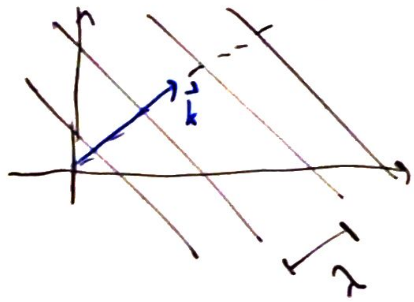
Week 5

How to write a 2D plane wave



$\Psi(x,y,t)$

when $|k| = \frac{2\pi}{\lambda}$

wavefronts defined by $\vec{k}\cdot\vec{r} = \text{const.}$

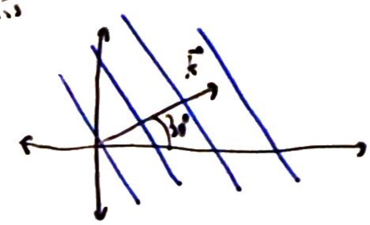
$$\Psi(x,y,t) = A\exp\left(i(\vec{k}\cdot\vec{r} - \omega t)\right)$$

write in components: $\vec{r} = x + y$ $\qquad \vec{k} = k_x + k_y$

$$\Psi(x,y,t) = A\exp\left(i(k_x x + k_y y - \omega t)\right)$$

Example: 2D wave travelling at 30° to x-axis

$\lambda = 20\,m \qquad v = 10\,m/s$



$\Psi(x,y,t)$: $k = \frac{2\pi}{\lambda} = \frac{\pi}{10}\,m^{-1}$

$$\omega = 2\pi f = 2\pi\frac{v}{\lambda} = \pi\,s^{-1}$$

$k_x = \frac{\pi}{10}\cos 30° = \frac{\sqrt{3}\pi}{20} \qquad k_y = \frac{\pi}{10}\sin 30° = \frac{\pi}{20}$

$$\Psi(x,y,t) = A\exp\left(i\left(\frac{\sqrt{3}\pi}{20}x + \frac{\pi}{20}y - \pi t\right)\right)$$

Spherical wave: $\Psi(r,t) = \frac{A}{r}e^{i(kr - \omega t)}$

Electromagnetic waves

$$\vec{E} = \vec{E}_0 \sin(\vec{k}\cdot\vec{r} - \omega t)$$
$$\vec{B} = \vec{B}_0 \sin(\vec{k}\cdot\vec{r} - \omega t)$$

$$|E_0| = c|B_0|$$

There is an energy density to do with E and B fields

$u_E = \frac{1}{2}\epsilon_0 E^2$ $\qquad$ $u_B = \frac{1}{2}\frac{1}{\mu_0}B^2$ $\qquad$ (woah! $E \geq m$)

$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$

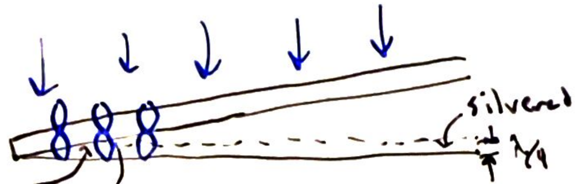
Irradiance of the EM wave $\quad I = \frac{1}{2}\epsilon_0 c E_0^2$ watts/m^2

this is what we observe $\qquad$ $I \propto E_0^2$

why so excited about the electric field?

Wiener experiments

get standing waves



silvered

$\lambda/4$

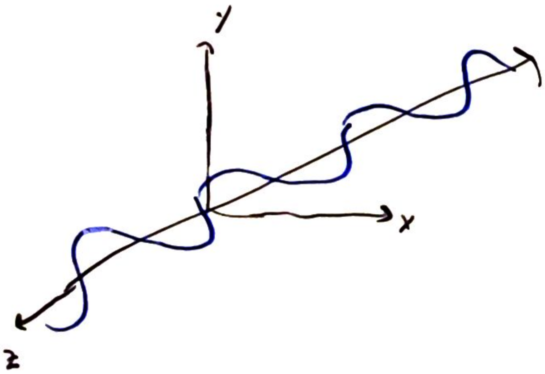
antinode on the photo emulsion

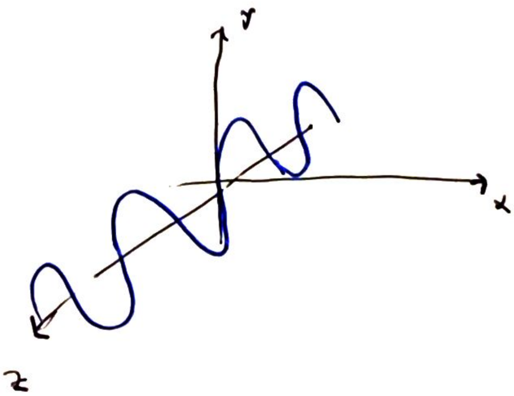
photo emulsion (very thin)

you then get dark lines on the paper spaced out

↳ the electric field does the exposure

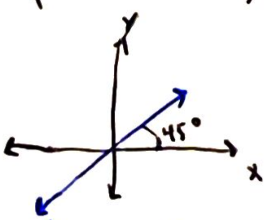
<u>polarization</u>: $\vec{E} = E_0 \sin(kz - \omega t)$

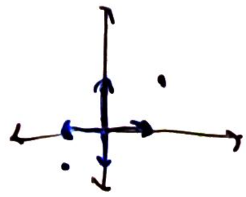




we define the direction of polarization by looking into the beam

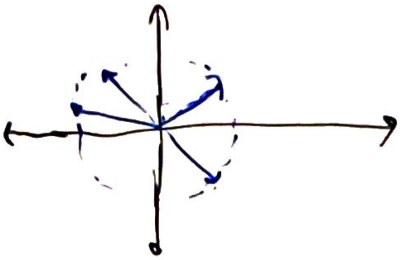
e.g. 45° linear polarization





we would write this as $\vec{E} = E_0 \sin(kz - \omega t)\hat{x} + E_0 \sin(kz - \omega t)\hat{y}$

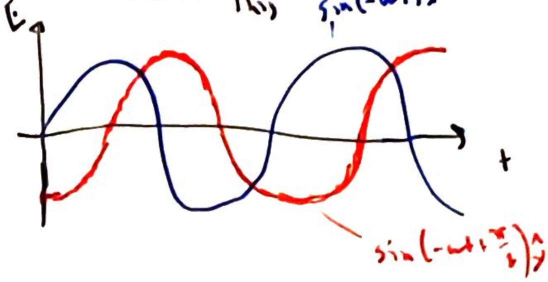
circular polarization - e.g. left circularly polarized



moves around a full circle in 1 wavelength

$$\vec{E} = E_0 \sin(kz-\omega t)\hat{x} + E_0 \sin(kz - \omega t + \tfrac{\pi}{2})\hat{y}$$

check this



read about dopler effect

$$\frac{\lambda'}{\lambda} = 1 - \frac{v}{c}$$

Ch 7:

Light interference

We pretty much treat the light as a scalar waves ; we use just one polarization

$$E_{1,p} = E_{0,1}\cos(ks_1 - \omega t + \phi_1) \qquad k = \frac{2\pi}{\lambda}$$

$$E_{2,p} = E_{0,2}\cos(ks_2 - \omega t + \phi_2)$$

ϕ_1 ; ϕ_2 are the phase constants

$E_p = E_{1,p} + E_{2,p}$ - instantaneous E-field at p

; we want the irradiance at p , power/area e.g. $\frac{\text{watts}}{m^2}$

Irradiance $\propto$ amplitude^2

$$\tilde{E}_{1,p} = E_1 \exp(i(ks_1 + \phi_1))$$

$$\tilde{E}_p = \tilde{E}_{1,p} + \tilde{E}_{2,p}$$

$$\tilde{E}_{2,p} = E_2 \exp(i(ks_2 + \phi_2))$$

$$\tilde{E}_p \tilde{E}_p^* = (\tilde{E}_{1,p} + \tilde{E}_{2,p})(\tilde{E}_{1,p} + \tilde{E}_{2,p})^*$$

$$\tilde{E}_p \tilde{E}_p^* = I$$

$$= I_1 + I_2 + E_1 E_2 \exp(i(k(s_1 - s_2) + (\phi_1 - \phi_2)))$$

$$+ E_1 E_2 \exp(i(k(s_2 - s_1) + (\phi_2 - \phi_1)))$$

$$E_1 = \sqrt{I_1} \qquad E_2 = \sqrt{I_2}$$

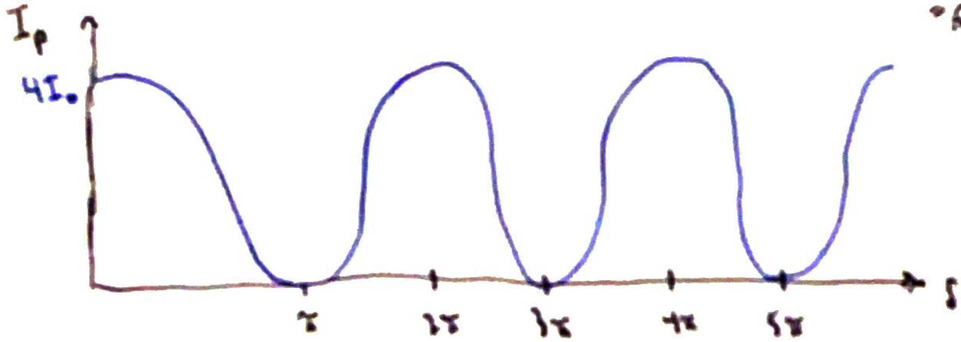
$$I = I_1 + I_2 + \sqrt{I_1}\sqrt{I_2}\, 2\cos(\delta)$$ when $\delta = k(s_2 - s_1) + \phi_2 - \phi_1$

$$\boxed{I_p = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\delta)}$$

let $I_1 = I_2 = I_0$

then $I_p = 2I_0 + 2I_0\cos(\delta)$

we refer to this as "fringes"



use double angle formula

$$I_p = 2I_0 \cdot 2\cos^2(\delta/2)$$

$$= 4I_0\cos^2(\delta/2)$$ thus "cosine squared" fringes

– typical of 2 beam interference

If $I_1 \neq I_2$

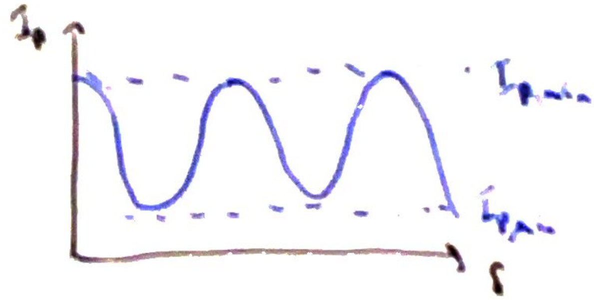
↳ $I_p = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\delta)$

$$I_{p,max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$I_{p,min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

define visibility: $V = \dfrac{I_{max} - I_{min}}{I_{max} + I_{min}}$

$0 < V < 1$



So far have assumed that ϕ_1 & ϕ_2 are fixed ($\phi_2 - \phi_1$ constant)

But light from different sources (two different lasers) will have ϕ_1 and ϕ_2 varying rapidly

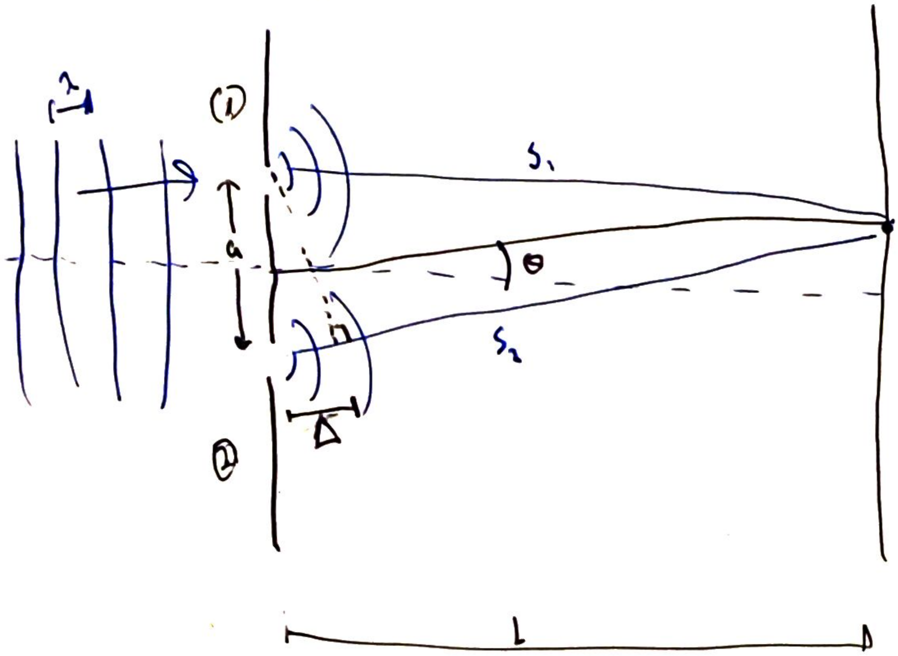
$$I_p = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\delta)$$

← will average out to zero

$I_p = I_1 + I_2$ we call the sources incoherent

average $I_p = I_1 + I_2$

young's 2 slits - an example of a wavefront division interferometer



slits of equal size

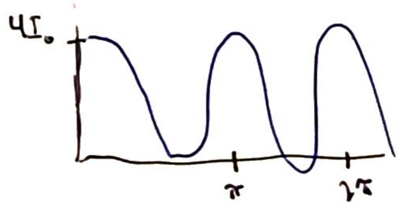
↳ give the same contribution

· $s_2 - s_1$ is not so big that the amplitudes at P do not differ significantly

$\Delta = s_1 - s_2 \approx a\sin\theta$

· let $\phi_1 = \phi_2$

then $\delta = k(s_2 - s_1) = k\Delta$ so $I_p = 4I_0\cos^2\left(\frac{\delta}{2}\right) = 4I_0\cos^2\left(\frac{k}{2}a\sin\theta\right)$



so we get maxima when

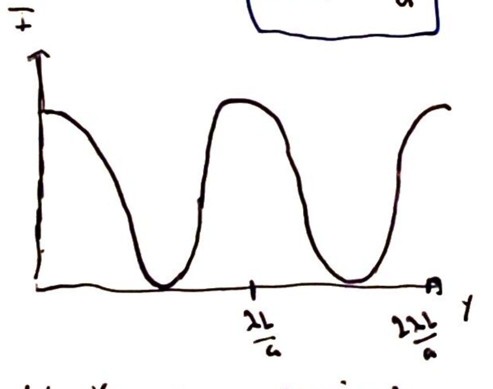
$\frac{k}{2}a\sin\theta = 0, \pi, 2\pi$

so $\frac{\pi}{\lambda}a\frac{y_{max}}{L} = m\pi$ $m = 0, 1, 2, \ldots \Rightarrow y_{max} = \frac{mL\lambda}{a}$

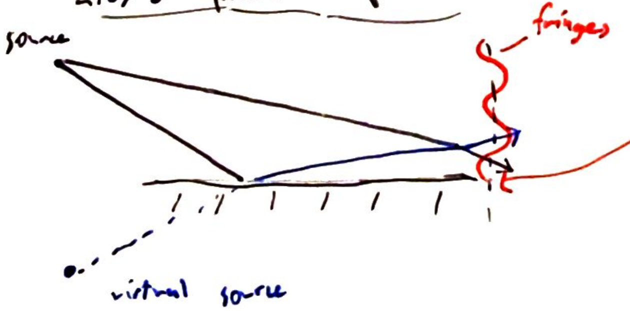
so $\boxed{\Delta y_{max} = \frac{\lambda L}{a}}$

so larger $\lambda \rightarrow$ spots spread apart

decrease slit spacing $\rightarrow$ spots spread apart



Lloyd's mirror experiment



destructive interference

- because reflected and transmitted light are 180° out of phase

↳ phase change due to reflection