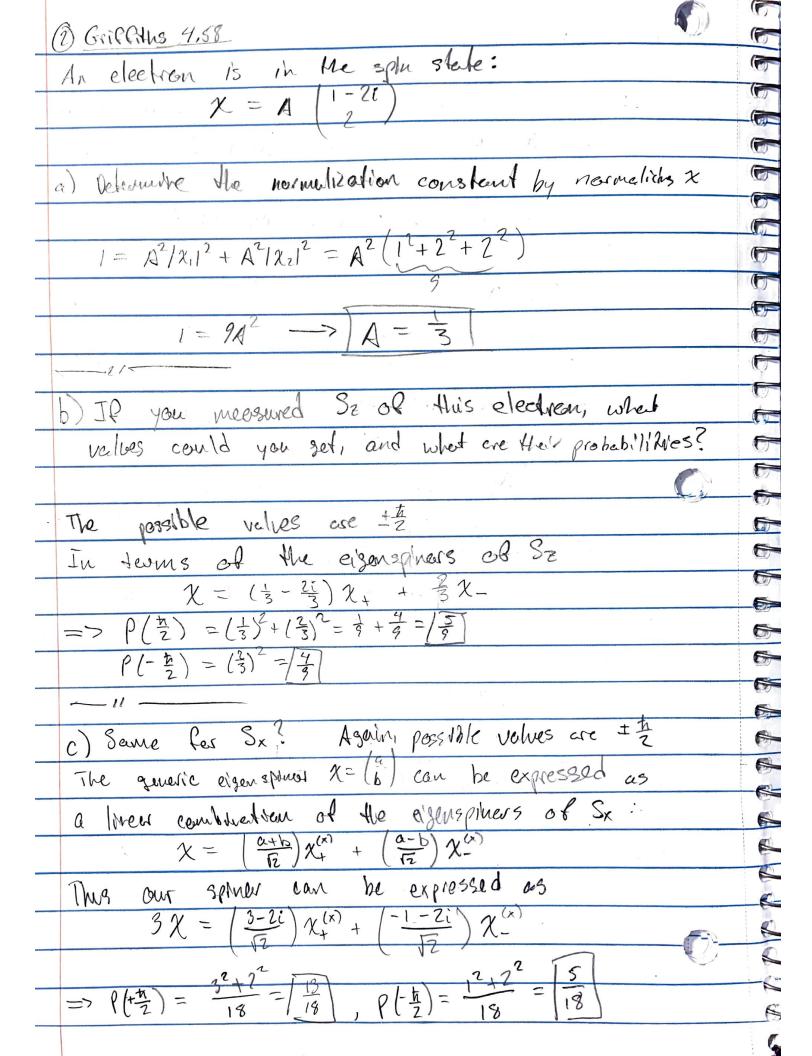
Phys 327 HW4

Trever Melaffrey 2/10/2022

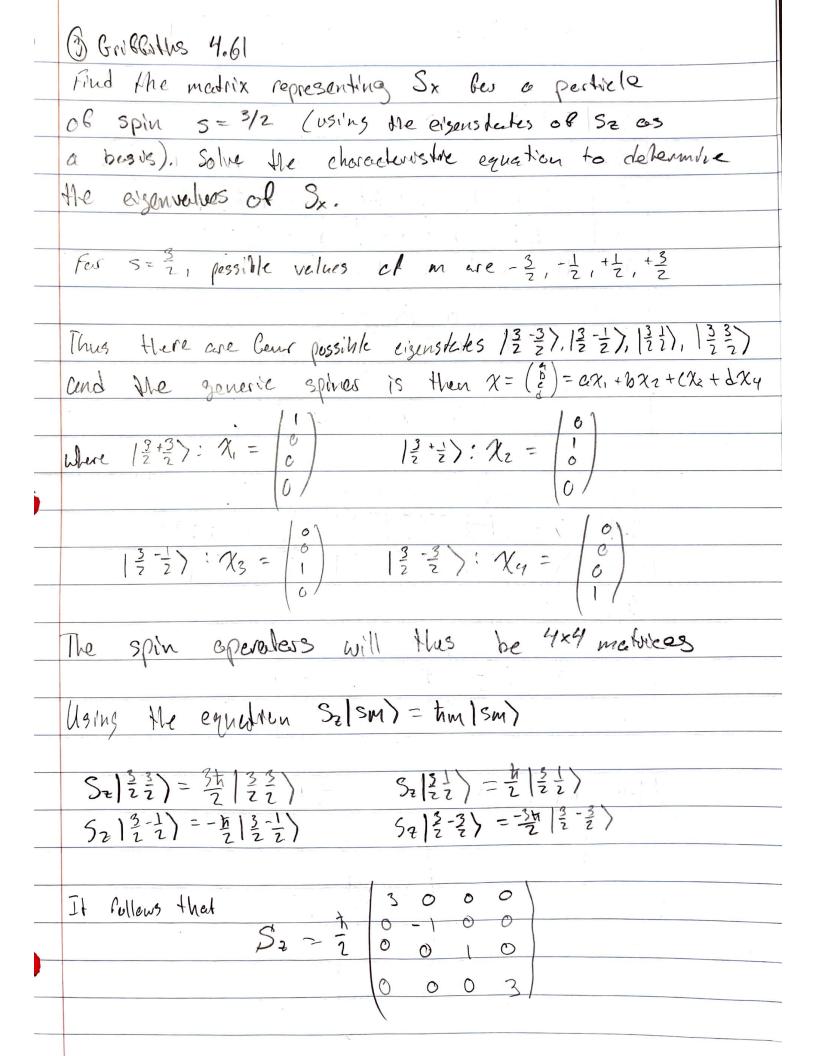
1) GNERILLS 4.37 a) Apply S- 10 (10), and confoun West you get 12/11-1) 110> = 1/2 (111) ; (1-1) = 111)  $S_{-}(10) = \frac{1}{12} (S_{-}^{(1)} + S_{-}^{(2)}) (11) + 111)$ = = (S(1) (1) + S(1) (1) + S(2) (1) + S(2) (1) Using the property S\_X\_=:0 and S\_X+= tix-S- (50) = = (50) (1) (50) (4) + (5(1) (1)) (1) + (1) (5(2) (1)) 1117+0+1117 S\_ 1107 = Et / W> = 511-17 6) Apply S+ to the state 1007 and conform you set o  $|00\rangle = \frac{1}{12}(114) - 141)$ ;  $S_t = S_t^{(1)} + S_t^{(2)}$ S+1007 = = (S(1)+S(2))(111)-111) = = ((S+147)12) + 197 (8+127) - (S+128)117 - 127 (S+147) = 100 + 111> - 111> -0

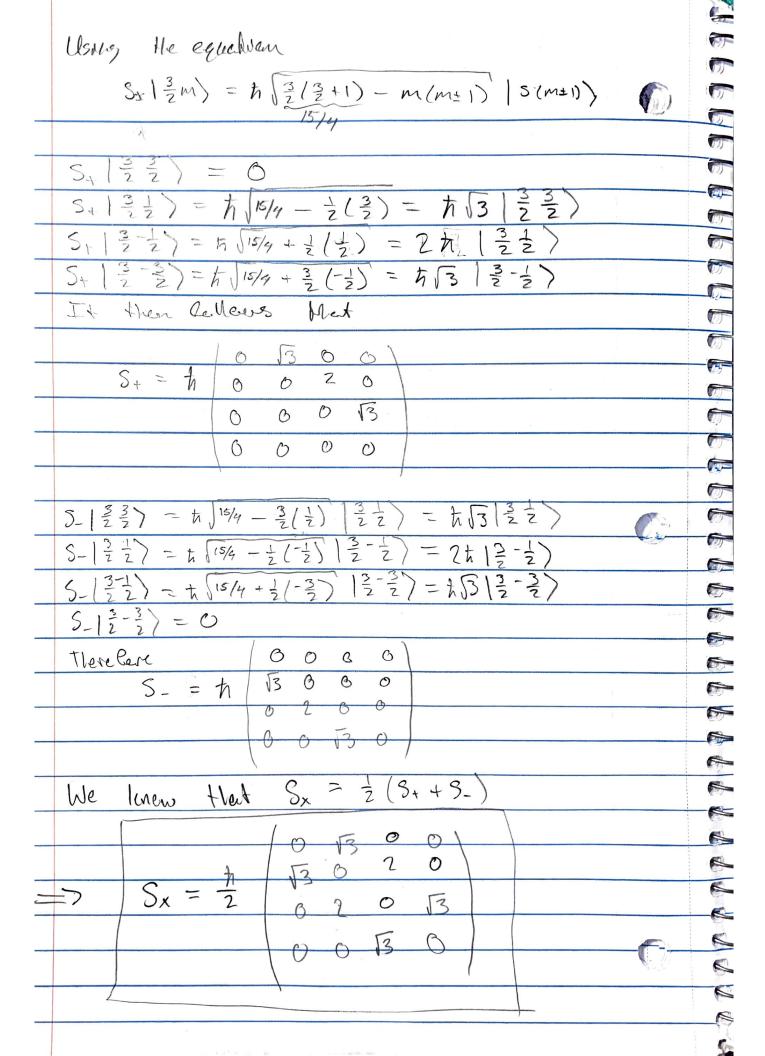
c) Show that 1117 and 11-17 are eigenstates of 5° with the appropriate eigenvalue. 52/11) = 2t2/11); 1177 = 117) We have  $S^2 = S_1^2 + S_2^2 + 2S_1 \cdot S_2$ It College Heat S. = 52 17 1) = (Sx, 19) (Sx2 17) + (Sy, 17) (Sy2 17) + (Sz, 17) (Sz2 17) こ(を1か)(き1か) + (き1か)(き1な)+(を1か)(き1か) - #2/11) + #2/11) - #2/11) = \$2/11) We now have the equation [since eigenvalues of 52 52 = 3 t2  $S^{2}|11\rangle = \left(\frac{3t^{2}}{4} + \frac{3t^{2}}{4} + 2\frac{t^{2}}{4}\right)|11\rangle$ = 2t2/11) => 2t2 eigennelie for 52 in eigenstate 1117 ii\ s2/1-1> = 2t2/11-17 This is He some process as before: らいらいり = ないり + ない - ない = ない) And we have the same solution. 52/1-12 = (3t2 + 3t2 + 2t2) 11-17 = 2k2/1-17 => 2+2 is also on eigenverthe Per eigenstate 11-1).



| d) What about So?   |
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| I la mand A de la manda de la |
| De need to knew the generic ferm of an eigenspiner  |
| $\chi = \binom{n}{b}$ in terms of the eigenspires of Su   |
|   |
| First determine the characteristive equection:  |
| $\frac{ -\rangle -i\hbar/2}{ i\hbar/2 -2} = 0 \longrightarrow \lambda^2 - \frac{\hbar^2/4}{4} \longrightarrow \lambda = \pm \frac{\hbar}{2}$  |
| 11/2 -2   |
| Again, ne can mausare ± 1/2 les Sy.   |
|   |
| An eisenvecter v will follow: Syv = 2v  |
| 1/1 0 -1) / V, ) = # / V, )   |
| $\frac{1}{2} \left( \begin{array}{c} 0 & -1 \\ 1 & 0 \end{array} \right) \left( \begin{array}{c} V_1 \\ V_2 \end{array} \right) = \frac{1}{2} \left( \begin{array}{c} V_1 \\ V_2 \end{array} \right)$   |
|   |
| $-iv_2 = v_1  \text{and}  iv_1 = v_2$   |
| $-iv_{2} = v_{1}  \text{and}  iv_{1} = v_{2}$ $= 7  \chi_{+}^{(9)} =  1/\overline{z}   \sqrt{ 2+ ^{2}} = \sqrt{z}$  |
| i/√z/   |
|   |
| The other exerceter:  |
|   |
| $\frac{i\hbar}{2}\begin{pmatrix}0&-1\\1&0\end{pmatrix}\begin{pmatrix}V_1\\V_2\end{pmatrix}=-\frac{i\hbar}{2}\begin{pmatrix}V_1\\V_2\end{pmatrix}$   |
|   |
| $-> -i V_2 = -V_1 \qquad i V_1 = -V_2$  |
| $= v_1 = iv_2 \qquad = v_2 = -iv_1$   |
| <br>$\chi_{-3}^{(5)} = \begin{pmatrix} 1 \\ -i \end{pmatrix} \implies \chi_{-}^{(5)} = \begin{pmatrix} 1/\sqrt{2} \\ -i/\sqrt{2} \end{pmatrix}$   |
| $\lambda_{-} = (-i) \Rightarrow \lambda_{-} = (-i)\sqrt{52}$  |
|   |

We now have the engineprinary of Sy:  $\chi_{+}^{(6)} = \begin{pmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix}$  and  $\chi_{-}^{(9)} = \begin{pmatrix} 1/\sqrt{2} \\ -i/\sqrt{2} \end{pmatrix}$ The general spokes  $\chi = \binom{a}{b}$  can be represented as a linear commention of each of Hem:  $\frac{a}{b} = A\left(\frac{1}{\sqrt{2}}\right) + B\left(-\frac{1}{\sqrt{2}}\right)$  $a = \frac{1}{12}(A+B)$   $b = \frac{1}{12}(A-B)$   $-ib = \frac{1}{12}(A-B)$ -> a-ib = \( \overline{\gamma} (2A) = \overline{\gamma} A = \overline{\sigma} \overline{\gamma} = \overline{\gamma} = \overline{\gamma} \overline{\gamma} = \overline{\gamma} \overline{\gamma} = \overline{ a+ib = 1/2 (2B) = 1/2 B => B = arib This  $\chi = \left(\frac{a-ib}{\sqrt{2}}\right)\chi_{+}^{(v)} + \left(\frac{a+ib}{\sqrt{2}}\right)\chi_{-}^{(v)}$ Ow spires 3 (1-2i) can be express as  $3\chi = \left(\frac{1-4i}{2}\right)\chi_{+}^{(9)} + \left(\frac{1}{2}\right)\chi_{-}^{(9)}$ 





| The usual characterostre equation is det(sx-21)=0   |
|---|
| $S_{x} - \lambda I = \frac{1}{2} \left[ \begin{array}{cccc} -\lambda & \sqrt{3} & 0 & 0 \\ \sqrt{5} & -\lambda & 2 & 0 \end{array} \right]$ |
| 2 ο 2 - λ [3]   |
| 0073-2  |
| Aleng Rist column:  |
| $ S_{x}-\lambda I  = -\lambda  2 - \lambda  3 - 3    3 - 3    3    3    3    3$   |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$   |
| $D_1 = -\lambda (\lambda^2 - 3) - 2(-2\lambda) = -\lambda^3 + 7\lambda$<br>$D_2 = \sqrt{3}(\lambda^2 - 3) = \sqrt{3}\lambda^2 - \sqrt{3}3$  |
| <br>$-5 S_{x}-\lambda I =\lambda^{4}-7\lambda^{2}-3\lambda^{2}+9=\lambda^{4}-10\lambda^{2}+9=0$   |
| $\Rightarrow (\lambda^2 - 9)(\lambda^2 - 1) = 0$  |
| <br>Thus the eigenvalues of Sx are  |
| $\int_{0}^{\pi} A = \pm \frac{3\pi}{2} \text{ and } A = \pm \frac{\pi}{2}$  |
|   |