A Crash Course in Growth Theory

See Barro Chapter 3-5

Trevor S. Gallen

Growth Facts

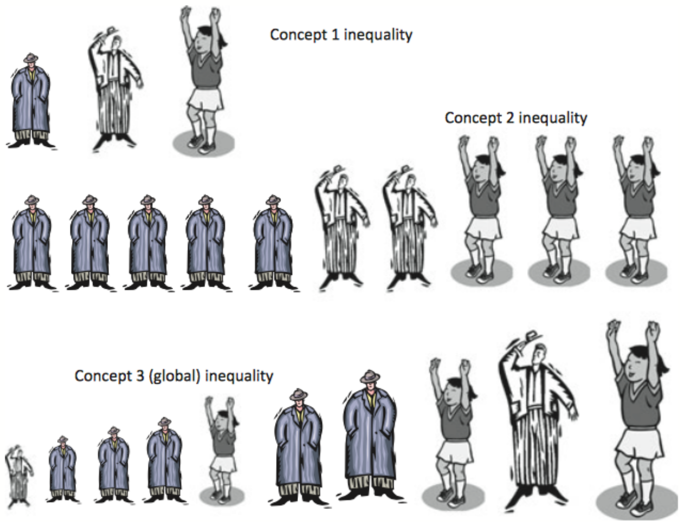
Growth

- We're interested in Economic growth
- An introductory macro class typically spends about eight of sixteen weeks on this.
- We are going to *zoom* through it to get to applications
- The readings may help
- I'll keep some slides in that might be interesting to look at, but you aren't responsible for them.

Inequality

- What's a good measure of global inequality?

Inequality



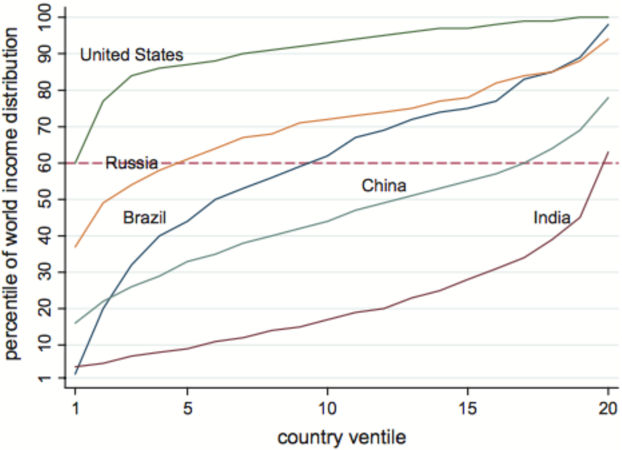
Milanovic, 2013

Inequality

- Different measures of inequality
  - Could take the inequality between countries (inequality 1)
  - Could take inequality between countries weighted by population (inequality 2)
  - Could forget countries, treat each person as an individual (inequality 3)

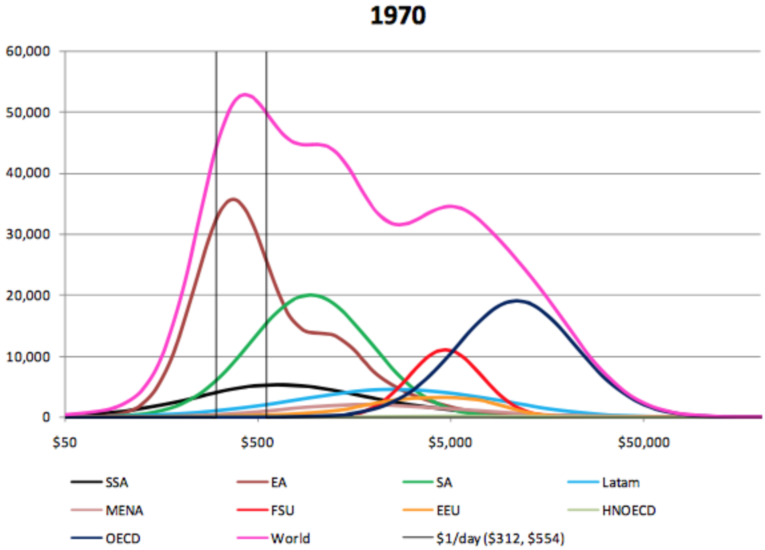
Percentiles of Income Distribution

Figure 7. Different countries and income classes in global income distribution, 2005.



Note: the line drawn at y = 60 shows the global position of the poorest 5 per cent of the US population.

World Income Distribution: 1970



Pinkovskiy & Sali-i-Martin, 2009

World Income Distribution: 1970

- Why are poor people poor?
- 80% of the answer is they live in poor countries
- 20% of the answer is the distribution of income within countries is uneven
- Economic growth is determinant of income inequality

Put another way...

- In 1981, 63% of China made below \$1.25 a day

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- To put in perspective, population of sub-Saharan Africa is 1.1 billion and 533 million live below $1.25 a day.

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- Note also that China had devastated its intellectual ($>$ middle school) class in the late 1960's and focused on agriculture, not built-up industry!

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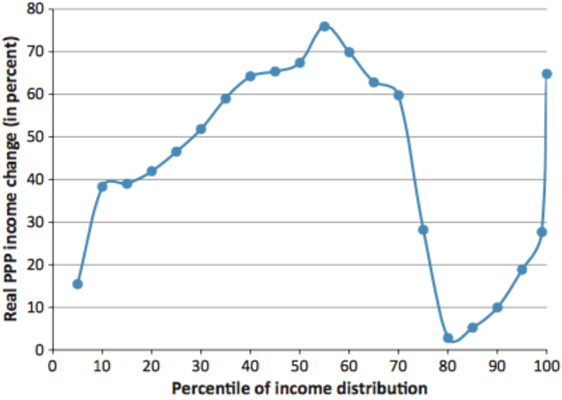
- We'll come back to growth

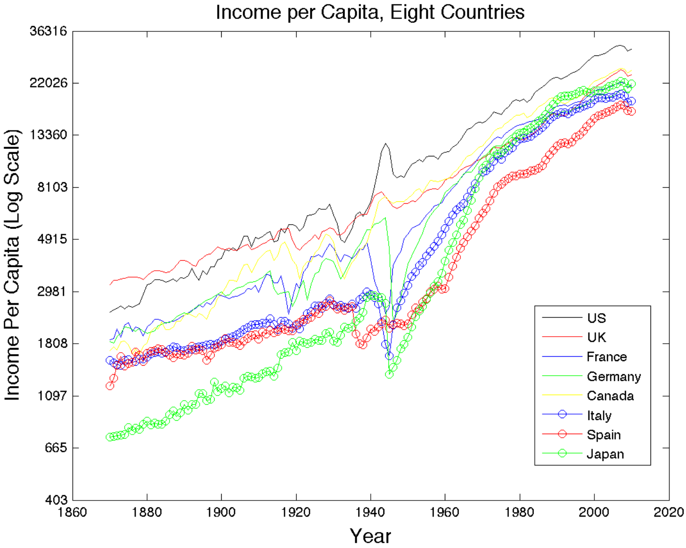
Inequality

Figure 4. Change in real income between 1988 and 2008 at various percentiles of global income distribution (calculated in 2005 international dollars).



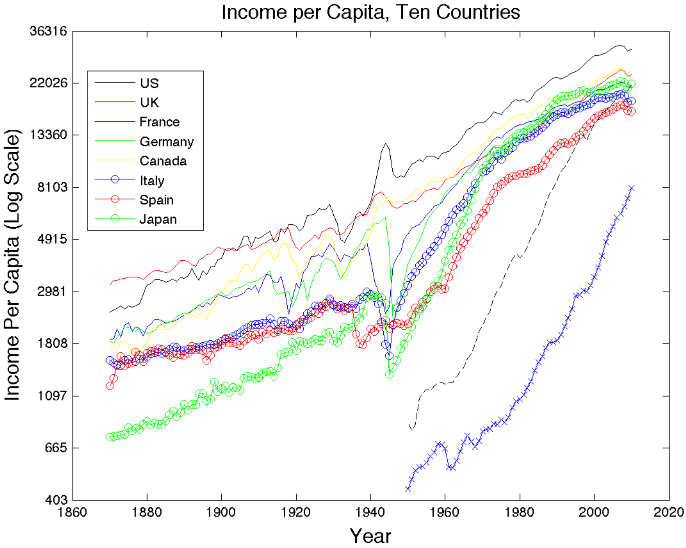
Milanovic, 2013

Cross-Country Per-Capita GDP



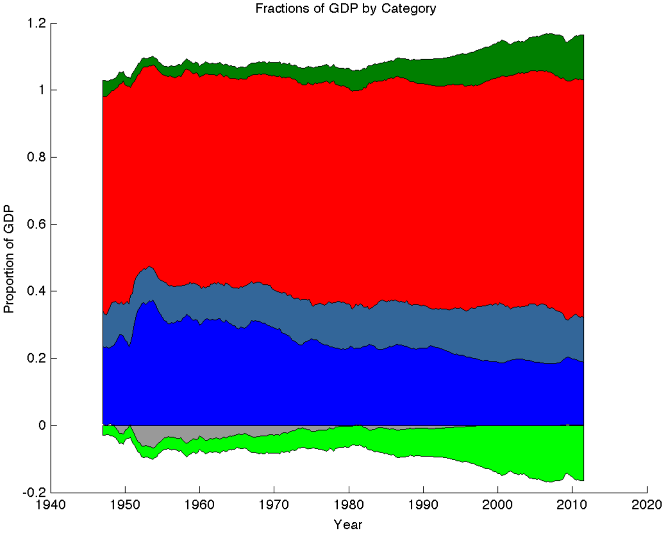
Note the log scale!

Cross-Country Per-Capita GDP

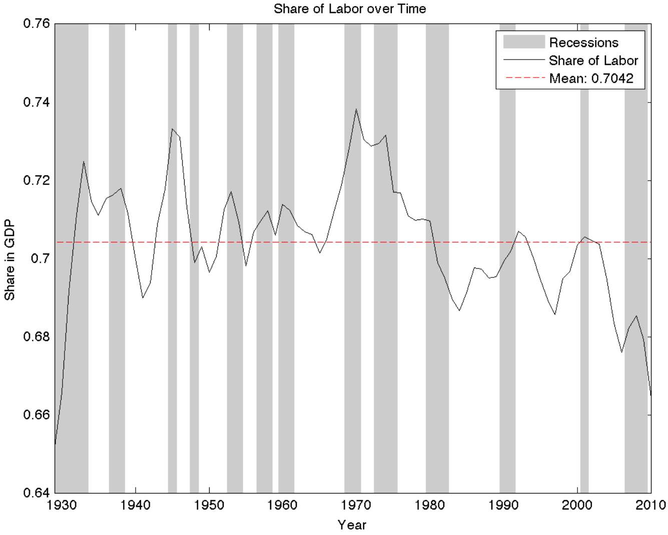


Add South Korea and China

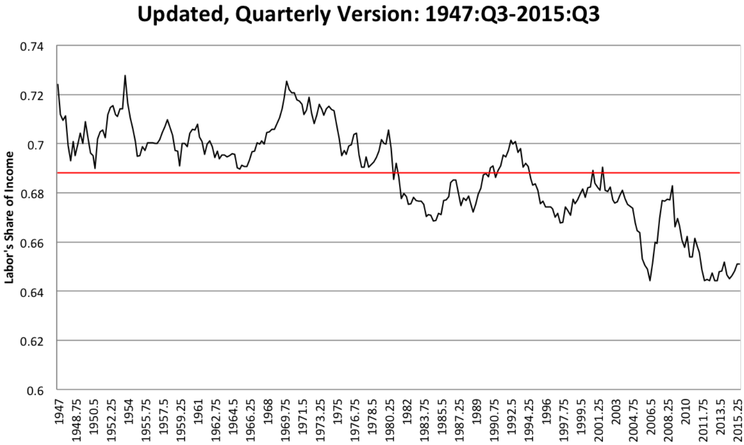
Great Ratios



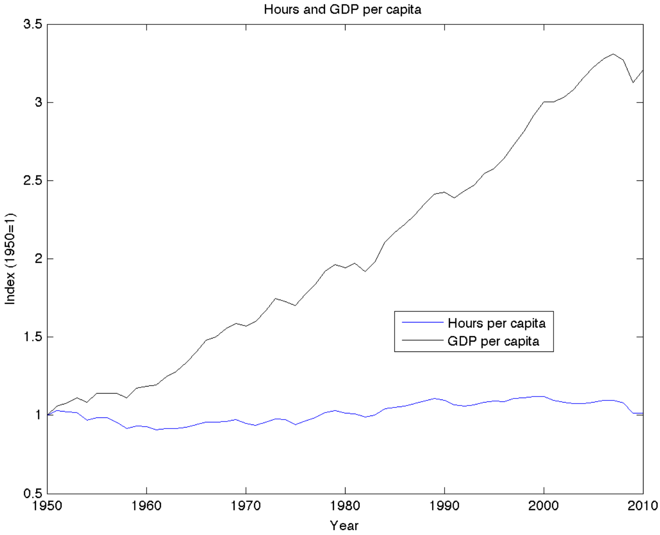
Labor's Share



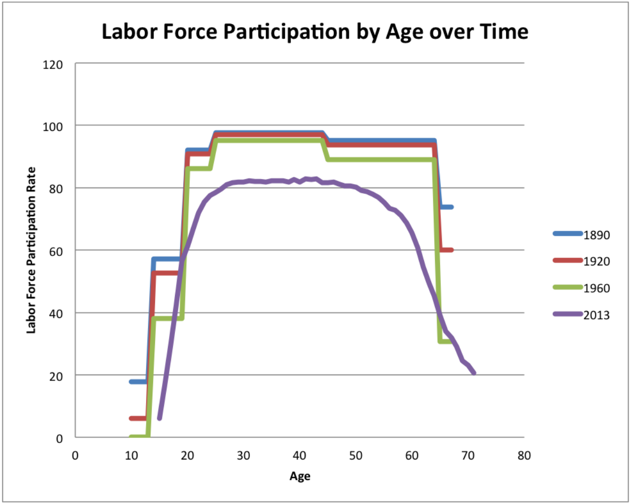
Labor's Share, Updated



Labor hours and GDP

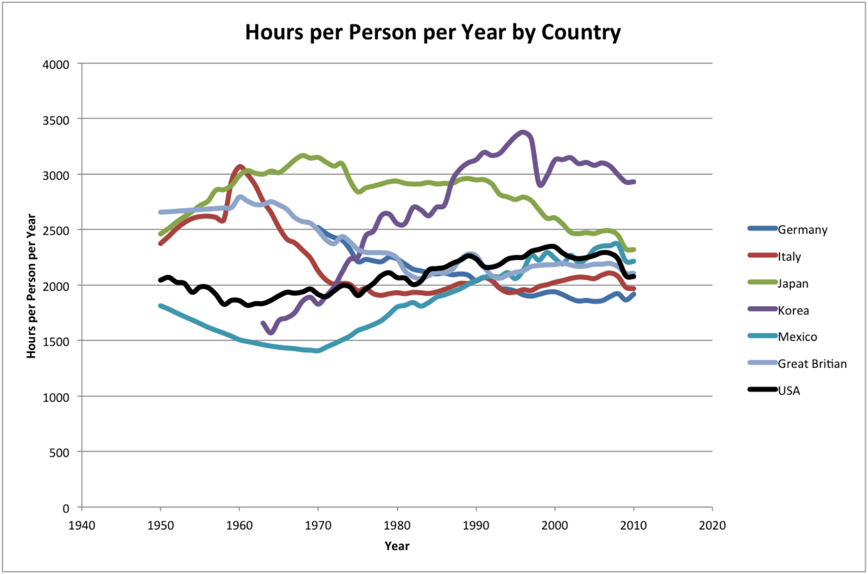


Labor hours



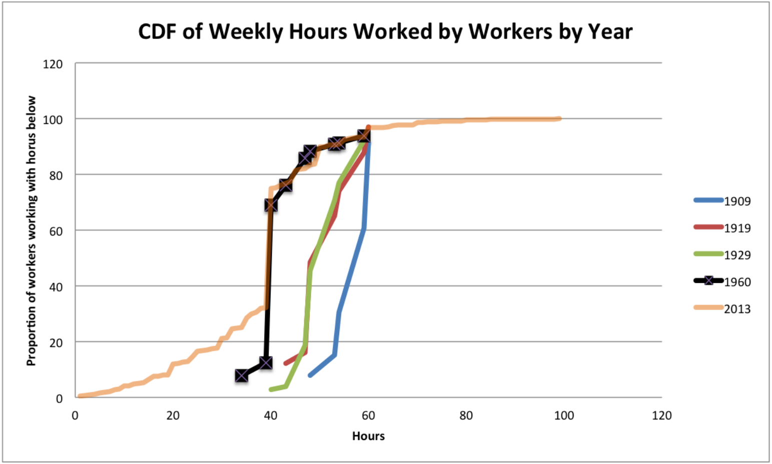
Labor Force Participation by Age over Time
Labor Force Participation Rate
120
100
80
60
40
20
0
0
10
20
30
40
50
60
70
80
Age
1890
1920
1960
2013

Labor hours



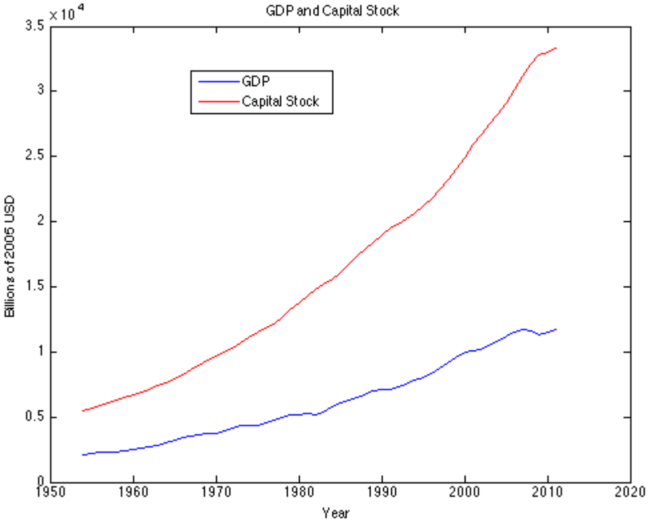
Hours per Person per Year by Country
Hours per Person per Year
4000
3500
3000
2500
2000
1500
1000
500
0
1940
1950
1960
1970
1980
1990
2000
2010
2020
Year
Germany
Italy
Japan
Korea
Mexico
Great Britian
USA

Labor hours

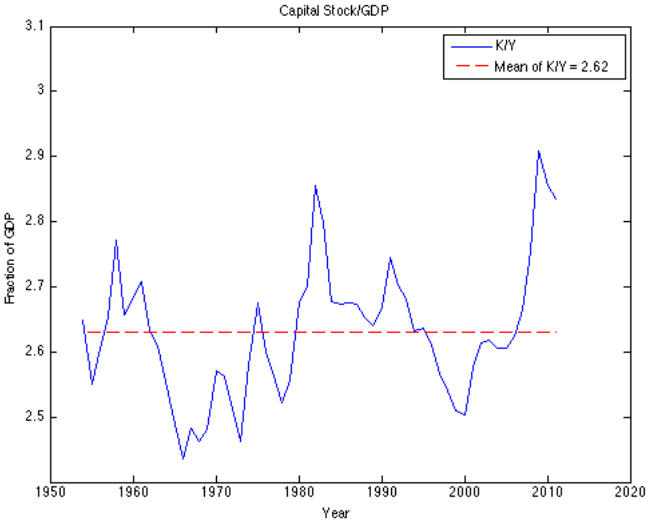


CDF of Weekly Hours Worked by Workers by Year
Proportion of workers working with horus below
120
100
80
60
40
20
0
0
20
40
60
80
100
120
Hours
1909
1919
1929
1960
2013

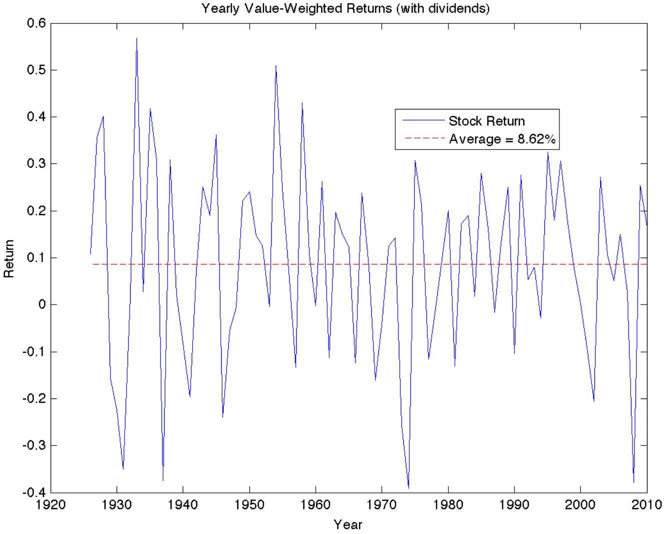
GDP and Capital Stock



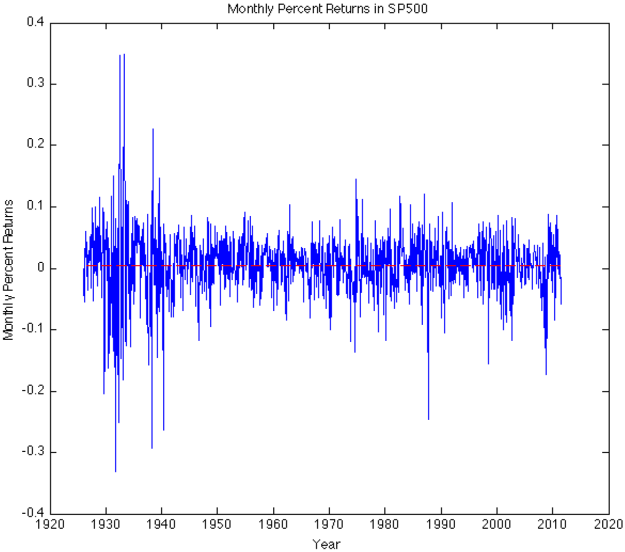
GDP and Capital Stock



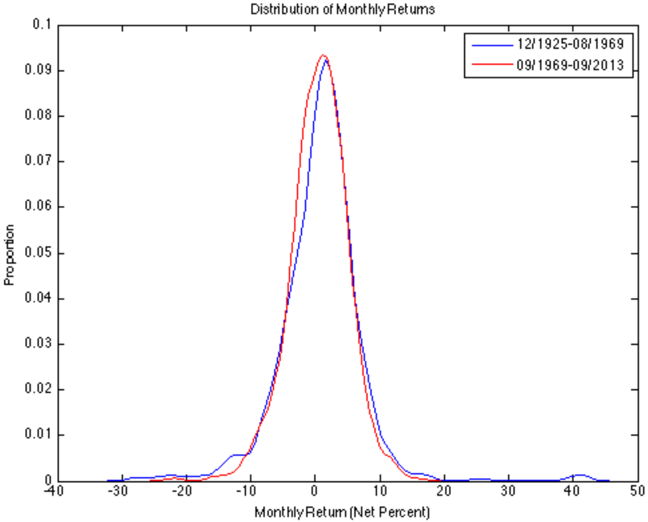
Interest Rates-I



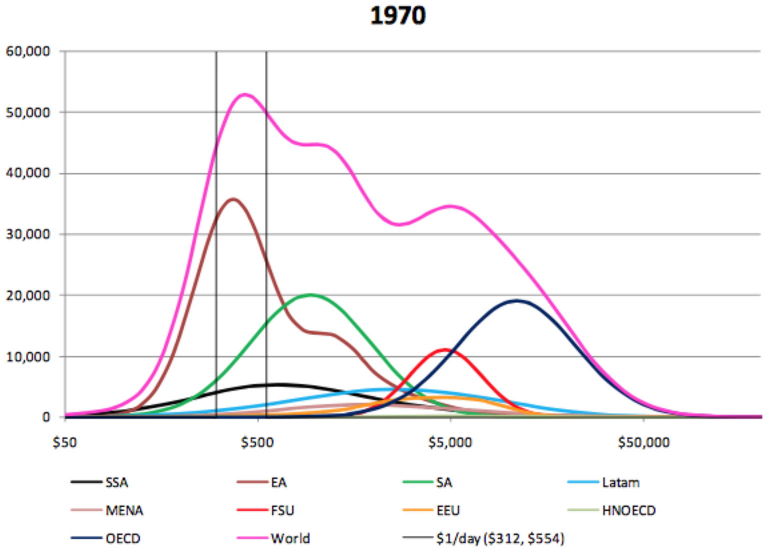
Interest Rates-II



Interest Rates-III

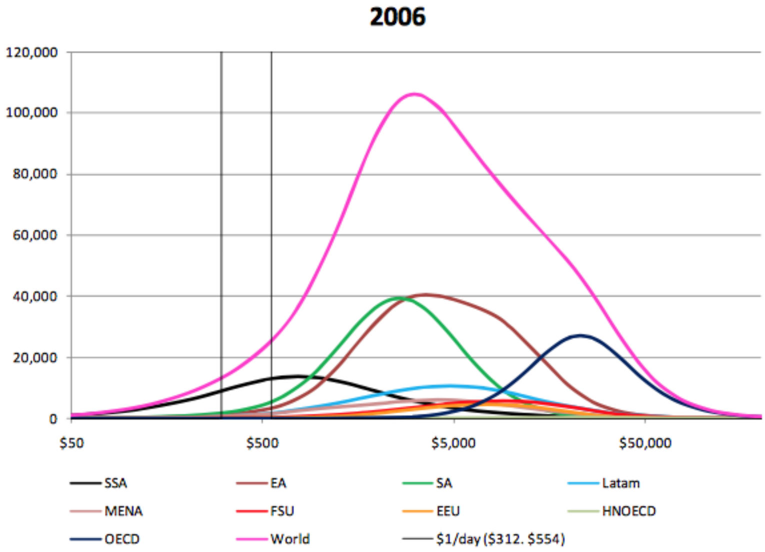


World Income Distribution: 1970



Pinkovskiy & Sali-i-Martin, 2009

World Income Distribution: 2006



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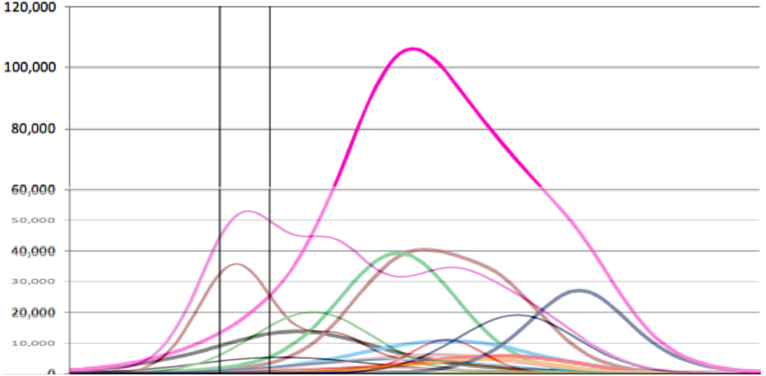
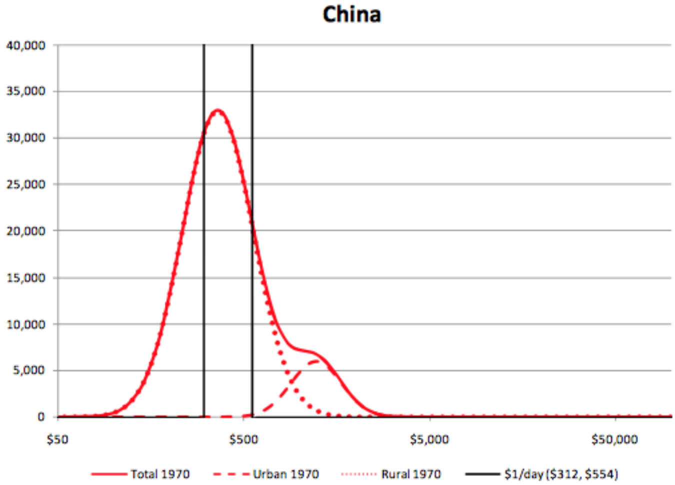
World Income Distribution: 1970 & 2006



Pinkovskiy & Sali-i-Martin, 2009

Chinese Income Distribution: 1970



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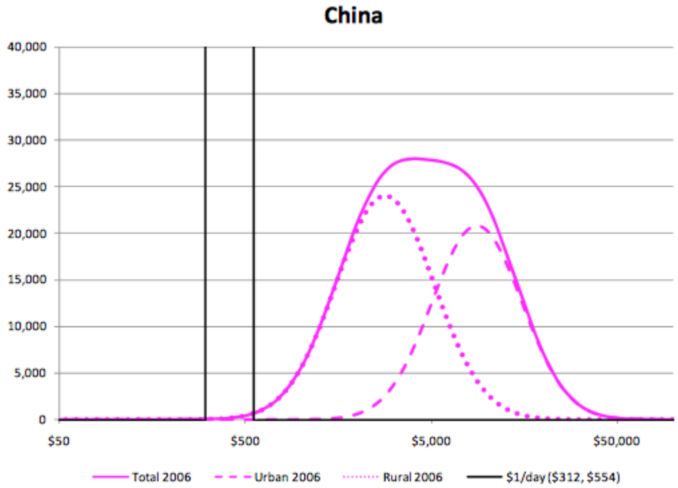
Chinese Income Distribution: 2006



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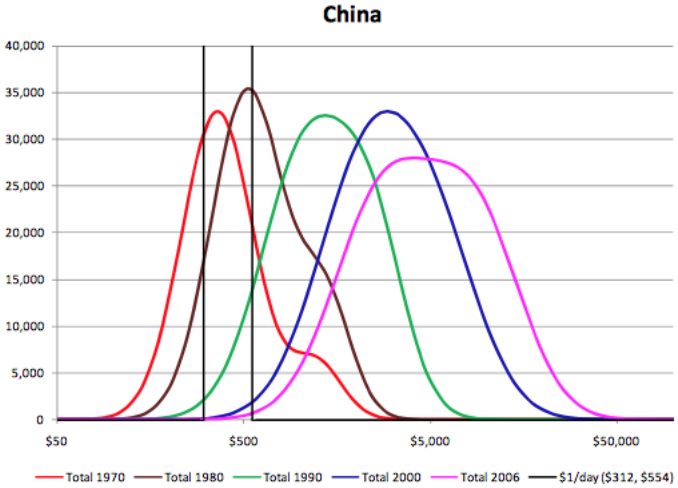

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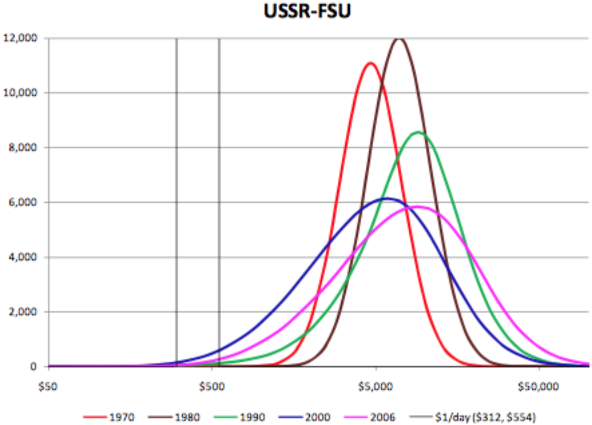
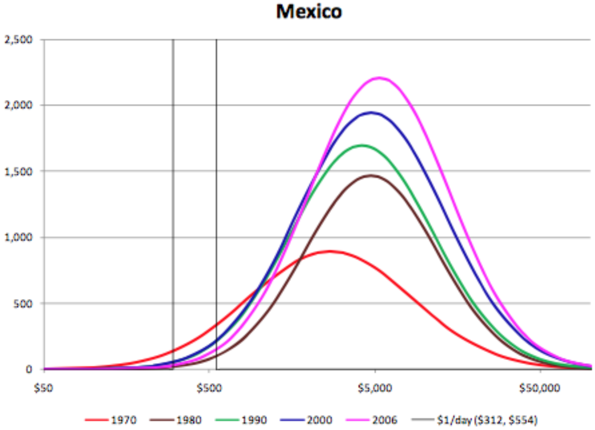
Soviet Income Distributions



Pinkovskiy & Sali-i-Martin, 2009

Mexican Income Distributions



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U.S. Growth Rates

Years	Growth Rate	GDP/Capita
1800-1820	0.24	2,159
1820-1840	1.09	2,682
1840-1860	1.42	3,555
1860-1880	1.77	5,052
1880-1900	1.26	6,490
1900-1920	1.54	8,809
1920-1940	1.17	11,121
1940-1960	2.43	17,974
1960-1980	2.50	29,475
1980-2000	2.20	45,538
2000-2010	1.41	50,857

1990 G-K, inflated to 2010.

- After start of Industrial Revolution, fairly constant over long periods.

Some Takeaways

1. World income distribution is very uneven
2. Most inequality comes from between-country inequality, not within-country
3. Advanced countries seem to grow at roughly the same rate
4. Some countries catch up to that
5. Consumption, investment, etc. are fairly constant
6. Growth is small, but has grown
7. Labor's share has been constant, compared to growth
8. Capital vs. GDP ratio has been constant, compared to growth
9. Labor hours have been constant, compared to wages
10. Interest rates are volatile, but stationary
11. U.S. growth from 1800-2010 has been about 1.5% per year
12. This has generated a per-capita increase in GDP of 25x
13. Labor hours haven't gone down over last 60 years in U.S.
    - But they have in Europe...
    - And they have for the elderly and young...
    - And they have for men...
    - And they have for workers...

Lots of facts!

1. Labor is constant: $L_t = \bar{L}$
2. Wages as a fraction of GDP are constant:
$$\frac{w_t L_t}{Y_t} = s_L$$
3. GDP grows at a constant rate: $Y_{t+1} = (1+\gamma) Y_t$
4. The interest rate is a constant: $r_t = \bar{r}$
5. Capital as a fraction of GDP is constant
$$\frac{K_t}{Y_t} = \bar{\Theta}$$
6. Capital income as a fraction of GDP is constant:
$$\frac{r_t K_t}{Y_t} = s_K$$
7. Growth rates between advanced countries are constant:
$$\frac{Y_{i,t+1}}{Y_{i,t}} = \frac{Y_{j,t+1}}{Y_{j,t}}, \quad j \neq i$$

Let's start Macro...

- We want to build a model to explain all these facts together
- Note nothing in the above should be able to explain business cycles because they're all "constant" growth relationships
- If we can use it to understand or even predict the outcome of business cycles as well as growth, this is a success.

Introduction to a Simple Model of Growth

Production Function-Intro

- Q: How do we summarize the production of five million firms all taking in different capital and labor types and producing different goods?
- A: “The” production function
- Assume a function that takes in capital and labor and spits out produced goods
- This will, and won’t, be as offensive as it sounds

Production Function-I

- We typically write the production function:

$$Y \;=\; A \;\cdot F(\;\; K \;\;,\;\; L \;\;)$$

- Y is production, GDP
- A is total factor productivity, productivity, or technology
- K is capital: all capital
- L is all labor hours
- Moreover, we frequently assume it to be "Cobb-Douglas"

$$Y = AK^{\alpha}L^{\beta}$$

- Where α and β are the contributions to GDP of capital and labor, respectively
- We'll see this will be equivalent to something Barro writes

Production Function-I

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Production Function-Cobb-Douglas

- The Cobb-Douglas production function is:

$$Y_t = A_t K_t^{\alpha} L_t^{\beta}$$

- Taking logs,

$$\begin{aligned}\log(Y_t) &= \log(A_t K_t^{\alpha} L_t^{\beta}) \\ &= \log(A_t) + \alpha \log(K_t) + \beta \log(L_t)\end{aligned}$$

- Taking differences,

$$\begin{array}{rcl}\log(Y_t) & = & \log(A_t) + \alpha \log(K_t) + \beta \log(L_t) ... \\ -\log(Y_{t-1}) & & -\log(A_{t-1}) + \alpha \log(K_{t-1}) + \beta \log(L_{t-1}) \\ & = & \log\left(\frac{A_t}{A_{t-1}}\right) + \alpha \log\left(\frac{K_t}{K_{t-1}}\right) + \beta \log\left(\frac{L_t}{L_{t-1}}\right) \\ \Delta^* Y_t & \approx & \Delta^* A_t + \alpha \Delta^* K_t + \beta \Delta^* L_t\end{array}$$

- Where Δ^* is "percent change in..."

Cobb-Douglas vs. Barro

- Our:

$$\Delta^* Y_t \approx \Delta^* A_t + \alpha \Delta^* K_t + \beta \Delta^* L_t$$

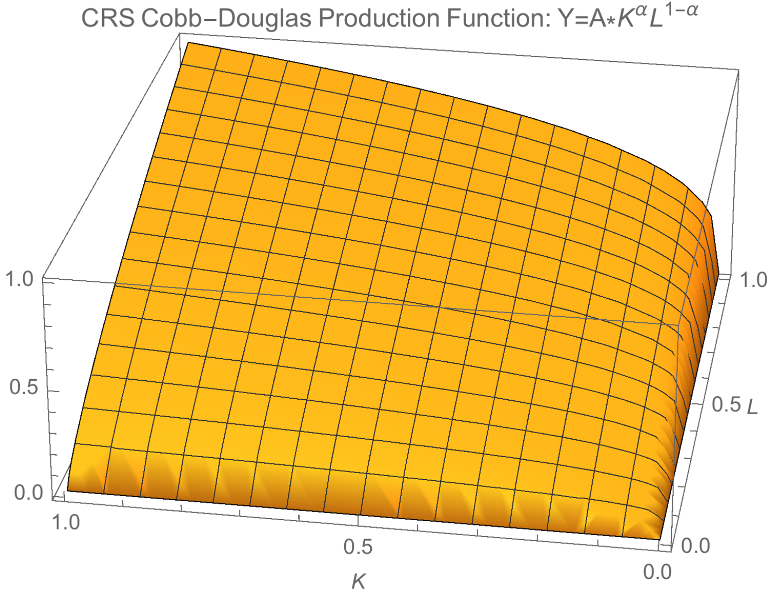
  is Barro's

$$\frac{\Delta Y}{Y} = \frac{\Delta A}{A} + \alpha \frac{\Delta K}{K} + \beta \frac{\Delta L}{L}$$

Cobb-Douglas vs. Barro

- If you're confused about what Barro says, use C-D. Everything will go through.
- Let's take a look at the Cobb-Douglas production function where $\alpha + \beta = 1$
- Three important pieces of information

Cobb-Douglas



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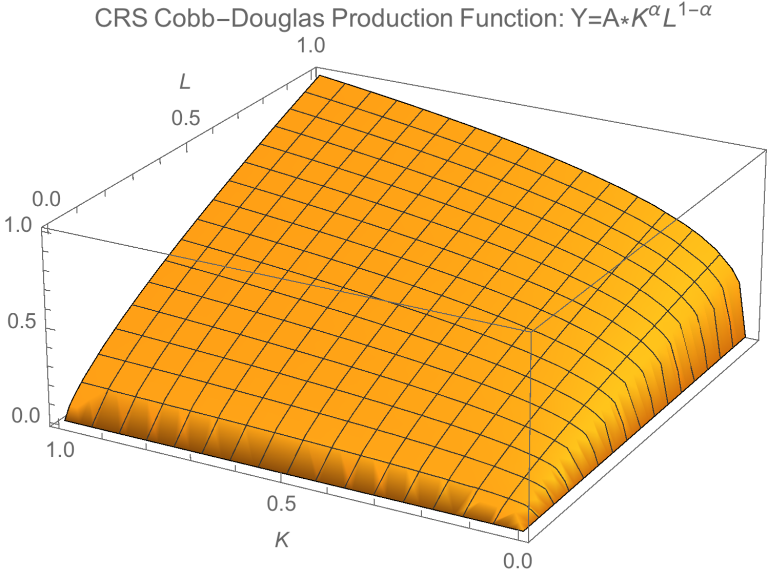

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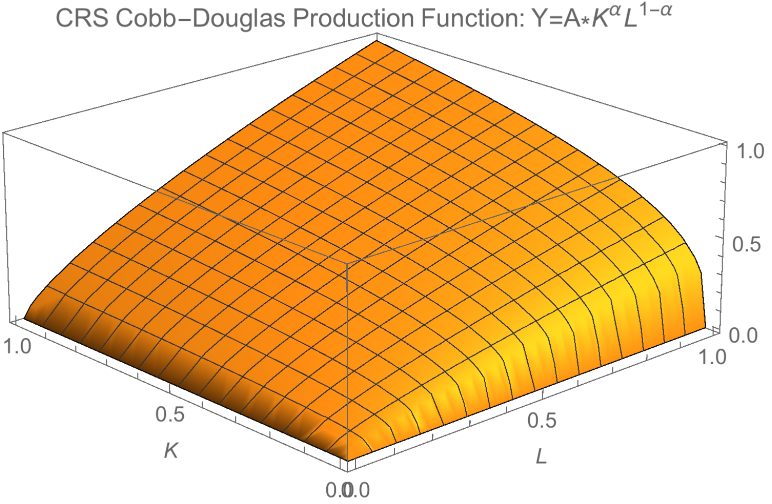

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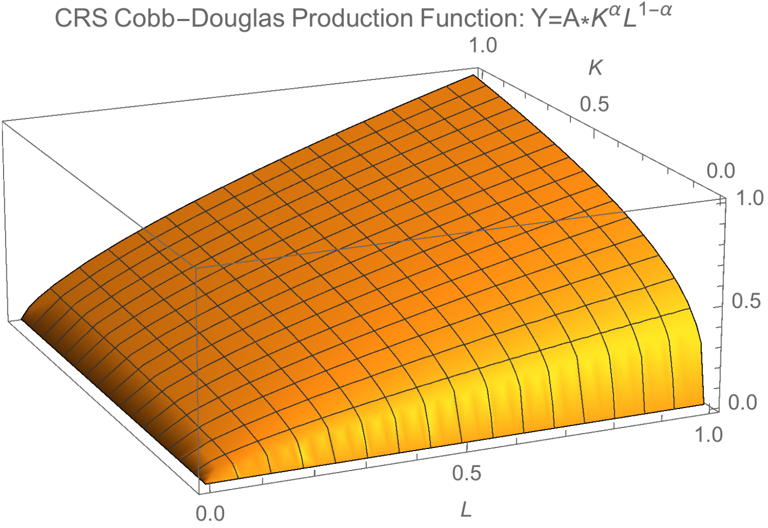

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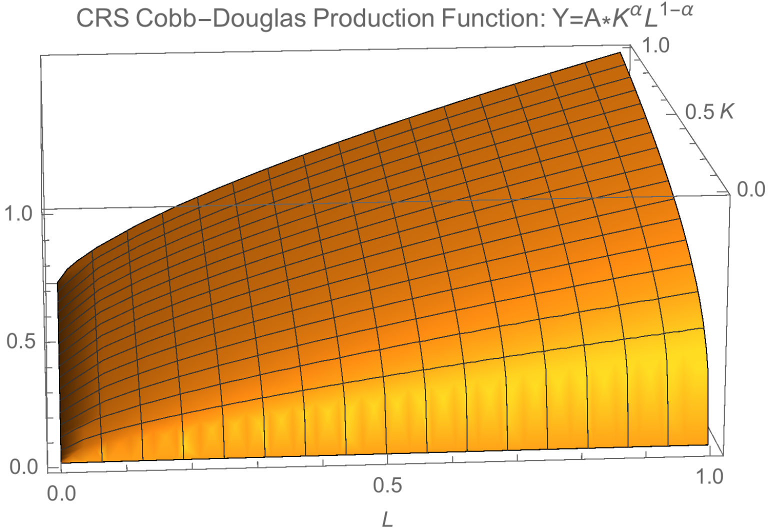

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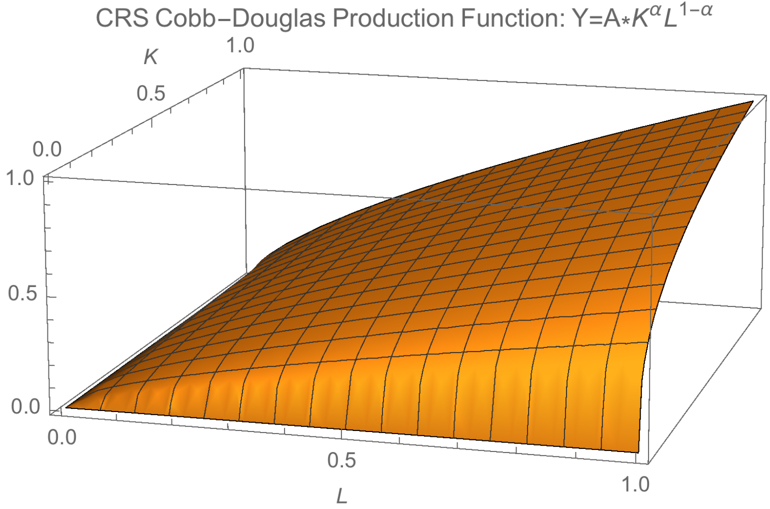

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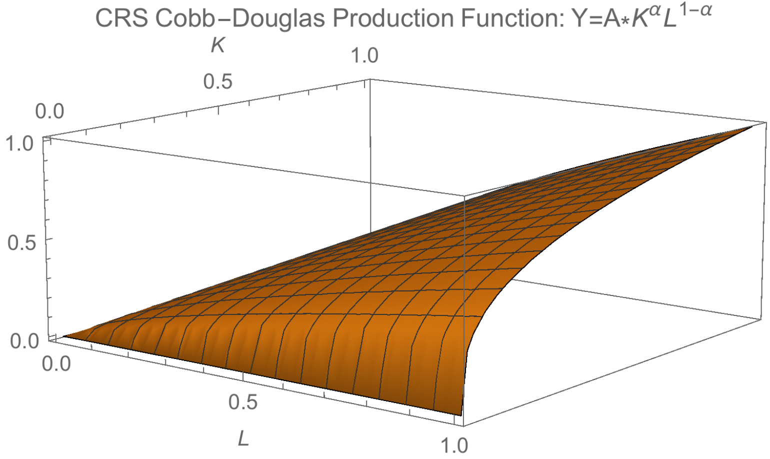

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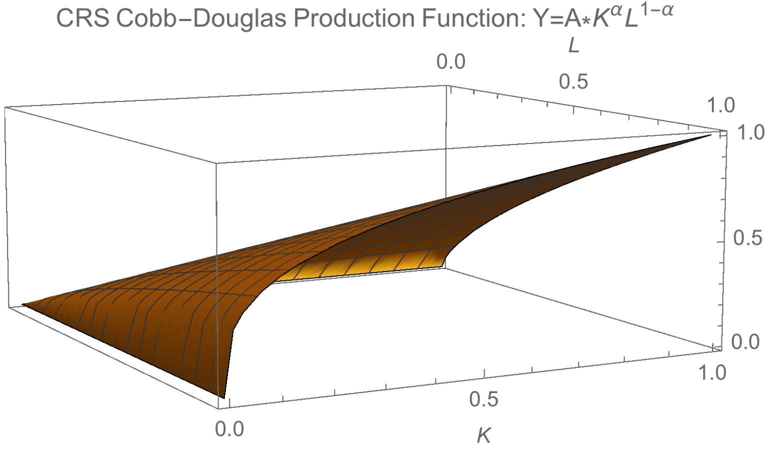

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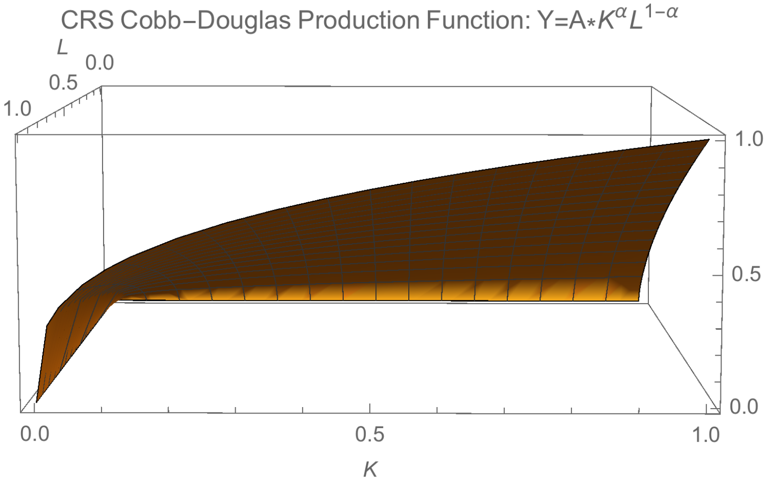

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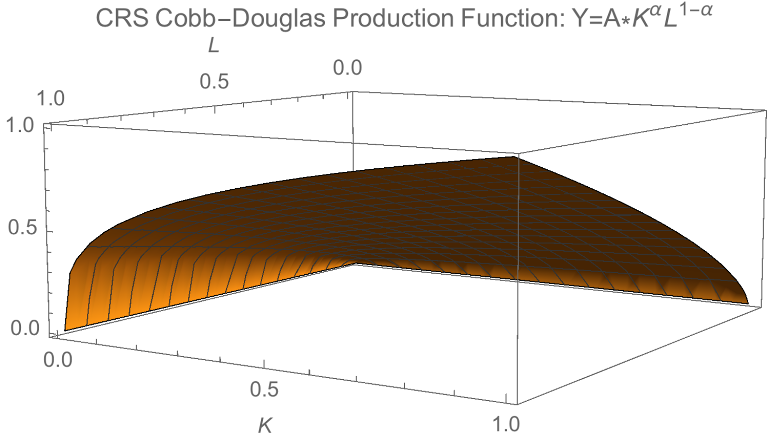

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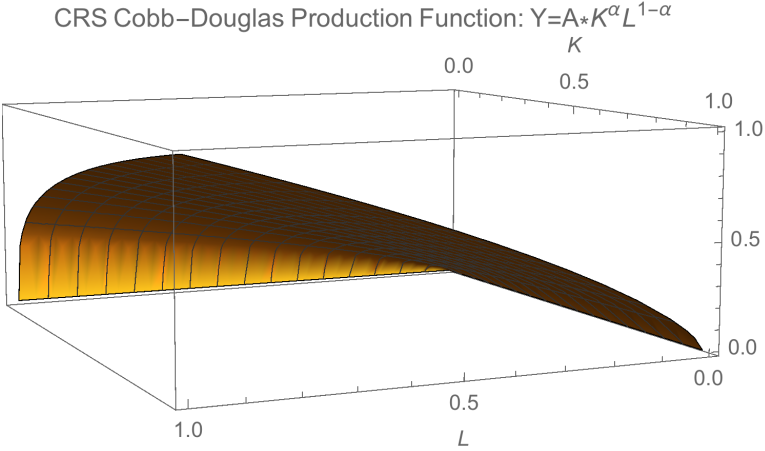

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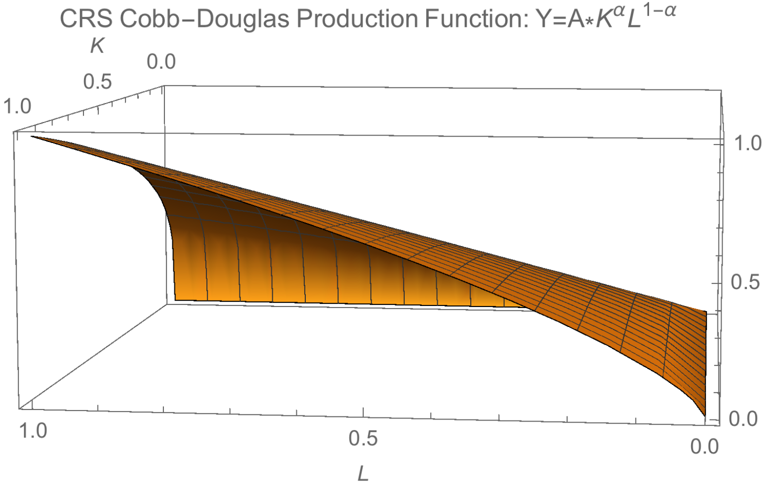

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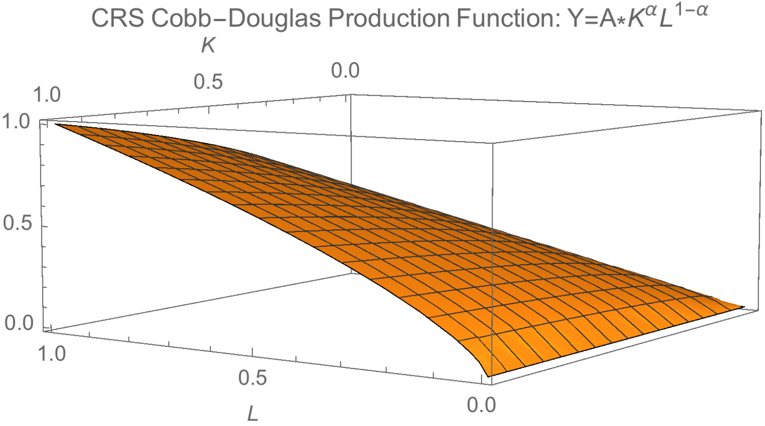

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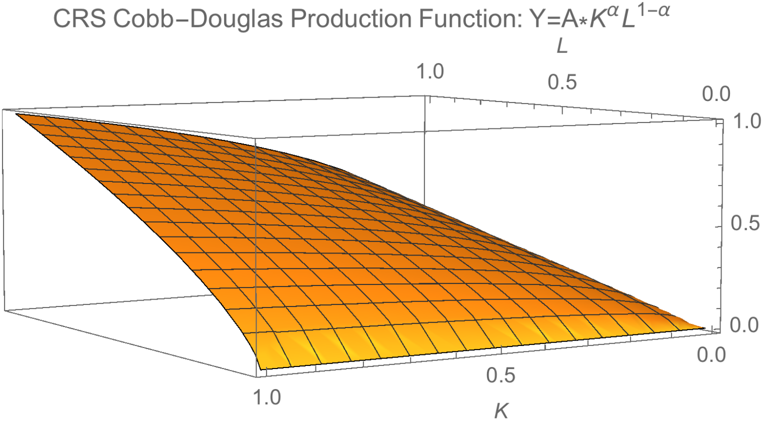

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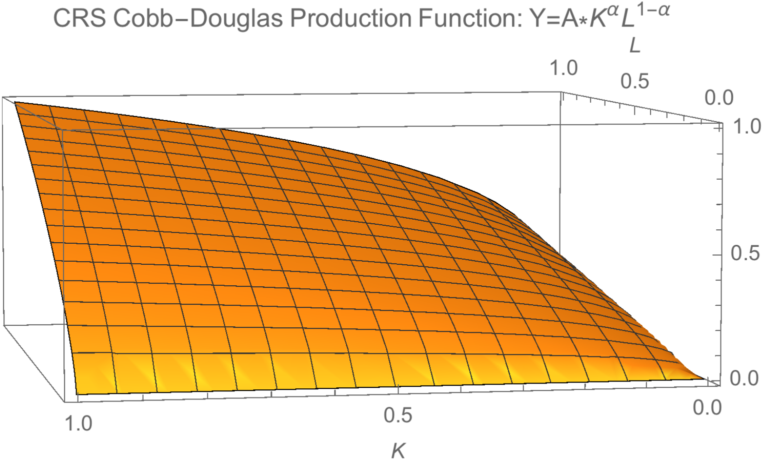

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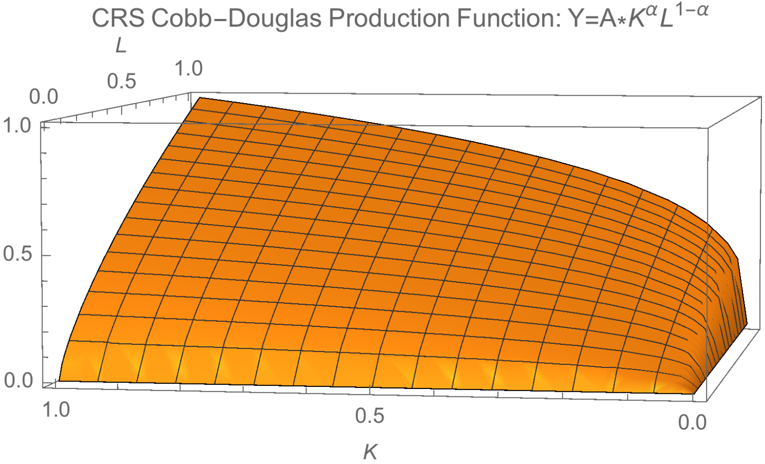

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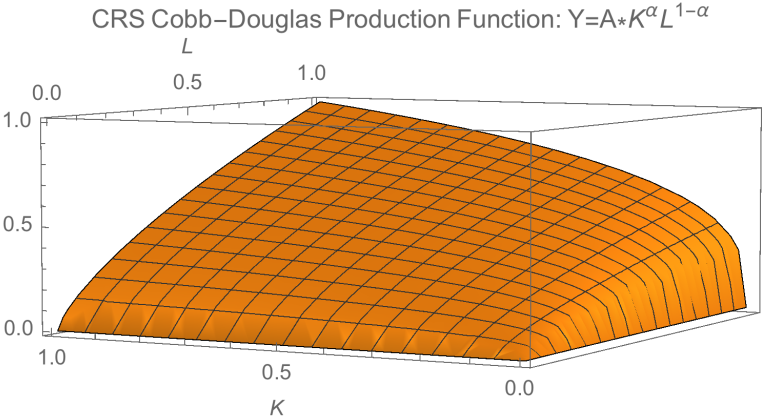
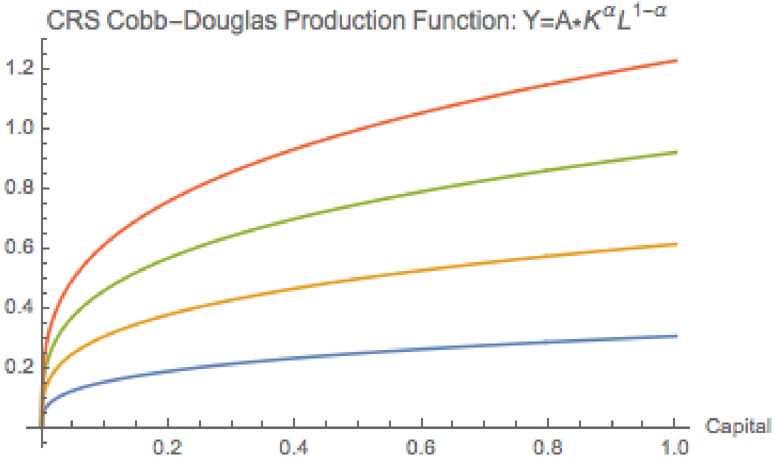
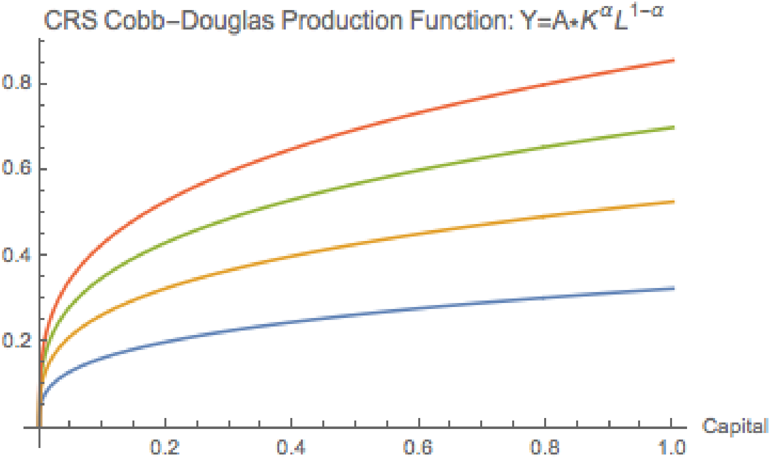


CD: L Constant, Raise A



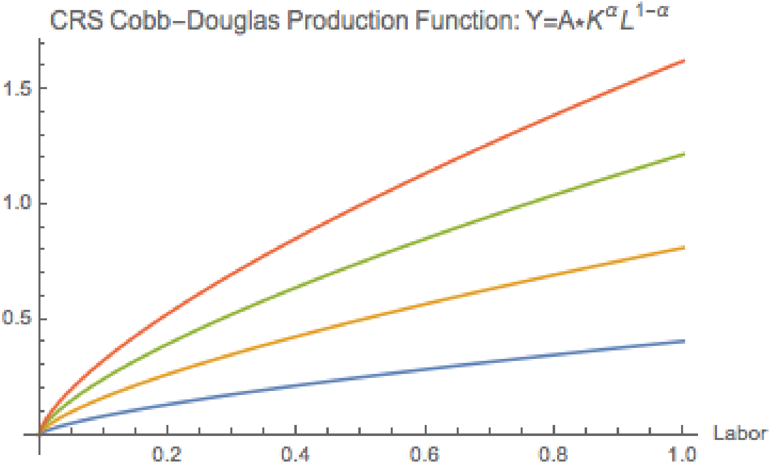
Note DRS as K rises, and A increases slope at a given K & increases level of Y at a given K.

CD: L Constant, Raise K



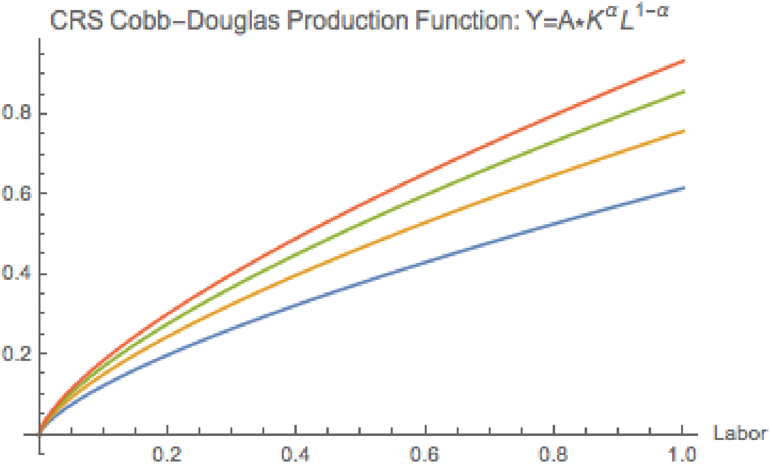
Note DRS as K rises, and L increases slope at a given K & increases level of Y at a given K.

CD: K Constant, Raise A



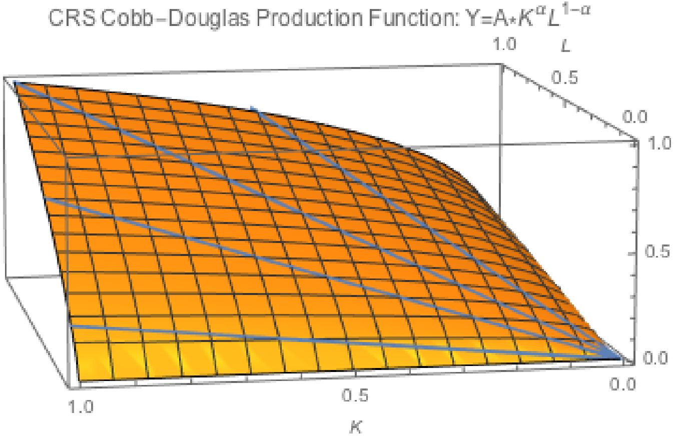
Note DRS as L rises, and A increases slope at a given L & increases level of Y at a given L.

CD: K Constant, Raise L



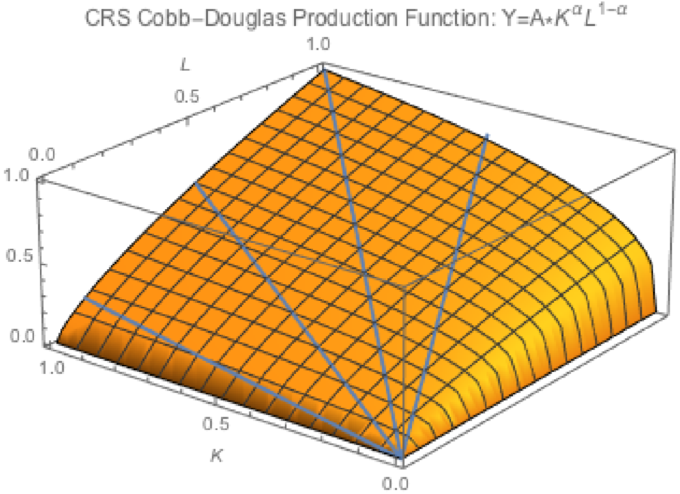
Note DRS as L rises, and K increases slope at a given L & increases level of Y at a given L.

CD: K/L Constant



Note that when K and L are increased in the same proportion, we are CRS.

CD: K/L Constant



Note that when K and L are increased in the same proportion, we are CRS.

Terminology

- For any given K, you could describe how a small change in L increases Y: this is the slope of Y with respect to L, holding K constant.
  - We can write this like: $\frac{Y(\bar{K},\bar{L})-Y(\bar{K},\bar{L}-\epsilon)}{\epsilon}$ for some small ϵ
  - Note that this is just $\left(\frac{\text{Rise}}{\text{Run}}\right)$
  - We call this slope the **Marginal product of labor** (MPL)
- For any given L, you could describe how a small change in K increases Y: this is the slope of Y with respect to K, holding L constant.
  - We can write this like: $\frac{Y(\bar{K},\bar{L})-Y(\bar{K}+\epsilon,\bar{L})}{\epsilon}$ for some small ϵ
  - We call this slope the **Marginal product of capital** (MPK)

Think about our pictures again

- Holding L and K constant, an increase in A increases the MPL and MPK
- Holding L constant, an increase in K increases the MPL and decreases the MPK
- Holding K constant, an increase in L increases the MPK and decreases the MPL
- A doubling of L and K doubles production and holds MPK and MPL the same as before!

Rewriting Cobb-Douglas

- Recall our functional form:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

- We can divide both sides by L to get:

$$\frac{Y_t}{L_t} = A_t \left(\frac{K_t}{L_t}\right)^{\alpha} \left(\frac{L_t}{L_t}\right)^{1-\alpha}$$

- Or, simplifying and writing per-unit labor as lowercase:

$$y_t = A_t k_t^{\alpha}$$

- This is really saying that it's all about the *per worker*, nothing special about larger populations

Growth Accounting

- Why are poor countries poor?
- Why is some Y high, some Y low?
- Cobb-Douglas points to big possibilities:
  - Poor countries are poor because they don't work (low L)
  - Poor countries are poor because they don't have capital (low K)
  - Poor countries are poor because they aren't productive (low A)
- Let's see

Growth Accounting-Roadmap: Next 10 Slides

1. Get an expression for percentage change in GDP in terms of percentage change in capital and percentage change in labor

2. Use that to get an expression for percentage change in GDP/worker as a function of percentage change in capital per worker

3. Combine savings behavior and capital's law of motion to get the percentage change in capital per worker as a function of current GDP, savings, and current capital.

4. Assume a growth rate of labor supply and find the growth rate of capital per worker

5. This gives us how GDP will grow per worker in the Solow Growth Model

Growth Accounting-I

- Recall that[1]

$$\Delta^* Y_t \approx \Delta^* A_t + \alpha \Delta^* K_t + \beta \Delta^* L_t$$

- If we are to declare Cobb-Douglas to be CRS, it must be that doubling both K_t and L_t doubles Y_t, holding A constant:

$$2 \approx 0 + 2\alpha + 2\beta$$

- Which gives:

$$\alpha + \beta = 1$$

- So CRS requires that the two exponents add to one
- Barro discusses how these mean that payments to factor inputs exhaust product.
- Let's understand why

[1] Where again, Δ^* are percentage changes!

Aside-I

- Let's imagine a firm has access to the production function:

$$Y_t(K_t, L_t) = A_t K_t^{\alpha} L_t^{1-\alpha}$$

- And has to pay labor wages wL_t and capital rental rates rK_t, so profit is:

$$\begin{aligned}\pi_t(K_t, L_t) &= Y_t(K_t, L_t) - w_t L_t - r_t K_t \\ &= A_t K_t^{\alpha} L_t^{1-\alpha} - w_t L_t - r_t K_t\end{aligned}$$

- Then first order conditions $\left(\frac{\partial \pi}{\partial K_t} = 0, \quad \frac{\partial \pi}{\partial L_t} = 0\right)$ will imply that:

$$(1-\alpha) = \frac{w_t L_t}{Y_t} \qquad \alpha = \frac{r_t K_t}{Y_t}$$

- Let's see why

Aside-II

$$(1-\alpha)A_t K_t^{\alpha} L_t^{-\alpha} - w_t = 0 \qquad \alpha A_t K_t^{\alpha-1} L_t^{1-\alpha} - r_t = 0$$

So:

$$(1-\alpha)\frac{L_t}{L_t} A_t K_t^{\alpha} L_t^{-\alpha} = w_t \qquad \alpha \frac{K_t}{K_t} A_t K_t^{\alpha-1} L_t^{1-\alpha} = r_t$$

Or:

$$(1-\alpha)\frac{Y_t}{L_t} = w_t \qquad \alpha \frac{Y_t}{K_t} = r_t$$

Rearranging:

$$(1-\alpha) = \frac{w_t L_t}{Y_t} \qquad \alpha = \frac{r_t K_t}{Y_t}$$

This is saying all production goes to someone! In this, we won't worry about entrepreneurs, they're either capital, labor, or both.

Aside-III

Recall that:

$$\log(x) - \log(\bar{x}) \approx \frac{x - \bar{x}}{\bar{x}}$$

For small deviations of x from $\bar{x}$.

Then:

$$\begin{aligned}
\Delta^* \left(\frac{y}{x}\right) &= \log\left(\frac{y}{x}\right) - \log\left(\frac{\bar{y}}{\bar{x}}\right) \\
&= \log(y) - \log(x) - \log(\bar{y}) + \log(\bar{x}) \\
&= \log\left(\frac{y}{\bar{y}}\right) - \log\left(\frac{x}{\bar{x}}\right) \\
&= \Delta^* y - \Delta^* x
\end{aligned}$$

That is, for small changes, the percentage change in a ratio is the percentage change in the numerator divided by the percentage change in the denominator.

Growth Accounting-II

- With our asides in mind, let's look at $\Delta^* y$, $y = \frac{Y}{L}$:

$$\Delta^* y \approx \Delta^* Y - \Delta^* L$$

- Similarly, for $k = \frac{K}{L}$,

$$\Delta^* k \approx \Delta^* K - \Delta^* L$$

- In other words, changes in GDP/worker comes either from changes in GDP, or changes in labor. Similarly, changes in capital per worker comes either from changes in capital, or changes in workers.
- Let's apply this

Growth Accounting-III

- We have, assuming no changes in productivity, that:

$$\Delta^* Y_t \approx \alpha \Delta^* K_t + (1 - \alpha)\Delta^* L_t$$

- or:

$$\Delta^* Y_t \approx \alpha \Delta^* K_t + \Delta^* L_t - \alpha \Delta^* L_t$$

- Which becomes

$$\Delta^* Y_t - \Delta^* L_t \approx \alpha \left(\Delta^* K_t - \Delta^* L_t\right)$$

- Which becomes:

$$\Delta^* y_t \approx \alpha \Delta^* k_t$$

- Assuming no productivity growth, the percentage change in per-worker GDP is equal to the percentage change in per-worker capital!

Growth Accounting-IV

- Now we have an equation that relates GDP/worker as a function of capital/worker
- if the two line up, then we're done: we could state that rich countries are rich because they have a lot of productive capital
- So let's look at how we measure the capital stock

Growth Accounting-V

- We want to know what's driving changes in the stock of capital
- Assume that capital falls apart at the same rate δ
- That is, if $\delta = 0.01$, 1% of all capital fell apart each period
- An economy *makes* Y, and δK falls apart: net income is $Y - \delta K$
- So total real savings is: $s \cdot (Y - \delta K)$
- Where "s" is the real savings rate, the proportion of net income we save.

Growth Accounting-VI

- Net income $Y - \delta K$ either goes into consumption or real savings:

$$Y - \delta K = C + s(Y - \delta K)$$

- In a closed economy/no government, everything produced is either consumed or invested:

$$Y = C + I$$

- Combining the two, we get that:

$$\underbrace{I - \delta K}_{\text{Net investment}} = \underbrace{s(Y - \delta K)}_{Realsavings}$$

- This is a **crucial** insight!
- Demand=Supply
- Markets must clear
- There is no savings without investment

Growth Accounting-VII

- We have the relationship:

$$I - \delta K = s(Y - \delta K)$$

- We define the change in capital as:

$$\Delta K = I - \delta K$$

- We can put the two together:

$$\Delta K = s(Y - \delta K)$$

- Or, dividing by K, we get:

$$\Delta^* K = s\frac{Y}{K} - s\delta$$

- Now we have the growth rate of capital in terms of savings, GDP, and current capital.

- Let's turn to $\Delta^* L$

Growth Accounting-VIII

- We assume constant population growth in workers, so that:

$$\Delta^* L = n$$

- Recall that we can write the percentage change in per worker capital $\Delta^* k$ as:

$$\Delta^* k = \Delta^* K - \Delta^* L$$

- Plugging in our results and noting $Y/K = y/k$:

$$\Delta^* k = s\frac{y}{k} - s\delta - n$$

- This is our core result, so:

$$\Delta^* y_t \approx \alpha \Delta^* k_t$$

- Becomes:

$$\Delta^* y_t \approx \alpha \left(s\frac{y}{k} - s\delta - n \right)$$

Growth Accounting-IX

- We have our key Solow equation:

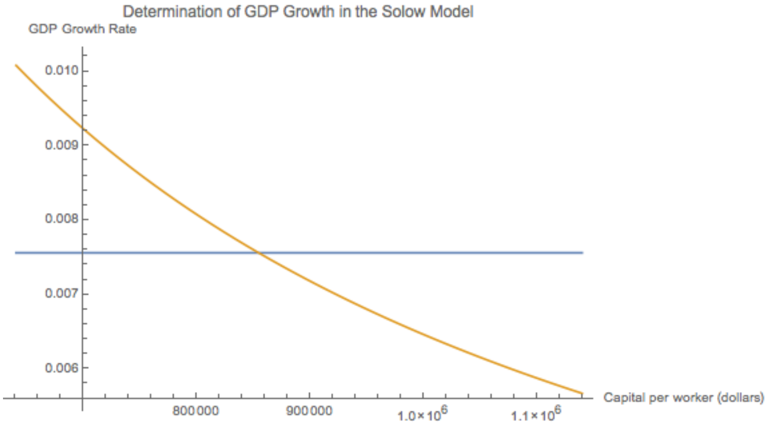
$$\Delta^* y_t \approx \alpha \left(s\frac{y}{k} - s\delta - n \right)$$

- Note that y/k is the *average* product of capital
- The average product of capital follows the marginal product of capital...
- So the average product of capital is diminishing as capital grows
- Let's graph out the two parts of the change in GDP/worker:

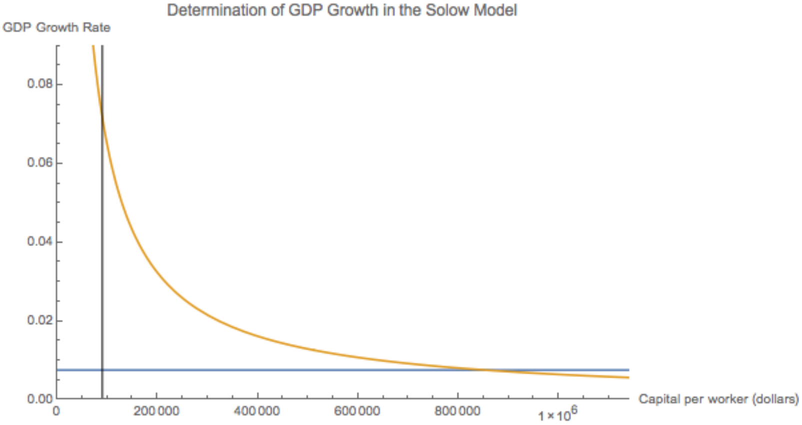
$$\alpha s\frac{y}{k} \quad vs. \quad \alpha(s\delta + n)$$

- For fun, $s = 0.19$, $\delta = 0.08$, $n = 0.01$, $\alpha = 0.30$, $y = 17$ trillion

Growth Accounting-X



Growth Accounting-XI



Growth Accounting-XII

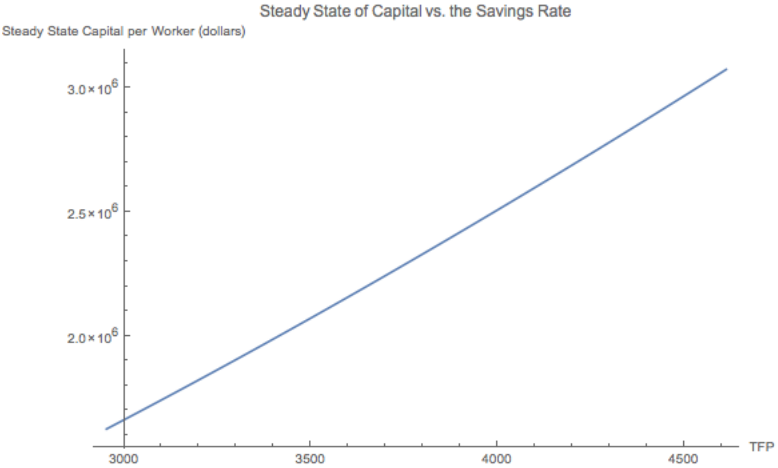
- Important points here
- First, understand the two influences:
  1. Depreciation of capital and population growth (decreasing the growth rate of GDP/capita) have a constant effect wrt how much capital there is
  2. Capital accumulation has a positive but declining effect increasing the growth rate of GDP/capita
  3. Where these two balance, we are at steady state, at k^* and therefore at y^*

Capital's Steady State-I

- There is a **steady state**, a level of capital after which all our savings are eaten up by depreciation
- We call the "steady" level of capital k^*.
- We can find it by setting growth equal to zero:

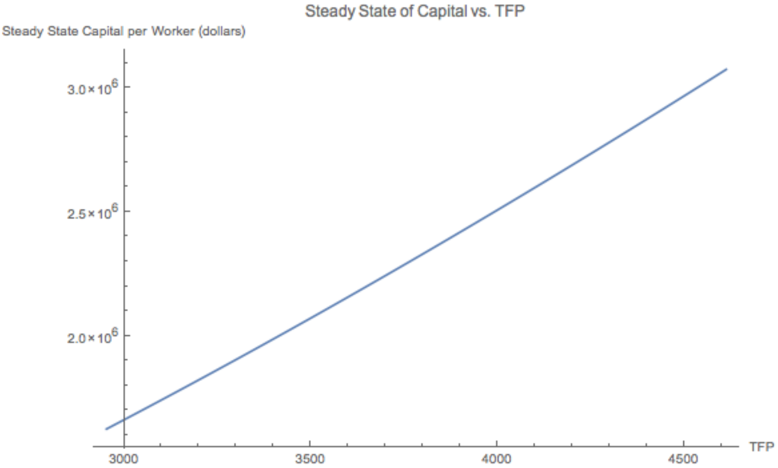
$$
\begin{aligned}
\Delta^* k &= s\frac{y^*}{k^*} - s\delta - n \\
0 &= s\frac{Ak^{*\alpha}}{k^*} - s\delta - n \\
k^* &= \left(\frac{s\delta + n}{sA}\right)^{\frac{1}{\alpha-1}}
\end{aligned}
$$

Capital's Steady State-II



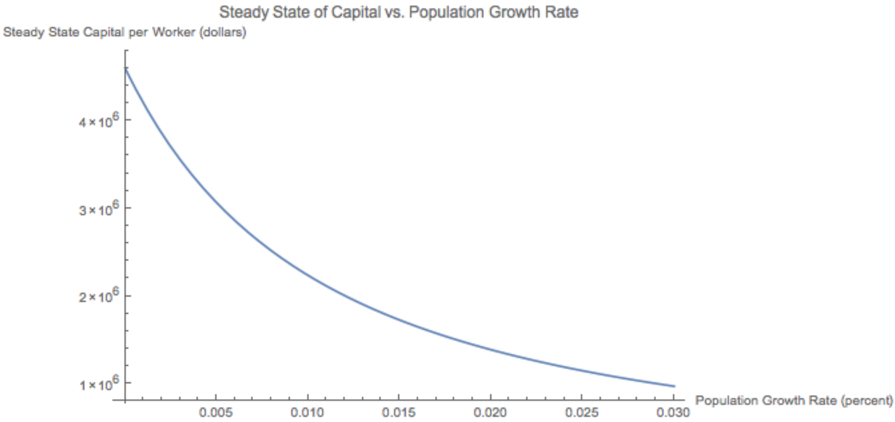
$$k^* = \left(\frac{s\delta + n}{sA}\right)^{\frac{1}{\alpha - 1}}$$

Capital's Steady State-III



$$k^* = \left(\frac{s\delta + n}{sA}\right)^{\frac{1}{\alpha - 1}}$$

Capital's Steady State-IIII



$$k^* = \left(\frac{s\delta + n}{sA} \right)^{\frac{1}{\alpha - 1}}$$

GDP/worker Steady State-I

- Note that GDP/capita closely mirrors capital per capita, with a slight change

Transitions to the Steady State-I

- We know what the steady state is
- How do we get there?
- Our whole economy is populated with unsurprised robots, so it's easy to calculate the path!
- Start with K_0, L_0, s, δ, and n
- Repeatedly plug these in to get evolution of paths

Transitions to the Steady State-II

- For Cobb-Douglas, we have:

$$\frac{y}{k} = \frac{Ak^{\alpha}}{k} = Ak^{\alpha-1}$$

- So our formula[2]:

$$\Delta^* k = s\frac{y}{k} - s\delta - n$$

- Becomes:

$$\Delta^* k = sAk^{\alpha-1} - s\delta - n$$

- The percentage change in k is decreasing in k

[2] Note I divide both Y and K by L

Transitions to the Steady State-III

- How long does it take to get half the distance to the steady state?
- We can show that the rate of convergence is controlled by δ and n

Transitions to the Steady State-IV

- Recall:

$$\Delta^* k = s\frac{y}{k} - s\delta - n$$

- Plugging in $y = Ak^\alpha$

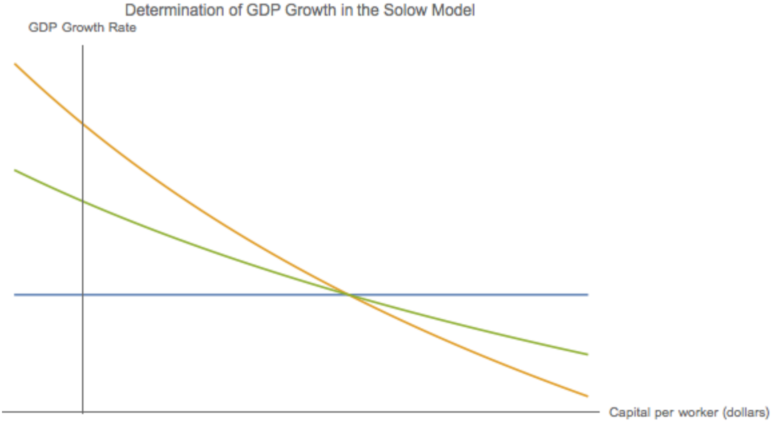
$$\Delta^* k = sAk^{\alpha-1} - s\delta - n$$

- Could alternatively write this as:

$$\frac{\partial \log k}{\partial t} = sA\exp(\log(k))^{\alpha-1} - s\delta - n$$

- This is a first-order differential equation that we can solve to get the time path of k wrt n, s, δ, A, and initial k.
- You don't have to know this

Capital's Steady State-IIII



What that causes convergence rates to differ here is α.

Applications of a Simple Model of Growth

Short Run and Long Run

- We have a framework for thinking about growth

$$k^* = \left(\frac{s\delta + n}{sA}\right)^{\frac{1}{\alpha-1}}$$

$$\Delta^* k = sAk^{\alpha-1} - s\delta - n$$

$$y = Ak^{\alpha}$$

$$c = (1-s)(Ak^{\alpha} - \delta k)$$

- We want to think about how changes in k, s, A, or n, will change consumption, production, and capital in the short and long run.

Shock to Capital

- What happens if, after being at k^*, we suddenly lose a bunch of capital?

$$k^* = \left(\frac{s\delta + n}{sA}\right)^{\frac{1}{\alpha-1}}$$

$$\Delta^* k = sAk^{\alpha-1} - s\delta - n$$

$$y = Ak^{\alpha}$$

$$c = (1-s)(Ak^{\alpha} - \delta k)$$

- In the long run, we know nothing has changed because k^* hasn't changed.

- In the short run, we can see that when capital goes down
  - $y \downarrow$
  - $y \downarrow \Rightarrow \Delta^* k \uparrow,\ c \downarrow,\ r \uparrow$

Short Run and Long Run

- We have a framework for thinking about growth

$$k^* = \left(\frac{s\delta + n}{sA}\right)^{\frac{1}{\alpha-1}}$$

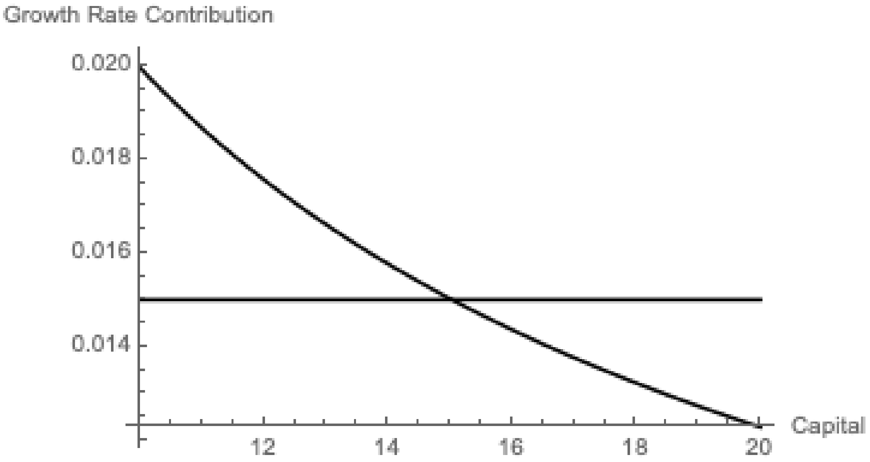
$$\Delta^* k = sAk^{\alpha-1} - s\delta - n$$

$$y = Ak^{\alpha}$$

$$c = (1-s)(Ak^{\alpha} - \delta k)$$

- We want to think about how changes in k, s, A, or n, will change consumption, production, and capital in the short and long run.

Shock to Level of Capital



Permanent Shock to Depreciation

- Depreciation moves from δ to δ', $\delta' < \delta$

$$k^* = \left(\frac{s\delta + n}{sA}\right)^{\frac{1}{\alpha - 1}}$$

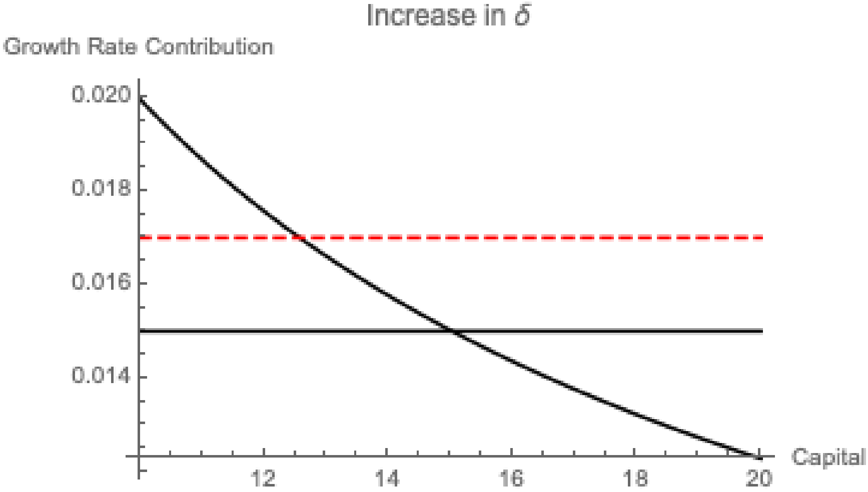
$$\Delta^* k = sAk^{\alpha-1} - s\delta - n$$

$$y = Ak^{\alpha}$$

$$c = (1 - s)(Ak^{\alpha} - \delta k)$$

- In long run, we now know the level of capital is lower
- We have "too much" capital and "too much" production
- Therefore, we save "too much" and have a lower level of capital tomorrow, higher than long run
- Higher depreciation means we converge more quickly to the steady state
- A period of declining consumption, declining capital, high interest rates

Permanent Shock to the Level of Depreciation



Permanent Shock to Savings Rate

- Savings moves from s to s', $s' > s$

$$k^* = \left(\frac{s\delta + n}{sA}\right)^{\frac{1}{\alpha-1}}$$

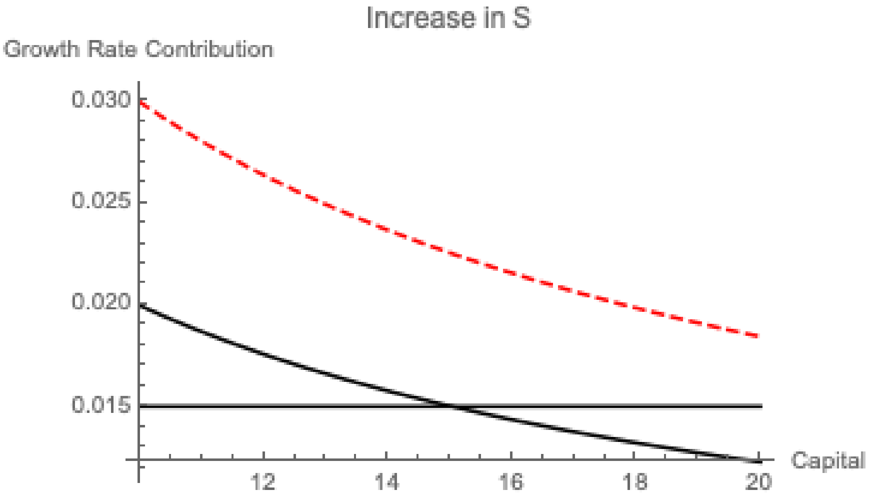
$$\Delta^* k = sAk^{\alpha-1} - s\delta - n$$

$$y = Ak^{\alpha}$$

$$c = (1-s)(Ak^{\alpha} - \delta k)$$

- In long run, we now know the level of capital is higher
- We have "too little" capital
- Our $\Delta^* k > 0$ compared to what it was
- Slowly converge to new steady state
- At first, lower consumption, then, higher consumption

Permanent Shock to the Level of Productivity



Permanent Shock to Labor Growth Rate

- Population growth moves from n to n', $n' > n$

$$k^* = \left(\frac{s\delta + n}{sA}\right)^{\frac{1}{\alpha-1}}$$

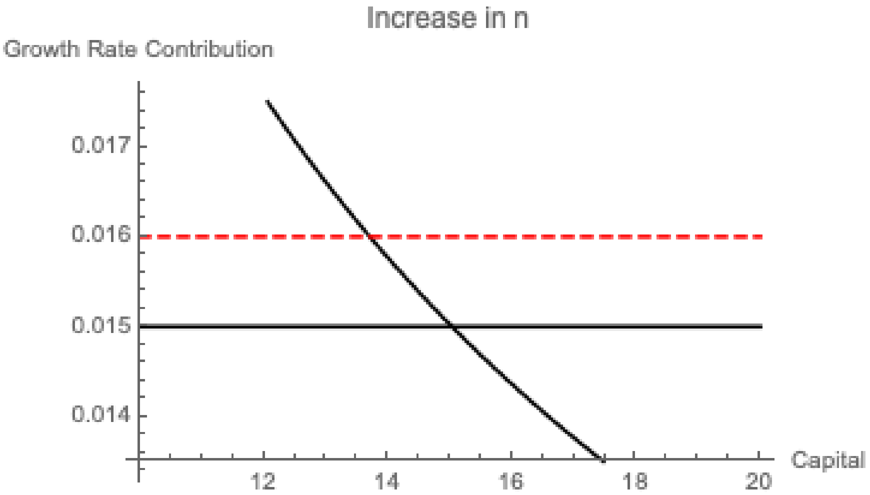
$$\Delta^* k = sAk^{\alpha-1} - s\delta - n$$

$$y = Ak^{\alpha}$$

$$c = (1-s)(Ak^{\alpha} - \delta k)$$

- In long run, capital (per worker) is lower
- We have "too much" capital
- Our choice of $\Delta^* k$ declines because n increases
- Very slowly converge to new steady state

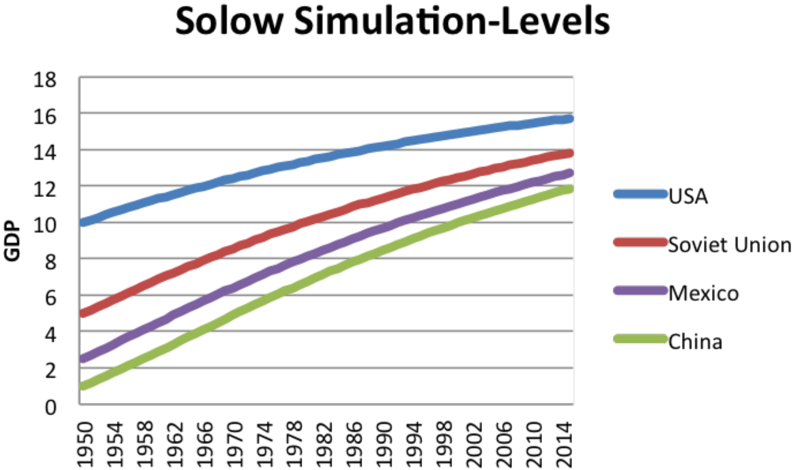
Permanent Shock to the Level of Productivity



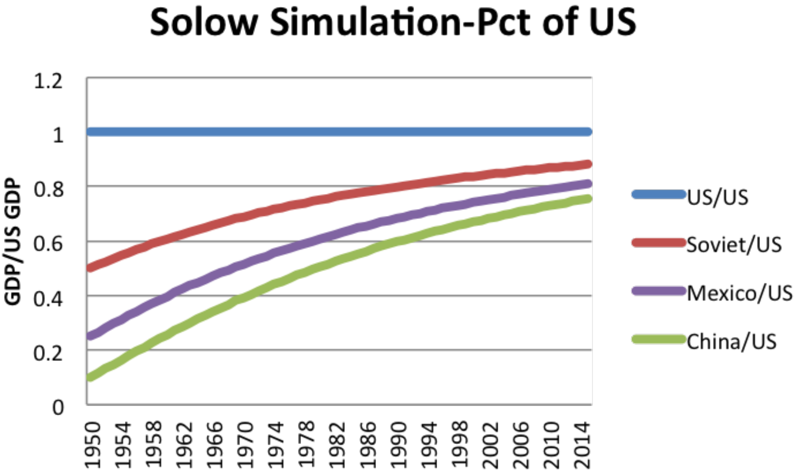
Convergence

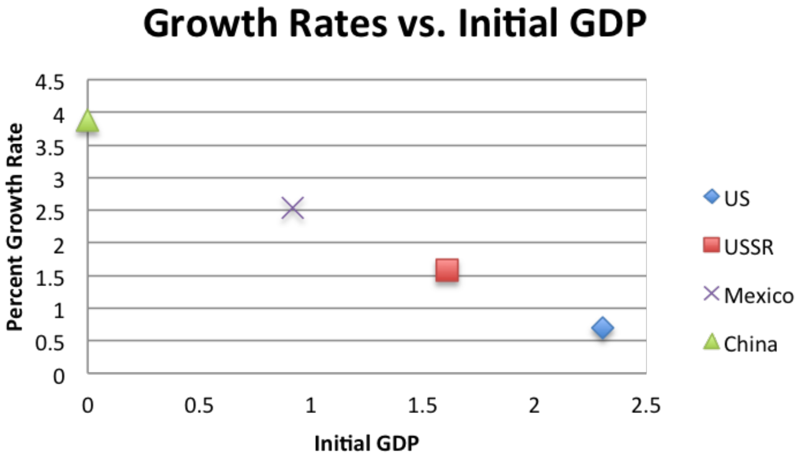
- $k^* = \left(\frac{s\delta+n}{sA}\right)^{\frac{1}{\alpha-1}}$

Parameter	k^*
s	$+$
A	$+$
n	-
δ	-
L(0)	0


Solow Simulation-Levels
GDP
0
2
4
6
8
10
12
14
16
18
1950
1954
1958
1962
1966
1970
1974
1978
1982
1986
1990
1994
1998
2002
2006
2010
2014
USA
Soviet Union
Mexico
China

Initial Capital




Growth Rates vs. Initial GDP
Percent Growth Rate
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5
Initial GDP
US
USSR
Mexico
China

Question

- Question: What does it take for convergence to hold?
- You have to have all the same parameters! Savings, technology/laws, depreciation rate, population growth rate, etc.

Question: Does convergence hold in reality?

a) Never

b) Always

c) For some subsets of countries

Does Convergence Hold Up?

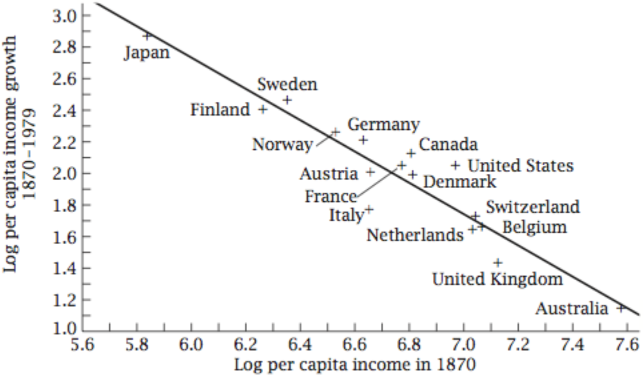


FIGURE 1.7 Initial income and subsequent growth in Baumol's sample (from DeLong, 1988; used with permission)

Romer figure 1.7

Q: What kind of countries will we have good historical data for?

Does Convergence Hold Up?

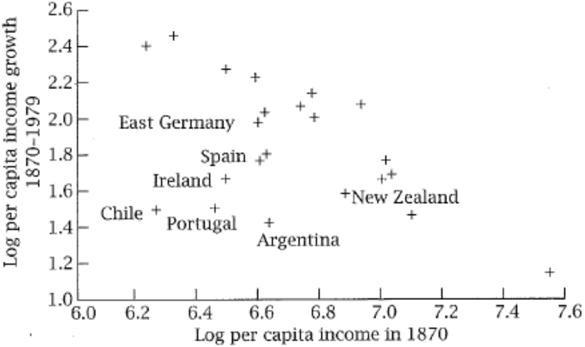


FIGURE 1.8 Initial income and subsequent growth in the expanded sample (from DeLong, 1988; used with permission)

Romer figure 1.8

Does Convergence Hold Up?

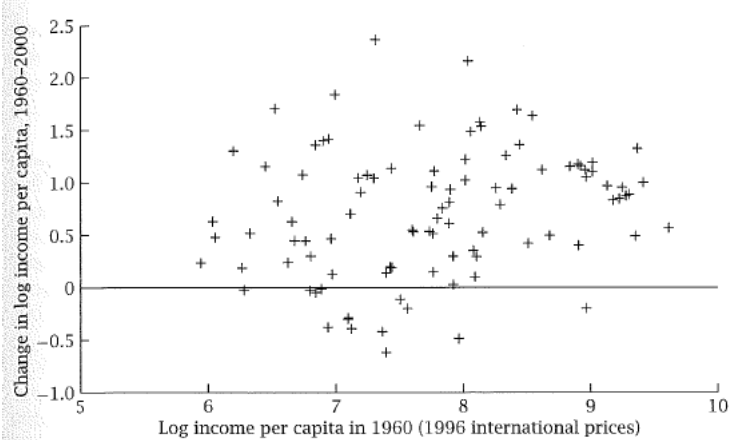


FIGURE 1.9 Initial income and subsequent growth in the postwar period

Romer (3rd ed) figure 1.9

Does Convergence Hold Up?

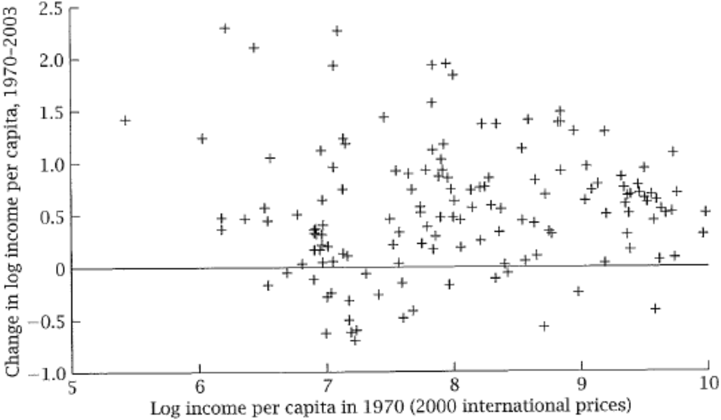
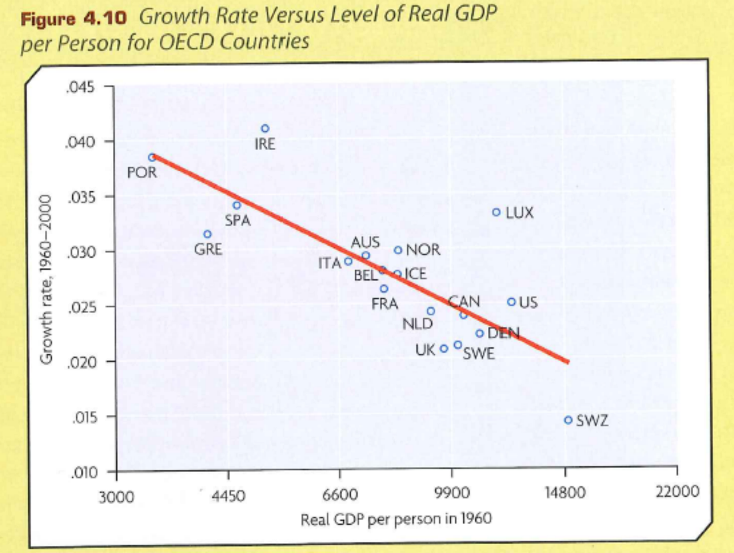


FIGURE 1.9 Initial income and subsequent growth in a large sample

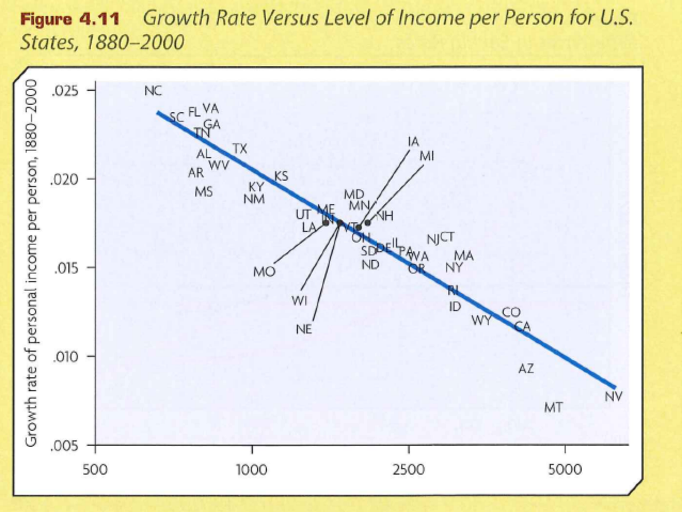
Romer (4th ed) figure 1.9

Does Convergence Hold Up?



Barro figure 4.10.

Does Convergence Hold Up?

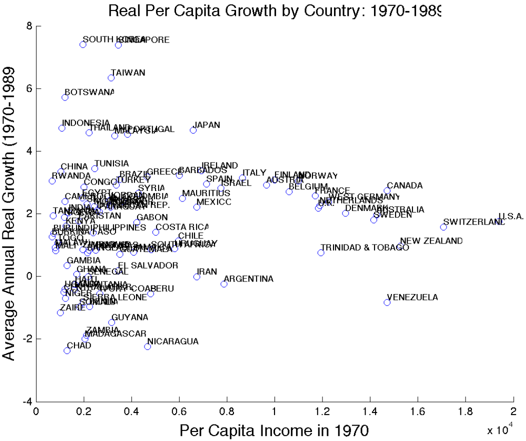


Barro figure 4.11.

Why Might Convergence Fail?

- Differences in everything we talked about, except initial capital!

An Explanation?

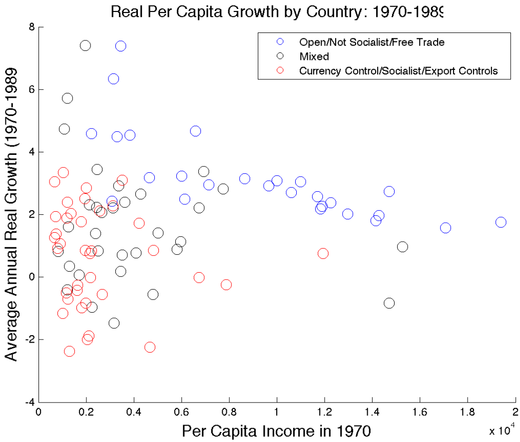


Sachs Warner 1995

An Explanation?

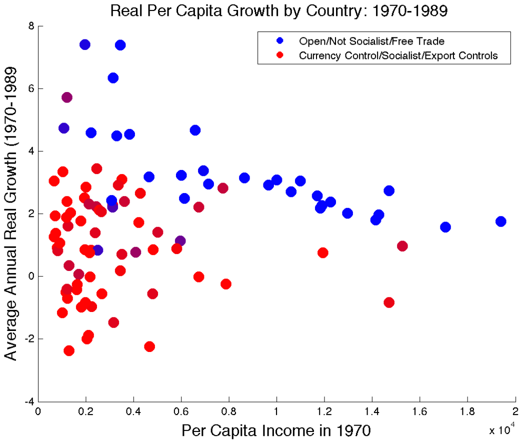
Sachs Warner 1995: Let's break things up by "free market"

An Explanation?



Sachs Warner 1995

An Explanation?



Sachs Warner 1995

An Explanation?

- Note that "socialism" isn't your father's socialism
- Typified by state ownership of labor, land, materials
- Typified by state planning and price setting
- What we might call with a broad brush "communism" or "command-and-control"
- **Not** what we mean by Northern Europe or original EU members

Conclusion

- We have our first scientific prediction from a model
  - Countries growth rates should be (roughly) linear in log-initial-capital
- Initial success based on bad data
- Failure looking at world at large
- Success looking at advanced/similar countries
- Success looking at states within the US
- Success looking at "free market" economies

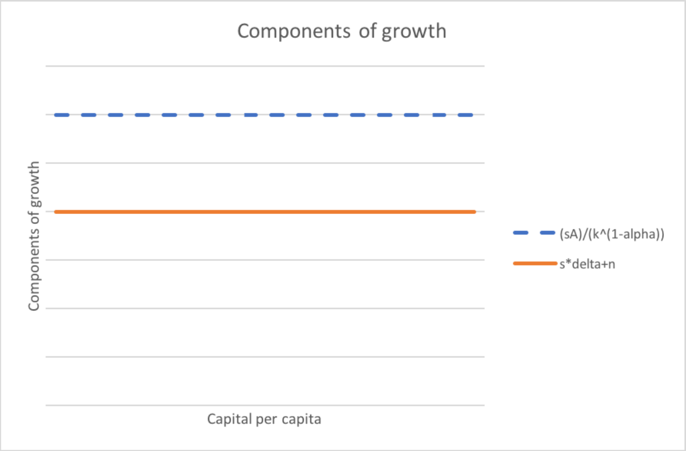
Introduction

- We have seen the Solow Growth model
- There were two important takeaways
- First, that diminishing returns bought us the concept of "convergence," that economies with the same parameters (depreciation, savings rate, technology/productivity) should in the long run converge to the same steady state GDP/capita
- Second, that it can't explain growth! People converge to a single steady state GDP/capita.
- Many might be confused by this second point...wasn't the point to explain growth?
- We did! We explained what *can't* explain growth (capital).

Moving on

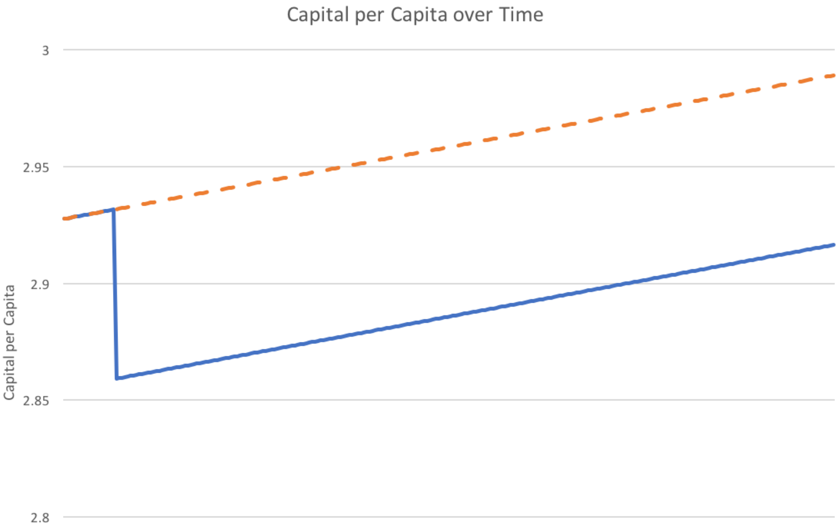
- So now what?
- How could we recover the Solow Growth model?
- One way is if $\alpha = 1$, constant returns to scale!
- Then we would have:

Constant Returns gives Constant Growth



Consequences?

Constant Returns gives Constant Growth



Logic

- We set out to explain growth using only capital accumulation
- But if it had decreasing returns to capital, it couldn't explain growth (but could explain catch-up)
- If it had constant returns to capital, it couldn't explain catch-up (but can explain growth)
- Conclusion: it's really about productivity, not about capital accumulation!

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