Topics and Sample Questions for EECS 302/MATH 304 Exam 1:

1. **Topics:** Sets, Cartesian Products, Relations and Functions

   Sample Problem: Given A = {1, 2, 4, 7} and B = {2, 4, 5}, check ALL boxes that are applicable in describing the following sets and briefly justify your answers

The set in each row represents…..	Relation	Function	Neither	Justification
V: {(1,2),(4,2},(7,5),(4,2)}				
W: {(1,2),(4,2),(2,5), (7,5)}				
X: {(1,2),(4,2),(2,3), (7,5)}				
Y: {(1,2),(4,2),(2,4), (7,5), (4,4)}				
Z: {(1,2),(4,2),(2,4), (7,5), (4,2)}				

2. **Topics:** Statements (or propositions) and logical operators ($\sim, \wedge, \vee, \rightarrow, \leftrightarrow$); Using logical operators to create negation of a statement, compound statements, and conditional statements; Truth values, truth tables and logical equivalences; Informal language and formal Language

   Sample Problems:

   2.1 Use logical identities (logical equivalence laws such as Associative laws, Commutative laws, Distributive laws, De Morgan's laws, Double negative laws, Idempotent laws, Absorption laws, and logical identities involving $\rightarrow$ and $\leftrightarrow$) to show logical equivalences of the following conditional statements: Let *p, q,* and *r* be proposition variables. Then

$$p\vee\sim q \rightarrow r\vee q \equiv (\sim p\wedge q)\vee(r\vee q) \equiv \text{negation of } \sim(\sim p\wedge q)\wedge(\sim r\wedge\sim q)$$

   2.2 Use a truth table to reconfirm the logical equivalence of the first two statements above

p	*q*	*r*							
T	T	T							
T	T	F							
T	F	T							
T	F	F							
F	T	T							
F	T	F							
F	F	T							
F	F	F							

3. **Topics:** Arguments: Proof of validity using (1) Truth Tables or (2) Rules of Inference/Valid Argument Forms (namely Modus Ponens, Modus Tollens,, Generalization, Specialization, Elimination, Transitivity, Proof by Division into Cases, and Contradiction Rule)

   Sample problems:
   Given the following argument in informal language (English)

   Sam is a CS major or an economics major.
   If Sam is a CS major, Sam is required to take EECS 302
   ∴ Sam is an economics major or Sam is not required to take EECS 302

   3.1 Write the above argument using formal language (using appropriate logical operators).

   3.2 Use appropriate rules of inference to deduce whether the argument is valid. (If you think it is invalid you can provide a case to show invalidity).

   3.3 Use a truth table to confirm the conclusion found in 3.2.

p	q	r				
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

4. **Topics:** Quantified statements (including quantified conditional statements and multiply-quantified statements) and negation of quantified statements

Sample problem:

Consider the following (multiply quantified) statement *S*:

S: $\forall e \in Z^+, \exists m \in N$ such that $\forall n \in N$, if $n \geq m$, then $\forall x \in R$, $|f_n(x) - f(x)| < e$.

4.1. What is the negation of the statement *S*?

4.2 What is the contrapositive form of the statement *S*? (Hint: Just concentrate on the if-then part)

5 **Topics:** Relationships among: i) logical conditioning quantifiers $\rightarrow$ and $\leftrightarrow$; (ii) conditional informal language operators ("if", "only if" and "if and only if"); (iii) implicit quantifiers $\Rightarrow$ and $\Leftrightarrow$; and (iv) necessary conditions, sufficient conditions and necessary and sufficient conditions

Sample problems:

Let $f(x)$ be a twice continuously differentiable real-valued function of a real variable x (i.e. the domain of f is R and the range (co-domain) of f is also R). The following results are well known:

a) A necessary condition for a real x^* to be a minimizer of f is that $f'(x^*) = 0$ and $f''(x^*) \geq 0$
b) A sufficient condition for a real x^* to be a strict local minimizer of f is that $f'(x^*) = 0$ and $f''(x^*) > 0$
c) If f is a convex function, then a necessary and sufficient condition for a real x^* to be a minimizer of f is that $f'(x^*) = 0$

Write each of the above statements in a formal form using first ($\rightarrow$ or $\leftrightarrow$) then ($\Rightarrow$ or $\Leftrightarrow$):

Answer:

a)
$\forall x \in R,\ x$ is a minimizer of $f(x) \rightarrow f'(x) = 0$ and $f''(x) \geq 0$
or
A real x is a minimizer of $f(x) \Rightarrow f'(x) = 0$ and $f''(x) \geq 0$

Now write similar formal statements for b) and c).

It is often easier to prove the above results by a contrapositive proof.

What is the contrapositive form of the necessary condition in part a) above?

5. **Topics:** Direct proof, counterexample, proof by contradiction, contrapositive proof

Sample problems:

5.1 Prove or disprove the following statements/claims formally and briefly justify each step in the body of your proof. Also clearly specify which method of proof you are using.

a) Claim: There exists an integer $m \geq 4$ such that $2m^2 - 5m + 2$ is prime

b) Claim: The difference between any odd integer and an even integer is odd

c) Claim: If a sum of two real numbers is less than 70, then at least one of the numbers is less than 35. (Hint use the fact that $p \rightarrow q \equiv \sim q \rightarrow \sim p$)

d) Claim: If x and y are rational numbers with $y \neq 0$, and if z is an irrational number, then $x + yz$ is irrational (Hint: Again, use the fact that $p \rightarrow q \equiv \sim q \rightarrow \sim p$)

5.2 Provide a brief justification for each claim made in the following body of proof

Theorem: There are infinitely many primes.

Proof:	Reason
Suppose there are finitely many prime with p being the largest prime.	This is the beginning statement of a proof by contradiction
Let $M = p! + 1$. Clearly every prime divides $p!$.	
But none divides M	
However, since M is an integer greater than 1, M is divisible by a prime	
Let q be a prime that divides M. So $q > p$.	
So the supposition that p is the greatest prime is false.	
Thus, there must be infinitely many primes. Hence the theorem is proved	

6. **Topics:** Key results and concepts in Number Theory: the quotient-remainder theorem, div and mod, floor and ceiling, the uniqueness factorization theorem

   Sample Problems:

   6.1 Show that if n mod 15 = 3, then $10n$ mod 15 = 0.

   6.2 If today is Tuesday, use the quotient-remainder theorem to find out the day of the week 127 days from now.

   6.4 Find the smallest value of k such that $2^5.5^1.7^7.11^3.k$ is a perfect cube.