Section 1.3 - Relations and Functions

Definition: Let A and B be sets, A relation R from A to B is a subset of $A \times B$

Example: $A = \{1,2\}$, $B = \{x,y\}$

$A \times B = \{(1,x), (1,y), (2,x), (2,y)\}$

$R_1 = \{(1,x), (2,y)\}$

Example: $A = \{1,2\}$, $B = \{1,2,3\}$

define a relation R from A to B

as: $(x,y) \in R$ if $\frac{x-y}{2} \in \mathbb{Z}$

Let: $A = \{1,2\}$, $B = \{1,2,3\}$

$A \times B = \{(1,1), (1,2), (1,3)$

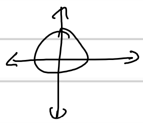
$(2,1), (2,2), (2,3)\}$

$\frac{1-1}{2} = 0$ ✓ $\quad \frac{1-3}{2} = -1$ ✓ $\quad \frac{2-2}{2} = 0$ ✓

$R = \{(1,1), (1,3), (2,2)\}$ for the relation $\frac{x-y}{2} \in \mathbb{Z}$

Let: a relation R as $(x,y) \in R$ if $x^2 + y^2 = 1$

$A = \mathbb{R}$ and $B = \mathbb{R}$



Think of a relation as mapping

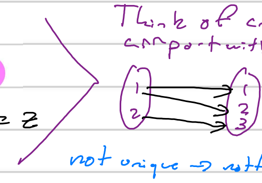
Neural Networking Connection

Relations can be anything that they are defined to be. These examples are defined by equations, not functions

Definition: A function F from A to B is a special relation

1. For every element $x \in A$, there is always $y \in B$ s.t. $(x,y) \in R$
   - every thing is mapped to at least one element in the other set
2. For every $x \in A$, $y \in B$, $z = B$ if $(x,y) \in F$, $(x,z) \in F$ then $y = z$
   - no element can have two separate mapped elements

Think of an airport units

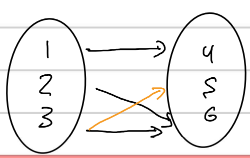


not unique → not function

Recall: $A \times B = \{(a,b) \mid a \in A, b \in B\}$ if $|A| = n$, $|B| = m$ then $|A \times B| = mn$

Functions Cont

Example $A = \{1,2,3\}$, $B = \{4,5,6\}$



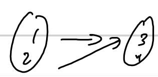
① Let $R_1 = \{(1,4), (2,6), (3,6)\}$ Function!

② Let $R_2 = \{(1,4), (2,6), (3,6), (3,5)\}$ Not a function!

Every airport "x" must have one flight, and can fly to any destination.

Definition: Let $F(1) = 3$

$F(2) = 3$

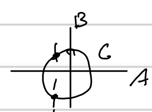


- 3 is the image of 1 or 2
- $\{1,2\}$ is the preimage of 3

The range of a function F is the set of all images. ← Set of preimages is the domain

Example $A = B = \mathbb{R}$ not a function if $A = B = \mathbb{R}$

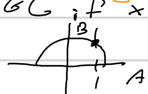
A relation C $(x,y) \in C$ if $x^2 + y^2 = 1$



X① Violates Function Criteria 1

But! If $A = [-1, 1]$

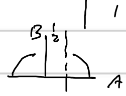
$B = \mathbb{R}^+$



This is a function

Or If $A = \mathbb{R}$

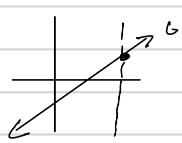
$B = [0, \frac{1}{2}]$



Not a function

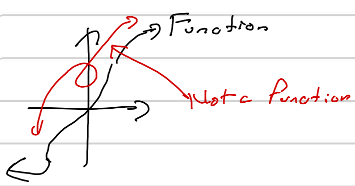
Example $A = B = \mathbb{R}$

A relation L $(x,y) \in L$ if $y = x - 1$



Function

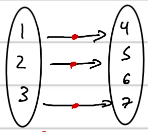
Example



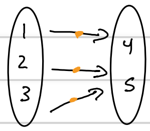
Note: Lemma 1, Lemma 2, Lemma 3 } Thm. 1

Sections 7.1-7.2

Definition: one-to-one (injective) function F: if all $x_1, x_2 \in X$, $F(x_1) = F(x_2)$ then $x_1 = x_2$



• One-to-one

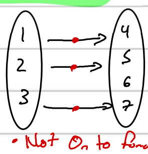


• Not one-to-one

← This is a function, though.

$|X| \le |Y|$ to be a one-to-one function ← Lemma 1

Definition: On to (surjective) function F: if every $y \in Y$ has a pre-image.



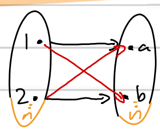
• Not On to function

$|X| \ge |Y|$ to be an on to function ← Lemma 2

Definition: Bijective function F is <u>both</u> one-to-one and onto

Theorem 1: If there exists a Bijective function $F: X \to Y$, then $|X| = |Y|$

Question: Suppose $|X| = |Y| = n$, How many Bijective functions can be created



There is $n!$ factorial possibilities

Proof:

$$\begin{bmatrix} 1 & \longrightarrow & a \\ 2 & & \\ 3 & & b \\ 4 & & c \\ & & d \end{bmatrix}$$

For 1: n ways

2: $n-1$ ways

3: $n-2$ ways