In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
```

7. Age of the universe and constraints on cosmological parameters

Make a 2D figure that shows the dependence of the age of the universe on $\Omega_{\rm matter}$ and Ω_Λ. Indicate the bounds on this 2D space that are set by age estimates of the Solar system, white dwarfs, and globular clusters. How does the region of $\Omega_{\rm matter}$, Ω_Λ that is allowed by these constraints compare with the region favored by other observations? (See your notes from class, figure 3.5 in the text, and the "Cosmic Triangle" article.) Use the equation

$$t_0 = \frac{2}{3} t_H (0.7\Omega_M + 0.3 - 0.3\Omega_\Lambda)^{-0.3}$$

$$t_0 = 6.52 h^{-1} \mathrm{Gyr}\ (0.7\Omega_M + 0.3 - 0.3\Omega_\Lambda)^{-0.3}$$

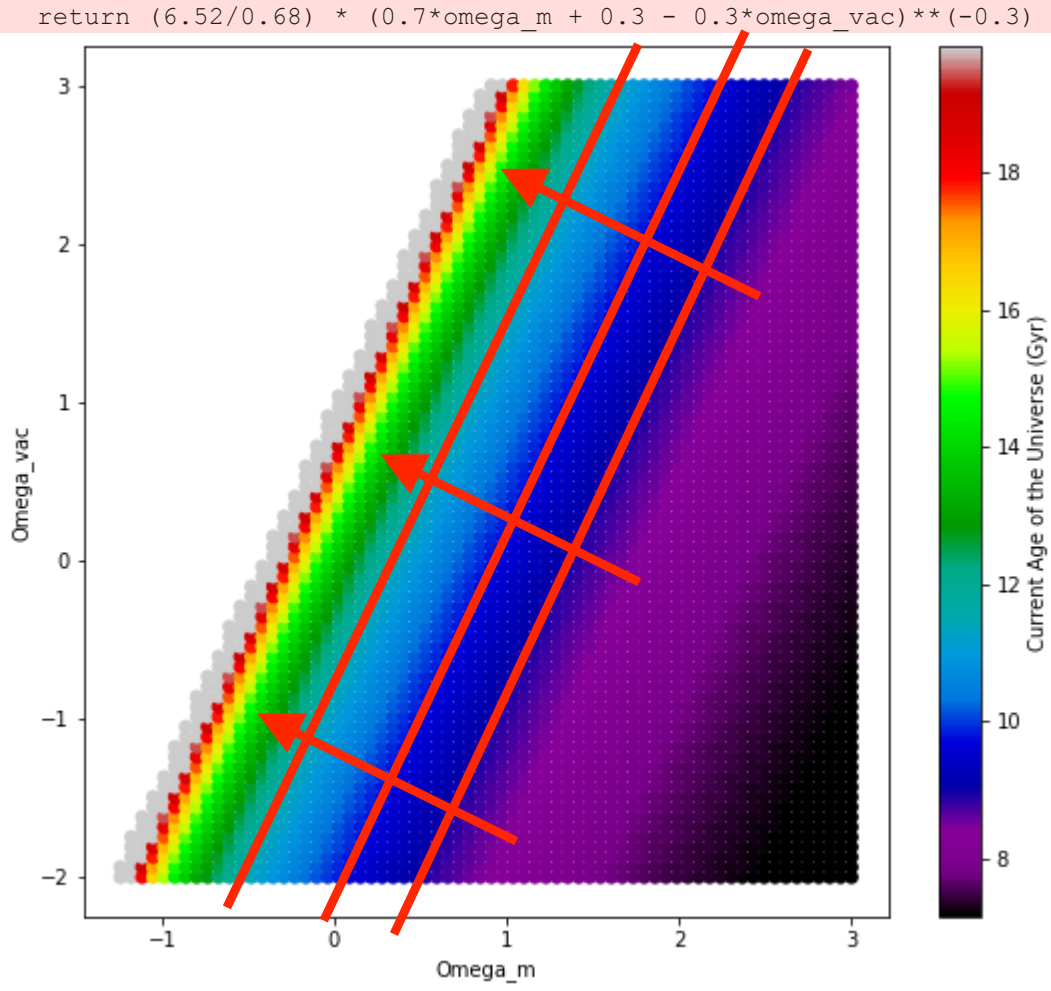
which relates the age of the universe now to the Hubble time ($t_H = 1/H$) and the cosmological parameters Ω_M and Ω_Λ.

Note that this equation is only valid if $\Omega_M > \frac{3}{7}(\Omega_\Lambda - 1)$

In [2]:
```
def t0(omega_m, omega_vac):
    return (6.52/0.68) * (0.7*omega_m + 0.3 - 0.3*omega_vac)**(-0.3) #take H0=68km/s/Mpc
```

In [3]:
```
fig = plt.figure(figsize=(9,8))

om_m   = np.linspace(-2, 3, 10)
om_vac = np.linspace(-2, 3, 10)

#Create mesh grid for plotting/colorbar
xmin, xmax = om_m.min(), om_m.max()
ymin, ymax = om_vac.min(), om_vac.max()
X, Y = np.mgrid[xmin:xmax:80j, ymin:ymax:80j]
positions = np.vstack([X.ravel(), Y.ravel()])
t0list = t0(positions[0,:], positions[1,:])

plt.scatter(positions[0,:], positions[1,:], c=t0list, cmap="nipy_spectral", \
            vmin=np.percentile(t0list[~np.isnan(t0list)], 2), vmax=np.percentile(t0list[~np.isnan(t0list)], 96))

cbar = plt.colorbar()
cbar.ax.set_ylabel('Current Age of the Universe (Gyr)')
plt.xlabel("Omega_m")
plt.ylabel("Omega_vac")
plt.show()
```

```
<ipython-input-2-2c511908b9c7>:2: RuntimeWarning: divide by zero encountered in power
  return (6.52/0.68) * (0.7*omega_m + 0.3 - 0.3*omega_vac)**(-0.3) #take H0=68km/s/Mpc
<ipython-input-2-2c511908b9c7>:2: RuntimeWarning: invalid value encountered in power
  return (6.52/0.68) * (0.7*omega_m + 0.3 - 0.3*omega_vac)**(-0.3) #take H0=68km/s/Mpc
```



Lines from right to left illustrate lower bounds from White Dwarfs, nuclear cosmochronology, and Globular clusters

In [ ]:

$$\Rightarrow H = \frac{cz - v_{pec}}{r}$$

Thus the error in H is $\boxed{\Delta H = \frac{1}{r}\Delta v_{pec}}$, if we take $\Delta z = \Delta r = 0$

Not given enough to determine distance for 5% accuracy. Need Δv and an estimate for H_0

$$\frac{\Delta H}{H} = \frac{\Delta v_{pec}}{Hr} = 0.05$$

$$\rightarrow \boxed{r = \frac{20\,\Delta v_{pec}}{H_0}}$$