

Some answers to student questions

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Noiseless coding

Question 1. Does there exist an algorithm to determine if a code is uniquely decipherable?

Answer. Yes! It is called the Sardinas-Patterson algorithm.¹

Question 2. What happens when equality is achieved in Kraft-McMillan inequality? In other words, given a sequence of potential lengths of code-words $(\ell_1, \ell_2, \dots, \ell_m)$, in an n -ary alphabet, what can we say when

$$\sum_{i=1}^m \frac{1}{n^{\ell_i}} = 1?$$

Answer. In the case equality is achieved, the code is exhaustive.² An exhaustive code $f : W \rightarrow \Sigma^*$ is one in which any sequence of letters is either a message $f^*(w_1 w_2 \cdots w_n)$ or the prefix of a message.³

References

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¹ Wikipedia. Sardinas-Patterson algorithm — Wikipedia, the free encyclopedia. <http://en.wikipedia.org/w/index.php?title=Sardinas%E2%80%93Patterson%20algorithm&oldid=1128726851>, 2023. [Online; accessed 24-January-2023]

² Mordecai J. Golin and Hyeon-Suk Na. Generalizing the Kraft-McMillan Inequality to Restricted Languages. In *Proceedings of the Data Compression Conference, DCC '05*, page 163–172, USA, 2005. IEEE Computer Society

³ L.S. Bobrow and S.L. Hakimi. Graph theoretic prefix codes and their synchronizing properties. *Information and control*, 15(1):70–94, 1969