Calculators and notes are not permitted during this exam. Show all relevant work, and present solutions clearly.

Name: ______________________________

Question	Points	Score
1	20	
2	20	
3	14	
4	16	
5	8	
6	10	
7	12	
Total:	100	

Common Derivatives

$f(x)$	$\frac{d}{dx}f(x)$
x^n, with n a real number	nx^{n-1}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\csc^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$
e^x	e^x
b^x	$b^x \ln b$
$\ln \lvert x \rvert$	$\frac{1}{x}$
$\log_b \lvert x \rvert$	$\frac{1}{x \ln b}$

1. (20 points) For each of the following limits, either compute its value or state that it does not exist. Show all work.

   (a) $\displaystyle\lim_{t\to 0} \frac{e^t - 1 - t}{5t^2}$

   (b) $\displaystyle\lim_{x\to 0^+} x^x$

   (c) $\displaystyle\lim_{x\to\infty} \frac{e^{3x}}{5e^{3x} + 2e^x}$

   (d) $\displaystyle\lim_{\theta\to\pi} \frac{\cos(\sin(\theta^2)) - 1}{\theta}$

2. (20 points) Evaluate the following definite and indefinite integrals. Show all work.

   (a) $\displaystyle\int \left(\sqrt{t} + \frac{5}{t} - \sec^2 t\right)\, dt$

   (b) $\displaystyle\int_{-1}^{1} (f(x) + 3g(x))\, dx$ where $\int_{-1}^{2} f(x)\, dx = -5$, $\int_{1}^{2} f(x)\, dx = -3$ and $\int_{-1}^{1} g(x)\, dx = 1$

   (c) $\displaystyle\int_{1}^{2} \frac{2s^2 - 4}{s^3}\, ds$

   (d) $\displaystyle\int_{e}^{e^2} \frac{1}{x \ln |x|}\, dx$ (Use u-substitution to evaluate)

3. (14 points) (a) Use the second derivative test to find the locations of the local extreme values of $p(x) = 2x^3 - 3x^2 - 12x + 7$, and for each of them determine if it is a local minimum or local maximum.

   (b) Find the critical points of $g(x) = x^{2/3}(5 - 2x)$ on the interval $[-1, \frac{5}{2}]$ and determine the locations and values of the absolute maximum and absolute minimum.

4. (16 points) For this problem, we will consider the function $f(x) = \sqrt{x}$ for $x \geq 0$.

(a) Use the definition of the derivative and evaluate the limit to confirm that

$$f'(x) = \frac{1}{2\sqrt{x}}$$

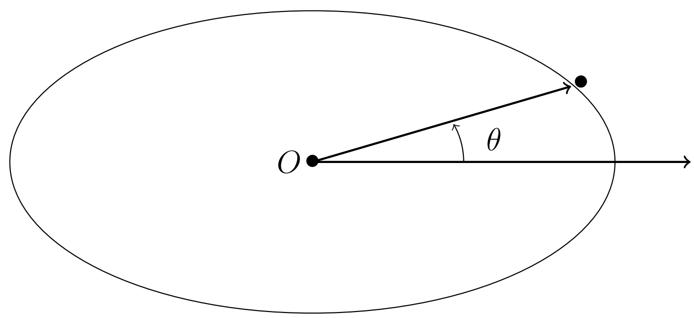
for $x > 0$. Do <u>not</u> use L'Hôpital's rule to evaluate the limit. Show all steps of your work.

(b) Compute the second derivative of f. (You do not need to set up a limit, just find the derivative of $f'(x)$ to get $f''(x)$).

(c) Find the linear approximations of f at $x = 1$ and $x = 25$. Use these to approximate the values of $\sqrt{2}$ and $\sqrt{26}$, respectively.

(d) Do these approximations overestimate or underestimate the exact values? Which approximation is closer to the exact value? Justify your answer. (Do not compute the exact values of $\sqrt{2}$ and $\sqrt{26}$.)

5. (8 points) Suppose that an ant is walking around the outside of an elliptic rock that is 4 inches wide and 2 inches tall. Assume the center of the rock is located at the origin, $O = (0, 0)$. Then the position of the ant in inches can be given by $(x, y) = (4\cos\theta, 2\sin\theta)$, where θ is the angle between the positive x-axis and the line from the origin to the ant. The angle of the ant changes at a constant rate of $1/3$ radians per second. What is the rate of change in the distance between the ant and the center of the rock when $\theta = \pi/6$? Include units in your answer.



6. (10 points) Let $g(x) = x^2 - 5$. We have the following values of g on the interval $[0, 4]$.

x	0	1	2	3	4
$g(x)$	-5	-4	-1	4	11

(a) Compute the following Riemann sums for $g(x)$ over the interval $[0, 4]$

(i) The midpoint Riemann sum with $n = 2$ subintervals.

(ii) The left Riemann sum with $n = 4$ subintervals.

(b) Use the Fundamental Theorem of Calculus to find the value of $\displaystyle\int_0^4 g(x)\,dx$.

(c) Without doing the computation, what could we say about the approximate value of the left Riemann sum of $g(x)$ over $[0, 4]$ when we take $n = 1000$ subintervals. Justify your answer.

(d) *Extra Credit* (2 points): Find the right Riemann sum for an arbitrary number of subintervals n, and then evaluate the limit of the Riemann sums as n goes to ∞. You can use the summation formula $\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$ to evaluate the sum. Show all steps in evaluating the limit.

7. (12 points) The number of foxes in a specific area is specified by the function $P(t)$, where t is the number of years after recording first began. Suppose that the rate of change in the number of foxes is given by

$$P'(t) = 1000e^{-0.5t}.$$

For this problem, you can use $\int e^{-0.5t}\,dt = -2e^{-0.5t} + C$.

(a) What is the change in the population of foxes in the first 2 years of recording?

(b) What is the net change in the population over the long-term? That is, what is $\lim_{T\to\infty}(P(T) - P(0))$?

(c) Suppose that the number of foxes at the start of recording is 1000. Solve the initial value problem and write the formula for $P(t)$.