Econ 352 - Consumer and Firm Behavior: The Work-Leisure Decision and Profit Maximization

(See Williamson Ch. 4)

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The Beginning

- Finally, we're going to start building up a model
- This model will look a lot like what you learn in micro!
- Start in a "static" (unmoving, one-period) model, then eventually go to "dynamic" (multi-period)
- We'll build up a basic consumer that chooses how much to consume (work), and how much to leisure (not work)
- Then we'll build up a firm that uses labor and capital to produce a consumption good
- In Chapter 5, we'll then put those two together to create a "representative agent" macroeconomy

Representative Consumer

- Start with a representative consumer
- Consumer likes consumption C and likes leisure ℓ
- The two inputs map into happiness (utility) $U(C, \ell)$
- Call a particular (C_1, ℓ_1) combination a "consumption bundle"
- Bundle (C_1, ℓ_1) is "strictly preferred" to bundle (C_2, ℓ_2) if:

$$U(C_1, \ell_1) > U(C_2, \ell_2)$$

- A consumer is "indifferent" between bundles (C_1, ℓ_1) and (C_2, ℓ_2) if:

$$U(C_1, \ell_1) = U(C_2, \ell_2)$$

- Now we'll think about the properties of consumer preferences

Representative Consumer-Principles

- We have a few beliefs about the consumer
  1. More is always preferred to less: $U(C_1, \ell_1) < U(C_1 + \epsilon, \ell_1)$ and $U(C_1, \ell_1) < U(C_1, \ell_1 + \epsilon)$, $\epsilon > 0$ (normal goods)

  2. Consumer likes diversity: take some optimal (C^*, ℓ^*) bundle. Then $U(\frac{1}{2}C^*, 2\ell^*) > U(\frac{1}{4}C^*, 4\ell^*)$ (imperfect substitution)

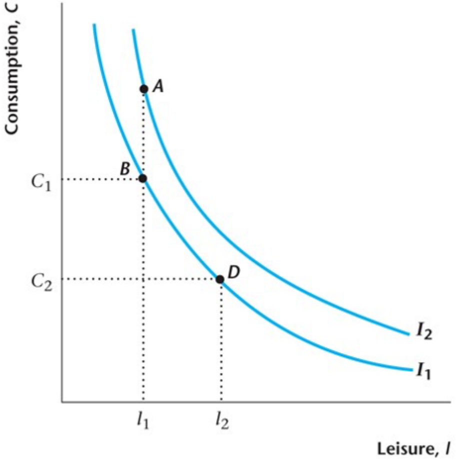
  3. Consumption and leisure are normal goods: as income increases, people want more of both consumption and leisure
     - Goods are "luxury" if you consume relatively more of them as income increases
     - Goods are "normal" if you consume more of them as income increases (even if share of spending falls)
     - Goods are "inferior" if you consume less of them as income increases

  4. Let's think about the all-important "indifference curve"

Indifference Curves

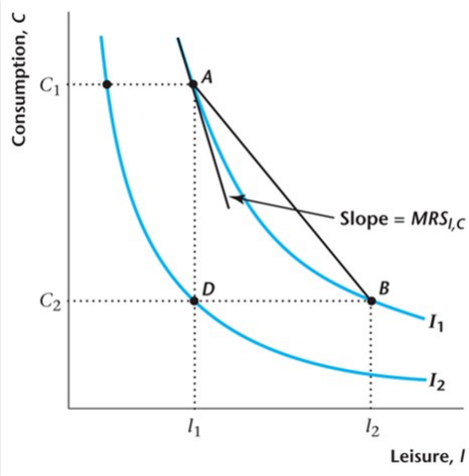
- Indifference curves are the curves along which a person is indifferent to one bundle or another. So, for some utility level $\bar{U}$, the indifference curve is defined over C and L by $\bar{U} = U(C, \ell)$.
- For instance, let's say you have 1.5 units of happiness from working 1 hour and consuming 3 candy bars. $U(3, 1) = 1.5 = \bar{U}$. The indifference curve tells us the set of bundles that would make you just as happy. For instance, if you had to deviate to leisuring only 0.5 hours, you might require not six but *seven* candy bars $U(7, 0.5) = 1.5$.
- Let's think about two indifference curves defined by $U(C, \ell) = I_1$ and $U(C, \ell) = I_2$

Indifference Curves



$U(C_B, \ell_B) = U(C_D, \ell_D) < U(C_A, \ell_A)$
Notice the bow/curve!

Properties of Indifference Curves



MRS=slope=willingness to trade off (price at which indifferent!)

Budget Constraint

- Now we know how the consumer feels about various bundles
- But how do they decide? Constrained optimization (otherwise choose infinite amounts!)
- Need a **budget constraint**
- Assume consumer is in a competitive market (I assume my wage is given/I can't change it)

Budget Constraint-II

- Time constraint, letting ℓ be leisure, N^s be work time, and h be total free hours:
$$\ell + N^s = h$$
- Great, h is some fixed amount, ℓ is leisure, but where does N^s play into all this?
- Workers get paid some real wage w for their hourly work, which they can use to buy C (trade off between ℓ and C now, via N^s!

Budget Constraint-II

- Letting N^S be work time, w be the real wage, π be "dividend income" and T be "lump-sum" tax:

$$C = wN^s + \pi - T$$

- Then, plugging in the time budget constraint:

$$C = w(h - \ell) + \pi - T$$

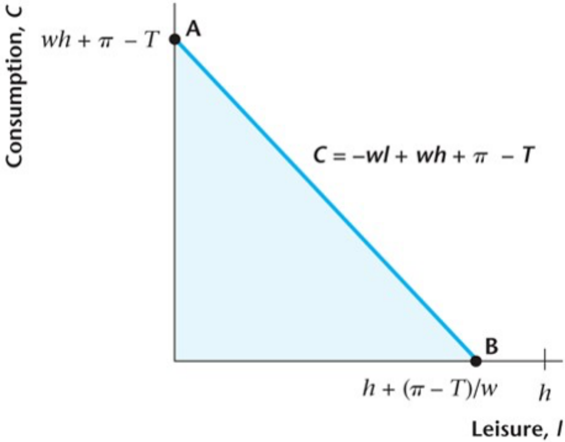
- Or:

$$C + w\ell = \underbrace{wh + \pi - T}_{\text{Disposable ``income''}}$$

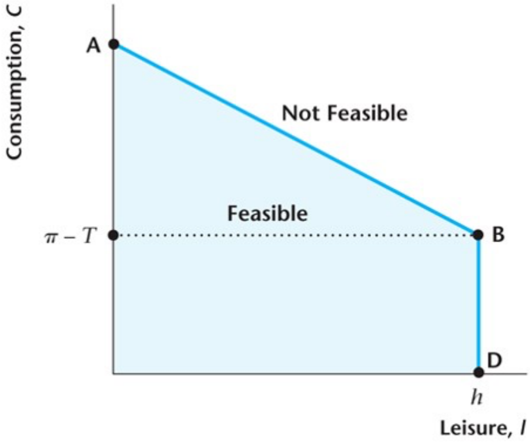
- Let's graph out the budget constraint in terms of C as a function of ℓ:

$$C = \underbrace{wh + \pi - T}_{\text{Intercept}} \underbrace{-w}_{\text{Slope}} \ell$$

BC when $T > \pi$



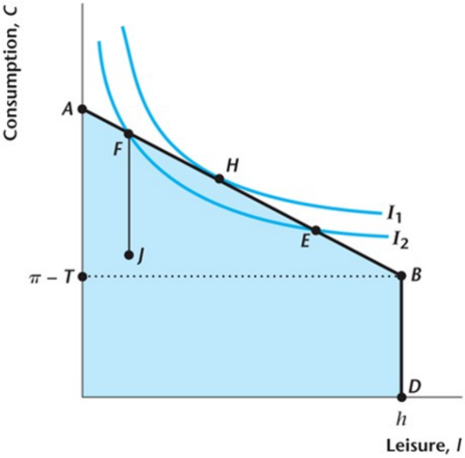
BC WHEN $T < \pi$



How does consumer optimize?

- Wants the highest possible indifference curve that fits along the budget constraint
- Intuitively, the indifference curve will just barely "kiss" the budget constraint
- If it didn't you could improve utility by moving up and to the right (increase both leisure and consumption)

Consumer optimization



F is better than J, E is the same as F, but H is the best

In math

- Constrained optimization problem (Lagrangian):

$$U(C,\ell) + \lambda(wh + \pi - T - w\ell)$$

- Unconstrained optimization problem:

$$U(wh + \pi - T - w\ell, \ell)$$

- Optimum happens when $\frac{\partial U}{\partial \ell} = 0$

$$-wU_1(C^*, \ell^*) + U_2(C^*, \ell^*) = 0$$

- Or:

$$w = -\frac{U_2(C^*, \ell^*)}{U_1(C^*, \ell^*)}$$

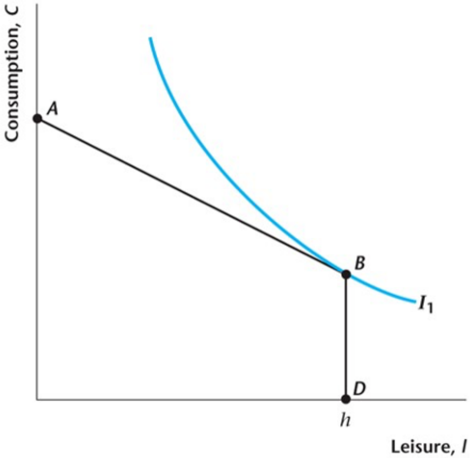
- But note that along $U(C,\ell) = \bar{U}$:

$$\frac{dC}{d\ell}|_{U(C,\ell)=\bar{U}} = -\frac{\frac{\partial U}{\partial \ell}}{\frac{\partial U}{\partial C}} = w$$

- The best indifference curve is tangent to our budget constraint (has slope -w wrt N^s)

At the optimum

- At the (interior!) optimum, we know that $MRS_{\ell,C} = w$
- Marginal rate of substitution of leisure for consumption is the wage
- If you were had to work one more hour, you would have to get w more consumption to be indifferent (at optimum, indifferent between more labor and more consumption, or less labor and less consumption)
- Now we'll have some predictions, which we'll mostly think of graphically

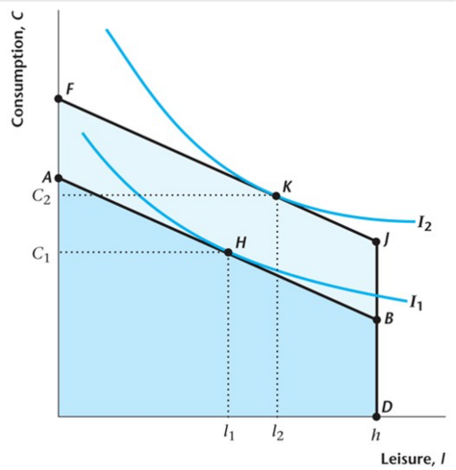


We could be at a "corner solution"

Changes in Real Dividends or Taxes

- Now let's think about a change in $\pi - T$, the "property income"
- Such a change is a "pure income effect" because it affects income but not the budget-constraint tradeoff between labor and leisure
- If leisure and consumption are both normal goods, then both should increase when $\pi - T$ increases

Non-labor income increase

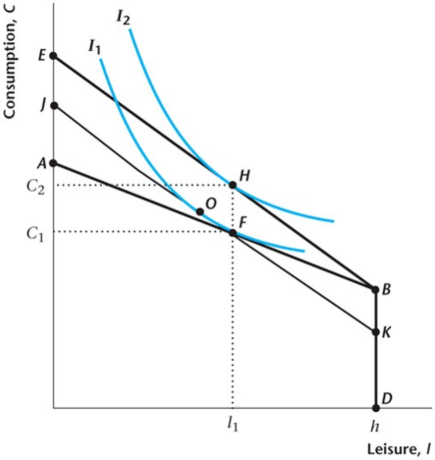


"Pure income effect:" consume more of everything ($c\uparrow$, $\ell\uparrow$, $N\downarrow$)

Changes in the Real Wage

- Changes in the real wage w are far more complicated
- If your wage increased, would you work more or less?
  - On one hand, leisure is a normal good, I'm richer, work less ("income effect")
  - On other hand, consumption is now "cheaper" in terms of labor, consume more of that! ("substitution effect")
- We call the substitution effect how we would change leisure if we were constrained to be on the same utility curve (no increase in "income" (happiness))
- We call the income effect the residual left over: the effect of moving to our higher indifference curve after the substitution effect
- Let's see it graphically

Wage Increase



When wages increase from B-A to B-E, define $F \to O$ as "substitution effect" and $O \to H$ as "income effect." K-J is the change in wage but no change in "income" (utility curve), which gives substitution effect.

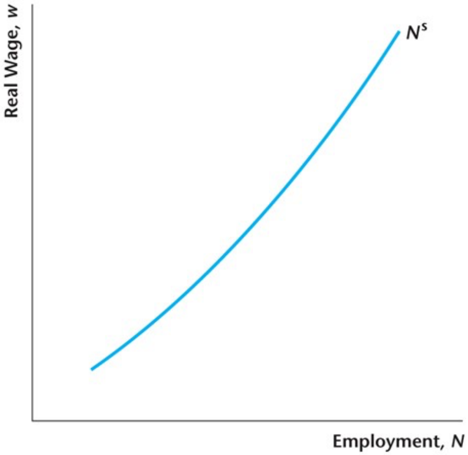
Labor supply curve

- While what we do builds from indifference curves, we sometimes use the result of them, the "labor supply curve"

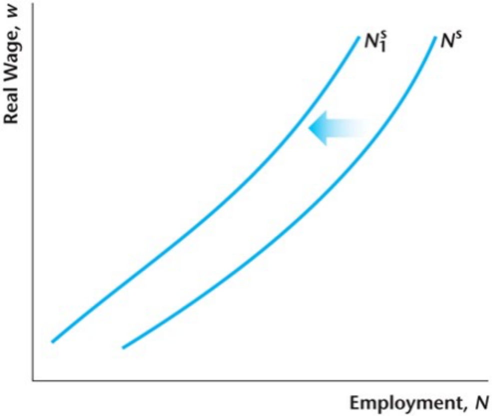
$$N^s(w) = h - \ell(w)$$

- Ordinarily, we think that the labor supply curve slopes upward (substitution effect dominates)
- While we'll talk about why, it's primarily because we're talking about short run changes in the wage that mostly mute the income effect (dynamics we haven't yet seen)
- But we'll see! In long run, it may be that income effect dominates substitution!
- But mostly, they're relatively close to balancing ("balanced growth preferences")

Labor supply curve



Effect of property income increase on labor supply



Closed-Form Solution

- Let's take the following example (Williamson does perfect complements in book):

$$U(C, \ell) = log(C) + \psi log(\ell)$$

- Such that: $C = wh + \pi - T - w\ell$
- Can write the Lagrangian as:

$$\mathcal{L} = \max C, \ell \left\{ log(C) + \psi \log(\ell) + \lambda \left(wh + \pi - T - C - w\ell \right) \right\}$$

FOC's:

$$\frac{1}{C^*} = \lambda^*$$

$$\frac{\psi}{w\ell^*} = \lambda^*$$

$$wh + \pi - T = C^* + w\ell^*$$

Solving these three equations for three unknowns (C^*, ℓ^*, λ^*):

$$C^* = \frac{\pi - T + hw}{1 + \psi} \qquad \ell^* = \frac{\psi}{1 + \psi} \frac{hw + \pi - T}{w}$$

- Can see that when $\pi - T \uparrow$, $C^* \uparrow$, and $\ell^* \uparrow$

Taking Stock

- Okay, great, we have the "representative agent" / "representative consumer"
- But where do wages and consumption goods come from?
- The "representative firm"
- Representative firm will have a **production function** that takes in capital K and labor demanded N^d:

$$Y = zF(K, N^d)$$

- We call z the "total factor productivity
- Crucial to understanding firm behavior is the **marginal product**.
- The marginal product of a factor of production is the additional output that can be produced with one additional unit of that factor input, holding constant the quantities of other factor inputs (e.g. MPL would be $\frac{\partial Y(K,N^d)}{\partial N^d}$, how much more Y you get if you add a unit of N^d holding K constant)

Properties of the production function

- As we did with the consumer, we start with five sensible properties
  1. Production function exhibits constant returns to scale (double inputs, double outputs):
  $$zF(xK, xN^d) = xzF(K, N^d)$$
  2. Output increases when either capital input or the labor input increases:
  $$\frac{\partial F(K, N^d)}{\partial K} > 0 \quad \text{and} \quad \frac{\partial F(K, N^d)}{\partial N^d} > 0$$
  3. The marginal product of labor decreases as the amount of labor increases:
  $$\frac{\partial^2 F(K, N^d)}{\partial (N^d)^2} < 0$$

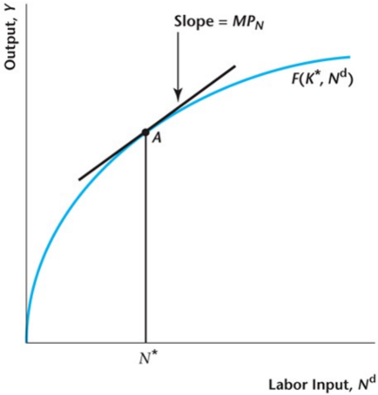
Properties of the production function-II

4. The marginal product of capital decreases as the amount of capital increases:
$$\frac{\partial^2 F(K, N^d)}{\partial (N^d)^2} < 0$$

5. The marginal product of capital increases as the amount of labor increases, and the marginal product of labor increases as the amount of capital increases:

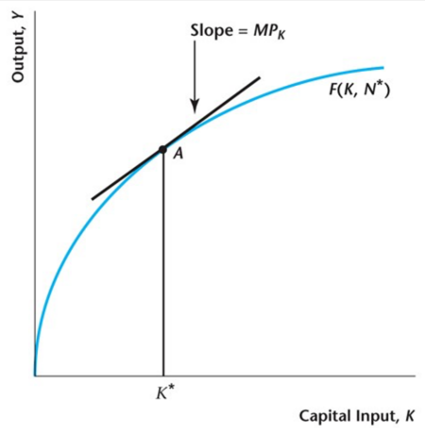
$$\frac{\partial^2 F(K, N^d)}{\partial (N^d) \partial K} = \frac{\partial^2 F(K, N^d)}{\partial K \partial (N^d)} > 0$$

Production Function, fixing Capital and Varying Labor



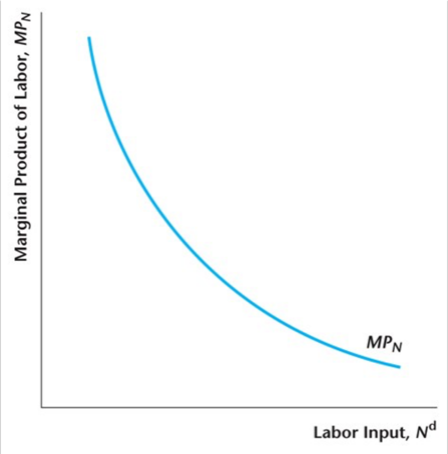
Note x-axis is capital

Production Function, fixing Labor and Varying Capital



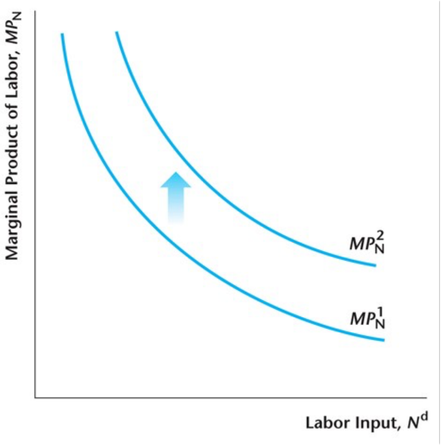
Note x-axis is labor

Marginal Product of Labor as Function of Labor



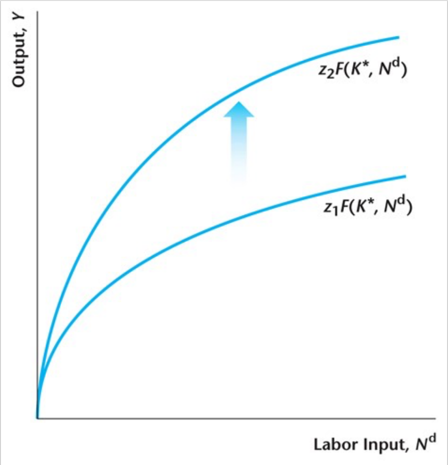
Labor is less productive as you get more of it (holding capital constant)

Marginal Product of Labor as Function of Labor, increasing Capital

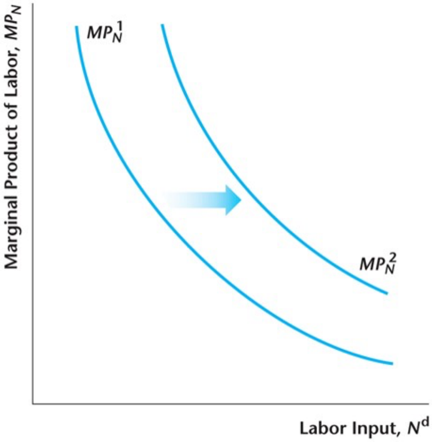


Marginal product of labor shifts up when capital increases

Total factor productivity increase on output as a function of labor



Total factor productivity increase on MPL as a function of labor



Profit Maximization

- We assume that firms choose K and N^d to maximize profit π:

$$\pi = zF(K, N^d) - wN^d - rK$$

- When choosing how much labor to use, take FOC with respect to labor (assume wage is given!):

$$\frac{\partial \pi(K, N^d)}{\partial N^d} = 0$$

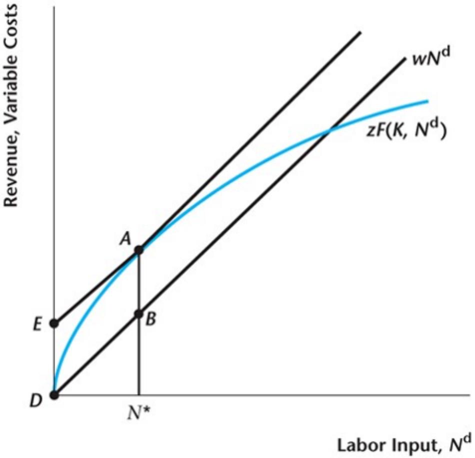
- Or:

$$z\frac{\partial F(K, N^d)}{\partial N^d} - w = 0$$

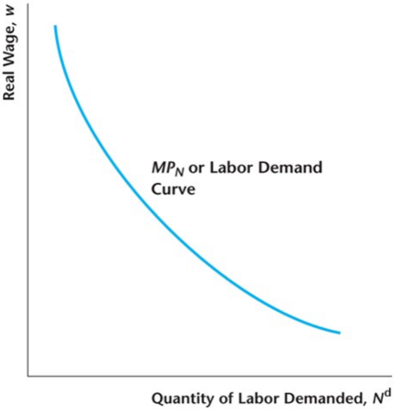
- Or, the marginal product of labor is equal to the wage:

$$MP_N = w$$

Revenue, variable costs, and profit maximization



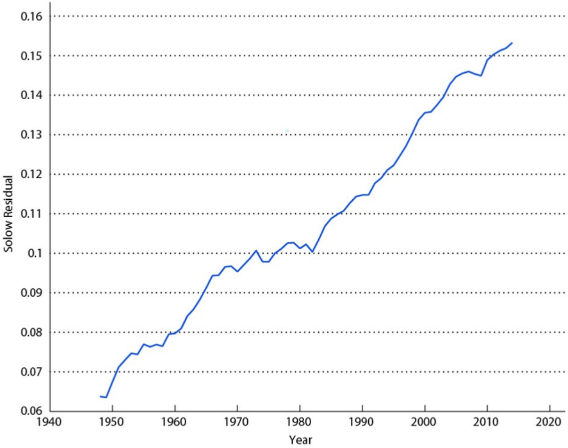
Marginal product of labor curve is labor demand curve



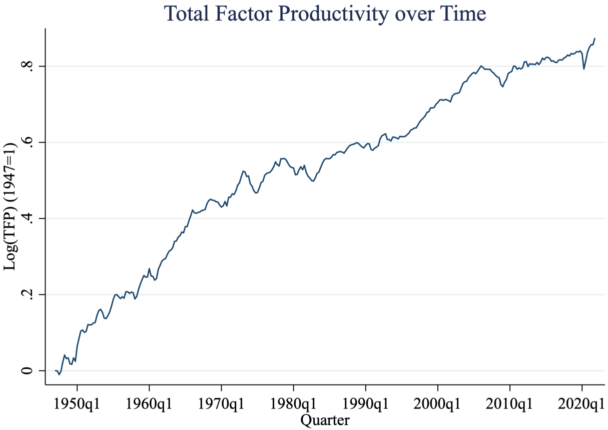
Measuring Z, the Solow Residual

- z is a measure of all the things we don't know and/or didn't model
- We can measure it for a specific production function if we know K, N^d, Y, and have the production function. For instance, if:
$$Y = zK^0.3L^{0.7}$$
- Then can measure z as:
$$z^{measured} = \frac{Y^{data}}{(K^{data})^0.3(L^{data})^{0.7}} = \frac{z^{true}K^{data})^0.3(L^{data})^{0.7}}{(K^{data})^0.3(L^{data})^{0.7}} = z^{true}$$
- Note that if true reality is actually: $Y = z^{true}K^{0.3}L^{0.7}O^{0.1}$ then:
$$z^{measured} = \frac{z^{true}K^{0.3}L^{0.7}O^{0.1}}{(K^{data})^{0.3}(L^{data})^{0.7}} = z^{true}O^{0.1}$$
- So the residual will move with not only the true residual but also O (oil).

Solow Residual



Fernald "Solow" Residual



Adjusts for inputs such as capital utilization, labor composition (richer model, like modeling oil above)

Next Steps

- Now, we have a consumer and a producer, we want to "close" the model, which we'll do next chapter
- This "closing" will be a big distinction from micro/labor