Phys 432 HW4

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① Distances to Surface of Last Scattering

a) What is the proper distance?

Note that the proper distance can be computed as:

$$d_p = d_{horizon,now} - d_{horizon\ at\ emission}$$

$$d_{horizon,now} = 3.2\frac{c}{H_0} = 3.2\left(\frac{3\times10^5\,km/s}{70\,km/s/Mpc}\right) = 13714\ Mpc$$

At large z, the horizon distance approaches

$$r_H \approx \frac{6000}{\sqrt{\Omega z}} h^{-1} \text{ Mpc}$$

At last scattering:

$$r_H \approx \frac{6000}{\sqrt{1090}} (.68)^{-1} \text{ Mpc} = 267 \text{ Mpc}$$

$$\Rightarrow d_p = 13714 \text{ Mpc} - 267 \text{ Mpc} = \boxed{13447 \text{ Mpc}}$$

b) What is its luminosity distance?

$$d_L = d_p(1+z) = 13447_{Mpc}(1+1090) \approx \boxed{1.5\times10^7\ Mpc}$$

c) What is the angular diameter distance?

$$d_A = \frac{d_p}{1+z} = \frac{13447\,Mpc}{1+1090} = \boxed{12.3\ Mpc}$$

2) He made in Milky Way Stars

The total luminosity of stars in our Galaxy is $L \approx 3\times10^{10} L_\odot$. Suppose that the luminosity of our Galaxy has been constant for the past ≈ 10 Gyr

a) How much energy (in Joules) has our galaxy emitted in the form of starlight during that time? Note that $L_\odot = 3.828\times10^{26}$ J/s

$$E = Lt = 3\times10^{10} L_\odot \left(\frac{3.828\times10^{26}\ J/s}{L_\odot}\right) 10\,Gyr \left(10^9\ yr/Gyr\right)\left(3.15\times10^7\ s/yr\right)$$

$$\Longrightarrow \boxed{E \approx 3.6\times10^{54}\ J}$$

b) Assume that fusion of H → He is the only fusion reaction in these stars. If 28.4 MeV are released for every He nucleus formed, how many He nuclei have been created?

$$N_{He} = E_{tot}[J] \Big/ \varepsilon_{He}[J/\text{nucleus}] = \frac{3.6\times10^{54}\,J}{28.4\,MeV/\#}\left(\frac{1\,MeV}{1.6\times10^{-13}\,J}\right)$$

$$\Longrightarrow \boxed{N_{He} \approx 8\times10^{65}\text{ He nuclei}}$$

c) If the baryonic mass of our Galaxy is $M \approx 10^{11} M_\odot$, by what amount has the Helium fraction Y of our Galaxy increased over the primordial value $Y = 0.24$?

One helium nucleus weighs $m \approx 4(1.67\times10^{-27}\,kg) = 6.64\times10^{-27}\,kg$

$$\rightarrow m_{He} = 3.32\times10^{-57} M_\odot$$

Thus the total mass of all Helium in our galaxy is

$$M_{He} = N_{He}\, m_{He} = 8\times10^{65}\,(3.32\times10^{-57} M_\odot) \approx 2.6\times10^{9} M_\odot$$

$$\Longrightarrow Y_+ = \frac{2.6\times10^{9} M_\odot}{10^{11} M_\odot} = .026$$

$$* \Longrightarrow \boxed{Y_{now} = Y + .026} \qquad (Y = 0.24)$$

③ Future of the Accelerating Universe

What will happen if the present-day acceleration of the universe is caused by a false vacuum that will eventually decay? We currently observe $\varepsilon_\Lambda = 0.69\varepsilon_{crit} = 3360\ MeV/m^3$, $\varepsilon_m = 0.31\varepsilon_{crit} = 1510\ MeV/m^3$.

a) What was the scale factor of the universe $a_{m\Lambda}$ in the past when the vacuum energy density became equivalent to matter density? The redshift?

$$\varepsilon_m = \varepsilon_{m,0}\, a^{-3}$$
$$\varepsilon_\Lambda = \varepsilon_{\Lambda,0} \quad \rightarrow \quad 1 = \frac{\varepsilon_{\Lambda,0}}{\varepsilon_{m,0}} a^3 \rightarrow a = \left(\frac{\varepsilon_{m,0}}{\varepsilon_{\Lambda,0}}\right)^{1/3} = \left(\frac{.31}{.69}\right)^{1/3}$$

$$\rightarrow \boxed{a_{m\Lambda} = 0.766} \quad \xrightarrow{a = \frac{1}{1+z}} \quad z = \frac{1}{.766} - 1 \Rightarrow \boxed{z = 0.306}$$

b) What will the Hubble parameter be in the future when Λ is strongly dominant and $a \gg a_{m\Lambda}$?

In the epoch when $a \gg a_{m\Lambda}$:

$$a(t) = a_{m\Lambda} \exp\left(\sqrt{1-\Omega_{m,0}}\, H_0 t\right)$$

$$H = \frac{\dot{a}}{a} = \frac{\sqrt{1-\Omega_{m,0}}\, H_0\, \cancel{a_{m\Lambda} e^{\sqrt{1-\Omega_{m,0}} H_0 t}}}{\cancel{a_{m\Lambda} e^{\sqrt{1-\Omega_{m,0}} H_0 t}}}$$

$$H = \sqrt{1-0.31}\ 70\ km/s/Mpc$$

$$\rightarrow \boxed{H = 58\ km/s/Mpc}$$

c) What is the Hubble time at that same epoch?

$$t = H^{-1} = \frac{1}{58 km/s/mpc} = \frac{1\ s\ Mpc}{58 km}\left(\frac{3.086\times10^{13} km}{10^{-6} mpc}\right)$$

$$t = 5.32\times10^{17} s\left(\frac{1\ Gyr}{3.15\times10^{16} s}\right) = \boxed{16.9\ Gyr}$$

d) If Λ is caused by a false vacuum that decays at $t_f = 50 t_0$, what is the scale factor in Gyr?

$$a(t) = a_{m\Lambda}\, e^{\sqrt{1-\Omega_{m,0}}\, H_0 (50)\, 0.955 H_0^{-1}}$$

$$a(t) = 0.766\, e^{(0.69)(50)(0.955)} = 1.56\times10^{14}$$

$$\boxed{a(670\ Gyr) = 1.6\times10^{14}}$$

e) If the vacuum energy instantaneously decays into blackbody radiation, what will be its temperature?

When Λ is strongly dominant $\varepsilon_\Lambda \approx \varepsilon_{c,r,t}$

$$\varepsilon_c(670\ Gyr) = \frac{3(3\times10^{8} m/s)^2}{8\pi(6.67\times10^{-11} m^3/kg s^2)} H^2$$

$58 km/s/mpc$
$= 1.88\times10^{-18} s^{-1}$

$$= 1.6\times10^{26} kg/m\left(1.88\times10^{-18} s^{-1}\right)^2 =$$

$$5.69\times10^{-10} J/m^3 \times \left(\frac{MeV}{1.6\times10^{-13} J}\right) = 3556\ MeV/m^3$$

Thus when Λ is converted to radiation $\varepsilon_r = 3556\ \text{meV/m}^3$

If it is all in the form of blackbody radiation

$$\varepsilon_r = \alpha T^4 \longrightarrow T = \left(\frac{5.7\times10^{-10}\,\text{J/m}^3}{7.6\times10^{-16}\,\text{J/m}^3/\text{K}^4}\right)^{1/4}$$

$$\Longrightarrow \boxed{T_f = 29.4\,\text{K}}$$

f) At that time, what will the matter energy density have reduced to? So, after decay of vacuum into radiation, what component of mass-energy would dominate the universe?

At this time, the universe consists nearly entirely of radiation and $\varepsilon_m = \varepsilon_{m,0}\,a^{-3} = \frac{1510\ \text{MeV}}{(1.6\times10^{14})^3} \approx \boxed{3.7\times10^{-40}\ \text{MeV/m}^3}$

g) Moving ahead from t_f, at what scale factor will matter again dominate? What will the radiation temperature be at that time?

Radiation will scale as $\propto a^{-4}$ and matter $\propto a^{-3}$

$$\Longrightarrow \frac{\varepsilon_r}{\varepsilon_m} = \frac{\varepsilon_{r,f}\,a^{-4}}{\varepsilon_{m,f}\,a^{-3}} \quad ; \text{RM equality at} \quad 1 = \frac{\varepsilon_{m,f}}{\varepsilon_{r,f}}\,a$$

$$\longrightarrow a = \left(\frac{3556\ \text{meV/m}^3}{3.7\times10^{-40}\ \text{MeV/m}^3}\right)$$

$$\longrightarrow \boxed{a_{rm} = 9.6\times10^{42}}$$

The radiation density is now $\frac{1}{2}\varepsilon_{r,f}$

$$\rightarrow \varepsilon_r \approx \frac{1}{2}\varepsilon_{CMB,0} = \frac{1}{2}\left(5.7\times10^{-10}\,J/m^3\right) = 2.3\times10^{-10}\,J/m^3$$

$$\rightarrow T = \left(\frac{2.3\times10^{-10}\,J/m^3}{7.6\times10^{-16}\,J/m^3/K^4}\right)^{1/4}$$

$$\boxed{T_{fin2} = 23.5\,K}$$