Week 2 pt. 2

$$\frac{dW_{tot}}{dt} = \int_V \vec{E}\cdot\vec{J}\, d\tau$$

energy needed per unit time

change of KE of charges inside V per unit time

- get rid of $\vec{J}$ in favor of $\vec{E}$ and $\vec{B}$ (use maxwell's (3))
  $\vec{J} = \frac{1}{\mu_0}\left[\vec{\nabla}\times\vec{B} - \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}\right]$
  (painful algebra)
- $$\frac{dW}{dt} = \int_V \left\{-\frac{\partial}{\partial t}\left[\frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{1}{\mu_0}B^2\right]\right\}d\tau - \underbrace{\int_V \frac{1}{\mu_0}\vec{\nabla}\cdot(\vec{E}\times\vec{B})\,d\tau}_{\text{use divergence theorem}}$$

  ↳ $\oint (\vec{E}\times\vec{B})\,d\vec{a}$ (surface integral)

Finally get
$$\frac{dW}{dt} = \underbrace{-\frac{\partial}{\partial t}\int_V \left[\frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{1}{\mu_0}B^2\right]d\tau}_{①} \underbrace{- \frac{1}{\mu_0}\oint_S (\vec{E}\times\vec{B})\cdot d\vec{a}}_{②}$$

① = decrease of energy already inside volume V per unit time

② = energy transfer through the boundaries, to/from volume V, per unit time

$$\frac{\text{energy}}{\text{volume}} = u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$$

(Rem: phys 3, electrical energy per volume in capacitor)
magnetic energy in inductor

⟹ Energy stored in fields

Define the Poynting vector

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} \quad \left(\frac{\text{energy}}{(\text{time})(\text{area})}\right)$$

$$\boxed{\frac{dW}{dt} = -\frac{d}{dt}\int_V u\,d\tau - \oint \vec{S}\cdot d\vec{a}}$$ Poynting Theorem

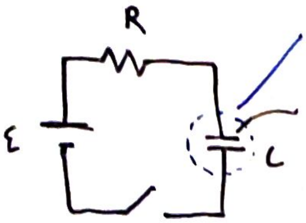
What if there are no charges inside V? ⟹ $\frac{dW}{dt} = 0$

$$\int_V \frac{\partial u}{\partial t}\,d\tau = -\oint_S \vec{S}\cdot d\vec{a}$$

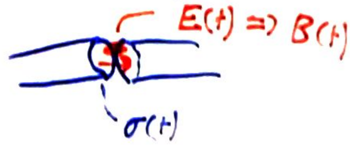
↳ divergence theorem $= -\int_V (\vec{\nabla}\cdot\vec{S})\,d\tau$

↳ $$\frac{\partial u}{\partial t} = -\vec{\nabla}\cdot\vec{S}$$

HW 8.2

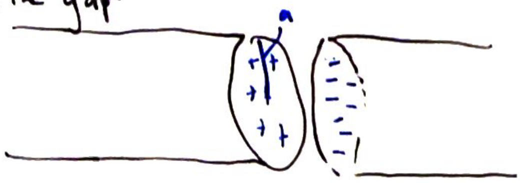


gap between 2 cylindrical wires



at $t=0$, close the switch $\Rightarrow I(t)$

in the gap:



goal: find $E(t)$, $B(t)$ inside the gap

check the continuity equations hold

find $\vec{E}$ and $\vec{B}$

$$\vec{E}_{cap} = \frac{\sigma(t)}{\varepsilon_0} = \frac{Q(t)}{\varepsilon_0 \pi a^2}$$

$$I(t) = \frac{dQ(t)}{dt} \Rightarrow Q(t) = \int I(t)\,dt$$

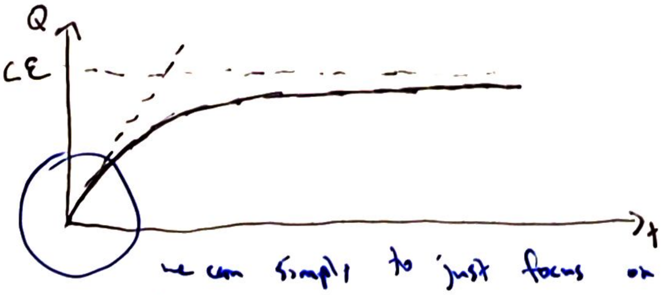
$$\sum_{loop} V = 0$$

$$\varepsilon - IR - \frac{Q}{C} = 0$$

$$\varepsilon - \frac{dQ}{dt}R - \frac{Q}{C} = 0 \quad \text{(first order ODE)}$$

$$\varepsilon - R\frac{dQ}{dt} - \frac{Q}{C} = 0$$

$\Rightarrow$ separation of variables



we can simply to just focus on the linear regime

$Q(t) \sim t$