⑧ find the characteristic polynomial and the eigenvalues of the matrix $\begin{pmatrix} 3 & 0 & -3 \\ 1 & 2 & 5 \\ 3 & 0 & 3 \end{pmatrix}$

$\det(A - \lambda I) = 0$

$$\det \begin{pmatrix} 3-\lambda & 0 & -3 \\ 1 & 2-\lambda & 5 \\ 3 & 0 & 3-\lambda \end{pmatrix}$$

$$= a_{22} c_{22} = (2-\lambda)(-1)^{2+2} \begin{vmatrix} 3-\lambda & -3 \\ 3 & 3-\lambda \end{vmatrix}$$

$$0 = (2-\lambda)\left[(3-\lambda)^2 + 9\right] = (2-\lambda)\left[9 + \lambda^2 - 6\lambda + 9\right]$$

Char. Polynomial: $\boxed{0 = (2-\lambda)(\lambda^2 - 6\lambda + 18)}$

$$\lambda = \frac{6 \pm \sqrt{36 - 4(18)}}{2} = 3 \pm \frac{i\sqrt{36}}{2} = 3 \pm 3i$$

Eigenvalues: $\boxed{\lambda = 2, \ \lambda = 3 \pm 3i}$