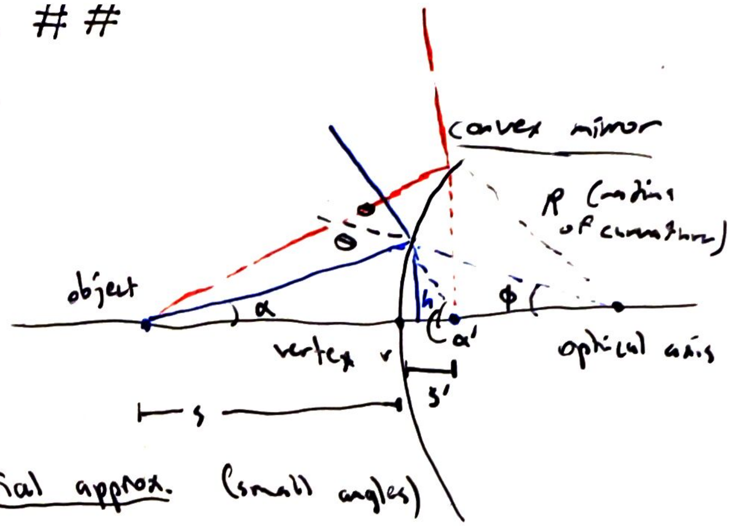
Week 2

Imaging by reflection and refraction

start with a spherical mirror



$$\left.\begin{aligned} \alpha + \alpha' &= 2\theta \\ \alpha + \phi &= \theta \end{aligned}\right] \rightarrow \alpha - \alpha' = -2\phi$$

$\tan\alpha = \frac{h}{s}$ $\quad \tan\alpha' = \frac{h}{s'}$

$\tan\phi = \frac{h}{R}$

make the paraxial approx. (small angles)

$\alpha = \frac{h}{s}$ $\quad \alpha' = \frac{h}{s'}$ $\quad \phi = \frac{h}{R}$

$\Downarrow$

$$\frac{h}{s} - \frac{h}{s'} = \frac{-2h}{R} \Rightarrow \frac{1}{s} - \frac{1}{s'} = -\frac{2}{R}$$

- note s' is independent of α

when $s \to \infty$, $\Rightarrow s' = \frac{R}{2}$ which is the focal length

we could repeat this for concave surfaces, but to keep the math straight, we need to adopt a sign convention

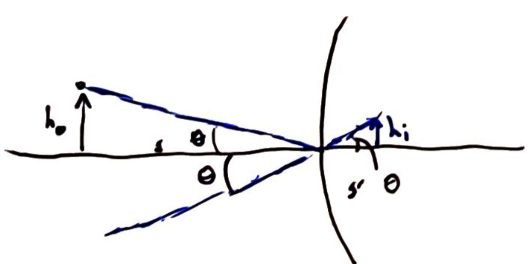
1. s is (positive) + if the object is to the left of the mirror vertex (real object)
2. s' is positive if the image is to the left of the vertex (real image)
3. R is + when the center of curvature is to the right of the vertex

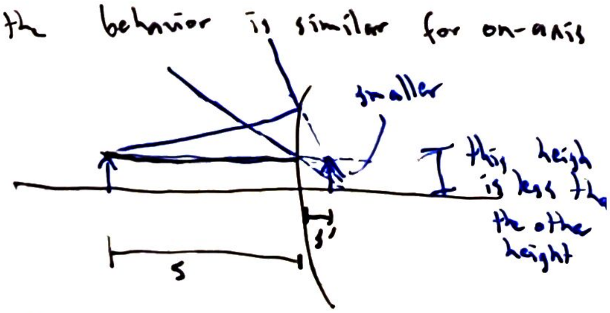
Conventionally, a concave lens has a positive focal length

$f > 0$ for concave

$f < 0$ for convex $\quad \Rightarrow \quad f = -\frac{R}{2}$

If we move our object point off-axis, the behavior is similar for on-axis





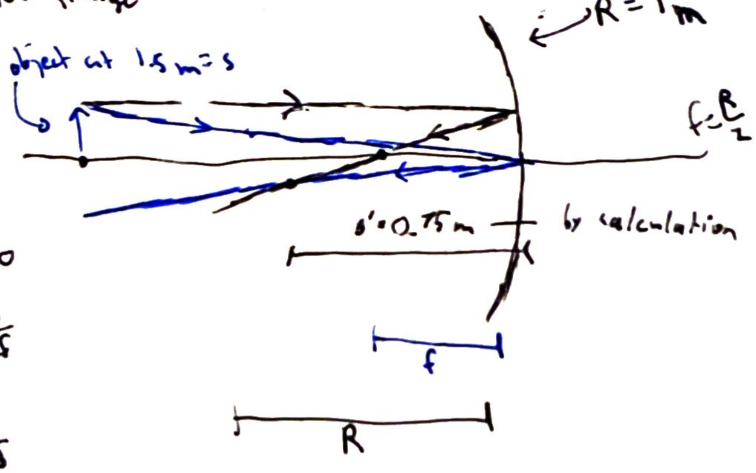
$$\frac{h_o}{s} = \frac{h_i}{s'} \Rightarrow m = \frac{h_i}{h_o} = \frac{s'}{s} \quad \text{magnification}$$

but typically $m = \frac{-s'}{s}$ to keep it with the previous sign convention

if $m > 0 \Rightarrow$ upright image

if $m < 0 \Rightarrow$ inverted image

ex:



$s = 1.5\ m$

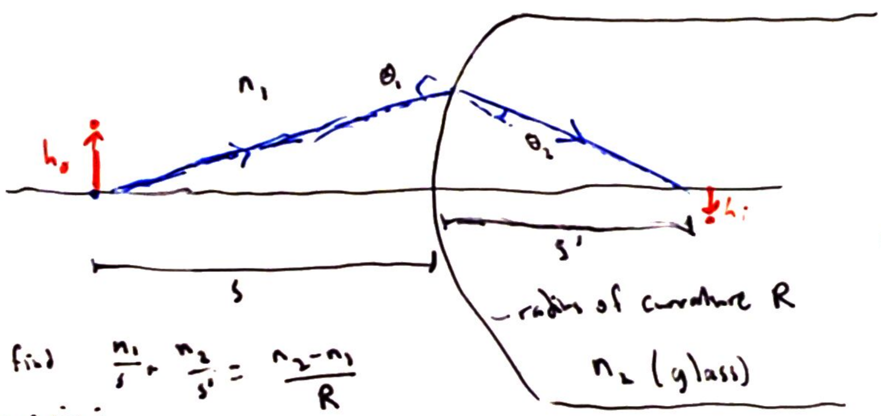
since R is to the left, $R < 0$

and $f = -\frac{R}{2} \Rightarrow \frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = 2 - \frac{2}{3} = \frac{4}{3}$$

so $s' = \frac{3}{4} = 75\ cm$

Refraction of spherical surfaces



$n_1 \sin\theta_1 = n_2 \sin\theta_2$

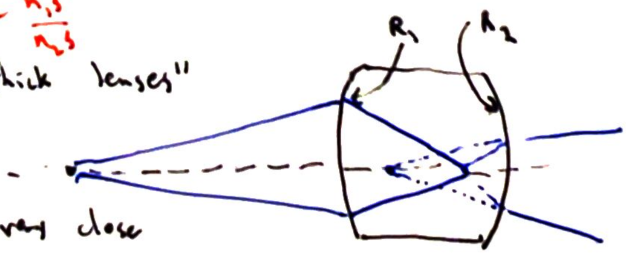
we would find $\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$

sign convention:

- s + for a real object (left of vertex)
- s' + for a real image (right of vertex)
- R + when center to the right of the vertex

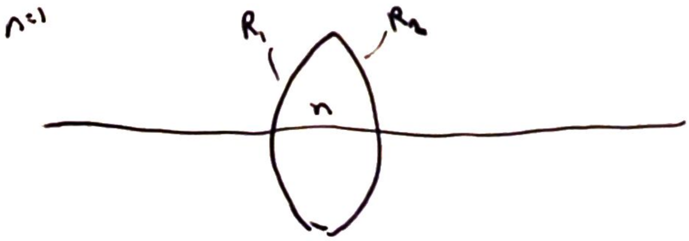
For a off-axis point $m = -\frac{n_1 s'}{n_2 s}$

We can use this to treat "thick lenses"



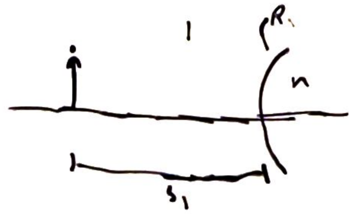
check out example 2.2 in text

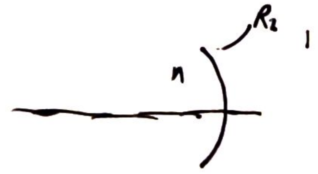
thin lenses: the two surfaces are very close



we have

$$\frac{1}{s}+\frac{n}{s'}=\frac{n-1}{R}$$





$$\frac{n}{s_2}+\frac{1}{s_2'}=\frac{1-n}{R_2}$$

$s_2=-s_1'$ because the lens is <u>thin</u>

the image of the first is the object of the second but on the opposite side

so $$\frac{1}{s_1}-\frac{n-1}{R_1}=\frac{1-n}{R_2}-\frac{1}{s_2'}$$

↳ location of the final image

$$\boxed{\frac{1}{s_1}+\frac{1}{s_2'}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}$$

sometimes written $\frac{1}{s_1}+\frac{1}{s_2'}=\frac{1}{f}$

when $\boxed{\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}$

lens-maker's formula

<u>Lens:</u> converging or diverging

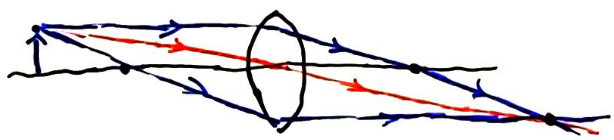
+f positive focal length

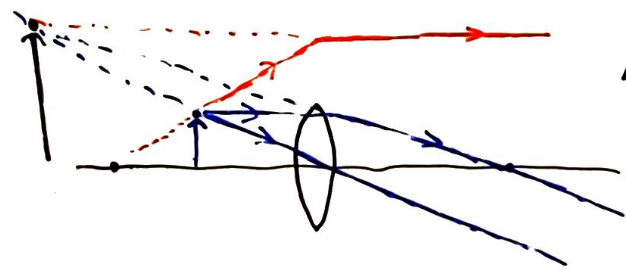
−f negative focal length

the focal length describes where parallel rays of light will be focused

ray tracing

object outside the focus

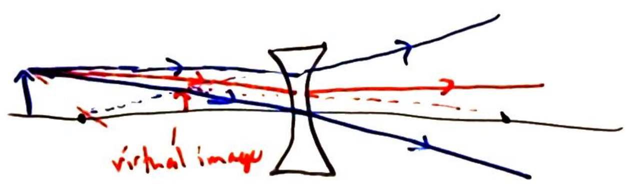




simple magnifier

<u>diverging</u> lens

object to the left



if the object is inside the focal length

↳ virtual, smaller, upright image