Numerical Methods-Lecture 11: Neoclassical Growth Model

(See Conesa, Kehoe, Ruhl 2007)

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Guidelines on Presentation & Paper

- Pick a topic
- Pick/write down a model that encompasses what you're talking about
  - Things in your model should be in there for a reason
  - Things not in your model should be excluded for a reason
- Solve your model
- *Answer a question.*
- The presentation & paper
  - Motivation (why important)
  - Brief lit review/alternative models
  - Model
  - Solution method
  - Solutions/interesting results
  - Conclusion

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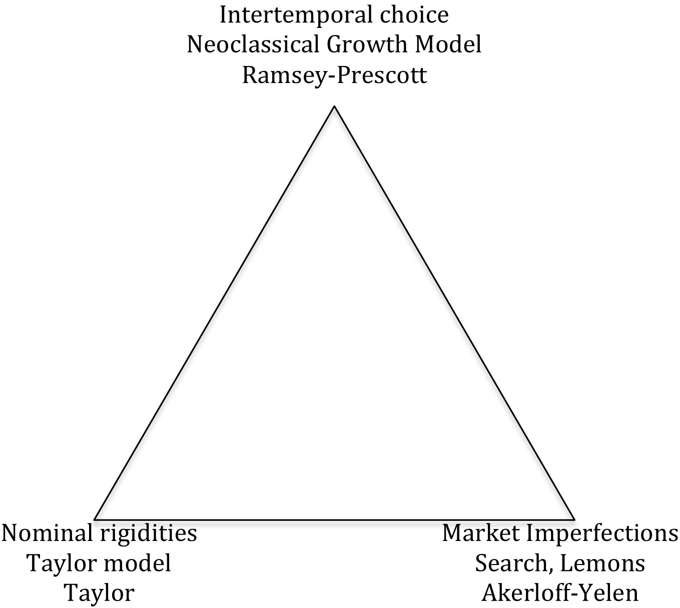
Blanchard's Triangle



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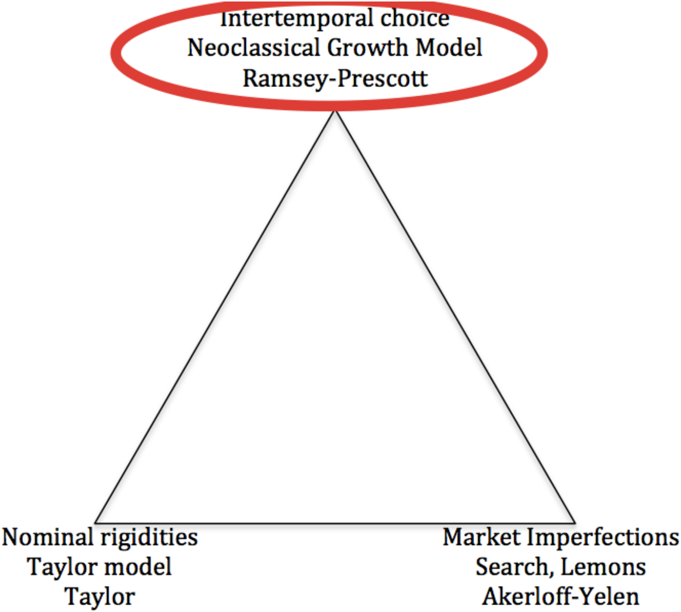


Neoclassical Growth Model

- This is a computational economics course
- It's worth understanding the NCG
- Focus on intertemporal choice and consequent dynamics mean an enormous amount of computational models center on it

Neoclassical Growth Model

- Utility over consumption and leisure:

$$\sum_{t=0}^{\infty} \beta^t u(C_t, 1 - L_t)$$

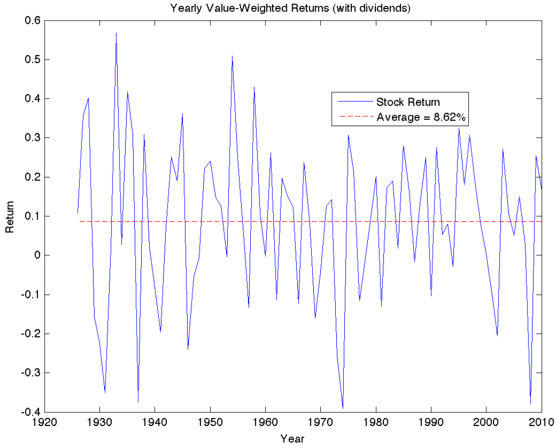
- Law of motion

$$C_t + I_t = Y_t = F(K_t, L_t)$$

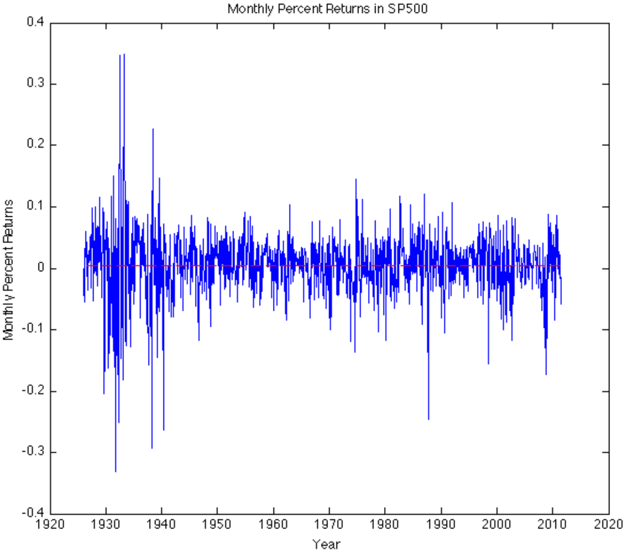
- Law of motion of capital

$$K_{t+1} = (1 - \delta)K_t + I_t$$

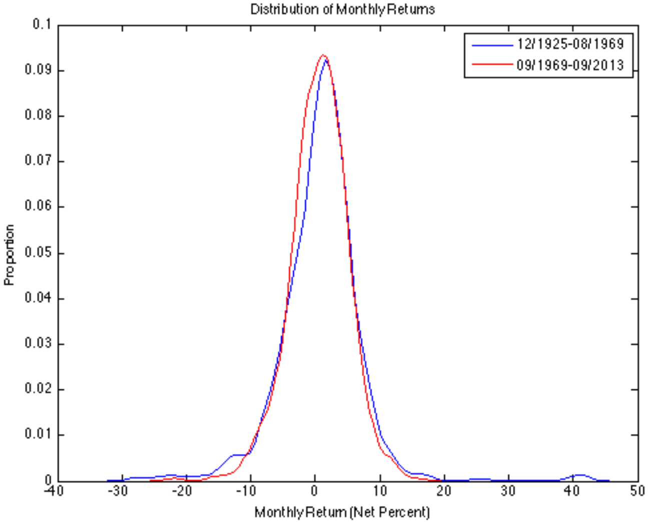
Motivating functional form-I



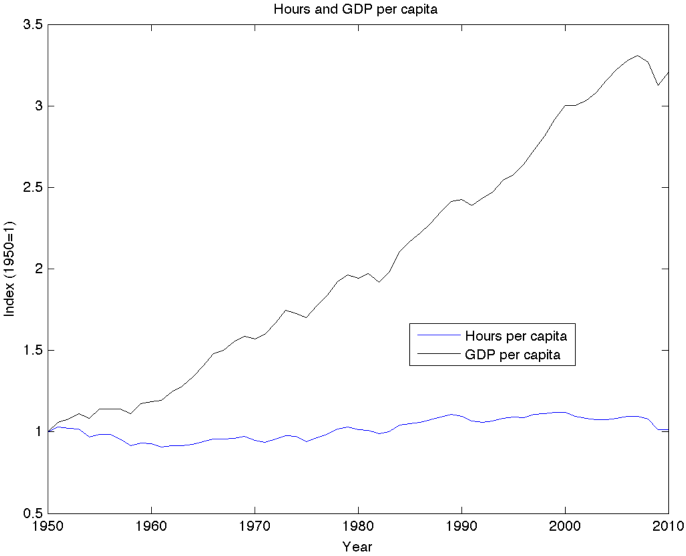
Motivating functional form-II



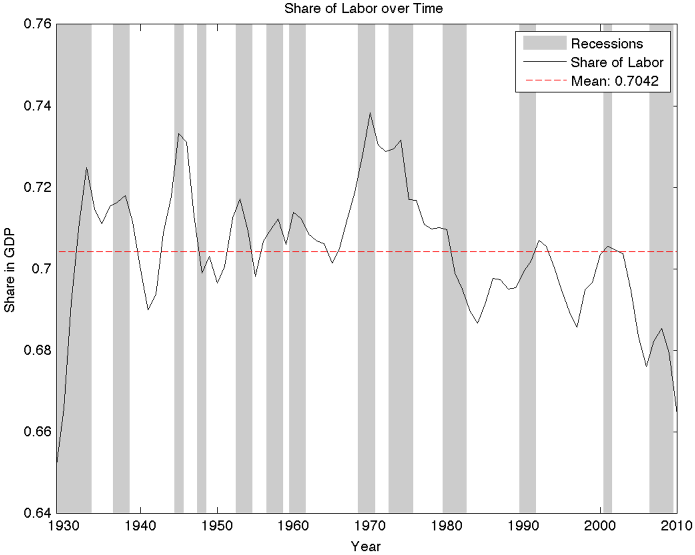
Motivating functional form-III



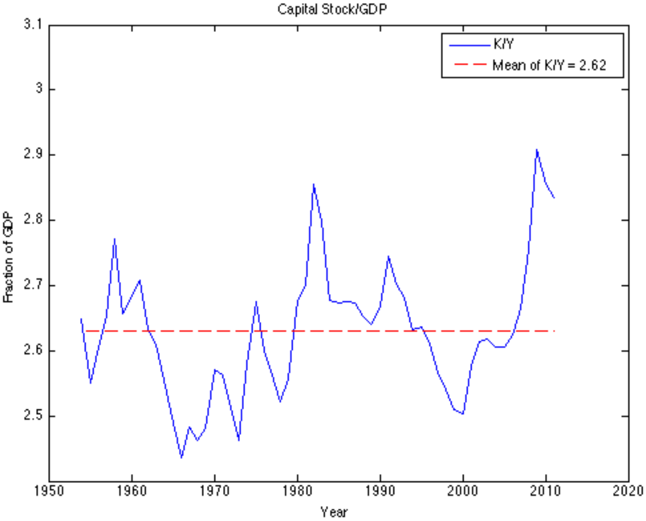
Motivating functional form-IV



Motivating functional form-V



Motivating functional form-VI



Discipline Functional Form

- Stylized/Kaldor's facts (per capita!):
  - Y_t, C_t, K_t, w_t have grown at a constant rate
  - r_t, L_t have not.
  - $(r_t - \delta)K_t/Y_t$, $w_t L_t/Y_t$ have been constant.
- This gives utility you normally see:

$$u(C_t, 1 - L_t) = \frac{(C_t \cdot g(1 - L_t))^{1-\sigma} - 1}{1 - \sigma}$$

or

$$u(C_t, 1 - L_t) = \log(C_t) + \psi \log(1 - L_t)$$

or:

$$u(C_t, 1 - L_t) = \log(C_t) - \psi \frac{\epsilon}{1 + \epsilon} L_t^{\frac{1+\epsilon}{\epsilon}}$$

- And production:

$$Y_t = A_t \left(\xi K_t^{\frac{\sigma-1}{\sigma}} + (1 - \xi) L_t^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

or:

$$Y_t = A_t L_t^{\alpha} K_t^{1-\alpha}$$

Simple NCG

$$\max \sum_{t=0}^{\infty} \beta^t \left[\log(c_t) + \psi\left(1 - L_t\right)\right]$$

$$C_t + I_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

$$K_{t+1} = (1-\delta)K_t + I_t$$

Simple NCG: FOC's

Given K_0, each period has 6 equations and 6 unknowns $(C_t, K_{t+1}, L_t, Y_t, w_t, r_t)$

$$
\begin{array}{rcll}
\frac{C_{t+1}}{C_t} & = & \beta(1-\delta+r_{t+1}) & \text{Euler/intertemporal}/K^S \\
\frac{\psi}{1-L_t} & = & \frac{w_t}{C_t} & \text{Intratemporal}/L^S \\
w_t & = & (1-\alpha)\frac{Y_t}{L_t} & L^D \\
r_t & = & \alpha\frac{Y_t}{K_t} & K^D \\
K_{t+1} & = & (1-\delta)K_t + I_t & \text{Law of motion of capital} \\
I_t + C_t & = & Y_t & \text{Feasibility}
\end{array}
$$

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I_t + C_t & = & Y_t & \text{Feasibility} \\
w_t L_t + r_t K_t & = & C_t + I_t & \text{Budget constraint}
\end{array}$$

Extra equation?

Simple NCG: FOC's

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$$

Extra equation?

Can reduce to two series.

Bring the model to data: issues?

- $Y_t \neq I_t + C_t$!
  - $Y_t \neq I_t + C_t + G_t + (I_t - X_t)$!
  - Could try to divvy up G and NX
  - Could attribute all to C
  - Does it matter?
- One type of capital
- One price for everything (constant relative price of capital and consumption)
- No terms of trade
- Need to calibrate parameters: β, δ, K_0, ψ, α, and A_t's

Bring the model to data: Step 1

- Calibrate: β, δ, K_0, ψ, α, and A_t's
- Given I_t data and δK_t data, choose δ and K_0 to target:

$$\underbrace{\frac{\overline{\delta K_t}}{Y_t}}_{Data} = \underbrace{\frac{\overline{\delta K_t}}{Y_t}}_{Model}$$

  And:

$$\frac{K_1}{Y_1} = \frac{1}{10}\sum_{t=1}^{10}\frac{K_t}{Y_t}$$

  That is, average depreciation in the model is the same as average depreciation in the data, and the starting capital/output ratio is comparable to the capital/output ratio over the next 10 periods.

- Calibrate: β, $\cancel{\delta}$, $\cancel{K_0}$, ψ, α, and A_t's

Bring the model to data: Step 2

- Given K_0 and δ, along with I_t, can construct K_t's in data
- Given constructed K_t, L_t and Y_t in data, can calculate TFP:

$$A_t = \frac{Y_t}{L_t^{\alpha} K_t^{1-\alpha}}$$

- Can calibrate α as the average wage share:

$$\alpha = \frac{1}{N} \sum_{t=1}^{N} \frac{w_t L_t}{Y_t}$$

- Calibrate: β, $\cancel{\delta}$, $\cancel{K_0}$, ψ, $\cancel{\alpha}$, and $\cancel{A_t}$'s

Bring the model to data: Step 3

- Set β so that the *expectation* of the intertemporal FOC in the data is true.

$$\beta = \frac{1}{N}\sum_{t=1}^{N} \frac{1}{(1-\delta+r_{t+1})}\frac{C_{t+1}}{C_t}$$

- Set ψ so that the *expectation* of the intratemporal FOC in the data is true.

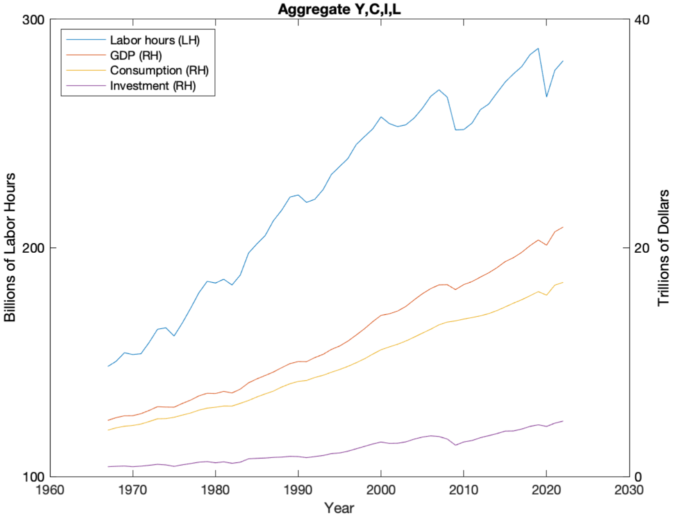
$$\psi = \frac{1}{N}\sum_{t=1}^{N} \frac{w_t(1-L_t)}{C_t}$$

- Calibrate: $\cancel{\beta}$, $\cancel{\delta}$, $\cancel{K_0}$, $\cancel{\psi}$, $\cancel{\alpha}$, and $\cancel{A_t}$'s

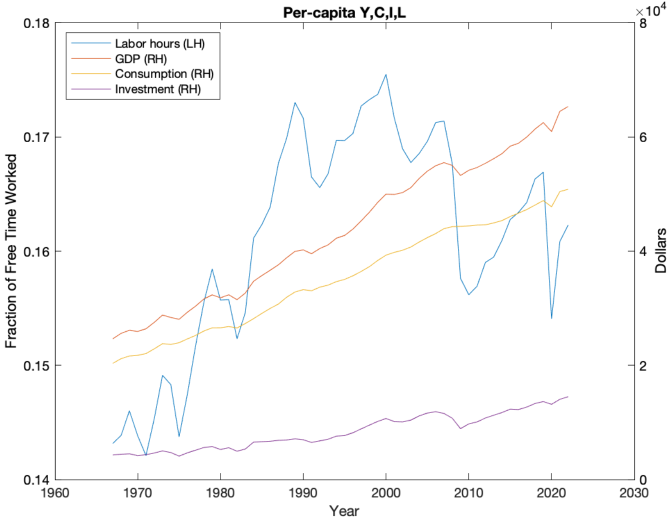
So actually take it to data

- I'm just going to use BEA data
- Download GDP (Y_t), Personal Consumption (C_t) from Table 1.1.6
- Download Gross Domestic Investment I_t and consumption of fixed capital δK_t from Table 5.2.6.
- Download Population from Table 7.1
- Download Hours and Employment from OECD
- Let's look at the series

GDP, Investment, Consumption, Labor hours

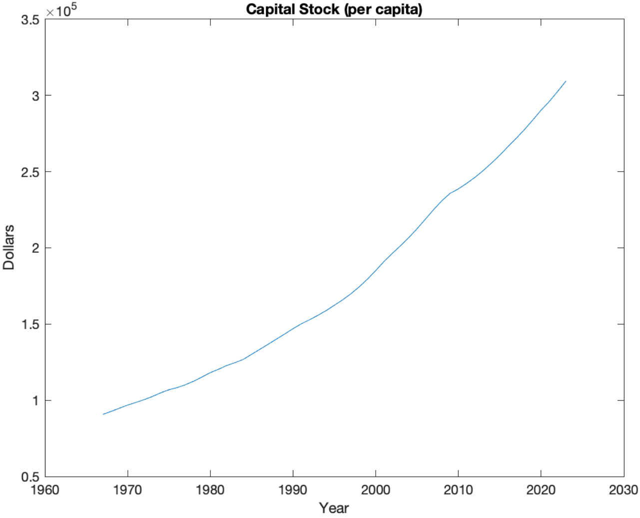


Per Capita

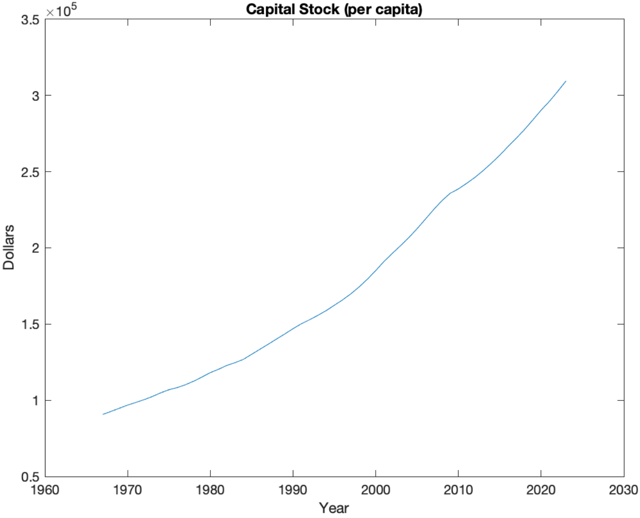


Per-capita Y,C,I,L
Labor hours (LH)
GDP (RH)
Consumption (RH)
Investment (RH)
Fraction of Free Time Worked
Dollars
$\times 10^4$
0.18
0.17
0.16
0.15
0.14
8
6
4
2
0
1960
1970
1980
1990
2000
2010
2020
2030
Year

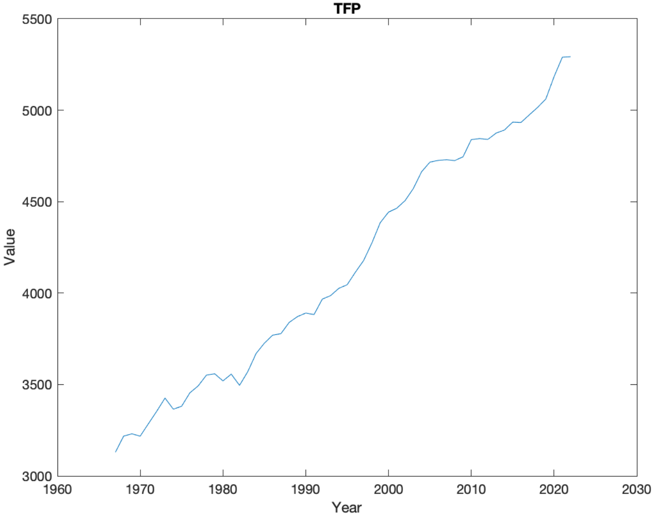
Create Capital Stock



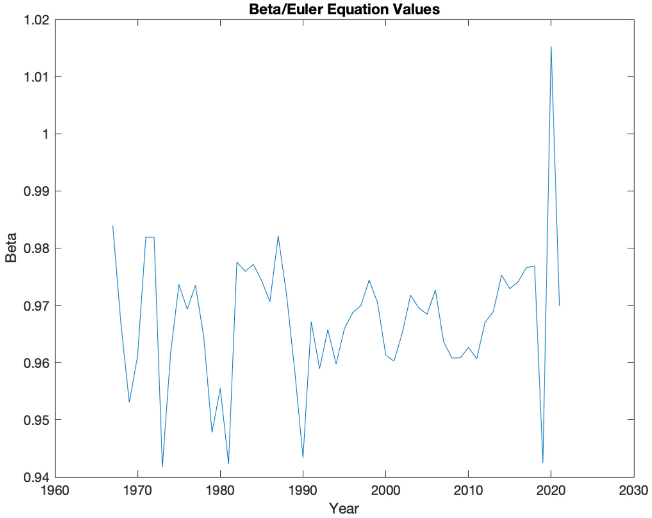
Create Capital Stock: $K_{t+1} = (1-\delta)K_t + I_t$



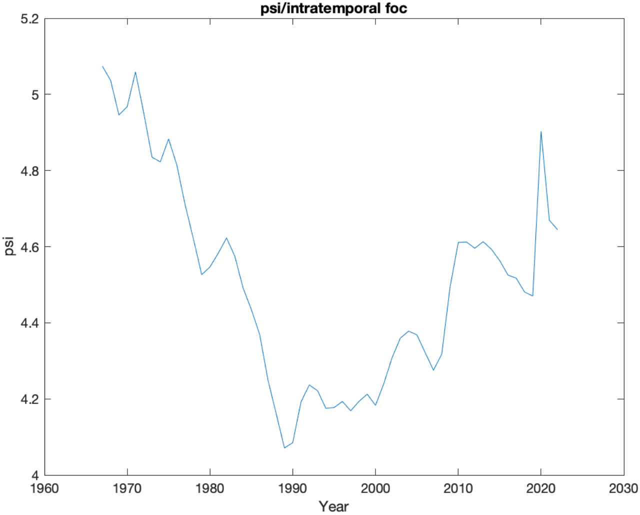
Find TFP: $A_t = \frac{Y_t}{L_t^{\alpha} K_t^{1-\alpha}}$



Find $\beta = \frac{1}{1-\delta+r_{t+1}} \frac{c_{t+1}}{c_t}$



FIND $\psi = \frac{w_t(1-L_t)}{c_t}$



Calibration

- $k_0 = 63396$
- $\delta = 0.043$
- $\beta = 0.963$
- $\psi = 4.58$
- $\alpha = 0.7$

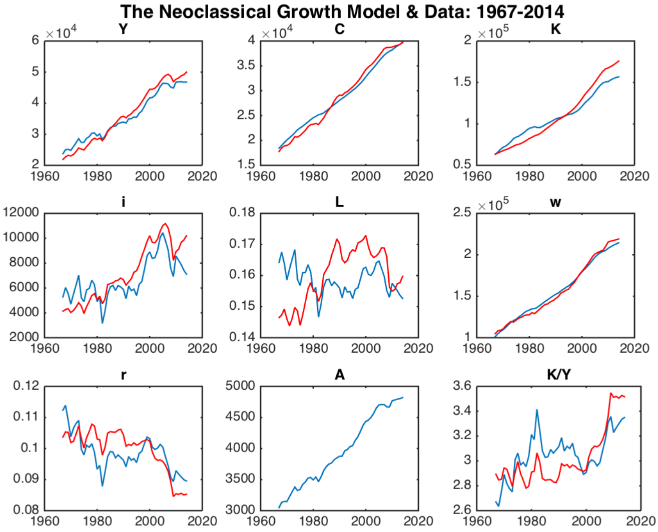
Solution and Assumptions

- There are many possible assumptions with respect to A_t
- A first step is complete knowledge of A_t
- Assume no labor, consumption, or capital taxes
- What about future A_t?
- Continue A_t growth constantly into the future
- Force to reach steady state growth path 20 years into constant future

Solution Method

- Given parameters, a path of A_t, and a k_0, I have two decisions to make:
  - $L_t \times T$ labor decisions from intratemporal FOC
  - $K_{t+1} \times T$ capital decisions from intertemporal FOC
- Solve for 40 existing years, then give 30 years to reach SS growth path
- Then, for 30 more periods, assume they're on autopilot
- They only get to choose 70×2 choices to solve $70 \times 2 + 30 \times 2$ FOC's!
- Make sure all FOC's are zero

Theory and Data



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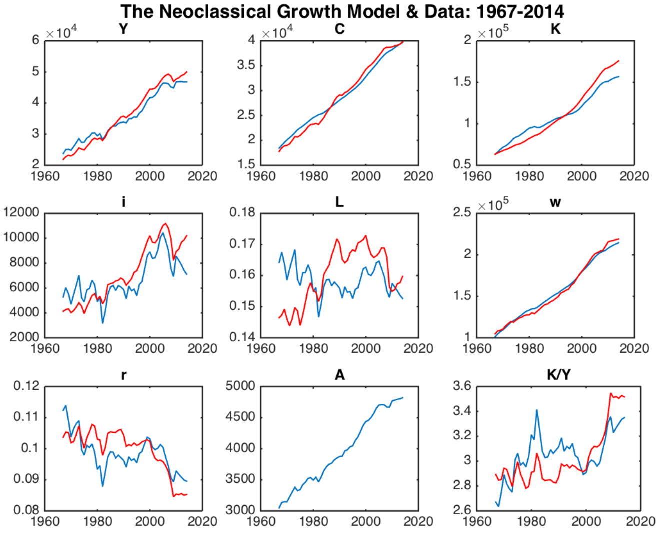
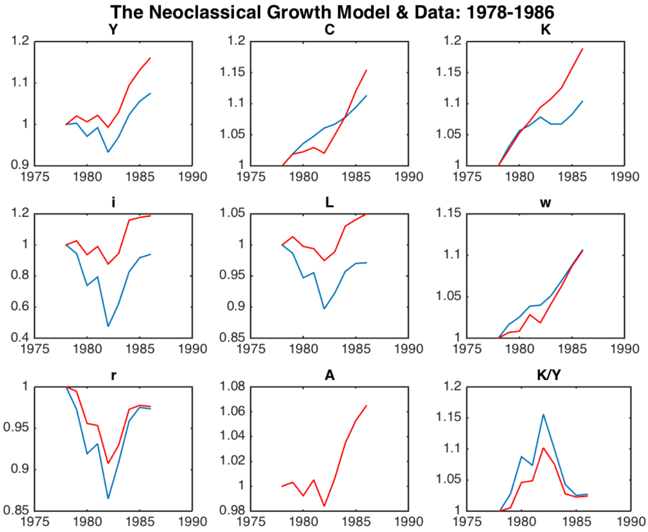
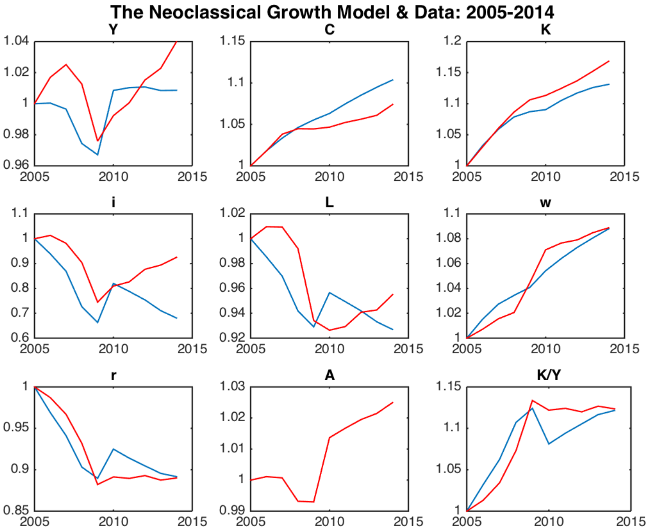


Which is theory, which is data?

Explaining the early 1980's



Explaining the late 2000's



(Macro)Economics as a science

- In similar position to meteorology, astronomy, geology
- No randomized experiments
- Data is too short for nonparametric analysis
- Ideal (and solution) a unified theory: explain all frequencies
- Parsimony keeps us honest
- Explaining multiple stochastic processes jointly keeps us honest
- Explaining dynamics gives more role for testable hypotheses