Math 387, Winter 2020, Homework assignment 1.

1. Find all solutions to the system

$$\begin{cases} x_1 + 2x_2 + 3x_3 = 8 \\ x_1 - x_2 + 3x_3 = 5 \end{cases}$$

2. Is $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \right\}$ a basis for $\mathbb{R}^3$?

3. Find bases of the column space, the row space and null space of $A = \begin{bmatrix} 1 & -1 & -3 & -12 \\ 0 & 4 & 5 & 19 \\ 1 & 3 & 2 & 7 \\ -1 & 1 & 3 & 12 \\ 3 & 8 & 9 & 29 \end{bmatrix}$.

   You may use that the row reduced echelon form of A is

$$\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

4. Find the inverse of $\begin{bmatrix} 1 & 3 & 0 & 0 & 0 \\ 2 & 5 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$

5. If it is given that $\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & j \end{bmatrix} = 5$, determine

$$\det \begin{bmatrix} 2a & 2b & 2c \\ d & e & f \\ g - 4d & h - 4e & j - 4f \end{bmatrix}.$$

   Provide an explanation for your answer.

6. Compute the determinant of $\begin{bmatrix} 1 & 4 & 0 \\ 1 & 2 & -1 \\ 3 & 0 & 1 \end{bmatrix}$.

7. Let $A = \begin{pmatrix} 0 & 0 & -1 \\ 1 & -4 & 0 \\ 4 & -13 & 0 \end{pmatrix}$? Find the eigenspace of A at $\lambda = -3$.

8. Find the characteristic polynomial and the eigenvalues of the matrix $\begin{pmatrix} 3 & 0 & -3 \\ 1 & 2 & 5 \\ 3 & 0 & 3 \end{pmatrix}$.

9. Given is that $A = PDP^{-1}$, with $P = \begin{pmatrix} -2 & 0 & -1 \\ 0 & 1 & 2 \\ 1 & 0 & 0 \end{pmatrix}$ and $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

   Compute A^{100}.

10. Show that $H = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \ : \ 2x_1 - 3x_2 + 5x_3 = 0 \right\}$ is a subspace of $\mathbb{R}^3$.