

③ Find bases of the column space,

the row space, and the null space of

$$A = \begin{bmatrix} 1 & -1 & -3 & -12 \\ 0 & 4 & 5 & 19 \\ 1 & 3 & 2 & 7 \\ -1 & 1 & 3 & 12 \\ 3 & 8 & 9 & 29 \end{bmatrix}$$

Given that

$$\text{ref}(A) = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{Basis for Col } A: \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ 3 \\ 1 \\ 8 \end{bmatrix}, \begin{bmatrix} -3 \\ 5 \\ 2 \\ 3 \\ 9 \end{bmatrix} \right\}$$

$$\text{Basis for Row } A: \{(1 \ 0 \ 0 \ -2), (0 \ 1 \ 0 \ 1), (0 \ 0 \ 1 \ 3)\}$$

$$\text{Null } A: \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \rightarrow \vec{x} = x_4 \begin{bmatrix} 2 \\ -1 \\ -3 \\ 1 \end{bmatrix}$$

$$\text{Basis is } \left\{ \begin{bmatrix} 2 \\ -1 \\ -3 \\ 1 \end{bmatrix} \right\}$$