Exercise 1.4

Let $\mathbb{F} = \mathbb{Z}_3$. Find the set of all solutions to:

$$\begin{cases} x_1 + x_2 = 2 \\ 2x_1 + x_2 + x_3 = 0 \end{cases}$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 2 & 1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 2 & 1 & 2 \end{array}\right) \cdot 2^{-1} = 2$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 \end{array}\right)$$

$x_1 = 1 - x_3$

$x_2 = 1 - 2x_3$

$$\rightarrow \left\{ \vec{x} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix} : x_3 \in \mathbb{Z}_3 \right\}$$