Final Project: Analysis of a Quarter-Car Suspension System with Varying Road Profiles and Vehicle Velocity

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System Dynamics (MECE-320)

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Introduction:

The purpose of this lab is to explore the myth of whether driving faster over rough terrain improves the occupant ride-quality, as initially explored in Episode 58 of *MythBusters*, titled *Shattering Sub-Woofer/Rough Road Driving*. To explore this myth, this lab utilizes MATLAB to model a quarter-car suspension system, then varies the road profile as well as the vehicle velocity. Plots of the y (road profile) vs. t, x_1 (displacement of the body) vs. t, and x_2 (displacement of the wheel) vs. t are generated in order to determine whether the myth is true or not. The road displacement, y(t), and the gravity forces, m_1g and m_2g, are all inputs. Additionally, the state variables, x_1, x_2, v_1, and v_2 as well as the compressive tire spring force, $F_{k_2} = k_2(y - x_2)$, are all outputs. The initial given values of the system include the 'sprung' weight $m_1g = 1000\ lb_f \approx 4448.22\ N$, 'un-sprung' weight $m_2g = 100\ lb_f \approx 444.82\ N$, 'tire-spring' constant $k_2 = 1000\ lb_f/in \approx 175127\ N/m$, suspension spring $k_1 = 125\ lb_f/in \approx 21890.88\ N/m$, and shock absorber coefficient $c_1 = 20.0\ lb_f * s/in \approx 3502.54\ N * s/m$. Additionally, the 'sprung' mass and 'un-sprung' mass were calculated to be $m_1 \approx 453.44\ kg$ and $m_2 \approx 45.34\ kg$, respectively. Next, initial values for the road profiles include the nominal amplitude $A = 2\sqrt{2}\ in \approx 0.0718\ m$, x-direction amplitude $P = 8\sqrt{2}\ in \approx 0.2874\ m$, mean-value offset $E = 2\sqrt{2}\ in \approx 0.0718\ m$, slow vehicle velocity $v_{slow} = 15\ mph \approx 6.71\ m/s$, and fast vehicle velocity $v_{fast} = 45\ mph \approx 20.12\ m/s$. As can be seen above, all given values were converted to metric units, so the remainder of the report will discuss results using metric units. Finally, based on the quarter-car suspension system, the governing equation of motion are as followed:

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 = c_1\dot{x}_2 + k_1x_2 - m_1g$$

$$m_2\ddot{x}_2 + c_1\dot{x}_2 + (k_1 + k_2)x_2 = c_1\dot{x}_1 + k_1x_1 - m_2g + k_2y(t)$$

The governing equations of motion were constructed in Simulink, which includes the outputs y(t), x_1, x_2, v_1, and v_2 as well as the compressive tire spring force, $F_{k_2} = k_2(y - x_2)$. A "Fixed-step" and "ode14x" solver, as well as a "Fixed-step size (fundamental sample time) of $0.001s$ were utilized. To vary the road profile, a "Step" block was used for the step input alone, while a combination of blocks including the "Clock," a "Constant" of zero, "Switch," and "Sine Wave Function" were used to vary the sinusoidal road profiles. All profiles "go live" at $t = 5s$, utilizing a step of $5s$, hence the addition of the other blocks for the sinusoidal road profiles. All given variables previously explained are defined in the MATLAB code. Once the Simulink model is run, the MATLAB code imports the outputs of the Simulink model, which are then used to plot all five cases. The code also trims the imported data in appropriate locations relative the time of the simulation in order to compute steady-state mass displacement, max-transient mass displacement, and sustained periodic mass displacement amplitude for all five cases. Finally, a combination of blocks including the "Switch" and a "Constant" of zero are used to set the compressive tire-spring-force equal to zero if tension in the 'tire-spring' is attempted at any time during the simulation.

Results and Discussion:

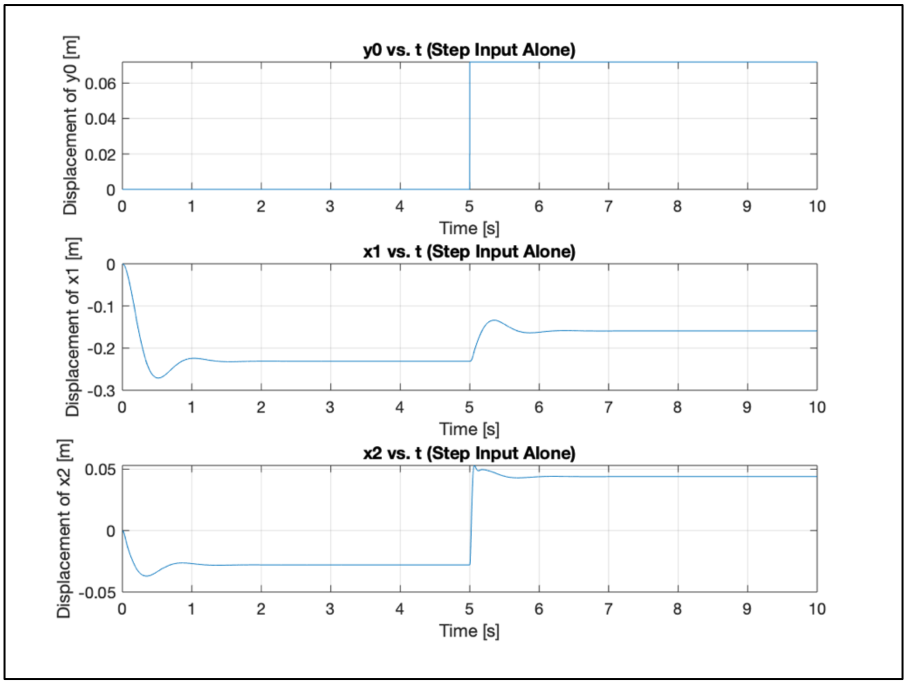

Figure 1: Plots of y_0, x_1, and x_2 Versus Time for the Step Input Alone

As shown in Figure 1, the first profile analyzed is just the step input along where $y_0 = E$. The top plot illustrates the change in height of the road profile, and at $t = 5s$, the displacement changes from $0\ m$ to $E \approx 0.0718\ m$ when the tire encounters the bump which then stabilizes at approximately $0.0718\ m$. The middle plot depicts a sharp decline in the displacement of the body due to the body's weight, which then stabilizes at a displacement of approximately $-0.2311\ m$ relative to its initial position. At $t = 5s$, the car again hits the bump of height E, reaching a max displacement of approximately $0.0972\ m$ before stabilizing at the new displacement of approximately $-0.1593\ m$ relative to its initial position. Lastly, the bottom plot again depicts a sharp decline, this time illustrating the displacement of the wheel due to both the weight of the body and the wheel, which then stabilizes at a displacement of approximately $-0.0279\ m$ relative to its initial position. At $t = 5s$, the car again hits the bump of height E, reaching a max displacement of approximately $0.0808m$ before stabilizing at the new displacement of approximately $0.0439\ m$ relative to its initial position.

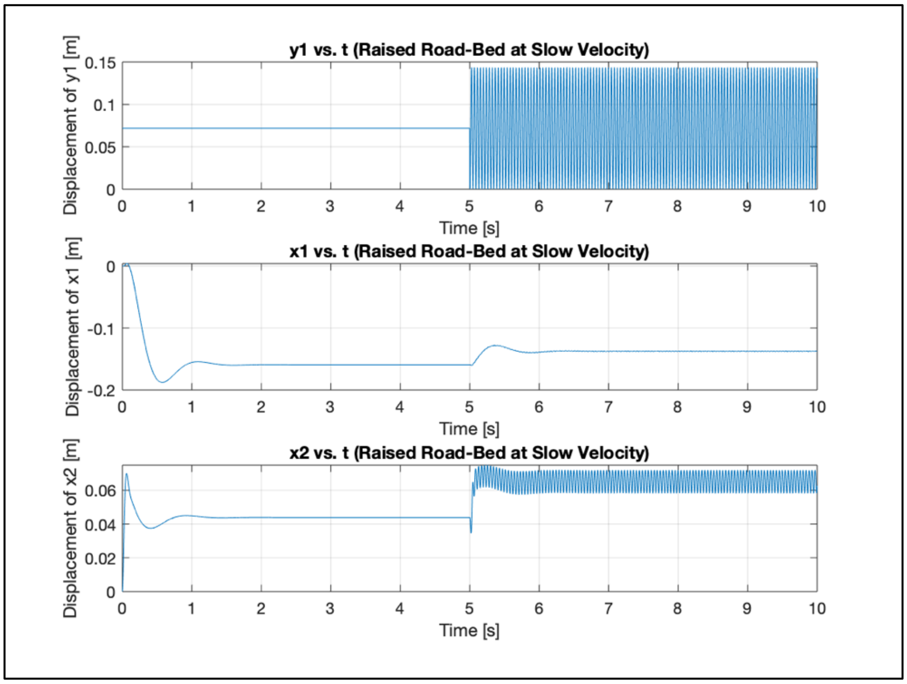

Figure 2: Plots of y1, x1, and x2 Versus Time for the Raised Road-Bed at Slow Velocity

As shown in Figure 2, the second profile analyzed is the raised road-bed at a slow velocity where $y_1 = Asin(((2\pi)/P) * v_{slow} * t) + E$, and as previously stated, $v_{slow} \approx 6.71\ m/s$. The top plot illustrates the change in height of the road profile, and at $t = 5s$, the displacement changes into a sinusoidal waveform, modeled by the equation previously stated, as the tire drives over the "washboard" surface, which then sustains its periodic mass displacement amplitude of approximately $0.0718\ m$. The middle plot depicts a sharp decline in the displacement of the body due to the body's weight, which then stabilizes at a displacement of approximately $-0.1593\ m$ relative to its initial position. At $t = 5s$, the car encounters the "washboard surface," reaching a max displacement of approximately $0.0324\ m$ before sustaining its periodic mass displacement amplitude of approximately $3.7219 * 10^{-4}\ m$. Lastly, the bottom plot first illustrates a sharp increase as a result of the addition of height E, then again depicts a sharp decline, which still illustrates the displacement of the wheel due to both the weight of the body and the wheel, which then stabilizes at a displacement of approximately $0.0439\ m$ relative to its initial position. At $t = 5s$, the car encounters the "washboard surface," reaching a max displacement of approximately $0.0403m$ before sustaining its periodic mass displacement amplitude of approximately $0.0069\ m$.

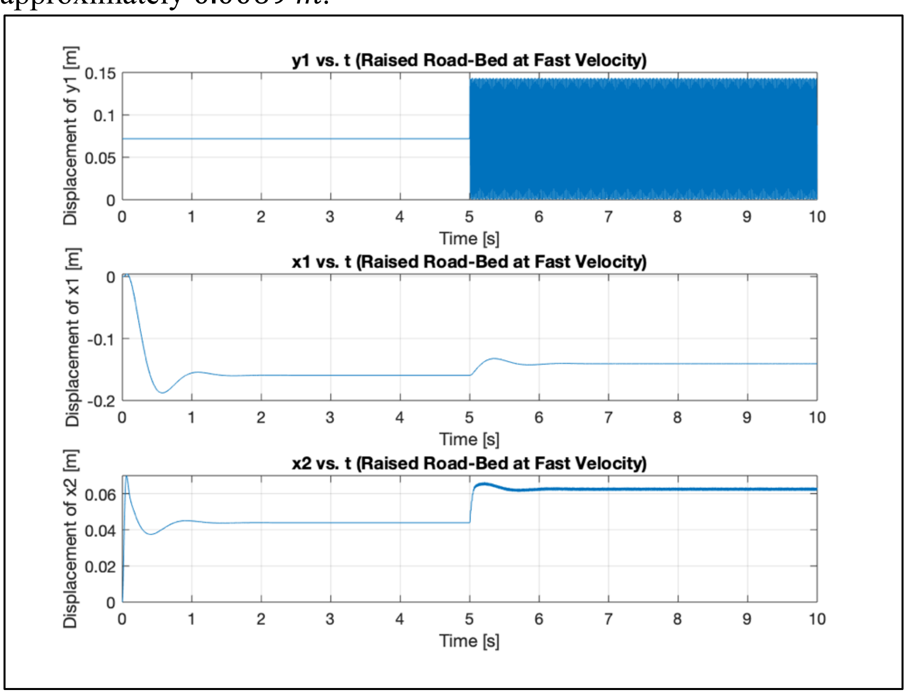

Figure 3: Plots of y1, x1, and x2 Versus Time for the Raised Road-Bed at Fast Velocity

As shown in Figure 3, the third profile analyzed is the raised road-bed at a fast velocity where $y_1 = Asin(((2\pi)/P) * v_{fast} * t) + E$, and as previously stated, $v_{fast} \approx 20.12\ m/s$. The top plot illustrates the change in height of the road profile, and at $t = 5s$, the displacement changes into a sinusoidal waveform, modeled by the equation previously stated, as the tire drives over the "washboard" surface, which then sustains its periodic mass displacement amplitude of approximately $0.0718\ m$. The middle plot depicts a sharp decline in the displacement of the body due to the body's weight, which then stabilizes at a displacement of approximately $-0.1593\ m$ relative to its initial position. At $t = 5s$, the car encounters the "washboard surface," reaching a max displacement of approximately $0.0268\ m$ before sustaining its periodic mass displacement amplitude of approximately $2.4731 * 10^{-5}\ m$. Lastly, the bottom plot first illustrates a sharp increase as a result of the addition of height E, then again depicts a sharp decline, which still illustrates the displacement of the wheel due to both the weight of the body and the wheel, which then stabilizes at a displacement of approximately $0.0439\ m$ relative to its initial position. At $t = 5s$, the car encounters the "washboard surface," reaching a max displacement of approximately $0.0223\ m$ before sustaining its periodic mass displacement amplitude of approximately $8.4354 * 10^{-4}\ m$. It should also be noted that the frequency is larger for all three plots as compared to Figure 2, which is expected since the larger frequency is caused by the increased velocity, which can be correlated to driving over the bumps more frequently. Additionally, the max-transient mass displacement and sustained periodic mass displacement amplitude appear smaller for the fast velocity case, indicating improved ride-quality.

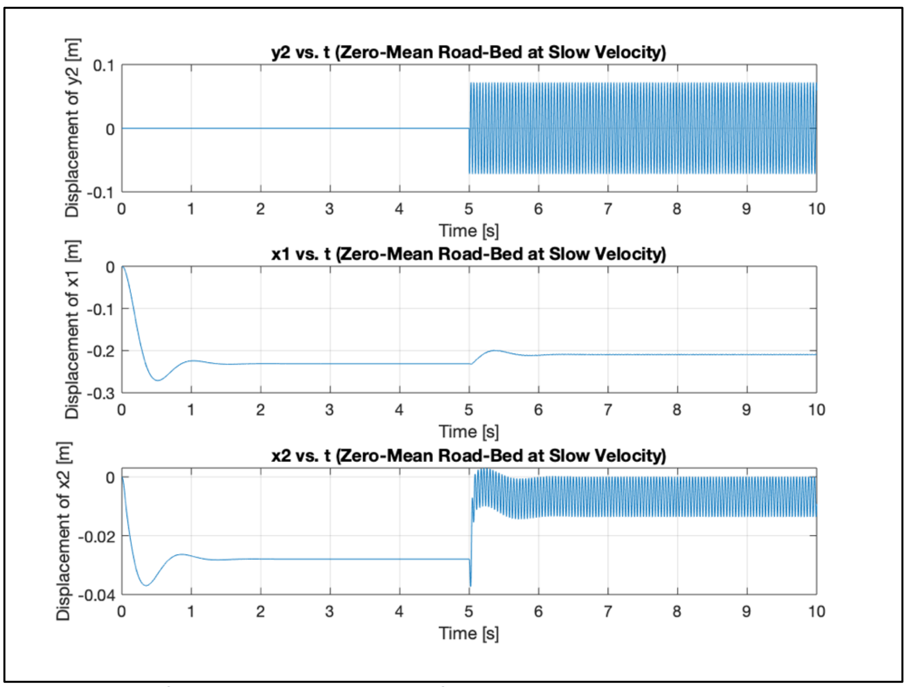

Figure 4: Plots of y2, x1, and x2 Versus Time for the Zero-Mean Road-Bed at Slow Velocity

As shown in Figure 4, the fourth profile analyzed is the zero-mean road-bed at a slow velocity where $y_2 = Asin(((2\pi)/P) * v_{slow} * t)$, and as previously stated, $v_{slow} \approx 6.71\ m/s$. The top plot illustrates the change in height of the road profile, and at $t = 5s$, the displacement changes into a sinusoidal waveform, modeled by the equation previously stated, as the tire drives over the "washboard" surface, which then sustains its periodic mass displacement amplitude of approximately $0.0718\ m$. The middle plot depicts a sharp decline in the displacement of the body due to the body's weight, which then stabilizes at a displacement of approximately $-0.2311\ m$ relative to its initial position. At $t = 5s$, the car encounters the 'washboard' surface, reaching a max displacement of approximately $0.0324\ m$ before sustaining its periodic mass displacement amplitude of approximately $3.7219 * 10^{-4}\ m$. Lastly, the bottom plot again depicts a sharp decline, which still illustrates the displacement of the wheel due to both the weight of the body and the wheel, which then stabilizes at a displacement of approximately $-0.0279\ m$ relative to its initial position. At $t = 5s$, the car encounters the "washboard surface," reaching a max displacement of approximately $0.0403\ m$ before sustaining its periodic mass displacement amplitude of approximately $0.0069\ m$.

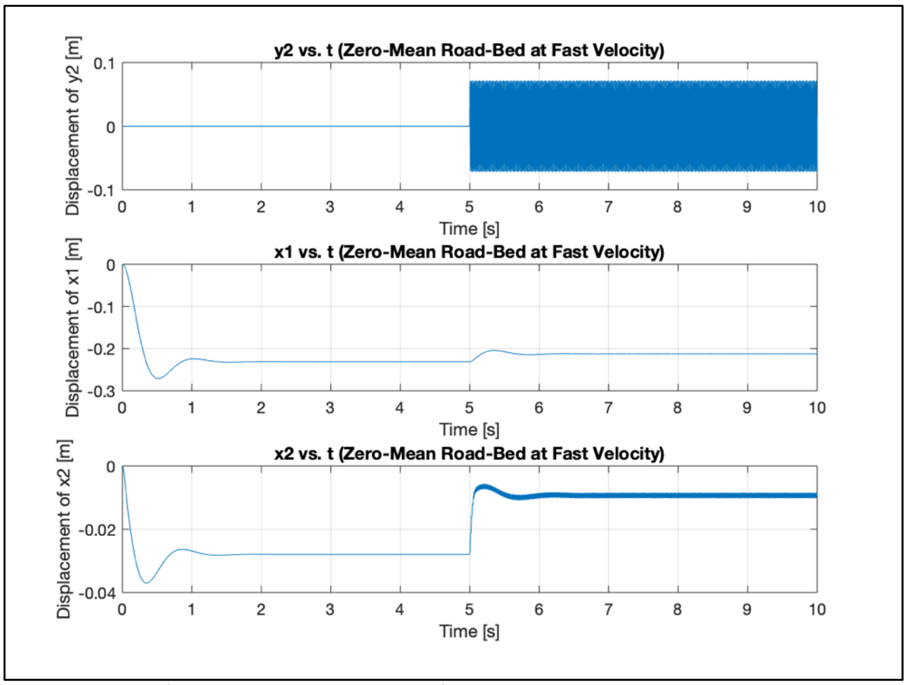

Figure 5: Plots of y2, x1, and x2 Versus Time for the Zero-Mean Road-Bed at Fast Velocity

As shown in Figure 5, the fifth profile analyzed is the zero-mean road-bed at a fast velocity where $y_2 = Asin(((2\pi)/P) * v_{fast} * t)$, and as previously stated, $v_{slow} \approx 20.12\ m/s$. The top plot illustrates the change in height of the road profile, and at $t = 5s$, the displacement changes into a sinusoidal waveform, modeled by the equation previously stated, as the tire drives over the 'washboard' surface, which then sustains its periodic mass displacement amplitude of approximately $0.0718\ m$. The middle plot depicts a sharp decline in the displacement of the body due to the body's weight, which then stabilizes at a displacement of approximately $-0.2311\ m$ relative to its initial position. At $t = 5s$, the car encounters the "washboard surface," reaching a max displacement of approximately $0.0268\ m$ before sustaining its periodic mass displacement amplitude of approximately $2.4731 * 10^{-5}\ m$. Lastly, the bottom plot again depicts a sharp decline, which still illustrates the displacement of the wheel due to both the weight of the body and the wheel, which then stabilizes at a displacement of approximately $-0.0279\ m$ relative to its initial position. At $t = 5s$, the car encounters the "washboard surface," reaching a max displacement of approximately $0.0223\ m$ before sustaining its periodic mass displacement amplitude of approximately $8.4354 * 10^{-4}\ m$. It should also be noted that the frequency is larger for all three plots as compared to Figure 4, which is expected since the larger frequency is caused by the increased velocity, which can again be correlated to driving over the bumps more frequently. Additionally, the max-transient mass displacement and sustained periodic mass displacement amplitude appear smaller for the fast velocity case, indicating improved ride-quality.

Table 1: List of Values for Position, Max-Transient Mass Displacement, and Sustained Periodic Mass Displacement Amplitude for x_1 and x_2 for All Five Cases

Case	Coefficient	Steady-State Mass Displacement [m]	Max-Transient Mass Displacement (Absolute) [m]	Sustained Mass Displacement [m]	Sustained Periodic Mass Displacement Amplitude (Absolute) [m]
Step Input Alone	x_1	-0.2311	0.0972	-0.1593	
	x_2	-0.0279	0.0808	0.0439	
Raised Road-Bed at Slow Velocity	x_1	-0.1593	0.0324		3.7219e-04
	x_2	0.0439	0.0403		0.0069
Raised Road-Bed at Fast Velocity	x_1	-0.1593	0.0268		2.4731e-05
	x_2	0.0439	0.0223		8.4354e-04
Zero-Mean Road-Bed at Slow Velocity	x_1	-0.2311	0.0324		3.7219e-04
	x_2	-0.0279	0.0403		0.0069
Zero-Mean Road-Bed at Fast Velocity	x_1	-0.2311	0.0268		2.4731e-05
	x_2	-0.0279	0.0223		8.4354e-04

As shown in Table 1, all steady-state mass displacements are as expected. Firstly, the weight of both the body and the wheel are only what initially “go live” and as a result, it can be observed that all mass displacements react downward. Additionally, it can be noted that the steady-state displacement of the wheel is larger for all cases as compared to the displacement of the body.

This is also expected since the 'tire-spring' is compressed by both the weight of the body and the wheel, while the suspension spring only is compressed by the weight of the body.

Furthermore, it can first be seen that when comparing the effect of slow versus fast vehicle velocity, every max-transient mass displacement and sustained periodic mass displacement amplitude was larger for the slow velocity case, indicating a poorer ride-quality. However, velocity had no impact on the steady-state mass displacement prior to encountering the 'washboard' surface. Additionally, it can be noted that there was no difference in results between the raised road-bed and zero-mean road bed, aside from the fact that all values were larger for the raised road-bed case due the addition of height E. Another interesting observation is that for the slow velocity cases, the max-transient mass displacement of x_1 is less than that of x_2, but for the fast velocity cases, the results are vice versa. This is a result that was not anticipated, but makes sense upon observation, since it's assumed that the fast velocity would cause a greater initial impulse, thus causing the displacement x_1 and x_2 to be impacted significantly due to both the 'tire-spring' and suspension spring needed to reduce the impact. However, when traveling at a slower velocity, only the displacement of x_2 would be impacted significantly since it's assumed that the 'tire-spring' could reduce the majority of the impact. This observation only holds true for the max-transient mass displacements since the results for sustained periodic mass displacement amplitude are consistent throughout. Finally, another interesting observation is the fact that max-transient mass displacement was greatest for the step input alone, even though the bump height for all five cases is the same. This is assumed to be due to the fact that the quarter-car suspension system is only affected by the single bump for the case of the step input alone. However, for the other cases, the quarter-car suspension system almost immediately experiences consecutive bumps after the initial, thus likely reducing the max-transient mass displacement caused by the first bump.

Conclusions:

Overall, this experiment successfully illustrated the effects of different road profiles and vehicle velocity on the mass displacements of a quarter-car suspension system. As previously explained, it can be seen that every max-transient mass displacement and sustained periodic mass displacement amplitude was larger for the slow velocity cases, indicating a poorer ride-quality. Specifically, traveling at a fast velocity reduced the max-transient mass displacement of x_1 and x_2 by 17.28% and 44.67%, respectively. Additionally, traveling at a fast velocity reduced the sustained periodic mass displacement amplitude of x_1 and x_2 by 93.36% and 87.77%, respectively. Thus, driving at a fast velocity, as compared to a slow velocity, reduced both the initial displacement of the quarter-car suspension system upon impact of the first bump at $t = 5s$, as well as the displacement of the quarter-car suspension system while driving over the consecutive bumps of the 'washboard' surface. To further support the evidence by visually observing the fast velocity cases in Figures 3 and 5, it can be seen that both their max-transient mass displacements and sustained periodic mass displacement amplitudes appear significantly smaller than those of their respective slow velocity cases in Figures 2 and 4. Therefore, it can be concluded from this experiment that it is better to drive fast across a 'washboard' surface if the intent is to improve occupant ride-quality.

References:

1. Hoosier_tire_info. (n.d.). Retrieved May 03, 2020, from https://mycourses.rit.edu/d2l/le/content/812737/viewContent/6412515/View

2. Koni_shock_article. (n.d.). Retrieved May 03, 2020, from https://mycourses.rit.edu/d2l/le/content/812737/viewContent/6412514/View

3. Mythbusters_web_article_washboard_road. (n.d.). Retrieved May 03, 2020, from https://mycourses.rit.edu/d2l/le/content/812737/viewContent/6412513/View

4. RIT_Speed_Bump. (n.d.). Retrieved May 03, 2020, from https://mycourses.rit.edu/d2l/le/content/812737/viewContent/6413242/View\

5. Roadbed_geometry_updated. (n.d.). Retrieved May 03, 2020, from https://mycourses.rit.edu/d2l/le/content/812737/viewContent/6420801/View

6. Suspension_system_static_equil. (n.d.). Retrieved May 03, 2020, from https://mycourses.rit.edu/d2l/le/content/812737/viewContent/6412521/View

7. Systems_Mythbusters_Final_Project_April2020d. (n.d.). Retrieved from https://mycourses.rit.edu/d2l/le/content/812737/viewContent/6422584/View

8. Wong_vehicle_suspension_chapter. (n.d.). Retrieved May 03, 2020, from https://mycourses.rit.edu/d2l/le/content/812737/viewContent/6412512/View

Appendix:

Simulink Diagrams

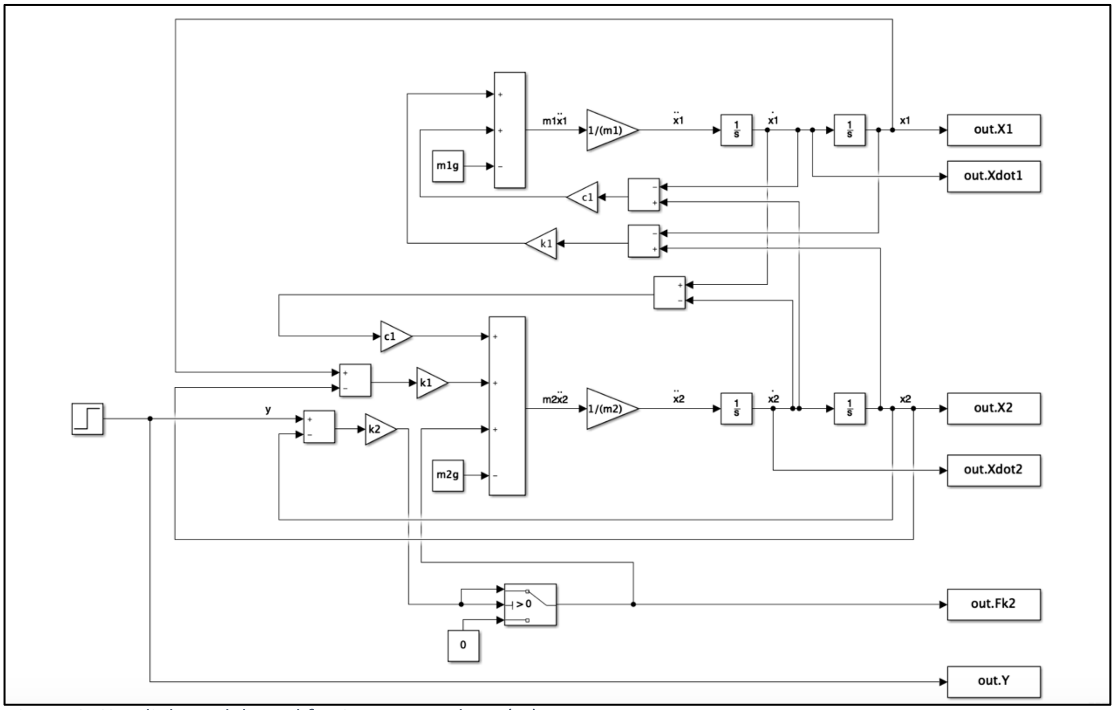

Figure 6: Simulink Model Used for Step Input Alone (y_0)

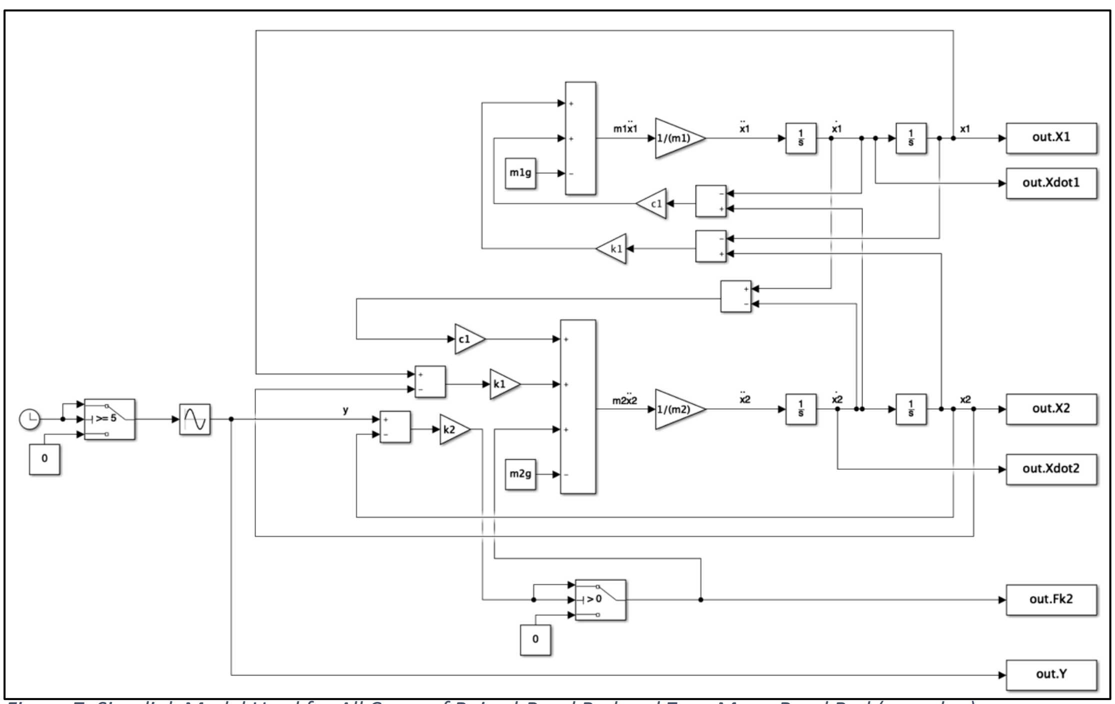

Figure 7: Simulink Model Used for All Cases of Raised-Road Bed and Zero-Mean Road Bed (y_1 and y_2)

Matlab Code

```
%%
% PROGRAM DESCRIPTION: Models a quarter-car suspension with varying
road
% profiles and vehicle travel velocity to determine whether it's
better to
% drive slow or fast across a 'washboard' road surface to improve
occupant
% ride-quality.
%          CREATED BY: Trevor Hess
%        DATE CREATED: April 29, 2020
%           FILE NAME: tdh_FinalProject.m
%
%%
clear, clc

%Define initial variables
m1g_given     = 1000;             %[lbf] sprung weight
m2g_given     = 0.1*m1g_given;    %[lbf] un-sprung weight
k2_given      = 1000;             %[lbf/in] tire-spring constant
k1_given      = 0.125*k2_given;   %[lbf/in] suspension spring
c1_given      = 20.0;             %[lbf*s/in] shock absorber
coefficient
A_given       = 2*sqrt(2);        %[in] nominal amplitude
P_given       = 8*sqrt(2);        %[in] x-direction period
E_given       = 2*sqrt(2);        %[in] mean-value offset
v_slow_given  = 15;               %[mph] slow travel velocity
v_fast_given  = 45;               %[mph] fast travel velocity

%Convert intial values to metric units and define additional variables
m1g    = (m1g_given)*4.44822;    %[N] sprung weight
m2g    = (m2g_given)*4.44822;    %[N] un-sprung weight
k2     = (k2_given)*175.127;     %[N/m] tire-spring constant
k1     = (k1_given)*175.127;     %[N/m] suspension spring
c1     = (c1_given)*175.127;     %[N*s/m] shock absorber coefficient
m1     = (m1g)/9.81;             %[kg] sprung mass
m2     = (m2g)/9.81;             %[kg] un-sprung mass
A      = (A_given)*0.025400;     %[m] nominal amplitude
P      = (P_given)*0.025400;     %[m] x-direction period
E      = (E_given)*0.025400;     %[m] mean-value offset
v_slow = (v_slow_given)*0.44704; %[m/s] slow travel velocity
v_fast = (v_fast_given)*0.44704; %[m/s] fast travel velocity

%Runs Simulink file
sim('tdh_FinalProject_simulink');

%Import Simulink data
t     = ans.tout;
x1    = ans.X1;
xdot1 = ans.Xdot1;
x2    = ans.X2;
```

```
xdot2 = ans.Xdot2;
Fk2   = ans.Fk2;
y     = ans.Y;

%Plots 5 figures in total:
%1. y0
%2. y1 at v_slow
%3. y1 at v_fast
%4. y2 at v_slow
%5. y2 at v_fast
figure(1)

subplot(3,1,1); %plots y vs. t
plot(t,y);
grid on;
title('y2 vs. t (Zero-Mean Road-Bed at Fast Velocity)'); %adjust title
based on road profile
ylabel('Displacement of y2 [m]'); %adjust yn based on road profile
where n = 0, 1, or 2
xlabel('Time [s]');

subplot(3,1,2); %plots x1 vs. t
plot(t,x1);
grid on;
title('x1 vs. t (Zero-Mean Road-Bed at Fast Velocity)'); %adjust title
based on road profile
ylabel('Displacement of x1 [m]');
xlabel('Time [s]');

subplot(3,1,3); %plots x2 vs. t
plot(t,x2);
grid on;
title('x2 vs. t (Zero-Mean Road-Bed at Fast Velocity)'); %adjust title
based on road profile
ylabel('Displacement of x2 [m]');
xlabel('Time [s]');

%Analyze steady-state mass displacement, max-transient mass
displacement, and sustained periodic
%mass displacement amplitudes for all plots
y_max_displacement        = abs(max(y(5000:5500,:)) -
min(y(5000:5500,:))) %for y0 case
y_ss_position_1           = mean(y(3000:5000,:)) %for all cases
%y_stabilized_position_2   = mean(y(7500:9000,:)) %for y0 case
y_stabilized_amplitude    = abs((max(y(7500:9000,:)) -
min(y(7500:9000,:)))/2) %for y1 and y2 cases

x1_max_displacement       = abs(max(x1(5000:5500,:)) -
min(x1(5000:5500,:))) %for all cases
x1_ss_position_1          = mean(x1(3000:5000,:)) %for all cases
%x1_stabilized_position_2 = mean(x1(7500:9000,:)) %for y0 case
```

```
x1_stabilized_amplitude  = abs((max(x1(7500:9000,:)) -
min(x1(7500:9000,:)))/2) %for y1 and y2 case

x2_max_displacement      = abs(max(x2(5000:5500,:)) -
min(x2(5000:5500,:))) %for all cases
x2_ss_position_1         = mean(x2(3000:5000,:)) %for all cases
%x2_stabilized_position_2 = mean(x2(7500:9000,:)) %for y0 case
x2_stabilized_amplitude  = abs((max(x2(7500:9000,:)) -
min(x2(7500:9000,:)))/2) %for y1 and y2 cases
```