PHYSICS 432/532: COSMOLOGY
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Lecture Notes #2: Age and Distances Scales

Age and distance scales: why do we think the universe is as old and large as we believe?

1 Age of the universe

To lowest order, we guess that the age of the universe must be something like the inverse of the Hubble constant, which measures the current expansion rate. Recall that observers measure H_0 in units of $\mathrm{km\,s^{-1}Mpc^{-1}}$. Converting to normal units, the "Hubble time"

$$t_H = H_0^{-1} = 9.78h^{-1}\,\mathrm{Gyr}$$

where we've used $h \equiv H_0/100\,\mathrm{km\,s^{-1}Mpc^{-1}}$. For $h = 0.68$ (as quoted in Ryden, close to the Planck satellite measurement of 0.67 - see below), $t_H = 14.4$ Gyr.

Note carefully, that is NOT the age of the universe in most cosmological models. That age would be exact only if the expansion rate had always been the same. This is clear from recalling that $v = Hd$ and $d = vt$, thus $t = 1/H$.

A related quantity, the "Hubble distance," is defined by

$$d_H = \frac{c}{H_0} = 4380Mpc$$

which is the distance that light can travel in a Hubble time.

In general, the expansion could be slowing down (decelerating if $q_0 > 0$) or speeding up ($q_0 < 0$), implying that the expansion was faster or slower in the past, respectively. The exact result (neglecting radiation) involves solution of a complicated integral. A very useful approximation (good for most cosmologies that we'll consider) is

$$t_0 = \frac{2}{3}t_H(0.7\Omega_{matter} + 0.3 - 0.3\Omega_{vac})^{-0.3}$$

Note that for $\Omega_{matter} = 1$, $\Omega_{vac} = 0$, this is simply $t_0 = 2t_H/3$. $\Omega_{vac} > 0$ makes the universe older, as does $\Omega_{matter} < 1$.

For a "benchmark model" of $\Omega_{matter} = 0.3, \Omega_{vac} = 0.7$ (I'm rounding off Ryden's benchmark of $\Omega_{matter} \approx 0.31, \Omega_{vac} \approx 0.69$), $t_0 = 0.957t_H = 13.7$ Gyr, just a bit smaller than the "Hubble time."

Thus, given H_0, Ω_{matter}, and Ω_{vac}, we can predict the age of the universe. We'll get to measuring the densities later. For now, how do we measure the expansion rate H_0? Recalling $v = Hd$, we have to measure both recession velocity (really, the redshift) and distance for some set of objects.

And does this cosmologically inferred age of the universe make any sense when compared to other constraints on the age of the universe?

Let's follow the roughly historical order of things to see why we believe the universe is as old as claimed.

Methods of Age Determination

We can set a minimum age on the universe by several methods:

- Nuclear cosmochronology
- Stellar evolution
- White dwarf cooling

Nuclear cosmochronology

We can set a lower bound on the age by dating our own solar system using radioactive element abundances. Good candidates must have long half lives, including the following decays:
$^{232}\mathrm{Th} \rightarrow\ ^{208}\mathrm{Pb}$ 20.27 Gy
$^{238}\mathrm{Th} \rightarrow\ ^{206}\mathrm{Pb}$ 6.45 Gy
$^{235}\mathrm{U} \rightarrow\ ^{207}\mathrm{Pb}$ 1.02 Gy
Applying this dating method to meteorites gives an extremely precise age for the Solar system: $t = 4.57$ Gy. Rocks on Earth are only somewhat younger, $t = 3.7$ Gy.

Next, we can estimate how old the elements were when the Solar system formed, by looking at isotopic ratios of, for example, $^{235}\mathrm{U}/^{238}\mathrm{U}$ or $^{232}\mathrm{Th}/^{238}\mathrm{Th}$. These ratios tell us the abundances in the pre-Solar material. These heavy elements were produced in a previous generation of massive stars, that exploded into supernovae type II, which create heavy nuclei via the "r process" (r for rapid) due to the large flux of neutrons. Best analysis of these data lead to a pre-Solar Milky Way lifetime of $t = 5.4$ Gy. Added to the Solar system age, the Milky Way must be at least $t = 4.6 + 5.4 = 10.0$ Gy old.

Stellar Evolution

Another lower bound on the age is the age of the oldest stars in the Galaxy. Here, we use our knowledge of the theory of stellar evolution to infer the ages of the stars. In detail, we look at globular clusters, systems of roughly 10^5 stars that have low abundances of metals. These low metal abundances imply that these stars formed early, before the first generation of massive stars filled the interstellar medium with heavier nuclei.

[DRAW color-magnitude diagram as luminosity vs. temperature.]

All the stars in a cluster formed at nearly the same time (relative to the age of the universe), but with different initial masses. Initially, the stars only lie along the **main sequence**, differentiated by more massive stars being hotter and brighter, where they fuse hydrogen into helium in their cores. When all the hydrogen in the core is fused to helium, it starts to fuse hydrogen in a shell around the He core. The stars then leave the main sequence and enter the **red giant branch**. Then they ascend the RGB, drop down on the **horizontal branch**, later ascend the *asymptotic giant branch*. Most stars end as **white dwarfs**, supported against gravity only by electron degeneracy pressure in their cores.

Massive stars burn the H in their cores first, so the high mass end of the MS gets depopulated over time. The location on the MS where stars are just leaving is called the **turnoff point**. We can estimate the age of a cluster of stars by computing theoretical models for what this luminosity-temperature diagram should look like as the stars age and populate different parts of the diagram. The main diagnostic is the location of the turnoff point. The technique is called "isochrone fitting" where we compare the observed color-magnitude diagram with a family of models that predict this diagram for the cluster at different times (isochrone = same time).

Best estimates of the age of globular clusters are in the range $12 - 15$ Gy. But the theory is complicated (particularly the effects of convection) and there may be systematic effects. A reliable lower bound on the age of the universe (at the 95% confidence level) is around $t = 12$ Gy.

White Dwarf Cooling

I mentioned that most stars end their lives as white dwarfs, with degenerate cores surrounded by a non-degenerate shell. These dead stars cool with time. Comparison of observations of WD's with models for the cooling rate of WD's yields an age for the disk of our Galaxy of roughly $t = 9.3$ Gy.

To summarize, these methods yield lower bounds of $t = 10.0, 12, 9$ Gy. The universe must be old enough to allow the formation of a collapsed galaxy, an hospitable environment for stars to form, before this time.

2 The Cosmological Distance Ladder

Now back to our dynamical means of estimating the age of the universe, which requires that we measure the expansion rate. Again, to measure the expansion rate H_0, we have to measure two things: the distance to objects and their recession velocity.

Rungs in the nearby distance ladder:

- nearby stars: trigonometric parallax (just geometry)
- nearby star clusters (Hyades): angular motions
- Large Magellanic cloud: Cepheid variables, SN 1987a
- LMC to M31: Cepheid variables
- M31 to Virgo cluster: Cepheids, SBF, PN
- Virgo to Coma: Fundamental plane of galaxies ($D_n - \sigma$)

Those methods, in addition to newer methods such as Tip of the Red Giant Branch (TRGB) are used to calibrate the distance scale to galaxies. Then other methods such as supernovae type Ia (SNeIa) are used as standard candles to extend the distance scale to cosmological distances. Another recently used method to get cosmological distances is gravitational lensing time delays (more about this later).

To estimate distances to nearby stars, we simply use geometry: observe the position on the sky of a star at different times of year. We know the size of Earth's orbit, so we know the distance to the star. This works out to a few kiloparsecs if we use the GAIA satellite (for reference, recall that the Sun is roughly 8 kpc from the center of the Galaxy).

Next step is to use variable stars whose intrinsic brightness correlates with their period of variability. These stars, called **Cepheids** and their cousins **RR Lyrae** can be observed in other nearby galaxies. Cepheids are quite massive stars, thus very bright, which allows them to be seen at great distance; they are one of a few types of stars that can be seen individually in other galaxies. During a particular stage of their evolution, these stars enter a phase where they oscillate in brightness, so its easy to pick them out. By measuring the period of oscillation, we can infer their intrinsic brightness and thus measure their distance by comparing to their observed brightness.

First stop is the LMC (50 kpc), a small (but not the smallest) irregular galaxy that orbits our the Milky Way, then out to M31 ($D_{M31}/D_{LMC} = 15.3$), the nearest large galaxy. Using HST, we can observe Cepheids in galaxies in the Virgo cluster ($D_{Virgo}/D_{M31} = 20.65$).

Combining all those measurements, we get a distance to the Virgo cluster of roughly $D_{Virgo} = 16$ Mpc. The recession velocity of the Virgo cluster is about $v = 1186\,\mathrm{km\,s^{-1}}$, which yields an estimate of the Hubble constant

$$H_0 = 74\,\mathrm{km\,s^{-1}Mpc^{-1}} \pm 8\%$$

This estimate may be affected by motions in the local universe. For example, if our local region of the universe is less dense than the average universe, we might live in a "bubble" that expands more quickly than average, thus have larger H_0 than the cosmic mean. To be precise, we should make our estimates of the Hubble constant over a larger volume, one that is large enough for the universe to look nearly homogeneous.

Big caveat: There is a large range of published values of H_0, from low values of 39 to high values approaching 100. Modern estimates are typically in a range around $65 \pm 15\,\mathrm{km\,s^{-1}Mpc^{-1}}$ or so. My colleagues and I used a median statistics analysis to infer that $H_0 = 67 \pm 2\,\mathrm{km\,s^{-1}Mpc^{-1}}$.

The Planck satellite CMB anisotropy experiment finds a best-fit value of the Hubble constant of $H_0 = 67.4 \pm 0.5\,\mathrm{km\,s^{-1}Mpc^{-1}}$. Note that this measurement depends on assuming the ΛCDM cosmology for the fitting.

3 Supernovae and the Accelerating Universe

To extend distance measurements to cosmological distances, one can construct the SNe Ia Hubble diagram. The idea is simple: SNe Ia are nearly standard candles, which means that their intrinsic brightness is nearly uniform. They're not exactly uniform, but there is a strong correlation between their peak brightness and the timescale of their light curve, so the variation can be accounted for.

SNe Ia are nearly standard candles because they all result from gravitational collapse of white dwarf stars when accretion of extra mass causes them to exceed the maximum mass ("Chandrashekar mass" of $\approx 1.4 M_{odot}$) that can be supported by electron degeneracy pressure.

[Draw light curve of SNe Ia. Rise time of few days, decay over couple of weeks.]

We can find SNe Ia out to very large distances, currently out to a maximum of $z = 1$ and beyond. We find SN by looking at the same distant galaxies night after night, looking for the appearance of a bright spot that wasn't there before. We then follow up by getting a spectrum of the object, which tells us whether or not it's a SN, as well as the recession velocity. We observe the SN over many nights to accurately calibrate its distance. That distance and the velocity yield a H_0 estimate for each SN.

This allows us to do two important things: (1) measure the Hubble constant over a volume much larger than fluctuations caused by galaxies, clusters of galaxies, or even superclusters of galaxies, (2) measure the change in the Hubble "constant" with redshift, thus measuring the deceleration parameter q_0.

For nearby $z < 0.1$ SNe Ia, we find an almost perfectly linear Hubble flow (meaning no large deviations of H_0 at different distances), with a best fit of

$$H_0 = 73 \pm 1.4\,\mathrm{km\,s^{-1}Mpc^{-1}}$$

from the SHOES collaboration (Riess et al. 2021). This measurement uses Hubble Space Telescope observations of Cepheids and GAIA satellite parallax measurements to calibrate—it's not independent of the distance ladder.

[Draw Hubble diagram figure from Reiss et al.]

When we compare these nearby SN to more distant SN, we find that the distant SN appear fainter than we expect if the universe has only matter in it, in which case the universe should be decelerating. This result could be explained if the universe is actually accelerating with time. Think about it: compared to a constant expansion rate or deceleration, acceleration makes the SN further away and fainter.

Caveat: there could be systematic effects that make the distant SN appear fainter: evolution, dust in the universe. But astronomers have tested for all these affects and so far the result stands up. The cosmological model that best explains the data is roughly $\Omega_{matter} \sim 0.3, \Omega_{vac} \sim 0.7$.

A nice feature of this model is that it makes a testable prediction: at redshifts beyond $z = 1$, the apparent brightness difference (between this model and others) should decrease. This would be hard to explain (certainly contrived) by systematic effects. Enough supernovae have been observed at redshift $z > 1$ to confirm that the brightness of this SN does indeed agree with the prediction of the $\Lambda > 0$ model.

4 Summary of Age, Parameter Estimates

Based on nuclear decay and white dwarfs, universe must be at least 10 Gy old. Based on star clusters, lower bound is probably about 12 Gy, perhaps as high as 15 Gy. How does this compare to the age predicted by the FRW cosmology, using our best estimates of the parameters?

Recall

$$t_0 = \frac{2}{3} t_H (0.7\Omega_{matter} + 0.3 - 0.3\Omega_{vac})^{-0.3}$$

Where

$$t_H = H_0^{-1} = 9.78h^{-1}\,\mathrm{Gyr}$$

Above, we got $t_0 = 13.8$ Gy for $h = 0.68$, $\Omega_{matter} = 0.3$, $\Omega_{vac} = 0.7$. This is just enough to allow everything to form!

But look what happens if $\Omega_{matter} = 1, \Omega_{vac} = 0$ for the same Hubble constant. In that case, $t_0 = (2/3)(9.78/0.67) = 9.7$. $h < 0.5$ and a small systematic bias that makes the star clusters appear too old might save this model. But it looks like the preponderance of evidence rules out this model.

A pure matter open universe with $\Omega_{matter} = 0.3$ and $h = 0.68$ yields $t_0 = (2/3)(9.78/0.67)(1.22) = 11.9$, so this model might just squeek in, if we can lower H_0 to $h = 0.6$ and if the star cluster estimates are systematically high. Perhaps still in the running, but just barely.

Thus, the evidence shows that, based only on age considerations, the data favor a model with positive vacuum energy.