Alternating Minimization Setup Verification

Trevor S.

July 2025

1 Verifying GPT's Formulation

First note that the equation given by ChatGPT:

$$v_i(t) = \sum_{j=1}^{m} \sum_{k=1}^{K_j} c_{jk} s_i^j (t - t_{jk}) \tag{1}$$

does indeed match Eq. (1b) in Dr. Vinjamuri's paper.

1.1 Transforming Eq. (1)

Want to use Eq. (1) to arrive at:

$$\mathbf{v}^g = \mathbf{S}\mathbf{c}^g \tag{2}$$

Dr. Vinjamuri's paper tell us that the duration of the synergies, T_s, is 39 (Note that the paper actually uses t_s, but GPT uses T_s in its formulation, i.e. $t_s = 39 = T_s$). Then the jth synergy with the ith joint can be written as a vector, $\mathbf{s}_i^j \in \mathbb{R}^{T_s}$.

The time shift t_{jk} in Eq. (1) must be accounted for, and GPT solves this problem by introducing a convolution matrix to shift s_i^j by t_{jk}, where $\mathbf{D}_{jk} \in \mathbb{R}^{39 \times T_s}$, $T_s \leq 39$. Then $\mathbf{D}_{jk} s_i^j$ is equivalent to $s_i^j(t - t_{jk})$ in Eq. (1) (which is Eq. (1b) in Dr. Vinjamuri's paper).

Note that the paper considers only 10 joints, so the result is:

$$s_j = \begin{bmatrix} s_1^j \\ s_2^j \\ \vdots \\ s_{10}^j \end{bmatrix} \in \mathbb{R}^{10T_s} \tag{3}$$

The matrix $\mathbf{S}$ is constructed by horizontally stacking each $\mathbf{D}_{jk} s_i^j$ (the time-shifted version of the ith joint's component of the jth synergy).

1.2 Checking Formulations are Identical

We must confirm that Equations (1) and (2) are identical formulations. Let $i = 1$, $m = 2$, $K_1 = 1$, $K_2 = 2$, $T_s = 3$, $T = 6$, $t_{11} = 2$, $t_{21} = 0$, $t_{22} = 1$, $c_{11} = 3$, $c_{21} = 1$, $c_{22} = 2$. Let

$$s^1 = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \text{ and } s^2 = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$$

For Eq. (1), we have

$$\begin{aligned}
c_{11}s^1(t - t_{11}) &= c_{11}s^1(t - 2) = 3\left[0, 0, -1, 2, 3, 0\right] = \left[0, 0, -3, 6, 9, 0\right], \\
c_{21}s^2(t - t_{21}) &= c_{21}s^2(t - 0) = 1\left[0, 1, -2, 0, 0, 0\right] = \left[0, 1, -2, 0, 0, 0\right], \\
c_{22}s^2(t - t_{22}) &= c_{22}s^2(t - 1) = 2\left[0, 0, 1, -2, 0, 0\right] = \left[0, 0, 2, -4, 0, 0\right].
\end{aligned}$$

Summing these, we get

$$v_1(t) = \begin{bmatrix} 0 \\ 1 \\ -3 \\ 2 \\ 9 \\ 0 \end{bmatrix} \tag{4}$$

For Eq. (2), we have

$$\mathbf{D}_{11} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{D}_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{D}_{22} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Then

$$\mathbf{S} = \left[\ \mathbf{D}_{11}s_1^1 \mid \mathbf{D}_{21}s_1^2 \mid \mathbf{D}_{22}s_1^2\ \right] = \left[\begin{array}{c|c|c} 0 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -2 & 1 \\ 2 & 0 & -2 \\ 3 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

This gives us

$$\mathbf{v}^g = \mathbf{S}\mathbf{c}^g = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -2 & 1 \\ 2 & 0 & -2 \\ 3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -3 \\ 2 \\ 9 \\ 0 \end{bmatrix}$$

1.3 Optimization

ChatGPT gives the following explicit optimization problem:

$$\min_{\{s^j\},\{c^g\}} \sum_{g=1}^{100} ||\mathbf{v}^g - \mathbf{S}(\{s^j\})c^g||_2^2 + \lambda \sum_{g=1}^{100} ||c^g||_1 \tag{5}$$

See *setup.pdf* for more info on the problem's setup and a breakdown of the alternating minimization solution.