Problem 2

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
```

2. Angular Sizes of Galaxies

Using HST, the Hubble Deep Field observation (two weeks of pointing at the same place on the sky!) reveals galaxies that range in redshift from $z = 0.2$ to $z = 4$ and in angular size from 0.1 to 5 arcseconds. Leave your answers to this problem in terms of the Hubble constant parameter h.

(a) For each of the two cosmologies in problem 1, make plots of $\theta(z)$ (the angular size on the sky in arcseconds) for several choices of lgal (the proper physical size) in the range $l = 0.5\,\text{kpc}$ to $l = 10\,\text{kpc}$ (recall that one parsec is 3 light years). Discuss the relationship between angular size and proper size of the galaxies. How is this complicated by cosmology?

For $\Omega_\Lambda = 0, \Omega_m = 1$ use Mattig's Formula to derive:

$$\theta(z, l_{\text{gal}}) = \frac{l_{\text{gal}}}{6 \times 10^6\,\text{kpc}} \frac{(1+z)^2}{z - \sqrt{1+z} + 1} h$$

This returns in radians; multiply by

$$\frac{180\,\text{degrees}}{\pi\,\text{radians}} \times \frac{60\,\text{arcmin}}{1\,\text{degree}} \times \frac{60\,\text{arcsec}}{1\,\text{arcmin}} = 206265\,\text{arcsec/radian}$$

to convert to arcseconds.

[2]:
```
def theta_matteronly(z, lgal):
    return (lgal/(6.e6)) * ((1+z)**2) / (z-np.sqrt(1+z)+1) * 206265
```

For $\Omega_\Lambda = 0.7, \Omega_m = 0.3$ use Pen's Approximation to derive:

$$\theta(z, l_{\text{gal}}) = \frac{l_{\text{gal}}}{3 \times 10^6\,\text{kpc}} [z - \frac{1+q_0}{2} z^2]^{-1} (1+z) h$$

where $q_0 = -0.55$ is the decceleration parameter.

[3]:
```
def theta_vacuum(z, lgal):
    return (lgal/(3.e6)) * (z - (1-0.55)/2*z**2)**(-1) * (1+z) * 206265
```

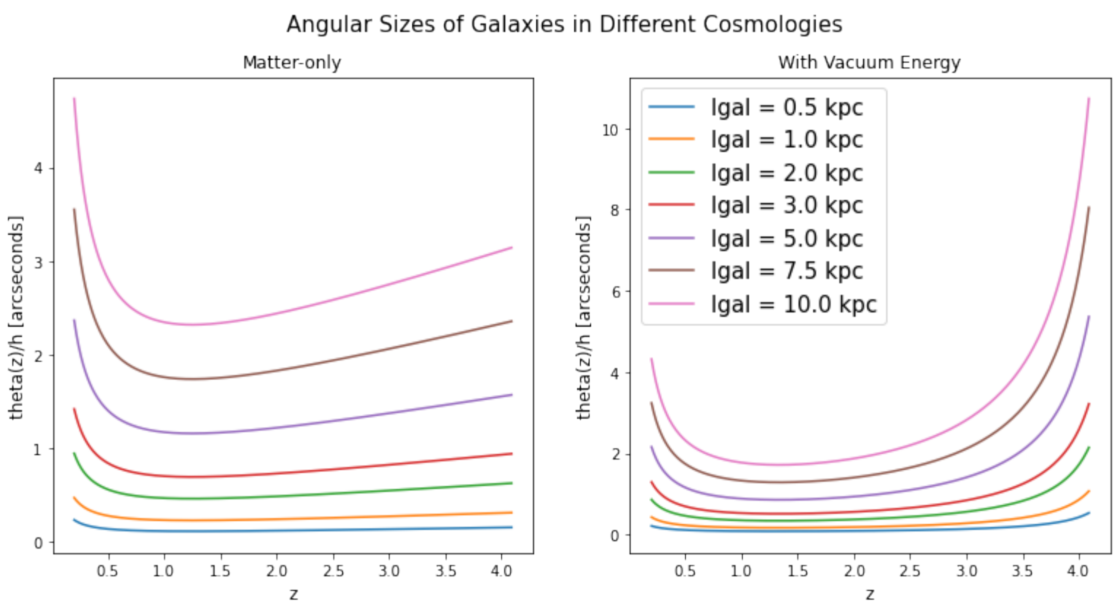
[5]:

```python
lgal = [0.5, 1, 2, 3, 5, 7.5, 10]
zrange = np.arange(0.2, 4+.1, 0.01)

fig, [ax1,ax2] = plt.subplots(1, 2, figsize=(13,6))
for l in lgal:
    ax1.plot(zrange, theta_matteronly(zrange, l), label="lgal = %.1f kpc"%l)
    ax2.plot(zrange, theta_vacuum(zrange, l), label="lgal = %.1f kpc"%l)
ax2.legend(loc="best", prop={"size":15})
ax1.set_ylabel("theta(z)/h [arcseconds]", fontsize=12)
ax2.set_ylabel("theta(z)/h [arcseconds]", fontsize=12)
ax1.set_xlabel("z", fontsize=12)
ax2.set_xlabel("z", fontsize=12)
plt.suptitle("Angular Sizes of Galaxies in Different Cosmologies", fontsize=15)
ax1.set_title("Matter-only", fontsize=12)
ax2.set_title("With Vacuum Energy", fontsize=12)
```

[5]:

```
Text(0.5, 1.0, 'With Vacuum Energy')
```



In both cases, the angular size of galaxies falls off with redshift, plateaus for a bit, then begins to increase again. The increase at higher redshift is much steeper in the case of vacuum energy.

Cosmology complicates observations in this regard because two galaxies of the same proper length at very different distances from us appear on the sky with the same angular size. In the case with vacuum energy, the galaxy at a farther distance may even have a larger angular size.

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b) What measurements or other information about these galaxies would allow us to disentangle this ambiguity?

Having the redshift of each galaxy would allow you determine the proper length as a function of angular size on the size, depending on your choice of cosmological model.