HOMEWORK 9

Problem 1

> *(a) Determine the binding energy per ion pair in LiF from the cohesive energy.*
> *(b) Determine the binding energy per ion pair in LiF from the minimum potential energy at equilibrium separation (i.e. based on lattice parameters).*

A) $B = \frac{E_{coh}}{N_A}$
$B = 1030\ kJ/mol/6.02 \cdot 10^{23}\ ions/mol = 10.7eV$

B)

$$B = \frac{\alpha e^2}{4\pi\epsilon_0 R_0}\left(1 - \frac{1}{n}\right)$$

$$B = \frac{1.7476 \cdot 1.44\ eV \cdot nm}{0.201nm}\left(1 - \frac{1}{6}\right)$$

$$B = 10.433eV$$

Problem 2

> *Copper has a density of 8.96 g/cm3 and molar mass of 63.5 g/mol. Calculate the center-to-center distance between copper atoms in the fcc structure.*

$8.96g/cm^3 \cdot \frac{1mol}{64.5g} = 138914.729mol/m^3 = 8.365 \cdot 10^{28}\ atoms/m^3$ The fcc structure contains 4 atoms, 1/2 per face and 1/8 per corner. If l is the length of one edge of the cube, then $5atoms/l^3 = 8.365 \cdot 10^{28} atoms/m^3$. Therefore,

$$l = \left(\frac{5\ atoms}{8.365 \cdot 10^{28}\ atoms/m^3}\right)^{1/3} = 0.391nm$$

Problem 3

> *Follow these steps to estimate the electrical conductivity of silver at temperature $T = 300$ K. The density of silver is $=$ $10.5\ g/cm^3$ and the molar mass is $M = 108\ g/mol$.*
> *(a) Determine the concentration of conduction electrons in silver assuming that each silver atom contributes one electron.*
> *(b) Use the equipartition of energy theorem (see e.g. Chapter 1 if you are unfamiliar with this) to estimate the average speed of conduction electrons.*
> *(c) Estimate the mean free path of conduction electrons by equating it to the nearest-neighbor distance in the lattice. The latter must be estimated based on the numbers provided and the fact that metallic silver forms an fcc lattice.*
> *(d) Combine the results of (a-c) to calculate the electrical conductivity.*

A)

If we treat the free electrons as a gas, given that there is one free electron per Au atom (the two 5s electrons instead fill the 4d level, leaving one left over),

$$10.5\ g/cm^3 \cdot 1\ mol/108\ g = 0.0972\ mol\ electrons/cm^3$$

B)

$$v_{av} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(0.0258\ eV)}{0.511 \cdot 10^6 eV/c^2}} = 116,675.785\ m/s = 1.167 \cdot 10^5\ m/s$$

C)

$10.5g/cm^3 \cdot \frac{1mol}{108g} = 97222mol/m^3 = 5.855 \cdot 10^{28}\ atoms/m^3$ Given that this is fcc just like problem 2,

$$l = \left(\frac{5\ atoms}{5.855 \cdot 10^{28}\ atoms/m^3}\right)^{1/3} = 0.440nm$$

D)

$$\sigma = \frac{ne^2\tau}{m}$$

$\tau = l/v_{av} = 0.440\ nm/1.167 \cdot 10^5\ m/s = 3.77 \cdot 10^{-15}s$
$n = 5.855 \cdot 10^{28}\ cm^{-3}$
$m = 9.109 \cdot 10^{-31}\ kg$

$$\sigma = \frac{(5.855 \cdot 10^{28}\ m^{-3})(1.60 \cdot 10^{-19}\ C)^2(3.77 \cdot 10^{-15}s)}{9.109 \cdot 10^{-31}\ kg} = 6.203\ MS/m$$

Problem 4

> *(a) Calculate the total nuclear binding energy of ^3He and ^3H.*
> *(b) Account for any difference of the calculated nuclear binding energies by considering the Coulomb interaction of the extra proton of ^3He.*

A) The total nuclear binding energy is the difference in mass between the constituent particles and the whole atom.
^3He:

$$B = \left(1(939.57\ MeV/c^2) + 2(938.28\ MeV/c^2) - 2809.4132MeV/c^2\right) c^2 = 6.716MeV$$

^3H:

$$B = \left(2(939.57\ MeV/c^2) + 1(938.28\ MeV/c^2) - 2809.4132MeV/c^2\right) c^2 = 7.988MeV$$

B) The difference is due to the two protons in He repelling each other.

Problem 5

> *Ordinary potassium contains 0.012% of the naturally occurring radioactive isotope K, which has a half-life of 1.3×10 year.*
> *(a) What is the activity of 1.0 kg of potassium?*
> *(b) What would have been the fraction of K in natural potassium 4.5×10 years ago?*

A)

$$t_{1/2} \approx \frac{0.693}{\lambda} \rightarrow \lambda = \frac{0.693}{1.3 \cdot 10^9 year} = 1.689 \cdot 10^{-17} Hz$$

1 kg of potassium is $1\ kg \cdot \frac{1\ mol}{39.098g} = 25.75mol\ K$. 25.75mol potassium will contain $25.75\ mol \cdot 0.00012\ mol\ ^{40}K/mol\ K = 0.00307\ mol\ ^{40}K$.

$$a = N = 1.689 \cdot 10^{-17} Hz \cdot 0.00307\ mol \cdot A_N = 31,218\ molecules/second$$

B)

$$N = N_0 e^{-\lambda t} = 0.00307\ mol \cdot e^{-1.689 \cdot 10^{-17}\ Hz \cdot (-4.5 \cdot 10^9\ years)} = 0.0338\ moles$$

The mole fraction is then $0.0338/25.75 = 0.13$ %.

Problem 6

> *What is the kinetic energy of the alpha particle that is emitted in the decay of the uranium isotope with mass number A = 234?*

$$^{234}U \rightarrow {}^{230}Th + {}^4He + KE$$

Replacing each element with its total energy (assuming the main atom is stationary before and after decay):

$$221,722.53MeV/c^2 \rightarrow 216,141.7MeV/c^2 + 3,728.4MeV/c^2 + KE/c^2$$

$$c^2(221,722.53MeV/c^2 - (216,141.7MeV/c^2 + 3,728.4MeV/c^2)) = 1,852MeV$$

Problem 7

> *Compare the recoil energy of a nucleus of mass 200 u that emits:*
> *(a) a 5.0-MeV alpha particle, and*
> *(b) a 5.0-MeV gamma ray?*

A)
The speed of an alpha particle with 5 MeV KE is (wolfram alpha) 0.05174 c. The momentum will be 192.9 MeV/c. By conservation of momentum, the nucleus will also recoil with the same energy. This one will not be even remotely relativistic, but it will lose 2 Amu by ejecting the alpha particle. 192.9 MeV/c / 198 u = 0.001046 c. Using the equation for kinetic energy, we get 0.1009 MeV.
B)
The momentum of a 5 MeV photon will be 5 MeV/c. Using the same calculation as above, we get KE = 67.8 eV.

Problem 8

> *A fossil bone fragment contains 50 g of carbon and registers a = 5 decays/s of carbon-14 activity. Assume that the carbon-14 to carbon-12 isotope ratio in the atmosphere, and consequently within living organisms, is constant over time at 1.3×10 ^{12}. Because carbon-14 decays radioactively with a half-life of 1.8×10^{11} s, the ratio of carbon-14 to carbon-12 in dead organisms decrease over time.*
> *(a) How many carbon-14 isotopes were in the bone fragment when the animal was alive?*
> *(b) Find the decay constant (λ) of carbon-14.*
> *(c) Determine the age of the bone fragment.*

A)
The 50 g of carbon will be split by mole ratio between 14 and 12 is $1.3 \cdot 10^{-12}$ $mol\ Carbon - 14/1\ mol\ Carbon - 12$. The average molecular weight will be (1-1.3e-12)*12 + (1.3e-12)*12 ok yeah it's 12, rounding to any number of meaningful digits.

$$50\ g\ carbon \cdot \frac{1\ mol\ C}{12\ g} \cdot \frac{1.3e-12\ mol\ ^{14}C}{1\ mol\ ^{12}C} = 3.262 \cdot 10^{12}\ molecules\ Carbon - 13$$

B)

$$t_{1/2} = 1/\lambda ln(2)$$

$t_{1/2} = 1.8 \cdot 10^{11}\ s$
$\lambda = 3.85081767e - 12 hertz$
C)
We can find the N number of molecules currently by the expression relating activity, N, and λ.

$$a = -\frac{dN}{dt} = \lambda N$$

$$3.85081767 \cdot 10^{-12} hertz \cdot N = 5\ decays/s \rightarrow N = 1.29842554 \cdot 10^{12}\ molecules\ ^{14}C$$

By

$$N = N_0 e^{-\lambda t}$$

given labmda, N, and N_0,

$$1.29842554 \cdot 10^{12} = 3.262 \cdot 10^{12} \cdot e^{-3.85081767 \cdot 10^{-12} t}$$

$$\lambda t = ln\left(\frac{N}{N_0}\right) \rightarrow t = 7,580\ years$$