

# ECON 352 - SEARCH AND UNEMPLOYMENT

(See Williamson Ch. 6)

Trevor S. Gallen

# INTRODUCTION

- ▶ We built up the “frictionless” (except taxes!) model of labor supply
- ▶ This model does not please many, and yields challenges
- ▶ Why is there unemployment? We'll build a model to think about it.
- ▶ Important note: we will start talking about something some find dry: firms and workers searching to fill jobs/vacancies and bargaining over wage. *if you find this boring*, please note that you can replace this with a male/female dating market! (more assumptions needed for homosexual dating market, feel free to ask when it comes up!)

## LABOR MARKET FACTS

- ▶ Let's review some of the facts and concepts.
- ▶ Let  $N$  be the working age population,  $Q$  be the labor force (employed+unemployed), and  $U$  be the number of unemployed. Then we define:

$$\text{unemployment rate} = \frac{U}{Q}$$

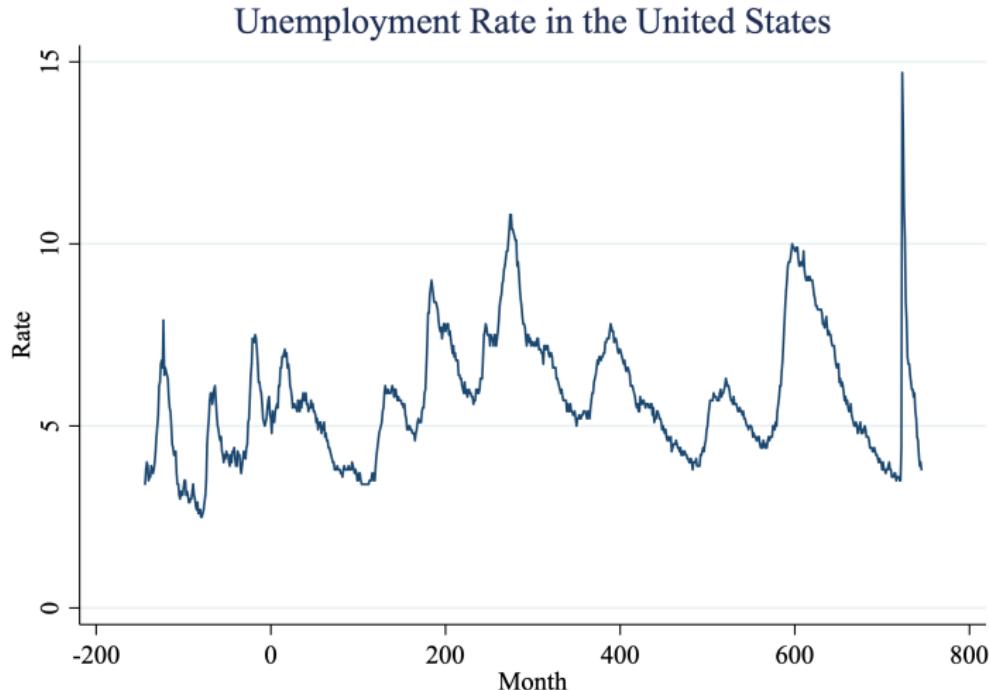
$$\text{participation rate} = \frac{Q}{N}$$

$$\text{employment/population ratio} = \frac{Q - U}{N}$$

- ▶ Let's look at the data first!

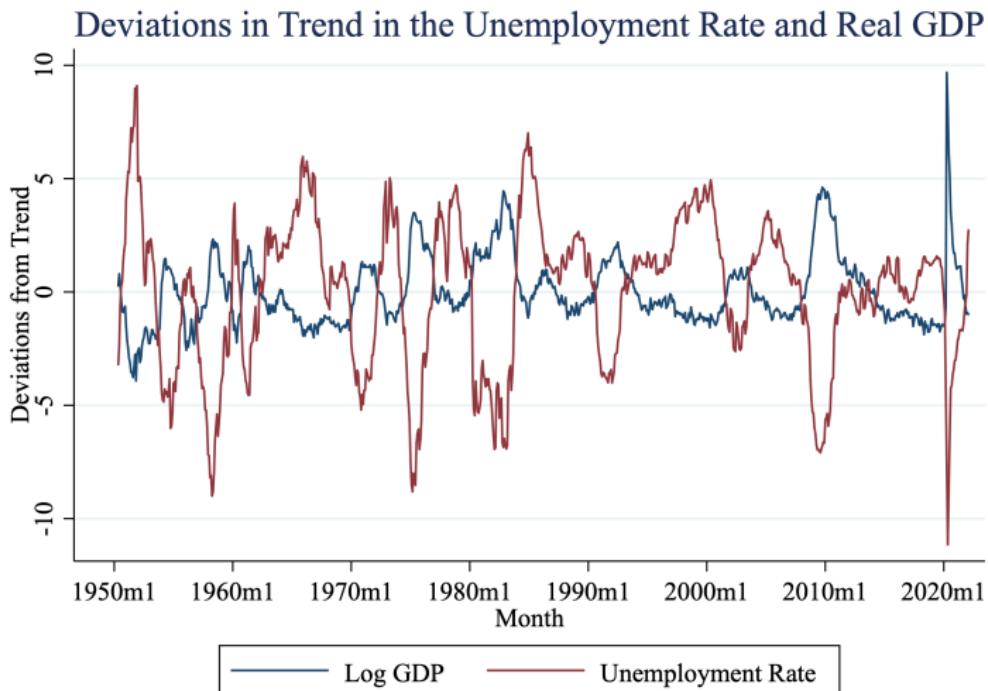
# UNEMPLOYMENT RATE IN THE US

## Figures



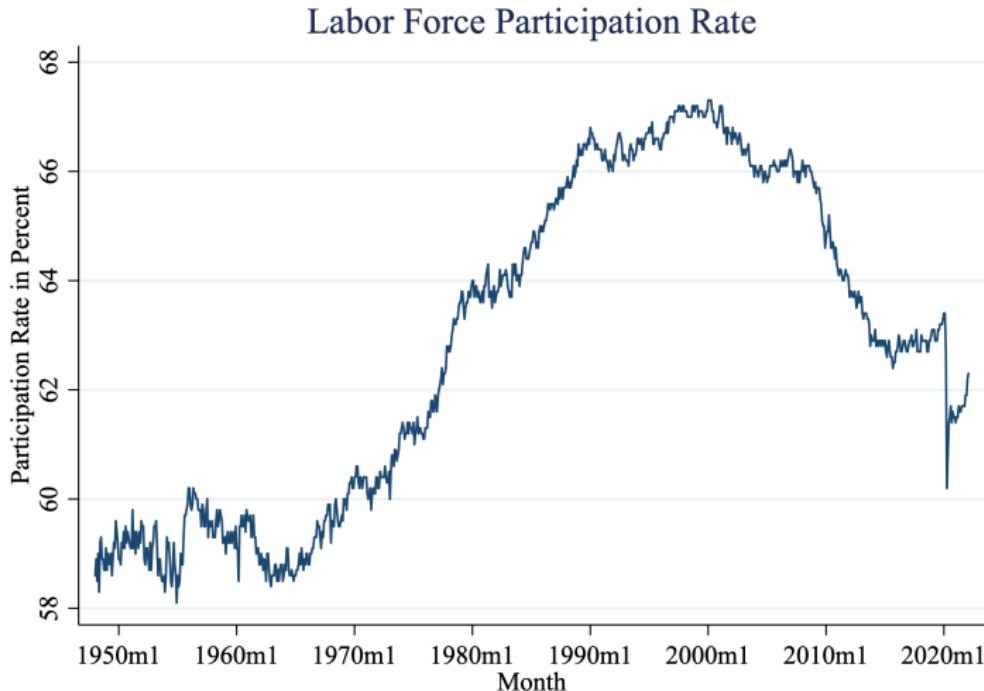
Unemployment is countercyclical, and has some long-run trends

# DEVIATIONS FROM TREND IN THE US UNEMPLOYMENT RATE



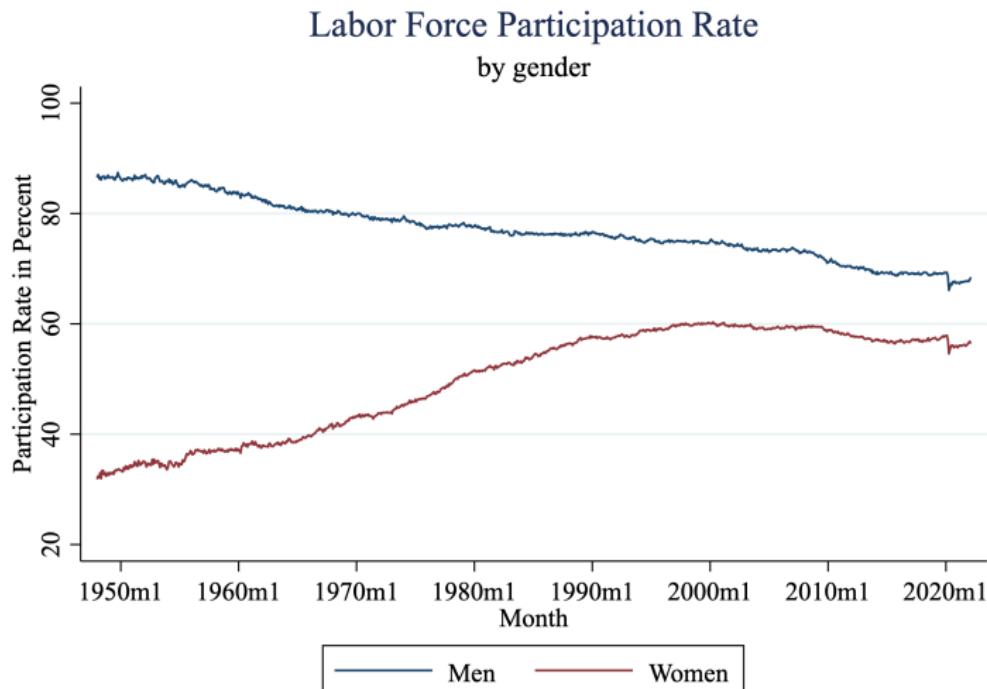
Unemployment is clearly countercyclical

# LABOR FORCE PARTICIPATION RATE



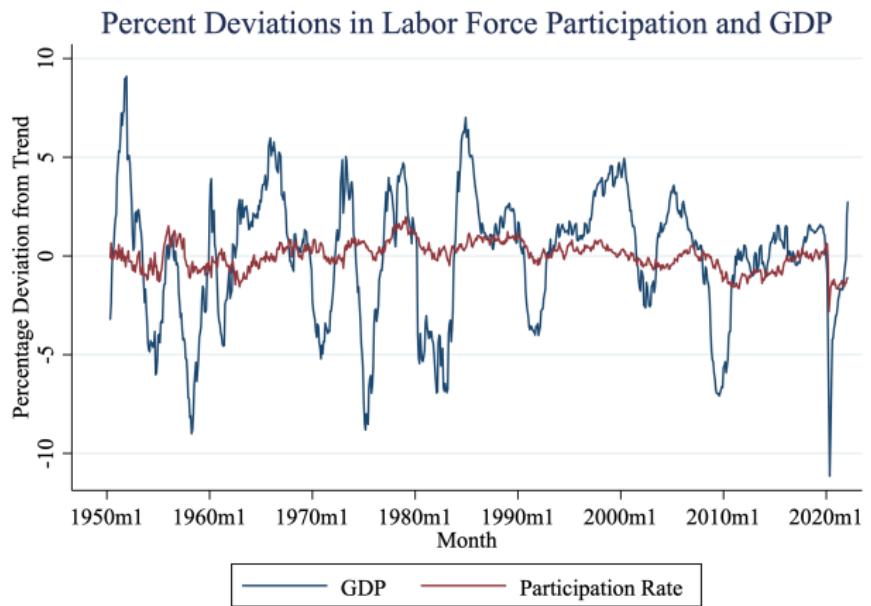
Stark long-run changes in the Labor Force Participation Rate

# LABOR FORCE PARTICIPATION RATES OF MEN AND WOMEN

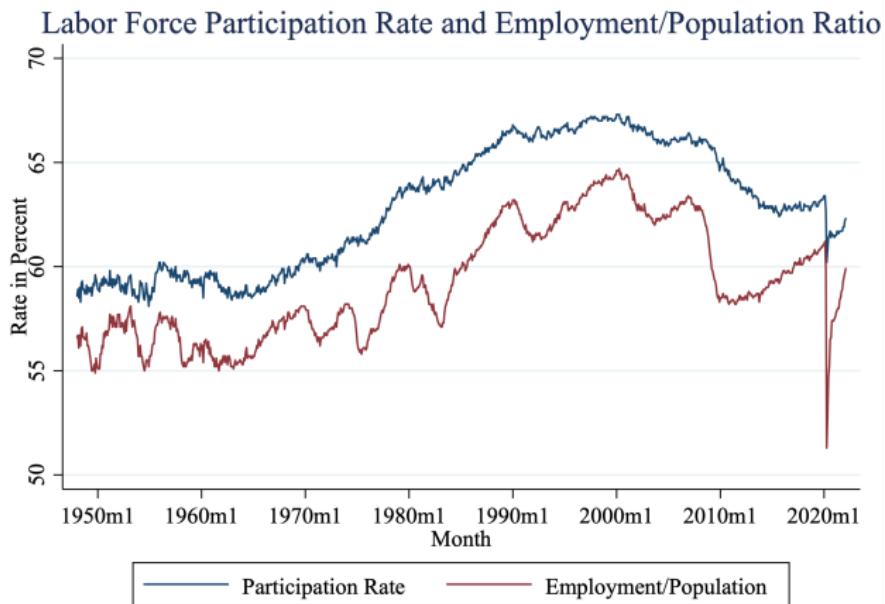


Highly heterogeneous long-run trends by gender!

# PERCENTAGE DEVIATIONS FROM TREND IN THE LABOR FORCE PARTICIPATION RATE AND REAL GDP



# LABOR FORCE PARTICIPATION RATE AND EMPLOYMENT/POPULATION RATIO



Participation rate is procyclical, but much less volatile than real GDP (unlike U)

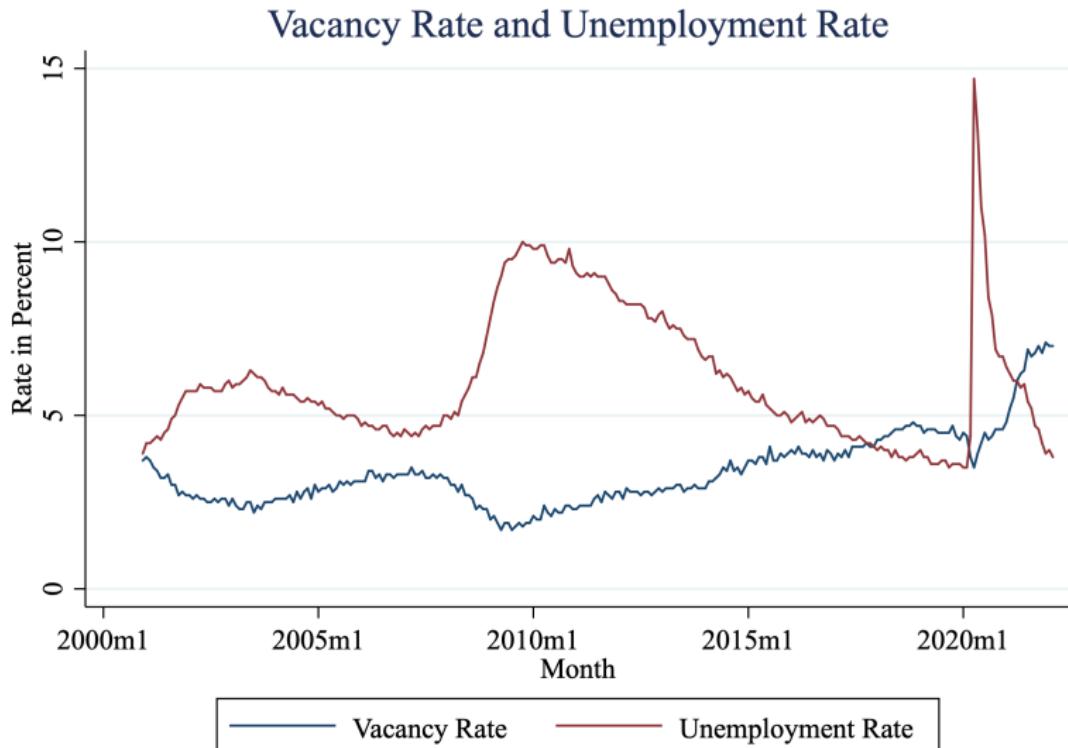
## VACANCY RATE AND THE BEVERIDGE CURVE

- ▶ We think of workers as the only “unemployed” people, but the problem is symmetric! (like dating!)
- ▶ Vacancies are “unemployed” firms: firms advertise job vacancies ( $A$ ) they wish to fill
- ▶ Unfilled jobs are costly to firms (otherwise, why hire?)
- ▶ Define the vacancy rate as:

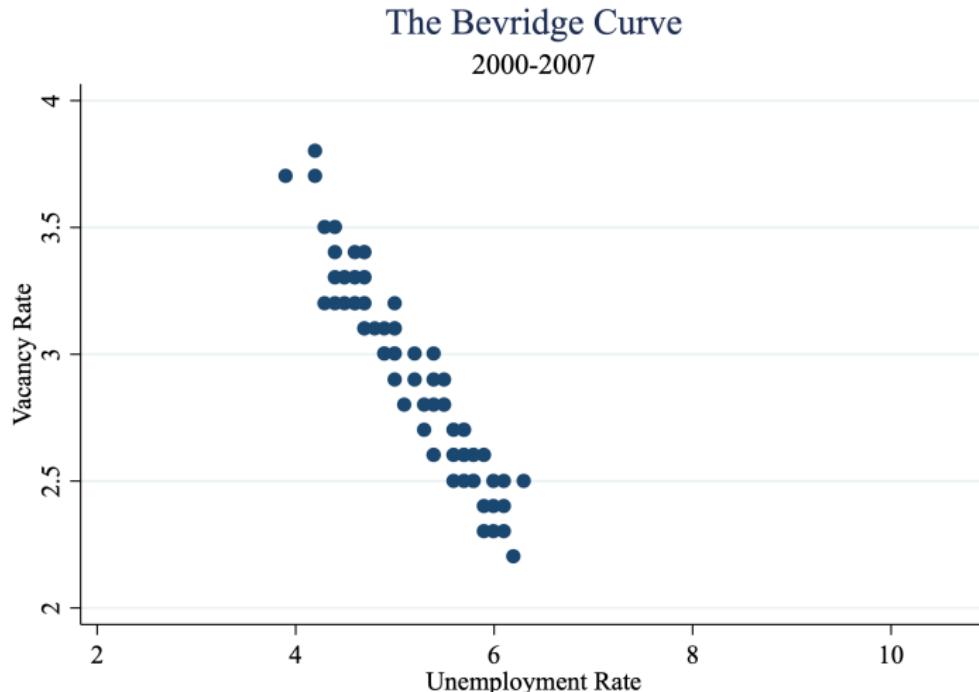
$$\text{vacancy rate} = \frac{A}{A + Q - U}$$

- ▶ Num vacancies / number of vacancies+employed, just like unemployment

# VACANCY RATE AND THE UNEMPLOYMENT RATE

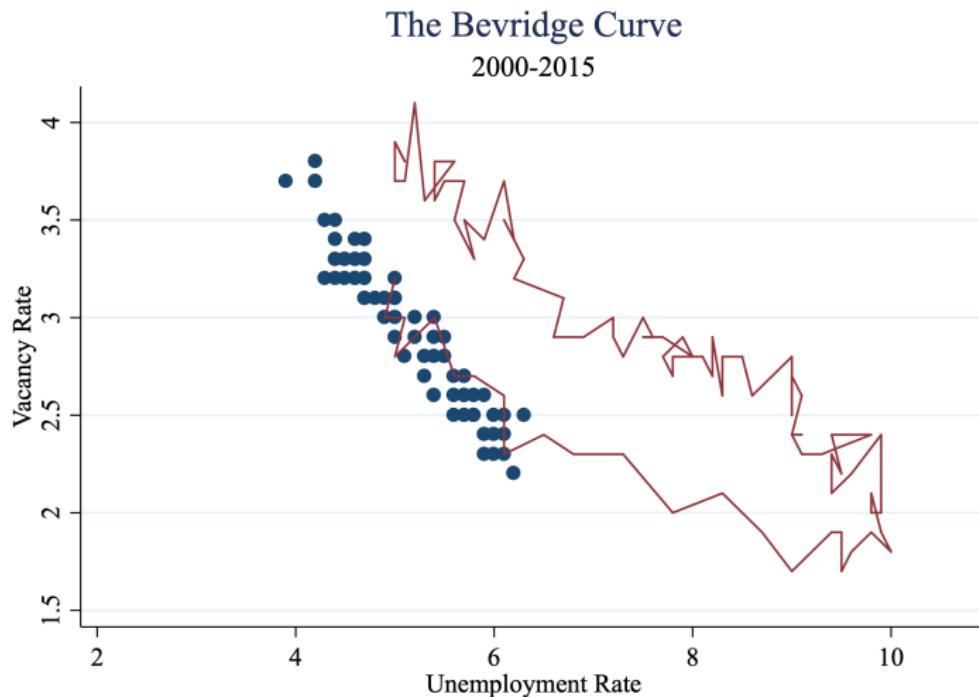


# VACANCY RATE AND THE UNEMPLOYMENT RATE



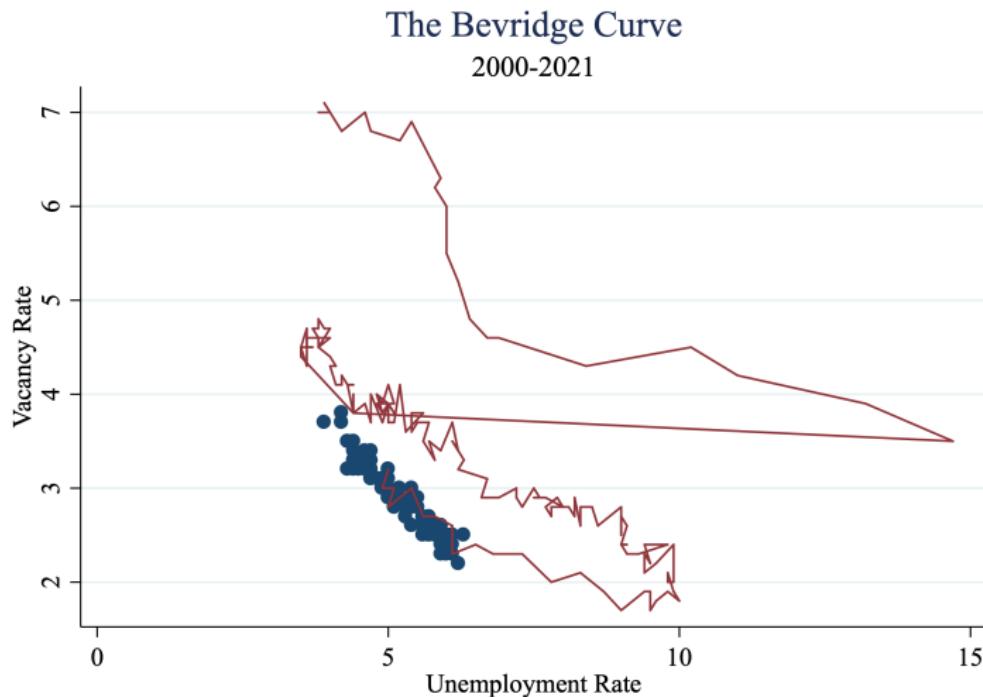
Pretty important! Tight empirical relationship we can really build a science on right?

# VACANCY RATE AND THE UNEMPLOYMENT RATE



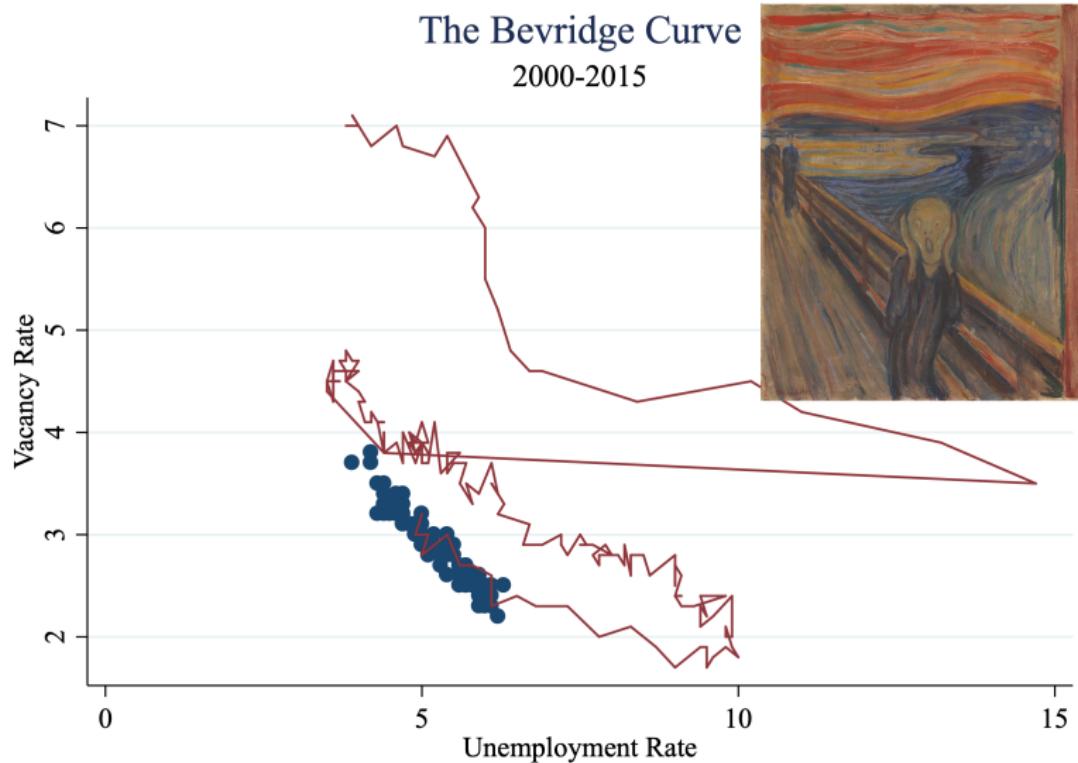
hmm...

# VACANCY RATE AND THE UNEMPLOYMENT RATE



Ah.

# VACANCY RATE AND THE UNEMPLOYMENT RATE



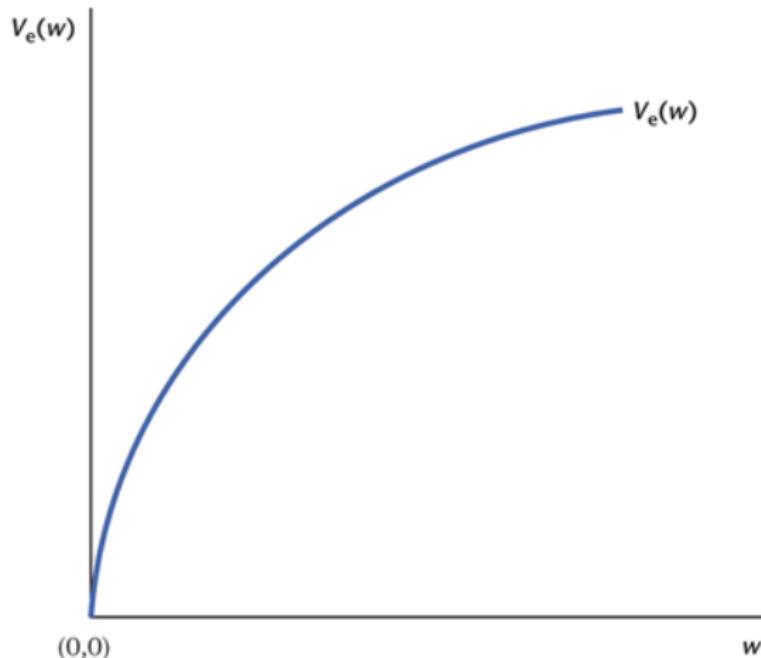
## ONE-SIDED SEARCH MODEL OF UNEMPLOYMENT

- ▶ We're going to focus on a worker, who is searching for work and receives job offers that pay particular wages, decides whether or not to accept a job and stop searching “McCall Search model”
- ▶ We want to understand why people might be unemployed, and search behavior
- ▶ Model: all workers are in labor force, either employed or unemployed
- ▶ Let  $V_e(w)$  denote the value of being employed at wage  $w$ .
- ▶ Let  $s$  be the probability of losing one's job
- ▶ Let  $V_u$  denote the value of being unemployed
- ▶ Let  $p$  be the probability of getting a job offer
- ▶ Let's make some predictions about  $V_e(w)$  and  $V_u$

## PREDICTIONS

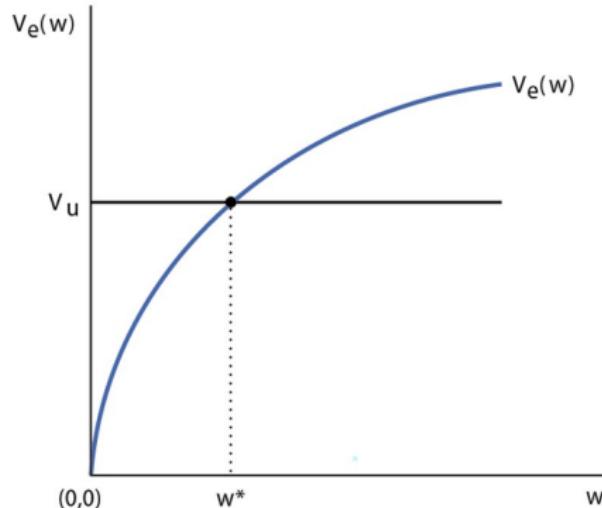
- ▶  $V_e(w)$  slopes up in the wage ( $\frac{\partial V_e(w)}{\partial w} < 0$ )
- ▶  $V_e(w)$ 's slope becomes less positive as wage increases  
 $(\frac{\partial^2 V_e(w)}{\partial w^2} < 0)$
- ▶  $V_e(w)$  shifts down when  $s$  increases
- ▶  $V_u$  is increasing in unemployment insurance benefits (+value of leisure) ( $b$ )
- ▶  $V_u$  is increasing in  $p$
- ▶ The  $w$  for which  $V_e(w) = V_u$  is called **the reservation wage  $w^*$** :
  - ▶ If offered wage is  $w \geq w^*$ , then accept offer
  - ▶ If offered wage is  $w < w^*$  then reject
- ▶ Let's graph

## WELFARE OF EMPLOYED WORKER $V_e(w)$



RESERVATION WAGE:  $V_e(w^*) = V_u$

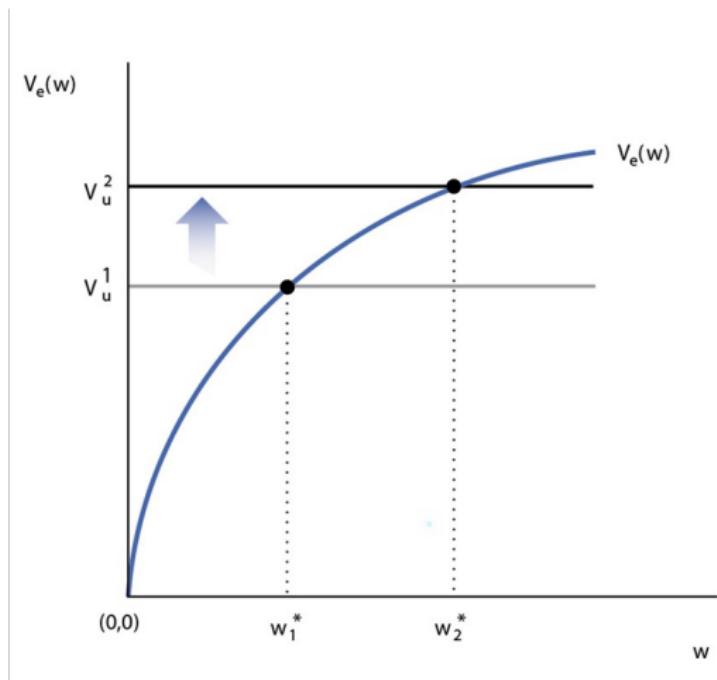
---



What happens if we shift the unemployment insurance benefit?

# INCREASE THE UNEMPLOYMENT INSURANCE BENEFIT

*b*



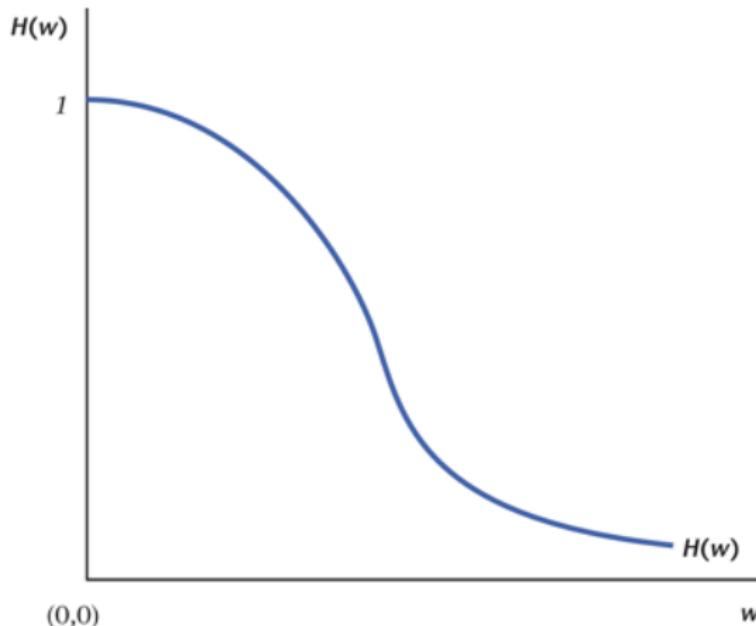
When  $b$  increases,  $V_u$  increases, so reservation wage  $w^*$  increases

## DETERMINING THE EQUILIBRIUM UNEMPLOYMENT RATE

- ▶ Let  $U$  be the unemployment rate,  $1 - U$  be employment rate, and  $s$  be the separation rate ( $\text{emp} \rightarrow \text{unemp}$ ).
- ▶ Then flow into unemployment is  $s \cdot (1 - U)$
- ▶ Let  $H(w^*)$  be the fraction of unemployed workers who (a) received a wage offer ( $w$ /prob  $p$ ), and (b) also had a wage offer greater than  $w^*$
- ▶ Then  $pH(w^*)U$  is the flow into employment ( $\text{unemp} \rightarrow \text{emp}$ )
- ▶ Equilibrium occurs when these two are equal (flows into employment are the same as flows out of unemployment)

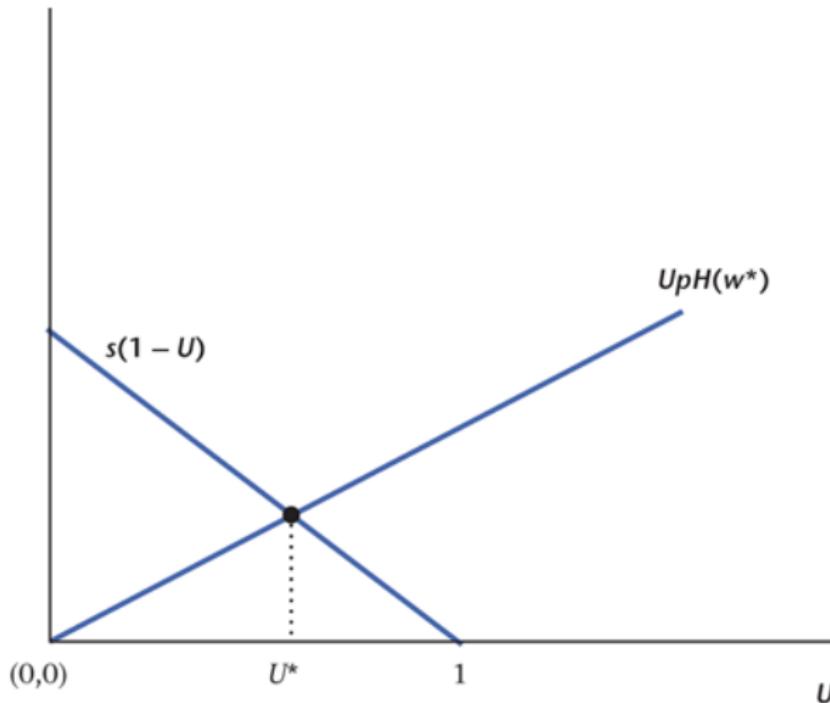
$$s(1 - U) = pUH(w^*)$$

## DISTRIBUTION OF WAGE OFFERS ABOVE THRESHOLD



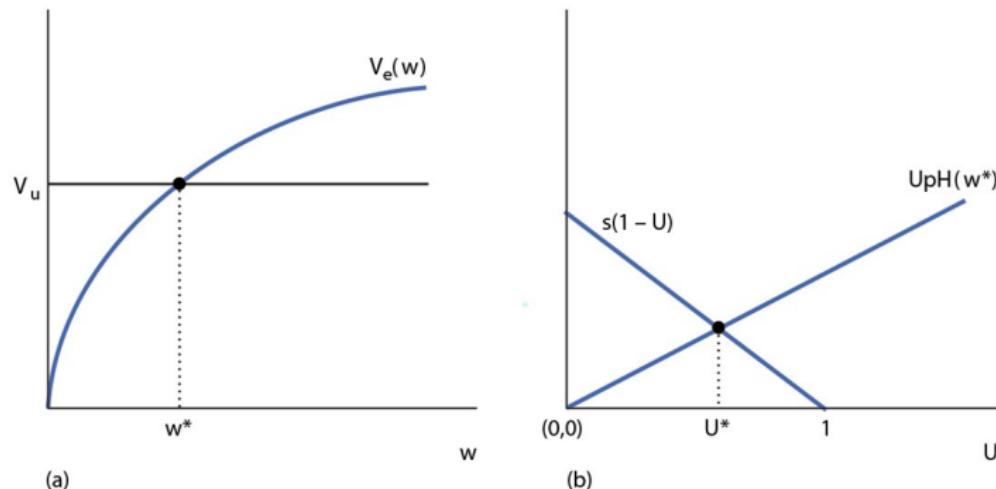
Idea: the higher the reservation wage, the smaller the fraction of workers getting that offer (this is just the “tail distribution” or 1-the CDF of wage offers)

# THE DETERMINANTS OF THE UNEMPLOYMENT RATE $U^*$ IN THE ONE-SIDED SEARCH MODEL



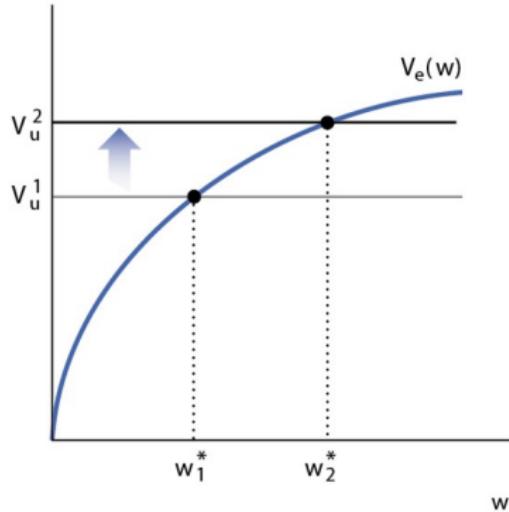
When  $b$  increases,  $V_u$  increases, so reservation wage  $w^*$  increases

# THE DETERMINATION OF THE RESERVATION WAGE AND THE UNEMPLOYMENT RATE IN THE ONE-SIDED SEARCH MODEL

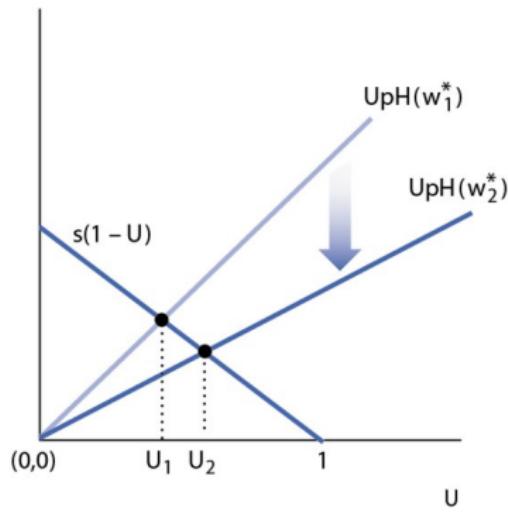


Putting both our previous figures together. Now let's use this to analyze increases in  $b$  and  $p$ .

# INCREASE IN UNEMPLOYMENT INSURANCE BENEFITS

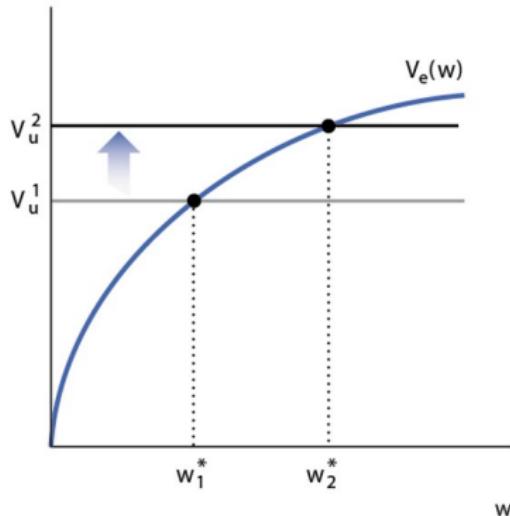


(a)

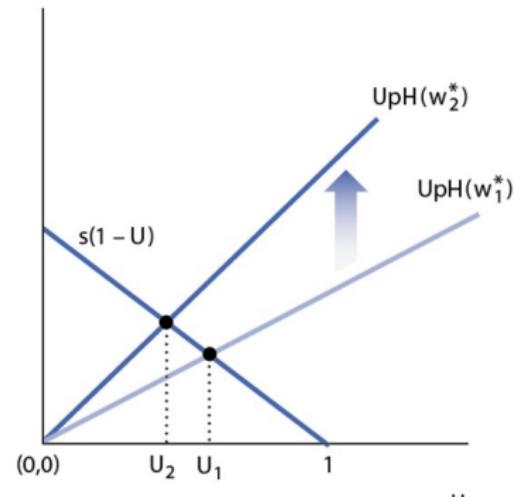


(b)

# INCREASE IN JOB OFFER RATE



(a)



(b)

## TWO-SIDED SEARCH

- ▶ This is cool beans and all, but what about the demand side of labor? Where does  $H(w)$  come from?
- ▶ Let's write down a two-sided model in which workers search for firms and firms search for workers

## TWO-SIDED SEARCH-SETUP

- ▶ Consumers
  - ▶ There are  $N$  consumers who can either work outside (home production or leisure) the market or search for market work
  - ▶  $Q$  is the quantity of consumers who look for work (labor force) so  $N - Q$  is the quantity who are out of the labor force
  - ▶  $P(Q)$  is the supply curve of workers who choose to work for market work: as the wage rises, we expect more workers to look for work
- ▶ Firms
  - ▶ It costs firms  $k$  (in units of consumption goods) to post a vacancy. If you don't hire, you shut down (this is a one-period, one-job-per-firm model).  $A$  will denote the number of active firms (number that post vacancies)
- ▶ Environment:
  - ▶ When  $Q$  workers search and  $A$  firms have vacancies, then the number of matches is determined by the “matching function”  $m$  with efficiency  $e$ :

$$M = e \cdot m(Q, A)$$

## MATCHING FUNCTION

The matching function is a catch-all that describes how searching workers and firms meet. We assume that:

1. The matching function is CRS  $e \cdot m(xQ, xA) = xe \cdot m(Q, A)$
2. If no workers search, no matches are made  $e \cdot m(0, A) = 0$
3. If no firms have vacancies, no matches are made  
 $e \cdot m(Q, 0) = 0$
4.  $m$  is increasing in both  $Q$  and  $A$
5. Marginal products are diminishing (increase  $Q$  and hold  $A$  constant, and the increase in matches slows)

## SUPPLY SIDE OF THE LABOR MARKET

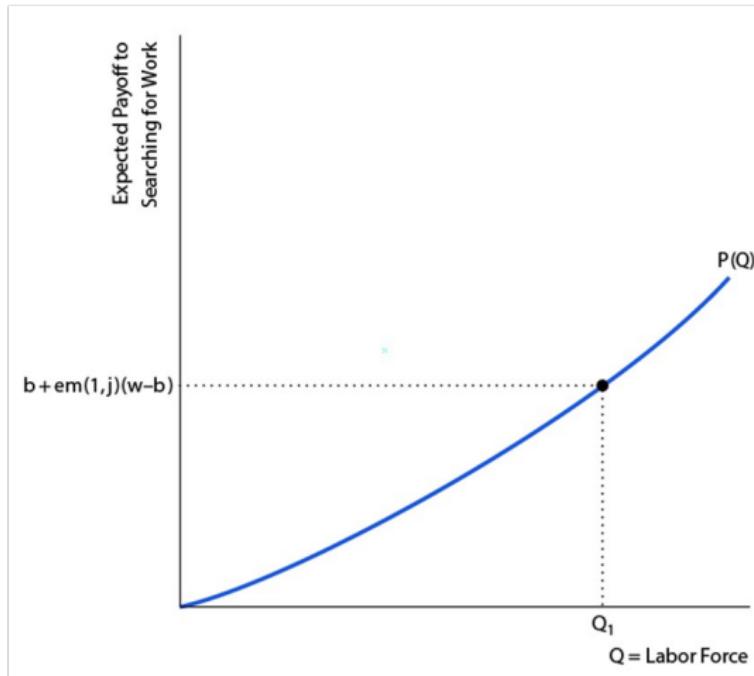
- ▶ This looks a lot like the one-sided market, but now depends on the matching function, rather than an exogenous matching probability  $p$
- ▶ Probability a consumer finds a job:  $p_c = \frac{e \cdot m(Q, A)}{Q}$
- ▶ Using constant returns to divide by  $Q$ :

$$p_c = m(1, j)$$

- ▶ Where  $j = \frac{A}{Q}$  is the “market tightness”
- ▶ This is useful: the probability of finding a job is dependent only on a single parameter  $j$ !
- ▶ Probability of staying unemployed is now  
 $1 - p_c = 1 - e \cdot m(1, j)$
- ▶ In equilibrium, supply curve of workers is expected payoff of searching:

$$P(Q) = p_c w + (1 - p_c)b = b + e \cdot m(1, j)(w - b)$$

# SUPPLY SIDE OF THE LABOR MARKET



Market wage, UI benefit, and labor market tightness determine the expected payoff to search, which gives the supply curve of search

## DEMAND SIDE OF THE LABOR MARKET

- ▶ Now we'll turn to the demand side, which will look similar, but will be simpler
- ▶ Probability a firm finds a worker:  $p_f = \frac{e \cdot m(Q, A)}{A} = e \cdot m\left(\frac{1}{j}, 1\right)$  using the same trick as before
- ▶ If a firm matches with a worker, they get  $z$ , but pay  $w$ , so on net keep  $z - w$ , but also pay the vacancy cost  $k$  no matter what
- ▶ If firms are free to enter the market, it must be that

$$\underbrace{p_f}_{\text{pr match}} \underbrace{(z - w)}_{\text{payoff if match}} \underbrace{- k}_{\text{vacancy cost}} = 0, \text{ the expected payoff is zero}$$

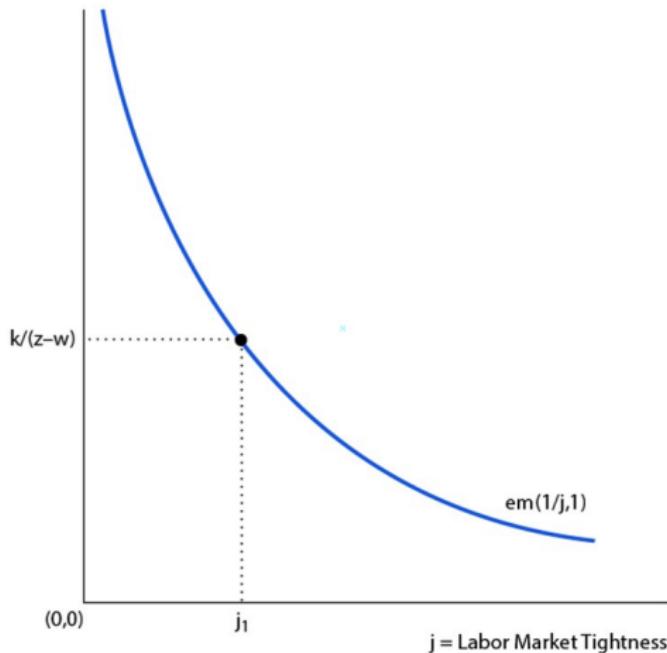
$$p_f = m\left(\frac{1}{j}, 1\right)$$

- ▶ Where  $j = \frac{A}{Q}$  is the "market tightness"
- ▶ Putting these two together we get:

$$e \cdot m\left(\frac{1}{j}, 1\right) = \frac{k}{z - w}$$

- ▶ Firms will post vacancies until this is true

## DEMAND SIDE OF THE LABOR MARKET



Firms post vacancies until the probability of matching with a worker is equal to  $k/(z - w)$

## HOW IS THE WAGE DETERMINED?

- ▶ When a firm matches with a worker, they produce  $z$
- ▶ How is  $w$  determined? Who gets the surplus from the match?
  - ▶ (This may also inform your understanding of seemingly dysfunctional relationship dynamics!)
  - ▶ Could be anything many rules. One Economists like is “Nash bargaining theory”
  - ▶ Think about the options of each party:
    - ▶ Worker’s next best option is  $b$ , so surplus is  $w - b$
    - ▶ Firm’s next best option is 0 ( $k$  is lost no matter what) so surplus is  $z - w$
    - ▶ Worker+firm surplus is  $z - b$
  - ▶ Nash bargaining predicts/dictates that each gets a constant share of total surplus.  $a$  will be the worker’s share:

$$w - b = a(z - b)$$

- ▶ Solving for the wage:

$$w = az + (1 - a)b$$

## HOW IS THE WAGE DETERMINED?

- ▶ Nash bargaining wage:

$$w = az + (1 - a)b$$

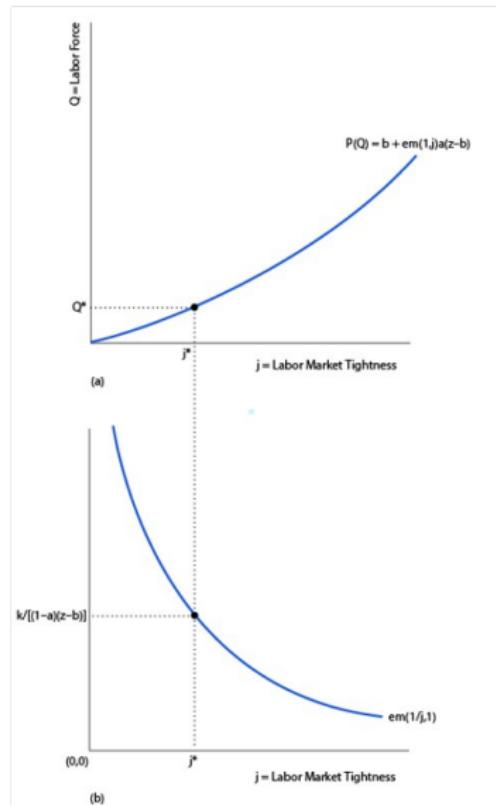
- ▶ Substituting into the supply and demand curves for labor we derived earlier:

$$P(Q) = b + e \cdot m(1, j)a(z - b)$$

$$e \cdot m\left(\frac{1}{j}, 1\right) = \frac{k}{(1 - a)(z - b)}$$

- ▶  $j$  will be determined by the second equation, which then determines  $Q$  in the first

# EQUILIBRIUM IN THE TWO-SIDED SEARCH MODEL



Demand side determines  $j^*$ , which then determines  $Q^*$

## EQUILIBRIUM IN THE TWO-SIDED SEARCH MODEL-CONTINUED

- ▶ With market tightness  $j^*$  and labor force  $\mathcal{Q}$ , can determine everything else
- ▶ With  $\mathcal{Q}$  we also know not in labor force  $N - \mathcal{Q}$ , and the unemployment rate:

$$U = \frac{\mathcal{Q}(1 - p_c)}{\mathcal{Q}} = 1 - e \cdot m(1, j)$$

- ▶ We also have the vacancy rate:

$$v = \frac{A(1 - p_f)}{A} = 1 - e \cdot m\left(\frac{1}{j}, 1\right)$$

- ▶ The total output in the economy is the number of matches  $M$  times productivity:

$$Y = Mz$$

- ▶ Or:

$$Y = e \cdot m(\mathcal{Q}, A)z = \mathcal{Q}e \cdot m(1, j)z$$

- ▶ Now let's apply it

## TWO-SIDED MODEL: INCREASING THE UI BENEFIT

- ▶ When the UI benefit  $b$  increases, the worker's outside option increases. But the wage is given by:

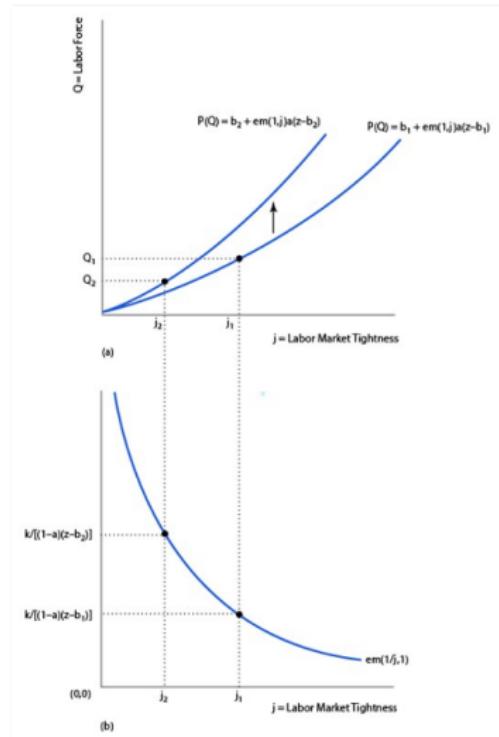
$$w = az + (1 - a)b$$

- ▶ So workers wages increase
- ▶ Firm surplus  $z - w$  falls, but it's still the case that:

$$e \cdot m\left(\frac{1}{j}, 1\right) = \frac{k}{z - w}$$

- ▶ Fixing vacancy cost  $k$  and productivity  $z$ , it must be that when the RHS rises, market tightness  $j$  must fall so the LHS rises commensurately
- ▶ In lay terms: firms now get less from each match, so the probability of matching must rise to make up for their loss (if it didn't, they would make negative profits, so firms leave until they make zero again)

# EQUILIBRIUM IN THE TWO-SIDED SEARCH MODEL



When  $b$  increases, firm search falls, and tightness increases.  
Meanwhile, supply shifts out, so that  $Q$  could increase or decrease

## TWO-SIDED MODEL: INCREASING THE UI BENEFIT-CONTINUED

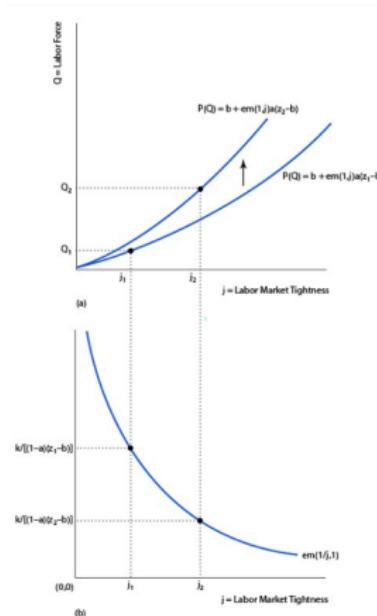
- ▶ Recall that  $P(Q) = b + e \cdot m(1, j)a(z - b)$ , so  $b$  rises (labor force rises) but  $j$  falls (so labor force falls). Ambiguous!
- ▶ But it's certain that  $U$  will rise and  $v$  will fall:

$$U = 1 - e \cdot m(1, j)$$

$$v = 1 - e \cdot m\left(\frac{1}{j}, 1\right)$$

- ▶ What happens to total output:  $Y = zM$ : ambiguous, depends on what happens to  $Q$ .
- ▶ In general, higher  $b$  has two effects: draws people in to searching for a job (increase GDP) but also pays people not to work (decrease GDP)
- ▶ Theoretically ambiguous affect on  $Y$ , but predicts high  $b$  countries should have higher "labor forces" but also higher unemployment

# EQUILIBRIUM IN THE TWO-SIDED SEARCH MODEL



When  $z$  increases, firm search rises, and tightness increases.  
Meanwhile, supply shifts out, so that  $Q$  increases unambiguously

## TWO-SIDED MODEL: INCREASING PRODUCTIVITY

- ▶ Now  $z$  increases. Again, start from wage:

$$w = az + (1 - a)b$$

- ▶ Wage increases by  $az$  what about firm side?

$$e \cdot m\left(\frac{1}{j}, 1\right) = \frac{k}{z - w}$$

- ▶ When  $z$  rises, then the RHS falls, so  $j$  must rise (labor tightness rise)
- ▶ And labor force (both  $j$  rises and  $z - b$  rises, so  $P(Q)$  definitely rises):

$$P(Q) = b + e \cdot m(1, j)a(z - b)$$

- ▶ Unemployment falls and vacancies rise

$$U = 1 - e \cdot m(1, j) \quad v = 1 - e \cdot m\left(\frac{1}{j}, 1\right)$$

- ▶  $Y = \overbrace{Q}^{\uparrow} e \cdot \overbrace{m(1, j)}^{\uparrow} \overbrace{z}^{\uparrow}$  increases

## DOES PRODUCTIVITY FIT WITH WHAT WE SEE IN THE BUSINESS CYCLE?

- ▶ Prediction: when  $z$  rises:
  - ▶ Labor force  $Q$  increases
  - ▶ Employment  $Q - U$  increases
  - ▶ Vacancies  $v$  increase
  - ▶ Wages  $w$  increase
  - ▶ Unemployment rate  $U$  falls
  - ▶ Total production  $Y$  increases
  - ▶ Concurrent increases in  $v$  and decreases in  $U$  are what the Beveridge curve shows
  - ▶ Good!
- ▶ Let's turn to our last application: efficiency of matching rate

## TWO-SIDED MODEL: DECREASE IN MATCHING EFFICIENCY

- ▶ Now  $e$  decreases. Again, start from wage:

$$w = az + (1 - a)b$$

- ▶ Wage is unchanged. Firm side?

$$e \cdot m\left(\frac{1}{j}, 1\right) = \frac{k}{z - w}$$

- ▶ When  $e$  falls, then  $j$  must fall to keep the LHS the same
- ▶ When  $j$  falls, then the supply curve shifts down for two reasons:

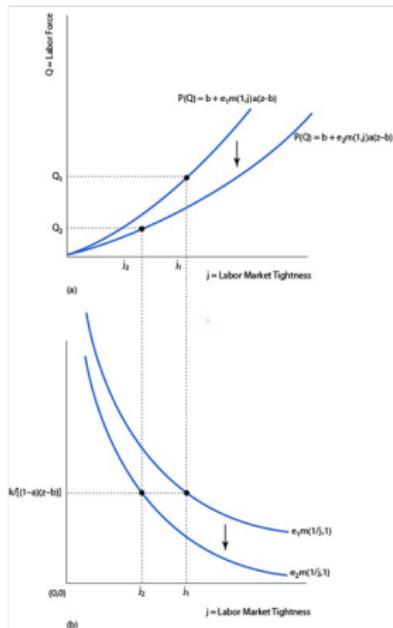
$$P(Q) = b + e \cdot m(1, j)a(z - b)$$

- ▶ So  $Q$  falls. Unemployment rises and vacancies fall

$$U = 1 - e \cdot m(1, j) \quad v = 1 - e \cdot m\left(\frac{1}{j}, 1\right)$$

- ▶  $Y = Qe \cdot m(1, j)z$  falls

# EQUILIBRIUM IN THE TWO-SIDED SEARCH MODEL



When  $e$  decreases, firm search falls, worker search falls

## TWO-SIDED MODEL: DECREASE IN MATCHING EFFICIENCY

- ▶ Decreases in matching efficiency might happen when we have “sectoral shocks” and “mismatch”
- ▶ Suddenly workers and firms are looking for different things
- ▶ Would be a shift *out* in the Beveridge curve

## SUMMARY: HOW TO THINK IN THE TWO SIDED MODEL

- ▶ We have two simple equations, so start with them:

$$w = az + (1 - a)b$$

$$e \cdot m\left(\frac{1}{j}, 1\right) = \frac{k}{z - w}$$

- ▶ The first gives you how  $w$  changes when  $z$  or  $b$  changes
- ▶ The second gives you how  $j$  changes when  $e$ ,  $k$ ,  $z$ , or  $w$  change
- ▶ Once you have how  $j$  changes, you can think about the labor supply side:

$$P(Q) = b + e \cdot m(1, j)a(z - b)$$

- ▶ And with that you can figure out unemployment, vacancies, production, etc.

## SIMPLE ONE-PERIOD MODELS

- ▶ Now you've seen our simple one-period models (for the market-clearing model and the unemployment/search-and-matching model)
- ▶ We'll turn to growth next section