Last time:

$$\vec{\nabla}\cdot\vec{D}=0$$
$$\vec{\nabla}\cdot\vec{B}=0$$
$$\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$
$$\vec{\nabla}\times\vec{H}=\frac{\partial\vec{D}}{\partial t}$$

Special case: linear media

$$\vec{D}=\varepsilon\vec{E} \qquad \vec{H}=\frac{1}{\mu}\vec{B}$$

$$\left.\begin{aligned}\vec{\nabla}\cdot\vec{E}&=0\\ \vec{\nabla}\cdot\vec{B}&=0\\ \vec{\nabla}\times\vec{B}&=\mu\varepsilon\frac{\partial\vec{E}}{\partial t}\\ \vec{\nabla}\times\vec{E}&=-\frac{\partial\vec{B}}{\partial t}\end{aligned}\right\} \rightarrow$$

note that these look exactly like maxwell's equations in a vacuum

but $\varepsilon_0\to\varepsilon$

$\mu_0\to\mu$

Therefore, if an electromagnetic wave travels in linear media, the only thing that happens is a different wave speed

$$v=\frac{1}{\sqrt{\varepsilon\mu}} \qquad \text{compare} \qquad c=\frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

$$v=\frac{c}{n} \qquad n=\text{refractive index}$$

note: $\varepsilon=\varepsilon_0(1+\chi_e)$ $\qquad \mu=\mu_0(1+\chi_m)$

$$\Rightarrow n\geq 1 \qquad n=\sqrt{(1+\chi_e)(1+\chi_m)}\geq 1$$

all previous results for $\vec{E}(z,t)$ and $\vec{B}(z,t)$ still hold with

$$\varepsilon_0\to\varepsilon \qquad \mu_0\to\mu \qquad c\to v=\frac{1}{\sqrt{\mu\varepsilon}}$$

$$u=\frac{1}{2}\left(\varepsilon E^2+\frac{1}{\mu}B^2\right) \qquad \vec{S}=\frac{1}{\mu}\left(\vec{E}\times\vec{B}\right)$$

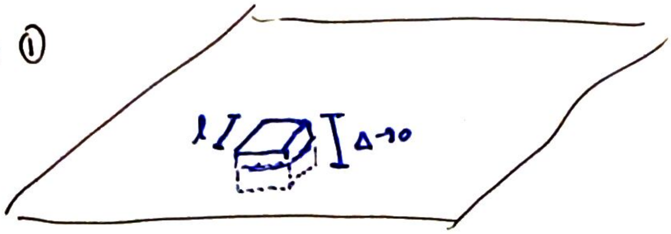
$$\omega=kv \qquad I=\frac{1}{2}\varepsilon vE_0^2$$

note usually $\chi_m \ll 1$ (so $\mu\approx\mu_0$)

$$n\approx\sqrt{1+\chi_e}$$

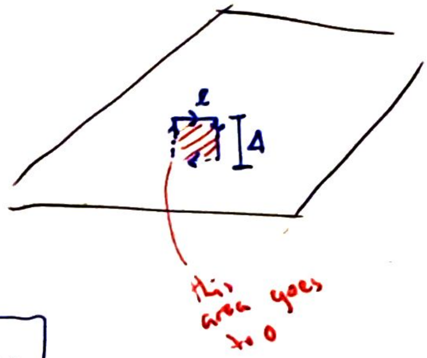
Boundary conditions, $\rho_F = 0$, $J_F = 0$

glass ①



plastic ②

Goal: find out if $\vec{D}, \vec{H}, \vec{E}, \vec{B}$ are continuous / discontinuous across boundary



1 $\oint_{surf} \vec{D} \cdot d\vec{a} = \cancelto{0}{Q_{free}}$ 2 $\oint_{surf} \vec{B} \cdot d\vec{a} = 0$

3 $\oint_{loop} \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int_{surf} \vec{B} \cdot d\vec{a}$ 4 $\oint \vec{H} \cdot d\vec{l} = \cancelto{0}{I_{free}} + \frac{\partial}{\partial t} \int_{surf} \vec{D} \cdot d\vec{a}$

from 1: the $D_\perp$ must be continuous $\boxed{D_{\perp,above} = D_{\perp,below}}$

and from 2: the $B_\perp$ must be continuous $\boxed{B_{\perp,above} = B_{\perp,below}}$

from 3:

$l(E_{\parallel,above} - E_{\parallel,below}) = 0$

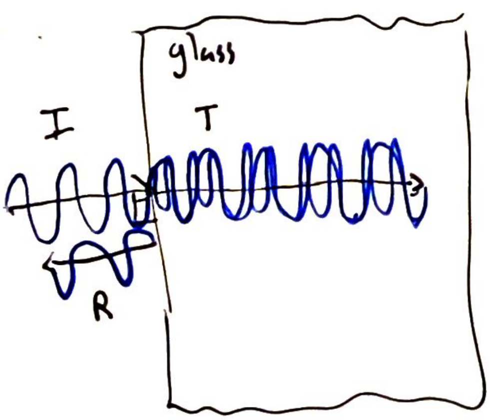
(because $\int B \cdot da \to 0$)

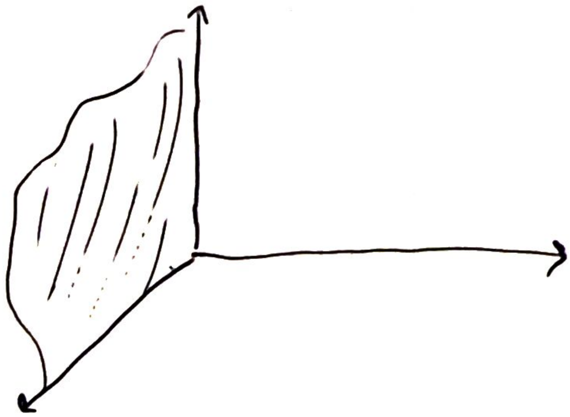
$\Rightarrow E_{\parallel,above} - E_{\parallel,below} = 0 \Rightarrow \boxed{E_{\parallel,above} = E_{\parallel,below}}$

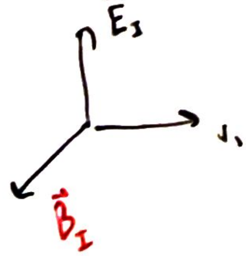
from 4: $\boxed{H_{\parallel,above} = H_{\parallel,below}}$

For linear media: $\boxed{\epsilon_{above} E_{\perp,above} = \epsilon_{below} E_{\perp,below}}$

$\boxed{\frac{1}{\mu_{above}} B_{\parallel,above} = \frac{1}{\mu_{below}} B_{\parallel,below}}$






$\vec{E}_I$
v_I
$\vec{B}_I$