

Exercise 1.3

Let $F = \mathbb{Z}_5$. Determine whether b is a linear combination of a_1, a_2 , where

$$\vec{a}_1 = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}, \quad \vec{a}_2 = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix}$$

$$\mathbb{Z}_5: \left(\begin{array}{cc|c} 1 & 2 & 4 \\ 3 & 4 & 4 \\ 1 & 1 & 0 \end{array} \right) \sim \left(\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & 3 & 2 \\ 0 & 4 & 1 \end{array} \right)$$

$$\sim \left(\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & 3 & 2 \\ 0 & 0 & 0 \end{array} \right) \sim \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 3 & 2 \\ 0 & 0 & 0 \end{array} \right)$$

$$\sim \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 3 \cdot 3^{-1} & 2 \cdot 3^{-1} \\ 0 & 0 & 0 \end{array} \right) \xrightarrow{3^{-1}=2} \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{array} \right)$$

$$\rightarrow \boxed{\vec{b} = \vec{a}_1 + 4\vec{a}_2}$$

$$= \begin{pmatrix} 1 + 4(2) \\ 3 + 4(4) \\ 1 + 4(1) \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} \quad \checkmark$$