AFT Lab

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Nothing is here

INTRODUCTION

Fourier analysis is the practice of viewing a time-domain signal in the frequency domain. Named after the mathemetician Joseph Fourier for his breakthrough discovery that most functions, even discontinuous functions like the step function, can be represented as an infinite sum of sine waves, the advancement was originally made in pursuit of more easily solving complex differential equations describing heat transfer. Some scientists and engineers, however, do Fourier wrong by simply referring to the technique by the anagram of its modern discrete implementation, by saying, for example, "plot the FFT".

The FFT, short for Fast-Fourier Transform, is as stated a method for calculating the discrete Fourier transform of a finite time-domain signal. One can forgive those who refer to Fourier analysis generally as "FFT", considering how this ingenious algorithm has paved the way for the discrete Fourier transform's widespread usage in signal processing across a wide range of fields and disciplines.

Utilizing a divide-and-conquer technique, the FFT could be compared to a sailboat that, rather than following the wind directly to its destination, turns sharply to the side and tacks back and forth across the wind, taking a significantly longer and more complicated path that arrives in a fraction of the time. To understand just how well this analogy works, consider that the first step the FFT performs on a signal of length 2049 is to add another 2047 zeros onto the end so that the signal length is an exact power of 2.

In this lab we will be conducting Fourier analysis, with the help of the FFT algorithm, of various acoustic signals. This analysis is particularly interesting in the context of speech and musical instruments, as noised that to our ears may sound like "one" note or vowel sound are in fact more often than not rich with multiple harmonics throughout the frequency spectrum.

We will first conduct a simplified test implemented entirely in software, examining the effect of longer observations on the linewidth, or uncertainty in the frequency domain, numerically. We will then conduct a simple test using an function generator with an oscilloscope to collect data for a sine wave and square wave, in order to compare their frequency responses.

In the next phase of the lab, acoustic signals will be examined. First, we will explore the interaction of two tuning forks as they constructively and destructively interfere due to slight differences in their frequency, producing what is known as a "beat". We will then explore characteristics of musical notes produced by a whistling sound, the human voice producing vowel sounds, and musical instruments.

Finally, we will conduct a numerical exploration of the breakdown of accurate signal reconstruction in the time-domain as a function of the nyquist frequency and the fundamental frequency of the signal.

APPARATUS

The apparatus consisted of the following.

BASIC FFT

Procedure and Results

In this section, a time-vector from 0 to 0.1 using 2048 equally spaced points was produced, and used to build a sine wave of equation $y = sin(2\pi 100x)$, shown in 4. The signal was then transformed into the frequency domain, using the FFT algorithm. With this signal, as with all signals for the duration of the lab, the signal was multiplied by a hamming window before performing the discrete Fourier transform to mitigate spectral leakage.

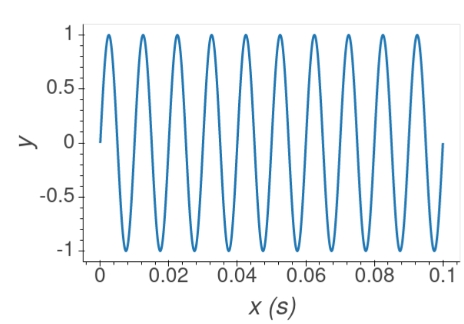

FIG. 1. Sine wave of frequency 100 Hz, observed for 0.1 seconds.

The Fourier analysis produced the expected frequency of 100 Hz, verifying that the technique was applied correctly, as seen in 2.

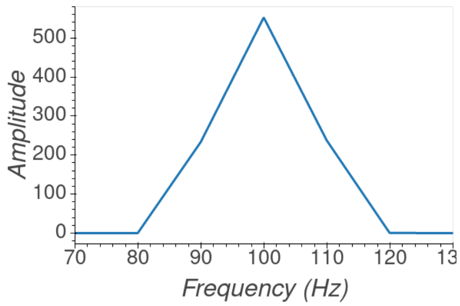

FIG. 2. Sine wave of frequency 100 Hz, observed for 0.1 seconds, in the frequency domain.

A second sine signal, calculated with the same sine function and N samples but longer observation window of 1 second, was produced, as seen in **??**.

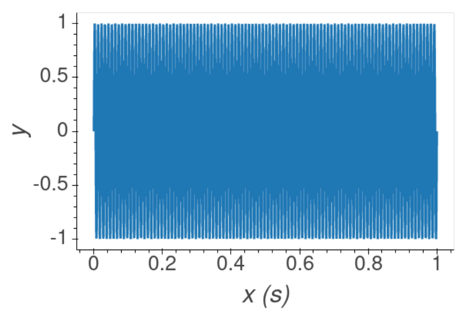

FIG. 3. Sine wave of frequency 100 Hz, observed for 1.0 seconds.

This longer sine wave was analyzed in the frequency domain as well, producing a 100 Hz peak, as seen in **??** with the same x-axis range as 2.

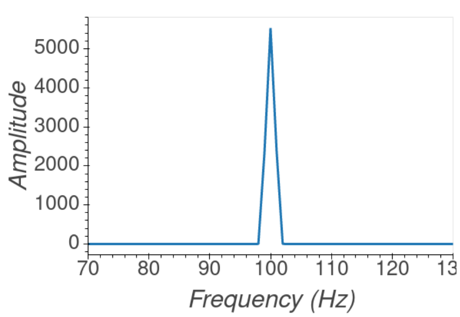

FIG. 4. Sine wave of frequency 100 Hz, observed for 1.0 seconds, in the frequency domain.

The characteristics of these two alike signals can be easily compared in 5, however quantitative characteristics are also given in I.

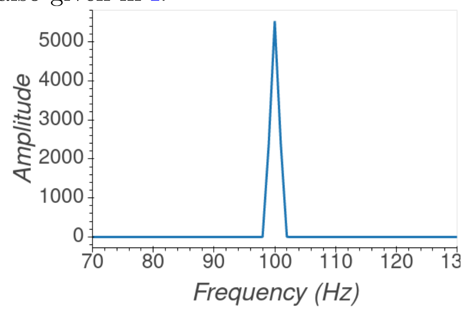

FIG. 5. Comparison of the short (0.1 second) and long (1.0 second) sine waves, given by the same equation, using Fourier analysis.

Conclusion

Uncertainty in the frequency of a signal, which manifests as the linewidth of a peak in the frequency domain, is reduced as a function of observation time. This is the result of a flavor of the Heisenberg Uncertainty Principle, where longer observations (greater N cycles) of a signal trade better certainty of the location of the signal in the frequency domain for worse certainty of the location of the signal in the time domain.

This simple experiment confirms that linewidth Γ is reduced with longer observations or more cycles. A higher precision FFT was produced in this experiment by collecting 100 cycles than by collecting 10 cycles. It also shows that if a longer observation time is used while the number N of samples is kept constant, sample rate decreases, which is seen in the frequency range as well. It is important to note that although the resolution of the frequency spectrum was reduced by a factor of ten (larger frequency bins), the uncertainty or linewidth was still reduced.

TABLE I. Frequency response and parameters given by a short and long observation of the same sine equation.

Time (s)	Cycles M	Samples N	FFT Range Δf	Resolving Power $\frac{f_0}{\delta f}$ (Hz)	Linewidth Γ (Hz)
0.1	10	2048	10225	10.00	17.4
1.0	100	2048	1022.5	1.000	1.74

Results

$$\eta_{PV} = P_E/P_L \tag{1}$$

TABLE II. Measured and accepted values of the speed of light and refractive index of various materials.

Apparatus	η (%)	Accepted η value	Refs.	Deviation
Photovoltaic Cell	15 ± 2	17 ± 2.5	[2]	0σ
Elecrolyzer	87 ± 6	80	[3]	2σ
Hydrogen Fuel Cell	49 ± 5	60	[4]	-3σ

Conclusions

SUMMARY

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[1] Wikipedia, Heat of Combustion: https://www.wikepedia.com
[2] Energysage, Most Efficient Solar Panels https://www.energysage.com/
[3] Carbon Commentary, Hydrogen made by Electolysis https://www.carboncommentary.com
[4] Energy.gov, Fuel Cell Fact Sheet https://www.energy.gov