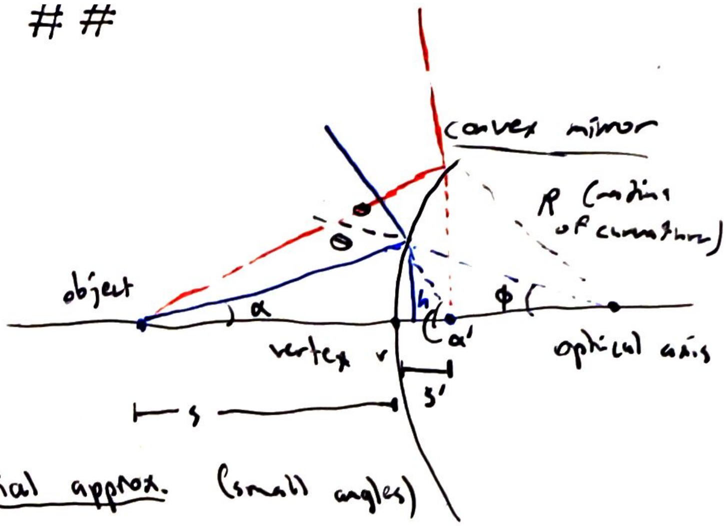
Week 2

Imaging by reflection and refraction

start with a spherical mirror



$$\left.\begin{aligned} \alpha + \alpha' &= 2\theta \\ \alpha + \phi &= \theta \end{aligned}\right] \rightarrow \alpha - \alpha' = -2\phi$$

$\tan\alpha = \frac{h}{s}$ $\quad \tan\alpha' = \frac{h}{s'}$

$\tan\phi = \frac{h}{R}$

make the paraxial approx. (small angles)

$\alpha = \frac{h}{s}$ $\quad \alpha' = \frac{h}{s'}$ $\quad \phi = \frac{h}{R}$

$\Downarrow$

$$\frac{h}{s} - \frac{h}{s'} = \frac{-2h}{R} \Rightarrow \frac{1}{s} - \frac{1}{s'} = -\frac{2}{R}$$

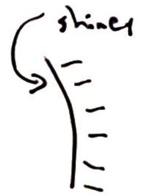
- note s' is independent of α

when $s \to \infty$, $\Rightarrow s' = \frac{R}{2}$ which is the focal length

we could repeat this for concave surfaces, but to keep the math straight, we need to adopt a sign convention

1. s is + (positive) if the object is to the left of the mirror vertex (real object)
2. s' is positive if the image is to the left of the vertex (real image)
3. R is + when the center of curvature is to the right of the vertex

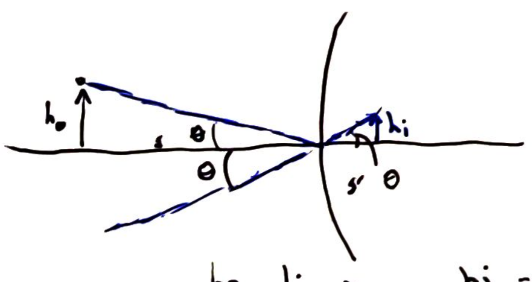
Conventionally, a concave lens has a positive focal length

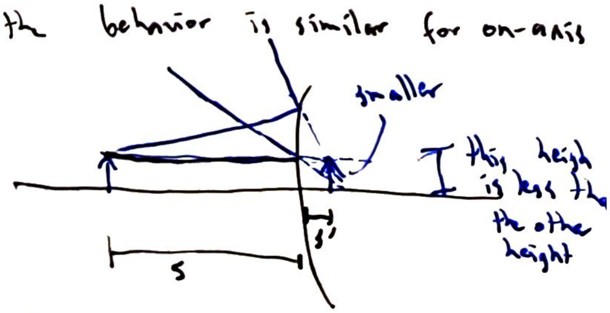


$f > 0$ for concave

$f < 0$ for convex $\Rightarrow f = -\frac{R}{2}$

If we move our object point off-axis, the behavior is similar for on-axis



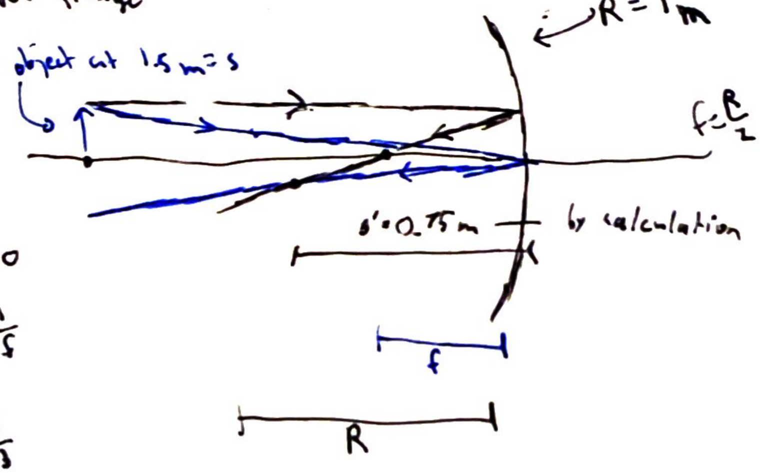


$\frac{h_o}{s} = \frac{h_i}{s'} \Rightarrow m = \frac{h_i}{h_o} = \frac{s'}{s}$ magnification but typically $m = \frac{-s'}{s}$ to keep it with the previous sign convention

if $m > 0 \Rightarrow$ upright image

if $m < 0 \Rightarrow$ inverted image

ex:



$s = 1.5\ m$

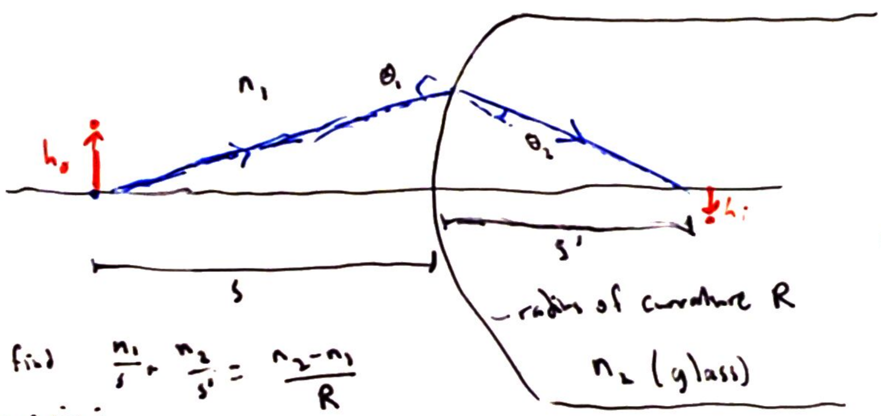
since R is to the left, $R < 0$

and $f = -\frac{R}{2} \Rightarrow \frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = 2 - \frac{2}{3} = \frac{4}{3}$$

so $s' = \frac{3}{4} = 75\ cm$

Refraction of spherical surfaces



$n_1 \sin\theta_1 = n_2 \sin\theta_2$

we would find $\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$

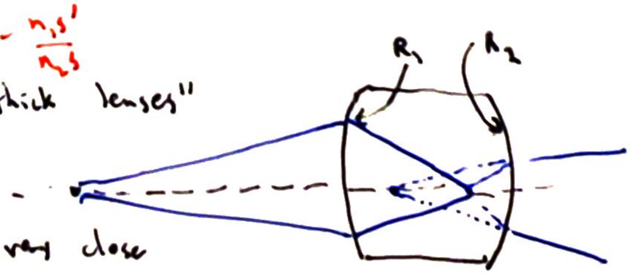
sign convention:

- s + for a real object (left of vertex)
- s' + for a real image (right of vertex)
- R + when center to the right of the vertex

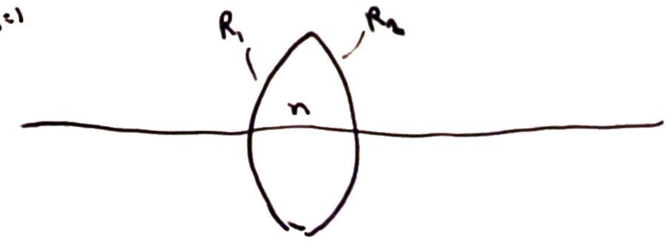
For a off-axis point $m = -\frac{n_1 s'}{n_2 s}$

We can use this to treat "thick lenses"

$\boxed{\text{check out example 2.2 in text}}$

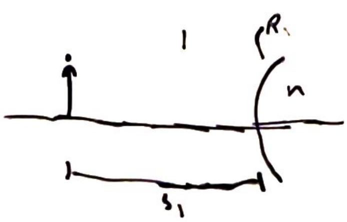


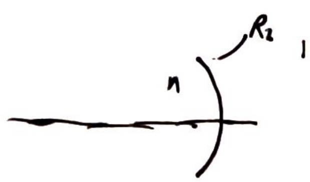
thin lenses: the two surfaces are very close



we have

$$\frac{1}{s}+\frac{n}{s'}=\frac{n-1}{R}$$





$$\frac{n}{s_2}+\frac{1}{s_2'}=\frac{1-n}{R_2}$$

$s_2=-s_1'$ because the lens is thin

the image of the first is the object of the second but on the opposite side

so

$$\frac{1}{s_1}-\frac{n-1}{R_1}=\frac{1-n}{R_2}-\frac{1}{s_2'}$$

location of the final image

$$\boxed{\frac{1}{s_1}+\frac{1}{s_2'}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}$$

sometimes written $\frac{1}{s_1}+\frac{1}{s_2'}=\frac{1}{f}$

when $\boxed{\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}$

lens-maker's formula

Lens: converging or diverging

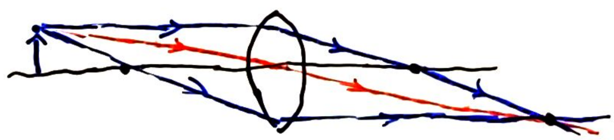
+f positive focal length

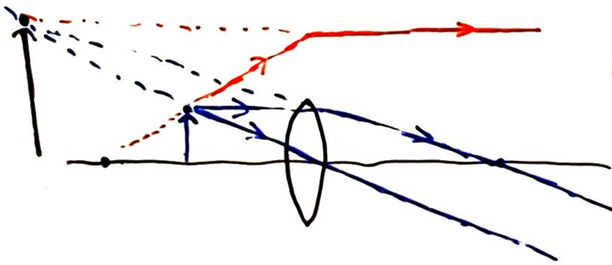
-f negative focal length

the focal length describes where parallel rays of light will be focused

ray tracing 

object outside the focus

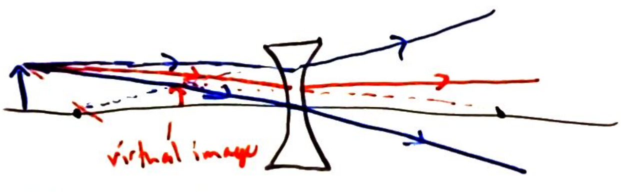




simple magnifier

diverging lens

object to the left

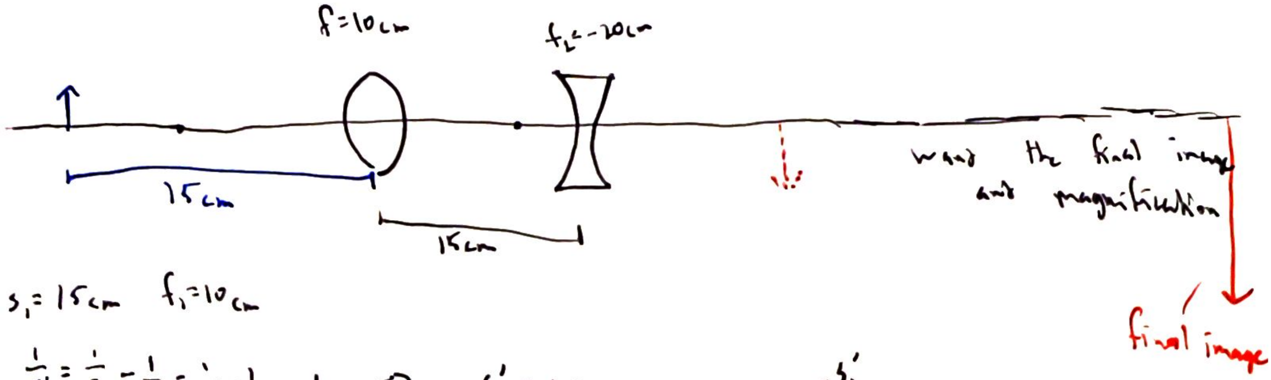


if the object is inside the focal length

↳ virtual, smaller, upright image

Example $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$

Week 2 pt. 2



$s_1 = 15\,cm \quad f_1 = 10\,cm$

$\frac{1}{s_1'} = \frac{1}{f_1} - \frac{1}{s_1} = \frac{1}{10} - \frac{1}{15} = \frac{1}{30} \Rightarrow s_1' = 30\,cm \qquad m = -\frac{s_1'}{s_1} = -2$

Lens 2: $s_2 = -15\,cm \quad f_2 = -20\,cm$

$\frac{1}{s_2'} = \frac{1}{f_2} - \frac{1}{s_2} = -\frac{1}{20} + \frac{1}{15} = -\frac{3}{60} + \frac{4}{60} = \frac{1}{60}$

$\Rightarrow s_2' = 60\,cm$

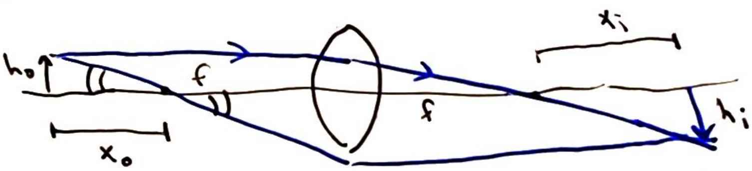
$m_2 = -\frac{s_2'}{s_2} = -\frac{60}{-15} = 4$

overall magnification: $m_1 m_2 = (-2)4 = -8$

Newton equation for thin lens:

$\frac{h_o}{x_o} = \frac{h_i}{f}$

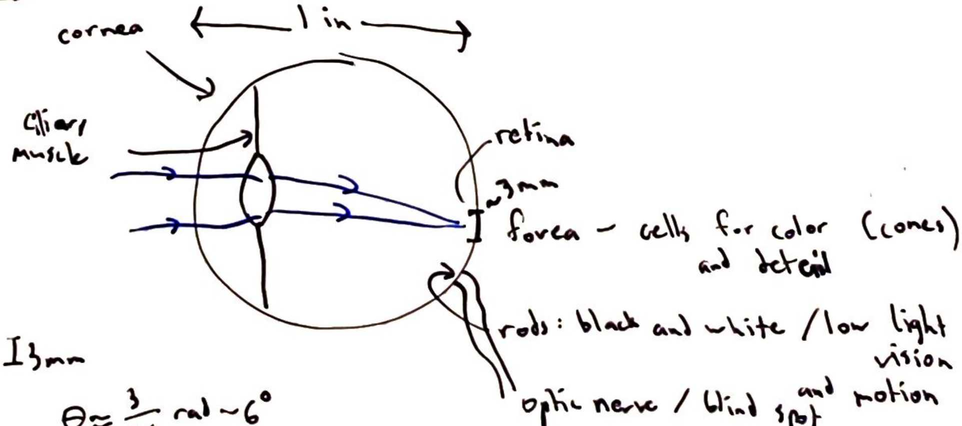
$\frac{h_o}{h_i} = \frac{x_o}{f}$

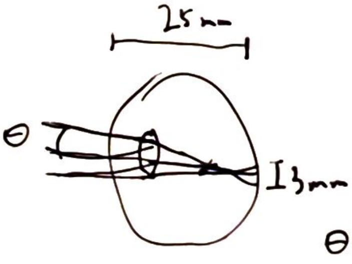


$\frac{h_i}{x_i} = \frac{h_o}{f} \qquad \frac{h_o}{h_i} = \frac{f}{x_i}$

so $\frac{x_o}{f} = \frac{f}{x_i} \Rightarrow x_o x_i = f^2$

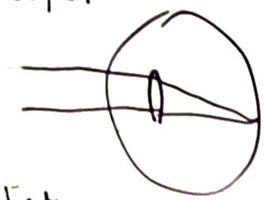
Chapter 19: The Eye





$\theta \approx \frac{3}{25}\,rad \sim 6°$

By changing lens thickness you can change the focal length far objects



close objects