Coupled Electric Oscillators

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Abstract

First we analyze a single LRC circuit. By curve fitting voltage responses for the circuit, the A and B-inductor decay natural frequencies is calculated as 14587.05± 0.03 rad/sec and 14578.44± 0.03 rad/s, and decay constants of 649.9± 0.1 s^{-1} and 651.3± 0.1 s^{1}. From these values the inductances is calculated to be 0.100±0.003 H for both of the inductors. When the A and B circuits were coupled via a coupling capacitor, each circuit produced a signal with a beat frequency of 170 ± 1 Hz. By using equations modeling a coupled LRC circuit, the coupling capacitance is calculated to be 4.1 ± 0.1 nF, which is one standard deviation away from its measured value of 3.9 ± 0.1 nF.

Introduction

Three essential components used in circuits are resistors, capacitors, and inductors. These components operate in distinct ways, which can be seen by the way we use inductance, resistance, and capacitance to calculate voltage.

Equations for Voltage

$$V = L \cdot \frac{d^2q}{dt^2}; \quad V = R \cdot \frac{dq}{dt}; \quad V = \frac{1}{C} \cdot q$$

Where V is voltage, L is inductance, R is resistance, C is capacitance, and q is charge. In other words, capacitors do work proportional to amount of charge buildup, resistors do work proportional to the flow of charge or current, and inductors do work proporional to the inflection of charge, or the change in current.

An LRC circuit combines these three components in series, and produces damped oscillation in response to changing voltage, via the above equations and Kirchoff's loop rule. We can understand why by relating these oscillations analagous to a spring-mass system.

Introduction, cont.

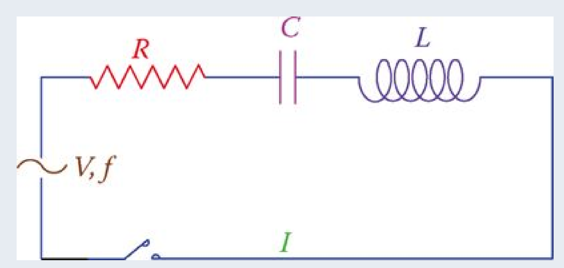

Figure 1: An LRC Circuit

Differential Equations

$$L \cdot \frac{d^2q}{dt^2} + R \cdot \frac{dq}{dt} + \frac{1}{C} \cdot q = 0$$
$$m \cdot \frac{d^2x}{dt^2} + \beta \cdot \frac{dx}{dt} + kx = 0$$

These circuits grow exponentially in complexity when two are copuled together. In the second phase of this lab we investigate the behavior of two coupled LRC circuits, operating with a 90 degree phase difference.

Apparatus

The apparatus consisted of the following.

- Coupled electronic oscillator circuit box
- Fixed capacitor, variable capacitor
- Two large inductor coils, labelled 'A' and 'B'
- Oscilloscope, Tektronix TDS210
- Digital Multimeter, Extech EX430A

Measure Inductance

In this phase of the lab we measure the basic parameters of our system, described in fig. 2, including L_A, L_B, R_{LA}, R_{LB}, full decay natural frequency ω_0, damping constant γ, time constant τ, quality factor Q and the standard capacitor. These values, save for the capacitance, are calculated by analyzing the voltage oscillations of the circuit. Values for R_{LA} and R_{LB} are also measured with an ohmmeter, and the capacitance is measured with a digital multimeter (DMM).

Measure Inductance, cont.

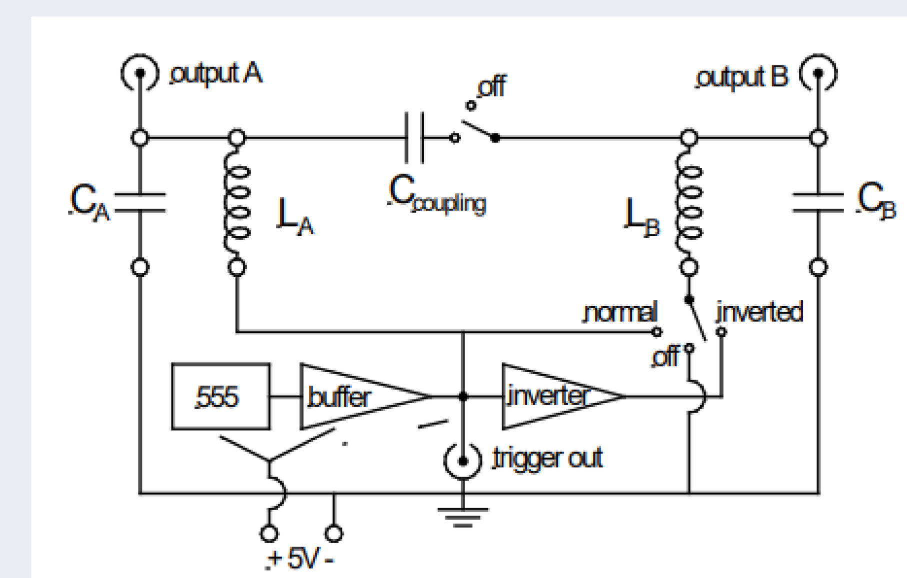

Figure 2: Test setup for the coupled electric oscillators.

The fit parameters will correspond to this solution of an LRC circuit.

$$q(t) = q_0 e^{-\gamma t/2} cos(\omega_0 t) \tag{1}$$

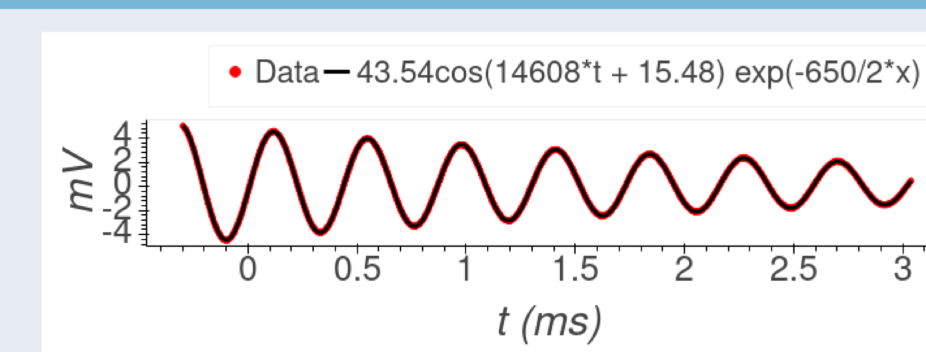

Figure 3: LRC Circuit response and fit, using B-inductor. The sign of the frequency is not relevant in this case.

The fit matched the data extremely well. The measuring precision of the DMM is responsible for virtually all uncertainty in calculated values in this lab.

Coupled Electric Oscillators

In the following section, a system with two coupled LRC circuits is analyzed. In this configuration, the square wave only excites the A-circuit, meaning that at the end of a pulse the A-capacitor is fully charged and the B-capacitor is empty. The decay is removed digitally by fitting the sum of both responses, and dividing each by the exponential, with the result shown in fig. 4

Coupled Oscillators, cont.

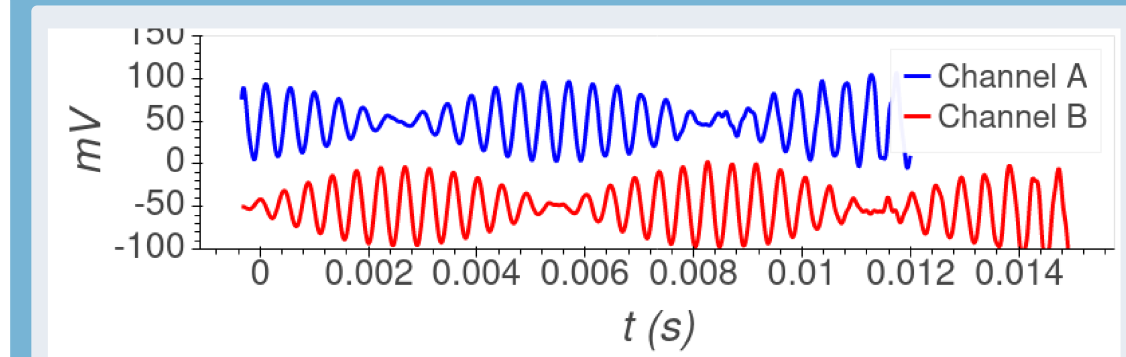

Figure 4: Two coupled LRC circuits, operating at a 90 degree phase difference, with the effect of damping removed.

The beat frequency of both waves are found using two methods of analysis, and the resulting average value for each wave is considered. This value was the same for both waves, at 170 ± 1 Hz.

This value, the difference between ω_+ and ω_-, can be used to calculate the coupling capacitance via the following equations, derived from differential analysis of the circuit.

$$\omega_+ = \frac{1}{2\pi\sqrt{LC}} \tag{2}$$

$$\omega_- = \frac{1}{2\pi\sqrt{L(C + C_{coup})}} \tag{3}$$

$$f_{beat} = \frac{1}{2\pi\sqrt{LC}} - \frac{1}{2\pi\sqrt{L(C + 2C_{coup})}} \tag{4}$$

$$C_{coup} = \frac{1}{2}\left[\frac{1}{L(\frac{1}{\sqrt{LC}} - 2\pi f_{beat})^2} - C\right] \tag{5}$$

Conclusion

The final measured and calculated values for C_{coup} are 3.9 ± 0.1 nF and 4.1 ± 0.1 nF respectively, which shows agreement within 1 sigma. This shows excellent agreement with measured values, and verifies the mathematical analysis and digital signal processing used to produce the result.