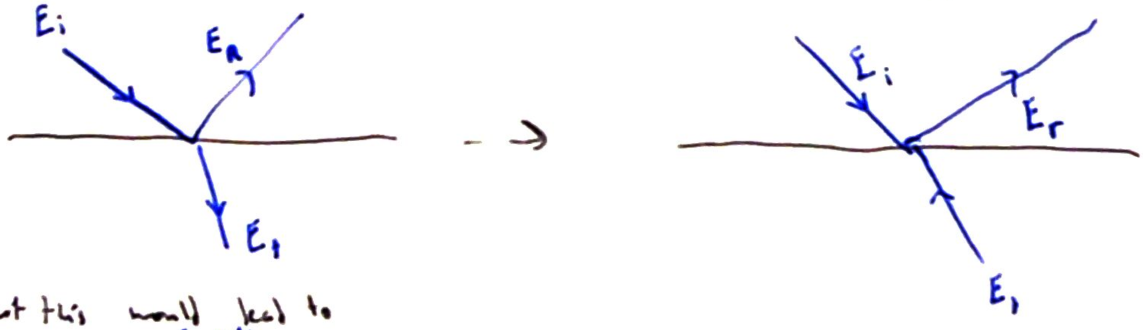
Week 6

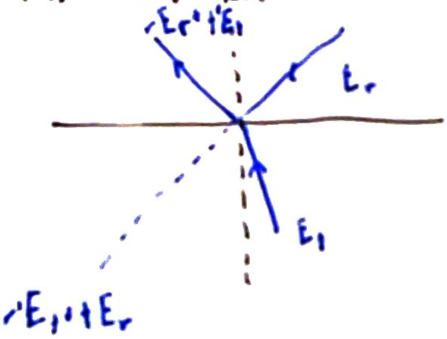
Stokes relations

reversibility

$r = \frac{E_r}{E_i}$ $\quad t = \frac{E_t}{E_i}$



but this would lead to



$E_i = rE_r + t'E_t$

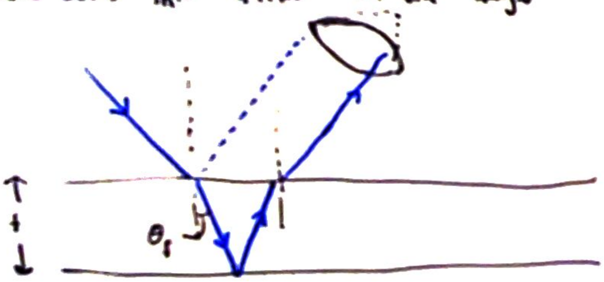
$= r^2E_i + tt'E_i$

$\boxed{1 = r^2 + tt'}$

$r'tE_i + trE_i = 0$

$\Rightarrow \boxed{r = -r'}$

Now come into film at an angle



The extra path taken through the film is

$\Delta = 2t\cos\theta_f$

The optical path is $\Delta = 2n_f t\cos\theta_f$

so the phase change acquired by the wave is

$\delta = k\Delta$

multiple beam interference

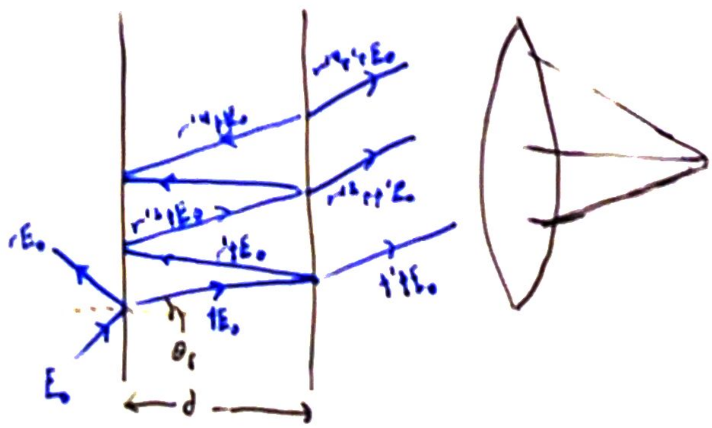
$\delta = k\Delta = k2dn_f\cos\theta_f$

Total field out is

$tt'E_0 + r'^2 t'tE_0 e^{-i\delta} + r'^4 t'tE_0 e^{-i2\delta}$

$= tt'E_0 \sum_{N=0}^{\infty} (r')^{2N} e^{-iN\delta}$

$E = \boxed{tt'E_0 \frac{1}{1 - r'^2 e^{-i\delta}}}$



to get irradiance $I = EE^*$

$$= (tt'E_0)^2 \left(\frac{1}{1-r'^2 e^{-i\delta}}\right)\left(\frac{1}{1-r'^2 e^{i\delta}}\right)$$

(using stokes relations)

so $$I = (tt'E_0)^2 \frac{1}{1-2r'^2\cos\delta + r'^4}$$

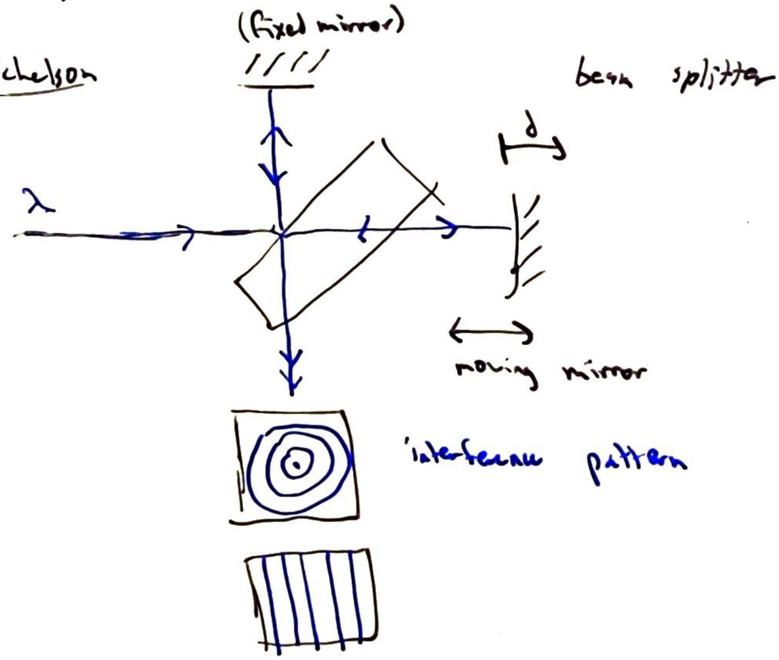
use $tt' = 1-r^2$, $r' = -r$ $\rightarrow$ $$I = I_0 \frac{(1-r^2)^2}{1-2r^2\cos\delta + r^4}$$

Ch 8: Interferometry

Taxonomy of interferometers

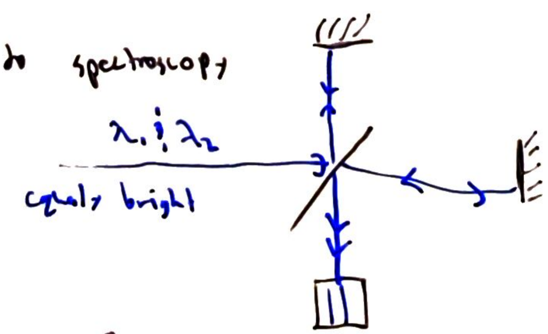
- Division of wavefront
  - 2 beam (Young's 2-slits) (Lloyd's mirror)
  - multiple beam (diffraction grating)
- Division of amplitude
  - 2 beam (Michelson)
  - multiple beam (Fabry Perot)

Michelson

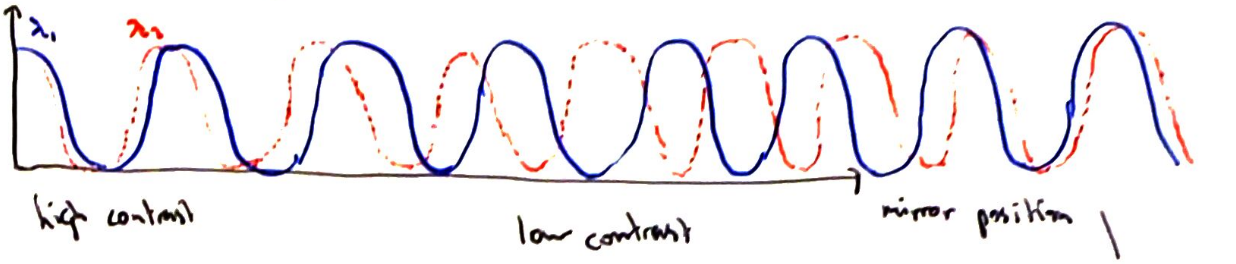


will see passage of 1 fringe if $d = \lambda/2$

Using michelson to do spectroscopy



if we look at a plot



we want this distance (high contrast to high contrast)

$2d_1 = m\lambda_1 \qquad 2d_1 = (m+n)\lambda_2$

$2d_2 = (m+p)\lambda_1 \qquad 2d_2 = (m+n+p+1)\lambda_2$

$2(d_2 - d_1) = p\lambda_1 = (p+1)\lambda_2 \quad \Rightarrow \quad p = \dfrac{\lambda_2}{\lambda_1 - \lambda_2}$

$2(d_2 - d_1) = \dfrac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$ but λ_1 & λ_2 are pretty close

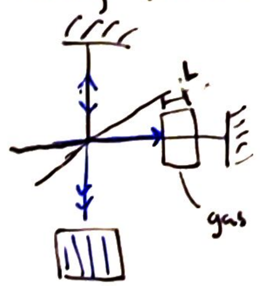
so $2\Delta d = \dfrac{\lambda^2}{\Delta\lambda}$ or $\Delta\lambda = \dfrac{\lambda^2}{2(\Delta d)}$

ex:

$\Delta d = \dfrac{\lambda^2}{2\Delta\lambda} = \dfrac{(589)^2}{2 \cdot 0.6} = 289000\ \text{nm} = 0.29\ \text{mm}$

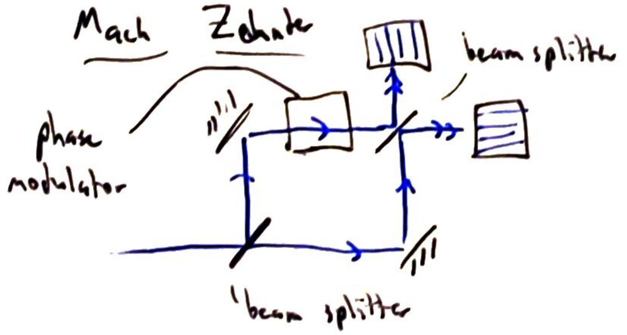
Another application

measuring refractive indexes of gasses

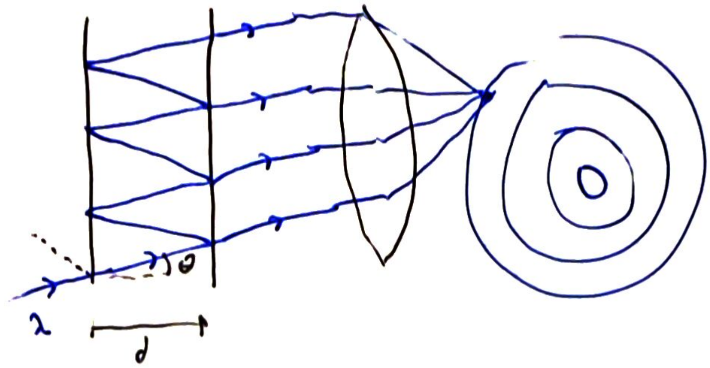


see hw 8.5

Mach Zehnder



Fabry Perot



$$T = \frac{I_t}{I_i} = \frac{1-r^2}{1+r^4-2r^2\cos\delta} \qquad \text{eq. 7.49}$$

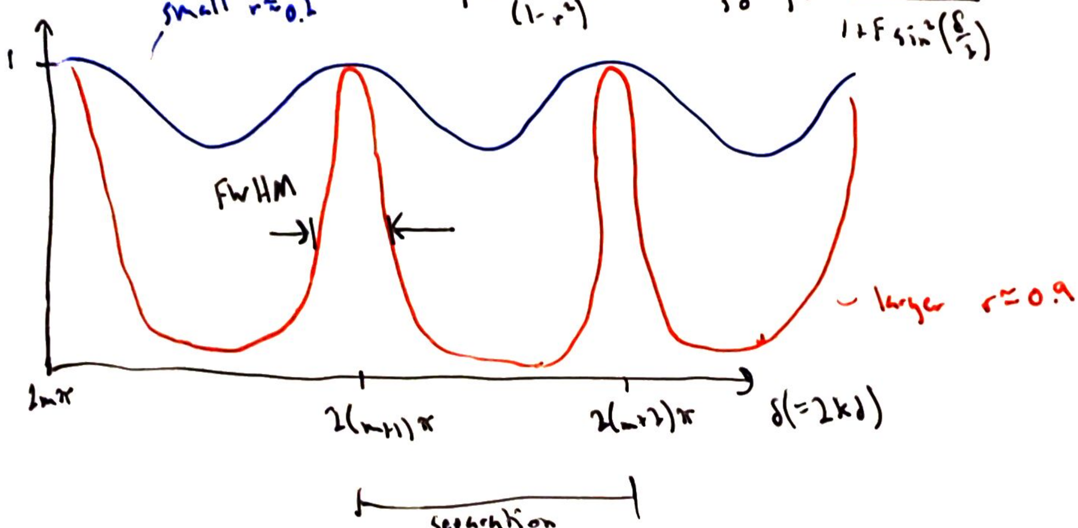
$$\delta = 2kd\cos\theta \approx 2kd$$

$$\cos\delta = 1 - 2\sin^2\frac{\delta}{2}$$

$$= \frac{1}{1+\frac{4r^2}{(1-r^2)^2}\sin^2\left(\frac{\delta}{2}\right)} \qquad \text{Airy function}$$

coefficient of finesse

$$F = \frac{4r^2}{(1-r^2)^2} \qquad \text{so } T = \frac{1}{1+F\sin^2\left(\frac{\delta}{2}\right)}$$



separation of peaks is the free spectral range (δ_{fsr})

$$\delta_{fsr} = \delta_{m+1} - \delta_m = 2\pi$$

What is the fsr in terms of frequency

$$\delta_{m+1} - \delta_m = 2k_1 d - 2k_2 d = 2d\Delta k$$

$$\text{but } k = \frac{2\pi f}{c} \qquad \frac{dk}{df} = \frac{2\pi}{c}$$

$$\text{so } \frac{2d\,2\pi\Delta f}{c} = 2\pi \quad \Rightarrow \quad \boxed{\Delta f = \frac{c}{2d}}$$

what is the ~~FWAHM~~ FWHM?

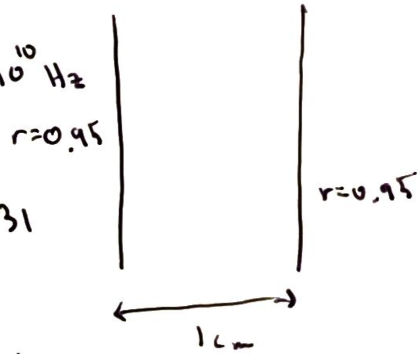
define the finesse, $\mathcal{F} = \frac{\pi\sqrt{F}}{2} = \frac{\pi r}{1-r^2}$

$$\mathcal{F} = \frac{\text{sep. between peaks}}{\text{FWHM}}$$

Finesse, gives the quality of the cavity (related to Q)

Interesting table: 8.2

Example:



then $\Delta f = \frac{c}{2d} = 1.5\times10^{10}$ Hz

since $\mathcal{F} = \frac{\pi\sqrt{F}}{2} = 31$

then FWHM $= \frac{\Delta f_{fsr}}{\mathcal{F}} = 4.8\times10^{8}$ Hz

Express this as wavelength shifts, near $\lambda = 640$ nm

$$f = \frac{c}{\lambda} \Rightarrow \frac{df}{d\lambda} = -\frac{c}{\lambda^2} \Rightarrow \Delta\lambda = \frac{\lambda^2 \Delta f}{c}$$

peak width is $\frac{(640\times10^{-9})^2 (4.8\times10^{8}\ \text{Hz})}{3\times10^{8}} = 5.5\times10^{-4}$ nm

so resolution is $\frac{\lambda}{\Delta\lambda} = \frac{640}{5.5\times10^{-4}} = 1.2\times10^{6}$