

# Solving a Lifecycle Model

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This homework is the first part of a multi-part homework. This homework starts us with a relatively simple deterministic lifecycle model and asks you to solve it in Matlab.

## Deliverables

- You should have a word/L<sup>A</sup>T<sub>E</sub>X document that has three sections:
  1. Discusses the model and answers the questions I pose throughout.
  2. Contains the tables and figures you will produce.
  3. Contains a discussion of your programming choices if you had to make any.
- You should have a Matlab file or set of files (zipped) that contain **all** your programs and raw data. There should be a file called “Main.M” that produces everything I need in one click.

## 1 Model

Households have period utility over consumption  $c_t$  and labor  $L_t$  at time  $t$ :  $u(c_t, L_t)$ , which takes the form of balanced growth preferences with constant Frisch elasticity of labor supply:

$$u(c_t, L_t) = \log(c_t) - \psi \frac{\epsilon}{1 + \epsilon} L_t^{\frac{1+\epsilon}{\epsilon}}$$

Letting  $s$  denote savings,  $w$  denote wages,  $r$  denote interest rates, households have period budget constraint:

$$c_t + s_t = w_t L_t + (1 + r_t) s_{t-1}$$

Households maximize the net present value of utility, discounted at a rate  $0 < \beta < 1$ . Assume that households live until period  $T$ .

**Question 1a:** Write out the household's net present value budget constraint.

**Question 1b:** Write out the household's sequence problem.

**Question 1c:** Write out the household's value function problem.

## 2 Basic Matlab

Now, let the following numerical assumptions hold:

Table 1: Calibration		
Concept	Parameter	Value
Lifespan	$T$	45
Disutility of labor	$\psi$	10
Frisch elasticity of labor supply	$\epsilon$	0.75
Discount factor	$\beta$	0.96
Wages: $w_t, t \neq 30$	$w_{t \neq 30}$	1
Wages: $w_{30}$	$w_{t=30}$	1.05
Interest rates: $r_t, t \neq 20$	$r_{t \neq 30}$	0.05
Interest rates $r_{20}$	$w_{t=20}$	1.1

**Question 2:** Write a Matlab program that does the following.

- Section 1a: Defines all exogenous parameters in Table 1, including making  $w_t$  and  $r_t$  45-entry column vectors. (It is important to be consistent in terms of column vs. row).
- Section 1b: Defines a NPV discount rate for all 45 periods (e.g. contains  $[1; 1/(1 + r_1); 1/((1 + r_1) * (1 + r_2))]$  and so on. You may find Matlab's cumulative product function "cumprod" useful.
- Section 2a: Define the utility function, which takes in  $c$  and  $L$  (as inputs) and spits out utility (given  $\psi$  and  $\epsilon$ , which are set by Table 1. Write it so you can give the function a vector of  $c$  and  $L$  and it will spit out a vector of utilities.

$$ut(c, L) = \log(c_t) - \psi \frac{\epsilon}{1 + \epsilon} L_t^{1 + \epsilon}$$

- Section 2b: A function of the lifetime excess savings, which takes in a vector of  $c_t$ 's and a vector of  $w_t$ 's and  $L_t$ 's, and spits out excess savings

$$bc\_lifetime(c, L) = \sum_{t=1}^T \frac{c_t}{\prod_{\tau=1}^t (1 + r_\tau)} - \sum_{t=1}^T \frac{w_t L_t}{\prod_{\tau=1}^t (1 + r_\tau)}$$

- Section 2c: A lifetime utility function which penalizes debt (a simple way of making constrained optimization unconstrained), e.g.:

$$ut\_lifetime(c, L) = \text{sum}(\text{beta}.[0 : T-1]ut(c, L)) - 10 * (bc\_lifetime(c, L) > 0) * bc\_lifetime(c, L)$$

- Section 3: Create an initial guess, and pass the negative of the life to a *minimizer*, such as *fminunc*. You may have to think about how to pass the anonymous arguments.

- Section 4: Graph out  $c_t$  and  $L_t$ . Clearly label your graphs.
- Section 5: Answer the following questions:
  - What is the intertemporal elasticity of substitution suggested by consumption's response in period 20?
  - What is the Frisch elasticity of labor supply suggested by labor's response in period 30?
  - Why is consumption rising over time?
  - What, in the calibration and functional form choice drove the numerical answers to the previous three questions? What parameterization of  $\beta$  would make  $c_{t+1}/c_t$  flat?

### 3 Research

You now have a very simple deterministic numerical model of people's labor over the lifecycle. However, the model is too simple to be particularly interesting. Change the model in an interesting way, and report your results. This is your opportunity to actually think, so please take time with this: how can you adjust the model, parameterization, etc. to make this model (a) more interesting and (b) more realistic?