Econ 352 - A Closed-Economy. One-Period Macroeconomic Model

(See Williamson Ch. 5)

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Taking stock

- We have a representative consumer/household that likes to consume and leisure, and works for wages, maximizing happiness
- We have a representative firm that produces/sells good while maximizes profits by choosing labor
- Now we put the together!

Some Terms and Assumptions

- Right now we'll have a "closed economy" which is a model of a single country with no interactions (no imports or exports: everything that's produed will be consumed or invested)
- This is different from an "open economy" which is when international trade is allowed
- The results for this chapter won't change much if we go to open economy, but the closed framework is useful for now
- Adding in government: for now, we're going to have an "exogenous" amount of G that is determined outside the model (not endogenous, determined within the model, like labor)
- Government budget constraint will be simple:

$$G = T$$

- This will allow for some basic analysis of **fiscal policy**, government spending

Competitive Equilibrium-Informal

- Here we define a "competitive equilibrium"
- We have three exogenous variables: G, z, K, which are exogenous, e.g. we take them as given
- We want to solve for the endogenous variables: C, N^s, N^d, T, Y, and w
- Our assumptions are that households and firms are maximizing, and that "markets clear"
- For instance, the labor market clears at a wage w^* when, given that wage, the amount of labor households wish to supply is equal to the amount of labor firms demand.

Competitive Equilibrium-Formal

1. Representative consumer chooses C (consumption) N^s (labor supply) to maximize utility subject to w (real wage) T (taxes) and π (dividend income
2. Representative firm chooses N^d (quantity of labor demanded) to maximize profits, with output $Y = zF(K, N^d)$ and maximized profits $\pi = Y - wN^d$, taking z and K as given
3. Market for labor clears: $N^d = N^s$
4. The government budget constraint is satisfied: $G = T$

- Usefully, all this implies $Y = C + G$, the income-expenditure identity

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Finding our Equilibrium

- Start with the production function:

$$Y = zF(K, N)$$

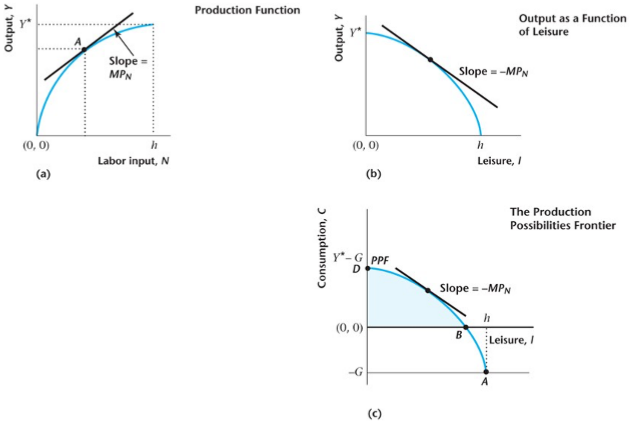
- Then, use the fact that $N = N^s = N^d = h - \ell$

$$Y = zF(K, h - \ell)$$

- Related to consumption via feasibility :

$$C = zF(K, h - \ell) - G$$

Production Function and Production Possibilities Frontier



Putting Supply & Demand Together

- Recall from last class that the marginal rate of substitution of leisure for consumption for the household will be equal to the wage:

$$MRS_{\ell,C} = w$$

- Also recall that the profit-maximizing amount of labor sets the marginal rate of transformation (the marginal product of labor) equal to the wage:

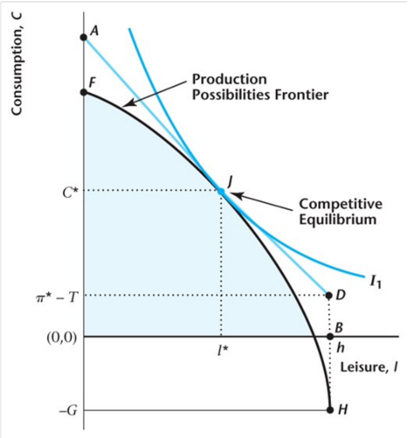
$$MP_N = w$$

- This means that:

$$MRS_{\ell,C} = MRT_{\ell,C} = MP_N$$

- It will turn out that this implies efficiency of the labor market, which we'll now discuss

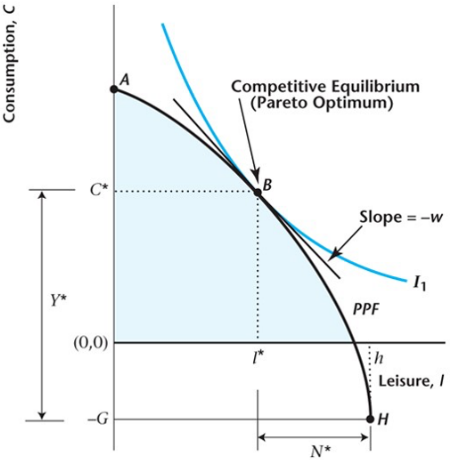
Production Function and Production Possibilities Frontier



Efficiency

- For economists, efficiency just means that people aren't "unnecessarily" unhappy
- Definition: A competitive equilibrium is "Pareto optimal" if there is no way to rearrange production or to reallocate goods so that someone is made better off without making someone else worse off
- No transactions that would make both parties happier are left on the table
- This will occur when the PPF, the frontier of possibilities, is tangent to preferences
- If it weren't, then could improve!

Production Function and Production Possibilities Frontier



First Welfare Theorem

- The **first fundamental theorem of welfare economics** states that, under certain conditions, a competitive equilibrium is Pareto Optimal
  - An old idea! But leaves room for preferences

- The **second fundamental theorem of welfare economics** states that, under certain conditions, a Pareto optimum is a competitive equilibrium
  - You could achieve any Pareto optimum given starting conditions (free redistribution)
- What breaks these? The biggest is **externalities**

Sources of Social Inefficiencies-Externalities

- Competitive equilibrium may not be Pareto optimal because of externalities
- If my private benefit for doing something is greater than the total public benefit (which includes my private) then I may do too much of it
- For instance, let's say that I enjoy being warm, so I have preferences over log burning of:

$$U(burn) = \log(logs) - n$$

- Where logs cost 1 work (so $logs = n$)
- On my own, I maximize utility when $x = 1$, which will set the PPF slope (1) equal my MRS (1/1=1).
- Efficient! But if society doesn't like squared pollution, so that social costs are: $Scost = burn^2$, then while I would pick 1, the socially optimal thing (including my own benefit and costs to others) would be to burn only 0.5 logs.

Sources of Social Inefficiencies-Taxes

- Competitive equilibrium may not be Pareto optimal because of distorting taxes
- I will set the marginal benefit of doing something equal to the **full** marginal cost (including taxes)
- But that means that if taxes increase marginal cost, I will have to increase marginal benefit, which I can do (for things with diminishing utility) by reducing them
- Let's take working for consumption, where I dislike working and like consumption:

$$U(C) = \sqrt{C} - n$$

- Where my budget constraint is: $C = (1 - \tau)n$ (wage is one)
- Then, maximizing my utility, I get:

$$n = \frac{1 - \tau}{4} \quad C = \frac{(1 - \tau)^2}{4}$$

Sources of Social Inefficiencies-Taxes Example

- Let's take working for consumption, where I dislike working and like consumption:

$$U(C) = \sqrt{C} - n$$

- Where my budget constraint is: $C = (1 - \tau)n$ (wage is one)
- Then, maximizing my utility, I get:

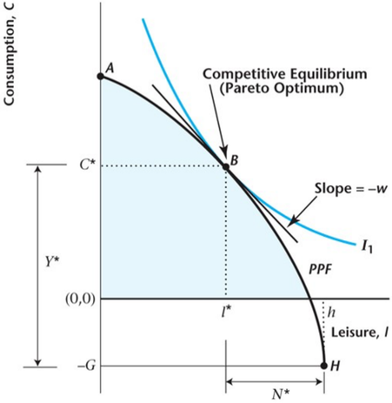
$$n = \frac{1-\tau}{4} \quad C = \frac{(1-\tau)^2}{4}$$

- For instance, if $\tau = 0$, $C = n = 0.25, U = -1.64$, $MRS_{\ell,c} = 2\sqrt{C} = 1 = w = MP_n$
- If $\tau = 0.5$, $C = 0.0625$, $n = 0.125$, $U = -2.90$ $MRS_{\ell,c} = 0.5 = (1-\tau)w < w = MP_n$
- There's a "wedge" between MRS and MP_n (of τ)

Sources of Social Inefficiencies-Firms are not Price Takers

- If firms have monopoly power, or if unions/workers have monopsony power, then inefficiency can result
- In both, there is another "wedge." Rather than the government artificially raising the cost of a good via a tax, a firm might artificially raise the cost of a good by restricting supply
- Similarly, rather than a government increasing the cost of labor to firms via a tax, a union might do it via negotiation
- In both, private benefit may mean leaving good (Pareto-improving) trades on the table

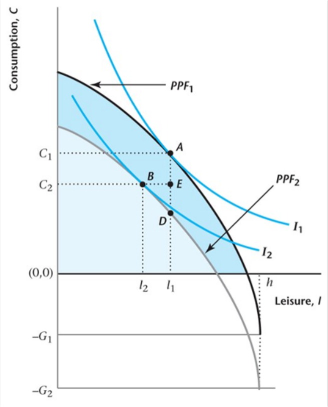
Production Function and Production Possibilities Frontier



Applying the model: changes in government purchases

- Now we're going to apply our model in a very simple way: we're going to have G increase and ask what this does to Y, C, n, and w.
- What happens? Recall that $G = T$, so $\Delta G = \Delta T$, a shift out of government spending will be a shift out of taxes and a shift in of the PPF
- We're poorer! So, consume less of all normal goods (less consumption, less leisure (more work)). $\Delta C < 0$, $\Delta n > 0$.
- If $\Delta n > 0$, and we know $w = MP_n = \frac{\partial zf(K,n)}{\partial n}$, and $\frac{\partial^2 zf(K,n)}{\partial n^2} < 0$, w falls (but by less than n increases, from CRS)
- From the budget constraint: if we're working more, then $-\Delta C < \Delta T$, and because $Y = C + G = C + T$, it must be that $\Delta Y > 0$.

Equilibrium Effects of an Increase in Government Spending



Concrete Example

- Production function: $Y = zn^{\alpha}$
- Preferences+time budget constraint $n = 1 - \ell$:
$$U(C, n) = \sqrt{C} + \sqrt{1 - n}$$
- Budget constraint: $C = nw + \pi - T$
- Profit: $\pi = Y - nw$
- Government b.c. : $G = T$
- We have 3 exogenous variables: z, α, G
- If we plug in the budget constraint into preferences to get rid of C, we have 7 endogenous variables: Y, n, ℓ, T, π, w
- And 7 equations: (1) firm FOC wrt n ("labor demand") (2) production function ("consumption supply"), (3) HH foc wrt n ("labor supply"), (4) time budget constraint (5) HH budget constraint ("consumption demand") (6) government budget constraint and (7) profit condition

Seven Equations, Seven Unknowns

Seven Equations, Seven Unknowns	
Firm FOC wrt n	$\alpha z n^{\alpha-1} = w$
Production function	$Y = z n^{\alpha}$
HH foc wrt n:	$n = \frac{w^2+T-\pi}{w(1+w)}$
Time budget constraint	$n = 1 - \ell$
Government budget constraint	$G = T$
Profit definition	$\pi = Y - nw$
HH Budget constraint	$C = nw + \pi - T$

- While we could solve for this "closed form" that's not what modern macroeconomists do
- Algebra is for suckers(?!)
- Instead we hand off to solvers...let's look at excel!

Excel

- See SimpleMacro.xlsx

Effects of an Increase in G

Variable	Baseline	G↑33%	% Change
z	1	1	0.00%
α	0.7	0.7	0.00%
g	0.3	0.4	33.33%
n	0.53	0.58	10.12%
w	0.85	0.82	-2.85%
Y	0.64	0.68	6.98%
ℓ	0.47	0.42	-11.32%
π	0.19	0.21	6.98%
T	0.30	0.40	33.33%
C	0.34	0.28	-16.31%

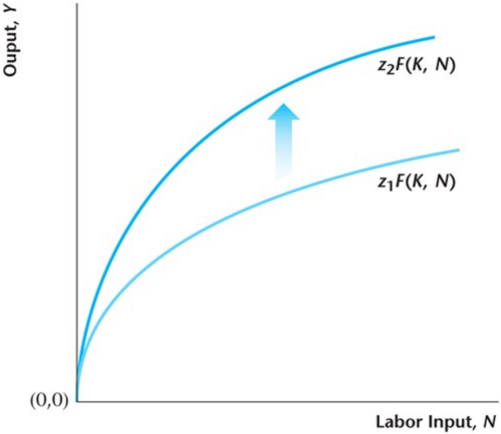
Summarizing the Simple Model

- Simple excel model hopefully useful in a few ways
- Never simplify again! Just throw all your equations and solve numerically
- Confirms Williamson/our intuitive explanation of what should happen
- Modern macro isn't so far from this...just make this an infinite period model/capital and you have Modern macro representative agent models: change G, how do things change?
- Now let's move on to changes in total factor productivity (z), rather than changes in G

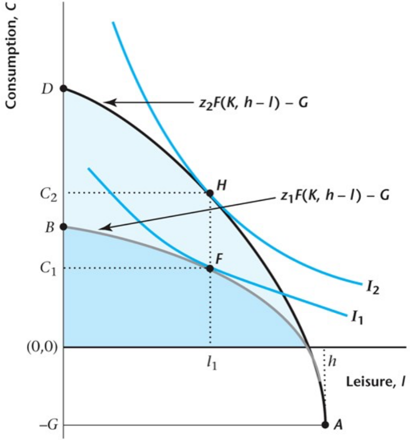
Shift in total factor productivity

- Must break up into short and long-run.
- Let's think about short run for now.
- When z increases, the PPF increases *and* slope changes (income and substitution effects)
- We know that C will increase, but n may rise or fall
- $Y = C + G$, G constant, so Y increases (could also see from n increasing)
- If labor stayed the same, then w increases ($MP_n \uparrow$)
- Let's see this graphically

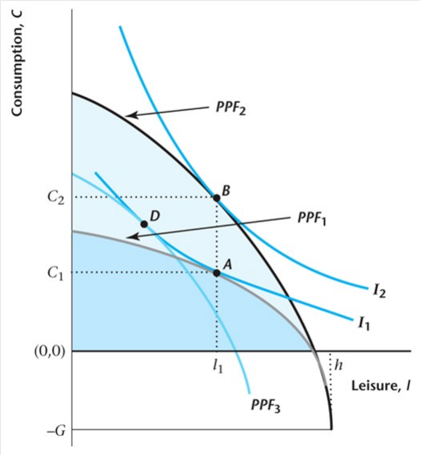
Production Function and Production Possibilities Frontier



Competitive Equilibrium Effects of an Increase in TFP



Income and Substitution Effects of an Increase in TFP



Another Concrete Example

- Let's go through the same example, but now z will increase rather than G
- Production function: $Y = zn^{\alpha}$
- Preferences+time budget constraint $n = 1 - \ell$:
$$U(C, n) = \sqrt{C} + \sqrt{1 - n}$$
- Budget constraint: $C = nw + \pi - T$
- Profit: $\pi = Y - nw$
- Government b.c. : $G = T$
- We have 3 exogenous variables: z, α, G
- If we plug in the budget constraint into preferences to get rid of C, we have 7 endogenous variables: Y, n, ℓ, T, π, w
- And 7 equations: (1) firm FOC wrt n ("labor demand") (2) production function ("consumption supply"), (3) HH foc wrt n ("labor supply"), (4) time budget constraint (5) HH budget

Excel

- See SimpleMacro.xlsx

Effects of an Increase in z

Variable	Baseline	z↑5%	% Change
z	1	1.05	5.00%
α	0.7	0.7	0.00%
g	0.3	0.3	0.00%
n	0.53	0.53	0.25%
w	0.85	0.89	4.92%
Y	0.64	0.67	5.19%
ℓ	0.47	0.47	-0.28%
π	0.19	0.20	5.19%
T	0.30	0.30	0.00%
C	0.34	0.37	9.77%

Takeaways

- What can we do with this simple model?
- Since WWII, z has increased dramatically, while Y, w, C, have gone up, and n has remained roughly constant
- Our simple prediction is that when z increases, Y, w, C, will go up, and we've noted an ambiguity in n (for our specific calibration, it increased)
- Idea: if we had preferences where income and substitution effects that cancel (so $w \uparrow$, $n\cdot$, then our model would be consistent
- But our model helps give an idea about short- and long-run. We saw that in the short run, L moves with w, so short run substitution effect should dominate, but long run should cancel.
- As we'll discuss, this would make sense if temporary shocks didn't increase *lifetime* income much, but did increase wages

Let's look at some data

- First we'll look at TFP & GDP, then at
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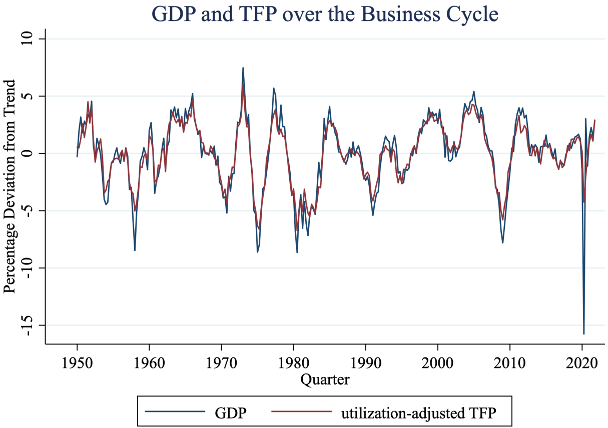
TFP and GDP over the Business Cycle

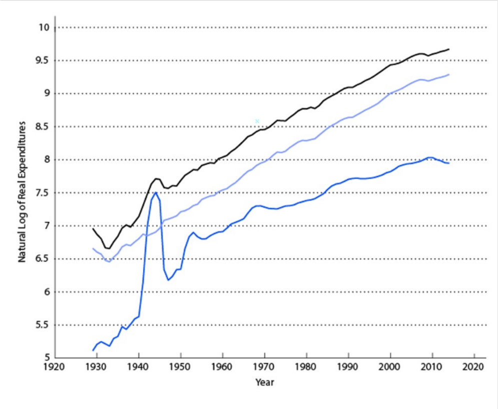


TFP and GDP move together–it explains growth, what about the business cycle?

Let's look at some more data

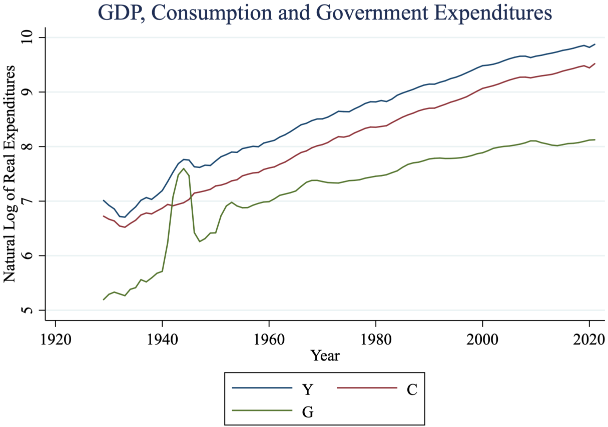
- Now let's look at what's happened to government spending over time, to link back to our discussion of government purchases

Y, C, and G over time

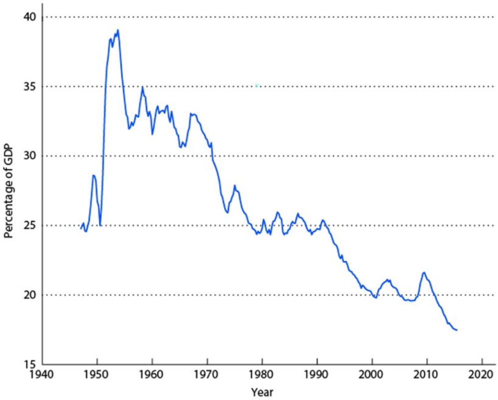


What about Covid times?

Y, C, and G over time-updated

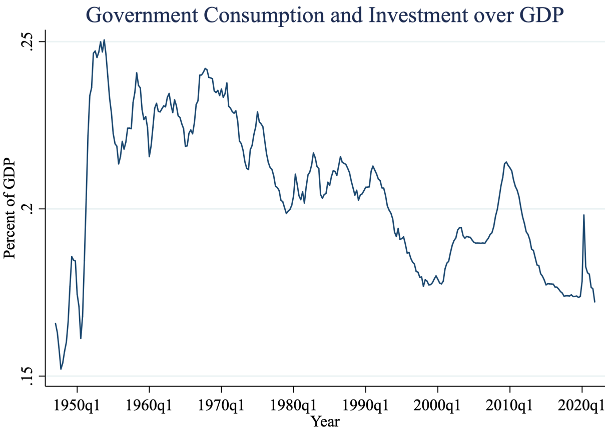


Government expenditures over time

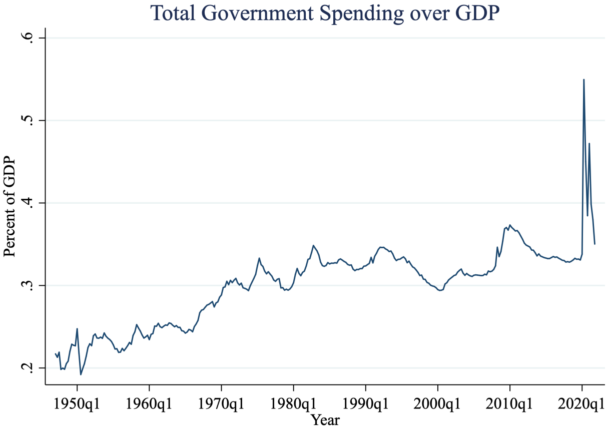


Is government declining as a fraction of GDP?

Government expenditures over time-updated

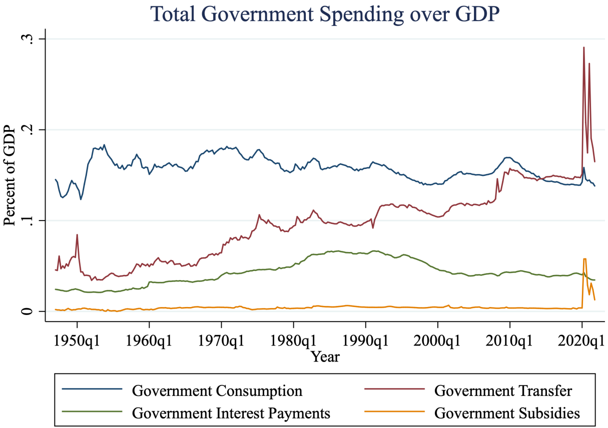


Government outlays over time-updated



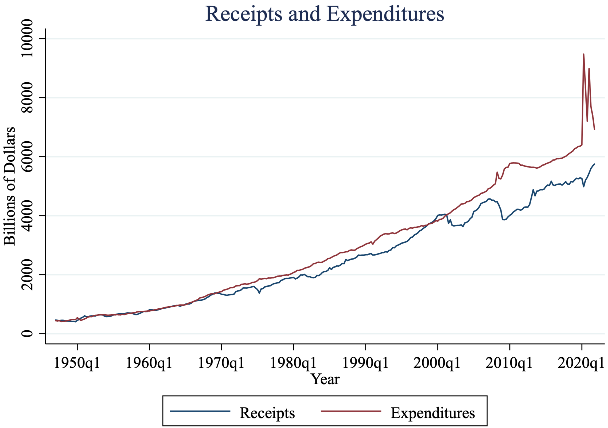
No! It's...expanding? (also...Covid!!)

Four Components of Government Expenditure



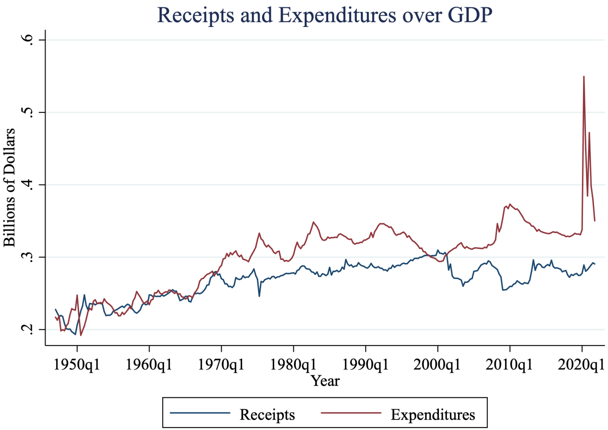
Rise in government spending comes from transfers, rather than consumption or subsidies or interest payments

$G \neq T!$



Also note that our $G = T$ assumption is wrong

Same thing, but over GDP



Taxes

- So far, the household budget constraint has been $C = wn + \pi - T$
- But T really depends on wn: make more, pay more
- This distorts behavior (income and substitution effects, where the other was just income)
- Instead, we'll change the budget constraint so that:

$$C = w(1-\tau)(h-\ell) + \pi$$

- Where, as a reminder, C is consumption, w is wages, τ is the (new!) "proportional tax rate" h is how much free time you have, ℓ is leisure, and π is profits
- We'll simplify the production function so that it's linear:

$$Y = zN^d$$

Labor Demand

- Profit of the firm:

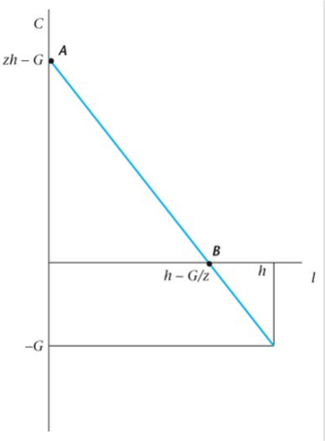
$$\pi = Y - wN^d = zN^d - wN^d$$

- FOC of the firm: $\frac{\partial \pi}{\partial N^d} = 0$:

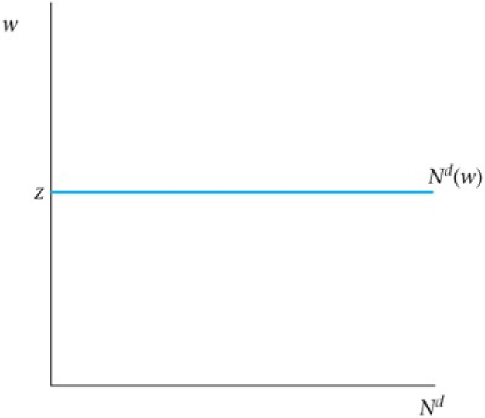
$$w = z$$

- Note that it does not depend on labor! Horizontal "perfectly elastic" demand curve

PPF with a Linear Production Function



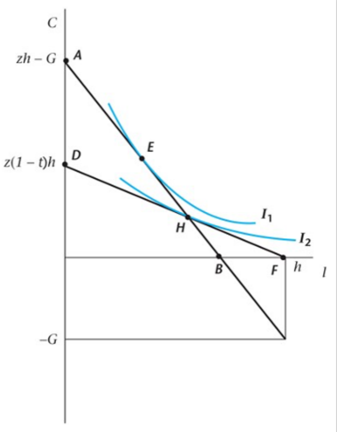
Labor Demand in Simplified Model



Shifting with a proportional tax

- Now, our PPF shifts with a proportional tax on labor income
- So let's compare two experiments: one in which we graph the old, lump-sum tax, and the other in which we have a proportional tax

Labor Demand in Simplified Model



Laffer Curve

- Rather than fixing G and examining what happens for a lump-sum vs a distortionary tax, we could instead think about G as a function of τ
- Revenue from a proportional tax on labor is:

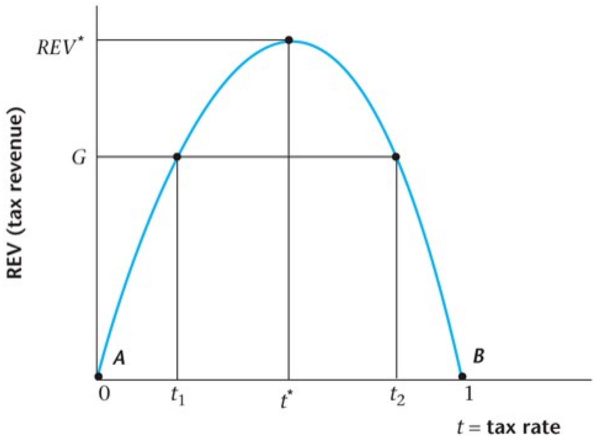
$$Rev = \tau n$$

- But, as we saw, n is a function of τ! So more clearly:

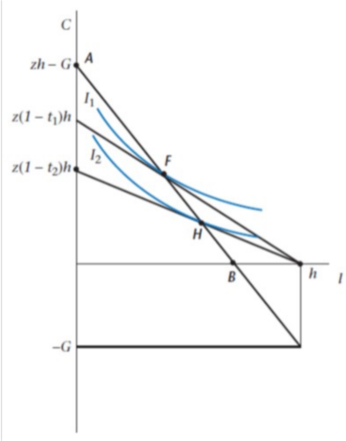
$$Rev = \tau n(\tau)$$

- There's a tension: when $\tau \uparrow$, $n(\tau) \downarrow$
- When $\tau = 1$, $L(\tau) =$, so $Rev = \tau L(\tau) = 0$, and when $\tau = 0$, $Rev = \tau L(\tau) = 0$. But presumably $Rev > 0$ for some τ in between–this is the "Laffer Curve"

Idealized Laffer Curve



Two Equilibria



Concrete Example

- Let's go back to our tax example, where we found:

$$n = \frac{1-\tau}{4} \quad C = \frac{(1-\tau)^2}{4}$$

- Then in that example ($w = 1 \Rightarrow Y = N^d$):

$$Rev = \tau \frac{1-\tau}{4}$$

- Or:

$$Rev = \frac{\tau}{4} - \frac{\tau^2}{4}$$

- There are two worlds that generate the same revenue! Let's look when $Rev = 0.03$

Our Laffer Curve

temp

Concrete Example-Two Equilibria

Laffer Curve Example

Variable	Notation	Regime 1	Regime 2
Revenue	Rev	0.03	0.03
Tax rate	τ	0.139	0.861
Labor	n	0.215	0.034
Consumption	C	0.185	0.005
Utility	U	0.215	0.035
Equivalent Variation	CV	-0.1	$\cdot$

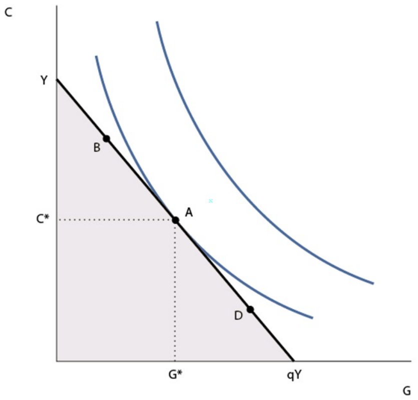
- Equivalent variation solves the problem, roughly $U(C_1+\delta, n_1) = U(C_2, n_2)$, e.g. how much would person in Regime 1 would have to lose (lump sum!) in order to be just as badly off as the person in Regime 2.

- The interpretation here is that a person in Regime 1 would be willing to give up 45% of their consumption just to stay in that regime!

How Large should Government Be?

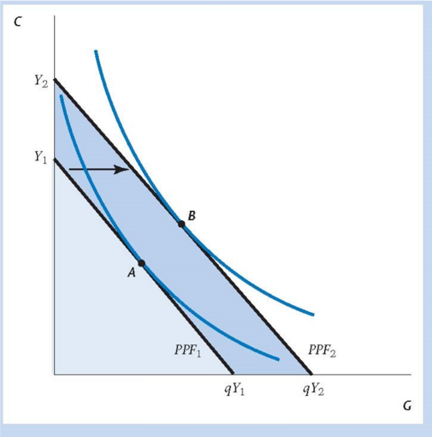
- Up until now, we've taken the size of government as given
- But obviously that's someone's choice as well!
- Recall that: $Y = C + T$
- Let's say that the government takes 1 unit of private consumption good and turns it into q units of public goods, so that:
$$C = Y - \frac{G}{q}$$
- q is some measure of government efficiency.
- Then we can write out the "societal" PPF of C vs G

Optimal Choice of Government Spending



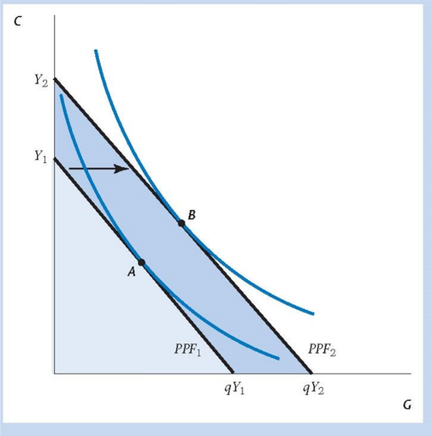
How do shifts in z, q affect our decision?

Optimal Choice of Government Spending



An increase in z clearly suggests an increase in C and G (this assumes G is normal)

Optimal Choice of Government Spending



An increase in q suggests G must increase, what happens to C is ambiguous (income and substitution effects)

- Presents interesting dilemma for a small-government type: do I want government to be efficient or not?

Conclusions

- This was a big chapter!
- Now we can take consumer & firm behavior and **start** thinking about various choices, such as:
  - When the government increases flat tax T, should I work more or less? Consume more or less?
  - When the government increases proportional tax τ, should I work more or less? Consume more or less?
  - When TFP z increases, what happens to C, n, etc.
  - What is the optimal size of government?
- We also saw our first simple numerical models of consumer behavior, cousins to what modern macroeconomists really use
- Next we'll turn to search & unemployment: frictions