Numerical Methods-Lecture VI: Applying Newton's Method

Trevor Gallen

Motivation

- We've seen the theory behind Newton's Method
- How can we apply it to Value Function Iteration?
  - Solving systems of equations
  - Maximization!
  - Caveat: we'll linearly interpolate for now.

Monopolistic Competition-I

We might have a system of equations describing agent behavior. Given elasticity of substitution σ, Income I, marginal cost of production ϕ, and fixed cost of entry ν, a monopolistically competitive system's equilibrium is given by:

- Consumption aggregation

$$C = \left(\int_0^n c_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

- Idiosyncratic demand curves:

$$c_i = \frac{I p_i^{-\sigma}}{\int_0^n p_i^{1-\sigma} di} \tag{2}$$

- Aggregate price:

$$P = \int_0^n \left(p_i^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \tag{3}$$

- Profit definition:

$$\pi_i = p_i c_i - \phi c_i - \nu \tag{4}$$

- Zero profit (free entry):

$$\pi_i = 0 \tag{5}$$

- Optimal markup:

$$p_i = \frac{\sigma}{\sigma - 1} \phi \tag{6}$$

Monopolistic Competition-II

- We won't bother simplifying, though we can in this case.
- We want to solve these six equations as a function of n, p_i, P, c_i, C, π_i, and we'll assume a symmetric equilibrium.
- To solve, we'll:
  - Step 1: Write all FOC's as a vectorized function of those six variables
  - Step 2: Write the Jacobian of the vector of FOC's as a function of those six variables
  - Step 3: Apply Newton's Method until we converge

Monopolistic Competition-III

For code, see Lecture_6_NewtonsMethod_DixitStiglitz.m

Alternative Use of Newton's Method: Estimation

- Linear regression of the type:

$$y_i = X_i \beta + \epsilon_i$$

is easy. (Where y is an $n \times 1$, X_i is an $n \times j$, β is a $j \times 1$, and ϵ_i is a $n \times 1$ matrix).

- $\beta = (X'X)^{-1}X'Y$
- What if we had a slightly different problem? (Nonlinear least squares, for instance).
- Newton's method helps us find a minimum.

Some Data

City	Crack Index	Crime Index
Baltimore	1.184	1405
Boston	3.129	835
Dallas	2.103	675
Detroit	2.057	2123
Indianapolis	0.858	1186
Philadelphia	4.087	1160

Some Data

- Given $\beta = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ we can calculate ϵ_i:

$$\epsilon_i = \begin{bmatrix} 1.18 \\ 3.13 \\ 2.10 \\ 2.06 \\ 0.86 \\ 4.09 \end{bmatrix} - \begin{bmatrix} 1 & 1405 \\ 1 & 835 \\ 1 & 675 \\ 1 & 2123 \\ 1 & 1186 \\ 1 & 1160 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- We can try to minimize $\sum \epsilon_i^2$.

In Matlab

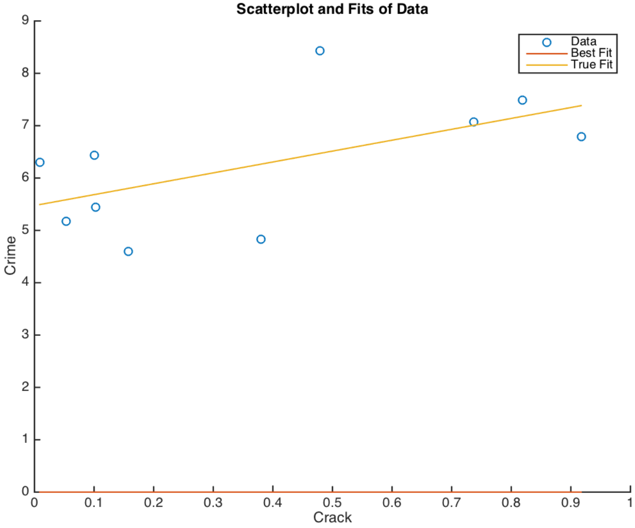
- I assume all the data is already in Y and X as it is listed.

```
f = (beta) sum((y-X'*beta).^2)
d1 = [d,0]
d2 = [0,d]
d3 = [d,d]
f_grad = (beta) [f([b(1)+d;b(2)]-f(b))/d ;
f([b(1);b(2)+d]-f(b))/d]
f_hess = (b) [(f(b+d1)-2.*f(b)+f(b-d))/d ,
 f(b+d3)-f(b+d1-d2)-f(b-d1+d2)+f(b-d3) ;
 f(b+d3)-f(b+d1-d2)-f(b-d1+d2)+f(b-d3) ;
 f(b+d2)-2*f(b)-f(b-d2)]/d^2
```

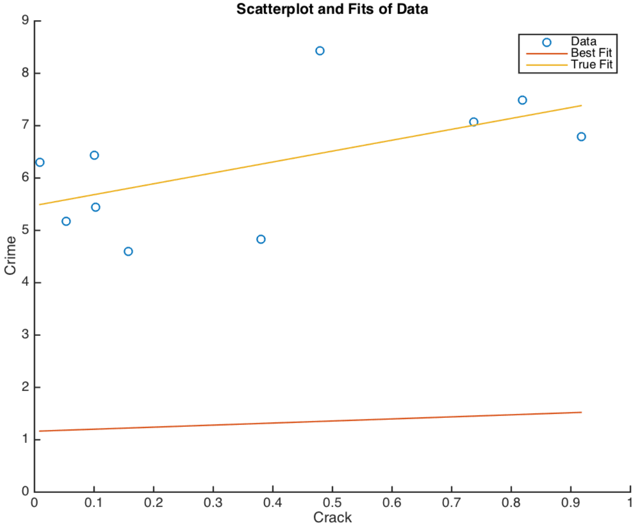
In Matlab

For code, see Lecture_6_NewtonsMethod_LinReg.m

Initial Conditions



Step 1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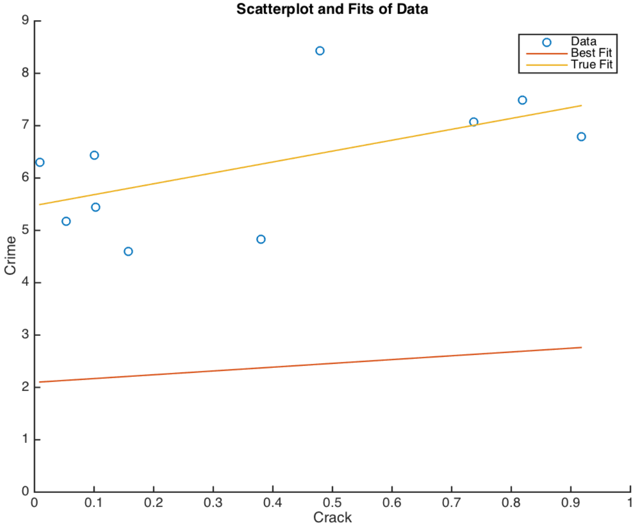

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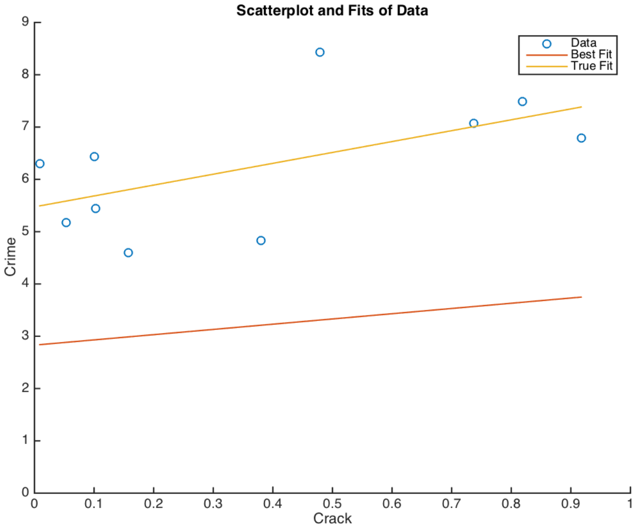
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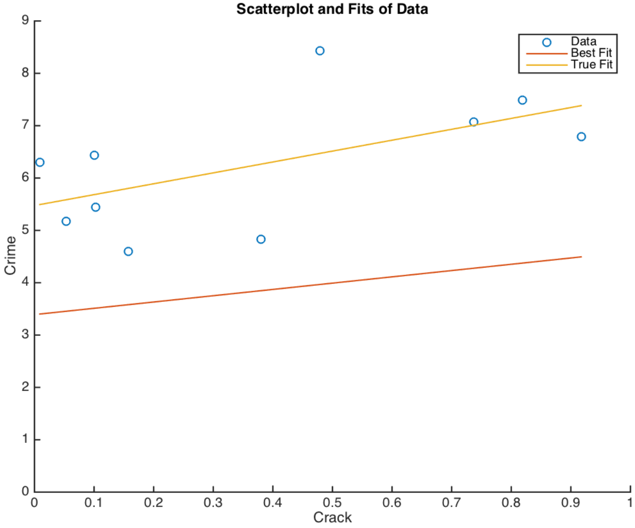

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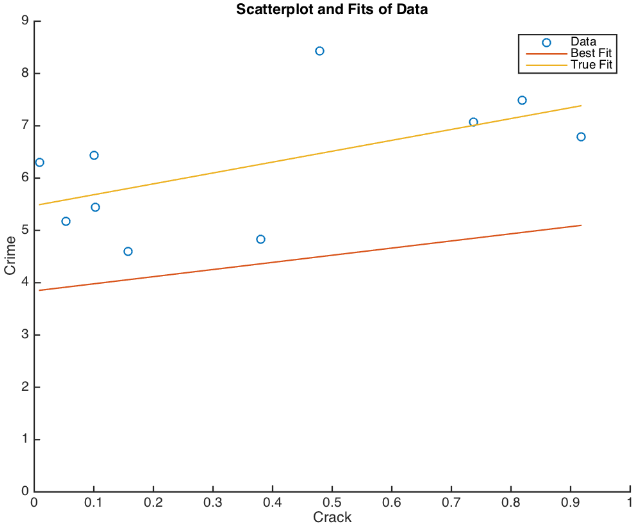

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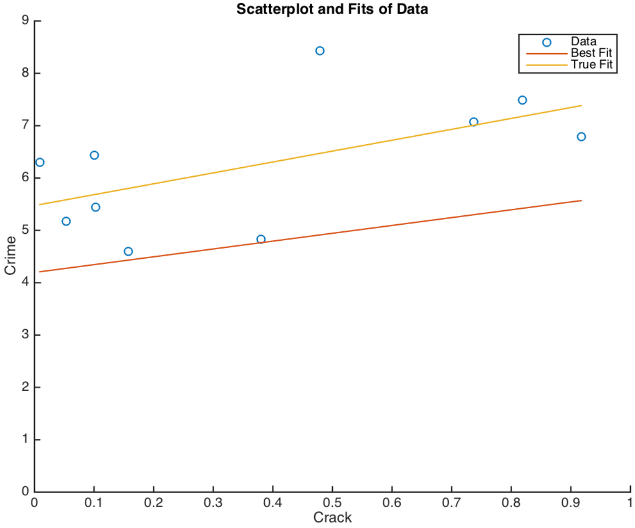
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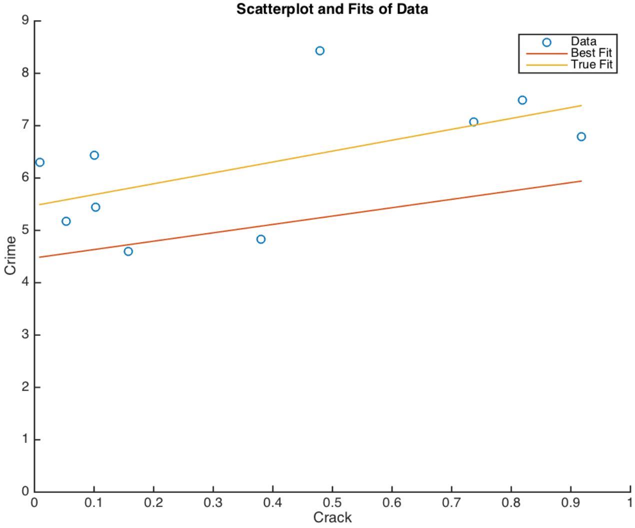
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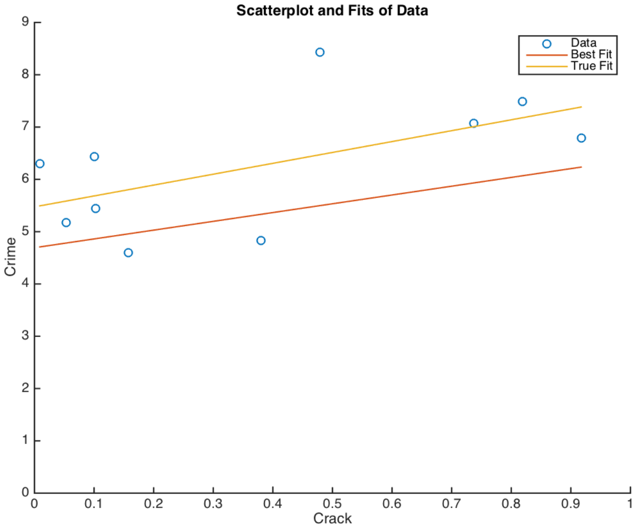

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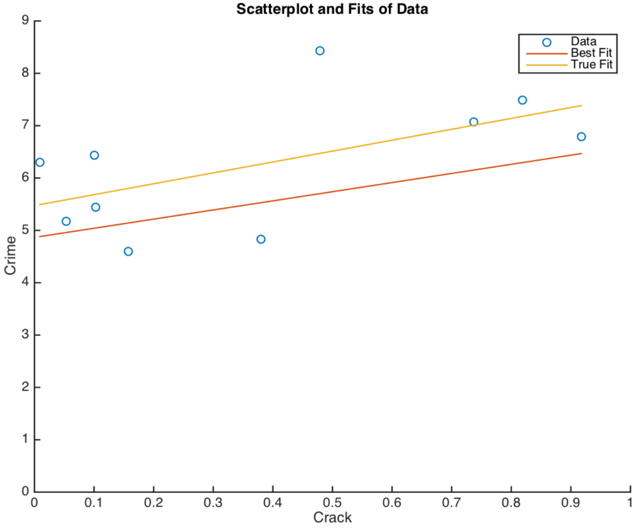
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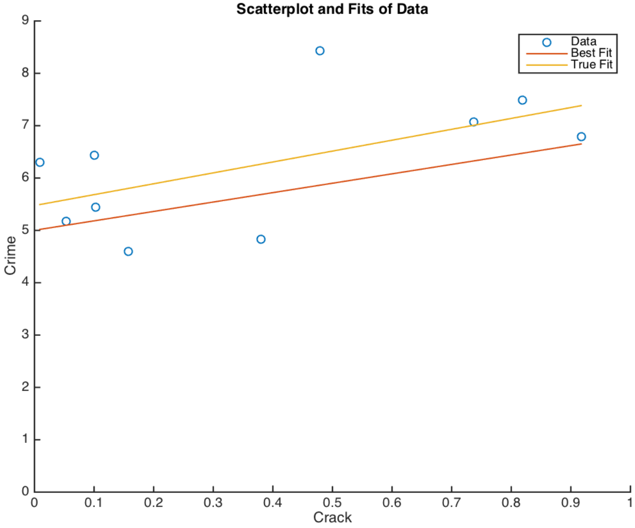
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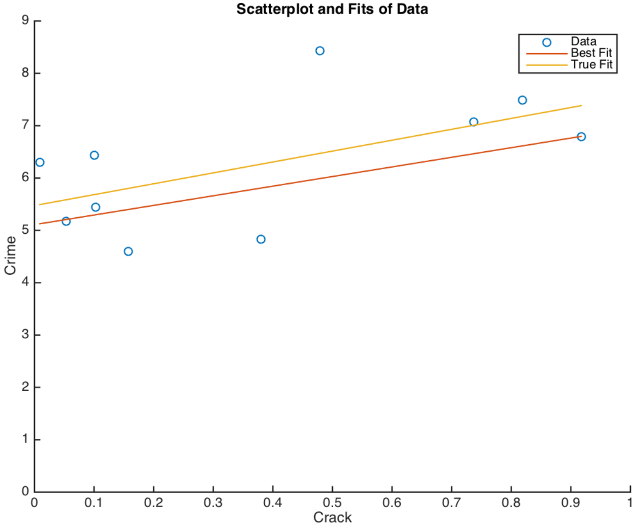

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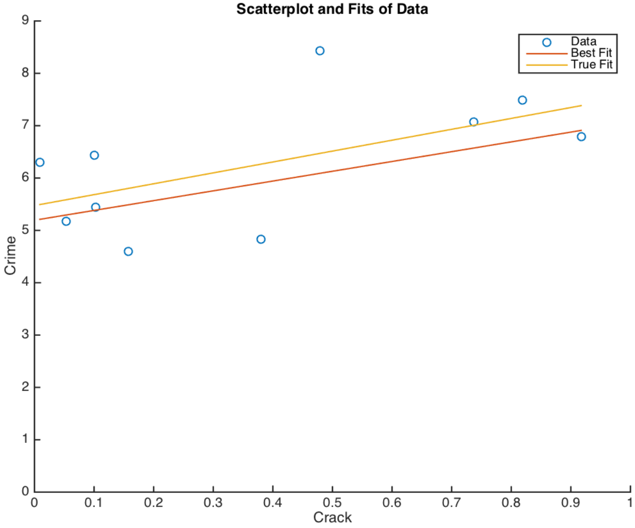

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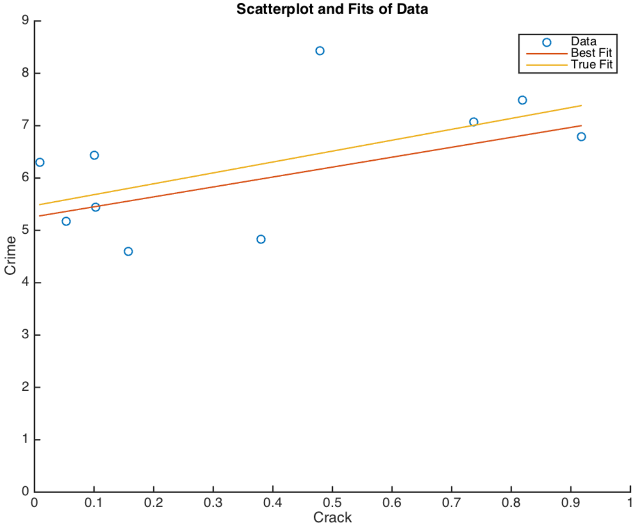
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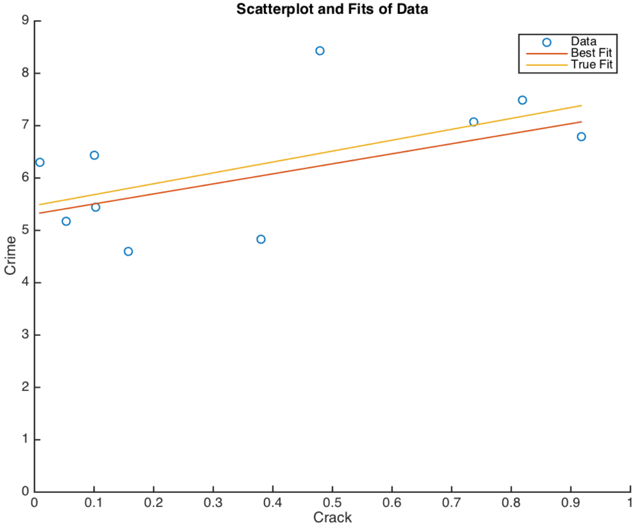

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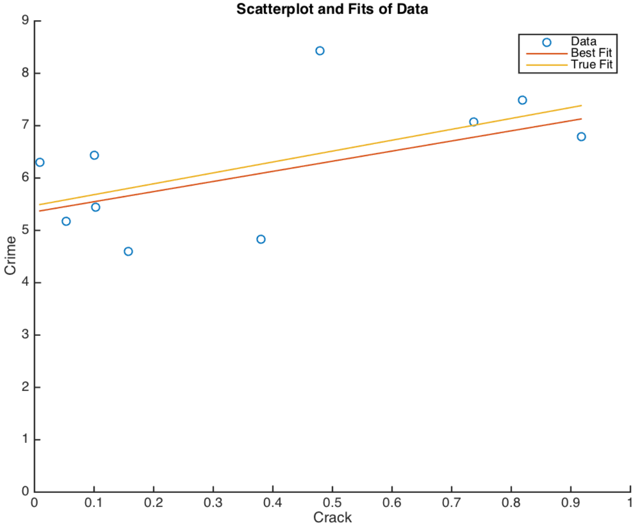
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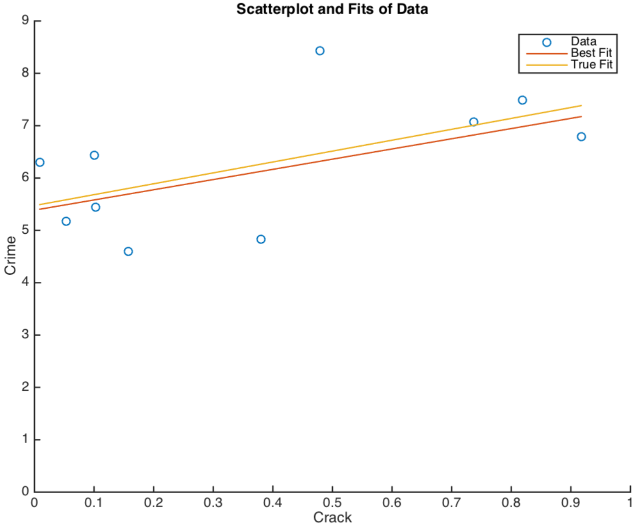

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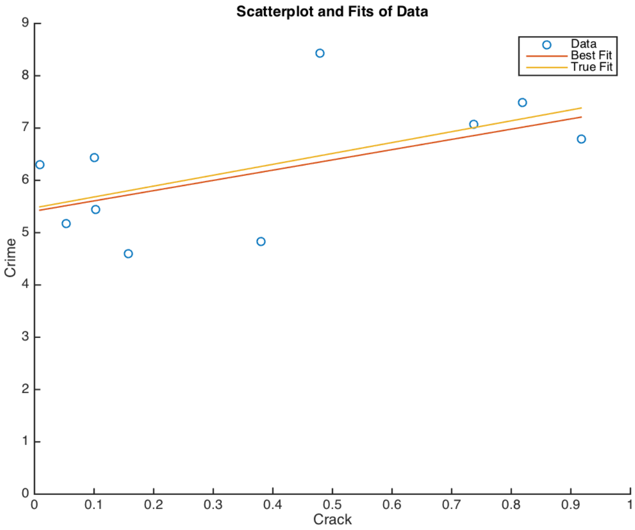

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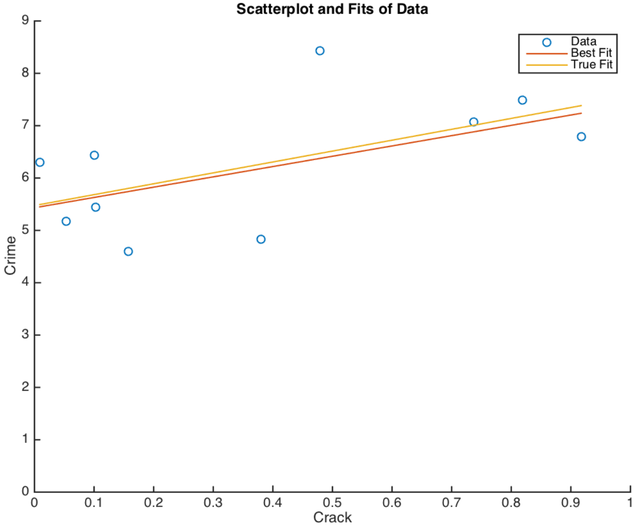

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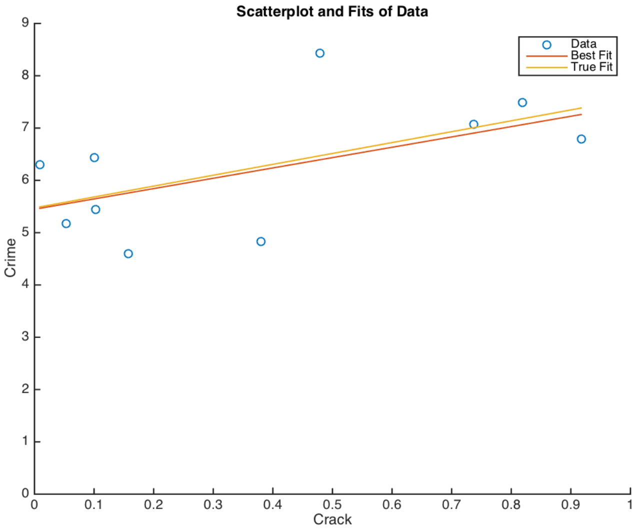
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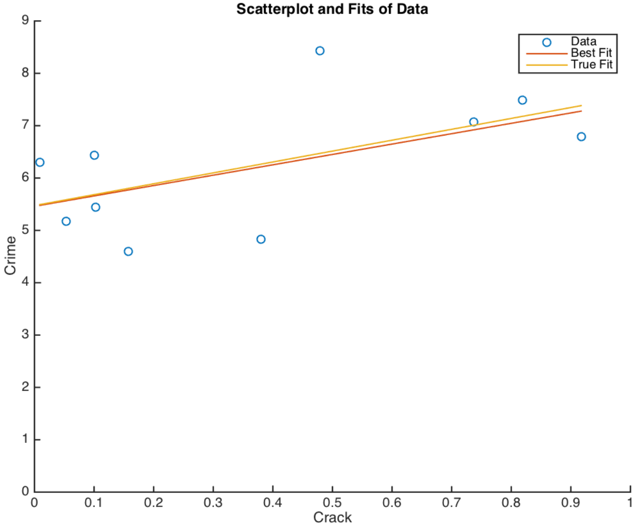
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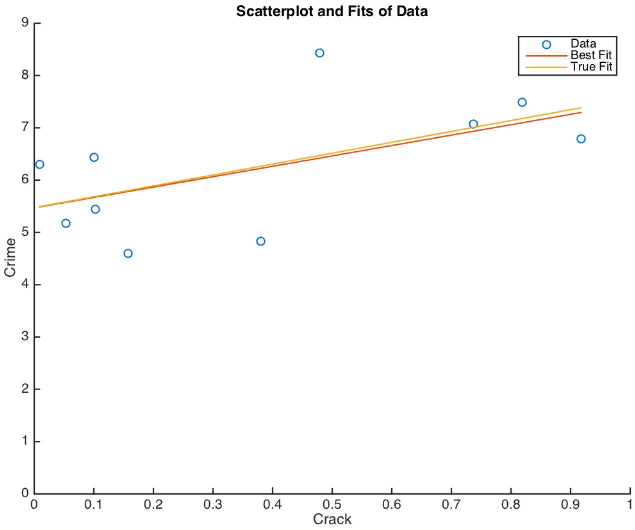
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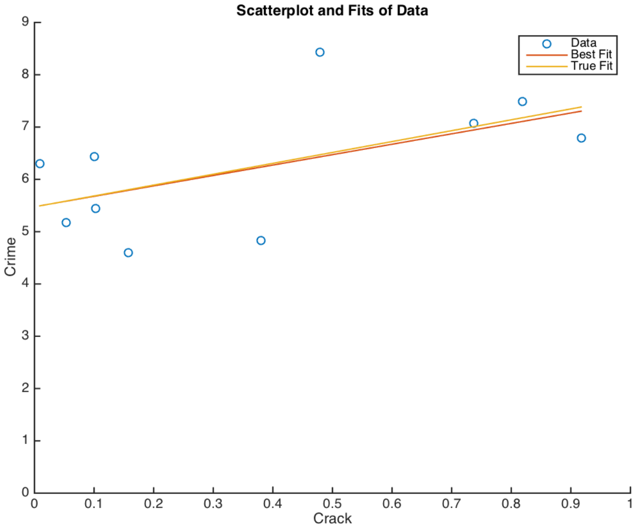

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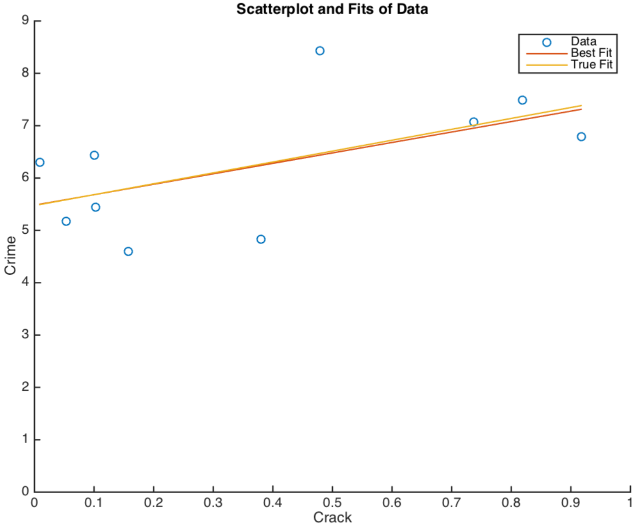
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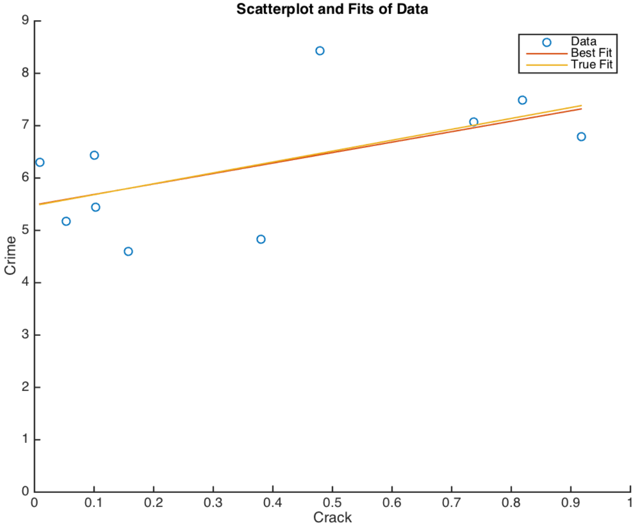

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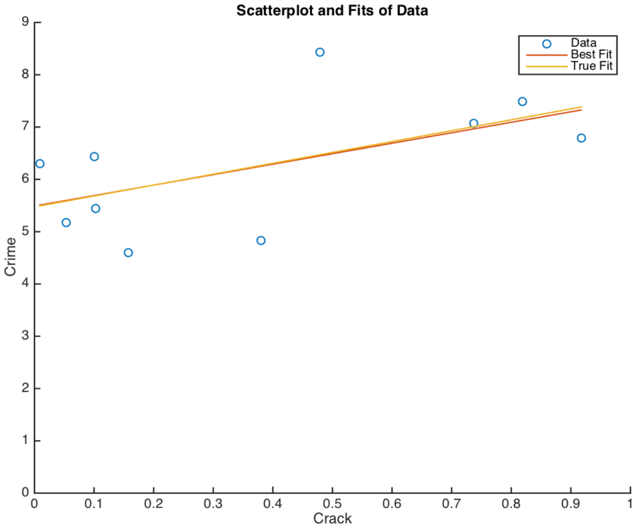
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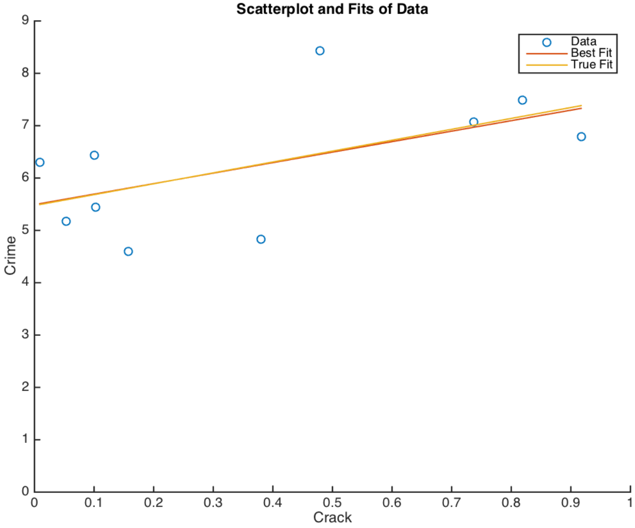

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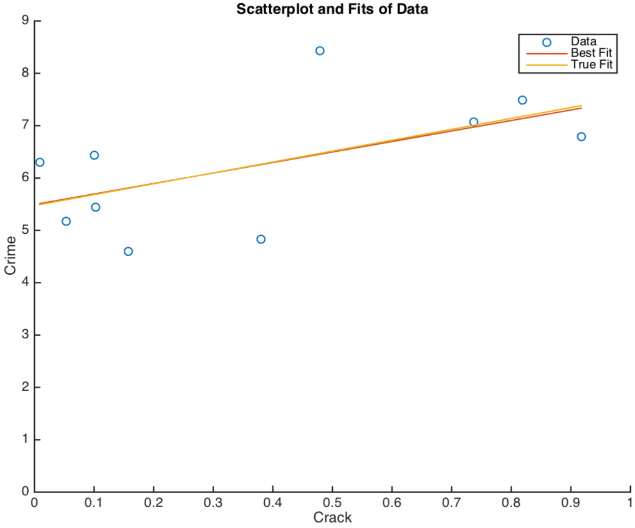

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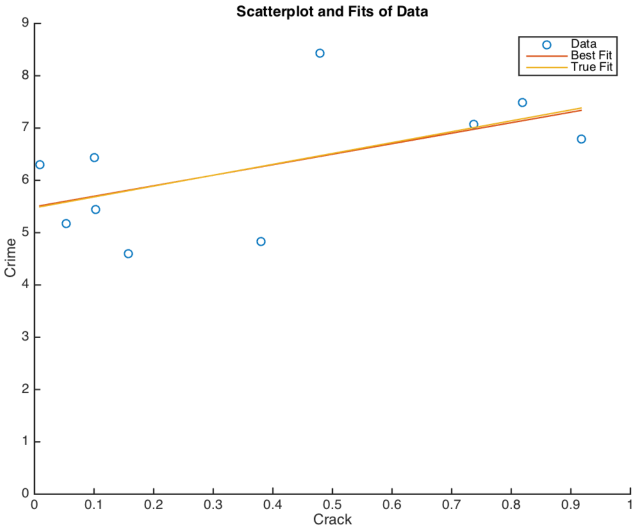
28TH STEP...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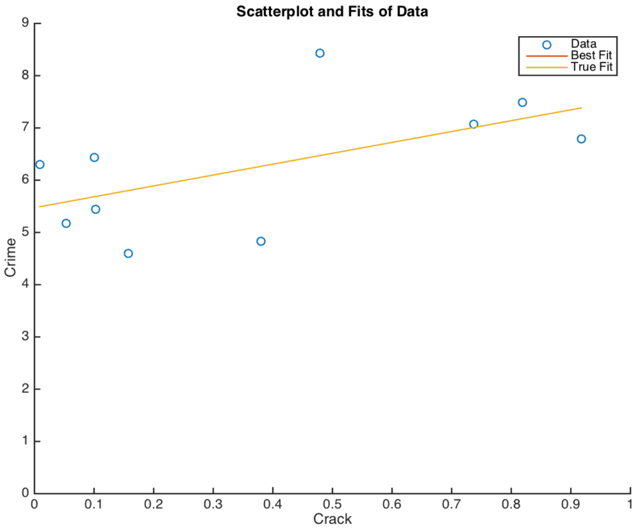
Last step!



Note: took about 863 steps to get both gradients to $< 10^{-10}$.