(10) Show that $H = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : 2x_1 - 3x_2 + 5x_3 = 0 \right\}$
is a subspace of $\mathbb{R}^3$

Clearly $H$ is a subset of $\mathbb{R}^3$. If it is a vector space — closed under addition and multiplication — then it is also a subspace.

1) Two vectors $v_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $v_2 = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$
in $H$

$$V_1 + V_2 = \begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \\ x_3 + y_3 \end{bmatrix}$$

$$2(x_1 + y_1) - 3(x_2 + y_2) + 5(x_3 + y_3) \overset{?}{=} 0$$

$$\underbrace{(2x_1 - 3x_2 + 5x_3)}_{=0} + \underbrace{(2y_1 - 3y_2 + 5y_3)}_{=0} = 0$$

→ Closed under addition

$$CV_1 = \begin{bmatrix} cx_1 \\ cx_2 \\ cx_3 \end{bmatrix}$$

$$2(cx_1) - 3(cx_2) + 5(cx_3) \overset{?}{=} 0$$

$$c\underbrace{[2x_1 - 3x_2 + 5x_3]}_{=0} = 0$$

→ Closed under scalar mult. ⟹ H is a subspace of $\mathbb{R}^3$