Kaldor Facts

Trevor Gallen

Winter, 2014

Labor and Capital

Break all inputs into two: labor (L) and capital (K), paid wage w and rental rate r:

$$\pi = Y - wL - rK$$

If Y is CRS and competitive, then "factor payments exhaust product":

$$\pi = 0 \Rightarrow Y = wL + rK$$

We're going to look at all of these pieces.

Labor and Capital

Break all inputs into two: labor (L) and capital (K), paid wage w and rental rate r:

$$\pi = Y - wL - rK$$

If Y is CRS and competitive, then "factor payments exhaust

product":

$$\pi = 0 \Rightarrow Y = wL + rK$$

We're going to look at all of these pieces.

Given this production function for a firm, does CRS require perfect competition?

Labor and Capital Growth

$$\overbrace{Y}^{\uparrow} = \overbrace{w}^{\uparrow}\overbrace{L}^{\uparrow} + \overbrace{r}^{\cdot}\overbrace{K}^{\uparrow}$$

Divide both sides by the number of people: ($l = \frac{L}{N}$, etc.)

$$\overbrace{y}^{\uparrow} = \overbrace{w}^{\uparrow}\overbrace{l}^{\cdot} + \overbrace{r}^{\cdot}\overbrace{k}^{\uparrow}$$

Stylizing this:

$$y_{t+1} = (1+\gamma)y_t$$

$$w_{t+1} = (1+\gamma)w_t$$

$$K_{t+1} = (1+\gamma)K_t$$

This has a lot of implications!

1) The share of wages and the share of profits in the national income has shown a remarkable constancy in “developed” capitalist economies of the United States and the United Kingdom since the second half of the nineteenth century.

1) The share of wages and the share of profits in the national income has shown a remarkable constancy in “developed” capitalist economies of the United States and the United Kingdom since the second half of the nineteenth century.

$$\frac{w_i N_i}{Y_i} = c_1 \qquad \frac{r_i K_i}{Y_i} = c_2$$

1) The share of wages and the share of profits in the national income has shown a remarkable constancy in "developed" capitalist economies of the United States and the United Kingdom since the second half of the nineteenth century.

$$\frac{w_i N_i}{Y_i} = c_1 \qquad \frac{r_i K_i}{Y_i} = c_2$$

1*) Capital and labors share are constant over time

2) The value of the capital equipment per worker (at constant prices) is steadily rising.

2) The value of the capital equipment per worker (at constant prices) is steadily rising.

$$\frac{K_{i,t+1}}{N_{i,t+1}} = (1+\gamma_{K/N})\frac{K_{i,t}}{N_{i,t}}$$

2) The value of the capital equipment per worker (at constant prices) is steadily rising.

$$\frac{K_{i,t+1}}{N_{i,t+1}} = (1 + \gamma_{K/N})\frac{K_{i,t}}{N_{i,t}}$$

2*) Capital per capita grows at a constant rate.

3)The value of the annual output per worker (at constant prices) is steadily rising.

3)The value of the annual output per worker (at constant prices) is steadily rising.

$$\frac{Y_{i,t+1}}{N_{i,t+1}} = (1 + \gamma_{Y/N})\frac{Y_{i,t}}{N_{i,t}}$$

3)The value of the annual output per worker (at constant prices) is steadily rising.

$$\frac{Y_{i,t+1}}{N_{i,t+1}} = (1+\gamma_{Y/N})\frac{Y_{i,t}}{N_{i,t}}$$

3^*) Output per capita grows at a constant rate.

4)The trend rates in both capital and annual output are about the same, so the capital/output ratio is the same.

4)The trend rates in both capital and annual output are about the same, so the capital/output ratio is the same.

$$\gamma_{K/N} = \gamma_{Y/N} \text{ so } \frac{K_{i,t+1}}{Y_{i,t+1}} = \frac{K_{i,t}}{Y_{i,t}}$$

4)The trend rates in both capital and annual output are about the same, so the capital/output ratio is the same.

$$\gamma_{K/N} = \gamma_{Y/N} \text{ so } \frac{K_{i,t+1}}{Y_{i,t+1}} = \frac{K_{i,t}}{Y_{i,t}}$$

4*)Capital-Output ratio grows over time.

4)The trend rates in both capital and annual output are about the same, so the capital/output ratio is the same.

$$\gamma_{K/N} = \gamma_{Y/N} \ \text{ so } \ \frac{K_{i,t+1}}{Y_{i,t+1}} = \frac{K_{i,t}}{Y_{i,t}}$$

~~4*)Capital-Output ratio grows over time.~~ (Pay attention!)

4)The trend rates in both capital and annual output are about the same, so the capital/output ratio is the same.

$$\gamma_{K/N} = \gamma_{Y/N} \text{ so } \frac{K_{i,t+1}}{Y_{i,t+1}} = \frac{K_{i,t}}{Y_{i,t}}$$

~~4*)Capital-Output ratio grows over time.~~ (Pay attention!)

4*)Capital-Output ratio is constant.

4)The trend rates in both capital and annual output are about the same, so the capital/output ratio is the same.

$$\gamma_{K/N} = \gamma_{Y/N} \quad \text{so} \quad \frac{K_{i,t+1}}{Y_{i,t+1}} = \frac{K_{i,t}}{Y_{i,t}}$$

4*)Capital-Output ratio is constant over time.

5)Constancy in the share of profit and in the capital/output ratio also involves constancy in the rate of profit earned on investments.

5)Constancy in the share of profit and in the capital/output ratio also involves constancy in the rate of profit earned on investments.

$$r_t = c_3$$

5)Constancy in the share of profit and in the capital/output ratio also involves constancy in the rate of profit earned on investments.

$$r_t = c_3$$

5*) Return to capital is constant.

6)Some societies grow faster than others.

6)Some societies grow faster than others.

$$\gamma_{i,K/N} \neq \gamma_{j,K/N}$$

6)Some societies grow faster than others.

$$\gamma_{i,K/N} \neq \gamma_{j,K/N}$$

6*)Growth in output per capita differs across countries.

Measuring GDP & Share of GDP

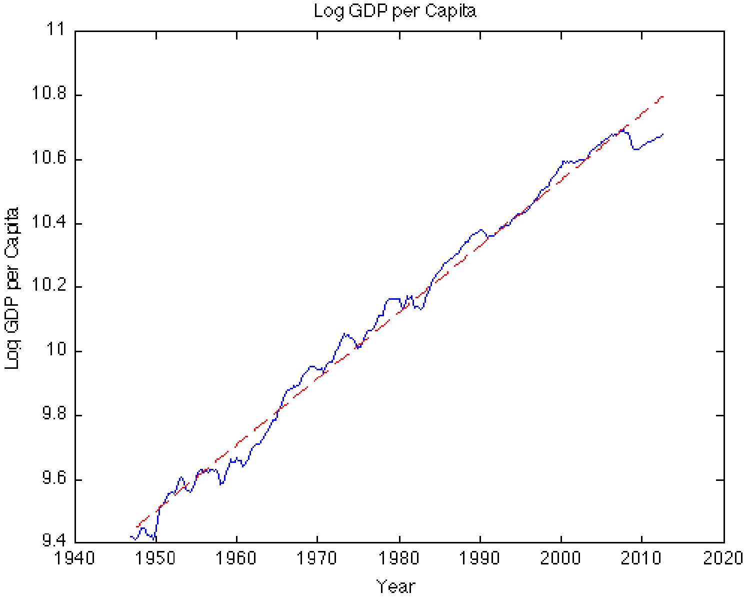
Table 2.A. Summary National Income and Product Accounts, 2008

[Billions of dollars]

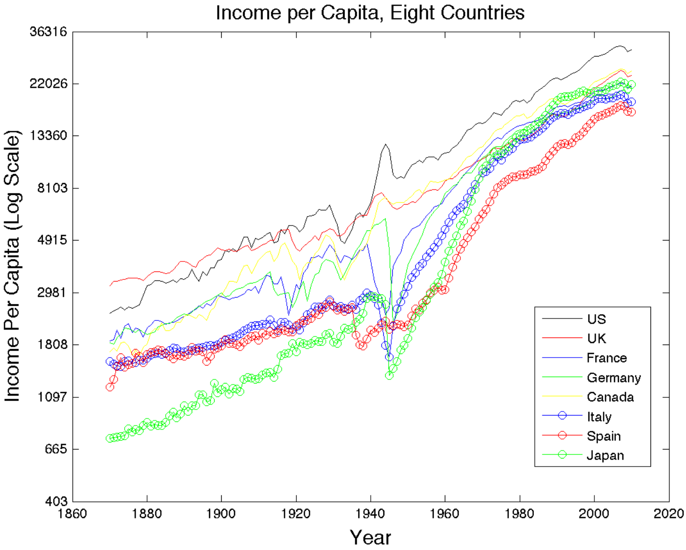
Account 1. Domestic Income and Product Account

Line			Line		
1	Compensation of employees, paid	8,044.8	15	Personal consumption expenditures (3–3)	10,129.9
2	Wage and salary accruals	6,548.2	16	Goods	3,403.2
3	Disbursements (3–12 plus 5–11)	6,553.2	17	Durable goods	1,095.2
4	Wage accruals less disbursements (4–9 plus 6–13)	–5.0	18	Nondurable goods	2,308.0
5	Supplements to wages and salaries (3–14)	1,496.6	19	Services	6,726.8
6	Taxes on production and imports (4–16)	1,047.3	20	Gross private domestic investment	2,136.1
7	*Less:* Subsidies (4–8)	53.5	21	Fixed investment (6–2)	2,170.8
8	Net operating surplus	3,454.8	22	Nonresidential	1,693.6
9	Private enterprises (2–19)	3,461.7	23	Structures	609.5
10	Current surplus of government enterprises (4–26)	–6.9	24	Equipment and software	1,084.1
11	Consumption of fixed capital (6–13)	1,847.1	25	Residential	477.2
			26	Change in private inventories (6–4)	–34.8
12	**Gross domestic income**	**14,340.4**	27	Net exports of goods and services	–707.8
			28	Exports (5–1)	1,831.1
13	Statistical discrepancy (6–21)	101.0	29	Imports (5–9)	2,538.9
			30	Government consumption expenditures and gross investment (4–1 plus 6–3)	2,883.2
			31	Federal	1,082.6
			32	National defense	737.9
			33	Nondefense	344.7
			34	State and local	1,800.6
14	**GROSS DOMESTIC PRODUCT**	**14,441.4**	35	**GROSS DOMESTIC PRODUCT**	**14,441.4**

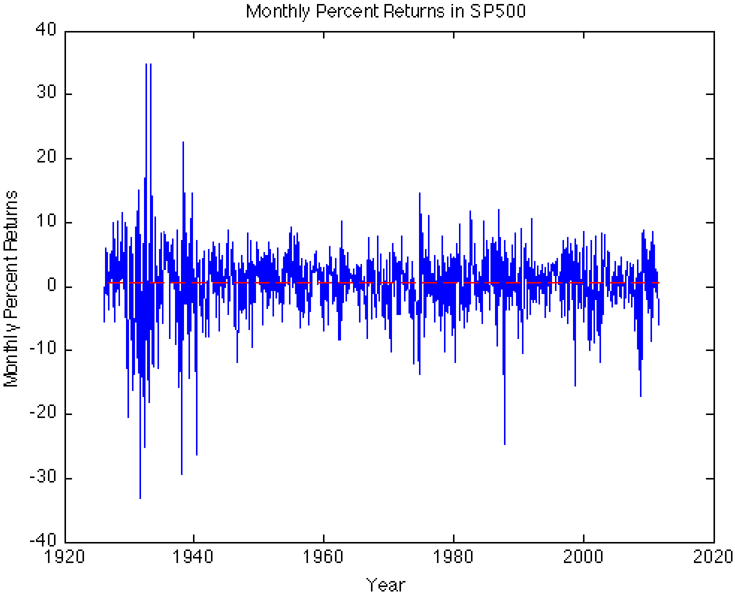
GDP has a Constant Growth Rate



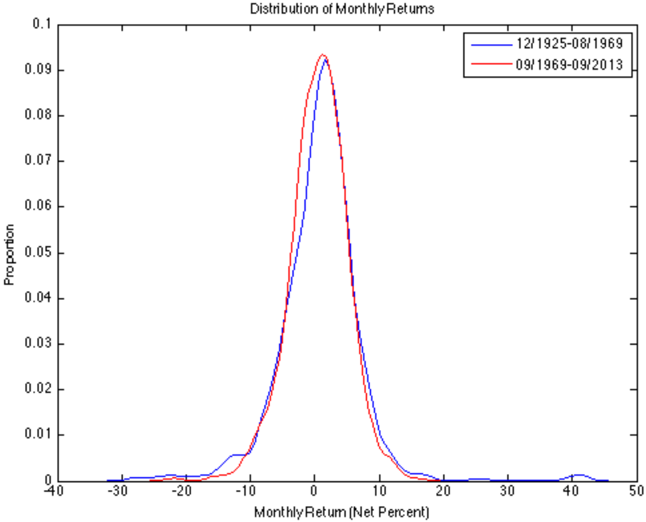
GDP has an Underlying Trend



Capital Returns come from a Stationary Distribution

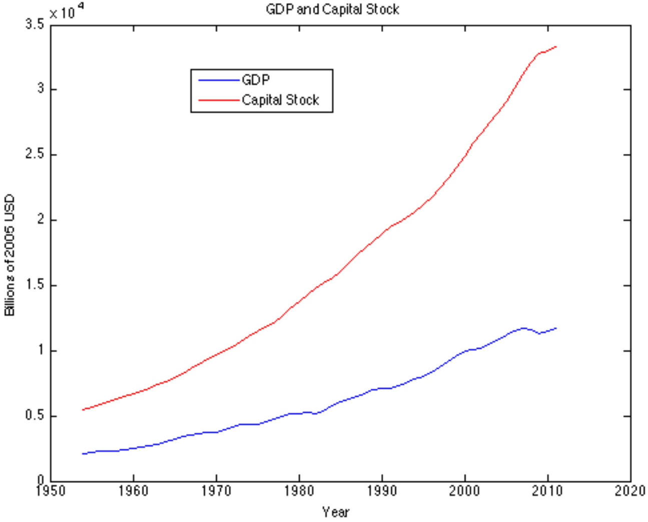


Capital Returns come from a Stationary Distribution



K-S test: p-value of 0.54: fail to reject.

GDP and Capital Stock have a Constant Relationship



Aside: How do we measure capital stock?

We typically use the "permanent inventory method," letting K_t be the capital stock in period t and i be gross investment

$$K_{t+1} = f(K_t, i_t)$$

Aside: How do we measure capital stock?

We typically use the "permanent inventory method," letting K_t be the capital stock in period t and i be gross investment

$$K_{t+1} = f(K_t, i_t)$$

The most common form for the law of motion of capital is:

$$K_{t+1} = (1 - \delta)K_t + i_t$$

What is δ?

Aside: How do we measure capital stock?

We typically use the "permanent inventory method," letting K_t be the capital stock in period t and i be gross investment

$$K_{t+1} = f(K_t, i_t)$$

The most common form for the law of motion of capital is:

$$K_{t+1} = (1 - \delta)K_t + i_t$$

What is δ?

Where might assuming constant depreciation δ cause problems?

Aside: How do we measure capital stock?

We typically use the "permanent inventory method," letting K_t be the capital stock in period t and i be gross investment

$$K_{t+1} = f(K_t, i_t)$$

The most common form for the law of motion of capital is:

$$K_{t+1} = (1 - \delta)K_t + i_t$$

What is δ?

Where might assuming constant depreciation δ cause problems?

Cambridge capital controversy

Aside: How do we measure capital stock?

We typically use the "permanent inventory method," letting K_t be the capital stock in period t and i be gross investment

$$K_{t+1} = f(K_t, i_t)$$

The most common form for the law of motion of capital is:

$$K_{t+1} = (1 - \delta)K_t + i_t$$

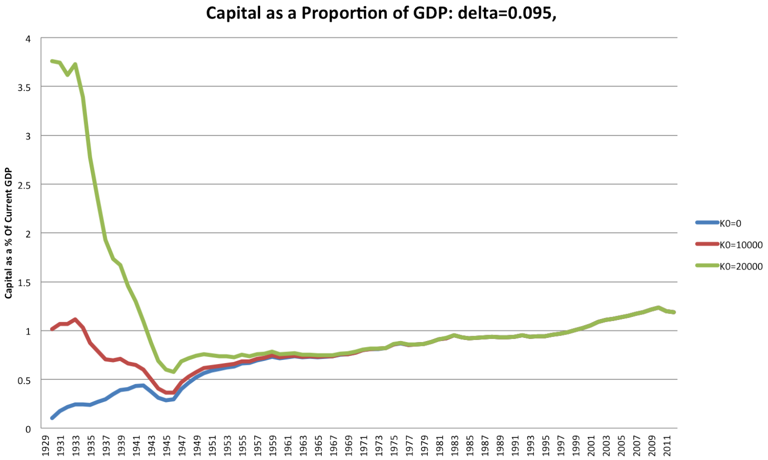
What is δ?

Where might assuming constant depreciation δ cause problems?

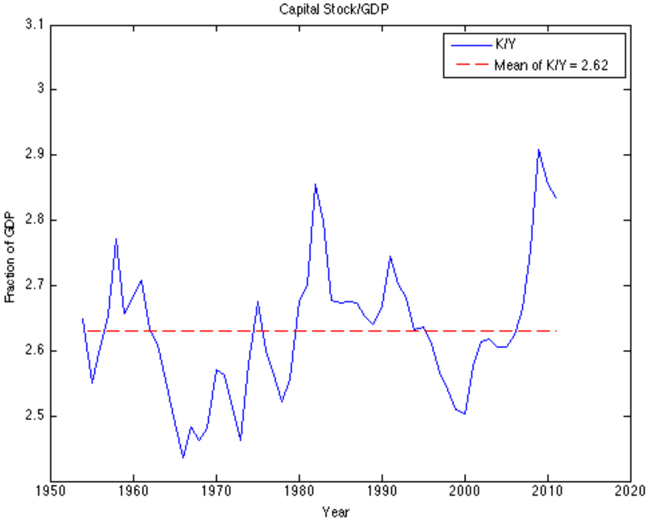
Cambridge capital controversy

Do we need to know K_0?

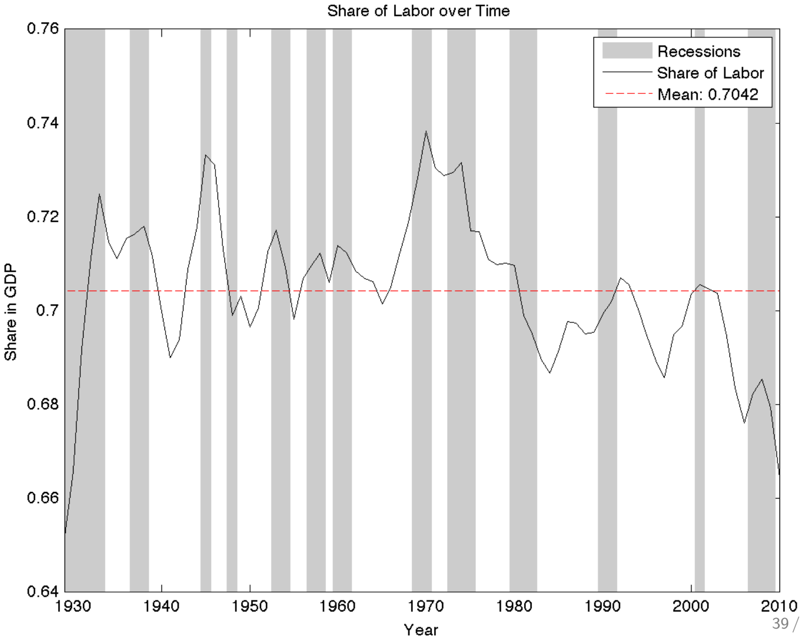
We don't really need to know K_0!



GDP and Capital Stock have a Constant Relationship-II



Share of Labor over Time is Constant



Economic possibilities for our grandchildren

- In 1930, Keynes wrote "Economic Possibilities for our Grandchildren"

Economic possibilities for our grandchildren

- In 1930, Keynes wrote "Economic Possibilities for our Grandchildren"
- By 2030, a "fifteen hours week" just for those who want to work!

Economic possibilities for our grandchildren

- In 1930, Keynes wrote "Economic Possibilities for our Grandchildren"
- By 2030, a "fifteen hours week" just for those who want to work!
- In 1953, Marxist James Cannon:

  Even at the present stage of economic development, if everybody worked and there was no waste, a universal four-hour day would undoubtedly be enough to provide abundance for all in the advanced countries.

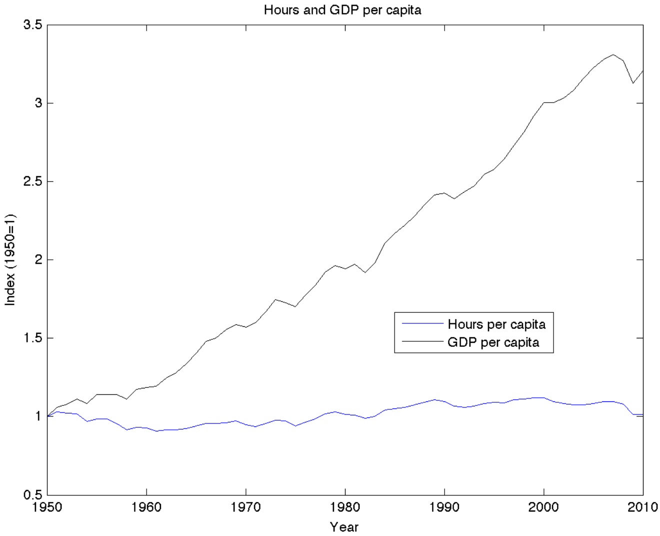
Economic possibilities for our grandchildren

- In 1930, Keynes wrote “Economic Possibilities for our Grandchildren”
- By 2030, a “fifteen hours week” just for those who want to work!
- In 1953, Marxist James Cannon:

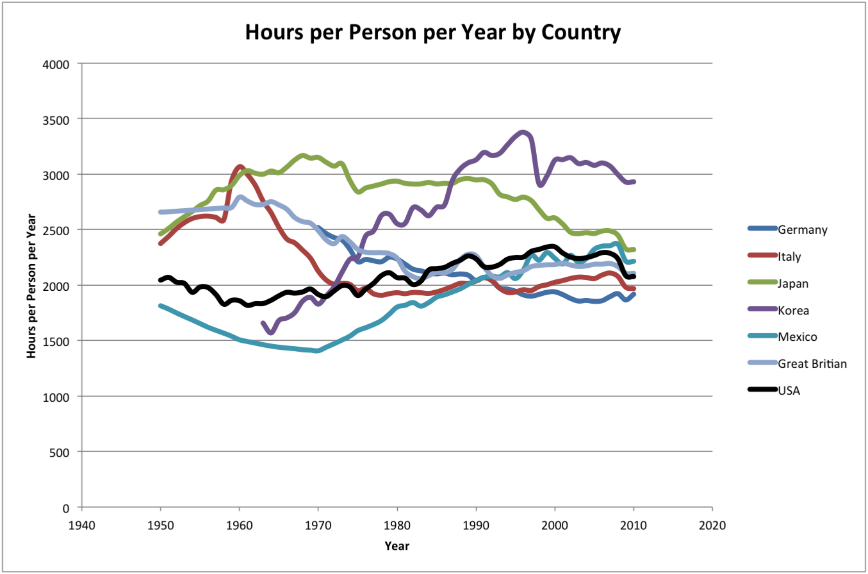
  > *Even at the present stage of economic development, if everybody worked and there was no waste, a universal four-hour day would undoubtedly be enough to provide abundance for all in the advanced countries.*

- What happened?

Hours constant, GDP up

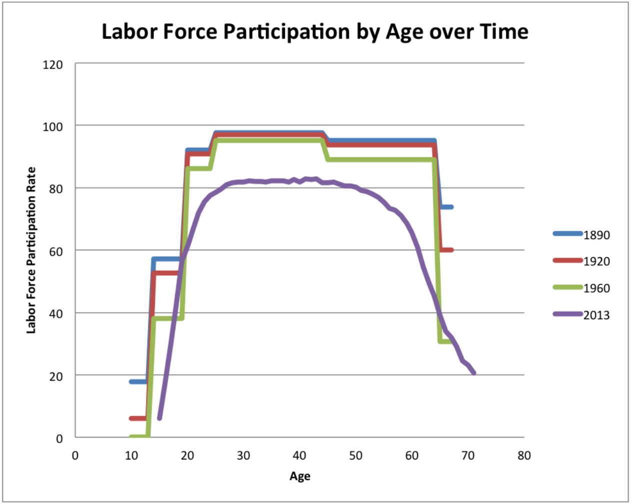


…KINDA…

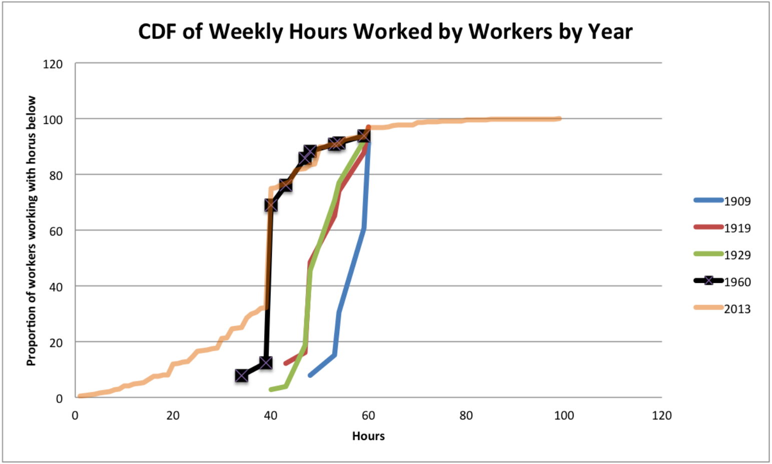


Hours per Person per Year by Country
Hours per Person per Year
4000
3500
3000
2500
2000
1500
1000
500
0
1940
1950
1960
1970
1980
1990
2000
2010
2020
Year
Germany
Italy
Japan
Korea
Mexico
Great Britian
USA

And for men, it's another story



Distribution of hours worked shifting down



Tallying Up...

- In U.S., total hours per person haven't gone down too much
- In other countries, they have
- In U.S., total hours worked per man have gone down
  - In U.S., total hours worked per man have gone down
  - Offset by women's increase

Tallying Up...

- In U.S., total hours per person haven't gone down too much
- In other countries, they have
- In U.S., total hours worked per man have gone down
  - In U.S., total hours worked per man have gone down
  - Offset by women's increase
  - If *households* taken together, not unreasonable...should they be?
- Labor force participation has been shrinking at the edges
- Hours per working worker shrinking a little too
- Either way, we're going to say household hours didn't move much
  - Keep in perspective the wage gains!

Convergence!

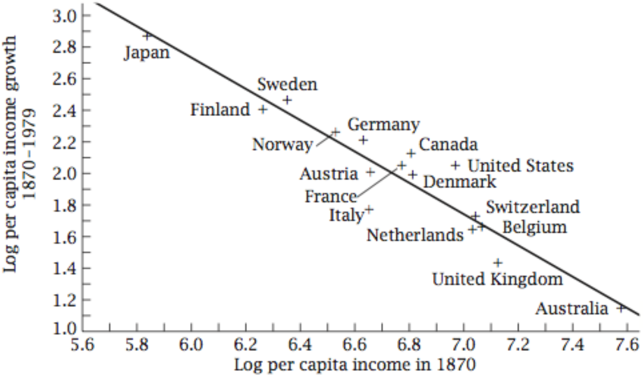


FIGURE 1.7 Initial income and subsequent growth in Baumol's sample (from DeLong, 1988; used with permission)

Here, the idea behind convergence explains >90% of growth heterogeneity.

No Convergence!

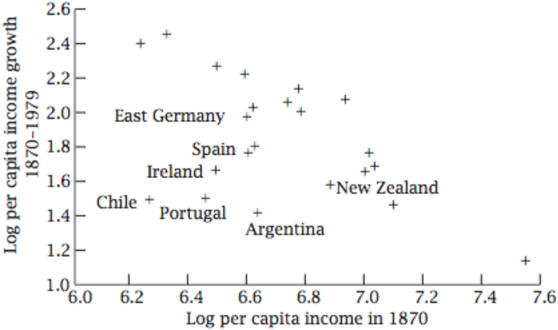
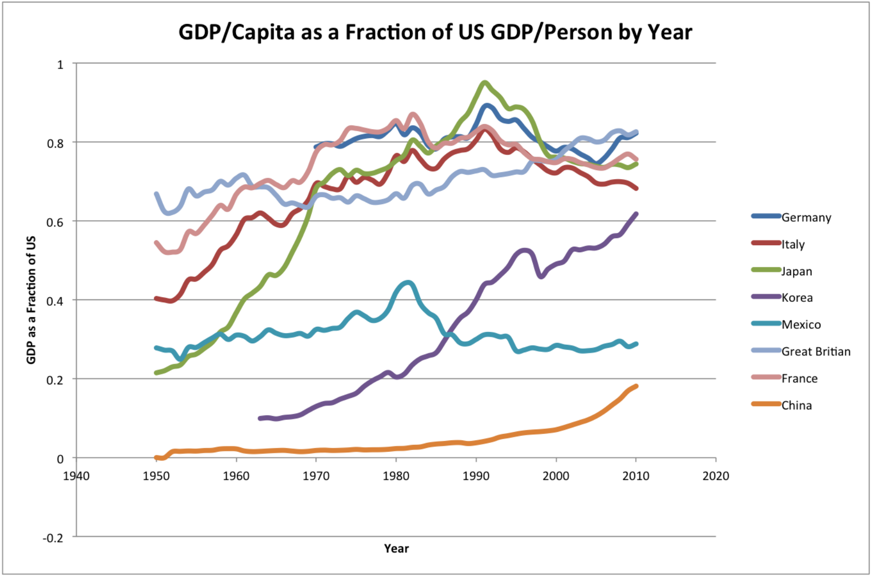


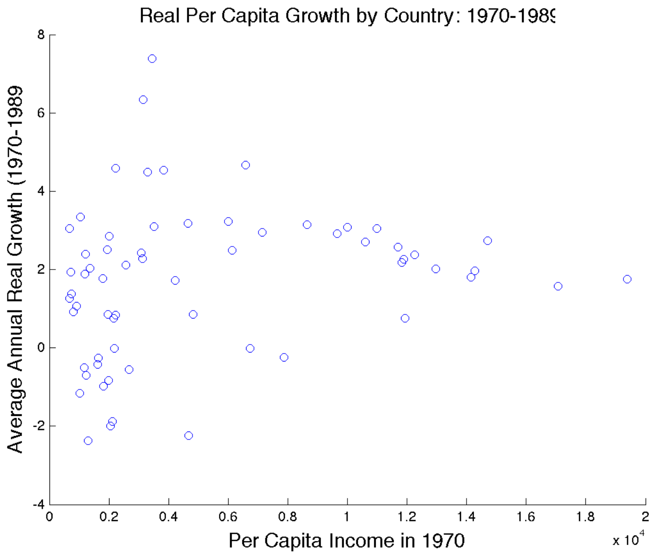
FIGURE 1.8 **Initial income and subsequent growth in the expanded sample (from DeLong, 1988; used with permission)**

Here, the idea behind convergence explains little growth heterogeneity.

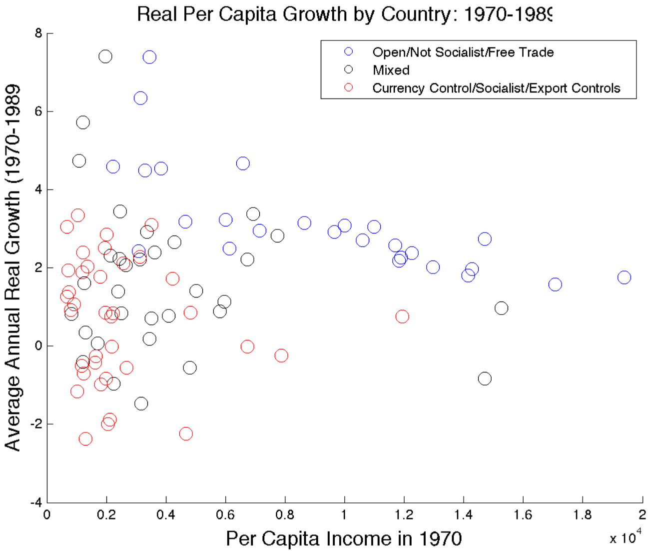
Stability among Developed Countries



No Convergence, but some constancy

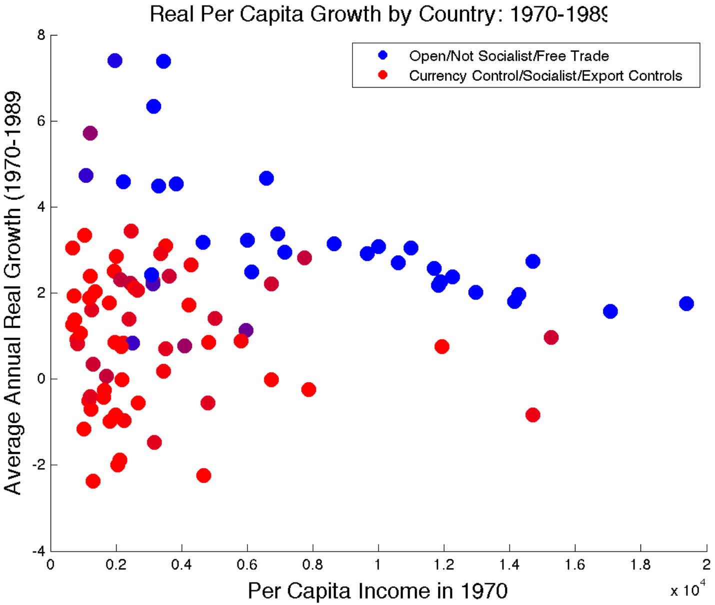


Understanding Growth Heterogeneity-I

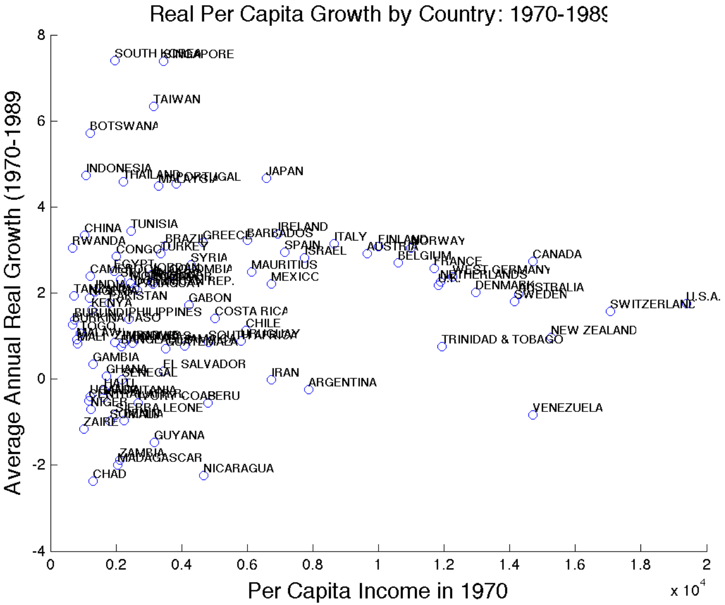


"Closed" economies in red, "open" economies in blue. (Sachs Werner definition).

Understanding Growth Heterogeneity-II



Understanding Growth Heterogeneity-III



Summarizing some alternative thinking...

- Malthus: if $w \uparrow$, $N \uparrow$, and all gains lost to population.
- Marx: . "profits" (entrepreneurial labor income(?) fall but unearned rents r increase
- Piketty: $\frac{rK}{Y} \uparrow$

Piketty: call K wealth, γ, g gross growth rates:

$$\frac{r_{t+1}K_{t+1}}{Y_{t+1}} = r_{t+1}\frac{\gamma_K}{g}\frac{K_t}{Y_t}$$

Claim:

$$\frac{r_{t+1}K_{t+1}}{Y_{t+1}} > r_t\frac{K_t}{Y_t}$$

For that to be true,

$$r_{t+1}\frac{\gamma_K}{g} > r_t$$

$$r_{t+1}\gamma_K > r_t g$$

If r is constant, it would suffice to say that:

$$\gamma_K > g$$

$r > g$ AND $\gamma_K > g$

Why would we think $\gamma_K > g$?

Imagine if we invested all returns into making new capital:

$$K_{t+1} = (1 - \delta)K_t + i_t$$

and, letting r_t^* be the gross return on capital:

$$i_t = r_t^* K_t$$

So, letting $r_t = r_t^* - \delta$ be the net return on capital:

$$K_{t+1} = (1 + r_t)K_t$$

Therefore, $r_t = \gamma_K$.

Are we done?

1. Print $r > g$ on shirts
2. ???
3. Profit!

No way!

- Savings behavior is theoretically/empirically unlikely!
- As $\frac{K}{Y} \uparrow$, $r \downarrow$
- Suggested r constant (or increasing) is empirically unlikely!

Savings behavior is theoretically/empirically unlikely!

- Think about $g \to 0$, as Piketty does.
- If K is growing ($\bar{r} > 0$ + savings behavior), we have:

$$i_t = rK_t$$

$$\frac{i_{t+1}}{Y_{t+1}} > \frac{i_t}{Y_t}$$

Eventually society uses up all its product just to try to make more capital.

The opposite is true, historically.

Savings behavior is theoretically/empirically unlikely!

- Also, we aren't reinvesting all capital benefits anyway!

- Housing services (including imputed rents) are about 18% of GDP.

As $\frac{K}{Y} \uparrow$, $r \downarrow$!

- Elasticity of substitution between labor and capital σ defines the curvature of the isoquant: how easy it is to substitute the two.
- Note: $\frac{\partial \log r}{\partial K/Y} = -\sigma$
- How does $\frac{rK}{Y}$ change (in percent) when K/Y changes (in percent)?

$$\frac{\partial rK/Y}{\partial K/Y}\frac{K/Y}{rK/Y} = \left(r + \frac{K}{Y}\frac{\partial r}{\partial K/Y}\right)\frac{1}{r} = 1 - \frac{1}{\sigma}$$

If $\sigma > 0$ (easy to substitute) then $\frac{rK}{Y} \uparrow$ as $\frac{K}{Y} \uparrow$ (r shrinks more slowly than K grows)

What is σ? Are labor and capital complements or substitutes?

Rognlie 2014 (remarkably readable!)

- 30/31 sources: $\sigma < 2$
- 29/31 sources: $\sigma < 1.5$
- 26/31 sources: $\sigma < 1$
- 14/31 sources: $\sigma < 0.5$
- Because of quirks of definitions (net σ vs. gross), for increases in net capital share Piketty probably actually needs $\sigma \approx 1.52$.
- Basic idea: r goes down, and $r - \delta$ goes down faster, in percentage terms, as δ stays the same.

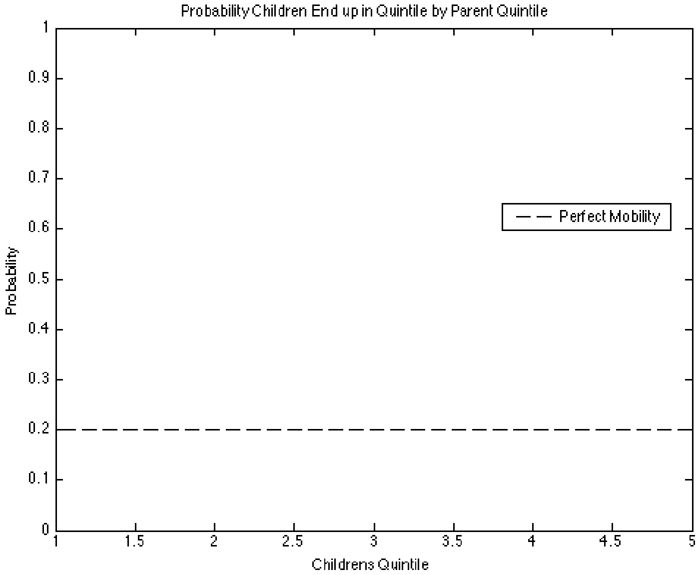
Residual Issues

- Rognlie: with computers, $K\uparrow$, but $\delta\uparrow\uparrow$, $p\downarrow$
- Rognlie: Recent rise in capital wealth and income almost all from housing
- Ray, everyone: it's all about *savings* behavior!
- Krusell/Smith: Savings model is wrong (we saw this)
- Acemoglu/Robinson: $r-g$ doesn't explain historical inequality
- Acemoglu/Robinson: Intermarriage means that r would have to be 8.5% when g is 1%
- Chetty/Hendren/Kline/Saez (via A/R): Child with parents in top 1% has 9.6% chance of being in top 1%
- Summers, McBride: Of Forbes 400 richest people in 1987, 81% weren't on the list 27 years later. Alternatively 70% are "self-made" rather than being heirs.

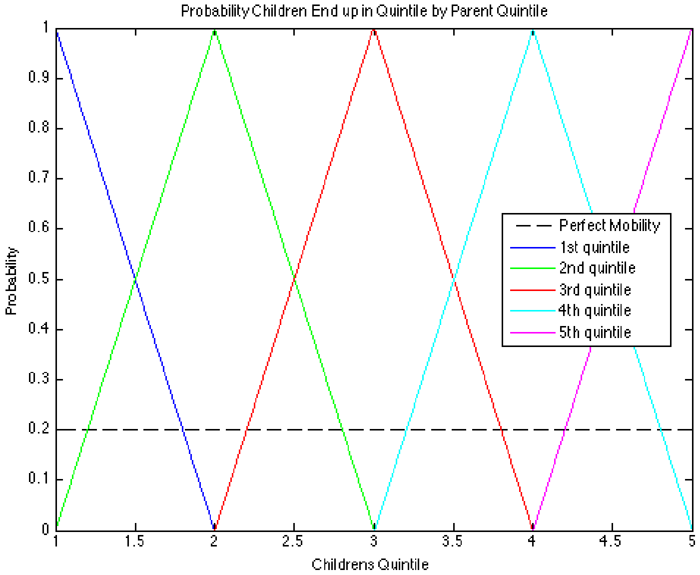
Let's talk about inequality

- Chetty Hendren Kline Saez (2014) Where is the Land of Opportunity? The Geography of Intergenerational Mobility in the United States
- 40 million linked parent-child tax records
- For children born 1980-1985, compare parent rank when child is 15 to child rank when 30
- Chetty/Hendren/Klein/Saez: 10% increase in parent income yields 3.4% increase to children income

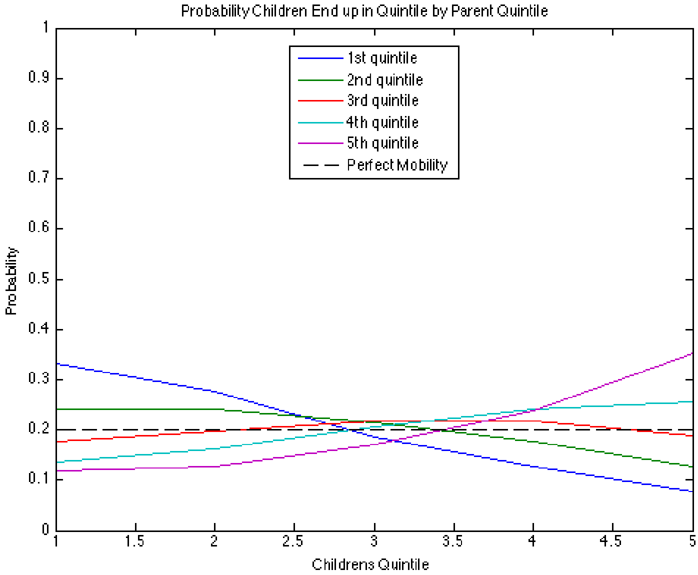
Transition Matrix: Perfect Mobility



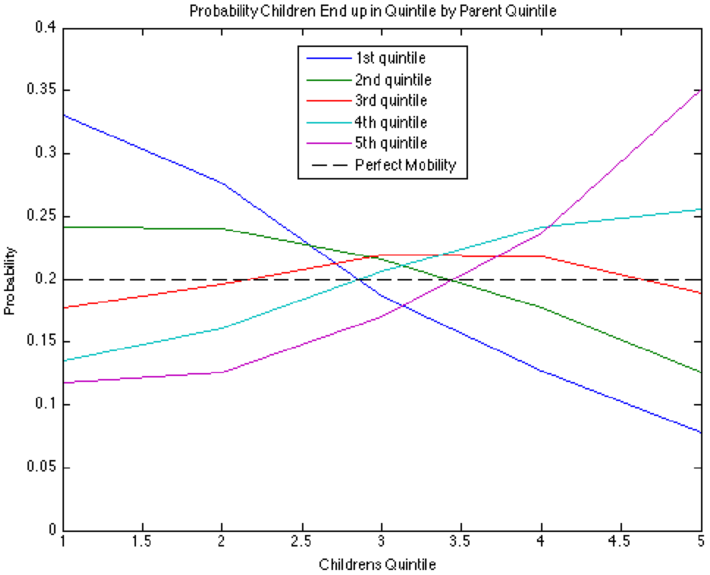
Transition Matrix: Perfect Nonmobility



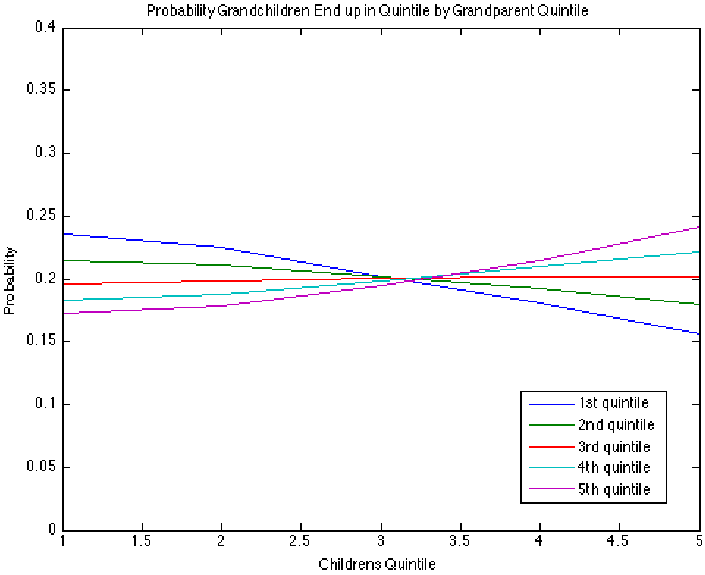
Transition Matrix: Children from Parents-I



Transition Matrix: Children from Parents-II



Transition Matrix: Gchild from Gparent-II



Summing Up Inequality

-