Heterogeneous Agents

(See Krusell Smith 1998)

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Introduction

- We spend an enormous time on representative agents
- Model has been quite fruitful
- But there are theoretical reasons to think that a RA model wouldn't capture everything
- What about heterogeneity? Income constraints?

Krusell Smith

- Take same basic NCG model we've been using
- We don't care who owns what: only the *total* income and capital in the society matter
- It's plausible to think that the *distribution* matters
- Now, people not only face aggregate uncertainty but also idiosyncratic incomeemployment shocks, and that they can't borrow past an exogenously-set lower bound.
- Because you can't insure your shocks, there's a wealth distribution

The Environment

- People have preferences over their stream of consumption c_t:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t)$$

- With:

$$U(c) = \lim_{\nu \to \sigma} \frac{c^{1-\nu} - 1}{1 - \nu}$$

- Aggregate production y:

$$y = c + k' - (1 - \delta)k$$

- Labor supplied is $\epsilon \tilde{l}$, where $\epsilon \in \{0, 1\}$ is exogenous

- Also have aggregate shock $z \in \{b, g\}$, correlated with ϵ

The Environment: Shocks

- Probability transition $\pi_{ss'\epsilon\epsilon'}$, denotes your probability of moving to state s' from state s and at the same time to state ϵ' from state ϵ.

- All inflows/outflows are balanced, so that (conditioning on z), we have independence across individuals

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- In addition, the aggregate number of households in the bad state is always u_g or u_b, depending on the state:

$$u_s \frac{\pi_{ss'00}}{\pi_{ss'}} + (1 - u_s) \frac{\pi_{ss'10}}{\pi_{ss'}} = u_{s'}$$

State Variables-I

- There is only one asset: aggregate capital
- Denoting aggregate capital as $\bar{k}$ and aggregate labor as $\bar{l}$:

$$w(\bar{k}, \bar{l}, z) = (1-\alpha)z\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha} \qquad r(\bar{k}, \bar{l}, z) = \alpha z\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha-1}$$

- In order to know what w and r will be, I need to know...
- ...what $\bar{k}$ and $\bar{l}$ will be!
- $\bar{k}$ and $\bar{l}$ come from everyone...I need to know the distribution of capital by employment status, called Γ, as well as my standard z.

State Variables-II

- I need to know the distribution of capital, Γ
- To plan for tomorrow, I need to know the *law of motion* of the distribution, to find Γ'.
- Call this law of motion of the distribution $H(\gamma, z, z')$
- Then for an individual, he needs to know his own capital, his own employment, the distribution of capital, and aggregate productivity: (k, ϵ, Γ, z)

Optimization Problem

- The agent's optimization problem is therefore:

$$V(k,\epsilon,\Gamma,z) = \max_{c,k'} \left[U(c) + \beta E(V(k',\epsilon';\Gamma,z')) \right]$$

- Subject to:

$$c + k' = r(\bar{k},\bar{l},z)k + w(\bar{k},\bar{l},z)\tilde{l}\epsilon + (1-\delta)k$$

$$\Gamma' = H(\Gamma,z,z')$$

$$k' \geq 0$$

- Solving this problem, we get:

$$k' = f(k,\epsilon,\Gamma,z)$$

Equilbrium

Equilibrium is:

1. H, the law of motion for Γ, consistent with f
2. V and f, the individual's value and policy functions
3. r and w, pricing functions that clear markets given the consumer's V and f

Do you see the problem?

A solution(?)

- How can we characterize a distribution?
- Only give the agents the first m (statistical!) moments of the distribution and make their best guess
- But then...we still don't have a good law of motion, consistent with f?

A solution algorithm

1. Summarize distribution by first m statistical moments
2. *Assume* a law of motion for agents
3. Solve and simulate behavior (inner loop)
4. From simulated behavior, solve for new law of motion.
5. If new law of motion is different, go back to step (3). Otherwise, proceed.
6. If result is different from with $m-1$ moments, add a moment. If not, end.

Model parameters

Assume some parameters

- Period of one quarter
- $\beta = 0.99$
- CRRA $\sigma = 1$
- Capital share $\alpha = 0.36$
- Good and bad shock: $z_g = 1.01$ & $z_b = 0.99$
- Unemployment rates: $u_g = 0.04$ & $u_b = 0.10$
- Choose process for (z, ϵ) so:
  - Average duration of good and bad times is 8 quarters
  - Average duration of an unemployment spell is 1.5 quarters in good times and 2.5 quarters in bad times

Results: Approximate Aggregation

Assume some parameters

- Only the mean of capital matters, predicts 99.9998% of variation in capital
- Better prediction techniques would mean nothing
- Caution: self-fulfilling approximate equilbiria *might* exist...
- But no evidence for this

Results: Why only the mean?

Assume some parameters

- Fundamentally, all that matters is your propensity to save out of wealth
- If everyone always saves the same proportion of wealth, it doesn't matter who has the wealth
- Savings behavior is only atypical for the very poor
- But the really poor don't matter for aggregate capital

Some issues

Assume some parameters

- Model distribution (entirely endogenous from labor) is not skewed enough
- Reality: poorest 20% have 0% wealth.
- Model: poorest 20% have 9% wealth
- Reality: richest 5% have 50% wealth.
- Model: richest 5% have 11% wealth
- How do we generate this?
  - Random discount factors
  - Differences in unemployed income
- These can nail the distribution
- With a more reasonable wealth distribution, nothing changes

Aggregate Time Series

- Lack of full insurance increases capital by 0.6% in the baseline.
- Up to 6.7% with high risk aversion
- Can get more hand-to-mouth with different β's, aggregate no longer looks like PHI
- Not many differences between representative agent and heterogeneous agent, except PIH-type behavior.

Conclusions

- Novel way to introduce interacting agents.
- Reminds us that bounded rationality w.r.t. expectations is very easy with Bellmans
- No change from heterogeneous agents is a result!

Computational Course Takeaways

- Economic intuition frequently more helpful than computational skill in solving computational problems
- Structural estimation on normal (well-behaved) problems is very easy (NFP)
  1. Make guesses at parameters
  2. Solve model
  3. Calculate difference between moments and reality (or likelihood)
  4. Go back to step 1 with this data, update parameters
- True in CGE models where you might (1) solve individual problems given w and θ, (2) clear markets given L(θ), then calculate moments
- True in dynamic choice models where you might (1) solve for Bellman given θ then choose θ to fit moments
- True in heterogeneous-agent models where you (1) assume parameters, a law of motion (2) solve VFI given parameters, law of motion (3) simulate law of motion (4) return to step #2 until law of motion is accurate description (5) return to step #1 to change parameters.