EECS 302/MATH 304 Discrete Mathematics

Final Exam Practice Problems covering topics after Exam 2

1.

Decision tree and CART, just use the last homework problems to practice

2. **Topics:** Cardinality of a set and probability of an equally-likely random event and all counting topics in Chapter 9

Sample Problems:

2.1 What is the probability that a 5-digit integer selected randomly will be divisible by 13?

Answer: A = set of 5-digit positive integers divisible by 13 $= \{13, 26, 39, 52, \ldots, 99996\}$

$B = \{1, 2, 3, 4, \ldots\ldots 7692\}$ where $7692 = \left\lfloor \frac{99999}{13} \right\rfloor$

Let $f: A \to B$ be defined as $\forall x \in A$, $f(x) = \frac{x}{13} \in B$

$\therefore$ f is obviously 1-to-1 and onto $\Rightarrow |A| = |B| = 7692$

$\therefore$ Prob that a 5-digit positive integer selected randomly will be divisible by 13

$= 7692/(99999-10000+1) = 7692/90000 = 0.085 \approx 8.5\%$

2.2 Given a group of 100 people, at minimum, how many people were born in the same month?

Answer: # of pigeons m = 100 people;

# of pigeonholes n = 12 months.

$\therefore$ At least $\left\lfloor \frac{100}{12} \right\rfloor + 1 = 9$ people will have the same birth month.

2.3 In a class 81 students, if there are five possible grades A, B, C, D, F, at least how many students will receive the same letter grade?

Answer: # of pigeons m = 81 students

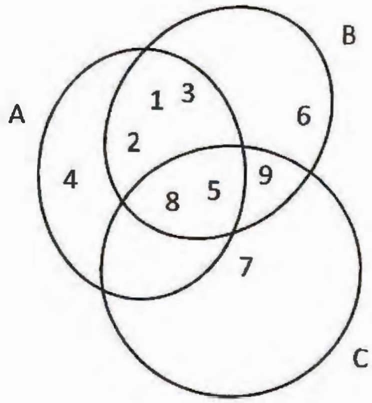
# of pigeonholes n = 5 grades

$\therefore$ At least $\left\lfloor \frac{81}{5} \right\rfloor + 1 = 17$ students will the same letter grade.

2.5 Consider the Venn diagram below for sets A, B and C, write down all members of the set

$$(A - B) \cup ((C - A) - B)$$

Then write down the power set of the resulting set



Answer: $A - B = \{4\}$

$C - A = \{7, 9\}$

$(C-A) - B = \{7\}$

$\therefore (A-B) \cup ((C-A)-B) = \{4, 7\}$

$\therefore$ Power set of $(A-B) \cup ((C-A)-B)$

$= \{\phi, \{4\}, \{7\}, \{4,7\}\}$

2.6. For a research study, how many ways a group of 6 mice can be selected from a large pool of mice aging from 1-6 weeks old?

Answer: # of 6-combinations from a set of 6 objects (mice of different ages) with repetition allowed

$$= \binom{6+6-1}{6} = \binom{11}{6} = \frac{11.\cancel{10}.\cancel{9}.\cancel{8}.7.\cancel{6!}}{\cancel{5}.\cancel{4}.\cancel{3}.\cancel{2}.1.\cancel{6!}} = 66$$

2.7 How many ways are there to distribute hands of 5 cards to each of four players from the standard deck of 52 cards?

Answer: # of ways to distribut 5 cards to

1^{st} player $= \binom{52}{5}$

2^{nd} player $= \binom{47}{5}$

3^{rd} player $= \binom{42}{5}$

4^{th} player $= \binom{37}{5}$

From the multiplicative rule, # of ways to distribute hands of 5 cards to each of 4 players $= \binom{57}{5}\binom{47}{5}\binom{42}{5}\binom{37}{5} = \dfrac{52!}{(5!)^4 32!}$

3. **Topics:** Graphs and Digraphs

Sample problems:

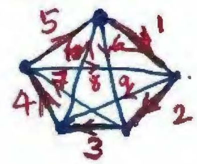
3.1 How many edges are in K_5?

Answer: $\binom{5}{2} = \frac{5 \cdot 4 \cdot \cancel{3!}}{2! \, \cancel{3!}} = 10$

3.2 Does K_5 have an Euler circuit? If yes show an example and if not justify why.

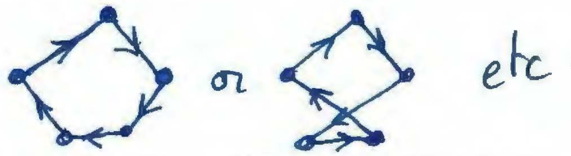
Answer: YES, since K_5 is connected and each vertex has an even degree (i.e 4)

An example of Euler circuit.



3.3 How about a Hamiltonian circuit? Justify your answer.

Answer: YES, for example

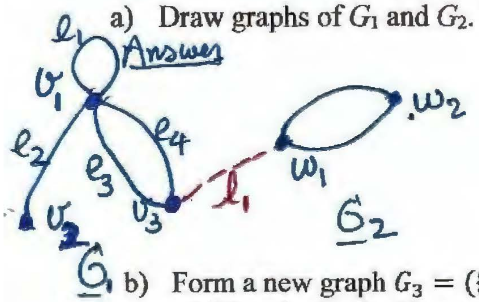


3.4 Consider graphs $G_1 = (\{v_1, v_2, v_3\}, \{e_1, e_2, e_3, e_4\})$ and $G_2 = (\{w_1, w_2\}, \{f_1, f_2\})$ with the following adjacency matrices respectively

$$A_1 = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

a) Draw graphs of G_1 and G_2. Is G_1 a connected graph? How about G_2? Explain.

Answer



G_1 is connected since there is no isolated subgraph.
G_2 is connected since there is no isolated subgraph
(G_1 and G_2 are not connected, however)

b) Form a new graph $G_3 = (\{v_1, v_2, v_3, w_1, w_2\}, \{e_1, e_2, e_3, e_4, f_1, f_2, l_1\})$, where l_1 is an additional edge incident on v_3 and w_1.
What is the adjacency matrix of G_3?

Answer:

$$\begin{array}{c|ccccc} & v_1 & v_2 & v_3 & w_1 & w_2 \\ \hline v_1 & 1 & 1 & 2 & & \\ v_2 & 1 & 0 & 0 & & \\ v_3 & 2 & 0 & 0 & 1 & \\ w_1 & & & 1 & 0 & 2 \\ w_2 & & & & 2 & 0 \end{array}$$

c) Is there an Euler circuit in G_3? Why or why not?

Answer: NO since there are vertices with odd degrees

e.g. $\deg(v_1) = 5$, degree$(v_2) = 1$, degree$(v_3) = 3$ + $\deg(w_1) = 3$.