Econ 352 - Economic Growth: Malthus and Solow

(See Williamson Ch. 7)

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Introduction

- We've seen some one-period models of Macro
- But we want to understand growth
- Let's start with growth facts and move on to a theory

Growth Facts

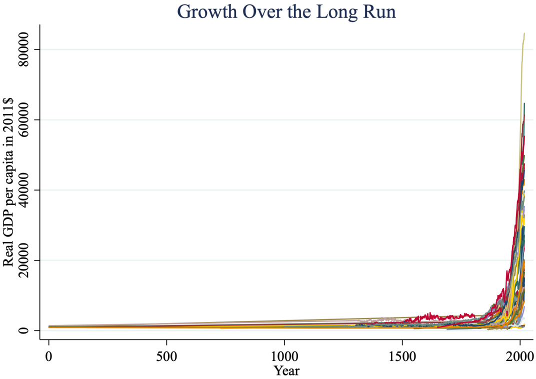
1. Before the Industrial Revolution (roughly 1800), living standards across time and countries did not vary much (everyone was poor)
2. Since then, per-capita income growth has bee sustained in the richest countries (roughly 2%/year in the US since 1900)
3. There's a positive correlation between investment and output per worker
4. There's a negative correlation between population growth and output per worker

Growth Facts

5. Differences in per-capita income exploded between 1800 and 1950
6. There is no correlation between level of output in 1960 and growth in per capita for the years 1960-2019
7. Richer countries are much more alike in terms of rates of growth of real per capita income than are poor countries

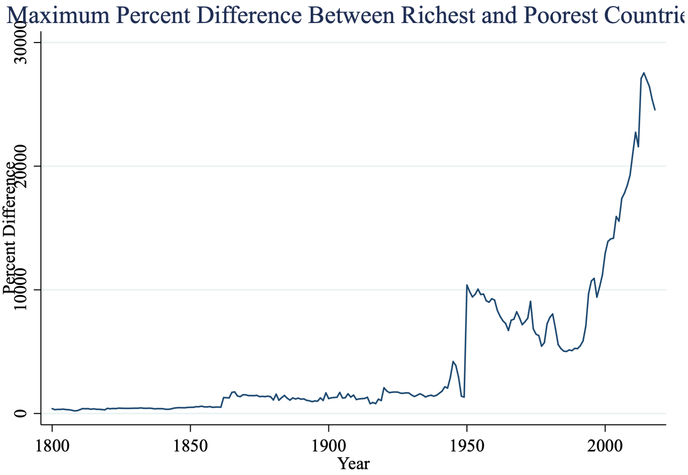
Let's see!

Growth Took Off!



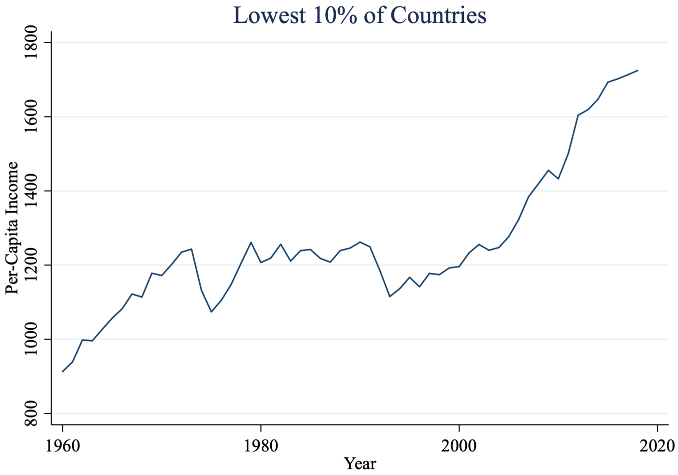
Growth is positively correlated with investment

As did disparities between countries



Growth is positively correlated with investment

Though almost all countries have grown



Growth is positively correlated with investment

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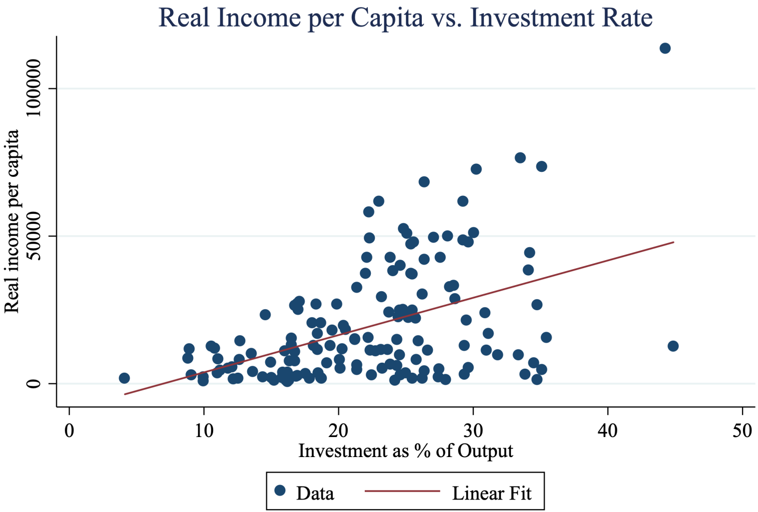
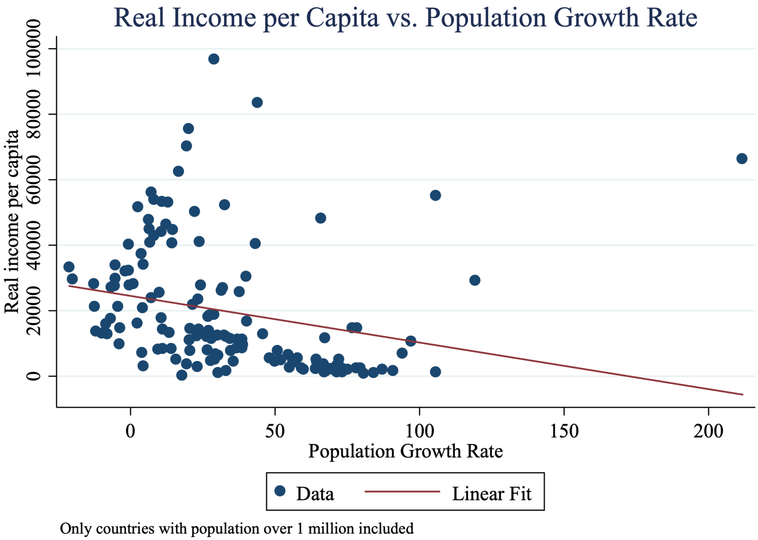
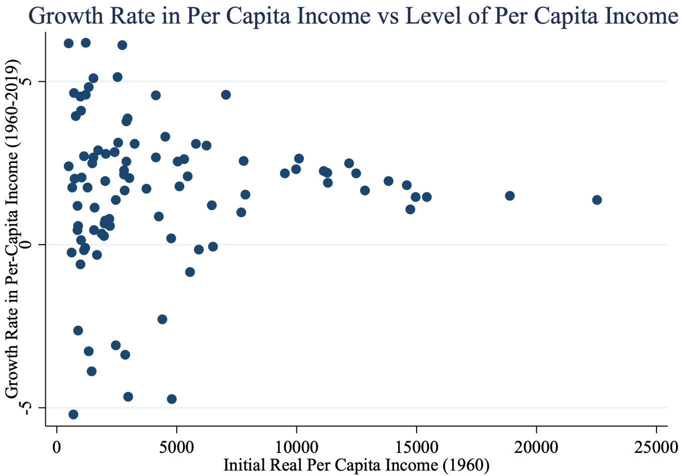


Only countries with population over 1 million included

Growth is negatively correlated with population growth

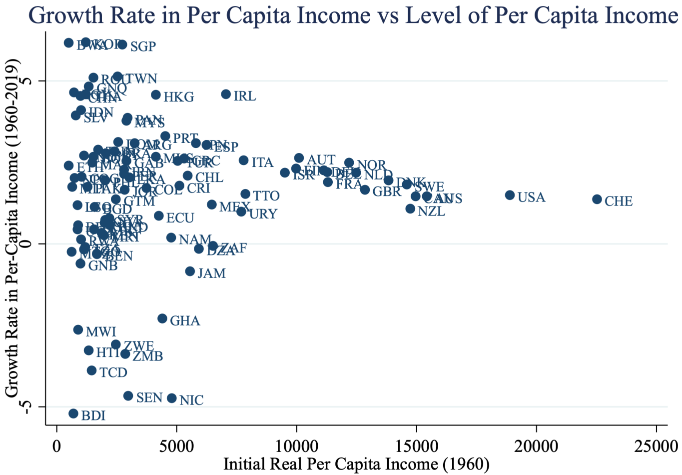


Growth is uncorrelated with initial level of GDP



(And is more homogenous among rich countries)

Growth is uncorrelated with initial level of GDP



Malthusian Model of Economic Growth-I

- This will be our first model, and it will be terrible
- Production function Y in terms of TFP z, capital (land!) L, and labor N:
$$Y = zF(L, N)$$
- Population growth (prime denotes next period):
$$N' = N + \text{Births} - \text{Deaths}$$
- Or:
$$N' = N + N(\text{birth rate} - \text{death rate})$$
- But the birth rate, we'll assume, is increasing in consumption per capita C/N (more food, more babies!)
$$\frac{N'}{N} = g\left(\frac{C}{N}\right)$$
- So need to think about C/N

Malthusian Model of Economic Growth-II

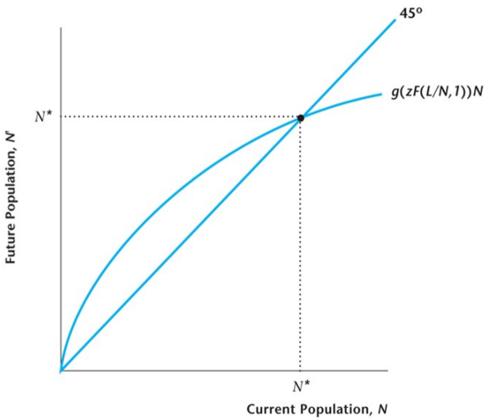
$$C = zF(L, N)$$

- Or:

$$N' = g\left(zF\left(\frac{L}{N}, 1\right)\right)N$$

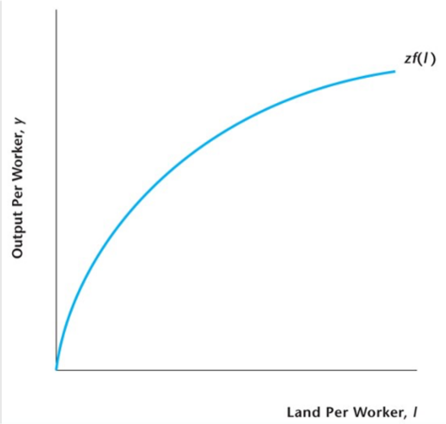
- Population next period is a concave function of per-capita consumption

Population Growth in the Steady State



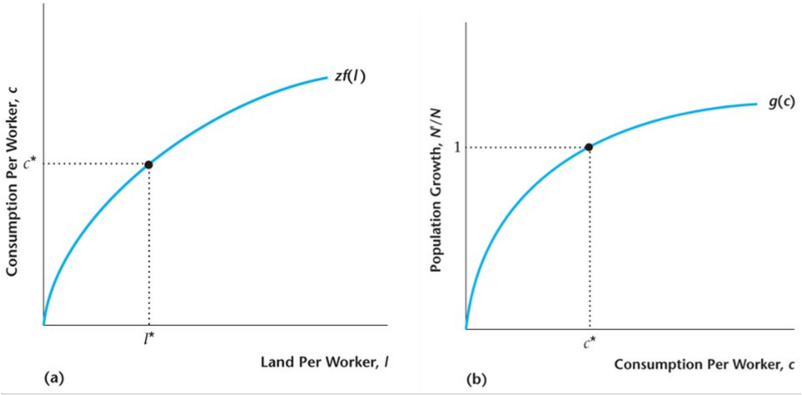
Consumption has a fixed point!

Per-Worker Production Function



$$\frac{F(L,N)}{N} = F\left(\frac{L}{N}, 1\right) = f(l)$$

Malthusian Model Steady State

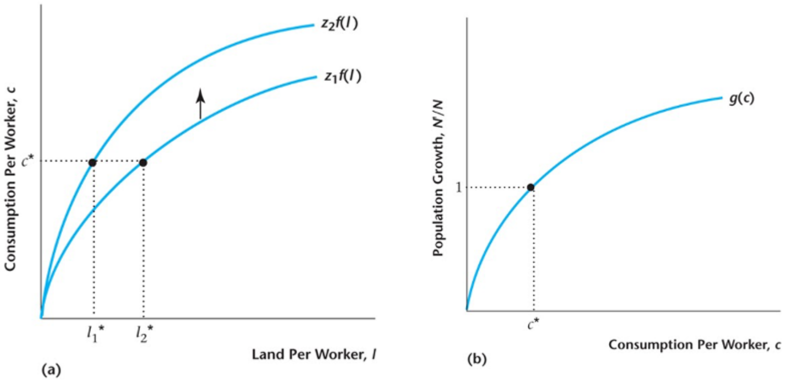


Now we can analyze shocks to the model: z and population control

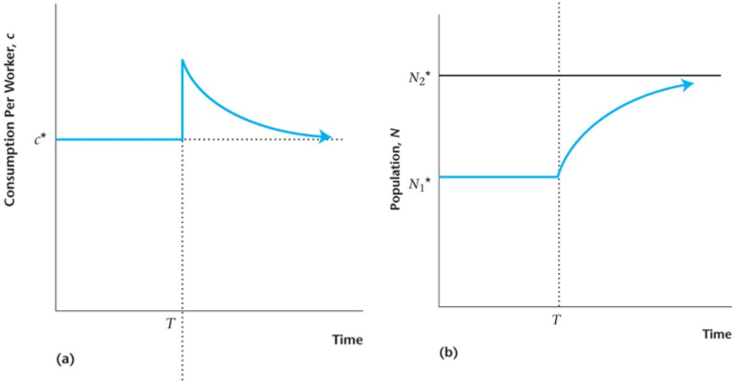
Malthusian Model Equilibrium Intuition

- What are these equations/this model telling us?
- We assume:
  1. land is fixed
  2. production as a function of labor alone is decreasing returns
  3. population growth depends on consumption
- Then if there aren't many people, they're very productive! Consumption is high, more children, consumption falls, still more children, until consumption falls enough that there is no more growth to the steady state
- Reverse if too many people: population growth goes negative, people grow more productive, and we converge from above to the steady state
- Now let's analyze what happens with an increase in total factor productivity (z) and how population control would work

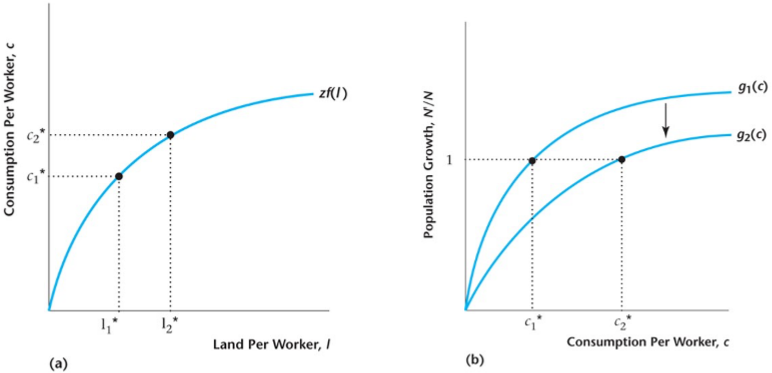
Increase in z



Increase in z



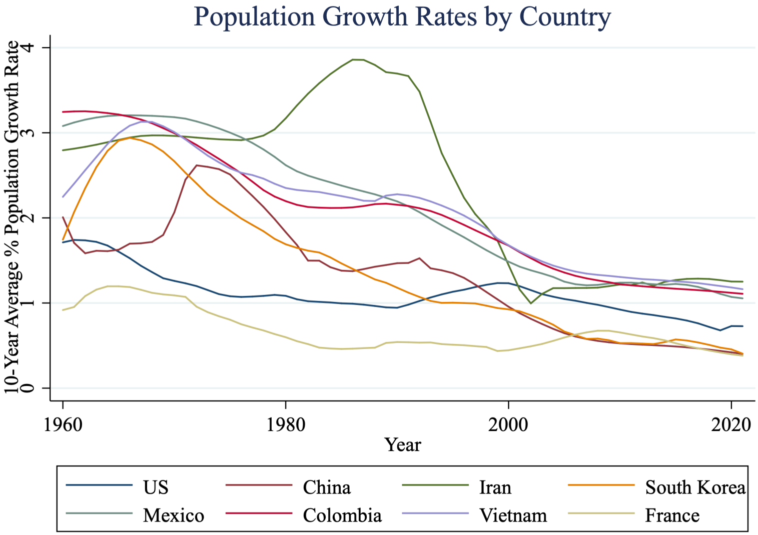
Population Control



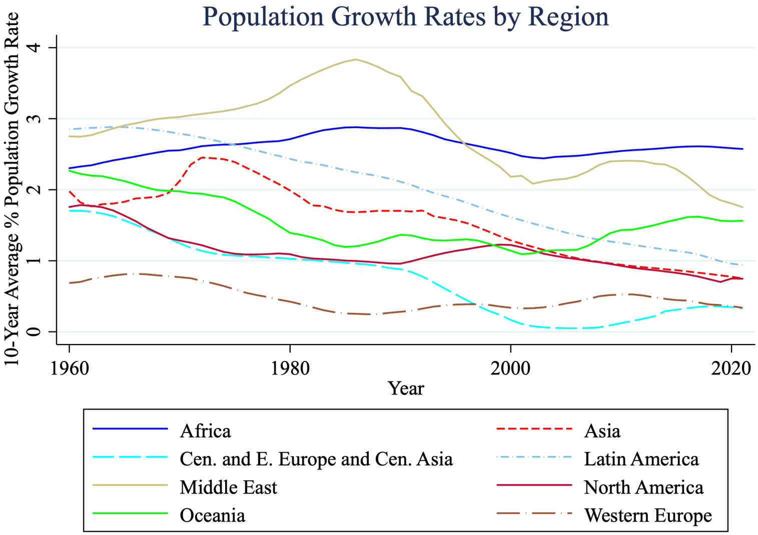
Is the Malthusian model any good?

- No! It's truly terrible
  - Aside: people aren't animals! There is **no** boom in babies after power-outs, **no** increase in births due to lockdowns. Every M.D. or nurse I've ever talked to about this gets this badly wrong! ($N > 8$)
- But it's a start on modellng growth
- Main change: land may be fixed, but capital isn't!
- Also, income and population growth are negatively correlated (at the country and the individual level)
- Issues with assumptions, but method/idea seems sound

Population Growth by Country



Population Growth by Region



Solow Growth Model-Overview

- We need a better model of growth than Malthusian
- What's the name of the game since 1800? Increasing productivity (z)!
- Let's say z is exogenously increasing (changing), human knowledge grows
- Population growth is fixed
- Savings/consumption are just constant fractions of production (simple assumption)
- Let it rip

Solow Growth Model-I

- Assumption 1: Population growth:

$$N' = (1 + n)N$$

- Assumption 2: s is (constant) fraction of product saved, $1 - s$ is fraction consumed

$$C = (1 - s)Y$$

- Assumption 3: Firms face the production function:

$$Y = zF(K, N)$$

- Or in per-capita terms, because CRS:

$$y = zf(k)$$

- Assumption 4: Capital has a law of motion:

$$K' = (1 - \delta)K + I$$

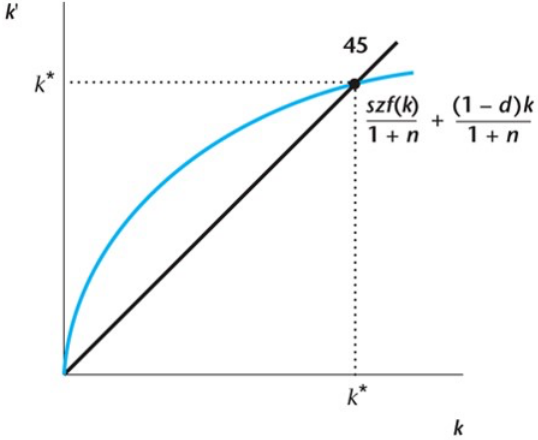
- Assumption 5: feasibility/social planner budget constraint:

$$Y = C + I$$

Solow Growth Model-II

-
- Now we can solve for growth and the "steady state" like we did with Malthus. Start with Assumption 5:
$$Y = C + I$$
- Solving for I in Assumption 4 and plugging in:
$$K' = sY + (1-\delta)K$$
- Using Assumption 3 and dividing by N:
$$\frac{K'}{N} = sz\frac{F(K,N)}{N} + (1-\delta)\frac{K}{N}$$
- Multiplying and dividing by N', we get in per-capita terms:
$$k'(1+n) = szf(k) + (1-\delta)k$$
- Or:
$$k' = \frac{szf(k)}{1+n} + \frac{(1-\delta)k}{1+n}$$
- Now we have how capital moves as a function of capital, can graph for equilibrium

Determination of the Steady State Quantity of Capital Per Worker



Analyzing the Steady State

- In the steady state, $k^* = k' = k$, so:

$$k^* = \frac{szf(^*)}{1+n} + \frac{(1-\delta)^*}{1+n}$$

- Or:

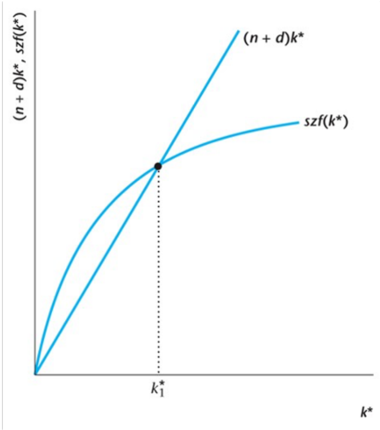
$$szf(k^*) = (n+\delta)k^*$$

- With this, we can ask: what happens to k^* when n, δ, s, or z increase?

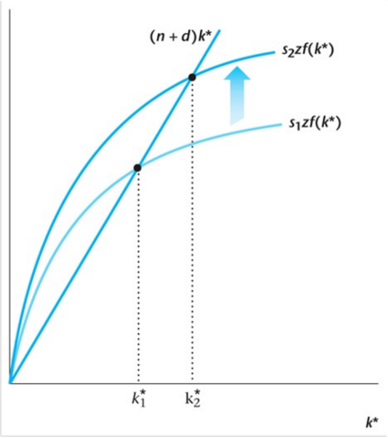
- Also note that $c^* = (1-s)y^* = (1-s)zf(k^*)$, so this also tells us about consumption

- The advantage is this will tell us what could and could not be causing growth!

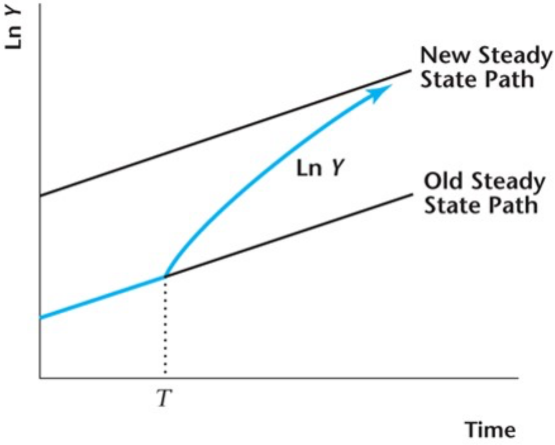
Determination of the Steady State Quantity of Capital Per Worker



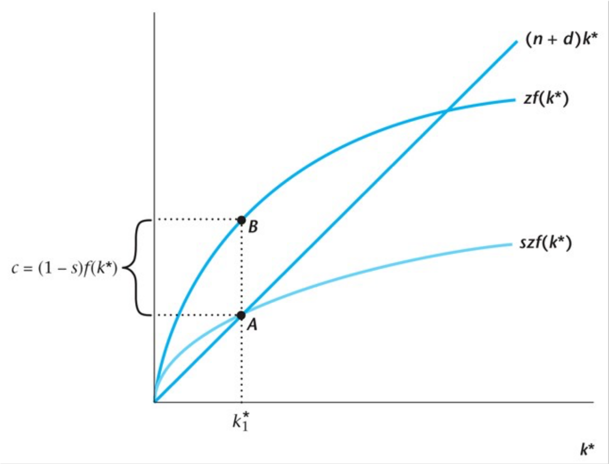
Increase in Savings Increases SS k^*



Effect of an Increase in the Savings Rate at time T



Steady State consumption per worker



Consumption per worker and the Golden Rule Capital Accumulation

- What is the “best” choice of savings rate (asterisk!)
- Let's maximize consumption c using s: on one hand, when it increases, more K, but less $(1 - s)Y$
- Find the maximize

Golden Rule Capital Accumulation

- Start with consumption as difference of production minus investment (see previous graph):

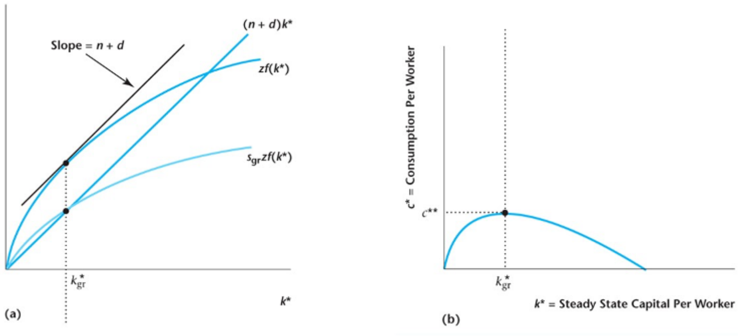
$$c = zf(k^*) - (n + \delta)k^*$$

- Where these two are farthest apart is the optimal (consumption-maximizing) level of capital
- That's where their slopes are equal! $\frac{\partial zf(k^*)}{\partial z^*} = \frac{\partial (n+\delta)k^*}{\partial k^*}$
- Or where:

$$MP_K = n + \delta$$

- Let's plot $s^* zf(k^*)$, $zf(k^*)$ and $(n + \delta)k^*$: best is when $s^* zf(k^*)$ intersects $(n + \delta)k^*$

Golden Rule Quantity of Capital per Worker



Labor Force Growth Rate

- What happens to GDP per capita when the labor force starts growing at a permanently higher rate? $(1+n) \rightarrow (1+n')$ where $n' > n$?

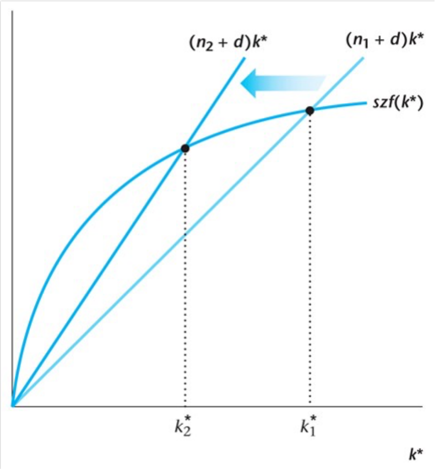
- We can draw it!

- But also recall we stop growing when $szf(k^*) = (n+\delta)k^*$

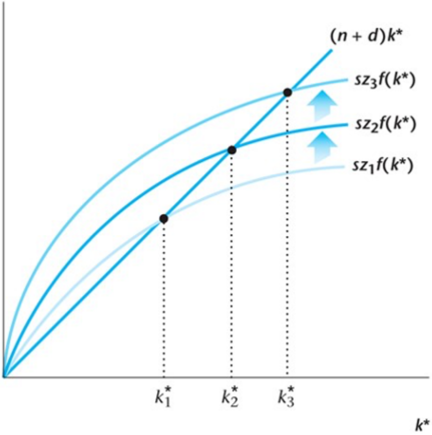
- If RHS increases because of n, then LHS must increase. How?

- By decreasing k^*!

Steady State Effects of an Increase in Labor Force Growth Rate



Steady State Effects of an Increase in Productivity



Measuring Z

- How can we measure z?

- Our model can help!
- Start with the production function:

$$Y = zF(K, N)$$

- Assume a Cobb-Douglas form:

$$F(K, N) = K^{\alpha} N^{1-\alpha}$$

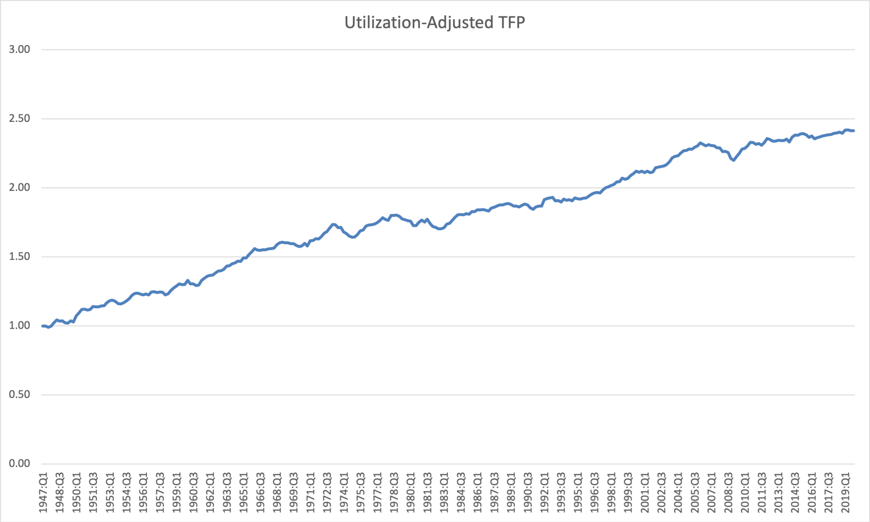
- Put numbers on α:

$$Y = zK^{0.3} N^{0.7}$$

- Then solve for z:

$$\hat{z} = \frac{\hat{Y}}{\hat{K}^{0.3} \hat{N}^{0.7}}$$

US TFP over Time



Chapter 7 Takeaways

- Growth facts!
- Malthusian model of growth: population growth a function of C
  - Predicts no per-capita growth, technological progress results in larger population
  - Can shock z, n
- Solow model of growth: population growth constant, productivity z and K can grow
  - Predicts steady-state capital/worker (or consumption),
  - Can analyze shocks to z, n, s, δ
  - Golden rule: what savings maximizes consumption?
- Growth accounting: what is z?