Estimating/Calibrating a Structural Model

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This homework is the first part of a multi-part homework. This homework starts us with a relatively simple deterministic lifecycle model and asks you to solve it in Matlab.

Deliverables

- You should have a word/LaTeXdocument that has three sections:
  1. Discusses the model and answers the questions I pose throughout.
  2. Contains the tables and figures you will produce.
  3. Contains a discussion of your programming choices if you had to make any.
- You should have a Matlab file or set of files (zipped) that contain **all** your programs and raw data. There should be a file called "Main.M" that produces everything I need in one click.

1 Model

Take the model from Homework 2, repeated here for convenience, with slightly different parts, highlighted in red: Households have Stone-Geary utility over consumption c_t:

$$u(c_t, L_t) = \sqrt{(}c_t - 1)$$

Their income each period is the sum of permanent income P_t and a transitory shock ϵ_t:

$$Y_t = P_t + \epsilon_t$$

Where:

$$\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$$

Permanent income is a random walk:

$$P_t = P_{t-1} + \zeta_t$$

Where:

$$\zeta_t \sim \mathcal{N}(0, \sigma_\zeta^2)$$

Households face the budget constraint:

$$Y_t + (1 + r)s_{t-1} = c_t + s_t$$

And the borrowing constraint: $s_t \geq \overline{s}$.

Households maximize the net present value of utility, discounted at a rate $0 < \beta < 1$. Assume that households live until period T.

Now, let the following numerical assumptions hold:

Table 1: Calibration

Concept	Parameter	Value
Lifespan	T	45
Discount factor	β	???
Borrowing constraint	$\overline{s}$	-0.2
Initial permanent income:	P_0	5
Variance of permanent income shock:	σ_ζ^2	0.02
Variance of transitory income shock:	σ_ϵ^2	0.04
Interest rates:	r_t	0.05
Initial savings:	s_0	$\sim \mathcal{N}(0,???)$

Table 1: Note that s_0 is truncated normal at $\overline{s}$

Question 1: Your goal is to calibrate this structural model by finding β and σ_ζ^2 such that, in expectation:

$$mean(\log(c_{30}^{sim})/\log(c_{29}^{sim})) = 1.001788$$

$$Var((s_2^{sim}))) = 0.00409$$

To do so, you need to have an estimating "outer loop" and a value-function solving and simulating "inner loop." Homework 2 created a file that 1) solved household models and (2) simulated households *given* β and $\sigma_{s_0}^2$. Your goal is to now (1) create a "wrapper" function that *takes* β and $\sigma^2_{s_0}$and spits out (for instance) the squared sum of difference between your simulated result and your target result, and chooses β and $\sigma^2_{s_0}$ to minimize that sum of squared errors.

This is the core of simulated method of moments/"calibration."