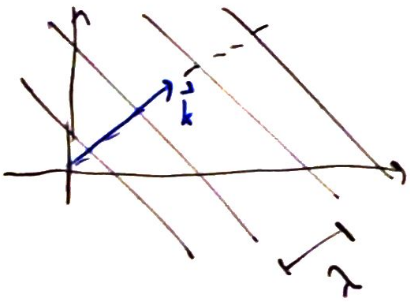
Week 5

How to write a 2D plane wave



$\Psi(x,y,t)$

where $|k| = \frac{2\pi}{\lambda}$

wavefronts defined by $\vec{k}\cdot\vec{r} = \text{const.}$

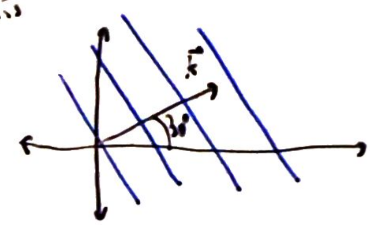
$$\Psi(x,y,t) = A\exp\left(i(\vec{k}\cdot\vec{r} - \omega t)\right)$$

write in components: $\vec{r} = x + y$ $\qquad \vec{k} = k_x + k_y$

$$\Psi(x,y,t) = A\exp\left(i(k_x x + k_y y - \omega t)\right)$$

Example: 2D wave travelling at 30° to x-axis

$\lambda = 20\,\text{m}$ $\qquad v = 10\,\text{m/s}$



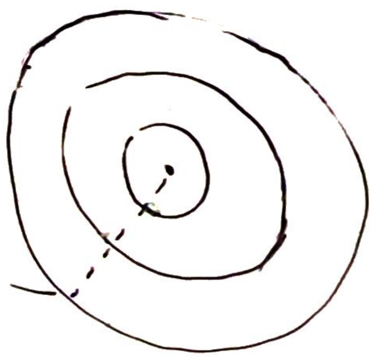
$$\Psi(x,y,t): k = \frac{2\pi}{\lambda} = \frac{\pi}{10}\,\text{m}^{-1}$$

$$\omega = 2\pi f = 2\pi\frac{v}{\lambda} = \pi\,\text{s}^{-1}$$

$k_x = \frac{\pi}{10}\cos 30° = \frac{\sqrt{3}\pi}{20}$ $\qquad k_y = \frac{\pi}{10}\sin 30° = \frac{\pi}{20}$

$$\Psi(x,y,t) = A\exp\left(i\left(\frac{\sqrt{3}\pi}{20}x + \frac{\pi}{20}y - \pi t\right)\right)$$

Spherical wave: $\Psi(r,t) = \frac{A}{r}e^{i(kr-\omega t)}$



at large r, the wave begins to look plane

↓ getting smaller as it goes out

Electromagnetic waves

$$\vec{E} = \vec{E}_0\sin(\vec{k}\cdot\vec{r} - \omega t)$$
$$\vec{B} = \vec{B}_0\sin(\vec{k}\cdot\vec{r} - \omega t)$$

$$|E_0| = c|B_0|$$

There is an energy density to do with E and B fields

$u_E = \frac{1}{2}\epsilon_0 E^2$ $u_B = \frac{1}{2}\frac{1}{\mu_0}B^2$ (woah! E&M)

$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$

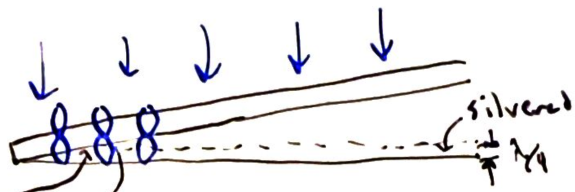
Irradiance of the EM wave $I = \frac{1}{2}\epsilon_0 c E_0^2$ watts/m^2

this is what we observe $I \propto E_0^2$

why so excited about the electric field?

Wiener experiments

get standing waves



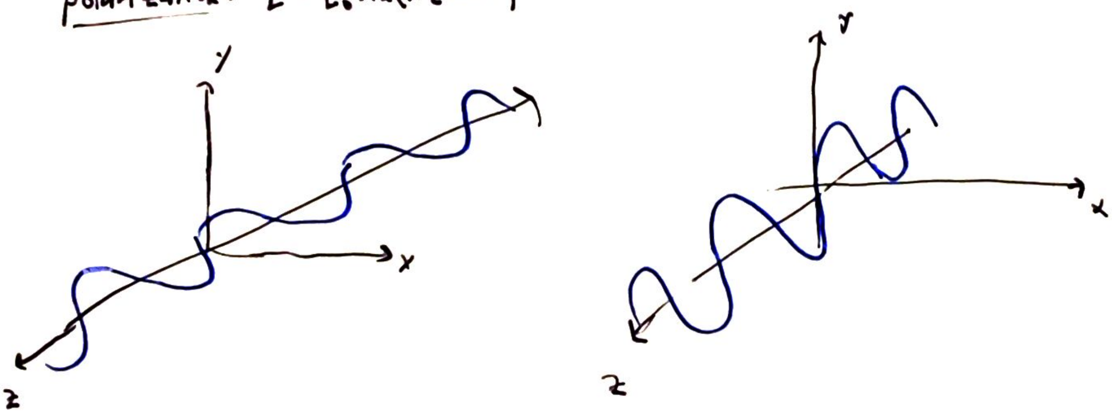
antinode on the photo emulsion

photo emulsion (very thin)

you then get dark lines on the paper spaced out

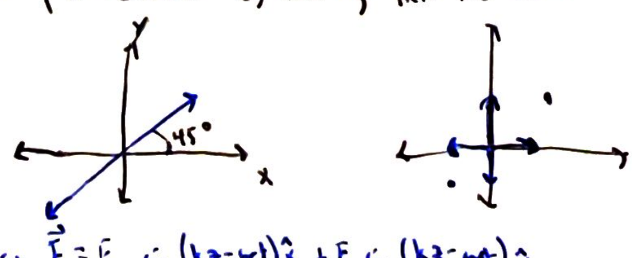
↳ the electric field does the exposure

polarization: $\vec{E} = E_0 \sin(kz - \omega t)$



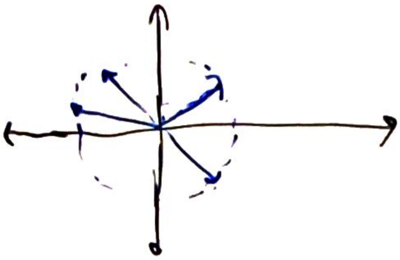
we define the direction of polarization by looking into the beam

e.g. 45° linear polarization



we would write this as $\vec{E} = E_0 \sin(kz - \omega t)\hat{x} + E_0 \sin(kz - \omega t)\hat{y}$

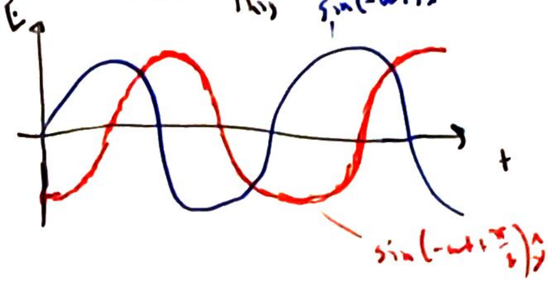
circular polarization - e.g. left circularly polarized



moves around a full circle in 1 wavelength

$$\vec{E} = E_0 \sin(kz - \omega t)\hat{x} + E_0 \sin(kz - \omega t + \tfrac{\pi}{2})\hat{y}$$

check this $\sin(-\omega t)\hat{x}$



read about dopler effect

$$\frac{\lambda'}{\lambda} = 1 - \frac{v}{c}$$

Ch 7:

Light interference

We pretty much treat the light as a scalar waves & we use just one polarization

$$E_{1,p} = E_{0,1}\cos(ks_1 - \omega t + \phi_1)$$

$$k = \frac{2\pi}{\lambda}$$

$$E_{2,p} = E_{0,2}\cos(ks_2 - \omega t + \phi_2)$$

ϕ_1 & ϕ_2 are the phase constants

$E_p = E_{1,p} + E_{2,p}$ - instantaneous E-field at p

& we want the irradiance at p, power/area e.g. $\frac{\text{watts}}{m^2}$

Irradiance $\propto$ amplitude^2

$$\tilde{E}_{1,p} = E_1 \exp(i(ks_1 + \phi_1))$$

$$\tilde{E}_p = \tilde{E}_{1,p} + \tilde{E}_{2,p}$$

$$\tilde{E}_{2,p} = E_2 \exp(i(ks_2 + \phi_2))$$

$$\tilde{E}_p \tilde{E}_p^* = (\tilde{E}_{1,p} + \tilde{E}_{2,p})(\tilde{E}_{1,p} + \tilde{E}_{2,p})^*$$

$$\tilde{E}_p \tilde{E}_p^* = I$$
$$= I_1 + I_2 + E_1 E_2 \exp(i(k(s_1 - s_2) + (\phi_1 - \phi_2)))$$
$$+ E_1 E_2 \exp(i(k(s_2 - s_1) + (\phi_2 - \phi_1)))$$

$$E_1 = \sqrt{I_1} \qquad E_2 = \sqrt{I_2}$$

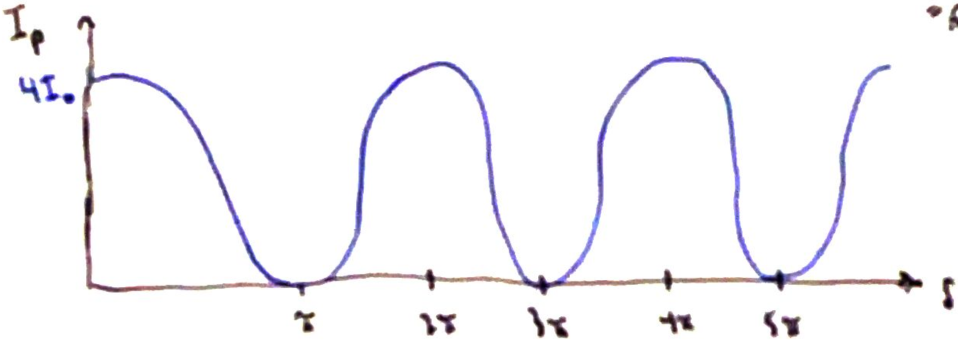
$I = I_1 + I_2 + \sqrt{I_1}\sqrt{I_2}\, 2\cos(\delta)$ when $\delta = k(s_2 - s_1) + \phi_2 - \phi_1$

$$\boxed{I_p = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\delta)}$$

let $I_1 = I_2 = I_0$

then $I_p = 2I_0 + 2I_0\cos(\delta)$

we refer to this as "fringes"



use double angle formula

$$I_p = 2I_0 \cdot 2\cos^2(\delta/2)$$

$$= 4I_0\cos^2(\delta/2)$$ thus "cosine squared" fringes

– typical of 2 beam interference

If $I_1 \neq I_2$

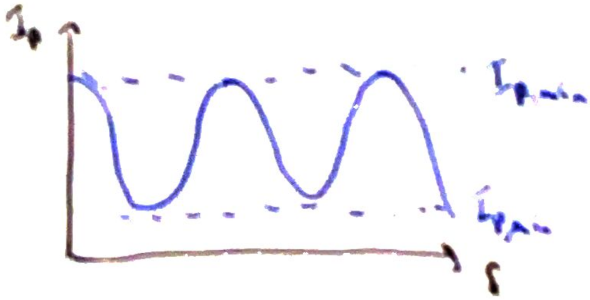
↳ $I_p = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\delta)$

$$I_{p,max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$I_{p,min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

define visibility: $V = \dfrac{I_{max} - I_{min}}{I_{max} + I_{min}}$

$0 < V < 1$



So far have assumed that ϕ_1 & ϕ_2 are fixed ($\phi_2 - \phi_1$ constant)

But light from different sources (two different lasers) will have ϕ_1 and ϕ_2 varying rapidly

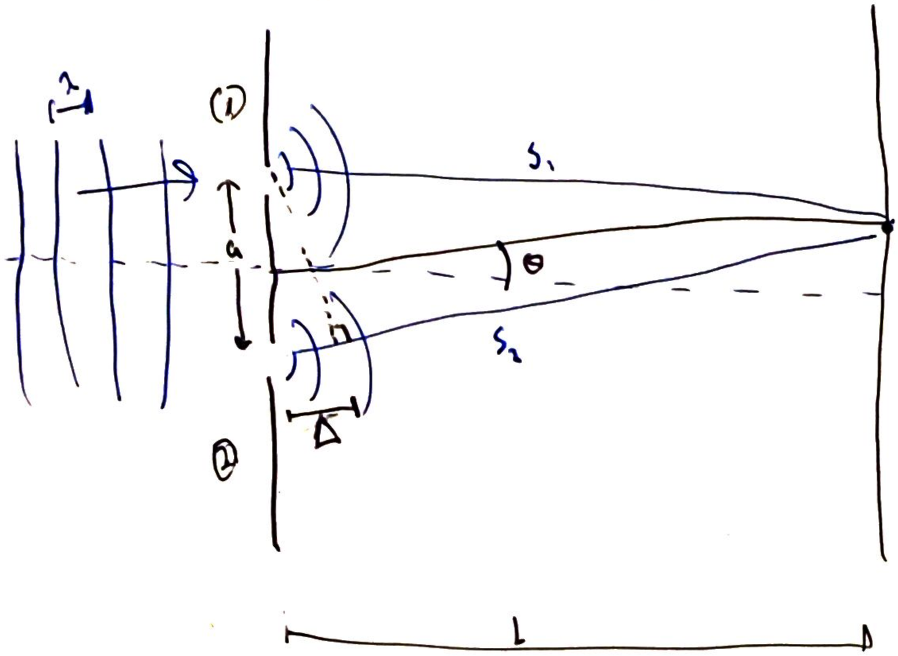
$$I_p = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\delta)$$

will average out to zero

$I_p = I_1 + I_2$ we call the sources incoherent

average $I_p = I_1 + I_2$

young's 2 slits - an example of a wavefront division interferometer



slits of equal size

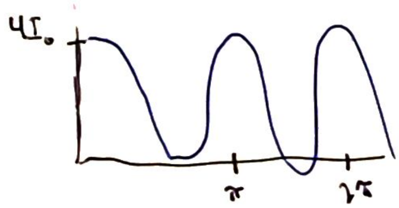
↳ give the same contribution

$s_2 - s_1$ is not so big that the amplitudes at P do not differ significantly

$\Delta = s_1 - s_2 \approx a\sin\theta$

let $\phi_1 = \phi_2$

then $\delta = k(s_2 - s_1) = k\Delta$ so $I_p = 4I_0\cos^2\left(\frac{\delta}{2}\right) = 4I_0\cos^2\left(\frac{k}{2}a\sin\theta\right)$



so we get maxima when

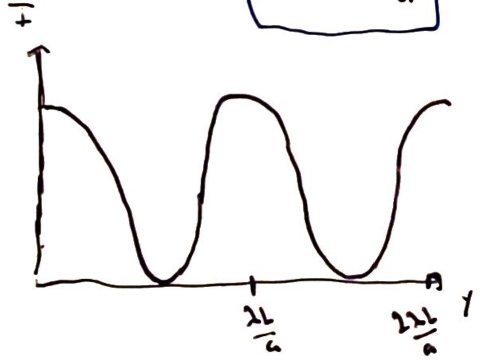
$\frac{k}{2}a\sin\theta = 0, \pi, 2\pi$

so $\frac{\pi}{\lambda}a\frac{y_{max}}{L} = m\pi$ $m = 0, 1, 2, \dots \Rightarrow y_{max} = \frac{mL\lambda}{a}$

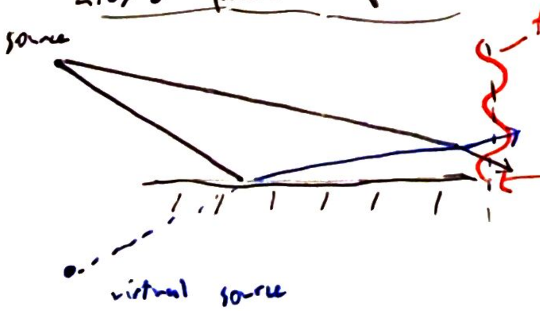
so $\boxed{\Delta y_{max} = \frac{\lambda L}{a}}$

so larger λ → spots spread appart

decrease slit spacing → spots spread appart



Lloyd's mirror experiment

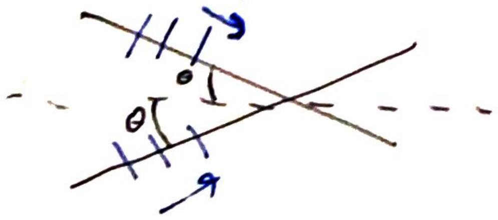


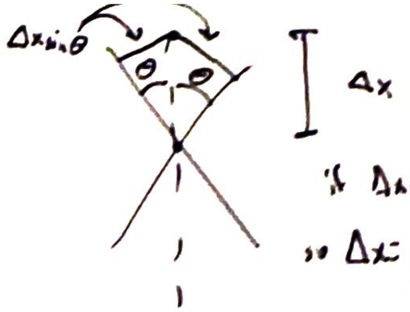
destructive interference

- because reflected and transmitted light are 180° out of phase

↳ phase change due to reflection

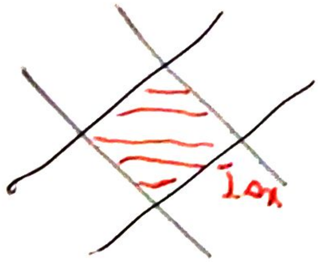
consider 2 plane waves





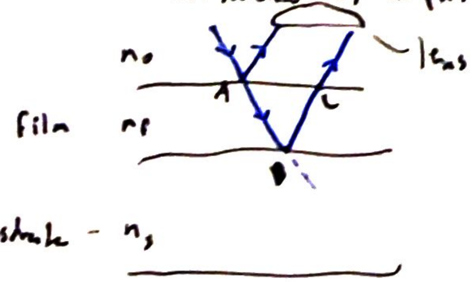
if $\Delta x \sin\theta = \frac{\lambda}{2} \Rightarrow$ const. interference

so $\Delta x = \frac{\lambda}{2\sin\theta}$



Interference in dielectric films

↳ interference by amplitude division



To begin, normal incidence, ignore multiple reflections

we figure the interference based on optical path difference

$\Delta = n_f(2t)$

$(K(s_1 - s_2))$

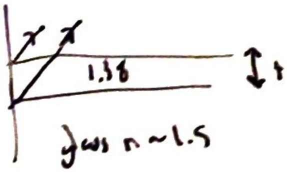
we may have phase change on reflection: $n_f > n_0 \Rightarrow \pi$ phase shift
$n_s > n_f \Rightarrow \pi$ phase shift

just like an extra half-wavelength $(\lambda/2)$

the phase shift due to the optical path difference is

$$\Delta\phi = k\Delta = k(2n_f t) = \frac{2\pi}{\lambda} 2 n_f t$$

e.g. if you wanted destructive interference with a film of MgF_2 on glass



$n_{MgF_2} = 1.38$ @ 550 nm

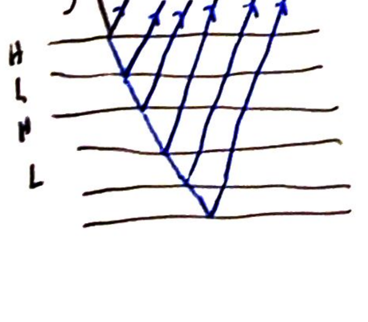
need $2t\left(\frac{2\pi}{\lambda} n_f\right) = \pi$

don't need to worry about phase changes from reflection because you get 2π

$\Rightarrow t = 99.6$ nm

using multilayer stacks of dielectrics can get very high reflectivities

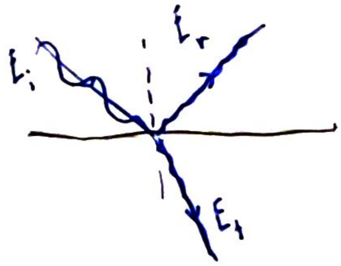
make these all interfere constructively



Start more complex thin film interferometers

but just before that

Stokes' relations:



reflection coefficients transmission coefficients

$$r = \frac{E_r}{E_i} \quad , \quad t = \frac{E_t}{E_i}$$