin\theta}{m} \qquad \frac{d\lambda}{d\theta} = \frac{a\cos\theta}{m}$

$\Delta\theta = \frac{m\Delta\lambda}{a\cos\theta}$

due to diffraction $\Delta\theta \approx \frac{\lambda}{w} = \frac{\lambda}{L\cos\theta}$

we will resolve λ and $\lambda + \Delta\lambda$

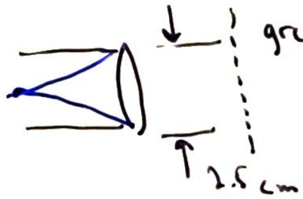
when $\frac{m\Delta\lambda}{a\cos\theta} = \frac{\lambda}{L\cos\theta}$ $\qquad R = \frac{\lambda}{\Delta\lambda} = \frac{Lm}{a}$

note that L/a is the number of slits

so the resolution $\boxed{R = Nm}$ where m is the order

no dependence on λ, just number of slits

In lab,



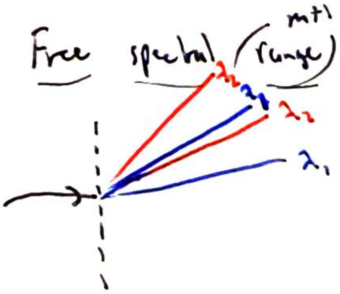
grating of 500 lines per mm

($a = 2\,\mu m$)

$N = 500 \times 25 = 12{,}500$

using $m = 2$, resolution would be $R = 25{,}000$ (pretty good)

Free spectral range



just overlapping

$m\lambda_2 = (m+1)\lambda_1$

$\Delta\lambda = \frac{\lambda_1}{m}$