Solving an Stochastic Infinitely-Lived Human Capital Accumulation Problem

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Econ 64200

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This homework is the first part of a multi-part homework. This homework takes the deterministic model we had in the first homework and adds (1) a stochastic state variable and (2) a second dimension of choice, and asks you to solve it in Matlab.

Deliverables

- You should have a word/LaTeXdocument that has three sections:
  1. Discusses the model and answers the questions I pose throughout.
  2. Contains the tables and figures you will produce.
  3. Contains a discussion of your programming choices if you had to make any.
- You should have a Matlab file or set of files (zipped) that contain **all** your programs and raw data. There should be a file called "Main.M" that produces everything I need in one click.

1 Model

Infinitely-lived households start each period with human capital endowment h_t, and have period utility over consumption c_t. They have one unit of time each period, which they use to either study i_t or work L_t . If they study, their human capital for next period grows subject to a stochastic productivity rate A_t known at time t, while if they work, consumption increases. Households have access to a savings technology which pays interest rate $1 + r$ when they save w_t. They maximize the net present value of utility discounted at a rate $0 < \beta < 1$:

$$\sum_{t=0}^{\infty} \log(c_t)$$

subject to the law of motion of human capital:

$$h_{t+1} = (1-\delta)h_t + exp(A_t)i_t^\gamma$$

Their budget constraint:

$$c_t + w_t = h_t L_t + (1+r)w_{t-1}$$

And the law of motion for human capital accumulation A_t:

$$A_{t+1} = \rho A_t + \epsilon_t$$

$$\epsilon_t \sim \mathcal{N}\left(0, \sigma_\epsilon^2\right)$$

2 Basic Matlab

Now, let the following numerical assumptions hold:

Table 1: Calibration

Concept	Parameter	Value
Discount factor	β	0.95
Ability to accumulate human capital:	A	1
Human capital depreciation rate:	δ	0.05
Decreasing returns to human capital investment:	γ	0.6
AR(1) parameter for productivity shocks:	ρ	0.95
Variance of productivity shocks:	σ_ϵ^2	0.01
Interest rate:	r	$\frac{1}{0.95} - 1$

Question 2: Write a Matlab program that takes in the parameterization of Table 1 and uses value function iteration to solve the agent's problem. To be clear, your problem is now $V(h_t, A_t, w_{t-1})$, and has a choice over not just i_t but also w_t. Also note that you will have to truncate the distribution of A_{t+1}: be clear about your justification for what you choose![1]

See Main.m for code.

Question 3: Plot the two-dimensional policy functions for i and for s as a function of A and w when $h = 8.46$.

[1] I suggest bounding state variable $h \in [7.5, 10]$, $w \in [0, 1.5]$, and finding your own bounds for A. Moreover, I suggest taking only five points in each space to start (125 points total), so that you "fail quickly" if something isn't working.

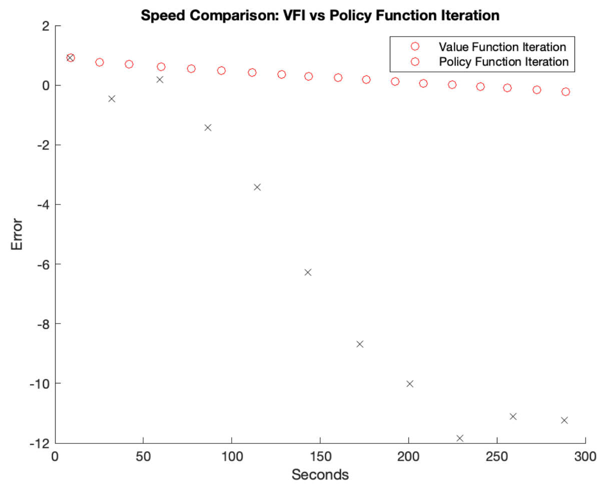

Figure 1: My code uses both Policy Function Iteration and Value Function Iteration. This figure compares the speed of the two, plotting on the x-axis seconds of run-time and on the y-axis the error between value function updates (when solving the actual value function). It omits the value function changes that do not involve maximization, to compare apples to apples. What a speed difference!

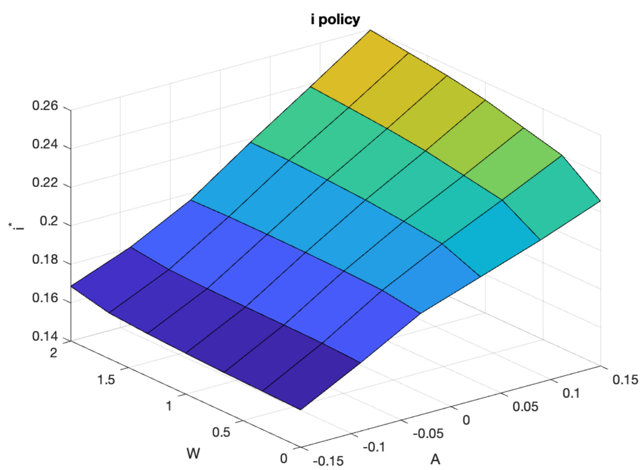

Figure 2: This figure plots the policy function for $i^*(s, A|h = 8.46)$.

Question 4: Assuming the productivity parameter is independently drawn across individuals, use your estimated policy functions for i and s, as well as the law of motion of A to simulate the savings over many periods, so your initialization doesn't matter for at least 1000 individuals. Using your simulated individuals, plot the stationary distribution of wealth in this society.

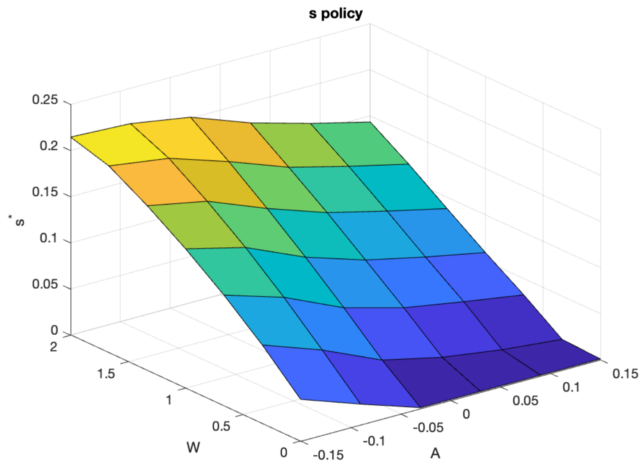

Figure 3: This figure plots the policy function for $s^*(s, A|h = 8.46)$.

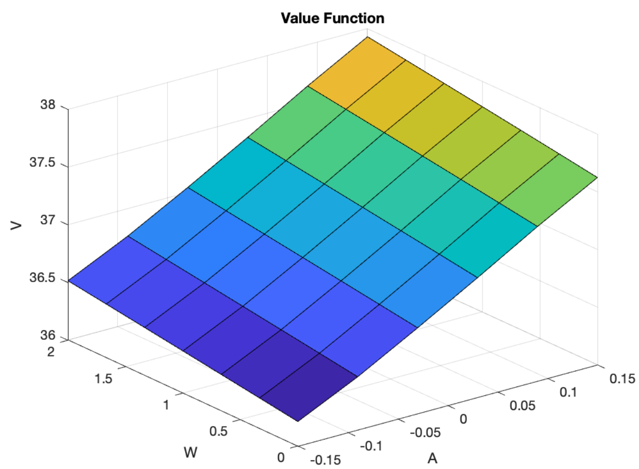

Figure 4: This figure plots the value function for $s^*(s, A|h = 8.46)$.

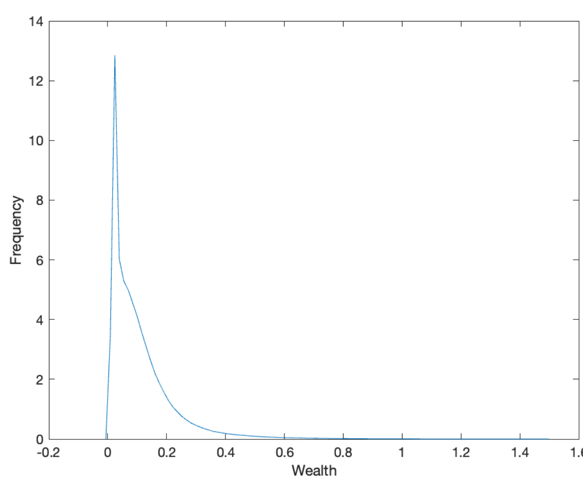

Figure 5: This figure plots the stationary wealth distribution.