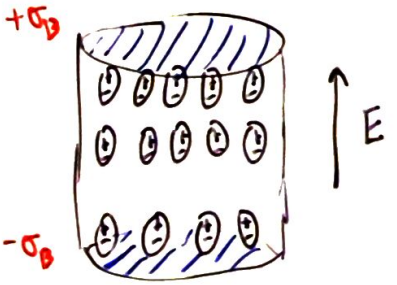
9.3 EM waves in matter

Maxwell's eq. in general

(1) $\vec{\nabla}\cdot\vec{E} = \frac{1}{\epsilon_0}\rho$ $\qquad$ $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

$\vec{\nabla}\cdot\vec{B} = 0$ $\qquad$ (4) $\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$

from Ch 4: $\rho = \rho_F + \rho_B$



$\rho_B = -\vec{\nabla}\cdot\vec{P}$

dipole moment per unit volume

(1) becomes $\vec{\nabla}\cdot(\epsilon_0\vec{E} + \vec{P}) = \rho_F$

let $\vec{D} = \epsilon_0\vec{E} + \vec{P}$ $\Rightarrow$ $\vec{\nabla}\cdot\vec{D} = \rho_F$

↳ we can control

Recall from Ch 6 $\vec{J} = \vec{J}_F + \vec{J}_B$ ← bound current due to electron spins

↑ we can control

with $\vec{J}_B = \vec{\nabla}\times\vec{M}$ ← magnetization

use (4)

$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$

$\vec{\nabla}\times\underbrace{\left(\frac{1}{\mu_0}\vec{B} - \vec{M}\right)}_{\equiv\vec{H}} = \vec{J}_F + \epsilon_0\frac{\partial\vec{E}}{\partial t}$

$\vec{\nabla}\times\vec{H} = \vec{J}_F + \epsilon_0\frac{\partial\vec{E}}{\partial t}$ but something is missing

↳ can control this one

when $E(t) \to P(t)$ (breathing dipoles)

$\frac{\partial\vec{P}}{\partial t} \neq 0$ then $\frac{\partial\vec{E}}{\partial t} \neq 0$

$\Rightarrow \frac{\partial\sigma_B}{\partial t} \neq 0$ "Breathing dipoles" $\Rightarrow$ effective current $I_P = \frac{\partial\sigma_B}{\partial t}\cdot da_\perp$

with $\sigma_B = \vec{P}\cdot\hat{n}$ $\qquad$ $\frac{dI_P}{da_\perp} = J_P = \frac{\partial(P\cdot\hat{n})}{\partial t}\frac{da_\perp}{da_\perp} \Rightarrow \vec{J}_P = \frac{\partial\vec{P}}{\partial t}$

so then $\vec{J} = \vec{J}_f + \vec{J}_b + \vec{J}_p$

and we get (in the end) $\vec{\nabla} \times \vec{H} = \vec{J}_f + \vec{J}_p + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

$$= \vec{J}_f + \frac{\partial}{\partial t}(\vec{P} + \varepsilon_0 \vec{E})$$

$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}}$$

Maxwell's equations in terms of free charges and free currents

$$\boxed{\begin{aligned} &1\ \vec{\nabla} \cdot \vec{D} = \rho_f \qquad \vec{D} = \varepsilon_0 \vec{E} + \vec{P} \\ &2\ \vec{\nabla} \cdot \vec{B} = 0 \\ &3\ \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ &4\ \vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t} \qquad \vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M} \end{aligned}}$$

Special case: linear matter

$\vec{P} \propto \vec{E} \Rightarrow \vec{P} = \varepsilon_0 \chi_e \vec{E}$

$\vec{M} \propto \vec{H} \Rightarrow \vec{M} = \chi_m \vec{H}$

also, $\vec{D} = \varepsilon \vec{E}$ $\quad \vec{H} = \frac{1}{\mu}\vec{B}$ $\quad \varepsilon = \varepsilon_0(1+\chi_e)$ $\quad \mu = \mu_0(1+\chi_m)$

Maxwell's equations in integral form

$$\int_V \vec{\nabla} \cdot \vec{D} = \int_V \rho_f$$

① $\oint_{surf} \vec{D} \cdot d\vec{a} = Q_{free}$

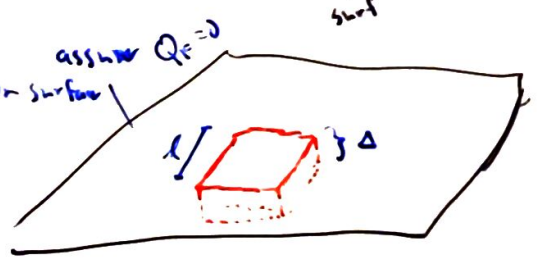
④ $\oint \vec{H} \cdot d\vec{\ell} = I_{free,encl.} + \frac{\partial}{\partial t} \int_{surf} \vec{D} \cdot d\vec{a}$

② $\oint_{surf} \vec{B} \cdot d\vec{a} = 0$

③ $\oint_{surf} \vec{\nabla} \times \vec{E} \cdot d\vec{a} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$

$\oint_{loop} \vec{E} \cdot d\vec{\ell} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$

1. assume $Q_f = 0$ on surface



we gonna take $\Delta \to 0$ at the end

2. find the B.C.'s $D_{\perp,1}$ vs. $D_{\perp,2}$