Physics 432/532: Cosmology
Winter 2022
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Problem Set 2

1. Field of View of the Hubble Space Telescope

The Hubble Space Telescope images an area on the sky that is approximately 2.5 minutes of arc across. Leave your answers to this problem in terms of the Hubble constant parameter h. [Recall units of angle: 60 arcseconds/arcminute, 60 arcminutes/degrees, 180 degrees/π radian.] It may be useful to employ "Mattig's formula" for the coordinate distance (see section 13.1 of lecture notes 1) for the matter-only case and/or Pen's approximation (see PDF posted in week 2 readings and note carefully that this formula is for the luminosity distance) for the more general case including a cosmological constant.

(a) What is the proper size in Mpc of the HST field of view at redshifts $0.5, 1, 2, 4$ for $\Omega_{matter} = 1, \Omega_{vac} = 0$ and for $\Omega_{matter} = 0.3, \Omega_{vac} = 0.7$? You may, if you choose, answer this by presenting plots for both cosmologies.

(b) What is the comoving size of the field at $z = 1$ for both cosmologies?

2. Angular Sizes of Galaxies

Using HST, the Hubble Deep Field observation (two weeks of pointing at the same place on the sky!) reveals galaxies that range in redshift from $z = 0.2$ to $z = 4$ and in angular size from 0.1 to 5 arcseconds. Leave your answers to this problem in terms of the Hubble constant parameter h.

(a) For each of the two cosmologies in problem 1, make plots of $\theta(z)$ (the angular size on the sky in arcseconds) for several choices of l_{gal} (the proper physical size) in the range $l = 0.5$ kpc to $l = 10$ kpc (recall that one parsec is 3 light years). Discuss the relationship between angular size and proper size of the galaxies. How is this complicated by cosmology?

(b) What measurements or other information about these galaxies would allow us to disentangle this ambiguity?

3*. Blackbody Radiation

An object emits blackbody radiation of temperature T in its own rest frame. We see it at redshift z and subtending a solid angle Ω (here Ω is solid angle, not the density parameter!).

(a) What flux do we measure? Here it helps to know that the phase-space density of photons is invariant:

$$\frac{I_\nu}{\nu^3} = constant$$

where I_ν is the surface brightness (flux per solid angle per unit frequency).

(b) What if the redshift is due to Doppler motion of a local object instead of cosmological?

4. Uncertainty in the distance scale

Suppose that a series of four different standard candles are used to step out along the cosmic distance ladder from our Galaxy to distances far enough to accurately measure the true expansion rate. Each standard candle has an uncertainty of $\Delta m = 0.2$ magnitudes in its intrinsic magnitude. Show that, by varying the calibration of each of the standard candles, it is possible for the measured Hubble constant to differ from its nominal value by a factor of ~ 0.7 to ~ 1.4. (Recall the definition of astronomical magnitudes, $m = -2.5 \log f + \text{constant}$ where f is the received flux.)

5. Limits on the parallax method

From Earth, optical telescopes can resolve differences in angle of at best 0.5 arcseconds. With this resolution limit, to what distance can we use trigonometric parallax to measure distances? What if we could measure differences in angle of 0.01 arcseconds? (Recall that there are 206,265 arcseconds in a radian.)

6. Peculiar velocities and the Hubble constant

The observed recession velocity of galaxies is the sum of their Hubble flow velocity, $v = Hr$, and the line-of-sight component of their "peculiar" velocity, due to local gravitational effects. Suppose that we estimate H_0 by measuring both the distance and redshift for a single object. How does the error in the Hubble constant depend on the r.m.s. peculiar velocity of this type of object? At what distance can we get 5% accuracy in H_0 if $v_{pec} = 200$ km s^{-1}?

7. Age of the universe and constraints on cosmological parameters

Make a 2D figure that shows the dependence of the age of the universe on Ω_{matter} and Ω_{vac}. Indicate the bounds on this 2D space that are set by age estimates of the Solar system, white dwarfs, and globular clusters. How does the region of $\Omega_{matter}, \Omega_{vac}$ that is allowed by these constraints compare with the region favored by other observations? (See your notes from class, figure 3.5 in the text, and the "Cosmic Triangle" article.) Use the equation

$$t_0 = \frac{2}{3} t_H (0.7\Omega_M + 0.3 - 0.3\Omega_{vac})^{-0.3}$$

which relates the age of the universe now to the Hubble time ($t_H = 1/H$) and the cosmological parameters Ω_M and Ω_{vac}. In addition to the bounds in $\Omega_{matter} - \Omega_{vac}$ space from the age estimates, also draw lines for the cases $t_0 = 6, 8, 10, 12, 14$Gyr.