Getting the Hang of Structural Estimation)

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This homework tries to lead you through a straightforward example of GMM to help you *actually* understand GMM better!

Deliverables

- You should have a word/LaTeXdocument that has three sections:
  1. Discusses the model and answers the questions I pose throughout.
  2. Contains the tables and figures you will produce.
  3. Contains a discussion of your programming choices if you had to make any.
- You should have a Matlab file or set of files (zipped) that contain **all** your programs and raw data. There should be a file called "Main.M" that produces everything I need in one click.

1 Model

Consider the simplest dynamic model that I can think of: a two-period model over labor and consumption, in which agents maximize utility subject to the standard budget constraint. Letting c_t, L_t, w_t be consumption, labor and wages in period t, ν be the initial property income of the agent, r be the interest rate between periods, ψ control the utility of leisure, and β be the discount rate, the agent maximizes:

$$U(c_1, c_2, L_1, L_2) = \log(c_1) + \beta \log(c_2) + \psi \log(1 - L_1) + \beta\psi \log(1 - L_2)$$

Subject to the NPV budget constraint:

$$w_1 L_1 + \frac{w_2 L_2}{1 + r} + \nu = c_1 + \frac{c_2}{1 + r}$$

We have a dataset of w_1, w_2, r, ν, c_1, c_2, and L_1 (L_2 can be found with the budget constraint).

You'll find that the closed-form solution to these three series is (feel free to check my work):

$$c_1 = \frac{\nu + r\nu + w_1 + rw_1 + w_2}{(1+r)(1+\beta)(1+\psi)}$$

$$c_2 = \beta\frac{\nu + r\nu + w_1 + rw_1 + w_2}{(1+\beta)(1+\psi)}$$

$$L_1 = \frac{-\nu\psi - r\nu\psi + w_1 + rw_1 + \beta w_1 + r\beta w_1 + \beta\psi w_1 + r\beta\psi w_1 - \psi w_2}{(1+r)(1+\beta)(1+\psi)w_1}$$

Question 1: Write a Matlab program that takes in parameters to be estimated $\theta = \{\beta, \psi\}$ and data on w_1, w_2, r, and ν and spits out c_1^*, c_2^*, and L_1^* as given in the formulae above.

Question 2: Use the data provided in data.csv on w_1, w_2, r, ν, c_1, c_2, and L_1 and the algorithm from Question 1 to "Calibrate" the model by choosing β and ψ so that the mean c_1, c_2, and L_1 in the data are equal to their mean model values. **The column order is** $\{c1, c2, L1, L2, w1, w2, r, nu\}$ This will give you three moments:

$$mom(\theta, data) = \begin{bmatrix} mean(c_1^{model} - mean(c_1^{data})) \\ mean(c_2^{model} - mean(c_2^{data})) \\ mean(L_1^{model} - mean(L_1^{data})) \end{bmatrix}$$

You can combine these to get an error by using the quadratic form with identity weighting matrix:

$$err = mom(\theta, data)' \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} mom(\theta, data)$$

Report your estimated coefficients. You have now (hopefully successfully) indirectly calibrated a structural model.

Question 3: You can turn calibration into estimation by reporting standard errors. This isn't as hard as it seems! Your moment matrix was a 3x1, formed from the means of an Nx3 set of individual conditions. All we need to do is find the (1) variance-covariance matrix of our moments, which is simply the covariance matrix of the individual moment conditions. So if:

$$mom_i(:,1) = c_1^{model} - c_1^{data}$$

$$mom_i(:,2) = c_2^{model} - c_2^{data}$$

$$mom_i(:,3) = L_1^{model} - L_1^{data}$$

Then you can calculate the optimal weighting matrix with:

$$S = cov(mom_i)$$

And weight by the inverse of S.

Then the formula for the asymptotic variance-covariance matrix of your estimates can be calculated as:

$$\hat{V} = \frac{\left(D' cov(mom_i)^{-1} D\right)^{-1}}{N}$$

Where N is the number of observations and D is the Jacobian (so D would look like:

$$D = E\left(\begin{bmatrix} \frac{\partial mom_1}{\partial \beta} & \frac{\partial mom_1}{\partial \psi} \\ \frac{\partial mom_2}{\partial \beta} & \frac{\partial mom_2}{\partial \psi} \\ \frac{\partial mom_3}{\partial \beta} & \frac{\partial mom_3}{\partial \psi} \end{bmatrix}\right)$$

You can calculate this with finite differences! Perturb $\hat{\beta}$ and see how the average moment condition changes for each of the three moment conditions for the first column, and similarly with $\hat{\psi}$ for the second column.

Report your estimated coefficients along with their standard errors.