

# NUMERICAL METHODS-LECTURE 11: NEOCLASSICAL GROWTH MODEL

(See Conesa, Kehoe, Ruhl 2007)

Trevor Gallen

# GUIDELINES ON PRESENTATION & PAPER

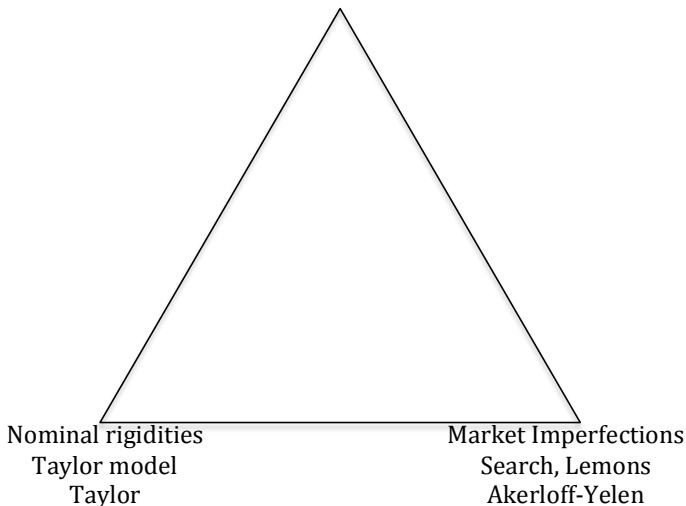
- ▶ Pick a topic
- ▶ Pick/write down a model that encompasses what you're talking about
  - ▶ Things in your model should be in there for a reason
  - ▶ Things not in your model should be excluded for a reason
- ▶ Solve your model
- ▶ *Answer a question.*
- ▶ The presentation & paper
  - ▶ Motivation (why important)
  - ▶ Brief lit review/alternative models
  - ▶ Model
  - ▶ Solution method
  - ▶ Solutions/interesting results
  - ▶ Conclusion

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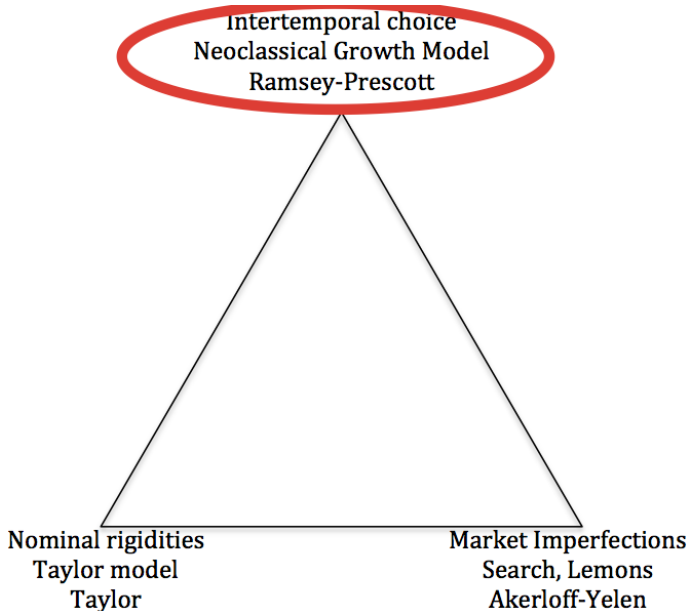
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# BLANCHARD'S TRIANGLE

Intertemporal choice  
Neoclassical Growth Model  
Ramsey-Prescott



# BLANCHARD'S TRIANGLE



# NEOCLASSICAL GROWTH MODEL

- ▶ This is a computational economics course
- ▶ It's worth understanding the NCG
- ▶ Focus on intertemporal choice and consequent dynamics mean an enormous amount of computational models center on it

# NEOCLASSICAL GROWTH MODEL

- Utility over consumption and leisure:

$$\sum_{t=0}^{\infty} \beta^t u(C_t, 1 - L_t)$$

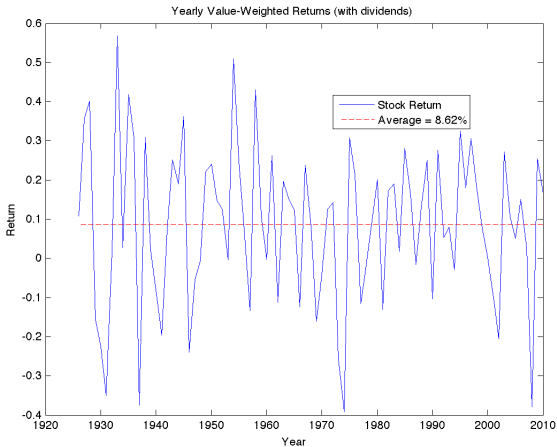
- Law of motion

$$C_t + I_t = Y_t = F(K_t, L_t)$$

- Law of motion of capital

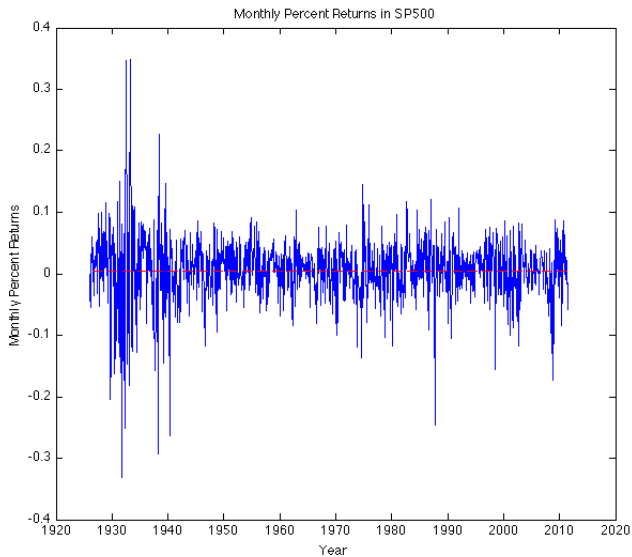
$$K_{t+1} = (1 - \delta)K_t + I_t$$

# MOTIVATING FUNCTIONAL FORM-I

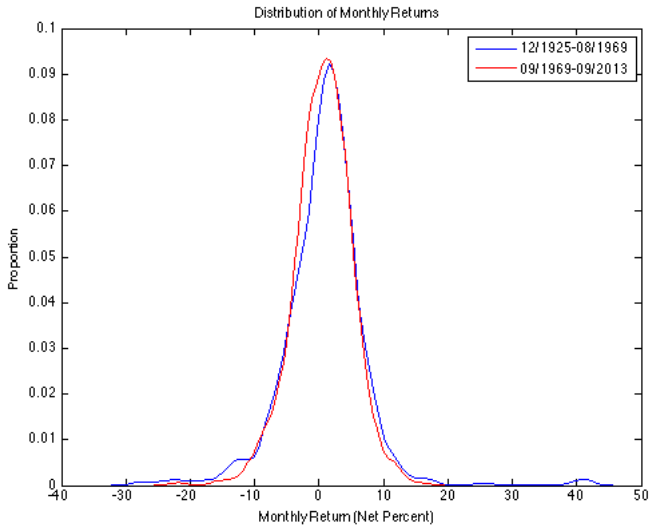




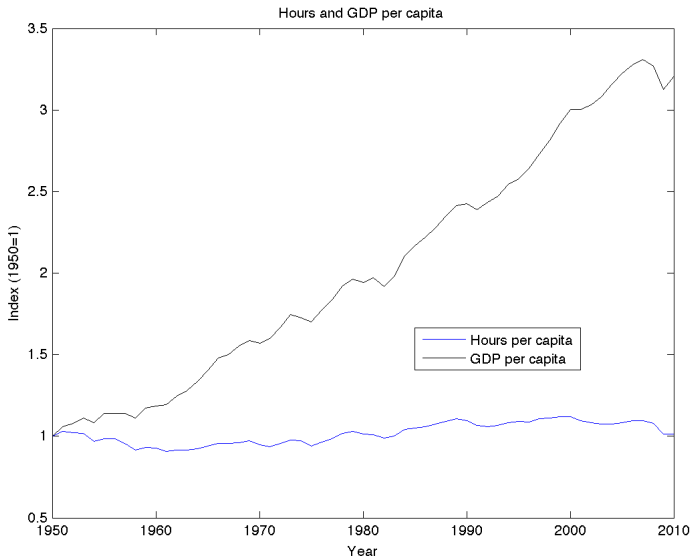
# MOTIVATING FUNCTIONAL FORM-II



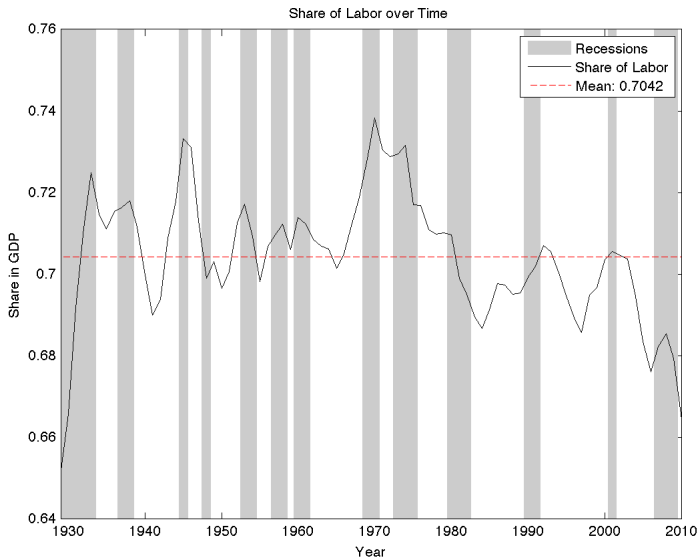
## MOTIVATING FUNCTIONAL FORM-III



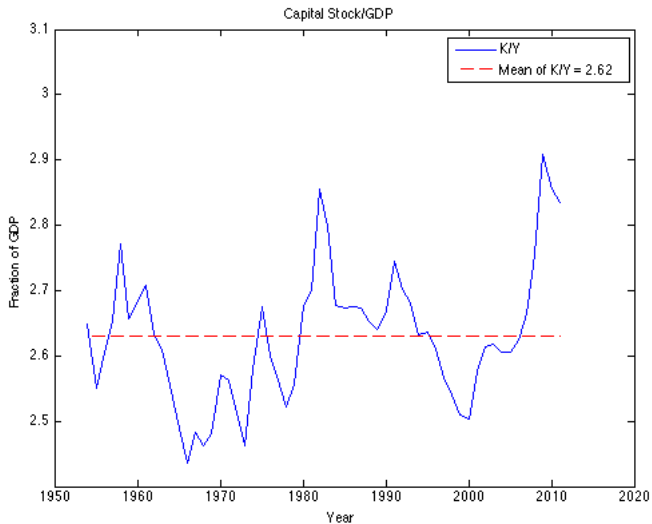
# MOTIVATING FUNCTIONAL FORM-IV



# MOTIVATING FUNCTIONAL FORM-V



# MOTIVATING FUNCTIONAL FORM-VI



## DISCIPLINE FUNCTIONAL FORM

- ▶ Stylized/Kaldor's facts (per capita!):
  - ▶  $Y_t, C_t, K_t, w_t$  have grown at a constant rate
  - ▶  $r_t, L_t$  have not.
  - ▶  $(r_t - \delta)K_t/Y_t, w_t L_t/Y_t$  have been constant.
- ▶ This gives utility you normally see:

$$u(C_t, 1 - L_t) = \frac{(C_t \cdot g(1 - L_t))^{1-\sigma} - 1}{1 - \sigma}$$

or

$$u(C_t, 1 - L_t) = \log(C_t) + \psi \log(1 - L_t)$$

or:

$$u(C_t, 1 - L_t) = \log(C_t) - \psi \frac{\epsilon}{1 + \epsilon} L_t^{\frac{1+\epsilon}{\epsilon}}$$

- ▶ And production:

$$Y_t = A_t \left( \xi K_t^{\frac{\sigma-1}{\sigma}} + (1 - \xi) L_t^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

or:

$$Y_t = A_t L_t^\alpha K_t^{1-\alpha}$$

# SIMPLE NCG

$$\max \sum_{t=0}^{\infty} \beta^t [\log(c_t) + \psi(1 - L_t)]$$

$$C_t + I_t = A_t K_t^\alpha L_t^{1-\alpha}$$

$$K_{t+1} = (1 - \delta)K_t + I_t$$

## SIMPLE NCG: FOC's

Given  $K_0$ , each period has 6 equations and 6 unknowns  
( $C_t, K_{t+1}, L_t, Y_t, w_t, r_t$ )

$\frac{C_{t+1}}{C_t}$	$= \beta(1 - \delta + r_{t+1})$	Euler/intertemporal/ $K^S$
$\frac{\psi}{1-L_t}$	$= \frac{w_t}{C_t}$	Intratemporal/ $L^S$
$w_t$	$= (1 - \alpha) \frac{Y_t}{L_t}$	$L^D$
$r_t$	$= \alpha \frac{Y_t}{K_t}$	$K^D$
$K_{t+1}$	$= (1 - \delta)K_t + I_t$	Law of motion of capital
$I_t + C_t$	$= Y_t$	Feasibility



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$I_t + C_t$	$= Y_t$	Feasibility
$w_t L_t + r_t K_t$	$= C_t + I_t$	Budget constraint

Extra equation?

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$I_t + C_t = Y_t$	Feasibility
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Extra equation?

Can reduce to two series.

## BRING THE MODEL TO DATA: ISSUES?

- ▶  $Y_t \neq I_t + C_t!$ 
  - ▶  $Y_t \neq I_t + C_t + G_t + (I_t - X_t)!$
  - ▶ Could try to divvy up  $G$  and  $NX$
  - ▶ Could attribute all to  $C$
  - ▶ Does it matter?
- ▶ One type of capital
- ▶ One price for everything (constant relative price of capital and consumption)
- ▶ No terms of trade
- ▶ Need to calibrate parameters:  $\beta$ ,  $\delta$ ,  $K_0$ ,  $\psi$ ,  $\alpha$ , and  $A_t$ 's

## BRING THE MODEL TO DATA: STEP 1

- Calibrate:  $\beta$ ,  $\delta$ ,  $K_0$ ,  $\psi$ ,  $\alpha$ , and  $A_t$ 's
- Given  $I_t$  data and  $\delta K_t$  data, choose  $\delta$  and  $K_0$  to target:

$$\underbrace{\frac{\delta K_t}{Y_t}}_{Data} = \underbrace{\frac{\delta K_t}{Y_t}}_{Model}$$

And:

$$\frac{K_1}{Y_1} = \frac{1}{10} \sum_{t=1}^{10} \frac{K_t}{Y_t}$$

That is, average depreciation in the model is the same as average depreciation in the data, and the starting capital/output ratio is comparable to the capital/output ratio over the next 10 periods.

- Calibrate:  $\beta$ ,  $\delta$ ,  ~~$K_0$~~ ,  $\psi$ ,  $\alpha$ , and  $A_t$ 's

## BRING THE MODEL TO DATA: STEP 2

- ▶ Given  $K_0$  and  $\delta$ , along with  $I_t$ , can construct  $K_t$ 's in data
- ▶ Given constructed  $K_t$ ,  $L_t$  and  $Y_t$  in data, can calculate TFP:

$$A_t = \frac{Y_t}{L_t^\alpha K_t^{1-\alpha}}$$

- ▶ Can calibrate  $\alpha$  as the average wage share:

$$\alpha = \frac{1}{N} \sum_{t=1}^N \frac{w_t L_t}{Y_t}$$

- ▶ Calibrate:  $\beta$ ,  $\delta$ ,  $K_0$ ,  $\psi$ ,  $\alpha$ , and  $A_t$ 's

## BRING THE MODEL TO DATA: STEP 3

- Set  $\beta$  so that the *expectation* of the intertemporal FOC in the data is true.

$$\beta = \frac{1}{N} \sum_{t=1}^N \frac{1}{(1 - \delta + r_{t+1})} \frac{C_{t+1}}{C_t}$$

- Set  $\psi$  so that the *expectation* of the intratemporal FOC in the data is true.

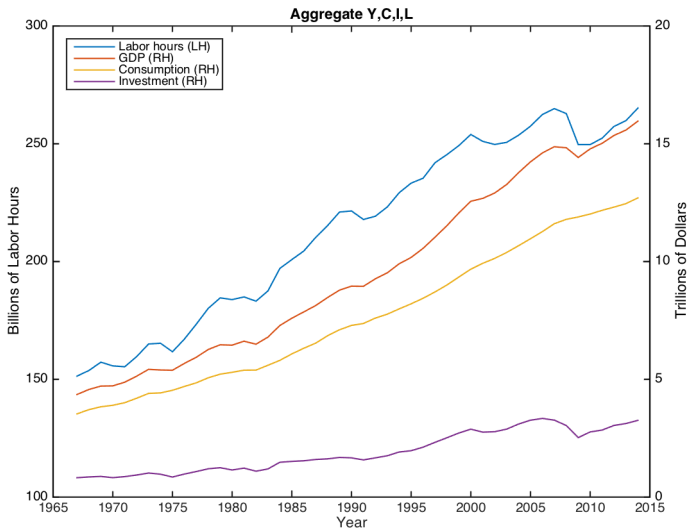
$$\psi = \frac{1}{N} \sum_{t=1}^N \frac{w_t(1 - L_t)}{C_t}$$

- Calibrate:  $\beta$ ,  $\delta$ ,  $K_0$ ,  $\psi$ ,  $\alpha$ , and  $A_t$ 's

## SO ACTUALLY TAKE IT TO DATA

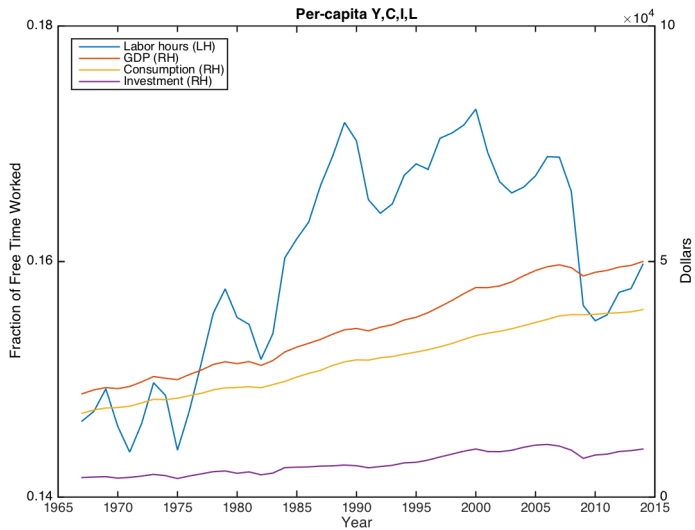
- ▶ I'm just going to use BEA data
- ▶ Download GDP ( $Y_t$ ), Personal Consumption ( $C_t$ ) from Table 1.1.6
- ▶ Download Gross Domestic Investment  $I_t$  and consumption of fixed capital  $\delta K_t$  from Table 5.2.6.
- ▶ Download Population from Table 7.1
- ▶ Download Hours and Employment from OECD
- ▶ Let's look at the series

# GDP, INVESTMENT, CONSUMPTION, LABOR HOURS

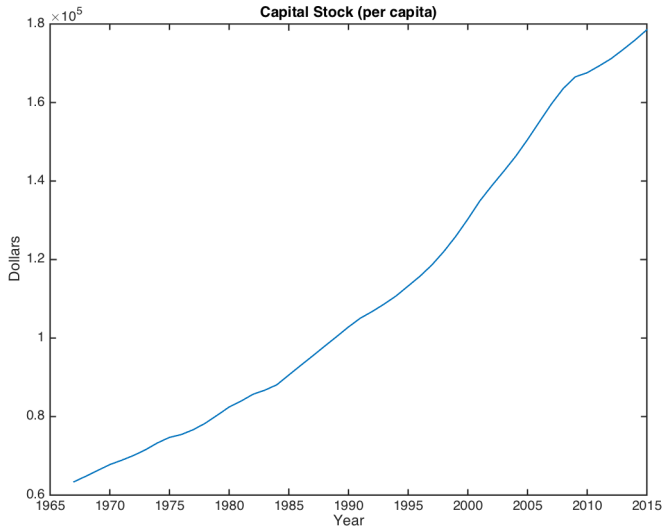




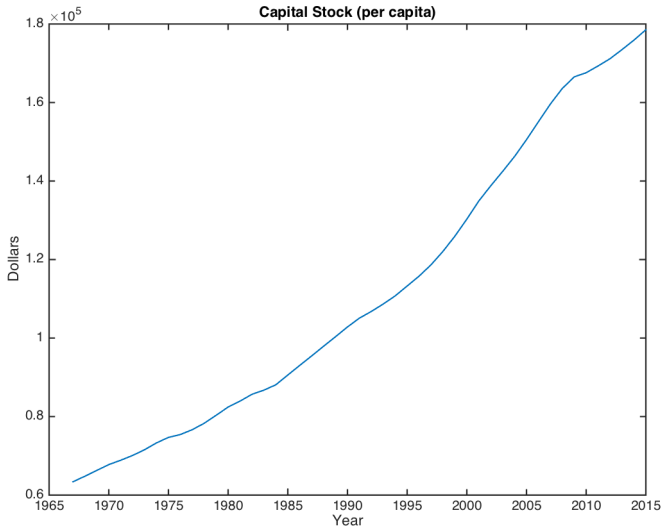
# PER CAPITA



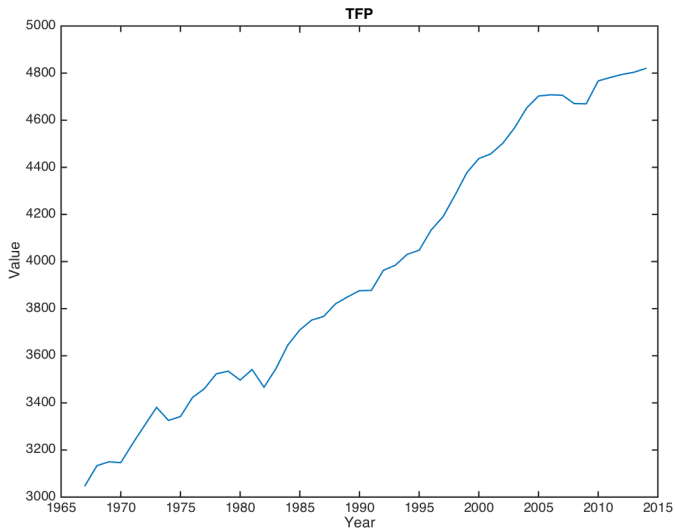
# CREATE CAPITAL STOCK



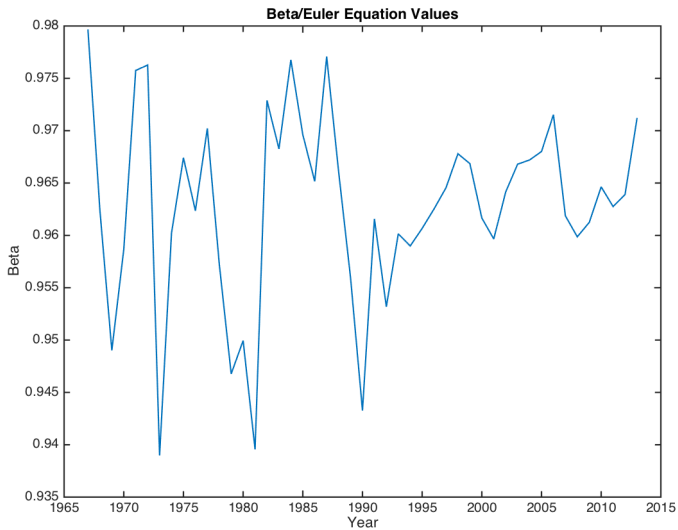
# CREATE CAPITAL STOCK: $K_{t+1} = (1 - \delta)K_t + I_t$



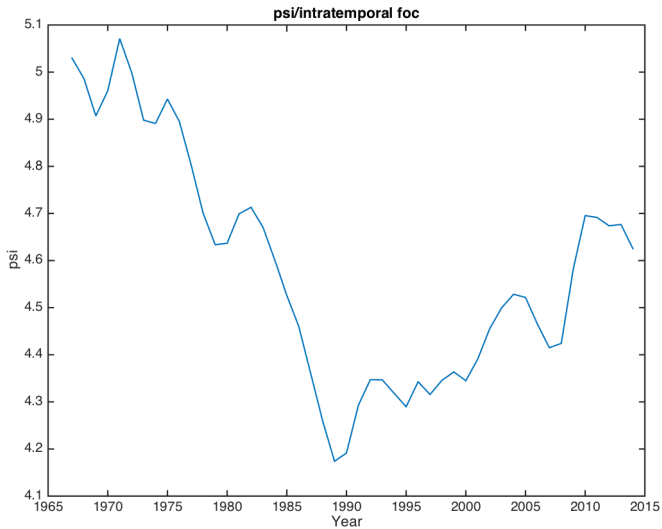
FIND TFP:  $A_t = \frac{Y_t}{L_t^\alpha K_t^{1-\alpha}}$



$$\text{FIND } \beta = \frac{1}{1-\delta+r_{t+1}} \frac{c_{t+1}}{c_t}$$



$$\text{FIND } \psi = \frac{w_t(1-L_t)}{c_t}$$



# CALIBRATION

▶  $k_0 = 63396$

▶  $\delta = 0.043$

▶  $\beta = 0.963$

▶  $\psi = 4.58$

▶  $\alpha = 0.7$

## SOLUTION AND ASSUMPTIONS

- ▶ There are many possible assumptions with respect to  $A_t$
- ▶ A first step is complete knowledge of  $A_t$
- ▶ Assume no labor, consumption, or capital taxes
- ▶ What about future  $A_t$ ?
- ▶ Continue  $A_t$  growth constantly into the future
- ▶ Force to reach steady state growth path 20 years into constant future

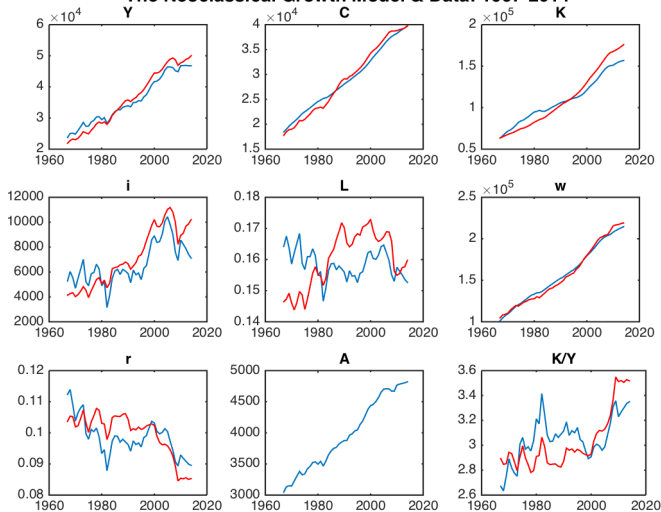


## SOLUTION METHOD

- ▶ Given parameters, a path of  $A_t$ , and a  $k_0$ , I have two decisions to make:
  - ▶  $L_t \times T$  labor decisions from intratemporal FOC
  - ▶  $K_{t+1} \times T$  capital decisions from intertemporal FOC
- ▶ Solve for 40 existing years, then give 30 years to reach SS growth path
- ▶ Then, for 30 more periods, assume they're on autopilot
- ▶ They only get to choose  $70 \times 2$  choices to solve  $70 \times 2 + 30 \times 2$  FOC's!
- ▶ Make sure all FOC's are zero

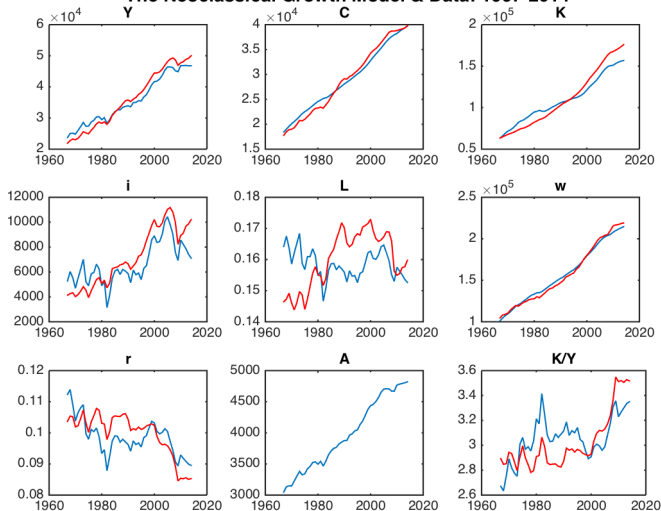
# THEORY AND DATA

The Neoclassical Growth Model & Data: 1967-2014



# THEORY AND DATA

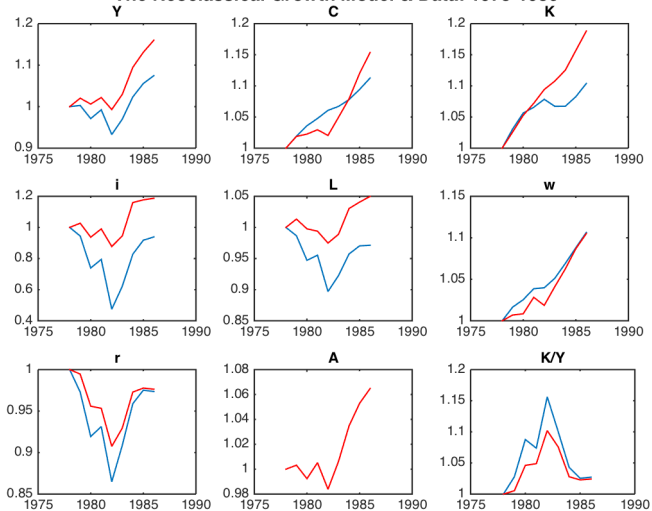
The Neoclassical Growth Model & Data: 1967-2014



Which is theory, which is data?

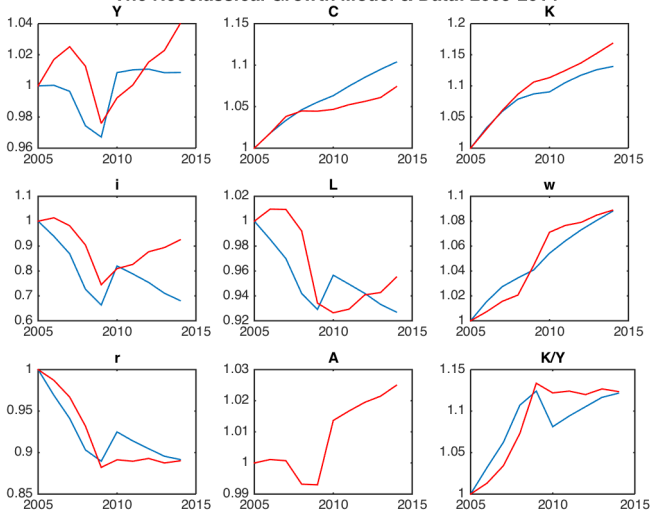
# EXPLAINING THE EARLY 1980's

The Neoclassical Growth Model & Data: 1978-1986



# EXPLAINING THE LATE 2000's

The Neoclassical Growth Model & Data: 2005-2014



# (MACRO)ECONOMICS AS A SCIENCE

- ▶ In similar position to meteorology, astronomy, geology
- ▶ No randomized experiments
- ▶ Data is too short for nonparametric analysis
- ▶ Ideal (and solution) a unified theory: explain all frequencies
- ▶ Parsimony keeps us honest
- ▶ Explaining multiple stochastic processes jointly keeps us honest
- ▶ Explaining dynamics gives more role for testable hypotheses