

# Week 2 Notes

Trevor Stepp

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## 1 Verifying GPT's Formulation

First note that the equation given by ChatGPT:

$$v_i(t) = \sum_{j=1}^m \sum_{k=1}^{K_j} c_{jk} s_i^j(t - t_{jk}) \quad (1)$$

does indeed match Eq. (1b) in Dr. Vinjamuri's paper.

### 1.1 Transforming Eq. (1)

Want to use Eq. (1) to arrive at:

$$\mathbf{v}^g = \mathbf{S} \mathbf{c}^g \quad (2)$$

Dr. Vinjamuri's paper tell us that the duration of the synergies,  $T_s$ , is 39 (Note that the paper actually uses  $t_s$ , but GPT uses  $T_s$  in its formulation, i.e.  $t_s = 39 = T_s$ ). Then the  $j$ th synergy with the  $i$ th joint can be written as a vector,  $\mathbf{s}_i^j \in \mathbb{R}^{T_s}$ .

The time shift  $t_{jk}$  in Eq. (1) must be accounted for, and GPT solves this problem by introducing a convolution matrix to shift  $s_i^j$  by  $t_{jk}$ , where  $\mathbf{D}_{jk} \in \mathbb{R}^{39 \times T_s}$ ,  $T_s \leq 39$ . Then  $\mathbf{D}_{jk} s_i^j$  is equivalent to  $s_i^j(t - t_{jk})$  in Eq. (1) (which is Eq. (1b) in Dr. Vinjamuri's paper).

Note that the paper considers only 10 joints, so the result is:

$$\mathbf{s}_j = \begin{bmatrix} s_1^j \\ s_2^j \\ \vdots \\ s_{10}^j \end{bmatrix} \in \mathbb{R}^{10T_s} \quad (3)$$

The matrix  $\mathbf{S}$  is constructed by horizontally stacking each  $\mathbf{D}_{jk} s_i^j$  (the time-shifted version of the  $i$ th joint's component of the  $j$ th synergy).

## 1.2 Checking Formulations are Identical

We must confirm that Equations (1) and (2) are identical formulations. Let  $i = 1$ ,  $m = 2$ ,  $K_1 = 1$ ,  $K_2 = 2$ ,  $T_s = 3$ ,  $T = 6$ ,  $t_{11} = 2$ ,  $t_{21} = 0$ ,  $t_{22} = 1$ ,  $c_{11} = 3$ ,  $c_{21} = 1$ ,  $c_{22} = 2$ . Let

$$s^1 = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \text{ and } s^2 = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$$

For Eq. (1), we have

$$c_{11}s^1(t - t_{11}) = c_{11}s^1(t - 2) = 3 [0, 0, -1, 2, 3, 0] = [0, 0, -3, 6, 9, 0],$$

$$c_{21}s^2(t - t_{21}) = c_{21}s^2(t - 0) = 1 [0, 1, -2, 0, 0, 0] = [0, 1, -2, 0, 0, 0],$$

$$c_{22}s^2(t - t_{22}) = c_{22}s^2(t - 1) = 2 [0, 0, 1, -2, 0, 0] = [0, 0, 2, -4, 0, 0].$$

Summing these, we get

$$v_1(t) = \begin{bmatrix} 0 \\ 1 \\ -3 \\ 2 \\ 9 \\ 0 \end{bmatrix} \quad (4)$$

For Eq. (2), we have

$$\mathbf{D}_{11} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{D}_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{D}_{22} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Then

$$\mathbf{S} = [ \mathbf{D}_{11}s_1^1 \mid \mathbf{D}_{21}s_1^2 \mid \mathbf{D}_{22}s_1^2 ] = \left[ \begin{array}{c|c|c} 0 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -2 & 1 \\ 2 & 0 & -2 \\ 3 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

This gives us

$$\mathbf{v}^g = \mathbf{S}\mathbf{c}^g = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -2 & 1 \\ 2 & 0 & -2 \\ 3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -3 \\ 2 \\ 9 \\ 0 \end{bmatrix}$$

### 1.3 Optimization

ChatGPT gives the following explicit optimization problem:

$$\min_{\{s^j\}, \{c^g\}} \sum_{g=1}^{100} \|\mathbf{v}^g - \mathbf{S}(\{s^j\})c^g\|_2^2 + \lambda \sum_{g=1}^{100} \|c^g\|_1 \quad (5)$$