Logical Equivalences

Given any statement variables, *p*, *q*, and *r*, a tautology **t** and a contradiction **c**, the following logical equivalences hold

1. Commutative laws:	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2. Associative laws:	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. Distributive laws:	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (q \vee r)$
4. Identity Laws:	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
5. Negation Laws:	$p \wedge \sim p \equiv \mathbf{c}$	$p \vee \sim p \equiv \mathbf{t}$
6. Double neagative law:	$\sim(\sim p) \equiv p$	
7. Idempotent Laws:	$p \wedge p \equiv p$	$p \vee p \equiv p$
8. Universal bound Laws:	$p \wedge \mathbf{c} \equiv \mathbf{c}$	$p \vee \mathbf{t} \equiv \mathbf{t}$
9. De Morgan's laws:	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
10. Absorption laws:	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11. Negation of **t** and **c**:	$\sim \mathbf{t} \equiv \mathbf{c}$	$\sim \mathbf{c} \equiv \mathbf{t}$

Valid Argument forms (Rules of Inference)

Rules of Inference for Statements

1. Modus Ponens:

$$\begin{array}{l} p \to q \\ p \\ \therefore\ q \end{array}$$

2. Modus Tollens:

$$\begin{array}{l} p \to q \\ \sim q \\ \therefore \sim p \end{array}$$

3. Generalization:

$$a)\ \begin{array}{l} p \\ \therefore\ p \vee q \end{array} \qquad b)\ \begin{array}{l} q \\ \therefore\ p \vee q \end{array}$$

4. Specialization:

$$a)\ \begin{array}{l} p \wedge q \\ \therefore\ p \end{array} \qquad b)\ \begin{array}{l} p \wedge q \\ \therefore\ q \end{array}$$

5. Conjunction:

$$\begin{array}{l} p \\ q \\ \therefore\ p \wedge q \end{array}$$

6. Elimination:

$$a)\ \begin{array}{l} p \vee q \\ \square\ q \\ \therefore\ p \end{array} \qquad b)\ \begin{array}{l} p \vee q \\ \square\ p \\ \therefore\ q \end{array}$$

7. Transitivity:

$$\begin{array}{l} p \to q \\ q \to r \\ \therefore\ p \to r \end{array}$$

8. Division into cases:

$$\begin{array}{l} p \vee q \\ p \to r \\ q \to r \\ \therefore\ r \end{array}$$

9. Contradiction rule:

$$\begin{array}{l} \square\ p \to \mathbf{c} \\ \therefore\ p \end{array}$$

Rules of Inference for Quantified Statements

1. Universal Instantiation:

$$\begin{array}{l} \forall x \in D,\ P(x) \\ a \in D \\ \therefore\ P(a) \end{array}$$

2. Universal Generalization:

$$\begin{array}{l} P(a) \text{ for every } a \in D \\ \therefore \forall x \in D,\ P(x) \end{array}$$

3. Existential Instantiation:

$$\begin{array}{l} \exists x \in D \text{ such that } P(x) \\ \therefore\ P(a) \text{ for some } a \in D \end{array}$$

4. Existential Generalization:

$$\begin{array}{l} P(a) \text{ for some } a \in D \\ \therefore\ \exists x \in D \text{ such that } P(x) \end{array}$$

Set Identities

Let *A, B* and *C* be sets of elements of a universal set *U*, and let ϕ be the empty set. The following set identities hold:

1. Commutative laws: $A \cap B = B \cap A$ $\quad$ $A \cup B = B \cup A$
2. Associative laws: $(A \cap B) \cap C = A \cap (B \cap C)$ $\quad$ $(A \cup B) \cup C = A \cup (B \cup C)$
3. Distributive laws: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ $\quad$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
4. Identity Laws: $A \cap U = A$ $\quad$ $A \cup \emptyset = A$
5. Complement Laws: $A \cup A^c = U$ $\quad$ $A \cap A^c = \emptyset$
6. Double Complement law: $(A^c)^c = A$
7. Idempotent Laws: $A \cap A = A$ $\quad$ $A \cup A = A$
8. Universal bound Laws: $A \cap \emptyset = A$ $\quad$ $A \cup U = U$
9. De Morgan's laws: $(A \cap B)^c = A^c \cup B^c$ $\quad$ $(A \cup B)^c = A^c \cap B^c$
10. Absorption laws: $A \cup (A \cap B) = A$ $\quad$ $A \cap (A \cup B) = A$
11. Complement of U and $\emptyset$: $U^c = \emptyset$ $\quad$ $\emptyset^c = U$
12. Set-difference Law: $A - B = A \cap B^c$