##Week 4 pt 3##

In electromagnetic wave in a vacuum

1) $\vec{B} \perp \vec{E}$
2) Transverse
3) $|\vec{B}| = \frac{|\vec{E}|}{c}$
4) $\delta_E = \delta_B$ (waves are in phase)

$k(\hat{z} \times \vec{E}_0) = \omega \vec{B}_0$ (from Faraday's)

$|\vec{B}_0| = \frac{k}{\omega} |\hat{z} \times \vec{E}_0|$

$= \frac{k}{\omega} |\hat{z}| |\hat{E}_0| \sin(\frac{\pi}{2})$

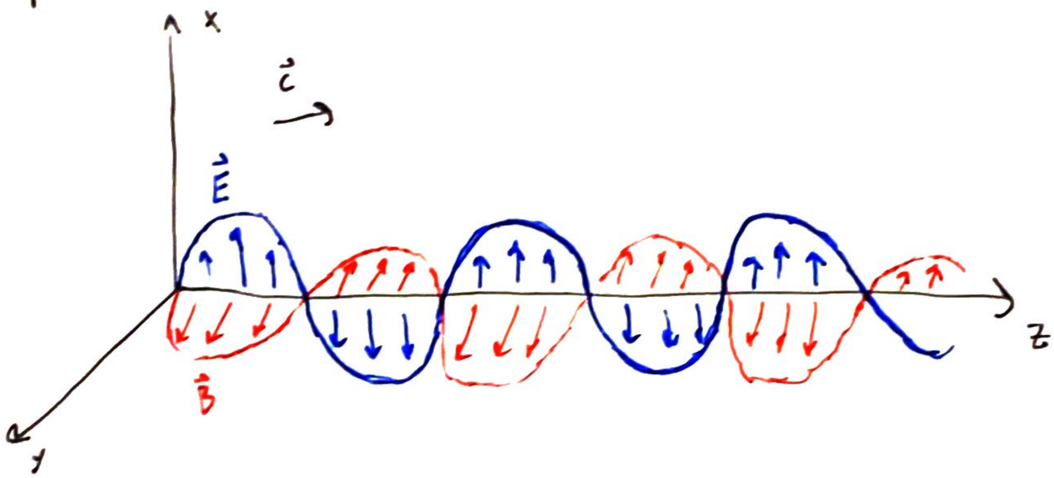
$= \frac{k}{\omega} |\vec{E}_0|$

$\tilde{B}_0 = \frac{k}{\omega} \tilde{E}_0$

$B_0 e^{i\delta_B} = \frac{k}{\omega} E_0 e^{i\delta_E}$

$\Rightarrow \delta_B = \delta_E$

Ampere's law → same shit



$\vec{E} \times \vec{B}$ shows direction of propagation

Energy and Momentum in EM waves

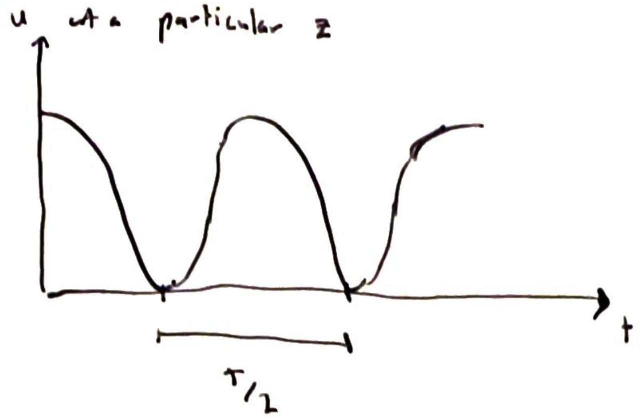
* $\vec{E}(z,t) = E_0 \cos(kz - \omega t + \delta)\hat{x}$

** $\vec{B}(z,t) = \sqrt{\mu_0 \epsilon_0} E_0 \cos(kz - \omega t + \delta)\hat{y}$ $\quad B_0 = \frac{1}{c} E_0 = \sqrt{\mu_0 \epsilon_0} E_0$

Recall: energy density: $u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{1}{\mu_0} B^2$

equal contribution due to E and B fields

$= \epsilon_0 E_0^2 \cos^2(kz - \omega t + \delta) = \frac{1}{\mu_0} B_0^2 \cos^2(kz - \omega t + \delta)$



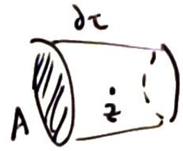
$T = \frac{2\pi}{\omega}$

$$\vec S = \frac{\text{energy}}{\text{time}\cdot\text{area}} = \frac{1}{\mu_0}\underset{*}{\vec E}\times\underset{*}{\vec B} = \frac{1}{\mu_0\epsilon_0}\frac{1}{c}\epsilon_0\cos^2(kz-\omega t+\delta)\,\hat z$$

we can check $\vec\nabla\cdot\vec S = -\frac{\partial}{\partial z}cu = \frac{\partial}{\partial t}u$

energy balance equation

wave carries energy $u\,d\tau = u A\,dz = uAc\,dt$



$$\frac{\text{energy}}{\text{area}\cdot\text{time}} = uc$$

$\vec g = \epsilon_0(\vec E\times\vec B)$ (momentum density in EM fields)

$$= \epsilon_0\mu_0\Big(\underbrace{\tfrac{1}{\mu_0}\vec E\times\vec B}_{\vec S}\Big) = \frac{1}{c^2}\vec S = \boxed{\frac{1}{c}u\hat z}$$

In experiments, only avg. values can be measured

Time-averaged value of <u>anything</u>

$$\langle x\rangle = \int_0^T x(t)\,dt\left(\frac{1}{T}\right)$$

$$\langle u\rangle = u_{max}\underbrace{\frac{1}{T}\int_0^T\cos^2(kz-\omega t+\delta)}_{\frac{1}{2}} = \frac{u_{max}}{2}$$

Trick: $\cos^2 x + \sin^2 x = 1$

integrate over 1 period $\Rightarrow$ $\underbrace{\frac{1}{2\pi}\int_0^{2\pi}\cos^2 x\,dx}+\underbrace{\frac{1}{2\pi}\int_0^{2\pi}\sin^2(x)\,dx} = 1$

these should be the same so each is $\frac{1}{2}$

so $\langle u \rangle = \frac{1}{2}\varepsilon_0 E_0^2$

$$\langle \vec{S} \rangle = \langle c u \hat{z} \rangle = c\hat{z}\langle u \rangle = \frac{1}{2} c \varepsilon_0 E_0^2 \hat{z}$$

$$\langle \vec{g} \rangle = \langle \frac{1}{c} u \hat{z} \rangle = \frac{1}{c}\hat{z}\langle u \rangle = \frac{1}{2c}\varepsilon_0 E_0^2 \hat{z}$$

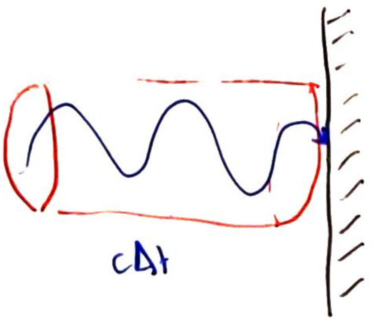
$$\text{Intensity} = \frac{\text{Avg. energy}}{(\text{area})\cdot\text{time}} = \frac{\text{Avg. power}}{\text{area}}$$

$$= |\langle \vec{S} \rangle|$$

$$I = \frac{1}{2} c \varepsilon_0 E_0^2$$

perfect absorber

Light falling on a surface exerts a force



$$F = \frac{|\Delta p|}{\Delta t}$$

$$\text{pressure} = \frac{F}{A} = \frac{1}{A}\frac{|\cancel{p_f}^{\,0} - p_i|}{\Delta t}$$

$$\vec{p}_i = \vec{g}(\text{volume}) = \vec{g} A c \Delta t$$

$$\text{pressure} = \frac{1}{A}\frac{\langle g \rangle c}{\Delta t} = c\langle g \rangle = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{I}{c} \quad \text{(Radiation pressure)}$$

compare with I