

Q Given is that $A = PDP^{-1}$ with $P = \begin{pmatrix} -2 & 0 & -1 \\ 0 & 1 & 2 \\ 1 & 0 & 0 \end{pmatrix}$

and $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. Compute A^{100}

$$A^{100} = PD^{100}P^{-1}$$

$$P^{-1} = \frac{\text{adj}(P)}{|P|} = \frac{[P_{ij}]^T}{|P|}$$

$$P_{ij} = \begin{pmatrix} 0 & 2 & -1 \\ 0 & +1 & +0 \\ 1 & 4 & -2 \end{pmatrix}$$

$i=3$ $|P| = p_{31}C_{31} = 1(1) = 1$

$$\rightarrow P^{-1} = [P_{ij}]^T = \begin{pmatrix} 0 & 0 & 1 \\ 2 & 1 & 4 \\ -1 & 0 & -2 \end{pmatrix}$$

$$A^{100} = \begin{pmatrix} -2 & 0 & -1 \\ 0 & 1 & 2 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} (1)^{100} & 0 & 0 \\ 0 & (-1)^{100} & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 2 & 1 & 4 \\ -1 & 0 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 2 & 1 & 4 \\ -1 & 0 & -2 \end{pmatrix}$$

$$\rightarrow A^{100} = \begin{pmatrix} 0 & 0 & -2 \\ 2 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix}$$