Topics and Sample Questions for EECS 302/MATH 304 Exam 2:
Sections of the text covered in Exam 2 are 4.8, 5.1. 5.2, 5.3, 5.4, 5.6, 6.1, 6.2, 8.1, 8.2, 8.3

1. **Topics:** Division Algorithm, Euclidean Algorithm GCD and LCM

   Sample Problem:
   Compute the greatest common divisor (GCD) of 264 and 126 using: a) the Prime Factorization Theorem and b) The Euclidean Algorithm.
   Then compute the least common multiple (LCM) of 264 and 126

2. **Topics:** Sequences and Ordinary, Mathematical Induction

   Sample Problems:

   2.1 Given the sequence $S = \{2, 6, 12, 20, 30, 42, \ldots\}$, write the sequence S in form $\{a_n\}_{n\geq 1}$, i.e. determine a formula for a_n

   2.2 Given the sequence $S = \{a_n\}_{n\geq 1}$ defined recursively as $a_1 = 1$, $a_2 = 1$ & $a_n = a_{n-1} + a_{n-2}$ for all $n \geq 3$
   Write S in the expanded list form.

   2.3 Prove by mathematical induction that $\sum_{i}^{n} i(i+1) = n(n+1)(n+2)/3$ for all $n \geq 1$

3. **Topics:** Strong Mathematical Induction and the Well Ordering Principle for Integers

Sample problems:

3.1 Use strong mathematical induction to prove the following statement:
Given $a_1 = 1, a_2 = 3$, and $a_k = a_{k-2} + 2a_{k-1}$ for all $k \geq 3$, then for all integers $n \geq 1$, a_n is odd.

4 **Topics:** Set Theory, Equality of Sets, Operations on Sets, Empty set, Properties of Sets

Sample problems:

Let A, B, and C be any sets of elements of the universal set U

4.1. Show that $(A-B)-C = A\cap(B\cup C)^c$

4.2 Use appropriate "Properties of Sets" to show that $\left(A\cap(B\cup C)^c\right)\cap(B\cup A)\cap C = \phi$

5 **Topics:** Relations on Sets, Directed Graphs, Inverse Relations, Reflexivity, Symmetry, Transitivity, Transitive Closure, Equivalence Relations, Equivalence classes

Sample problems:

5.1 Let A = {a, b, c, d} and let R be a relation on A defined as
R = {(a, b), (a, c), (b, a), (b, d), (c, c), (c, d), (d, d)}
Draw a digraph (directed graph) representing R, and determine whether R is reflexive, symmetric, and/or transitive. If not transitive, find its transitive closure. Also find inverse relation R^{-1}.

5.2 Let A = {1, 2, 3, 4} and let R be a relation on A defined as For all $x, y \in A$, $xRy \Leftrightarrow 3 \mid (x - y)$

List all elements of R and R^{-1}.

Draw a digraph representing R, and determine whether R is an equivalence relation on A. If so find all equivalence classes induced by R