

⑦ Let $A = \begin{pmatrix} 0 & 0 & -1 \\ 1 & -4 & 0 \\ 4 & -13 & 0 \end{pmatrix}$. Find the eigenspace of A at $\lambda = -3$

$$A\vec{v} = -3\vec{v}$$

$$A\vec{v} + 3\vec{v} = 0$$

$$(A + 3I)\vec{v} = 0$$

$$\left(\begin{array}{ccc|c} 3 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 \\ 4 & -13 & 3 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -1/3 & 0 \\ 0 & -1 & -1/3 & 0 \\ 0 & -13 & 13/3 & 0 \end{array} \right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & -1/3 & 0 \\ 0 & 1 & -1/3 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \rightarrow \begin{aligned} x_1 &= \frac{1}{3}x_3 \\ x_2 &= \frac{1}{3}x_3 \\ x_1 &= x_2 = \frac{1}{3}x_3 \end{aligned}$$

$$\rightarrow \vec{v} = x_3 \begin{pmatrix} 1/3 \\ 1/3 \\ 1 \end{pmatrix}$$

Eigenspace of A at $\lambda = -3$ is

$$\text{span} \left\{ \begin{pmatrix} 1/3 \\ 1/3 \\ 1 \end{pmatrix} \right\}$$