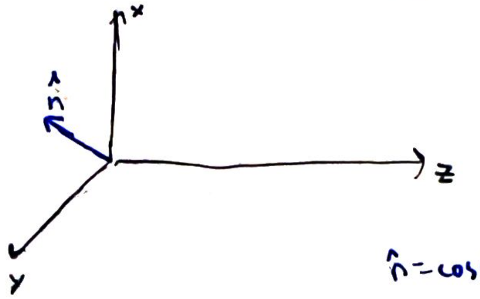
Week 4 pt 2



$\hat{n}$ - Polarization axis

for transverse waves here

$\hat{n}\cdot\hat{z}=0$

$\hat{n}=\cos\theta\,\hat{x}+\sin\theta\,\hat{y}$

In ch. 7 we derived (no ρ, no J, free space)

$$\nabla^2\vec{E}=\mu_0\varepsilon_0\frac{\partial^2}{\partial t^2}\vec{E}$$

$$\nabla^2\vec{B}=\mu_0\varepsilon_0\frac{\partial^2}{\partial t^2}\vec{B} \qquad c=\frac{1}{\sqrt{\mu_0\varepsilon_0}}=3\times10^8\frac{m}{s}$$

Monochromatic Plane waves

↳ single wavelength (single color)

↳ only dependance on one coordinate

will choose z

$$\nabla^2=\cancel{\frac{\partial^2}{\partial x^2}}^{0}+\cancel{\frac{\partial^2}{\partial y^2}}^{0}+\frac{\partial^2}{\partial z^2}$$

so we get
$$\frac{\partial^2}{\partial z^2}\vec{E}=\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\frac{\partial^2\vec{B}}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2}$$

⇒ solutions to above equations is $\vec{E}=\vec{E}_0\cos(kz-\omega t+\delta)$

(3) $\vec{E}=\mathrm{Re}\left[\tilde{\vec{E}}_0 e^{i(kz-\omega t)}\right]=\mathrm{Re}\left[\tilde{\vec{E}}\right]$

(4) $B=\mathrm{Re}\left[\tilde{\vec{B}}_0 e^{i(kz-\omega t)}\right]=\mathrm{Re}\left[\tilde{\vec{B}}\right]$

complex amplitudes

Goal: find additional information about 3 and 4

sub into maxwell's equations

$$\left.\begin{aligned}&\vec{\nabla}\cdot\vec{E}=0 \quad (\text{b/c } \rho=0)\\ &\vec{\nabla}\cdot\vec{B}=0 \quad (\text{always})\end{aligned}\right\}$$

$\vec{\nabla}\cdot\tilde{\vec{E}}=\frac{\partial}{\partial z}\tilde{E}_z=\tilde{E}_{0,z}(ik)e^{i(kz-\omega t)}=0$

Similarly $\vec{\nabla}\cdot\vec{B}(z,t)=\tilde{B}_{0,z}(ik)e^{i(kz-\omega t)}=0$

so $\tilde{E}_{0,z}=\tilde{B}_{0,z}=0$

so EM waves are not longitudinal, they are transverse

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \text{(Faraday's law)}$$

$$\vec{\nabla}\times\tilde{\vec{E}} = -\frac{\partial \tilde{\vec{B}}}{\partial t}$$

$$-\frac{\partial \vec{B}}{\partial t} = -i\omega \tilde{\vec{B}}_0 e^{i(kz-\omega t)}$$

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & \partial/\partial z \\ \tilde{E}_x & \tilde{E}_y & 0 \end{vmatrix} = \hat{x}\left(-\frac{\partial}{\partial z}\tilde{E}_y\right) - \hat{y}\left(-\frac{\partial}{\partial z}\tilde{E}_x\right)$$

$$= \hat{y}\frac{\partial}{\partial z}\tilde{E}_x - \hat{x}\frac{\partial}{\partial z}\tilde{E}_y$$

so $$\frac{\partial}{\partial z}\tilde{E}_x\hat{y} - \frac{\partial}{\partial z}\tilde{E}_y\hat{x} = -i\omega\tilde{\vec{B}}_0 e^{i(kz-\omega t)}$$

$$= -i\omega\tilde{\vec{B}}$$

so $-i\omega\tilde{B}_y = \frac{\partial}{\partial z}\tilde{E}_x$ and $-i\omega\tilde{B}_x = \frac{\partial}{\partial z}\tilde{E}_y$

$$\Rightarrow \quad -\omega\tilde{B}_{0,y} = k\tilde{E}_{0,x}$$

$$-\omega\tilde{B}_{0,x} = -k\tilde{E}_{0,y}$$

$\hookrightarrow$ $\tilde{\vec{B}}_0 = \frac{k}{\omega}\underbrace{(\hat{z}\times\tilde{\vec{E}}_0)}$ so $\boxed{|\vec{B}_0| = \frac{1}{c}|\vec{E}_0|}$

no phase shift between E and B waves

$\delta_E = \delta_B$ (in vacuum)