Numerical Methods-Lecture 11: Neoclassical Growth Model

(See Conesa, Kehoe, Ruhl 2007)

Trevor Gallen

Guidelines on Presentation & Paper

- Pick a topic
- Pick/write down a model that encompasses what you're talking about
  - Things in your model should be in there for a reason
  - Things not in your model should be excluded for a reason
- Solve your model
- *Answer a question*.
- The presentation & paper
  - Motivation (why important)
  - Brief lit review/alternative models
  - Model
  - Solution method
  - Solutions/interesting results
  - Conclusion

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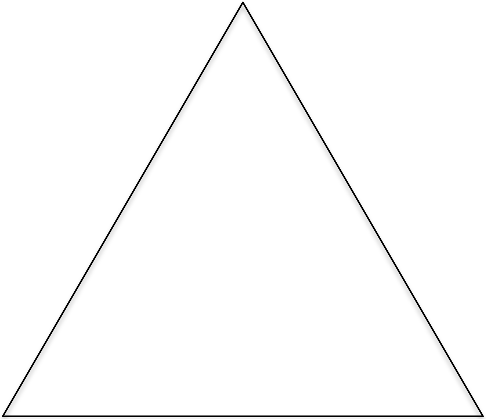
Blanchard's Triangle

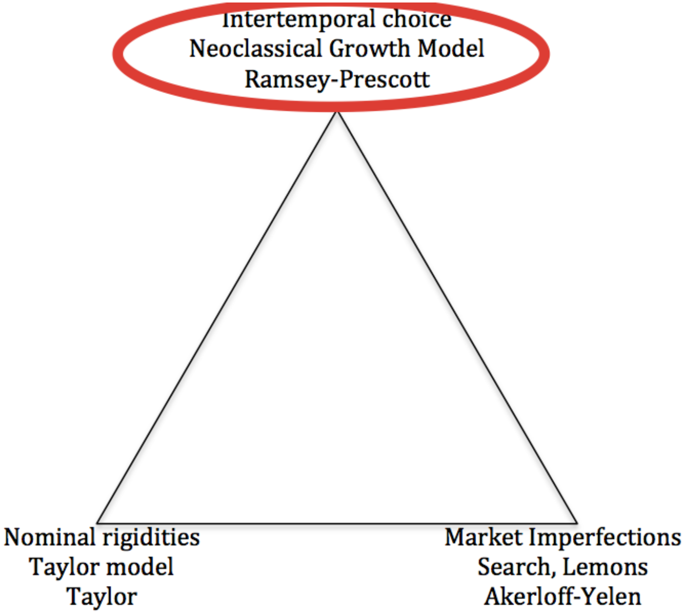
Intertemporal choice
Neoclassical Growth Model
Ramsey-Prescott



Nominal rigidities
Taylor model
Taylor

Market Imperfections
Search, Lemons
Akerloff-Yelen

Blanchard's Triangle



Neoclassical Growth Model

- This is a computational economics course
- It's worth understanding the NCG
- Focus on intertemporal choice and consequent dynamics mean an enormous amount of computational models center on it

Neoclassical Growth Model

- Utility over consumption and leisure:

$$\sum_{t=0}^{\infty} \beta^t u(C_t, 1 - L_t)$$

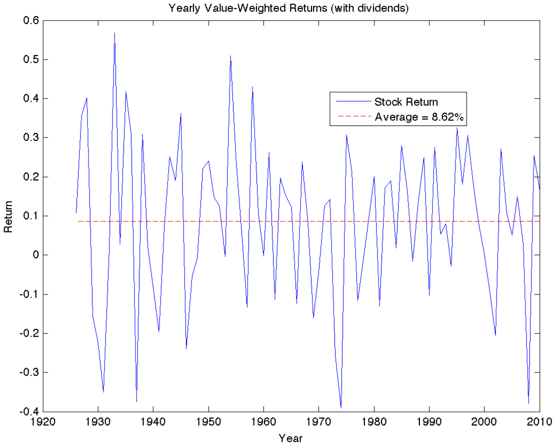
- Law of motion

$$C_t + I_t = Y_t = F(K_t, L_t)$$

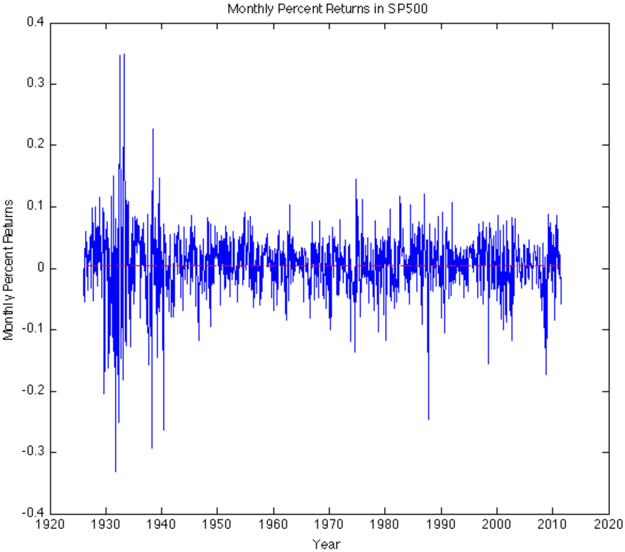
- Law of motion of capital

$$K_{t+1} = (1 - \delta)K_t + I_t$$

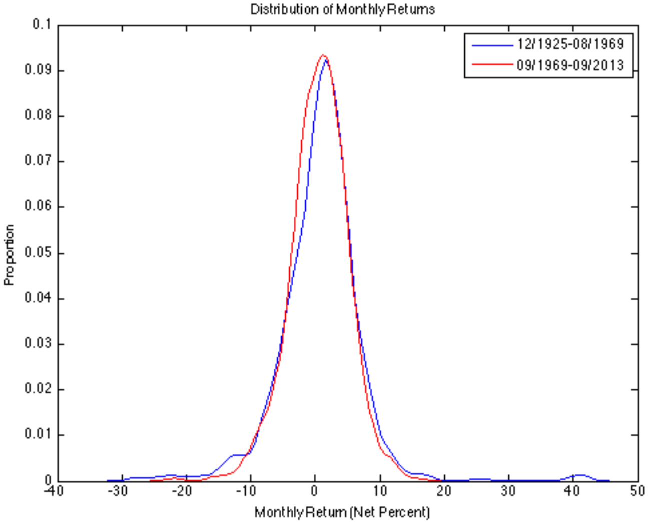
Motivating functional form-I



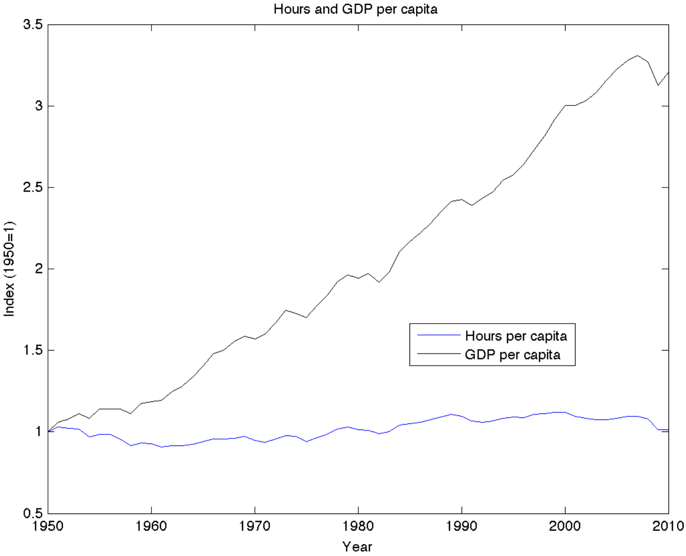
Motivating functional form-II



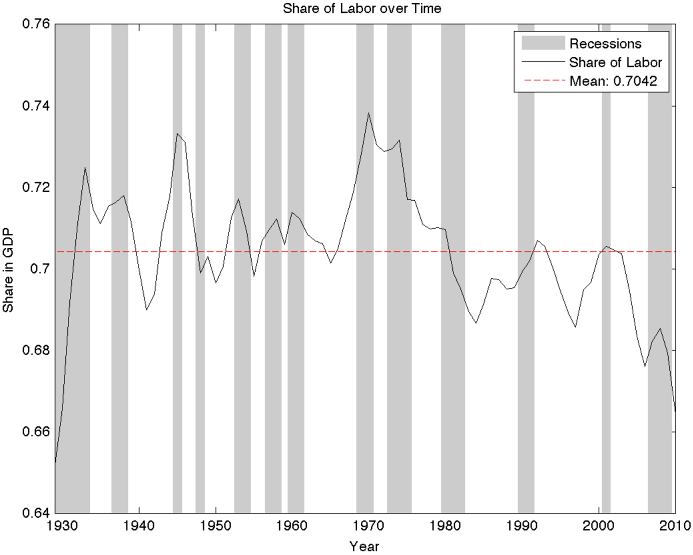
Motivating functional form-III



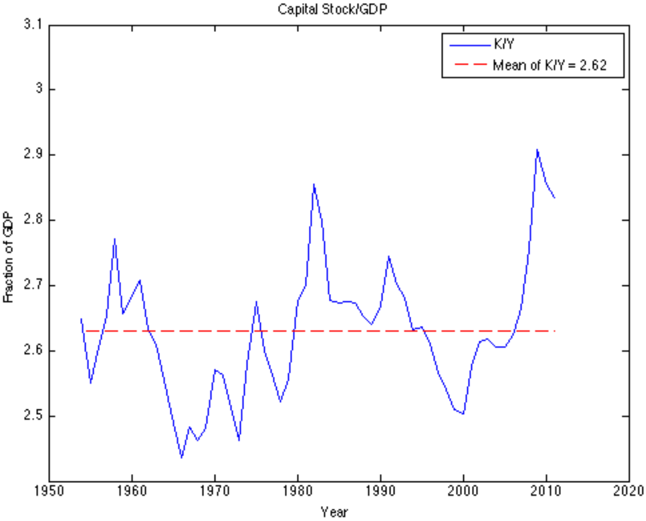
Motivating functional form-IV



Motivating functional form-V



Motivating functional form-VI



Discipline Functional Form

- Stylized/Kaldor's facts (per capita!):
  - Y_t, C_t, K_t, w_t have grown at a constant rate
  - r_t, L_t have not.
  - $(r_t - \delta)K_t/Y_t$, $w_t L_t/Y_t$ have been constant.
- This gives utility you normally see:

$$u(C_t, 1 - L_t) = \frac{(C_t \cdot g(1 - L_t))^{1-\sigma} - 1}{1 - \sigma}$$

or

$$u(C_t, 1 - L_t) = \log(C_t) + \psi \log(1 - L_t)$$

or:

$$u(C_t, 1 - L_t) = \log(C_t) - \psi \frac{\epsilon}{1 + \epsilon} L_t^{\frac{1+\epsilon}{\epsilon}}$$

- And production:

$$Y_t = A_t \left(\xi K_t^{\frac{\sigma-1}{\sigma}} + (1 - \xi) L_t^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

or:

$$Y_t = A_t L_t^{\alpha} K_t^{1-\alpha}$$

Simple NCG

$$\max \sum_{t=0}^{\infty} \beta^{t} \left[\log(c_t) + \psi\left(1 - L_t\right)\right]$$

$$C_t + I_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

$$K_{t+1} = (1-\delta)K_t + I_t$$

Simple NCG: FOC's

Given K_0, each period has 6 equations and 6 unknowns $(C_t, K_{t+1}, L_t, Y_t, w_t, r_t)$

$$
\begin{array}{rcll}
\frac{C_{t+1}}{C_t} & = & \beta(1-\delta+r_{t+1}) & \text{Euler/intertemporal}/K^S \\
\frac{\psi}{1-L_t} & = & \frac{w_t}{C_t} & \text{Intratemporal}/L^S \\
w_t & = & (1-\alpha)\frac{Y_t}{L_t} & L^D \\
r_t & = & \alpha\frac{Y_t}{K_t} & K^D \\
K_{t+1} & = & (1-\delta)K_t + I_t & \text{Law of motion of capital} \\
I_t + C_t & = & Y_t & \text{Feasibility}
\end{array}
$$

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w_t L_t + r_t K_t & = & C_t + I_t & \text{Budget constraint}
\end{array}
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Extra equation?

Simple NCG: FOC's

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$$

Extra equation?

Can reduce to two series.

Bring the model to data: issues?

- $Y_t \neq I_t + C_t$!
  - $Y_t \neq I_t + C_t + G_t + (I_t - X_t)$!
  - Could try to divvy up G and NX
  - Could attribute all to C
  - Does it matter?
- One type of capital
- One price for everything (constant relative price of capital and consumption)
- No terms of trade
- Need to calibrate parameters: β, δ, K_0, ψ, α, and A_t's

Bring the model to data: Step 1

- Calibrate: β, δ, K_0, ψ, α, and A_t's
- Given I_t data and δK_t data, choose δ and K_0 to target:

$$\underbrace{\overline{\frac{\delta K_t}{Y_t}}}_{\textit{Data}} = \underbrace{\overline{\frac{\delta K_t}{Y_t}}}_{\textit{Model}}$$

  And:

$$\frac{K_1}{Y_1} = \frac{1}{10}\sum_{t=1}^{10}\frac{K_t}{Y_t}$$

  That is, average depreciation in the model is the same as average depreciation in the data, and the starting capital/output ratio is comparable to the capital/output ratio over the next 10 periods.
- Calibrate: β, $\cancel{\delta}$, $\cancel{K_0}$, ψ, α, and A_t's

Bring the model to data: Step 2

- Given K_0 and δ, along with I_t, can construct K_t's in data
- Given constructed K_t, L_t and Y_t in data, can calculate TFP:

$$A_t = \frac{Y_t}{L_t^{\alpha} K_t^{1-\alpha}}$$

- Can calibrate α as the average wage share:

$$\alpha = \frac{1}{N} \sum_{t=1}^{N} \frac{w_t L_t}{Y_t}$$

- Calibrate: β, $\cancel{\delta}$, $\cancel{K_0}$, ψ, $\cancel{\alpha}$, and $\cancel{A_t}$'s

Bring the model to data: Step 3

- Set β so that the *expectation* of the intertemporal FOC in the data is true.

$$\beta = \frac{1}{N} \sum_{t=1}^{N} \frac{1}{(1 - \delta + r_{t+1})} \frac{C_{t+1}}{C_t}$$

- Set ψ so that the *expectation* of the intratemporal FOC in the data is true.

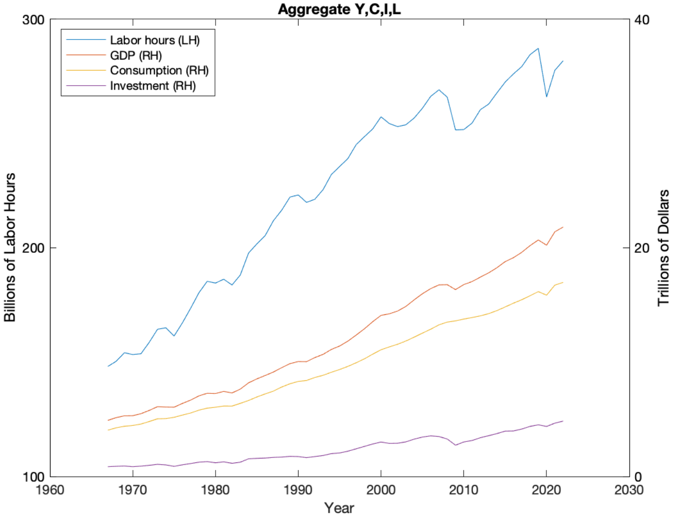
$$\psi = \frac{1}{N} \sum_{t=1}^{N} \frac{w_t(1 - L_t)}{C_t}$$

- Calibrate: $\cancel{\beta}$, $\cancel{\delta}$, $\cancel{K_0}$, $\cancel{\psi}$, $\cancel{\alpha}$, and $\cancel{A_t}$'s

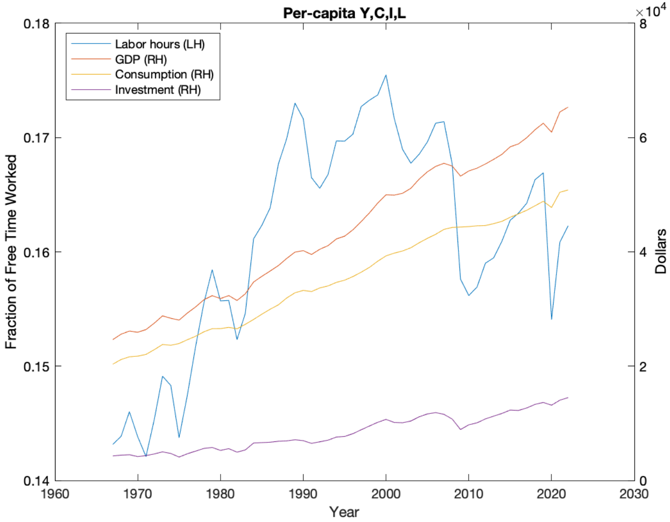
So actually take it to data

- I'm just going to use BEA data
- Download GDP (Y_t), Personal Consumption (C_t) from Table 1.1.6
- Download Gross Domestic Investment I_t and consumption of fixed capital δK_t from Table 5.2.6.
- Download Population from Table 7.1
- Download Hours and Employment from OECD
- Let's look at the series

GDP, Investment, Consumption, Labor hours

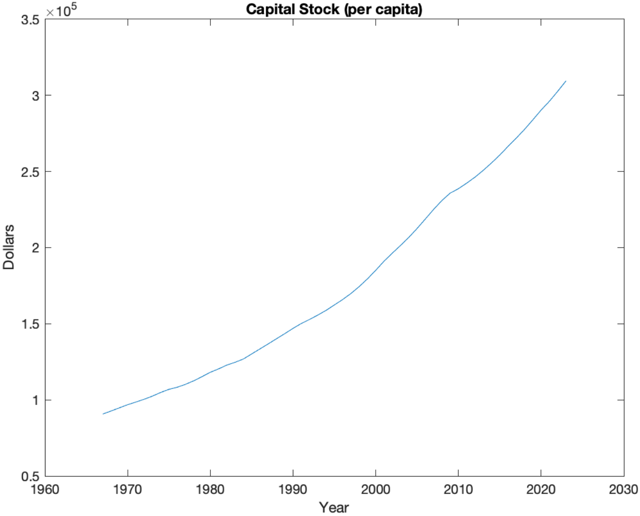


Per Capita

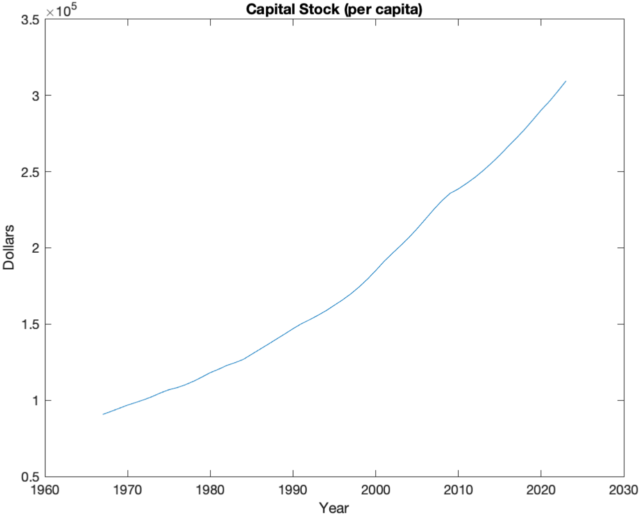


Per-capita Y,C,I,L
Labor hours (LH)
GDP (RH)
Consumption (RH)
Investment (RH)
Fraction of Free Time Worked
Dollars
Year
×10^4
0.18
0.17
0.16
0.15
0.14
8
6
4
2
0
1960
1970
1980
1990
2000
2010
2020
2030

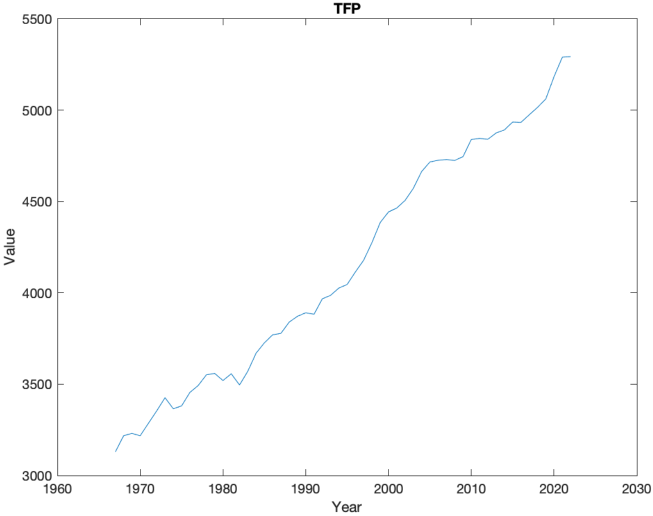
Create Capital Stock



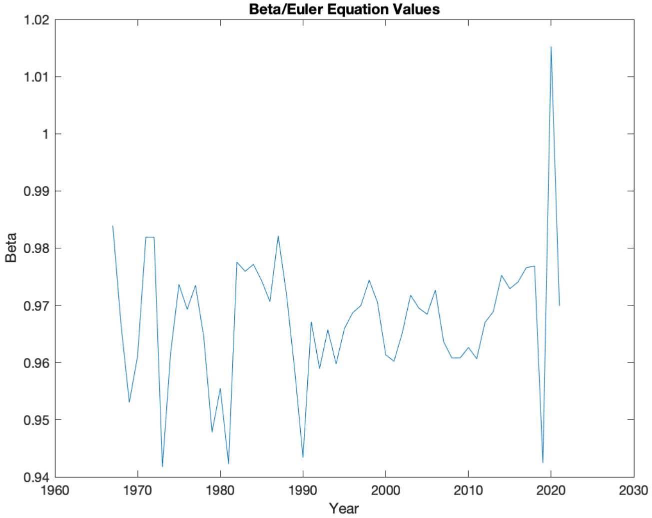
Create Capital Stock: $K_{t+1} = (1 - \delta)K_t + I_t$



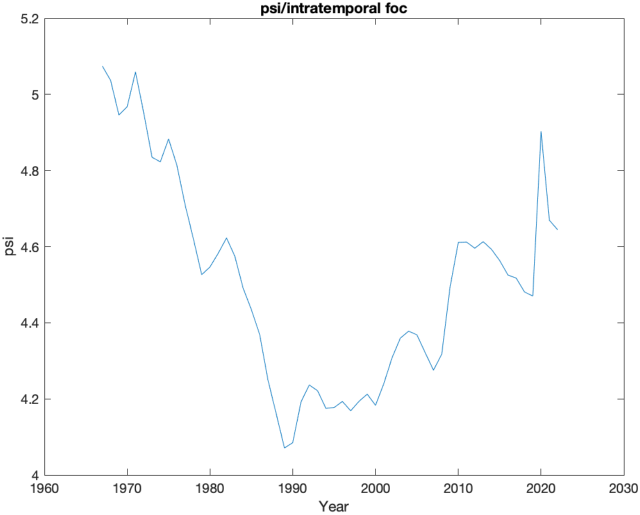
Find TFP: $A_t = \frac{Y_t}{L_t^{\alpha} K_t^{1-\alpha}}$



Find $\beta = \frac{1}{1-\delta+r_{t+1}} \frac{c_{t+1}}{c_t}$



FIND $\psi = \frac{w_t(1-L_t)}{c_t}$



Calibration

- $k_0 = 63396$
- $\delta = 0.043$
- $\beta = 0.963$
- $\psi = 4.58$
- $\alpha = 0.7$

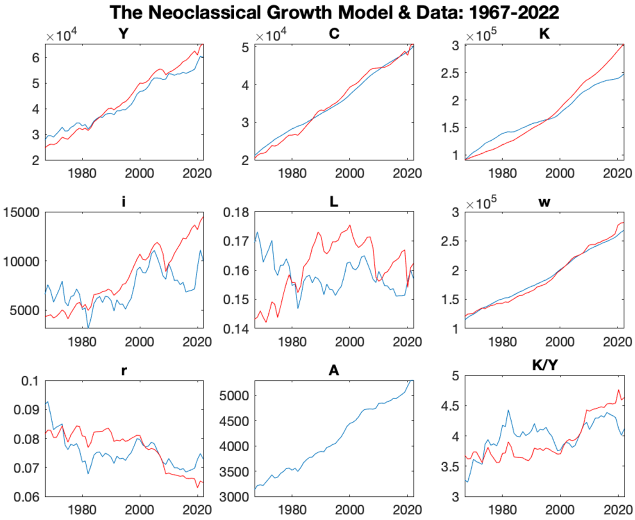
Solution and Assumptions

- There are many possible assumptions with respect to A_t
- A first step is complete knowledge of A_t
- Assume no labor, consumption, or capital taxes
- What about future A_t?
- Continue A_t growth constantly into the future
- Force to reach steady state growth path 20 years into constant future

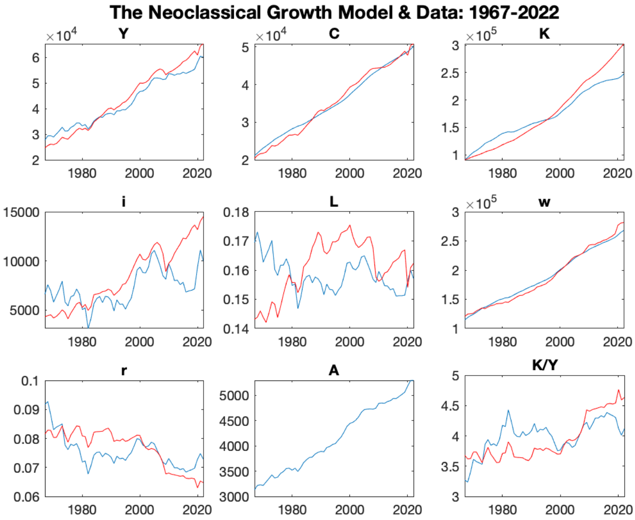
Solution Method

- Given parameters, a path of A_t, and a k_0, I have two decisions to make:
  - $L_t \times T$ labor decisions from intratemporal FOC
  - $K_{t+1} \times T$ capital decisions from intertemporal FOC
- Solve for 40 existing years, then give 30 years to reach SS growth path
- Then, for 30 more periods, assume they're on autopilot
- They only get to choose 70×2 choices to solve $70 \times 2 + 30 \times 2$ FOC's!
- Make sure all FOC's are zero

Theory and Data

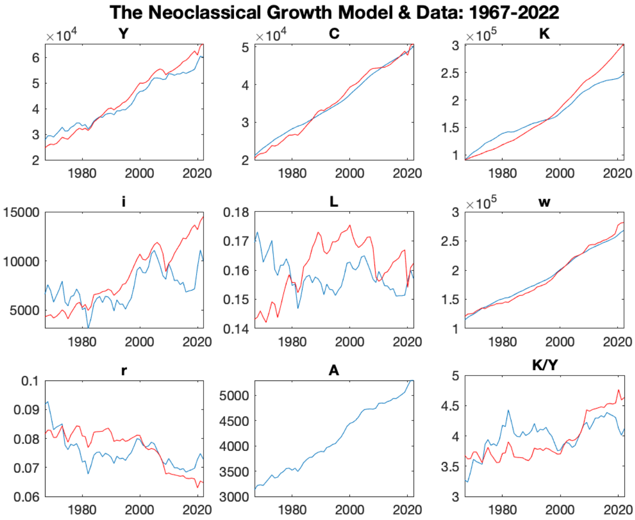


Theory and Data



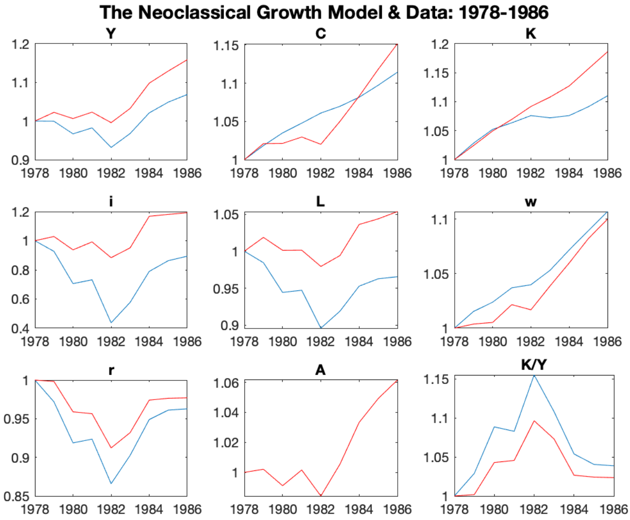
Which is theory, which is data?

Theory and Data

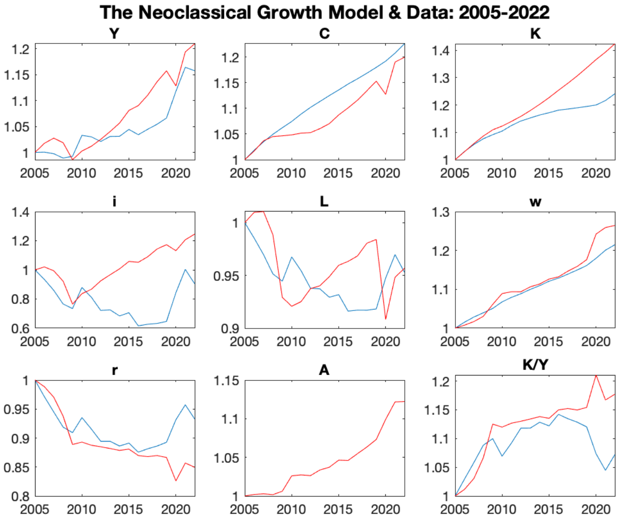


Which is theory, which is data? Red is data! (Look at L)

Explaining the early 1980's



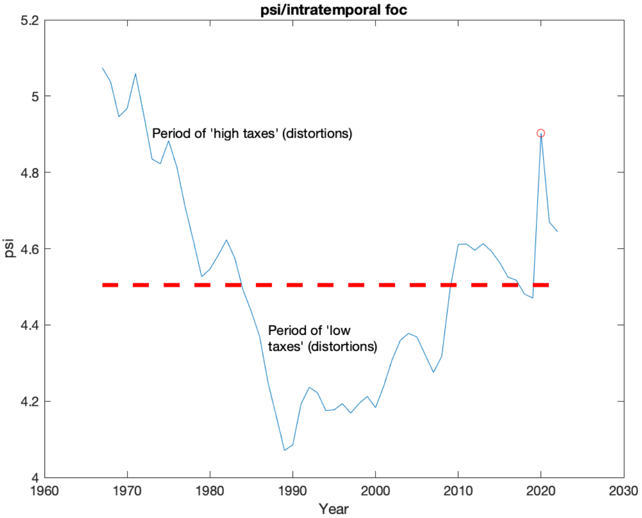
Explaining the late 2000's



Aside on Wedges

- Macro, IO, trade, and much of labor take models+data seriously
- Can ask "what (invisible) tax would explain the behavior I see?"
- Put another way, feed in some labor tax τ that makes our model behave like the data
- This is "wedge accounting" a la Mulligan (2002) Chari Kehoe McGrattan (2004/2007)
- Same concept as a Solow residual
- This is also how MLE estimation of these processes works: given a time series process for residual, choose parameters to maximize likelihood of seeing observations

Find $\psi = (1 - \tau_t)\frac{w_t(1-L_t)}{c_t}$



Choose ψ to be mean, then $\psi = (1 - \tau_t)\frac{w_t(1-L_t)}{c_t}$. Note 2020!

Aside on Wedges-II

- Many don't like wedges, why have invisible process that reconciles theory with data? Theory is always true!?
- Misses the point!
- Wedges are diagnostics: measurements of what in the data doesn't make sense
- If your change to the model helps make wedges small, it's progress!
- **Wedges are a quantitative way of describing puzzles and their (sometimes partial) resolution**
- Finally, note that "wedges" aren't always tax wedges–could be preference shocks, etc.

(Macro)Economics as a science

- In similar position to meteorology, astronomy, geology
- No randomized experiments
- Data is too short for nonparametric analysis
- Ideal (and solution) a unified theory: explain all frequencies
- Parsimony keeps us honest
- Explaining multiple stochastic processes jointly keeps us honest
- Explaining dynamics gives more role for testable hypotheses