HMWK1: DUE VIA EMAIL Jan 13, PDF file
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Subject: Math-323-hmw1-FirstName-LastName
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1. ODE Review Problems (although we did not review these basic problems in class, they should be pretty easy review)
   (a) Find the general solution to $y' = -4y$ and then solve the initial value problem with $y(0) = 2$.
   (b) Find the general solution to $y' = -4y + 1$ and then solve the initial value problem with $y(0) = 2$.
   (c) Find the general solution to $y'' = 9y$ and then solve the initial value problem with $y(0) = 1$, $y'(0) = 0$.
   (d) Find the general solution to $y'' = -9y$ and then solve the initial value problem with $y(0) = 1$, $y'(0) = 0$.

2. Partial Derivatives Review Problems
   (a) For the function $f(x, y) = x^2e^y - y\sin(x + y)$, compute f_x, f_y, f_{xx}, f_{xy}, f_{yx}, and f_{yy}.
   (b) In part (a), you should have obtained $f_{xy} = f_{yx}$. In all of the examples from Math 323, mixed partials will always be equal, but this does not hold in general.

3. Differential equation system: Consider the system of equations :
$$u_1'(t) = 11u_1(t) - 30u_2(t), \quad u_2'(t) = 4u_1(t) - 11u_2(t)$$
   with initial conditions $u_1(0) = 11,\ u_2(0) = 4$.

   Solve this system as described in class: rewrite it as a matrix differential system $(\vec{u})' = \vec{A(u)}$ for the appropriate two-by-two matrix; find the eigenvectors and eigenvalues for A; and use the eigenvector-eigenvalue calculation to write down solutions to matrix differential system; use the initial conditions to find the particular solution of interest.

4. Differential equation system 2: Consider the system of equations:
$$u_1'(t) = -7u_1(t) - 24u_2(t), \quad u_2'(t) = -24u_1(t) + 7u_2(t)$$
   with initial conditions $u_1(0) = 17,\ u_2(0) = -6$.

   Solve as in the previous problem. Note that the matrix is symmetric, so you can find an orthonormal basis of eigenvectors for A; that it makes it easier to find the unknown constants c_1 and c_2.

5. By substituting into the equation, determine whether the these functions are solutions of the indicated PDEs.
   (a) Is $v(x, t) = x + t$ a solution to $v_t + v_x = 0$?
   (b) Is $v(x, t) = x + t$ a solution to $v_t - v_x = 0$?
   (c) Is $v(x, t) = \sin(x + t) + \cos(x - t)$ a solution to $v_{tt} - v_{xx} = 0$?
   (d) Is $u(x, y) = x^2 - y^2$ a solution to $u_{xx} + u_{yy} = 0$?

6. In the first lecture, I introduce the basic problem of solving the heat equation using separation of variables, and end up with a solution which is a sum of an infinite number of solutions coming from the separation of variable. We did this for:
$$u_t = u_{xx},\ 0 \le x \le 1$$
   Redo the solution from the notes , this time with the slight modification
$$u_t = \alpha^2 u_{xx},\ 0 \le x \le L$$
   L is the new length of the interval, α^2 is a positive constant which determines how fast heat travels along the bar.