Speed of Light Lab

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The refractive indices of several materials were evaluated. The refractive index of air was calculated to be 0.99 ± 0.01, slightly lower than the accepted value of 1.00. The refractive index of glass was calculated to be 1.49±0.03 in a glass Fiber-Optic cable. The accepted value for the refractive index of glass is 1.52, slightly higher than the calculated value. The refractive index of water was calculated to be 1.34± 0.01, slightly higher than the accepted value of 1.33. An "effective" refractive index of a coaxial cable was also computed to be 1.56±0.02.

INTRODUCTION

The speed of light is foundational to our understanding of the universe, as it is in fact not just the speed photons travel at, but rather a universal speed limit. However, even though photons do always travel at the speed of light, we can measure the net wavefront in a material at lower speeds, due to the absorption and re-emission of new photons by atoms in a medium. This "speed of light in a medium" is usually thought of not as a speed, but as a ratio $v/c = n$, the refractive index. The refractive index is so called because light follows Snells law, where its direction will be refracted proportional to this refractive index depending on the refractive indeces of the two mediums.

It is, however, also useful to think of "speed of light in a medium" as a speed, because the speed of communications in our incredibly connected world is crucial. Even electrical signals, governed by the electromagnetic force which is mediated by photons, propogate through electrical circuits and cables not at the drift speed of electrons but rather at the speed an electric field propogates through them. A major advancement in communications, however, came from using photons themselves rather than electrical signals to communicate over long distances, using Fiber-Optic (FO) cables. These cables take advantage of differences in the refractive index of glass and another material to create total internal reflection of all photons input at the end of a long and thin glass tube. These cables are capable of transmitting great amounts of information in a very thin tube, as, while conductors permit more information to flow through them limited by their circumference, FO cables use their entire cross-sectional area to transmit information.

In this lab, we will measure the speed of light (and the effective speed of light) through different mediums, in order to calculate refractive index. The core technique will involve using a laser and a beam splitter to pass two beams through paths of very different lengths, collecting the signals on two photodiodes (PD), and measuring the relative delay of the longer path compared to the shorter one on an 0.5 GHz oscilloscope. This laser will be triggered on and off with a nonosecond pulse generator. All factors: the pulse generator, the beam path lengths, the sample rate, and the comparison between the two PD input signals for their time delay, will introduce sources of uncertainty that must be minimized by careful measurements and signal processing.

The mediums considered will be air, glass (FO cable), and water. The effective index of the coax cables connecting the PD and oscilloscope will also be measured.

APPARATUS

The apparatus consisted of the following.

- Oscilloscope, Tektronix TDS210
- OpenChoice Desktop, Oscilloscope Software
- Optical breadboard
- Beam splitter, 2 mirrors, focusing lens
- Focused laser diode
- 2 Amplified high-speed silicon photodiodes
- Water-Filled optical cell
- Fast edge nanosecond pulse generator
- Fiber Optic Cable with mount
- BNC cables and male-male BNC connector
- Spectrometer and Quantum software, Amadeus
- Jupyter, Python compiler

PROCEDURES AND RESULTS

Test Pulse, Scope and Laser Diode

The fast edge nanosecond pulse generator uses six74AC14 Schmitt Triggers as shown in fig. 1, hand mounted to a copper circuit board.

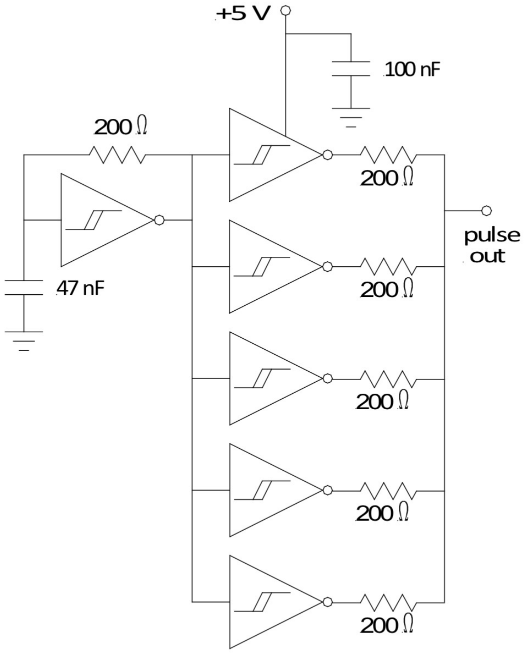

FIG. 1. Circuit diagram of the nanosecond pulse generator

The pulse generator was connected directly to the oscilloscope, and adjusted until several cycles were visible on the display. This output was analyzed to verify the setup. These square pulses are shown in fig. 2. We can observe an approximate amplitude of 5.60 mV, and using fourier analysis we can verify that the fundamental frequency is 90 kHz.

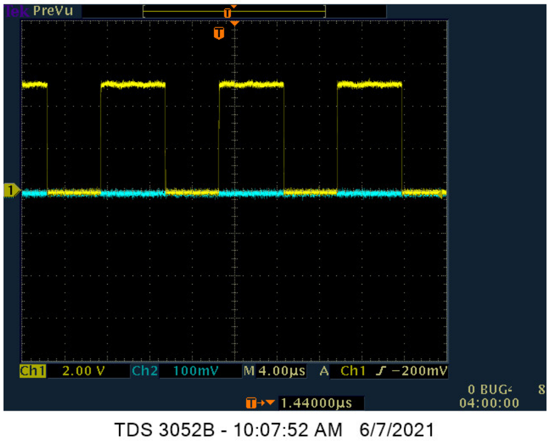

FIG. 2. Measured square waves on the oscilloscope.

The pulse generator will be used to power the laser, creating a wave front that propogates simultaneously through two paths of different lenghts.

Optical Setup

Next, the two paths were configured, based on 4. We will consider only the distances of the paths after the beam splitter, as the paths up until that point are equivalent. The path measured by PD1 is given by length 1 alone, and the path measured by PD2 is given by lengths 2-5. These lengths are given in I, with all measurements performed with a tape measure. After completing all measurements in the span of two days, a final redemonstration of the lab was performed in a single day, and it is these results that are shown below, with the geometry of the setup unchanged for the entirety of the lab.

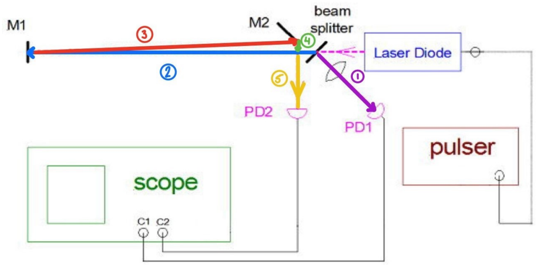

FIG. 3. Annotated test setup.

TABLE I. Lengths of various paths shown the optical setup

length #	Color	Length (mm)
1	Purple	168.5
2	Blue	917.5
3	Red	730.0
4	Green	5.0
5	Mustard	124.5

Before measuring distances PD1 and PD2 were connected to the oscilloscope, and the signal was isolated on the oscilloscope, triggered by PD1, as shown in 4, where a visible time shift can be observed. By referencing the oscilloscope output, the setup was optimized for the strongest signal by adjusting the beam splitter and M2.

Measurements were then recorded for analysis, as shown in fig. 5. It should be noted that for a perfect experimental setup, these signals would appear as simple step functions or square waves. However, producing such a sharp signal is very difficult, and as such more detailed analyis must be done to compare the two signals than simply looking at the step time. The "10-90" rise times

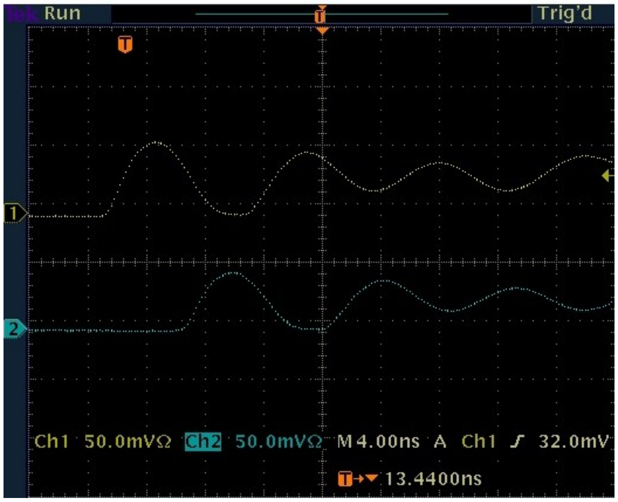

FIG. 4. Oscilloscope output for test setup with equal length BNC cables. Signals PD1 and PD2 are labeled on the left 1 and 2 respectively.

were used to compare flight times of the two signals, in addition to the min and max of the signals. These values are shown in table II.

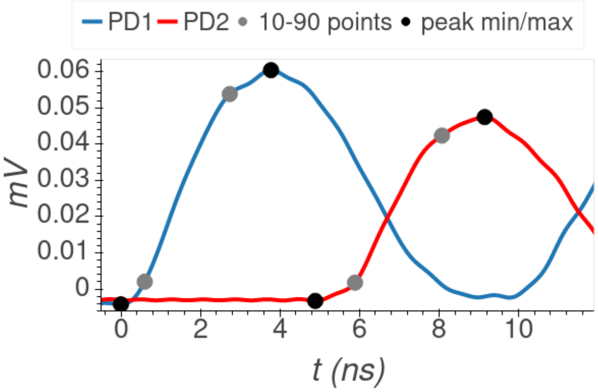

FIG. 5. Oscilloscope data comparing reference points for PD1 and PD2.

From table II, the time delay between PD1 and PD2 was calculated, with the mean and the standard deviation being taken as the recorded measurement and uncertainty, at 5.2 ± 0.2 ns. The high uncertainty in this measurement can be attributed to the difference in the peak shapes, where the full-width half-max (FWHM) for PD1 was calculated to be 4.657 ns, and the FWHM for PD2 was calculated to be 4.534 ns. This error is compensated for in future measurements, but in this case is less relevant.

Next, the BNC cable connecting PD2 to the oscilloscope, which originally was of equal length to the cable connecting PD1, was replaced with a significantly longer cable in order to explore such a scenario, and calculate the effective refractive index of the BNC cables. The scope output is shown in . 7, and the graph with reference points included is shown in fig. 6. The values for the 10-90 rise time, min, and max are shown in table III.

TABLE II. Time values of the first nanosecond peaks for equal length BNC cables

PD #	Start (ns)	End (ns)	10% (ns)	90% (ns)
1	0.0	2.47	0.67	2.02
2	5.4	7.79	5.97	7.27

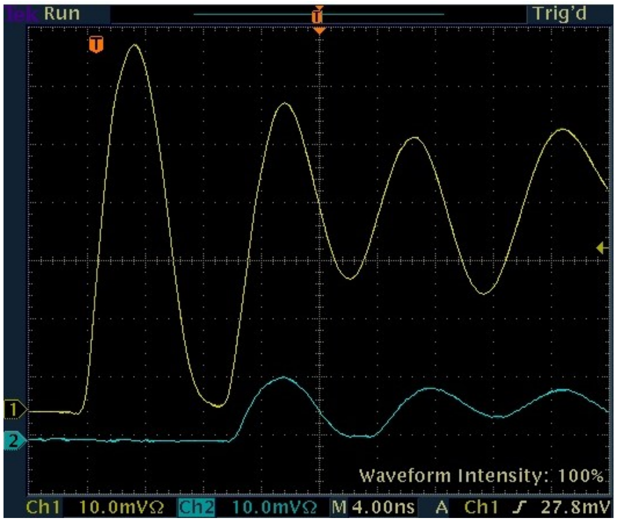

FIG. 6. Oscilloscope output for test setup with different length BNC cables. Signals PD1 and PD2 are labeled on the left 1 and 2 respectively.

TABLE III. Time values of the first nanosecond peaks for different length BNC cables

PD #	Start (ns)	End (ns)	10% (ns)	90% (ns)
1	0.00	4.04	0.89	3.01
2	9.82	14.39	11.01	13.31

From table III, the time delay between PD1 and PD2 with added cable length was calculated, with the mean and the standard deviation being taken again as the recorded measurement and uncertainty, at 10.1 ± 0.2 ns. This value can be directly compared to the previous time delay value, as the test setup did not change at all save the cable being replaced. By subtracting the time

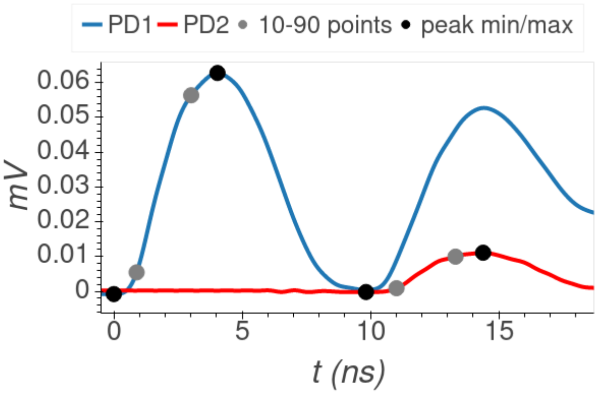

FIG. 7. Oscilloscope data comparing reference points for PD1 and PD2, where PD2 has been delayed using a longer BNC cable.

differences for each reference point without added cable length from the same differences for each reference point with added cable length, the effect of the cable alone could be isolated. Because of the nature of subtracting differences of differences, the uncertainty of this resulting measurement was significantly lower than the previous given values. This value was 4.93 ± 0.06 ns.

A plot showing both sets of captures, with extra cable and without extra cable, is shown in fig. 8. The extra delay due to the extra cable length can be seen clearly.

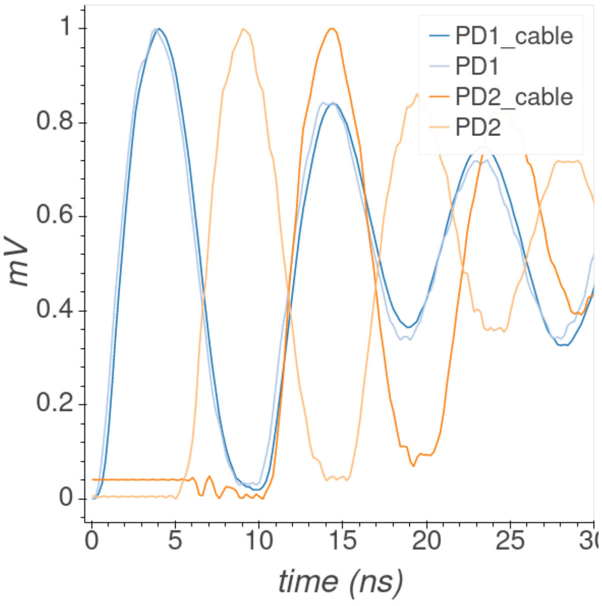

FIG. 8. Normalized comparison of the signals from PD1 and PD2 for equal length cables as well as the inequal length cables, denoted by the suffix "_cable".

$$v = \Delta x / \Delta T \tag{1}$$

Eq. 1 was used to calculate the speed of the electric signal in the BNC cable, where Δx is the added distance of cable in meters and Δt is the added time delay in seconds. This speed was found to be $1.92 \pm 0.02 \times 10^8$ m/s. The "effective" refractive index was calculated using eq. 2, where n is the refractive index, c is the speed of light in vacuum, and v is the speed of light in a given medium. The effective refractive index of the BNC cable was calculated to be 1.56 ± 0.02.

It is clear that if one cable was longer than the other then the time delay would be substantially altered. In the actual test setup, every part of the setup that both beams experience is not considered, as it does not factor into the cause of the time delay between the beams. This includes the path of the beam before it splits, but also the paths of the electric signals from the PD sensors to the oscilloscope. If the paths were not the same then this error would completely ruin our ability to record accurate data.

$$n = c/v \tag{2}$$

Speed of Light in Air

The speed of light in air was found using the same setup as in the previous section, after removing the extra length BNC cable. The raw output of the oscilloscope for this step is shown in fig. 9.

Because of the importance of precisely measuring peak times, some high frequency noise was filtered out with a simple square-cutoff low-pass filter, and cubic spline interpolation was used to increase measuring precision of the wave features. This spline interpolation is justified by the fact that the oscilloscope is averaging 512 waves in this single observation, improving measurement precision when projecting curves between points. The processed signal, with all peaks besides the first ones removed, is shown in fig. 10.

The values for the time shifts with respect to each signal reference point is shown in Table IV. The average and standard deviation of the time shift was found to be 5.32 ± 0.07 ns. By referencing the setup geometry and using eq. 1, the speed of light in air was calculated to be $3.01 \pm 0.04 \times 10^8$ m/s, and by using eq. 2 the corresponding refractive index was found to be 0.99 ± 0.01. The accepted value of 1.0003 is therefore within the uncertainty. This uncertainty mostly comes from

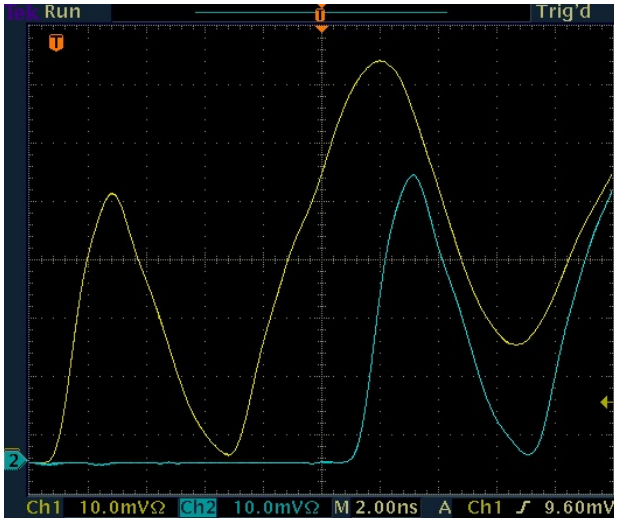

FIG. 9. Oscilloscope readout of the two signals PD1 (yellow) and PD2 (blue) for the two paths in air.

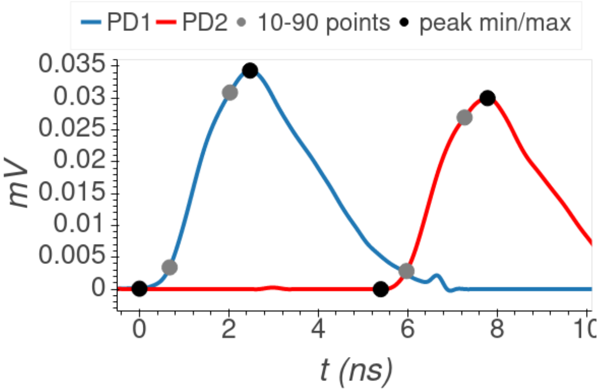

FIG. 10. Processed signals PD1 and PD2 in air, with calculated reference points included.

measuring distances with a measuring tape, lining up the markings on the tape with laser pulses that are below it, however some error also comes from the length of the nanosecond pulse and the difference in signal shapes between PD1 and PD2.

TABLE IV. Time values of the first nanosecond peaks for equal length BNC cables

PD #	Start (ns)	End (ns)	10% (ns)	90% (ns)
1	0.0	2.47	0.67	2.02
2	5.4	7.79	5.97	7.27

Speed of Light in Glass

The speed of light in glass was found using the same setup as in the previous section, but the PD2 mount was replaced with a holder for a FO cable, and this FO cable, measured at 2065.5 ± 0.5 m was plugged directly into PD2, such that a length of glass cable was added to the previous setup. The raw output of the oscilloscope for this step is shown in fig. 11.

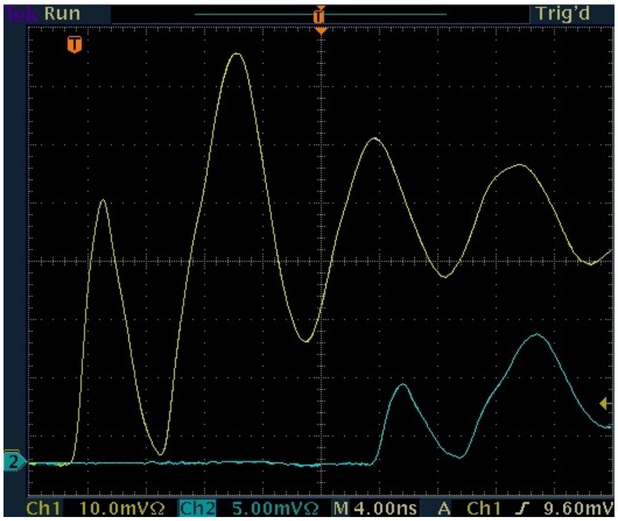

FIG. 11. Oscilloscope readout of the two signals PD1 (yellow) and PD2 (blue) for the two paths in glass.

The same pre-processing as in the previous step was used again. The processed signal, with all peaks besides the first ones removed, is shown in fig. 12.

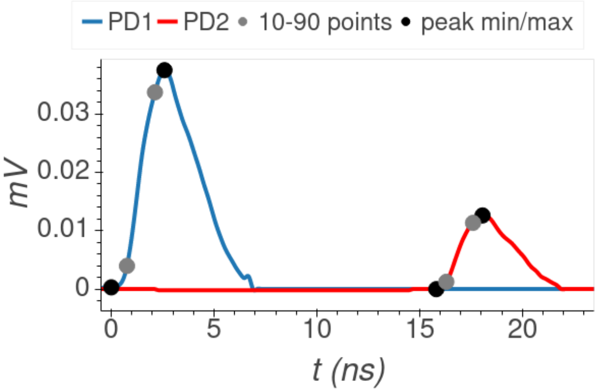

FIG. 12. Processed signals PD1 and PD2 in glass, with calculated reference points included.

The values for the time shifts with respect to each signal reference point is shown in Table V. The average and standard deviation of the time shift was found to be

15.6 ± 0.2 ns. In order to account for just the effect of the glass, the previous value for the time-shift due to air in the prior step was simply subtracted from this one, giving a 10.3 ± 0.2 ns time shift due to the FO cable. By using the equation for velocity 1, the speed of light in glass was calculated to be $2.01 \pm 0.03 \times 10^8$ m/s, and by using eq. 2 the corresponding refractive index was found to be 1.49 ± 0.03.

TABLE V. Time values of the first nanosecond peaks with an extra FO cable inserted into PD2

PD #	Start (ns)	End (ns)	10% (ns)	90% (ns)
1	0.00	2.59	0.76	2.12
2	15.81	18.06	16.30	17.60

The accepted value of 1.52 is therefore within the uncertainty. Again, this uncertainty mostly comes from measuring distances with a measuring tape, lining up the markings on the tape with laser pulses that are below it, however some error also comes from the length of the nanosecond pulse and the difference in signal shapes between PD1 and PD2.

Speed of Light in Water

The speed of light in water was found using the same setup as in the previous section, Speed of Light in Air, by placing a clear plastic tube full of water, measuring 614.0 ± 0.05 mm, in the PD2 beam path such that the beam would pass through it twice. Specifically, the tube was placed along paths 2 and 3 in the fig 4. Special care was taken when originally setting up the first experiment to make sure that the beam traveling along path 2 was 5mm away from the location where path 3 is incident on M2, specifically for this phase of the test, and as such no measurements were retaken here. The raw output of the oscilloscope for this step is shown in fig. 13.

The same pre-processing as in the previous step was used again. The processed signal, with all peaks besides the first ones removed, is shown in fig. 14.

The values for the time shifts with respect to each signal reference point is shown in Table VI. The calculation of the time-of-flight of light through water in this test setup is more complex than in the other phases of this experiment because in this case, water *replaces* air along a portion of the path length for PD2. We can assume that because the speed of light is constant, we can subtract out the length the light traveled through air in our water measurement by finding the proportion of the time-shift in air that is due to the length other than that covered by the water tube, as shown in eq. 3.

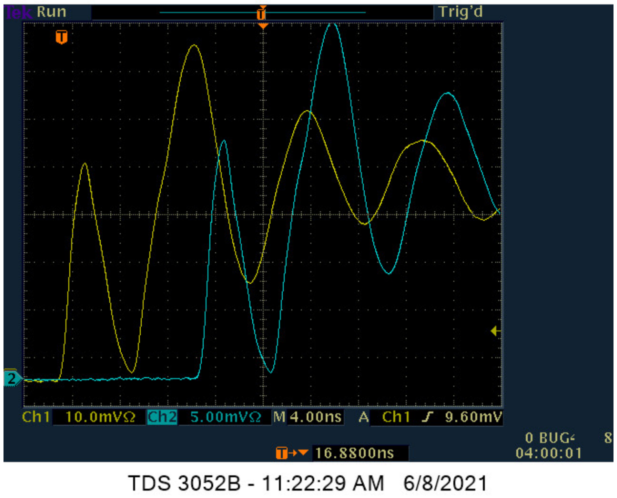

FIG. 13. Oscilloscope readout of the two signals PD1 (yellow) and PD2 (blue) for the two paths in water.

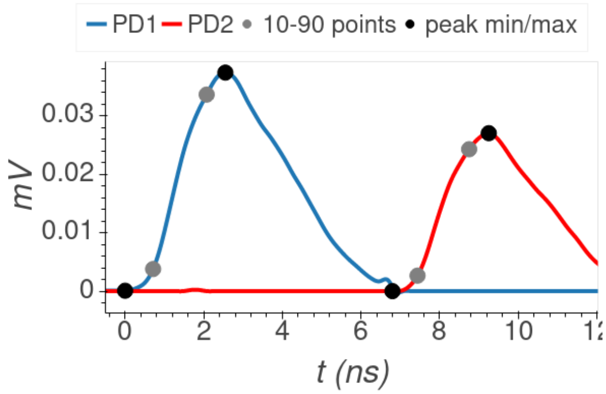

FIG. 14. Processed signals PD1 and PD2 in water, with calculated reference points included.

$$t_{airdelay} = \frac{L_2 + L_3 + L_4 + L_5 - 2L_{water}}{v_{air}} \qquad (3)$$

Where v_{air} is the speed of light in air calculated in the preceding section, L_2 to L_5 are the path lengths in Figure **??**, and L_{water} is the length of the water cell which is multiplied by 2 since the laser passes through it twice.

This value for time-of-flight through the water tube alone was calculated to be 6.72 ± 0.5 ns. By using the equation for velocity 1, the speed of light in glass was calculated to be $2.24 \pm 0.02 \times 10^8$ m/s, and by using

TABLE VII. Measured and accepted values of the speed of light and refractive index of various materials.

Material	v (m/s)	n	Accepted n value	Refs.	Deviation
Air	$3.01\pm0.04 \times 10^8$	0.99 ± 0.01	n=1.00	[2]	-1 σ
Glass	$2.01\pm0.03 \times 10^8$	1.49 ± 0.03	$n = 1.52$	[2]	-1 σ
Water	$2.24\pm0.02 \times 10^8$	1.34 ± 0.01	$n = 1.33$	[2]	-1 σ

eq. 2 the corresponding refractive index was found to be 1.34 ± 0.01.

TABLE VI. Time values of the first nanosecond peaks when a water-filled optical cell is in the path leading to PD2

PD #	Start (ns)	End (ns)	10% (ns)	90% (ns)
1	0.0	2.55	0.71	2.07
2	6.8	9.25	7.44	8.75

The accepted value of 1.33 is therefore within the uncertainty. Again, this uncertainty mostly comes from measuring distances with a measuring tape, lining up the markings on the tape with laser pulses that are below it, however some error also comes from the length of the nanosecond pulse and the difference in signal shapes between PD1 and PD2.

SUMMARY

In spite of the several sources of error and imperfect optimizations of the test setup, all of the calculated values matched accepted values within one standard deviation, which is quite excellent. The caluclated value for the refractive index of air, 0.99 ± 0.01 compares well to the accepted value of 1.0003, forgiving the mortal physics sin of suggesting an object is going faster than the speed of light, of course. The calculated value for the refractive index of glass, 1.49 ± 0.03 compares well to the accepted value of 1.52. And finally, the calculated value for the refractive index of water, 1.34 ± 0.01, compares well to the accepted value of 1.33 [2].

While these small uncertainties of around 1% each are quite small, and this can be attributed to very careful measurements of distance, a consistent test setup, a high sample-rate oscilloscope, and some careful signal processing, there are several methods that could be utilized to more precisely measure these refractive indeces. First, a longer time delay between PD1 and PD2 would minimize the relative error of the oscilloscope. Ideally, the delay would max out the sampling resolution of 2 microseconds on the Tektronix TDS210, which could be achieved by sending a light signal 250x farther than the length of a lab bench, perhaps reflecting off a nearby tower. On the other hand, distance measurement could be much more accurate by using a micrometer instead of a tape measure. Finally, an even shorter pulse generator would simplify signal processing and reduce the widths of the peaks that are compared to calculate a time shift.

Finally, it is quite interesting to compare the propogation of electric field through a cable to the propogation of light though a medium. Our calculated effective refractive index of the BNC cable, 1.56 ± 0.02, demonstrates that current is not mediated by electrons bumping into each other, or the flow of electrons down a wire, but rather by the nearly-fast-as-light (through some mediums) propogation of an electric field through a wire. We can see that the BNC cable was a comparable speed to the FO cable, but it is of course important to note that there are other advantages of FO cable besides the exact speed of the communication.

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[1] American Physical Society, Speed of Light History
https://www.aps.org

[2] BYJU's Educational, Refractive Index
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[3] Wikipedia, Refractive Index
https://en.wikipedia.org/wiki/RefractiveIndex