Math 387 HW2

Trevor McCaffrey
1/13/2022

Exercise 1.1

In this exercise we are working in the field $\mathbb{Z}_5$

i) Compute $\begin{pmatrix} 1 & 2 & 0 \\ 1 & 3 & 4 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 2 & 1 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 3 & 3 \end{pmatrix}$

ii) Find $A^{-1} = \begin{pmatrix} 2 & 2 & 0 \\ 1 & 4 & 3 \\ 1 & 0 & 0 \end{pmatrix}^{-1}$

$$A^{-1} = \frac{[A_{ij}]^T}{|A|}$$

$$A_{ij}^T = \begin{pmatrix} 2(0)-3(0) & -[0] & 2(3)-0(4) \\ -[1(0)-3(1)] & 0 & -[2(3)-0(1)] \\ 1(0)-3(1) & -[2(0)-2(1)] & 2(4)-2(1) \end{pmatrix}$$

$$\text{adj}(A) = \begin{pmatrix} 0 & 0 & 1 \\ 3 & 0 & 4 \\ 2 & 2 & 1 \end{pmatrix}$$

$$\det(A) = 2(3)-0(4) = 1$$

$$\rightarrow A^{-1} = \begin{pmatrix} 2 & 2 & 0 \\ 1 & 4 & 3 \\ 1 & 0 & 0 \end{pmatrix}$$