Excercise 1.2

In this excercise we are working in the field $\mathbb{C}$.

i) Find $\det \begin{pmatrix} 3+i & 1-2i \\ 1+2i & -i \end{pmatrix} = (3+i)(-i) - (1-2i)(1+2i)$

$= -3i - i^2 - [1 - 4i^2]$

$= -3i + 1 - 1 - 4 = \boxed{-4 - 3i}$

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ii) Find $A^{-1} = \begin{pmatrix} 2-i & 2+i \\ 4-i & 4 \end{pmatrix}^{-1} = \frac{1}{\det(A)} \begin{pmatrix} 4 & -2-i \\ -4+i & 2-i \end{pmatrix}$

$\det(A) = (2-i)(4) - (2+i)(4-i) = 8 - 4i - [8 - 2i + 4i - i^2]$

$= 8 - 4i - 9 - 2i = -1 - 6i$

$\frac{1}{-1-6i} \times \frac{-1+6i}{-1+6i} = \frac{-1+6i}{1^2 + 6^2} = -\frac{1}{37} + \frac{6}{37}i$

$\rightarrow A^{-1} = \frac{1}{37} \begin{pmatrix} 4(-1+6i) & (-2-i)(-1+6i) \\ (-4+i)(-1+6i) & (2-i)(-1+6i) \end{pmatrix}$

$\boxed{A^{-1} = \frac{1}{37} \begin{pmatrix} -4 + 24i & 8 - 11i \\ -2 - 25i & 4 + 13i \end{pmatrix}}$