CSDS302 HW1 - Trevor Swan (*tcs94*)

Section 1.2

Problem 3

a. Is $4 = \{4\}$?

No, because number while $\{4\}$ is a set containing 4.

b. How many elements are in the set $\{3, 4, 3, 5\}$?

There are 3 elements in the set. 3 is repeated, but it only counts as one element.

c. How many elements are in the set $\{1, \{1\}, \{1, \{1\}\}\}$?

This set has 3 distinct elements, 2 of which are sets while one is a number.

Problem 6

For any integer n, let $T_n = \{n, n^2\}$. How many elements are in each of T_2, T_{-3}, T_1, and T_0? Justify your answers.

There are 2 elements in T_2 and T_{-3}, $T_2 = \{2, 4\}$ and $T\_3 = \{-3, 9\}$. T_1 has one element, though it repeated once. T_0 is an empty set $\{0, 0\}$ containing one element.

Problem 7

Use the set-roster notation to indicate the elements in each of the following sets.

a. S = $\{n \in \mathbb{Z} \mid n = (-1)^k, for\ some\ integer\ k\}$.

$-1^1 = -1, -1^2 = 1, -1^3 = -1 \ \therefore S = \{-1, 1\}$.

e. W = $\{s \in \mathbb{Z} \mid 1 < t < -3\}$

$1 < t < -3$ is not valid as $-3 < 1$.

f. X = $\{u \in \mathbb{Z} \mid u \leq 4 \; or \; u \geq 1\}$

$u \leq 4$ and $u \geq 1$ encompasses all integer values $\therefore \; X = \mathbb{Z}$.

Problem 9

c. Is $\{2\} \in \{1, 2\}$?

No, $\{1, 2\}$ contains the numbers 1 and 2, but $\{2\}$ is a set $\therefore \{2\} \notin \{1, 2\}$.

g. is $\{1\} \subseteq \{1, 2\}$?

Yes, $\{1, 2\}$ contains the numbers 1 and 2, so it has the subset $\{1\} \therefore \{1\} \subseteq \{1, 2\}$.

Problem 10

b. Is $(5, -5) = (-5, 5)$?

No, mirrors of coordinates (a, b) where $a \neq b$ are not equivalent as they amp to different regions in a plane

d. Is $((\frac{-2}{-4}, (-2)^3 = (\frac{3}{6}, -8))$?

Yes, both simplify to $(\frac{1}{2}, -8)$.

Problem 12

Let $S = \{2, 4, 6\}$ and $T = \{1, 3, 5\}$. Use the set roster notation to write each of the following sets, and indicate the number of elements that are in each set.

a. $S \; X \; T$

$= \{(2, 1), (2, 3), (2, 5), (4, 1), (4, 3), (4, 5), (6, 1), (6, 3), (6, 5)\}$

c. $S \; X \; S$

$= \{(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)\}$

Section 1.3

Problem 2

Let $C = D = \{-3, -2, -1, 1, 2, 3\}$ and define a relation S from C to D as follows. For every $(x, y) \in C\ X\ D, (x, y) \in S$ means that $\frac{1}{x} - \frac{1}{y}$ is an integer.

a. Is 2 S 2? Is -1 S -1? Is $(3, 3) \in S$? Is $(3, -3) \in S$?

$(2, 2) : \frac{1}{2} - \frac{1}{2} = 0 \in \mathbb{Z}$, yes
$(-1, -1) : \frac{1}{-1} - \frac{1}{-1} = 0 \in \mathbb{Z}$, yes
$(3, 3) : \frac{1}{3} - \frac{1}{3} = 0 \in \mathbb{Z}$, yes
$(3, -3) : \frac{1}{3} - \frac{1}{-3} = \frac{2}{3} \notin \mathbb{Z}$, no

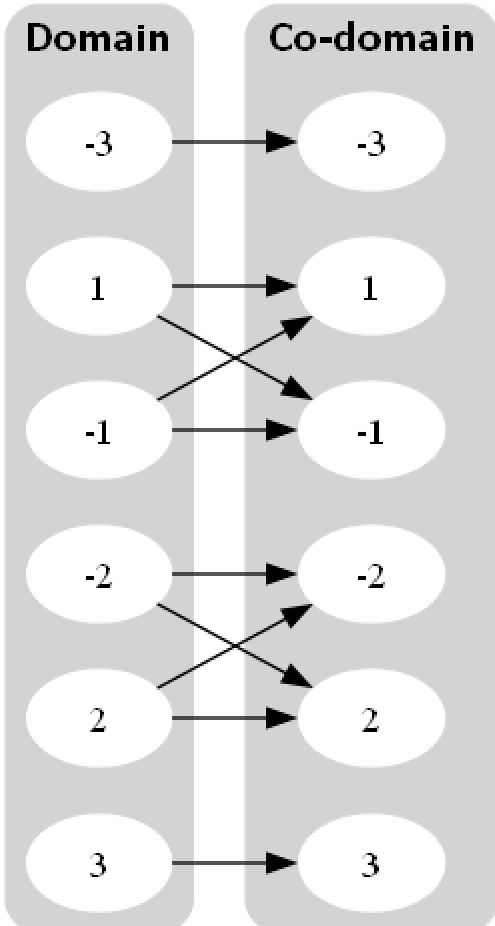
b. Write S as a set of ordered pairs.

$(x, y) \in C\ X\ D \therefore S =$
$\{(-3, -3),\ (-2, -2),\ (-1, -1),\ (1, 1),\ (2, 2),\ (3, 3),\ (-2, -2),\ (2, -2),\ (1, -1),\ (-1, 1)\}$

c. Write the domain and co-domain of S.

Domain: $\{-3, -2, -1, 1, 2, 3\}$
Co-domain: $\{-3, -2, -1, 1, 2, 3\}$

d. Draw an arrow diagram for S.



Problem 8

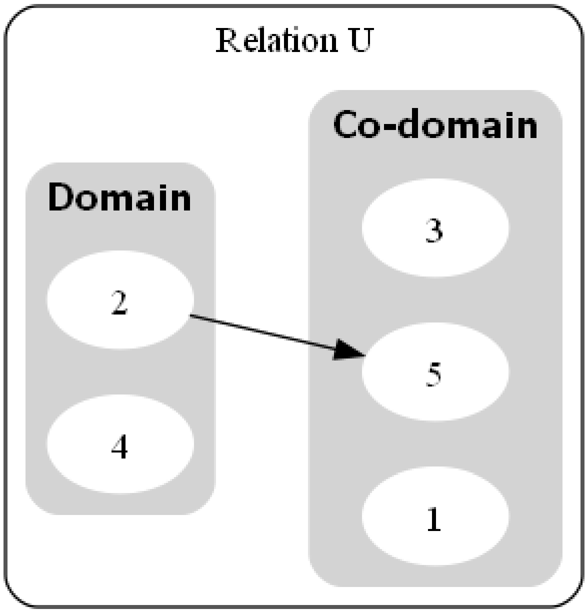
Let $A = \{2, 4\}$ and $B = \{1, 3, 5\}$ and define relations U, V, and W from A to B as follows:

For every $(x, y) \in A\ X\ B$

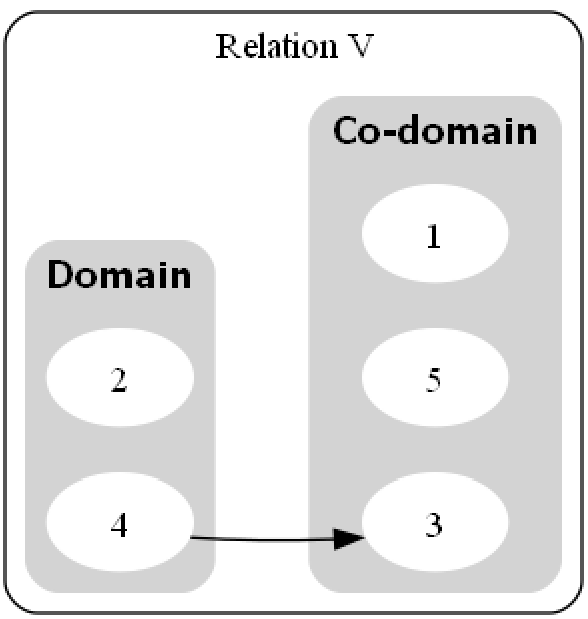
- $(x, y) \in U$ **means that** $y - x > 2$
- $(x, y) \in V$ **means that** $y - 1 = \frac{x}{2}$
- $W = \{(2, 5), (4, 1), (2, 3)\}$

a. Draw Arrow Diagrams for U, V, and W.

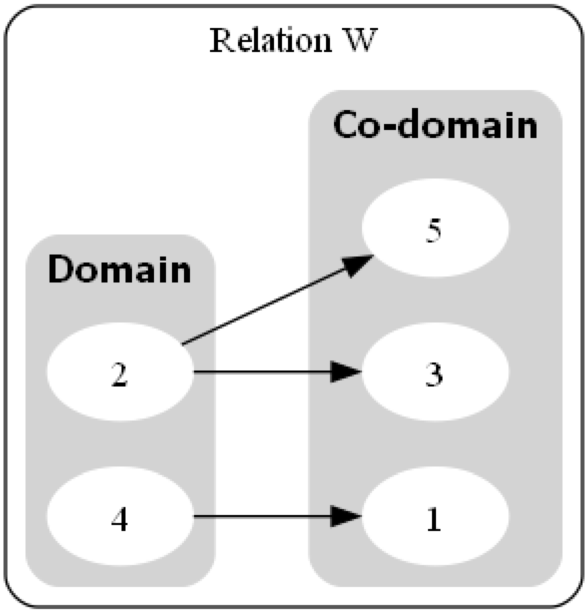
$A = \{2, 4\}$ and $B = \{1, 3, 5\}$
For Relation U: $y - x > 2 \therefore R_U = \{(2, 5)\}$.



For Relation V: $y - 1 > \frac{x}{2} \therefore R_V = \{(4, 3)\}$.



For Relation W: $\{(2,5),(4,1),(2,3)\}$.

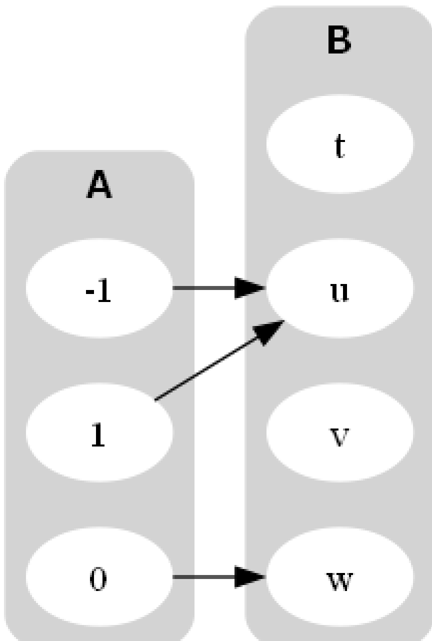


b. Indicate whether any of the relations U, V, and W are functions.

Functions require the elements of the domain to be mapped to an element in the co-domain. Relations U and V are not functions because they violate this rule. W is not a function because the element 2 has two mapped elements, $\{(2,3),(2,5)\}$.

Problem 13

Let $A = \{-1, 0, 1\}$ and $B = \{t, u, v, w\}$. Define a function $F : A \to B$ by the following arrow diagram:



a. Write the domain and co-domain of F.

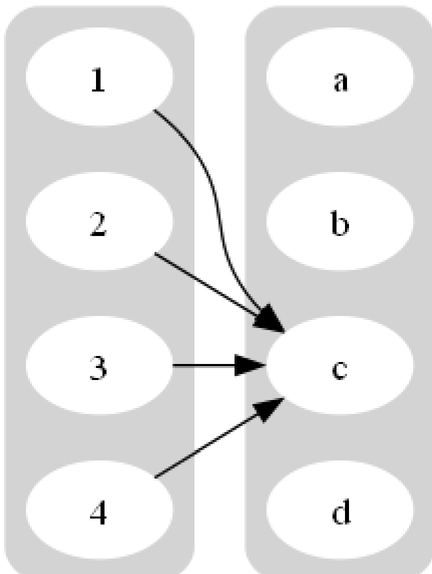
Domain = $\{-1, 0, 1\}$
Co-domain = $\{t, u, v, w\}$

b. Find $F(-1), F(0)$ and $F(1)$.

By the arrow diagram, we can see that $F(-1) = u$, $F(0) = w$ and $F(1) = u$. The input of F represents the preimage to a corresponding image.

Problem 14

Let $C = \{1, 2, 3, 4\}$ and $D = \{a, b, c, d\}$. Define a function $G : C \to D$ by the following arrow diagram:



a. Write the domain and co-domain of G.

Domain = $\{1, 2, 3, 4\}$
Co-domain = $\{a, b, c, d\}$

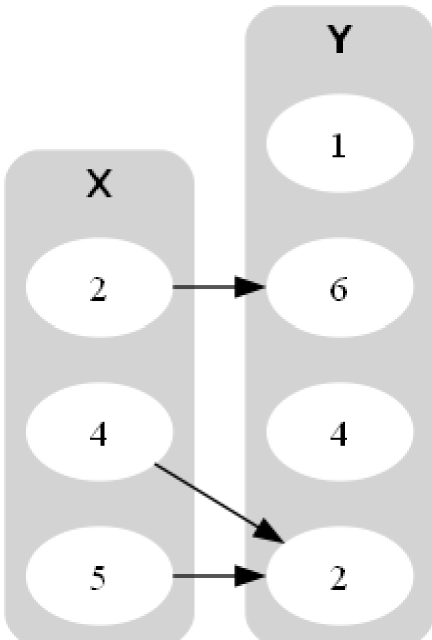
b. Find $G(1), G(2), G(3)$, and $G(4)$.

All elements of the domain are mapped to the same element in the co-domain
$\therefore G(1) = G(2) = G(3) = G(4) = c$.

Problem 15

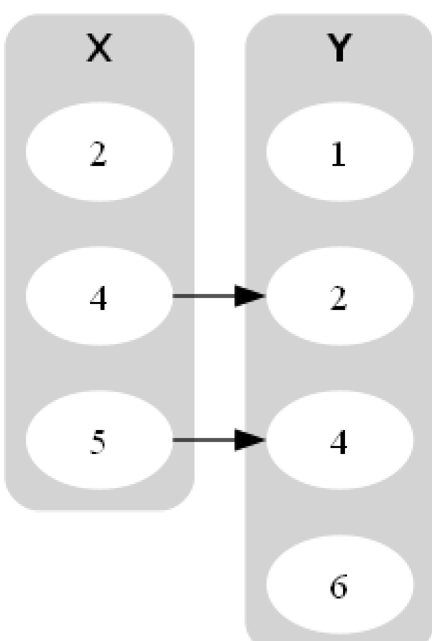
Let $X = \{2, 3, 5\}$ and $Y = \{1, 2, 4, 6\}$. Which of the following arrow diagrams determine functions from X to Y.

d.



Yes, all elements in x have one corresponding element in Y.

e.



No, not all of the elements in X are mapped to an element in Y.

Problem 18

Let h be the constant function defined in example $1.3.6$. Find $h(-\frac{12}{5}), h(\frac{0}{1})$ and $h(\frac{9}{17})$.

- **In $1.3.6$ the constant function is defined as $h(r) = 2$.**
  $h : r \to 2 \therefore h(\frac{-12}{5}) = h(\frac{0}{1}) = h(\frac{0}{1}) = 2.$

Problem 20

Define functions H and K from R to R by the following formulas:

- **For every $x \in R, H(x) = (x-2)^2$ and $K(x) = (x-1)(x-3)+1$**

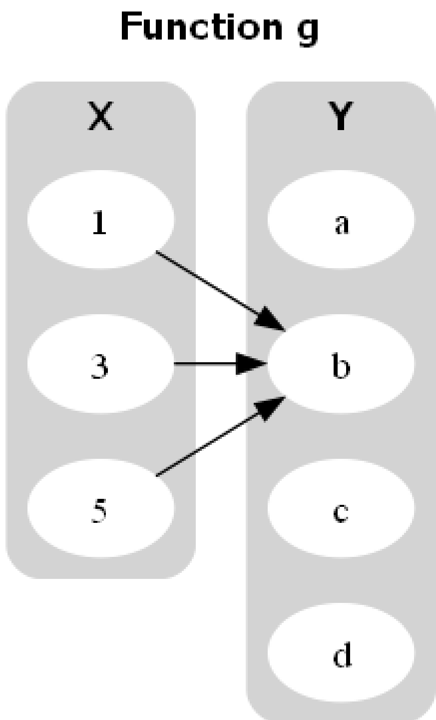
Does $H = K$? Explain.

$H(x) = (x-2)^2 \ \overrightarrow{algebraic} \ H(x) = x^2 - 4x + 4$
$K(x) = (x-1)(x-3)+1 \ \overrightarrow{FOIL} \ K(x) = x^2 - 3x - x + 3 + 1 \ \overrightarrow{simplify} \ K(x) = x^2 - 4x + 4$
H and K both simplify/expand out to be the same function, and since they have the same domain $x \in \mathbb{R}$ and co-domain $(x-2)^2, H = K$ as they are the same function.

Section 7.1

Problem 2

Let $X = \{1, 3, 5\}$ and $Y = Y = \{a, b, c, d\}$. Define $g : X \to Y$ by the following arrow diagram:



a.

Domain = $\{1, 3, 5\}$
Co-domain = $\{a, b, c, d\}$

b.

All elements in the domain map to the same element in the co-domain $\therefore g(1) = g(3 = g(5) = b$.

c.

Their is only one mapped image and it is b $\therefore$ Range = $\{b\}$.

d.

3 is an inverse image of b, not a.
1 is an inverse image of b, as b is the image of 1.

e.

$\{1, 3, 5\}$ is the inverse image of b.
c has no inverse as there is no preimage that corresponds to c. $\{0\}$

f.

$\{(1, b), (3, b), (5, b)\}$

Problem 4

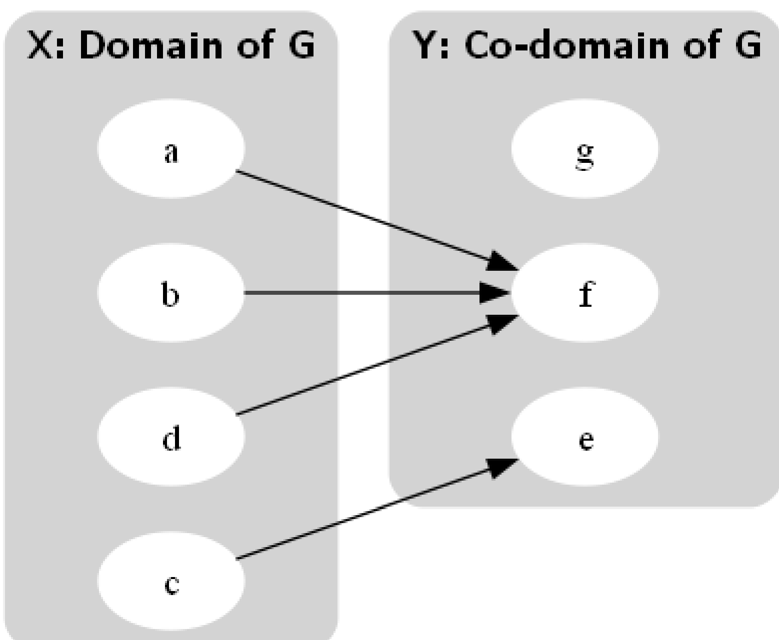
b. Find all functions from $X = \{a, b, c\}$ to $Y = \{u\}$.

There is only one function $f(X) = Y$ because there is only one element in the co-domain.
This function can be defined as: $x \in X, F(x) = u$.

Section 7.2

Problem 7

Let $X = \{a, b, c, d\}$ and $Y = \{e, f, g\}$. Define function G by the arrow diagram below.



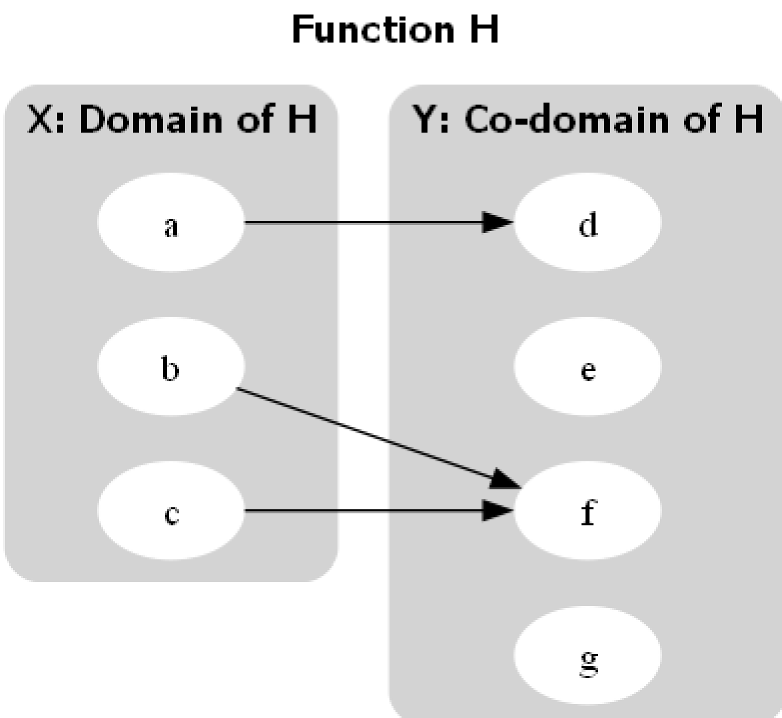
b. Is G one-to-one? Why or why not? Is it onto? Why or why not?

a and b have the same image and $a \neq b$, so it is not one-to-one. g does not have a pre-image, so the function is not onto either.

Problem 8

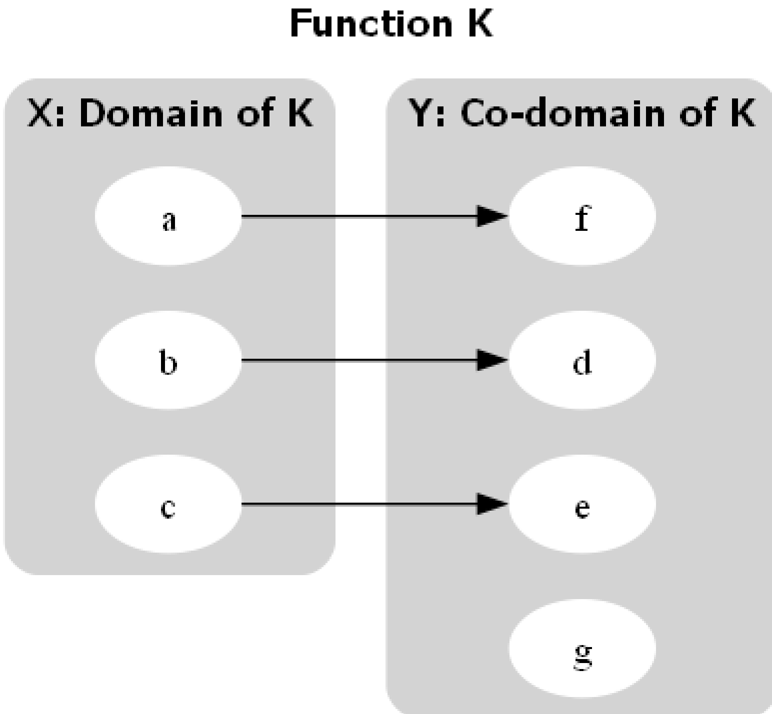
Let $X = \{a, b, c\}$ and $Y = \{d, e, f, g\}$. Define function H and K by the arrow diagrams below.

a. Is H one-to-one? Why or why not? Is it onto? Why or why not?



b and c have the same image, but $b \neq c$, so H is not one-to-one. g does not have a pre-image, so the function is not onto.

b. Is K one-to-one? Why or why not? Is it onto? Why or why not?



a, b and c all have their own image, so K is one-to-one. g has no pre-image, so K is not onto.