Econ 352 - A Real Intertemporal Model with Investment

(See Williamson Ch. 11)

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Introduction

- We have the intratemporal and intertemporal components of the household's problem
- Now we'll put them together and build a real intertemporal model with investment
- This model is at the core of all modern macro, which just adds bells & whistles (rigidity, search/matching, imperfections, heterogeneity).

Representative Consumer

- Faces budget constraint today and tomorrow:

$$C + S^P = w(h - \ell) + \pi - T$$

$$C = w'(h' - \ell') + \pi' - T' + (1 + r)S^P$$

- Combining as we have in past chapters:

$$C + \frac{C'}{1+r} = w(h - \ell) + \frac{w'(h - \ell)}{1+r} + \pi + \frac{\pi'}{1+r} - T - \frac{T'}{1+r}$$

- The household optimizes over ℓ, ℓ', C, and C', which we have shown can be summarized as:

$$MRS_{\ell,C} = w$$

$$MRS_{\ell',C'} = w'$$

$$MRS_{C,C'} = 1 + r'$$

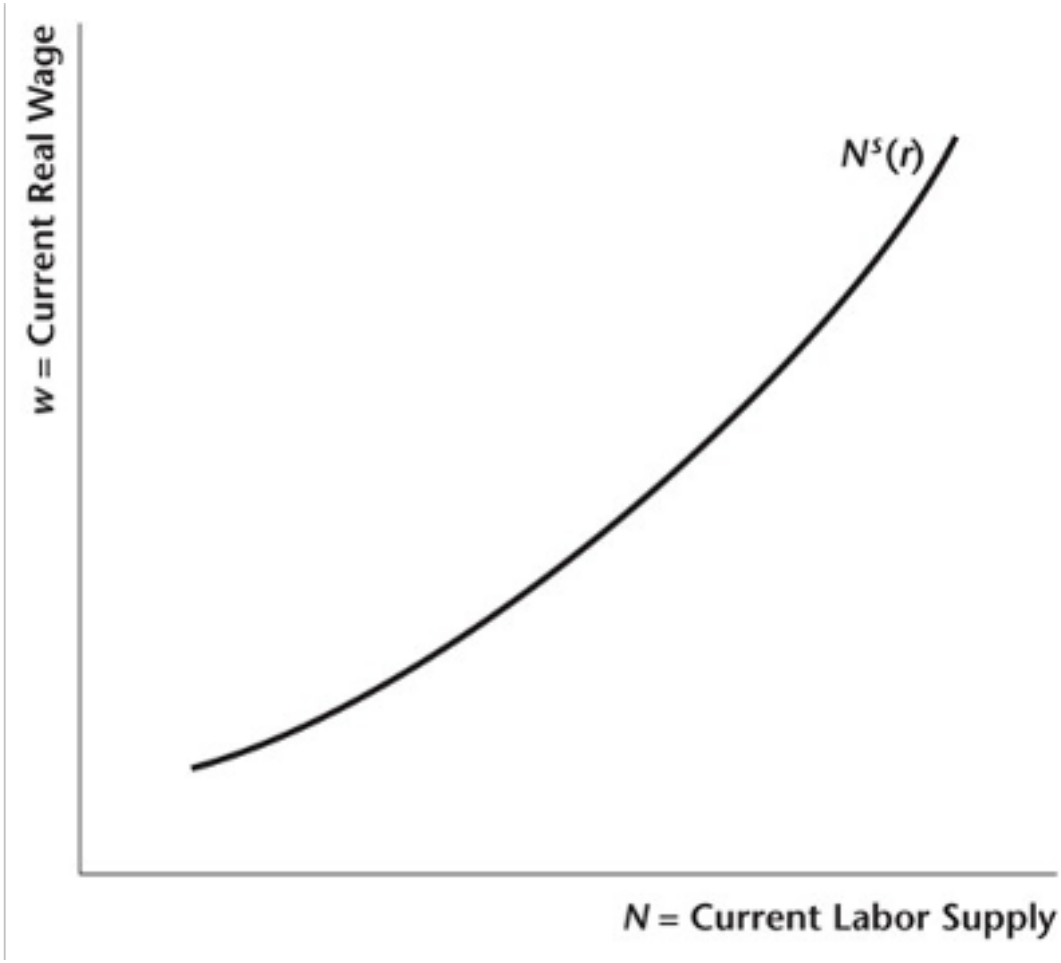
- The fourth equation is given by the budget constraint (four equations, four unknowns, given w and r)

Current Labor Supply

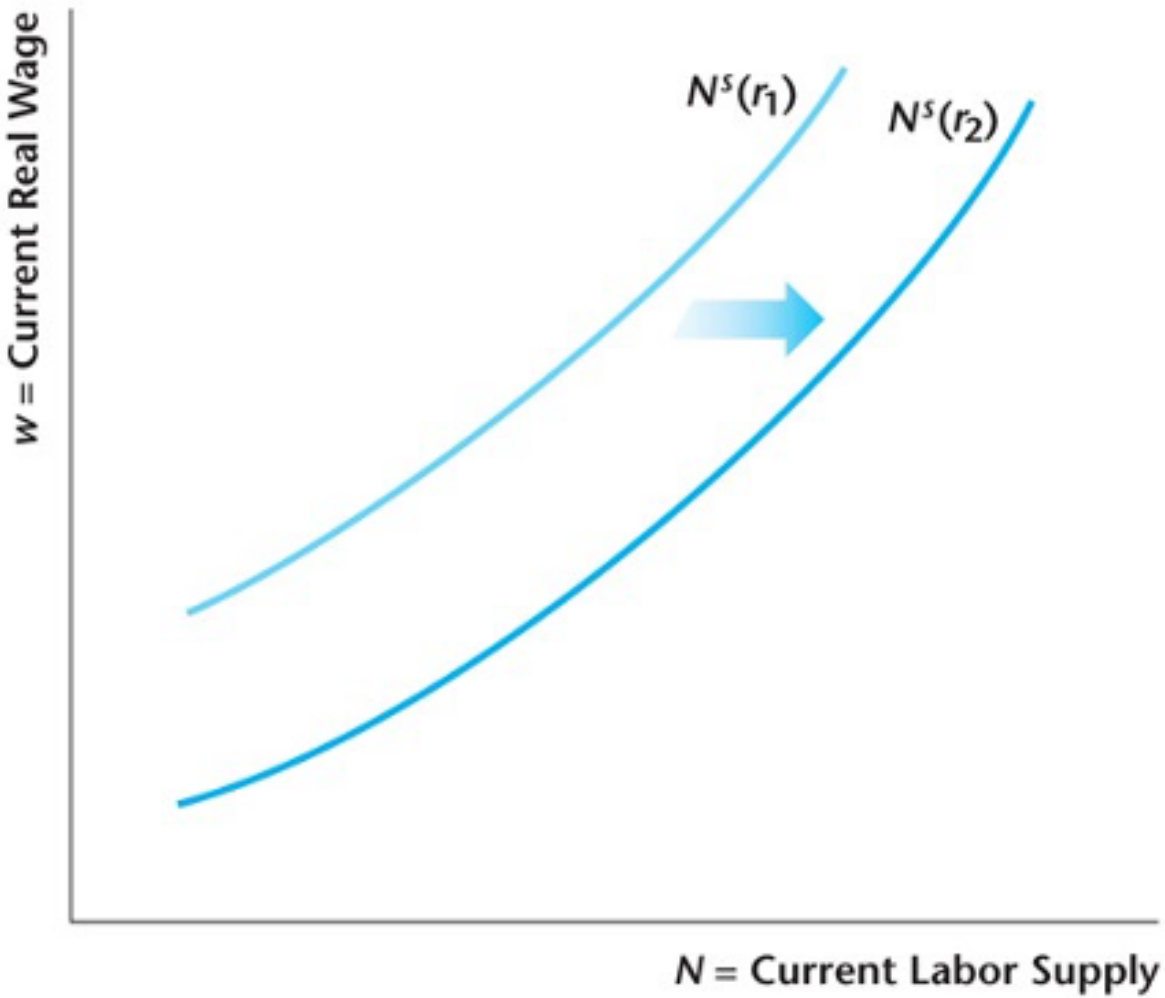
$$MRS_{\ell,C} = w$$

- From our condition, we think that:
  1. When $w \uparrow$,$N \uparrow$. Wages increase, labor increases. Income effects of a wage change today are muted because total lifetime income doesn't increase but substitution effect still in full force.
  2. When $r \uparrow$, $N \uparrow$. When the real interest rate changes, the real wage of today increases in terms of what it can buy tomorrow.
  3. When total lifetime wealth increases, labor today decreases (income effect).
- With these we can graph out the labor supply curve and shifts

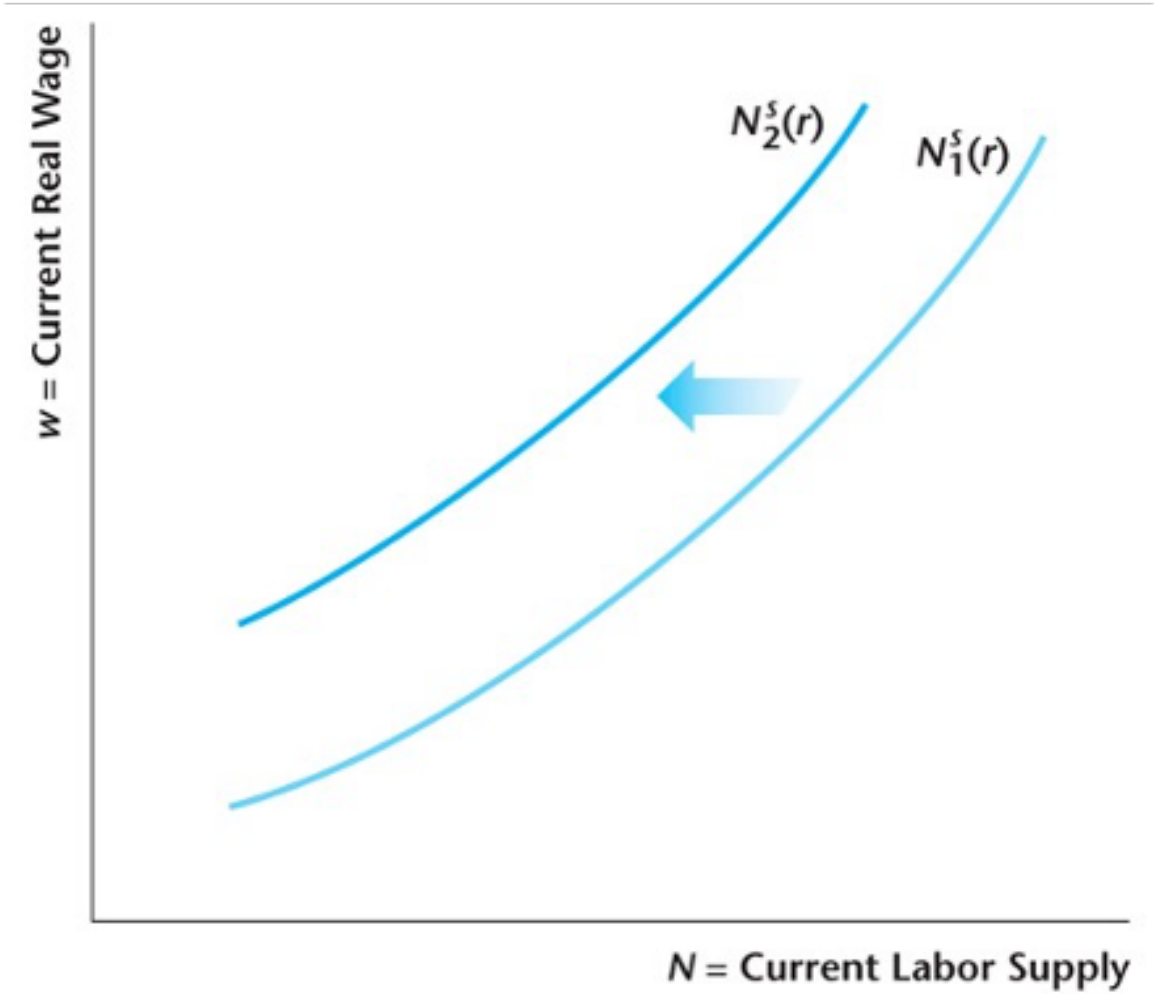
The Representative Consumer's Current Labor Supply Curve



Increase in the Real Interest Rate Shifts the Current Labor Supply Curve to the Right



Increase in Lifetime Wealth Shifts the Current Labor Supply Curve to the Left

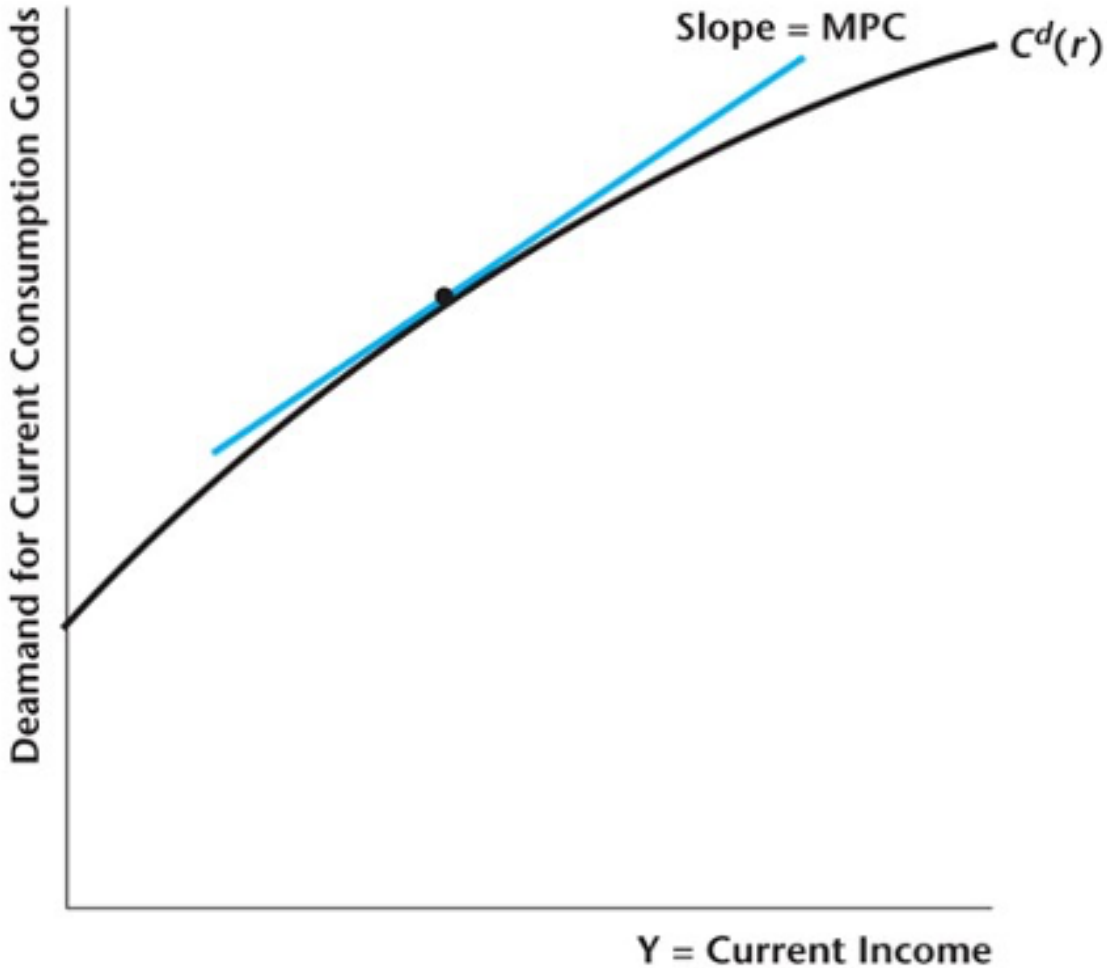


Current Demand for Consumption Goods

$$MRS_{\ell,C} = w$$

- Similarly for consumption:
  1. As current (total) income increases, consumption today increases
  2. When real interest increase, consumption demand shifts down (assuming again income effect is muted)
  3. When lifetime wealth increases, consumption demand shifts up
- With these we can graph out the labor supply curve and shifts

Increase in Lifetime Wealth Shifts the Current Labor Supply Curve to the Left



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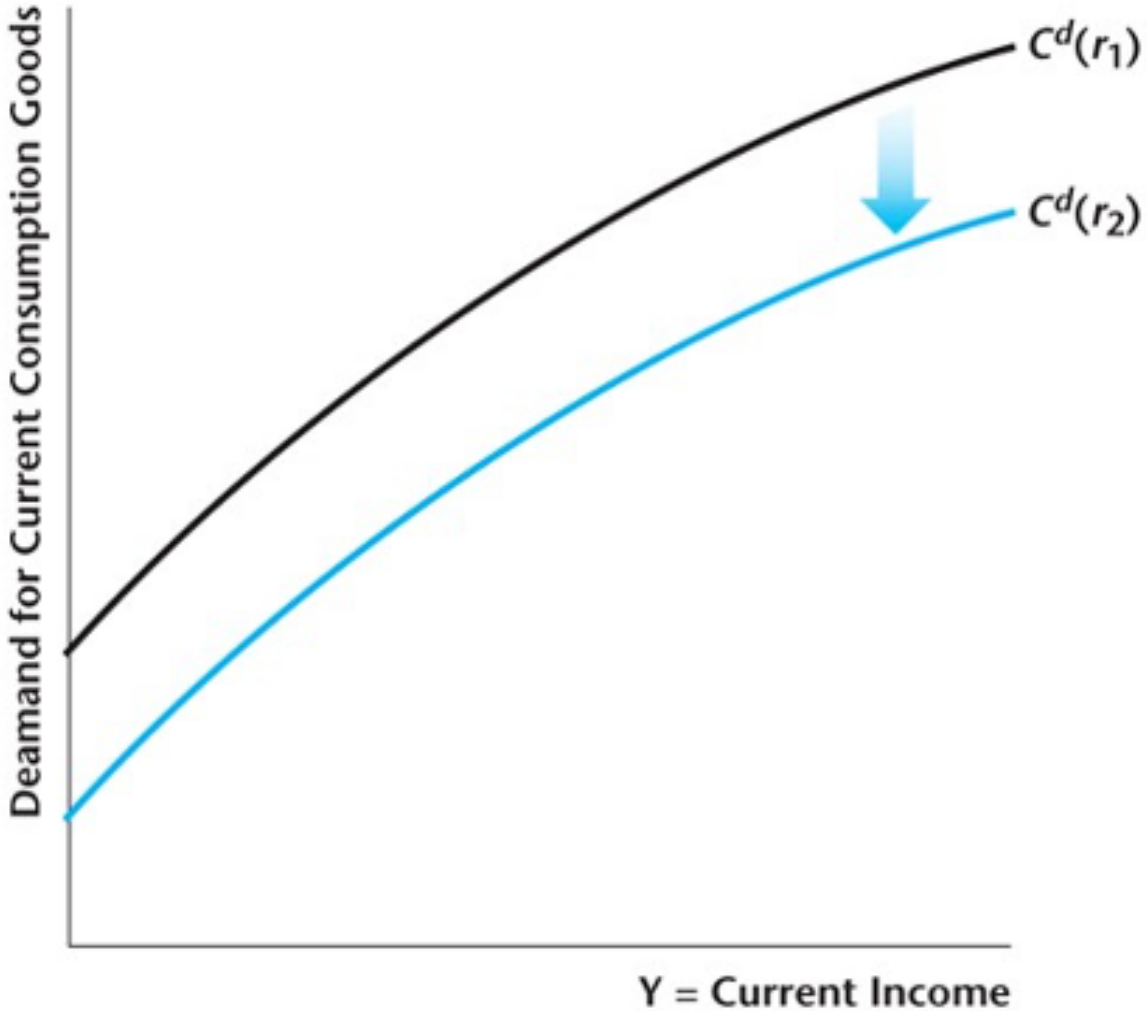

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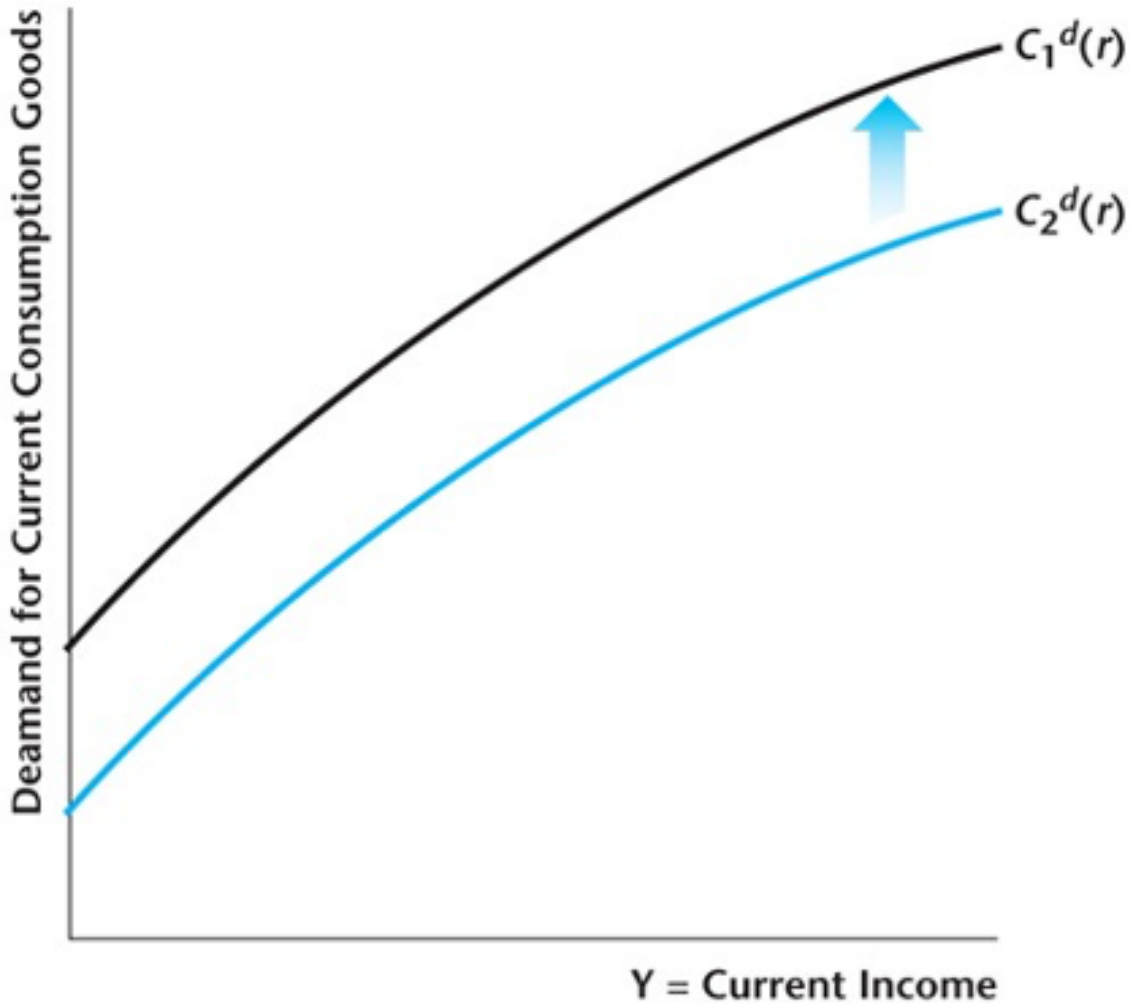


The Representative Firm

- We have the consumer, now let's get the firm. Production function(s):

$$Y = zF(K, N) \qquad Y' = z'F(K', N')$$

- Law of motion of capital:

$$K' = (1 - \delta)K + I$$

- Firm profits:

$$\pi = Y - wN - I$$

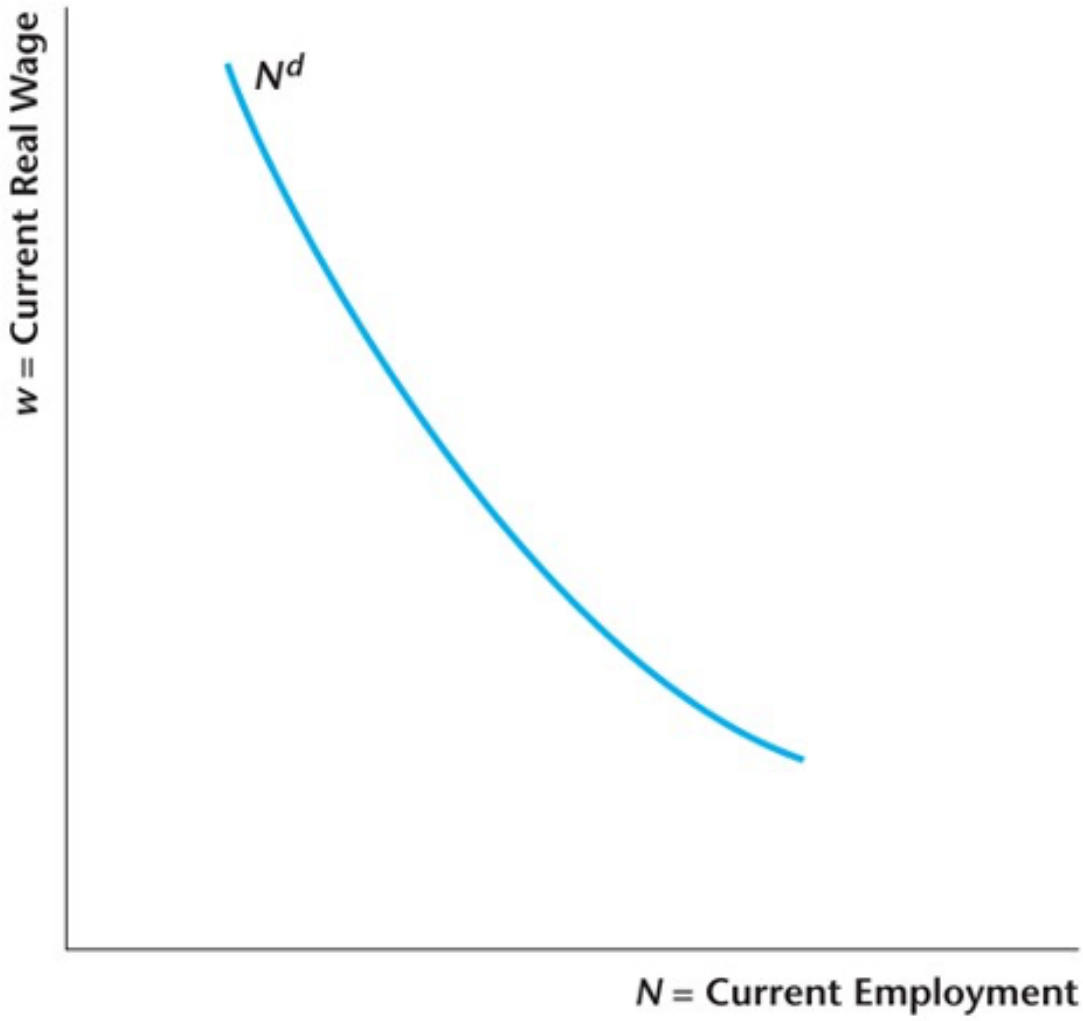
$$\pi' = Y' - w'N' - (1 - \delta)K'$$

- Firm maximizes NPV of profits:

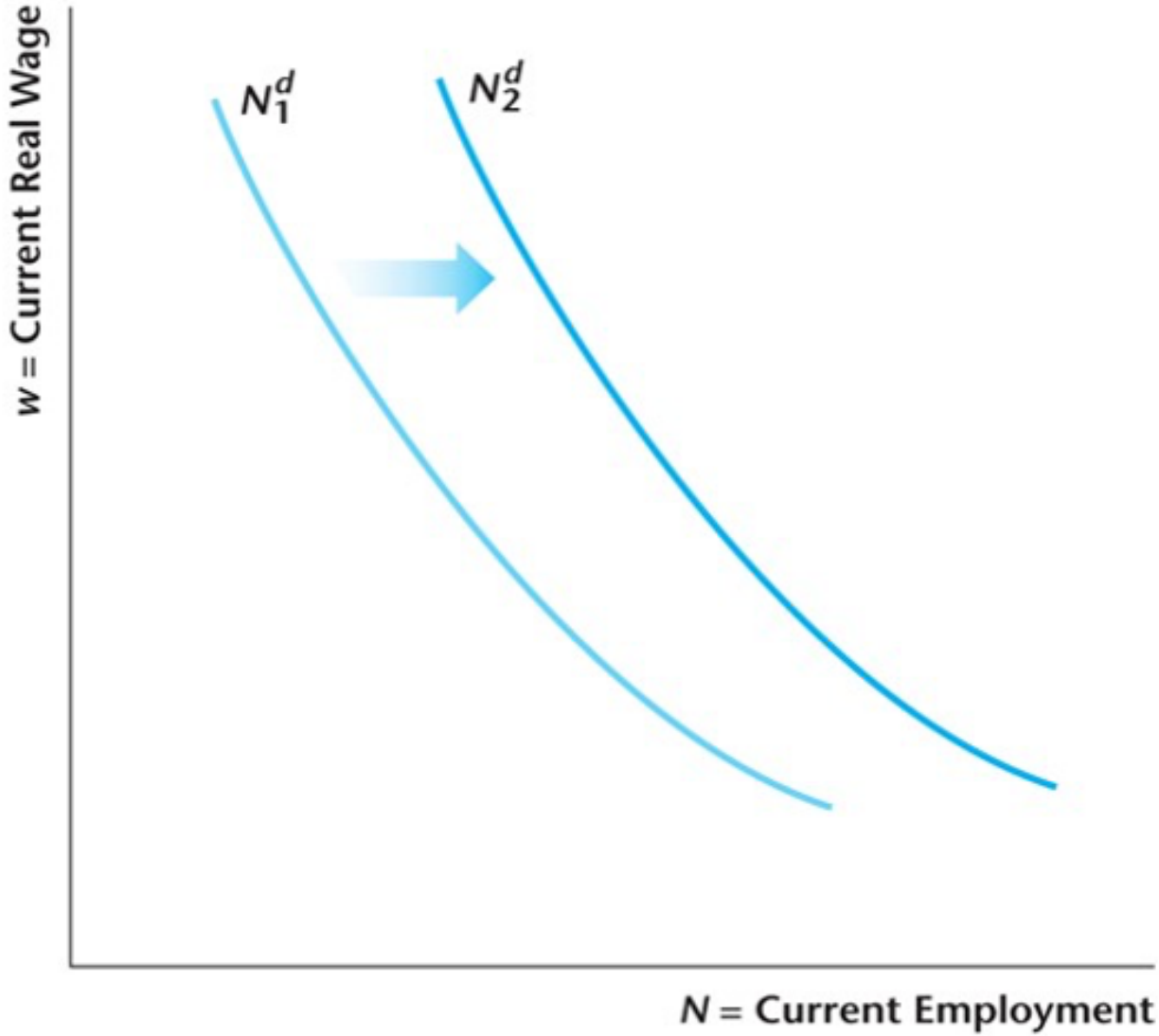
$$V = \pi + \frac{\pi'}{1 + r}$$

- Hires labor until $MP_N = w$

The Demand Curve for Current Labor is the Representative Firm's Marginal Product of Labor Schedule



The Demand Curve for Current Labor Shifts Out with TFP



The Representative Firm-Investment

- Marginal cost of investment is just one (numeraire) $MC(I) = 1$
- Marginal benefit of investment is complicated–discounted MPK of tomorrow for whatever's left:

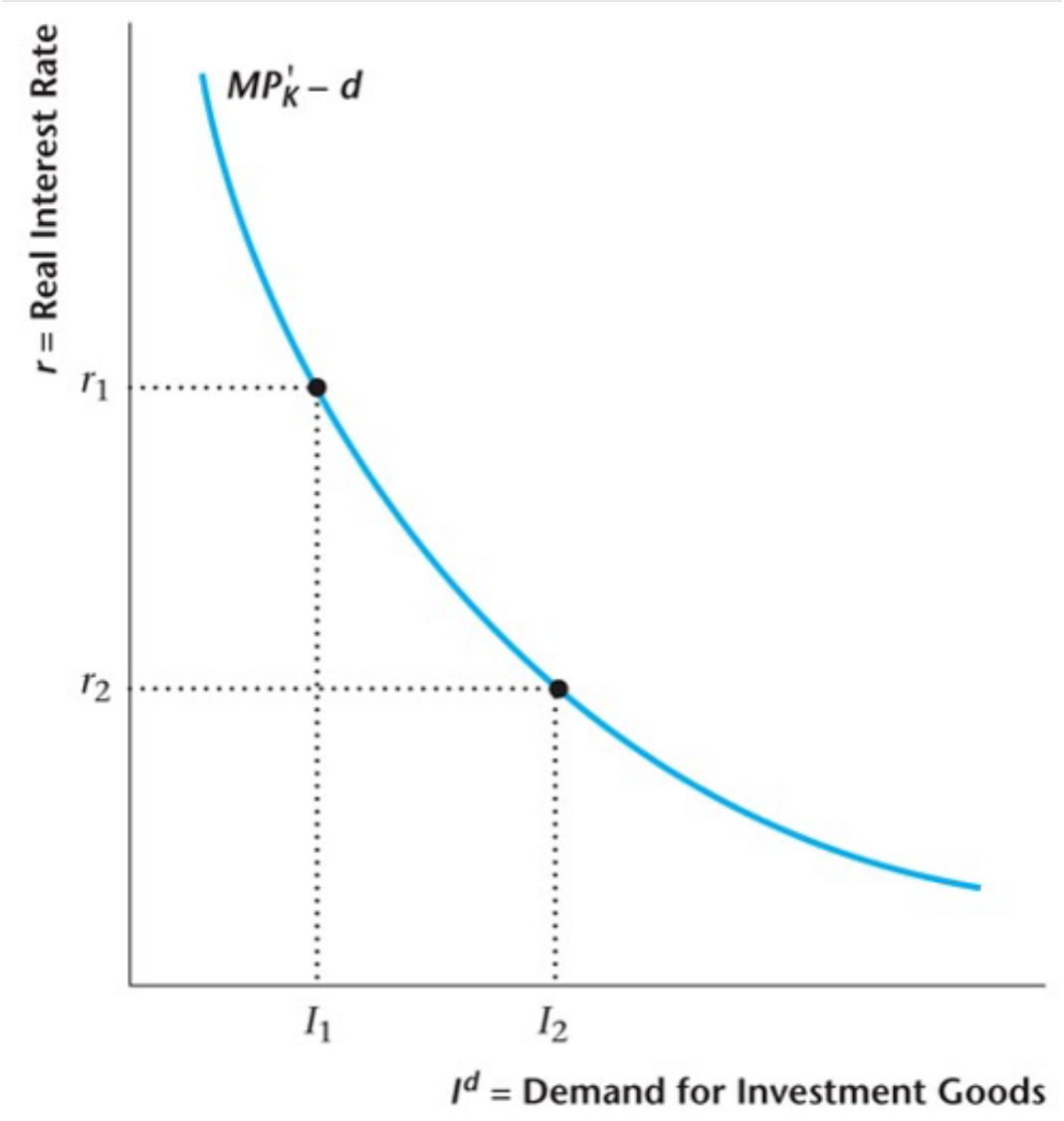
$$MB(I) = \frac{MP'_K + 1 - \delta}{1 + r}$$

- Which gives:

$$MP'_K = r + \delta$$

- Optimal investment rule: the real interest rate (including depreciation) should equal the marginal product of capital
- We could also have gotten this if households had loaned out the capital
- Optimal investment schedule:
  - Shifts to right when future TFP z' increases
  - Shifts to left when current capital stock K is higher

The Demand Curve for Current Labor Shifts Out with TFP



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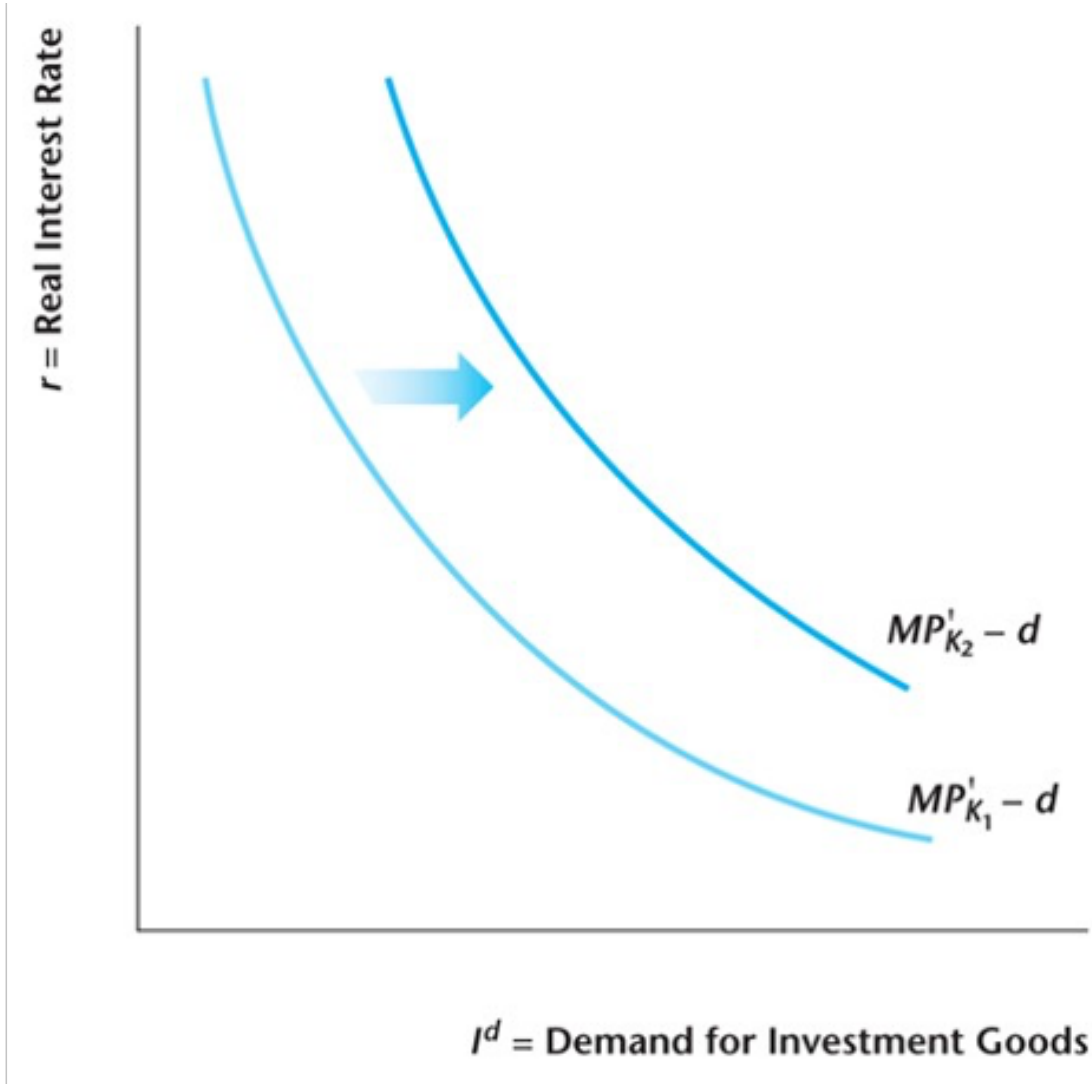


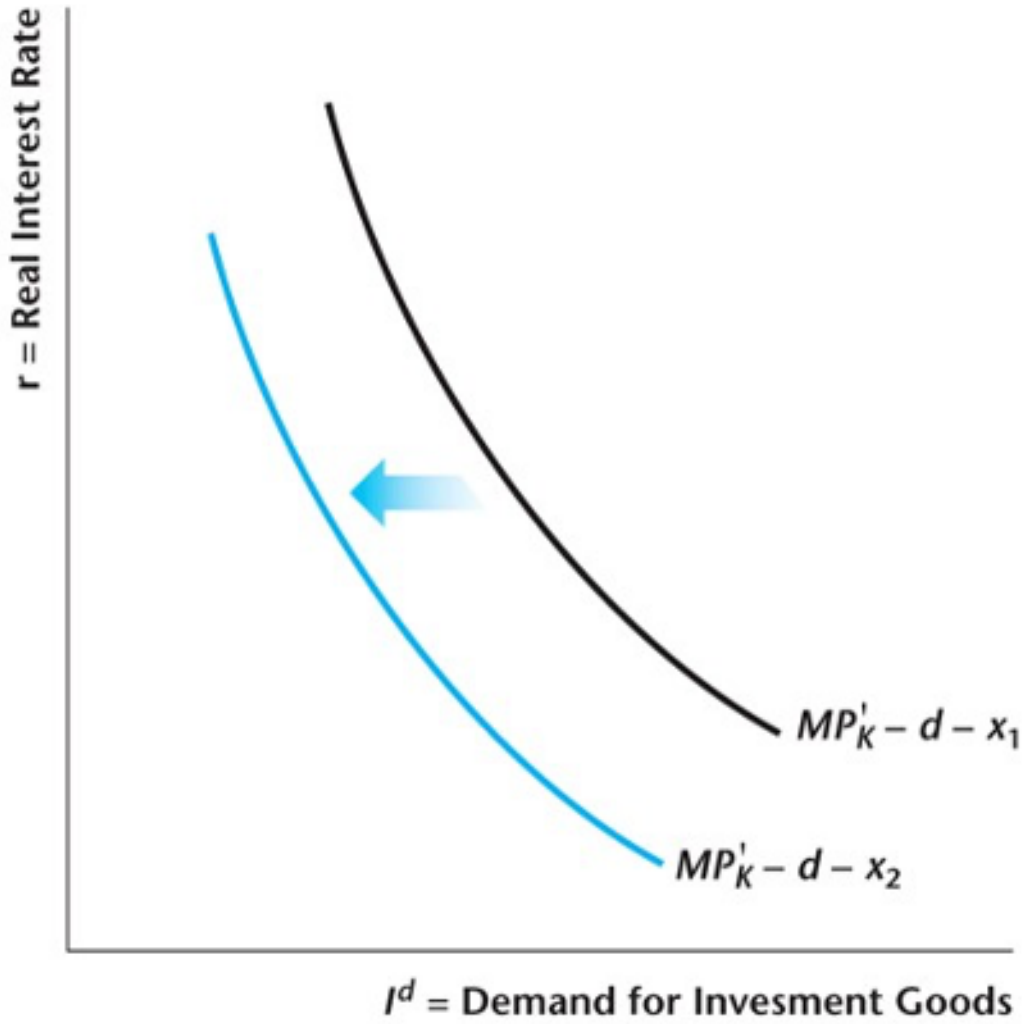
Investing with Asymmetric Information

- We assumed perfect capital markets before
- But in reality, some firms may be "riskier" and the risk premium on their loans may change over the business cycle
- Let r be the "safe" interest rate and r^l be a loan interest rate.
- Now the investment decision becomes:

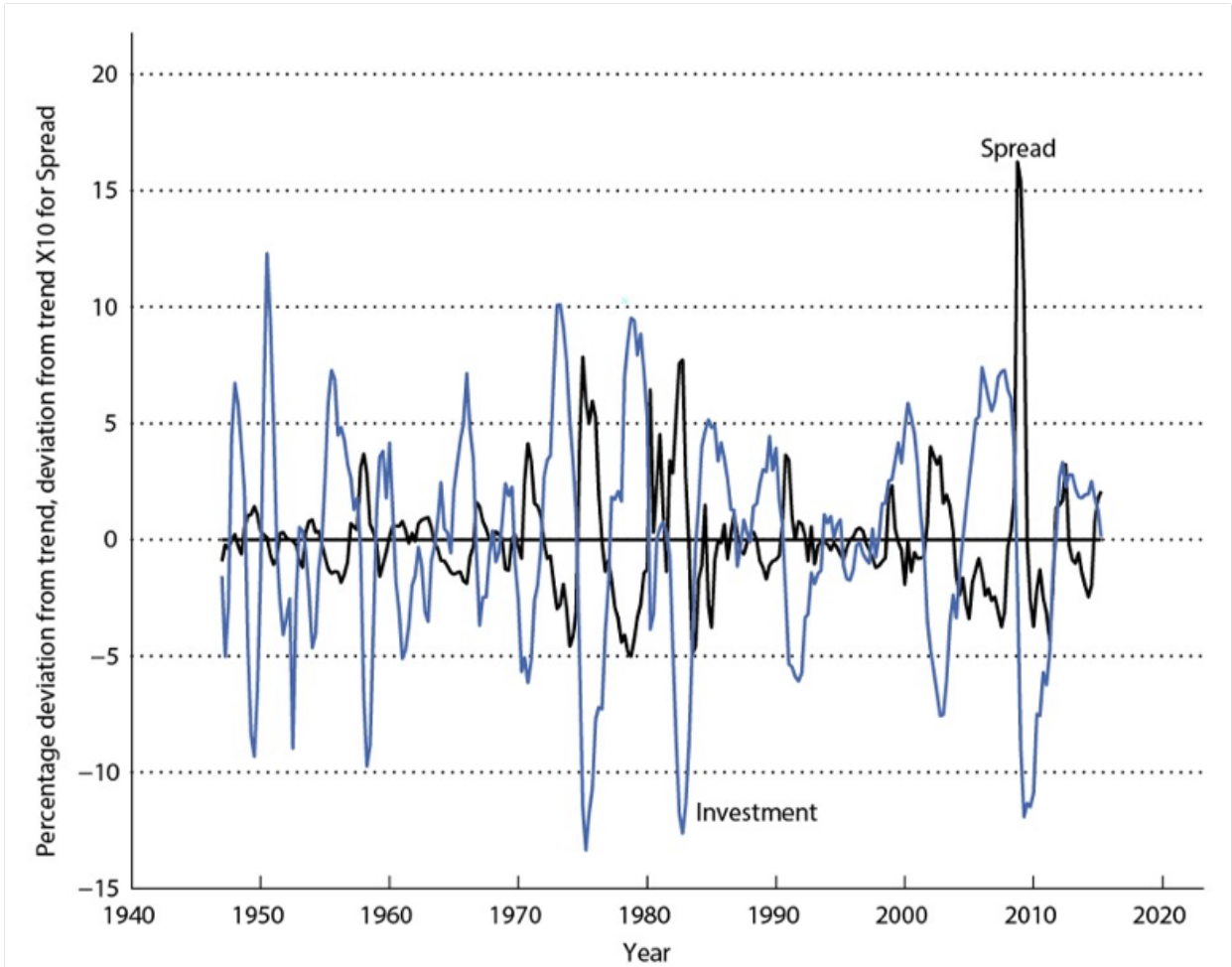
$$MP'_K - \delta = r + x$$

- This shifts up MPK, which shifts down investment

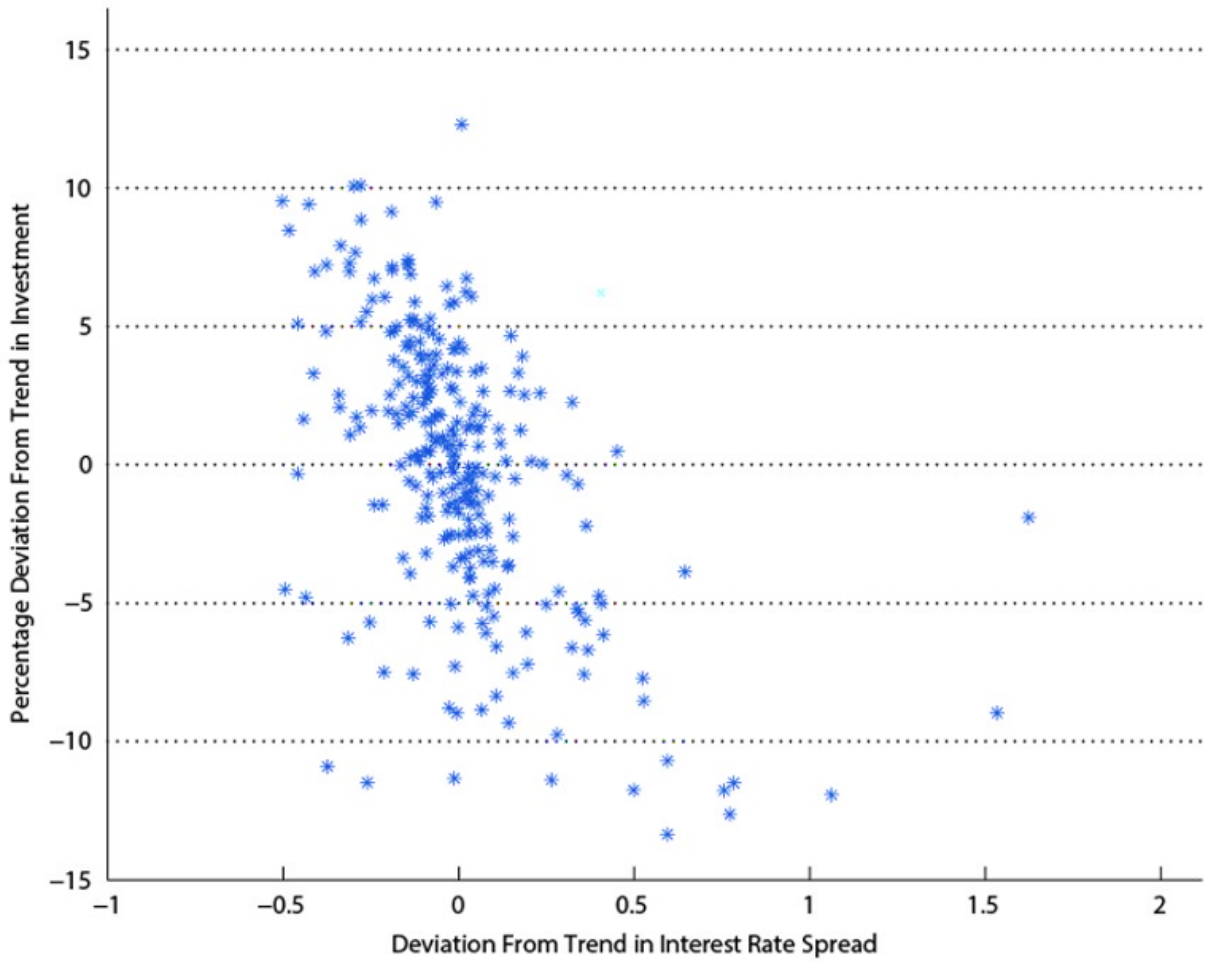
The Effect of an Increased Default Premium on a Firm's Optimal Investment Schedule



Investment and the Interest Rate Spread



Scatter Plot: Investment vs Interest Rate Spread



Government

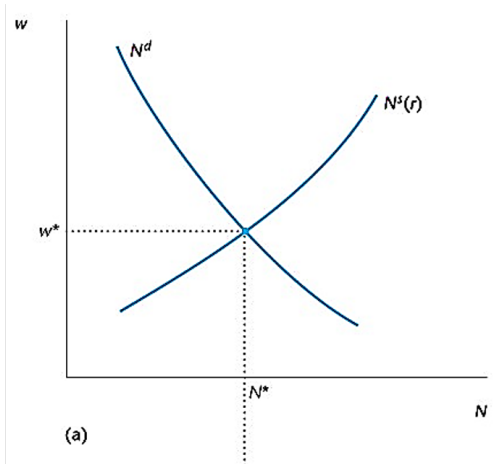
- Government is as it was before:

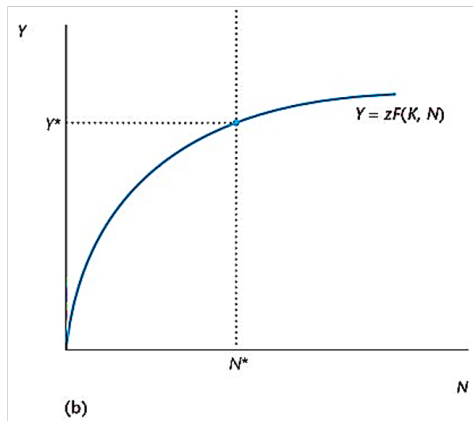
$$G + \frac{G'}{1+r} = T + \frac{T'}{1+r}$$

Competitive Equilibrium

- We have a:
  - Representative consumer that works, leisures, consumes, saves, and pays taxes
  - We have firms that take in labor, invest in capital, and produce
  - We have a government that taxes and spends
- Thus far, we ignore future markets in labor and goods, though they obviously would affect a consumer/worker/firm today!
- This won't change most of our qualitative results
- Equilibrium will be when supply=demand, budget constraints hold, and agents optimize

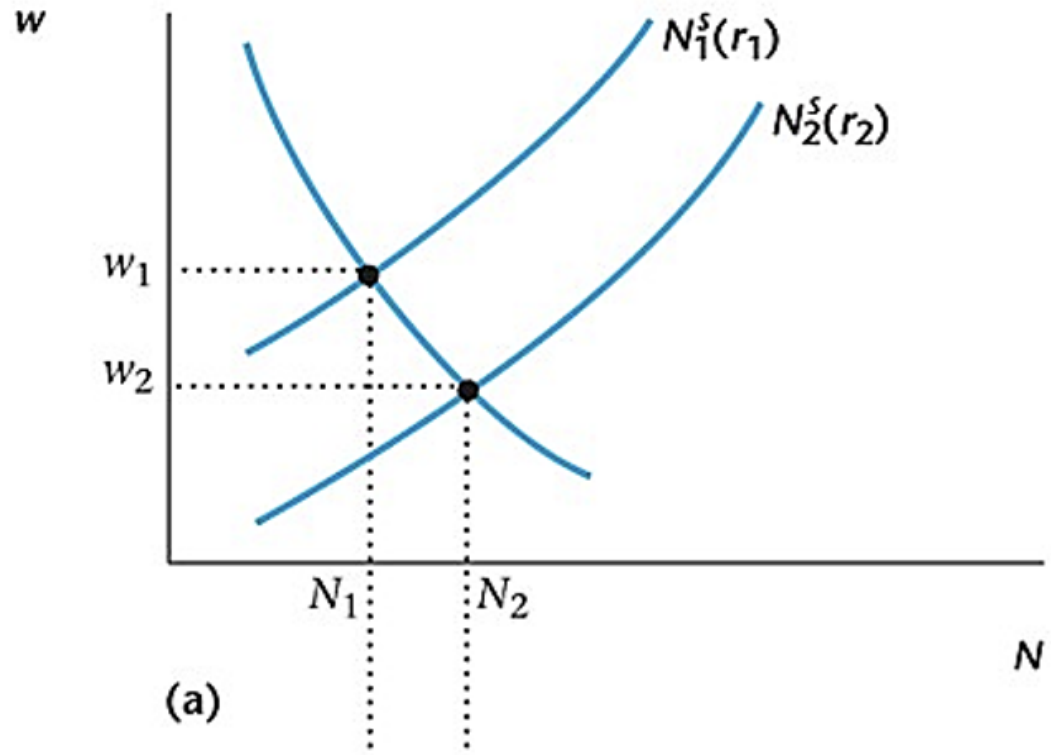
Determination of Equilibrium in the Labor Market Given the Real Interest Rate r

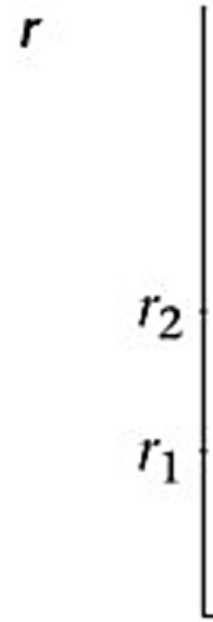




Given r and w, we have labor supply & demand, and wage. Given labor, we have production.

Construction of the Output Supply Curve $Y(r)$

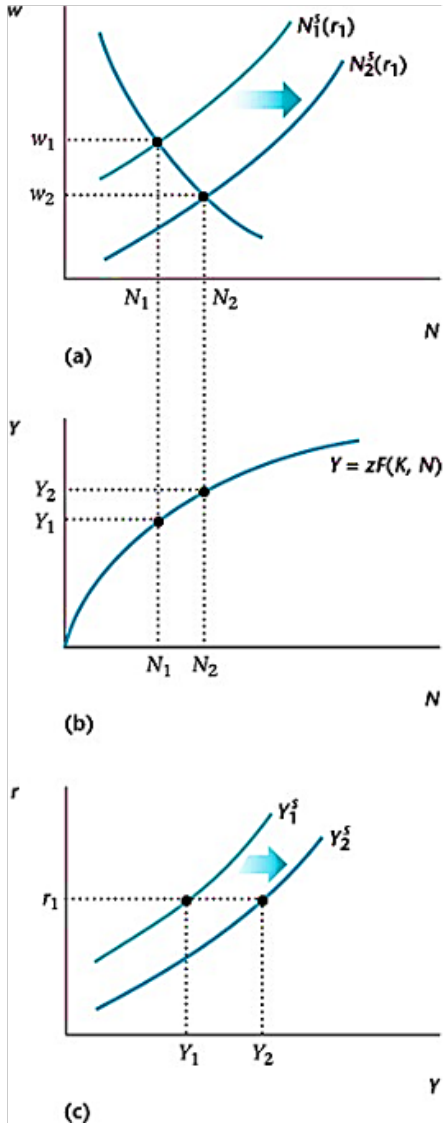




What shifts the output supply curve?

- Increase in lifetime wealth (such as from a decrease in government taxation) shifts the labor supply in, which causes the output supply curve to shift down
- Current total factor productivity increase shifts the labor demand out, which causes the output supply curve to shift up
- Current capital stock increase shifts the labor demand out, which causes the output supply curve to shift up

An Increase in Current or Future Government Spending Shifts the Y^S Curve

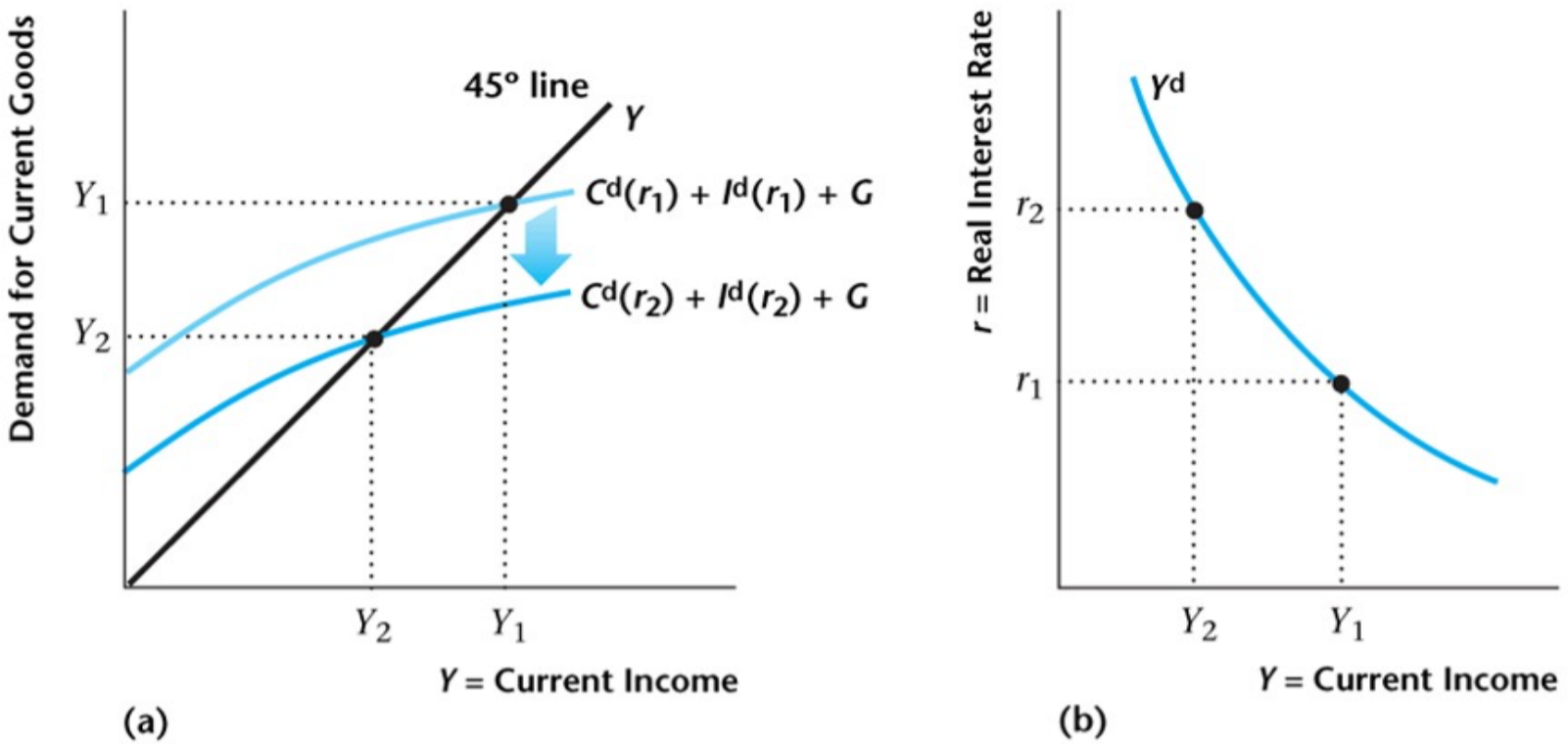


We can trace out how shifts in r affect Y.

Shifting to Demand

- Now we have the output supply curve, and how it reacts to changes in interest rates and government expenditure, we need the output demand curve
- Decreases in taxes shifts Y^D to the right (lifetime wealth increases)
- Increase in future income Y' shifts Y^D to the right (lifetime wealth increases)
- Increase in future TFP shifts Y^D to the right (lifetime wealth increases)
- Decrease in current capital stock shifts Y^D to the right (more demand for capital)

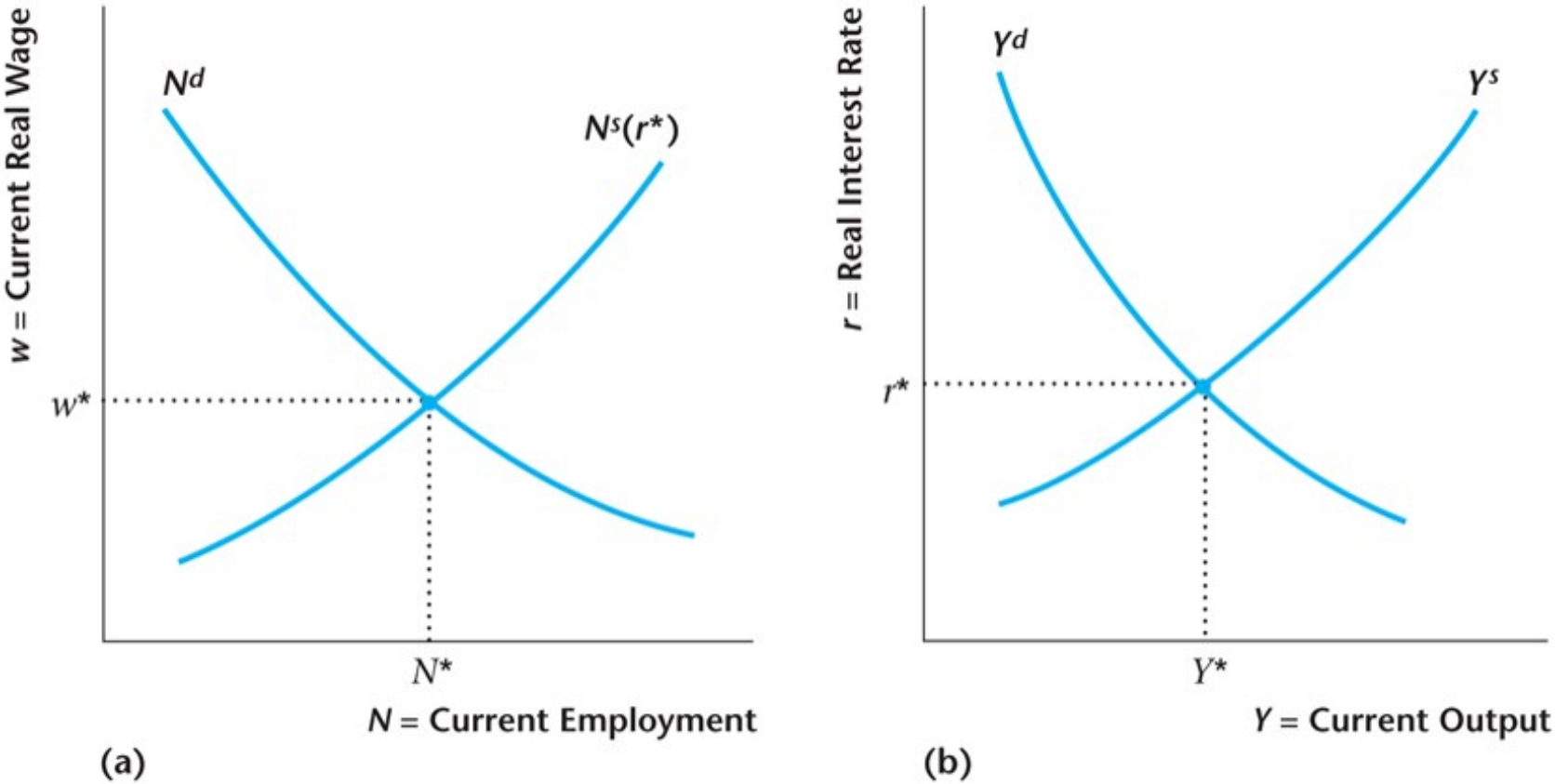
Output Demand



Putting it All Together

- We have output demand and supply
- Labor market clear via wages
- Output market clears via interest rates
- Putting them together graphically

Equilibrium in a Real Intertemporal Model



Now we can start analyzing various policies!

Policies

- Now that we have a real dynamic equilibrium model, we can use it to analyze various policies:
  1. How does an increase in current government purchases, anticipated to be temporary, affect current macroeconomic variables?
  2. What are the effects on current macroeconomic variables of a decrease in the current capital stock, brought about by a natural disaster or a war?
  3. How does a temporary increase in total factor productivity affect macroeconomic variables, and how does this fit the key business cycle facts?
  4. If total factor productivity is expected to increase in the future, how does this affect current macroeconomic variables?
  5. How do credit frictions affect macroeconomic activity?
  6. What are the effects of sectoral shocks on the economy?
- Let's go through one by one!

Example 1: Temporary Increase in Government Purchases

- We did this before in our one-period model and found G crowded out C
- But now we're intertemporal! We have an interest rate. Three new things to examine:
- As G increases, it will increase the interest rate, which will affect both investment and consumption
- As G increases, labor supply will be intertemporally substituted
- Now we have government spending multipliers!

Temporary Increase in Government Purchases

- When current period G shifts from G_1 to G_2, we need to know the change in output demand

- We will assume that MPC is a constant, and denote the shift in the output demand curve as Δ, so that:

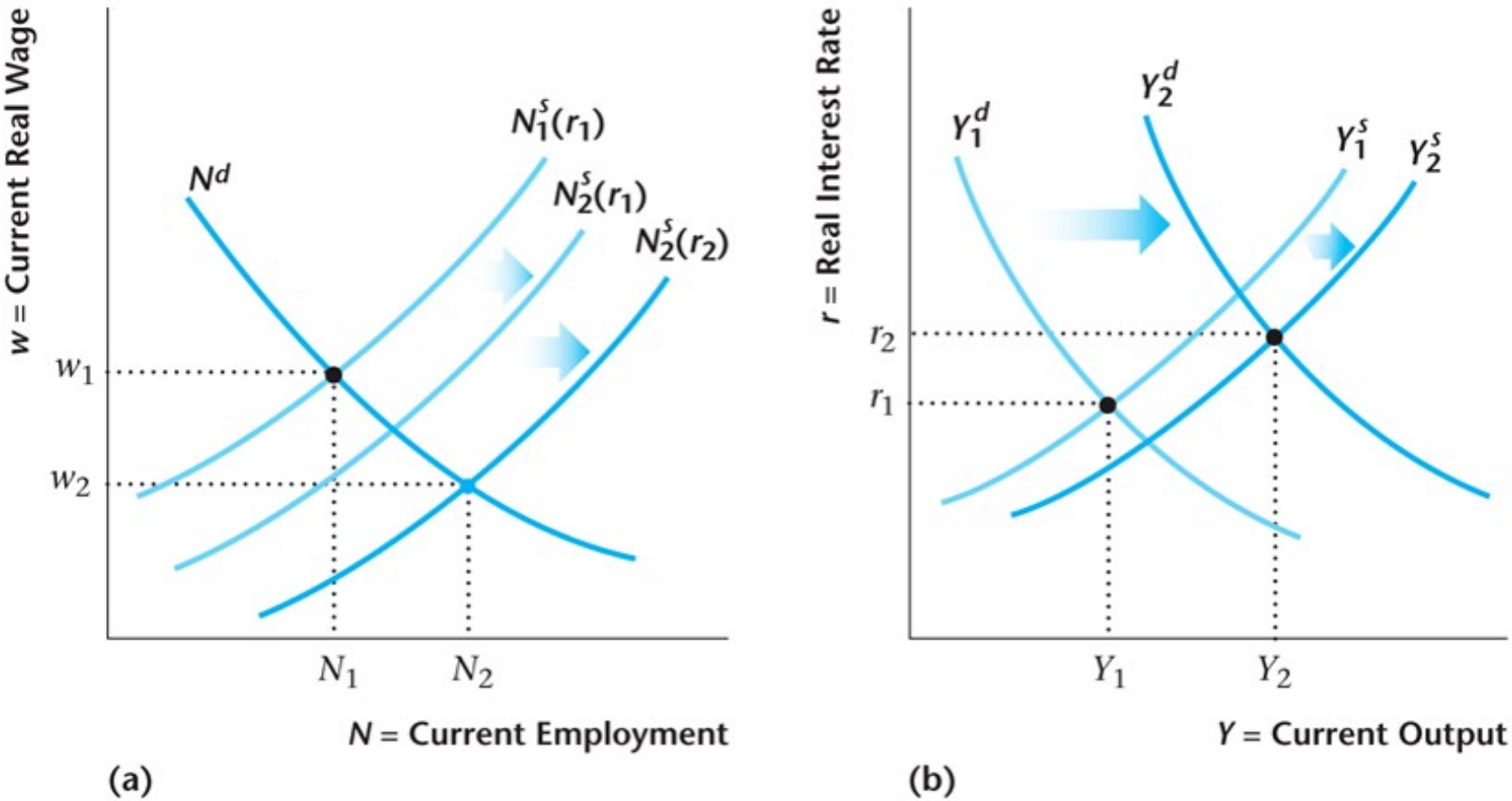
$$\Delta = G_2 - G_1 - MPC(G_2 - G_1) + MPC\Delta$$

- Where:
  - $G_2 - G_1$ is the direct effect of an increase in government expenditure on goods demanded
  - $MPC(G_2 - G_1)$ is the effect of an increase in taxes on consumer expenditure (crowd-out)
  - $MPC\Delta$ is the add-on effect of an increase in wealth on consumer expenditure
- Solving for Δ, we get $\Delta = 1$
- And the demand multiplier $m_d = \frac{\Delta}{G_2 - G_1} = 1$
- But now need affects on w, N, r, Y

Temporary Increase in Government Purchases

- G increases from G_1 to G_2
- Y^D shifts one-for-one with increase in expenditures
- Lifetime wealth decreases, so demand for leisure decreases, labor supply increases
- When labor supply increases, output supply increases
- Output supply typically shifts by less than output demand (small effect on wealth), so interest rates increase
- When interest rates increase, labor supply increases further
- Let's see it graphically

Example 1: Increase in Govt Purchases



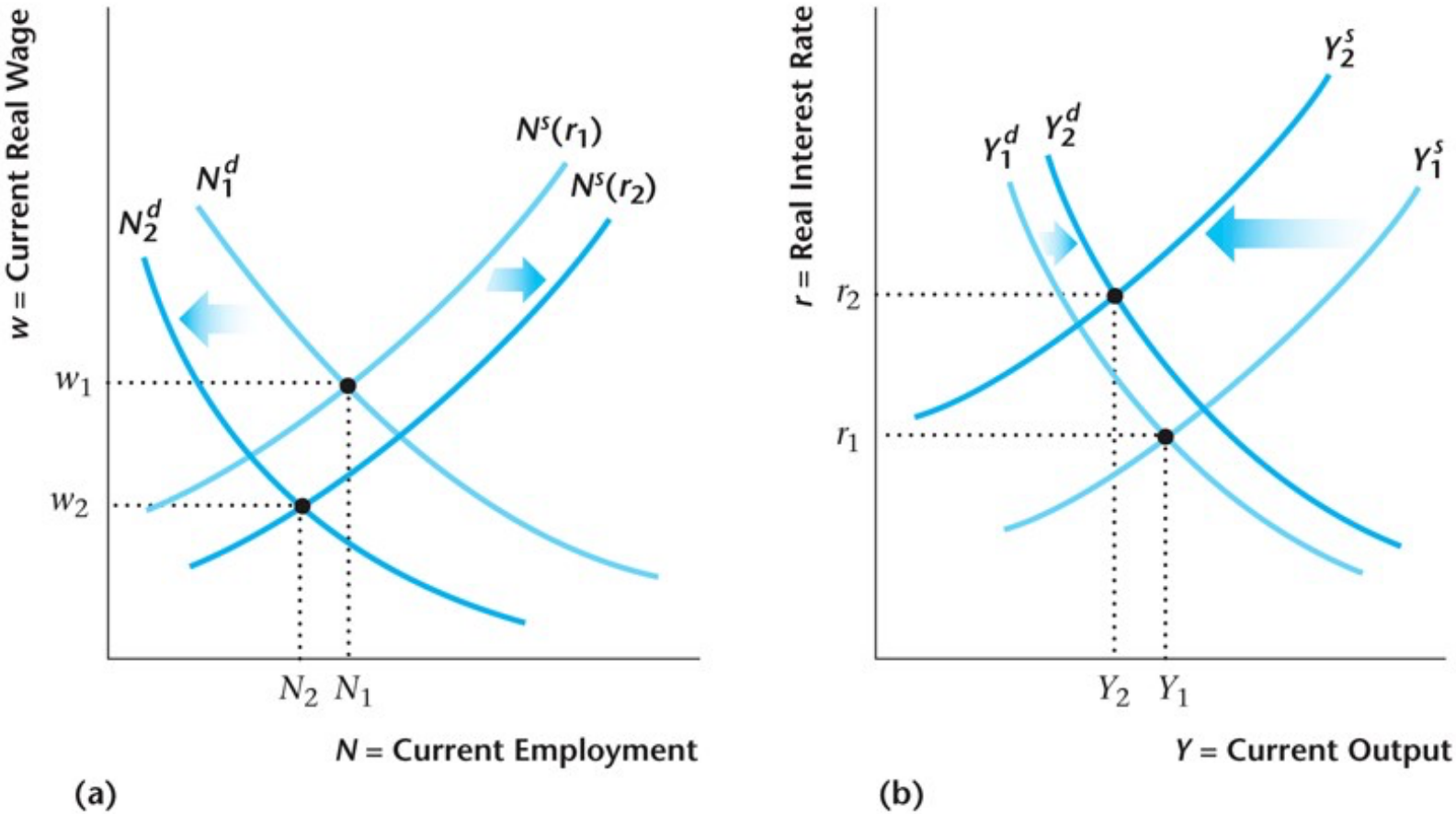
Government Expenditure Multiplier

- Total output shifts by less than the government's expenditure shift, as interest rates rise
- Consequently the expenditure multiplier is less than one.
- Multiplier is smaller if affect on labor supply via wealth decreases and interest rate increases are smaller
- Some argue its more than one! We'll tackle these models later, but we can see graphically that if interest rates didn't increase,
- For now, let's move to the effects of a decrease in capital stock K

Example 2: Decrease in K

- Now let's say that K shifts from K_1 to K_2, $K_2 < K_1$
- Firms are less productive, so demand for labor shifts downwards, and output supply shifts downwards
- Capital is in short supply, so output demand shifts outwards
- We see that both push up the interest rate r, but the affects on output are unclear
- As r increases, labor supply shifts out
- Workers are less productive, so w falls, but what happens to employment is similarly unclear

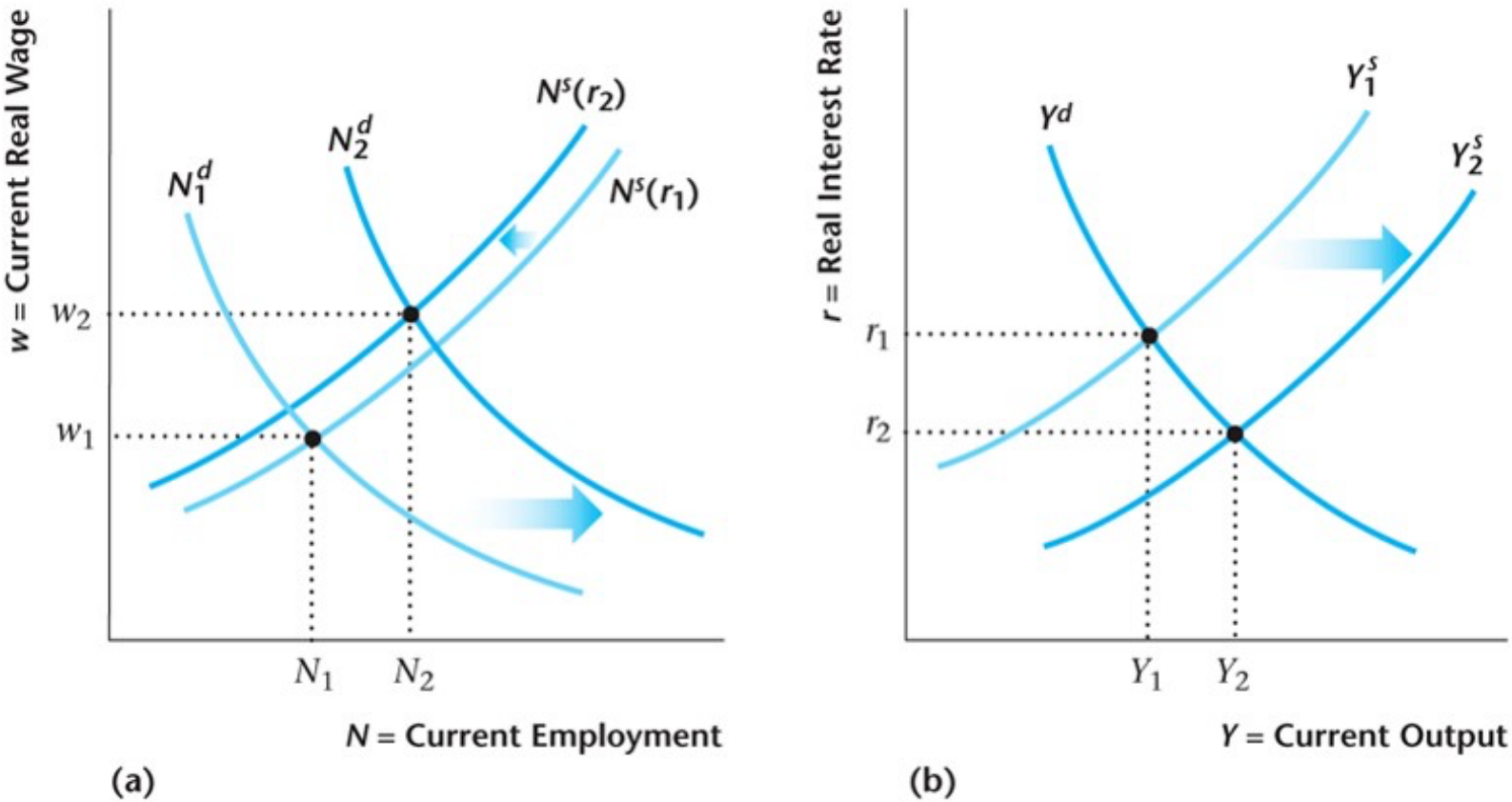
Decrease in Capital



Example 3: Increase in current TFP z

- Now let's say that z shifts from z_1 to z_2, $z_2 > z_1$
- Firms are more productive, so demand for labor shifts outwards, and output supply shifts outwards
- Interest rates fall as output shifts out, so supply of labor shifts in slightly
- Because the shift in labor demand dominates, wages rise and labor increases
- Importantly, consumption, real wages, investment, employment, and average labor productivity all move together, which is what we saw in the data

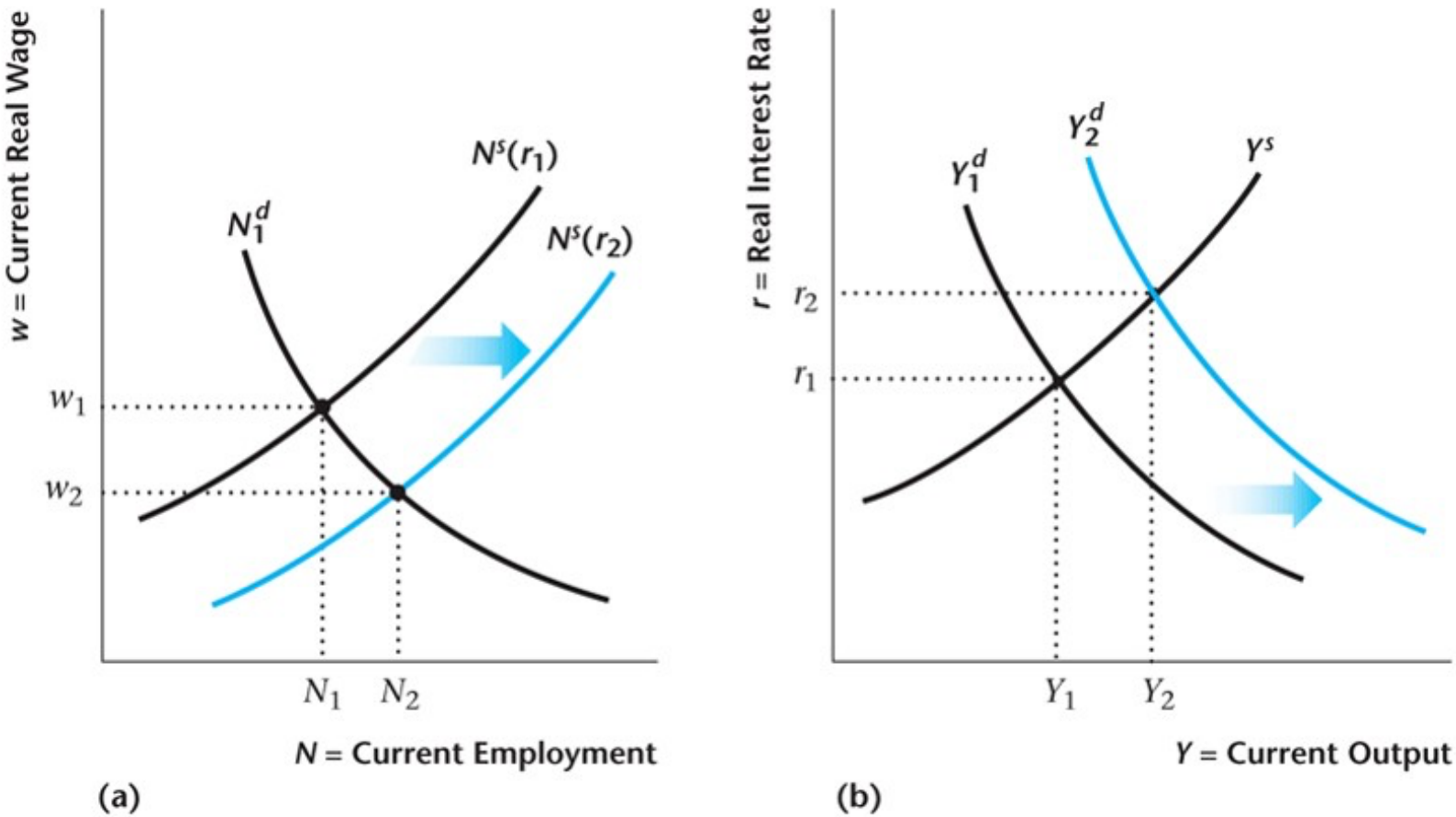
Decrease in Capital



Example 4: Increase in future TFP z'

- Now let's say that z' shifts from z'_1 to z'_2, $z'_2 > z'_1$
- Firms know future *MPK* is higher, so demand for investment increases, output demand increases
- This causes interest rates to rise, increasing labor supply, driving wages down and labor up

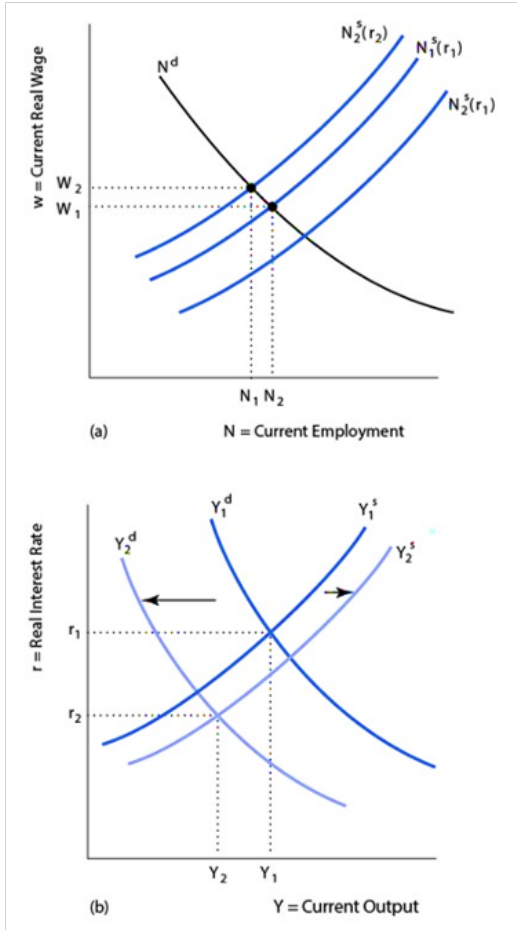
Increase in Future TFP



Example 5: Credit Frictions

- Credit frictions, due to asymmetric information and limited commitment, operate through the interest rate
- r increases due to wedge/risks, lowering output demand and increasing labor supply
- As labor supply increases, output supply increases
- Output could rise or fall–in practice, the effect on consumption falling is larger than the effect on labor, so output falls
- When it falls, labor demand falls

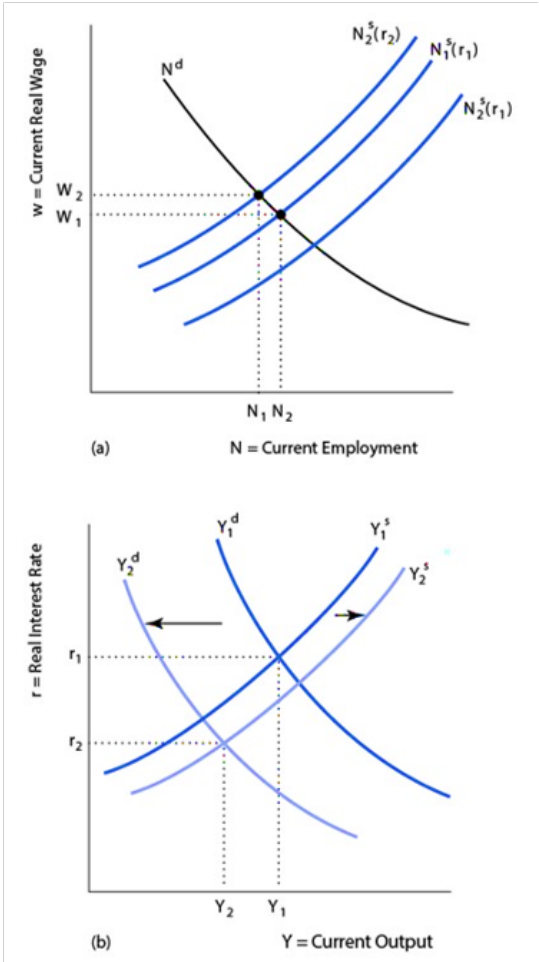
Increase in Future TFP



Example 6: Sectoral Shocks

- Sectoral shocks are disturbances to technology and preferences
- Idea is that the market is reallocating resources and experiencing "mismatch"
- We'll model a shock tat affects labor market mismatch–acts as a friction, so workers and firms both experience an extra non-market cost to matching (like a tax wedge!)
- Labor demand and supply shift down, so output supply shifts down
- This increases MPL/average labor productivity

Sectoral Shock/Mismatch



Summary

- We have a real, dynamic (two-period) model of the macroeconomy
- Lets us think about investment, consumption, output, real interest rates, and employment
- Two (linked) key markets we think through: labor supply/demand, and output supply/demand
- Interest rates clear output markets, and affect labor supply
- Can think through a variety of examples cleanly