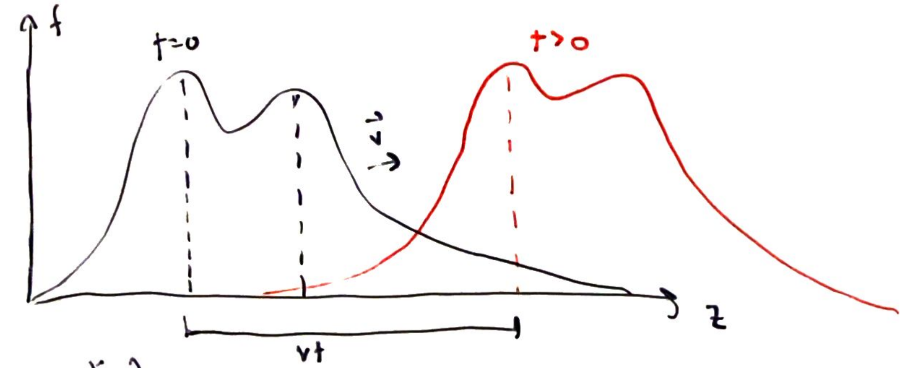
Week 4

9.1 <u>Wave</u> <u>basics</u>



1D (no spreading)

Fixed shape (no dispersion or absorbtion)

$$f(z,t) = f(z-vt, 0)$$

↳ z,t must appear together as combination $z-vt$

Wave equation in 1D (see Griffiths or Knight → derivation of wave equation)

$$\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

v = wave speed

$v = \sqrt{\frac{T}{\mu}}$ on a string

↳ solutions are of the form $f(z,t) = f(z-vt, 0) = g(v-zt)$

↳ velocity enters as v^2 → we can have a solution $f(z,t) = f(z+vt, 0) \equiv h(z+vt)$

travels to the left, negative z direction

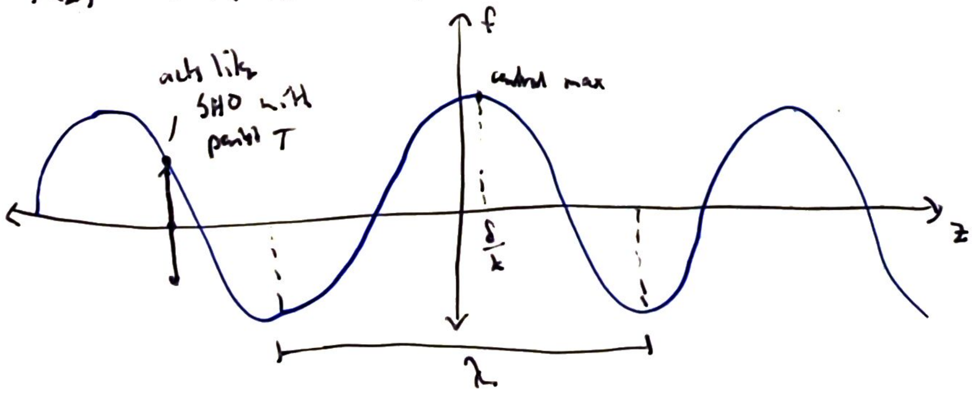
solutions $f(z,t) = g(z-vt) + h(z+vt)$

Note: g and h are not necessarily the same

if same, then we get standing waves

specific type of f

$$f(z,t) = A\cos(k(z-vt) + \delta)$$



phase $= k(z-vt)+\delta$

phase constant $=\delta$

Notice $f(z,t) = \max(\text{peak})$ when phase $=0 \Rightarrow k(z_{max}-vt)+\delta=0$

suppose $t=0 \Rightarrow$ $kz_{max} = -\delta$

$$z_{max} = -\frac{\delta}{k}$$

if $\delta > 0 \rightarrow$ step to the left by $\left|\frac{\delta}{k}\right|$

if $\delta < 0 \rightarrow$ step to the right by $\left|\frac{\delta}{k}\right|$

$\lambda = \frac{2\pi}{k}$ $\qquad T = \frac{\lambda}{v} = \frac{2\pi}{kv}$

wave number

$\nu = \frac{1}{T} = \frac{kv}{2\pi}$ (frequency) $\qquad \omega = 2\pi\nu = kv$

$(z-vt)$ must show up in this combination

to the right: $f(z,t) = A\cos(kz-kvt+\delta) = A\cos(kz-\omega t+\delta)$

wave travelling to the right

to the left: $f(z,t) = A\cos(k(z+vt)-\delta) = A\cos(kz+\omega t-\delta)$

$$= A\cos(-kz-\omega t+\delta)$$

*

In general, $f(z,t) = A\cos(kz-\omega t+\delta)$

travelling right $\Rightarrow$ $k>0$

travelling left $\Rightarrow$ $k<0$

$\lambda = \frac{2\pi}{|k|}$

$\omega = |k|v$

Complex notations! (we'll use them a lot)

Euler's formula: $e^{i\theta} = \cos\theta + i\sin\theta$

$Re(e^{i\theta}) = \cos\theta$

$Re\left[Ae^{i(kz-\omega t+\delta)}\right] = A\cos(kz-\omega t+\delta)$ → same old *

$$Ae^{i(kz-\omega t+\delta)} = \underbrace{Ae^{i\delta}}_{\tilde{A}} e^{i(kz-\omega t)}$$

$\tilde{A}$ (complex amplitude)

The wave equation is linear

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} \Rightarrow f_1, f_2 \text{ are possible solutions}$$

$f = c_1 f_1 + c_2 f_2 + c_3 f_3 + \dots + c_n f_n$ is also a solution of the differential equation

we can create a superior wave by superimposing a bunch of different waves

What makes waves 'different'?

$k, \delta, A, \cancel{\omega}$

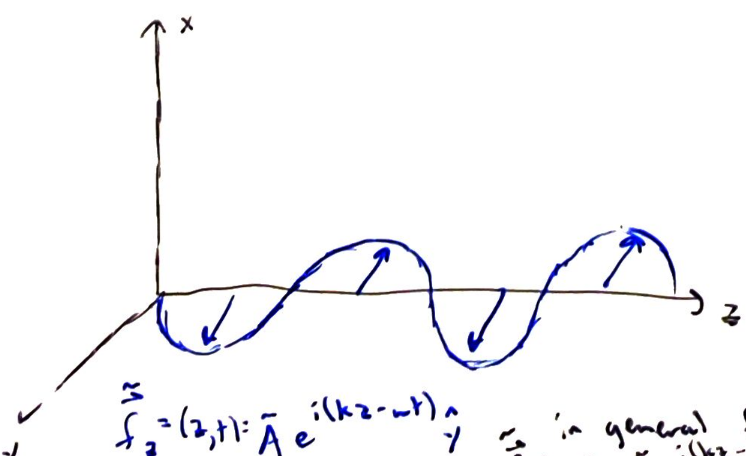
$\omega = kv$ → determined by medium

so ω and k are not independent

$$\text{so } f(z,t) = \sum_{\text{all } k} A_k \cos(kz - \omega t + \delta_k)$$
$$= \mathrm{Re}\left[\sum_{k=-\infty}^{\infty} \tilde{A}_k e^{i(kz-\omega t)}\right] = \mathrm{Re}\int_{-\infty}^{\infty} \tilde{A}(k) e^{i(kz-\omega t)}\, dk$$

Polarization

$\tilde{\vec{f}}(z,t) = \tilde{A} e^{i(kz-\omega t)} \hat{x}$ oscillates along x direction



$\tilde{\vec{f}}_2(z,t) = \tilde{A} e^{i(kz-\omega t)} \hat{y}$

in general for transverse waves, $\tilde{\vec{f}}(z,t) = \tilde{A} e^{i(kz-\omega t)} \hat{n}$