AFT Lab

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The relationship between both sample rate and number of cycles with FFT linewidth was established, and it was found that the number of cycles observed cycles was much more relevant in precisely calculating the frequency of a signal. It was observed that sampling a signal at a sample rate lower than twice that of the signal does not produce an accurate reconstruction of the signal in the time domain or the frequency domain. Several acoustic sounds were produced in the lab and analyzed. It was found that tuning forks tuned to the same note will produce an observable acoustic "beat" due to their slight differences in frequency. It was found that a human whistle produces only a single, pure tone, and the sound produced by two humans whistling the same note is indistinguishable. Several human vowel sounds were analyzed, and it was found that each vowel sound has a unique frequency signature, and that these vowel sounds can be differentiated from each other clearly. Finally, two string-based instruments were analyzed for the quality of their timbre, or the harmonics present in their music. A guitar was found to have much fewer harmonics, and lower overall harmonics than a single-string guitar, which has much higher harmonics and a great many harmonics.

INTRODUCTION

Fourier analysis is the practice of viewing a time-domain signal in the frequency domain. Named after the mathemetician Joseph Fourier for his breakthrough discovery that most functions, even discontinuous functions like the step function, can be represented as an infinite sum of sine waves, the advancement was originally made in pursuit of more easily solving complex differential equations describing heat transfer. Some scientists and engineers, however, do Fourier wrong by simply referring to the technique by the anagram of its modern discrete implementation, by saying, for example, "plot the FFT".

The FFT, short for Fast-Fourier Transform, is as stated a method for calculating the discrete Fourier transform of a finite time-domain signal. One can forgive those who refer to Fourier analysis generally as "FFT", considering how this ingenious algorithm has paved the way for the discrete Fourier transform's widespread usage in signal processing across a wide range of fields and disciplines.

Utilizing a divide-and-conquer technique, the FFT could be compared to a sailboat that, rather than following the wind directly to its destination, turns sharply to the side and tacks back and forth across the wind, taking a significantly longer and more complicated path that arrives in a fraction of the time. To understand just how well this analogy works, consider that the first step the FFT performs on a signal of length 2049 is to add another 2047 zeros onto the end so that the signal length is an exact power of 2.

In this lab we will be conducting Fourier analysis, with the help of the FFT algorithm, of various acoustic signals. This analysis is particularly interesting in the context of speech and musical instruments, as noised that to our ears may sound like "one" note or vowel sound are in fact more often than not rich with multiple harmonics throughout the frequency spectrum.

We will first conduct a simplified test implemented entirely in software, examining the effect of longer observations on the linewidth, or uncertainty in the frequency domain, numerically. We will then conduct a simple test using an function generator with an oscilloscope to collect data for a sine wave and square wave, in order to compare their frequency responses.

In the next phase of the lab, acoustic signals will be examined. First, we will explore the interaction of two tuning forks as they constructively and destructively interfere due to slight differences in their frequency, producing what is known as a "beat". We will then explore characteristics of musical notes produced by a whistling sound, the human voice producing vowel sounds, and musical instruments.

Finally, we will conduct a numerical exploration of the breakdown of accurate signal reconstruction in the time-domain as a function of the nyquist frequency and the fundamental frequency of the signal.

APPARATUS

The apparatus consisted of the following.

BASIC FFT

Procedure and Results

In this section, a time-vector from 0 to 0.1 using 2048 equally spaced points was produced, and used to build a sine wave of equation $y = sin(2\pi 100x)$, shown in 4. The signal was then transformed into the frequency domain, using the FFT algorithm. With this signal, as with all signals for the duration of the lab, the signal was multiplied by a hamming window before performing the discrete Fourier transform to mitigate spectral leakage.

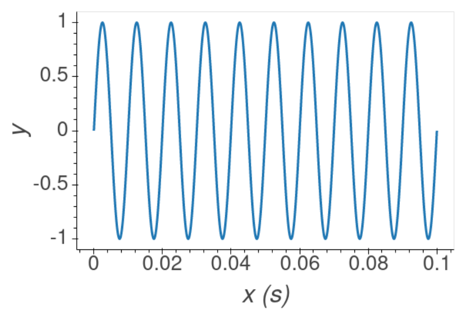

FIG. 1. Sine wave of frequency 100 Hz, observed for 0.1 seconds.

The Fourier analysis produced the expected frequency of 100 Hz, verifying that the technique was applied correctly, as seen in 2.

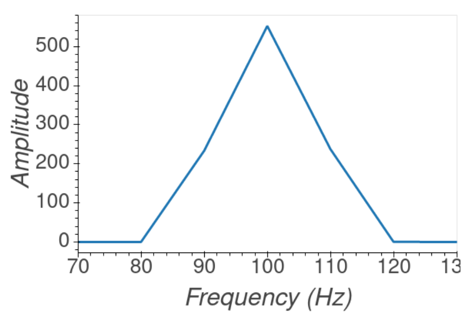

FIG. 2. Sine wave of frequency 100 Hz, observed for 0.1 seconds, in the frequency domain.

A second sine signal, calculated with the same sine function and N samples but longer observation window of 1 second, was produced, as seen in **??**.

This longer sine wave was analyzed in the frequency domain as well, producing a 100 Hz peak, as seen in **??** with the same x-axis range as 2.

The characteristics of these two alike signals can be easily compared in 7, however quantitative characteristics are also given in I.

A brief exploration into the effect of the sample rate on the appearance of a signal, and its corresponding Nyquist

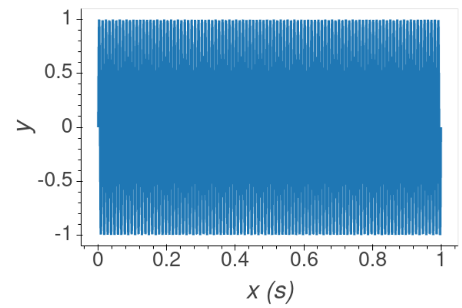

FIG. 3. Sine wave of frequency 100 Hz, observed for 1.0 seconds.

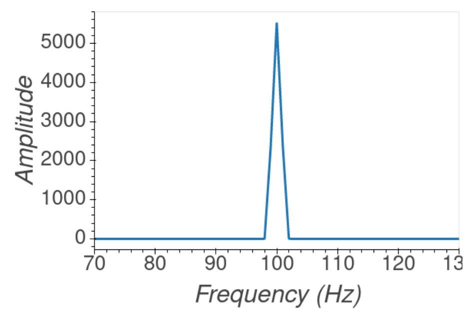

FIG. 4. Sine wave of frequency 100 Hz, observed for 1.0 seconds, in the frequency domain.

frequency, was conducted numerically. The same sine wave corresponding to the equation $y = sin(2\pi 100x)$ was sampled for 0.1 seconds, using 10 and 100 points. Using an understanding of the Nyquist frequency, which states that the sample rate must be at least twice that of the sine wave in order to truthfully reconstruct it, the lowest number of samples per 0.1 seconds that can truthfully reconstruct the sine wave was plotted as well, along with one less than that number. These four waves can be seen in 6.

While it is barely visible behind the other curves, it is still clear that the blue wave is the baseline, a sample rate high enough to reconstruct the sine wave very well. Because the frequency of the sine wave is 100 Hz, the lowest Nyquist frequency acceptable would be 200 Hz, or 20 samples over 0.1 seconds. From this it is clear why the red wave, sampled at a rate such that the Nyquist frequency of that wave is half of what is needed, does not match the true sine wave whatsoever and looks like a completely different signal. Observing that this red signal appears to make exactly one cycle in 0.1 seconds, we can see that it looks like a 10 Hz signal instead of a 100 Hz signal. It is also easy to make out that the green wave, sampled at 20 samples per second, contains almost exactly two samples per period or one sample per positive and negative peak - the Nyquist frequency corresponding to that sample rate is exactly the lowest possible value. Finally, comparing the purple wave, which is sampled with a Nyquist just

TABLE I. Frequency response and parameters given by a short and long observation of the same sine equation.

Time (s)	Cycles M	Samples N	FFT Range Δf	Resolving Power $\frac{f_0}{\delta f}$ (Hz)	Linewidth Γ (Hz)
0.1	10	2048	10225	10.00	17.4
1.0	100	2048	1022.5	1.000	1.74

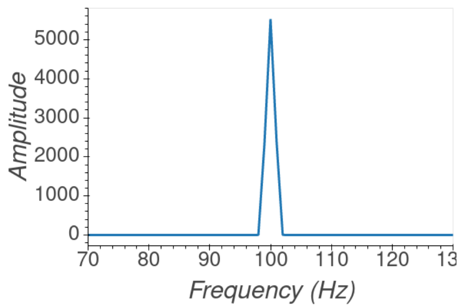

FIG. 5. Comparison of the short (0.1 second) and long (1.0 second) sine waves, given by the same equation, using Fourier analysis.

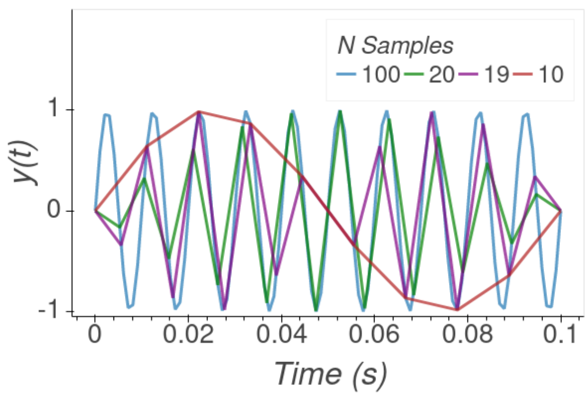

FIG. 6. The same equation, sampled with four different numbers of samples per one tenth of a second.

below the requisite for this sine wave, to the green and blue waves, it can be seen that in the middle of the signal it skips a peak, resulting in what looks like a similar frequency (but slightly lower in frequency) sine wave that is dampened in the middle.

Conclusion

Uncertainty in the frequency of a signal, which manifests as the linewidth of a peak in the frequency domain, is reduced as a function of observation time. This is the result of a flavor of the Heisenberg Uncertainty Principle, where longer observations (greater N cycles) of a signal trade better certainty of the location of the signal in the frequency domain for worse certainty of the location of the signal in the time domain.

This simple experiment confirms that linewidth Γ is reduced with longer observations or more cycles. A higher precision FFT was produced in this experiment by collecting 100 cycles than by collecting 10 cycles. It also shows that if a longer observation time is used while the number N of samples is kept constant, sample rate decreases, which is seen in the frequency range as well. It is important to note that although the resolution of the frequency spectrum was reduced by a factor of ten (larger frequency bins), the uncertainty or linewidth was still reduced.

TUNING FORKS

Procedure and Results

In this section, the beat frequency of two tuning forks tuned to the same note was analyzed. An acoustic "beat" is caused by a slight difference in the frequency of two sounds. Because sounds are waves and can constructively or destructively interfere with each other, two waves of constant frequency can interact with each other at a fixed point in space with a repeating pattern. Specifically, a "beat" usually occurs when the two constant frequencies are just slightly different. So, while two tuning forks are more or less exactly the same to most human ears, the frequency difference between them when they are played simultaneously may produce an audible "beat".

Two tuning forks were selected, and held at equal distances from a microphone connected through an amplifier to an oscilloscope. The resulting wave was recorded and is shown in **??**.

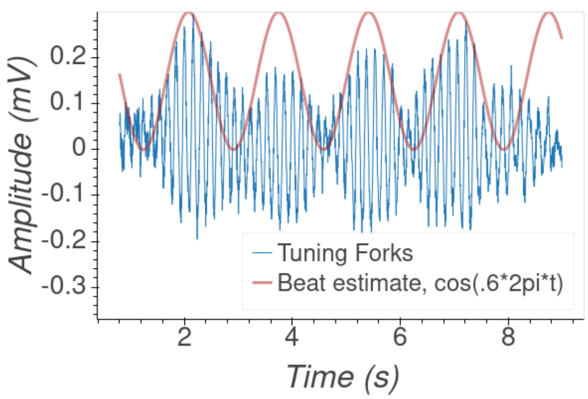

FIG. 7. Two tuning forks of nearly equal pitch interacting in a constructive/destructive pattern, or "beat", with an estimation of this beat frequency.

The frequency of the tuning forks can be seen as the waves of shorter wavelength, and the interaction of the two slightly different notes producing a "beat" can be seen as the oscillating amplitude of these peaks. It should be noted that the frequency of the tuning forks themselves may not be accurately represented at this sample rate, as the nyquist frequency is 120 Hz but the frequency of the tuning forks is in the thousands. The low sample rate was necessary to observe the beat pattern, as this was the priority of this experiment. The frequency of these beats was estimated using trial-and-error at about 0.6 Hz. This fit is not perfect, perhaps due to holding the forks at inconsitent distances from the microphone.

WHISTLE

Procedure

In the next phase of the lab, a human whistle was produced by two different subjects and analyzed in the frequency domain. As in the last section and in all following sections, the acoustic sounds were recorded with a microphone, and sampled with an oscilloscope before being saved and analyzed with Fourier analysis. Each human subject attempted to produce a constant whistling note during recording.

Results

In the first part of the test only one person whistled a higher-register note. The frequency signature of this whistle is shown in 8.

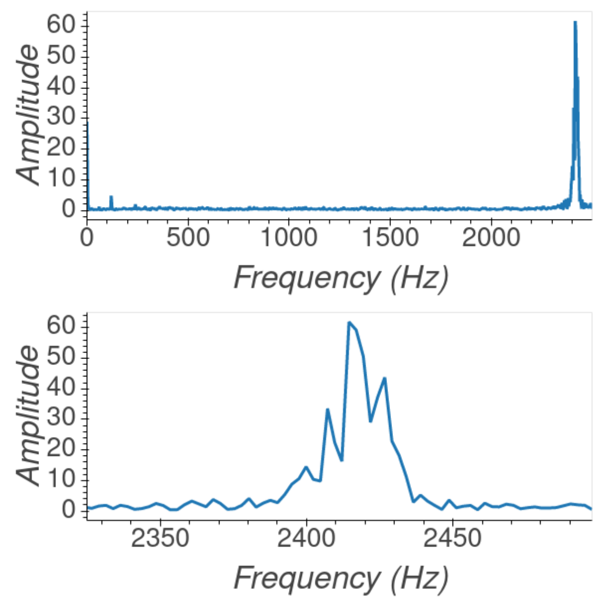

FIG. 8. The frequency response of a higher-register note produced by a human whistling, at full scale and zoomed in to the relevant area.

Only a single frequency is observed in a human whistle, similar to what may be expected from a tuning fork. While it is true that the zoomed in version of the FFT in 8 reveals a non-zero linewidth and seems to include a few frequency components, this is likely due to the subject's whistling skill being inhibited by wearing a face-mask, causing the note to waver in pitch slightly.

In the next part of this test, two subjects, Adrian and Trevor, whistled the same note, recording them separately. A lower note was used that was within both subjects' registers. This is shown in 9.

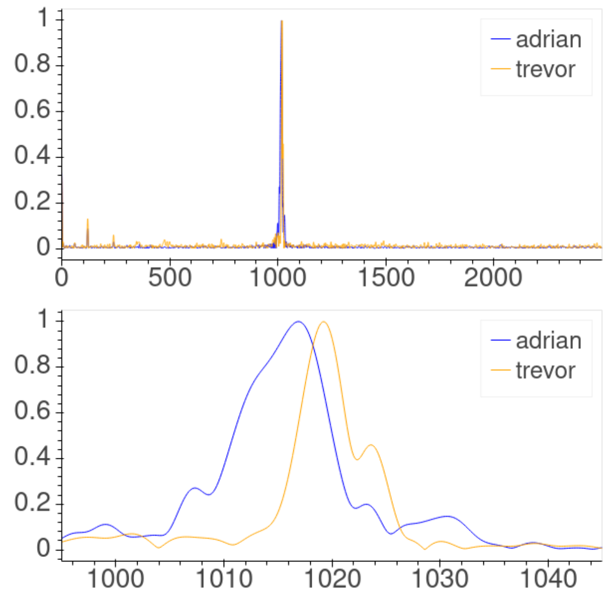

FIG. 9. The normalized frequency response of two subjects recording the same whistling note separately.

Again, as before, a single frequency was observed for both subjects' whistles. While a zoomed in view does seem to show that Adrian's whistle involves slightly more variation lower than the main note, this is likely simply a matter of Adrian not holding the exact same note during testing, while Trevor's note was more consistent. For both whistlers, there are no harmonics present in the Fourier analysis.

Conclusion

It appears for all intents and purposes that the whistle of one identical, constant note by two people are indistinguishable. This is indicative of the physiology of whistling.

In order to produce a whistle, there is at the core an interaction between high-speed air and still air that forms a "vortex ring". This occurs in the space between the tongue, pushed up against the roof of the mouth, and the lips, inside the mouth. As these rings of circulating air hit the lips, a pressure wave is sent back into the mouth. A whistler is able to shape their mouth such that this pressure wave resonates with the formation of new vortex rings.

It is this mechanism that amplifies the overall sound and produces a single, pure tone. This is because there is only one path that the sound travels, and nodes only at the back of the tongue and the lips.

It is true, though, that just because the same note held by two people is indistinguishable, doesn't mean that all whistling is the same. Because whistling is a musical instrument, artistic talent is required to produce music, and this artistic talent may be observable from one person to the next.

HUMAN VOICE

Procedure and Results

In this phase of the lab, several vowel sounds were produced, recorded with a microphone and oscilloscope, and analyzed in the frequency domain. These vowel sounds were 'a', 'e', 'o', and 'u', pronounced like in 'car' with a bostonian accent, 'cheese', 'coo coo ca choo', and 'choose'. These sounds were produced by a human male with a relatively low voice. The frequency response of these 5 vowel sounds are shown in **??**.

Conclusion

These frequency responses help describe the sounds we are used to hearing and making in a few ways.

First, beginning at the earliest peak, the '0' frequence or offset. There is a significant peak observed for 'e' and 'u', and a small one for 'o', but not for 'a'. The reason for this is that for 'e' and 'u', the sounds are produced by building up pressure at the very back of the throat and then releasing, while the 'a' sound is simply released without any effort. This pressure wave is recorded as an average amplitude greater than zero. Trying to make an 'e' sound without building up pressure feels very awkward.

Next, we can look at the number of peaks present. 'a', has the most activity in the frequency domain when judged by number of peaks. This relates to the way this sound is produced with an open mouth, with very many different paths and harmonics available for the sound to be shaped by. 'e' has very vew peaks, which relates to the more closed position of the mouth and the flat shape of the tongue. 'o' and 'u' have almost the exact same number of peaks and frequencies of said peaks, which relates to them being essentially the same sound but with a slight shift at the front of the mouth. This shift serves to shift the relative amplitude of the higher harmonics closer to the lower ones.

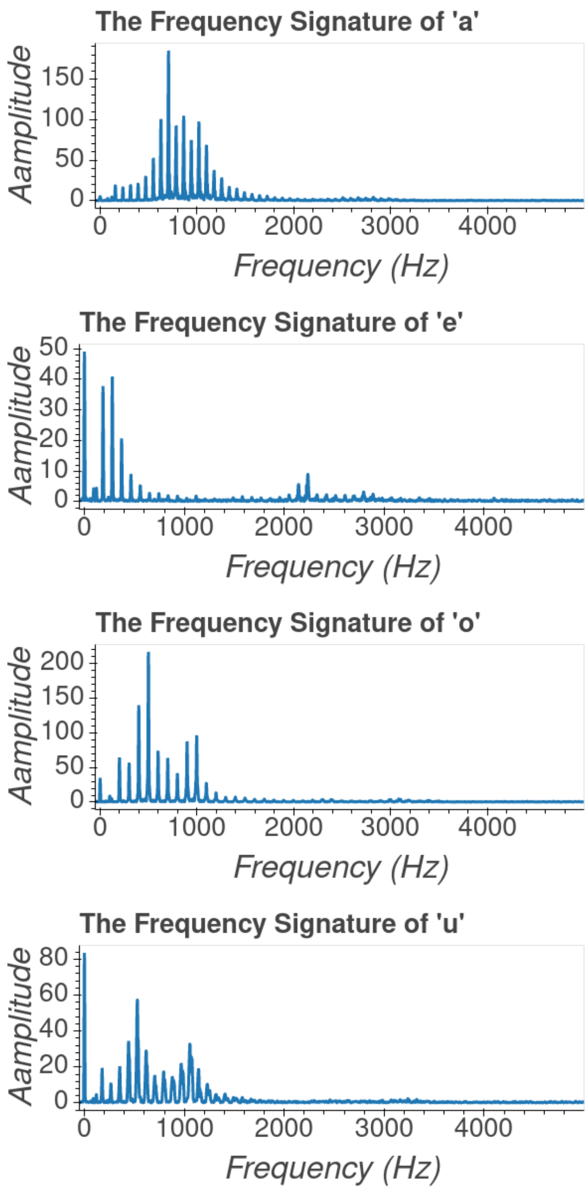

FIG. 10. The frequency signatures of various vowel sounds.

Next, we can look at the locations of the peaks. Perhaps the most interesting vowel in this regard is 'e'. 'e' has both the lowest peak frequency and the largest higher frequency in the 2 kHz range. This relates to the dropped jaw leaving more room for the vocal cord to oscillate at a lower frequency, and also to the higher pitch resonance created in the nasal cavities. 'a' has a peak frequency that is higher pitched, but has lower and higher amplitude harmonics, but has very little resonance in the nasal cavities producing higher frequency harmonics. This can be observed by saying 'e' and 'a' with one's nose plugged - the 'e' sounds much different but the 'a' sounds the same. Finally, for 'o' and 'u', the fundamental frequency is much more noticable as they are lower-sounding notes, but very many higher sounds are present, kind of the same sound that is the main one you hear when humming.

MUSICAL HARMONICS

Procedure and Results

Finally, the same recording setup as before was used to record two musical instruments playing the same note. The two selections were a classical guitar and a single-string. The frequency response of both of these instruments playing a C can be seen in 11.

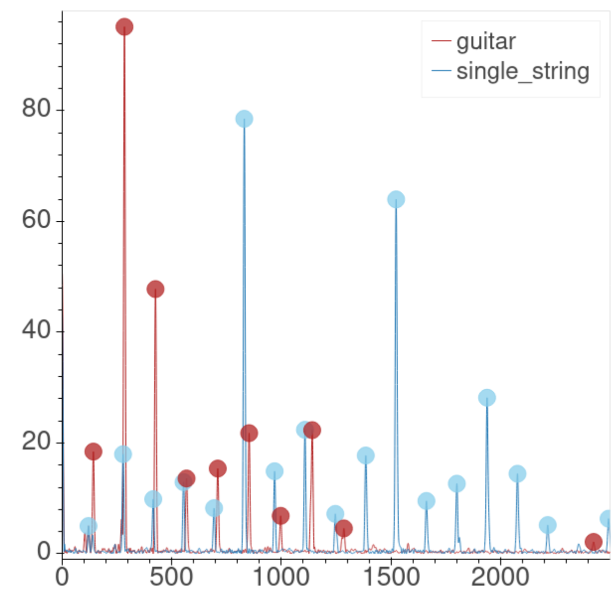

FIG. 11. The frequency response of the two selected instruments. The locations of peaks used in further analysis are marked with dots.

It is clear that these instruments are indeed quite different from their frequency response. Most prominent is the comparison between the most prominent peaks. The guitar's strongest harmonic is very low, the second harmonic. This makes sense given the construction of the instrument - there is a very large, hollow body situated just under the location where one strums, which serves to amplify lower frequencies and increase the richness of the sound. The single-string's strongest harmonic is much higher, the fifth harmonic. This makes sense given its construction is just a single string drilled into a 1/2x3 plank of wood. There is nothing to amplify the lower frequencies, and the result is a much tinnier sound.

The further point of comparison between the two instruments is the amount of harmonics. The guitar has a strong second harmonic (and the fundamental frequency is also observable), as well four or five more, much quieter higher harmonics. The single-string has a second, very significant harmonic at 1500 kHz, and another one at 2000 kHz. This is much higher than any of the harmonics with a significant amplitude for the guitar had. In addition, there are just a slew of harmonics, there's something present at essentially every single harmonic all the way up to 3000 kHz.

To clarify, all musical notes have a single fundamental frequency, the lowest frequency, and a series of harmonics which are integer multiples of the fundamental. It is doubtful that any musical instrument intentionally has more than one fundamental, as it would sound like playing two discordant notes at the same time.

A summary of the above FFT plot can be seen in 12, where each frequency was divided by the fundamental in order to find the integer factor, or the harmonic number. Each harmonic was also converted to a musical note.

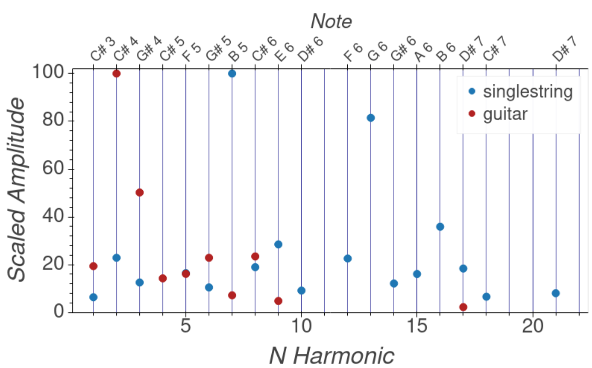

FIG. 12. Normalized plot of the harmonics of both instruments and their relative amplitudes.

This plot clarifies the difference between the two instruments. The beauty of the guitar as a musical instrument is really striking, with the C, the fundamental frequency, harmonizing with a C one octave up, a G which is the fifth half-step below the C, and again with another G and C in even higher registers. For the single-string, by contrast, while the tuner appropriately picked up the lowest note, the C to tune to, the most predominant notes are the B and G, where the B is a very awkward two half-steps below the C, and the G is a very awkward 6 half-steps below the C. This explains why no matter how tuned it was, the single-string always sounded out of tune and discordant.

For the purpose of demonstration, a long cardboard tube was used to create a flute-like instrument by cutting a hole near one end and rounding it with tape. A note was successfully produced with great effort, which was recorded using a laptop microphone and saved as an mp3. The A(N) is shown in 13.

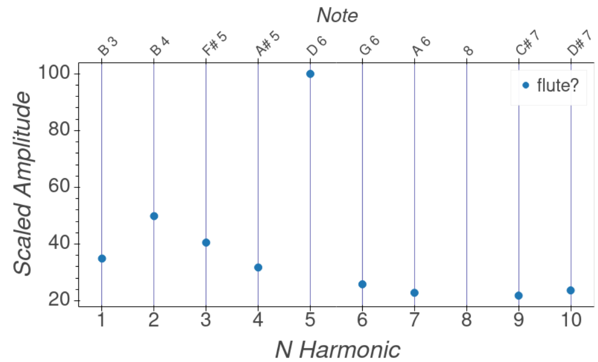

FIG. 13. Normalized plot of the harmonics of a makeshift flute and their relative amplitudes.

FIG. 14. Picture of the makeshift flute.

It is not clear from seeing the normalized scale, but only one of these notes was audible, the fifth harmonic. The sound of air rushing past did seem to produce measurable frequencies which did correspond to harmonics of a fundamental (corresponding to the length of the tube), but in reality these other harmonics just sounded like air blowing through a cardboard tube. It is unexpected that the main note is the fifth harmonic because the hole was located about one seventh of the way down the tube, not one fifth. From this it seems like the point of the tube where the note was produced was inside the tube. This perhaps could help design improvements, as the length could be cut down to match this length of five times the distance from the hole to the first end. An image is shown in 14.

SUMMARY

The characteristics of several acoustic signals, and the inherent constraints of Fourier analysis as it pertains to sample rate, Nyquist frequency, and spectral leakage, were explored. It was observed that a signal sampled at too low a sample rate will alias into the lower frequency ranges, making the signal look like a lower frequency signal. It was also observed that, as long as the Nyquist frequency corresponding to the sample rate is still high enough to observe the signal, more sine wave cycles is more important than more samples per second in determining frequency with very high certainty, or low linewidth.

Several acoustic noises were analyzed, and their frequency- domain characteristics were linked to the character of the sound that can be observed through listening. Two tuning forks were struck simultaneously, in order to observe the slight difference in their exact tuning through the pattern of constructive and destructive interference that it causes, known as acoustic "beat". This beat was observed with a period of about 0.6 seconds.

A human whistle was analyzed, and it was found to be very similar in the frequency domain to a single sine wave. Two subjects produced a whistle at the same note, and these whistles were sound to be more or less identical. This was linked to the physiological way that whistles are produced by humans, which inherently produces only a single oscillating pressure wave with no harmonics.

Several vowel sounds were found to be distinguishable through a series of frequency-domain features, including a plosive sound formed by releasing pressure to create the sound, nasal harmonics, numbers of peaks, and the frequency of the highest peak.

Several musical instruments were analyzed, two of which both create sound using strings and were playing the same note at time of recording, and their timbre was revealed through Fourier analysis, explaining the rich and deep sound of the guitar and the tinny and discordant sound of the single-string.

Fourier analysis is truly an insightful tool in the realm of acoustic physics, and it was all enabled by Fourier, and the incredible algorithm known as the Fast-Fourier-Transform.

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[1] Wikipedia, Fast Fourier Transform: https://en.wikipedia.org/wiki/FastFouriertransform

[2] Wall Street Journal, The Secret of Whistling https://www.wsj.com/?mod=wsjheader$_logo$

[3] Royal Society Publishing, Bioacoustics of Human Speech https://royalsocietypublishing.org/doi/10.1098/rspb.2019.1116