## Heat Transfer

- Energy Transfer through a temperature difference

**(i) Conduction** – Energy Transfer due to the direct interaction of particles

Simple Conduction: $\dot{Q}_{cond} = kA_c \frac{T_1 - T_2}{\Delta x}$

↳ From Fourier's Law: $\dot{Q}_{cond} = -kA_c \frac{dT}{dx}$

- Gasses << Solids due to crystallinity

**(ii) Convection**: Enhanced conduction through bulk fluid motion

↳ moving fluid → large temperature gradient

Described by: $\dot{Q}_{conv} = h \cdot A_s (T_s - T_\infty)$

↳ From Newton's Law of Cooling

Heat Transfer Coefficient: h↑ as $U_\infty$↑

**(iii) Radiation**: Energy transfer due to electromagnetic waves

- Does not require a medium and all matter at a non-zero temperature emit radiation.

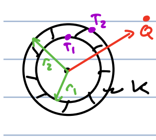
Described by: $\dot{Q}_{emitted} = \varepsilon \sigma A_s T_s^4$

↳ Stefan–Boltzmann Eq — ε: emissivity; $A_s$: surface area; $T_s$: absolute surface temp

where $\sigma$ = Stefan–Boltzmann constant = $5.67 \times 10^{-8} \frac{W}{m^2 K^4}$

## Conduction in Radial Systems

**(i) Cylindrical Coordinates**

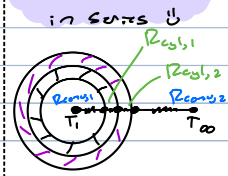



Still Fourier's Law:

$$\dot{Q}_r = \frac{2\pi L k}{\ln\left(\frac{r_2}{r_1}\right)}(T_1 - T_2)$$

$$R_{cyl} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi L k}$$

Radial can also be in series!




**(ii) Spherical Coordinates** ($A_c = 4\pi r^2$) ← still Fourier's

$$\dot{Q} = \frac{4\pi r_1 r_2 k}{r_2 - r_1}(T_1 - T_2) \qquad R_{sph} = \frac{r_2 - r_1}{4\pi r_1 r_2 k}$$

## Finned Heat Transfer

↳ Still uses Fourier's Law to Represent $\dot{Q}_{conduction}$

$$\dot{Q}_{cond} = -kA_c \frac{dT}{dx}$$

$$\Rightarrow 0 = \frac{d}{dx}\left(kA_c \frac{dT}{dx}\right) - hP[T(x) - T_\infty]$$

### Applications

(i) Electronics Cooling

(ii) Heat Exchangers

→ Car Radiator

→ HVAC Systems

(iii) Extremities (Toes, etc.)

## Two Different Boundary Condition Assumptions

**(i) Assume infinitely long fin tip**

a) $T(x \to \infty) \to T_\infty \Rightarrow \theta(x \to \infty) \to 0$

b) $T(x = 0) \to T_b \Rightarrow \theta(x = 0) \to \theta_b$

$\theta(x) = T(x) - T_\infty$

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-mx} \quad \text{for } m = \sqrt{\frac{hP}{kA_c}}$$

$$T(x) = (e^{-mx})(T_b - T_\infty) + T_\infty$$

$$\dot{Q} = \sqrt{hPkA_c}\,(T_b - T_\infty)$$

P is the Perimeter

**(ii) Assume Adiabatic Fin Tip, Finite length**

a) $\dot{Q}|_{x=L} = 0 = -kA_c \frac{dT}{dx}\Big|_{x=L} \Rightarrow \frac{d\theta}{dx}\Big|_{x=L} = 0$

b) $\theta(0) = \theta_b$

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh[m(L-x)]}{\cosh(mL)}$$

Hyperbolic cosine and tangent

$$\dot{Q} = \dot{Q}|_{x=0} = \sqrt{hPkA_c}\,(T_b - T_\infty)\tanh(mL)$$

$$T(x) = \frac{\cosh[m(L-x)]}{\cosh(mL)}(T_b - T_\infty) + T_\infty \;;\; m = \sqrt{\frac{hP}{kA_c}}$$

where...

$$\cosh(x) = \frac{1 + e^{-2x}}{2e^{-x}} \quad \text{and} \quad \tanh(x) = \frac{e^{2x} + 1}{e^{2x} - 1}$$

## Thermal Resistance Network

**(i) Conduction**

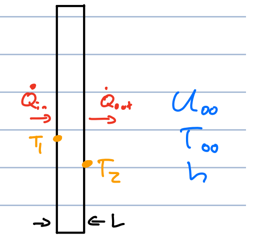
$$\dot{Q}_{cond} = kA_c \frac{(T_1 - T_2)}{L} = kA_c \frac{\Delta T}{L}$$

$$\dot{Q}_{cond,wall} = \frac{\Delta T}{R_{cond}} \rightsquigarrow R_{cond} = \frac{L}{kA}$$

**(ii) Convection**

$$\dot{Q}_{conv} = hA_s(T_2 - T_\infty) = hA_s \Delta T$$

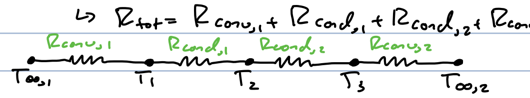
$$\dot{Q}_{conv} = \frac{\Delta T}{R_{conv}} \rightsquigarrow R_{conv} = \frac{1}{hA_s}$$




### Series Resistances

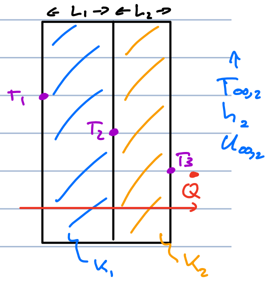
- $\dot{Q}$ is constant $\Rightarrow \Delta T = \dot{Q} R_{tot}$

↳ $R_{tot} = R_{conv,1} + R_{cond,1} + R_{cond,2} + R_{conv,2}$




$$\dot{Q} = \frac{T_{first} - T_{last}}{R_{tot}} = \frac{T_{\infty,1} - T_1}{R_{conv,1}} = \frac{T_1 - T_3}{R_{cond,1} + R_{cond,3}} = \dots$$

Resistances must correspond with temperatures

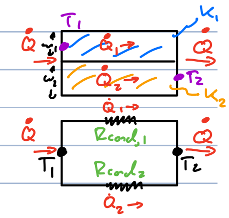


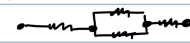

### Resistances in Parallel

- ΔT is constant, $\dot{Q}$s added individually $\dot{Q}_{tot} = \Sigma \dot{Q}_i$

$\Rightarrow \dot{Q} = \frac{\Delta T}{R}$

↳ $\frac{1}{R_{tot}} = \frac{1}{R_{cond,1}} + \frac{1}{R_{cond,2}} + \frac{1}{R_{cond,3}} + \dots$




- For some problems like , collapse parallel resistances to get $R_{middle}$, add to the ones in series!

## Transient Heat Transfer

**Lumped System Analysis**: Assume temp varies only with time $T = T(t)$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \quad \text{where } b = \frac{hA_s}{\rho V c} = \frac{h}{\rho L_c \cdot c}$$

& assumption holds (c: specific heat; $L_c$: characteristic length)

$\dot{Q}(t) = hA_s[T_\infty - T(t)]$ decreases with time

$Q_{total} = mc[T(t_{final}) - T_i]$ total heat transfer from t=0 to t=f

$Q_{max} = mc[T_\infty - T_i]$ maximum possible energy change

- Assumption holds for Biot # ≤ 0.1 ⇝ completely uniform temperature and can use Lumped System Analysis (LSA)

$$\text{Biot \#} = Bi = \frac{\text{Convection @ surface}}{\text{Conduction within}} = \frac{hL_c}{k} \quad \text{where } L_c = \frac{V}{A_s} \quad \begin{matrix}\text{volume} \\ \text{surface area}\end{matrix}$$

↳ If only given thickness (t) of object, assume t << width (w)

$$= L_c = \frac{V}{A_s} = \frac{Lwt}{2Lw + 2Lt} = \frac{wt}{2(w+t)} \approx \frac{t}{2}$$

can use in 'b' $b = \frac{h}{\rho \cdot L_c \cdot c}$

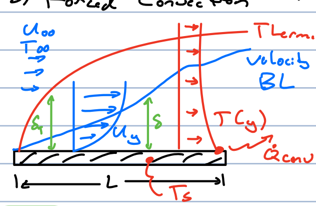
## Convection and External Flow

1) Natural / Free Convection

- Flow is induced by temp gradients

2) Forced Convection → Imposed Velocity

**Recall** $Re_L = \frac{\rho U_\infty L}{\mu}$, $Re_{D_h} = \frac{\rho U_\infty D}{\mu}$, $D_h = \frac{4A_c}{P}$ (characteristic diameter)




Boundary Layer

- Define BL thickness

$U_{BL} = 0.99\,U_\infty = y = \delta(x)$

$T_{BL} = T_s + 0.99(T_\infty - T_s) = y = \delta_T(x)$

**Flux**: $q''_{conv} = h\Delta T = \frac{\dot{Q}_{conv}}{A_s} \Rightarrow \dot{Q}_{conv} = q''_{conv} \cdot A_s$

$q''_{cond} = k_{fluid} \frac{\Delta T}{L} = \frac{\dot{Q}_{cond}}{A_s} \Rightarrow \dot{Q}_{cond} = q''_{cond} \cdot A_s$

### Prandtl Number

$$Pr = \frac{\text{viscosity}}{\text{diffusivity}} = \frac{\nu}{\alpha}$$

(dimless; ν: kinematic viscosity; α: thermal diffusivity) Tables!

**Typical Values**

(i) Pr << 1 ⇒ liquid metals

(ii) Pr ≈ 1 ⇒ gases (air)

(iii) Pr > 1 ⇒ oils, water

### Nusselt Number

$Nu_L = \frac{hL}{k_{fluid}}$ | $Nu_x = \frac{x h_x}{k_{fluid}}$

**(i) Local**

a) Laminar: $Nu_x = 0.332 \cdot Re_x^{1/2} \cdot Pr^{1/3}$

b) Turbulent: $Nu_x = 0.0296 \cdot Re_x^{4/5} \cdot Pr^{1/3}$

**(ii) Average**

Note: If $Re \geq 5 \times 10^5$, use T&L unless stated to be fully turbulent!

a) Laminar: $Nu_L = 0.664 \cdot Re_L^{1/2} \cdot Pr^{1/3}$

b) Turbulent: $Nu_L = 0.037 \cdot Re_L^{4/5} \cdot Pr^{1/3}$

c) L & T: $Nu_L = (0.037 Re_L^{4/5} - 871) \cdot Pr^{1/3}$

# Convection & Internal Flow

Energy Contained in the Fluid as:

entities $\begin{cases} \text{(i) Internal Energy} \\ \text{(ii) Kinetic (KE) \& Potential (PE) Energy} \\ \text{(iii) Flow Work Energy} \end{cases}$

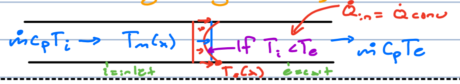
• For true liquids: $\Delta h = c_p \Delta T$

## Energy Analysis

$\dot{Q}_{in} = h A_s [T_s(x) - T_m(x)]$ $\dot{Q}_{in}$ due to convection

$q''_{in} = h_x [T_s(x) - T_m(x)]$ $\dot{Q}_{in}$ on a flux basis (per unit area)

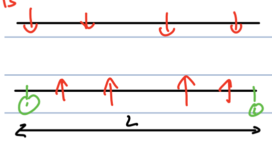
↳ generally constant, but can be local




## Consider Two Possible Cases

### (i) $q''_s$ constant (along x) → const $\dot{Q}$, like solar radiation

↳ constant surface heat flux




$\dot{Q}_{in} = q''_s A_s = \dot{m} c_p (T_e - T_i)$

$T_e = T_i + \dfrac{q''_s A_s}{\dot{m} c_p} = T_i + \dfrac{q''_s PL}{\dot{m} c_p}$

$T_m(x) = \dfrac{q''_s P x}{\dot{m} c_p}$

### What about $T_s$

Newton's Law of Cooling

$q''_s = h [T_s(x) - T_m(x)]$ constant for fully developed

$\Rightarrow T_s(x) = T_m(x) + \dfrac{q''_s}{h}$

### Determining h

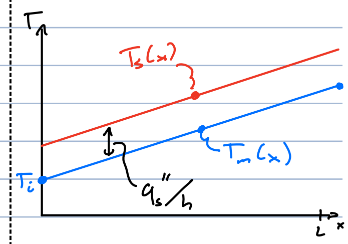
• For Laminar Pipe

• Circular Pipe

• $q''_s$ constant

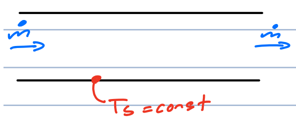
$Nu_D = \dfrac{hD}{k_{fluid}} = \dfrac{48}{11} \approx 4.36$

$\Rightarrow h = \dfrac{48}{11} \cdot \dfrac{k_{fluid}}{D}$




### (ii) $T_s$ constant (along x) → like ducts, submerged pipes

↳ constant surface temperature




$\dot{Q}_{conv} = h A_s [T_s - T_m(x)]$

$\dot{Q}_{in} = h A_s \Delta T_{lm} = \dot{m} c_p (T_e - T_i)$

↳ $T_s - T_m(x) = \Delta T_{avg} \approx \Delta T_{lm} = \dfrac{\Delta T_e - \Delta T_i}{\ln\left(\frac{\Delta T_e}{\Delta T_i}\right)}$

Use log mean diff bc $T_m$ changes with position x.

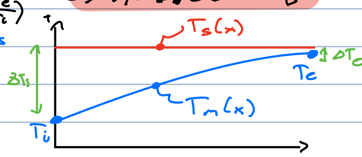
$T_e = T_s - (T_s - T_i) \cdot \left(\exp\left[-\dfrac{hPL}{\dot{m} c_p}\right]\right)$

$T_m(x) = T_s - (T_s - T_i) \cdot \left(\exp\left[-\dfrac{hPx}{\dot{m} c_p}\right]\right)$

### h for constant $T_s$, laminar flow, circular pipe

$Nu_D = 3.66$

$\Rightarrow h = 3.66 \cdot \dfrac{k_{fluid}}{D}$




### Laminar Flow for other cross-sections

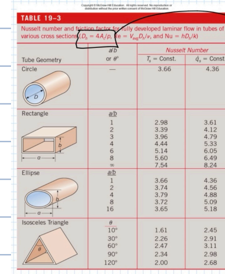


TABLE 19–3

Nusselt number and friction factor for fully developed laminar flow in tubes of various cross sections ($D_h = 4A_c/p$, $Re = V_{avg} D_h/\nu$, and $Nu = hD_h/k$)

| Tube Geometry | a/b or θ° | Nusselt Number $T_s$ = Const. | Nusselt Number $\dot{q}_s$ = Const. |
|---|---|---|---|
| Circle | — | 3.66 | 4.36 |
| Rectangle | a/b | | |
| | 1 | 2.98 | 3.61 |
| | 2 | 3.39 | 4.12 |
| | 3 | 3.96 | 4.79 |
| | 4 | 4.44 | 5.33 |
| | 6 | 5.14 | 6.05 |
| | 8 | 5.60 | 6.49 |
| | ∞ | 7.54 | 8.24 |
| Ellipse | a/b | | |
| | 1 | 3.66 | 4.36 |
| | 2 | 3.74 | 4.56 |
| | 4 | 3.79 | 4.88 |
| | 8 | 3.72 | 5.09 |
| | 16 | 3.65 | 5.18 |
| Isosceles Triangle | θ | | |
| | 10° | 1.61 | 2.45 |
| | 30° | 2.26 | 2.91 |
| | 60° | 2.47 | 3.11 |
| | 90° | 2.34 | 2.98 |
| | 120° | 2.00 | 2.68 |

$D_H = \dfrac{4A_c}{P}$

## Turbulent Flow ($Re_D > 2300$)

Dittus-Boelter Equation

$Nu_D = 0.023 \cdot Re_D^{4/5} \cdot Pr^n$

where: $\begin{cases} n = 0.4 \text{ if fluid is heated} \\ n = 0.3 \text{ if fluid is cooled} \end{cases}$

## Thermal Radiation

• Radiation Requires no medium

• All objects at non-zero (absolute) temperature emit radiation

• Amount of radiation proportional to temperature

• Thermal Radiation is a subset of the full EM spectrum (0.1 μm < λ < 100 μm) microns (μm)

• Effects that affect radiative transfer

(i) Temperature (iii) Wavelength

(ii) Geometry, direction (iv) Material Properties

## Blackbody Radiation

• Perfect emitter and absorber

a) Emits maximum radiation at a given T

b) Absorbs all incident radiation

c) Diffuse/uniform radiations

Examples: Sun @ 5800K, Snow & white paint in IR

Can be temperature or wavelength specific

## Two Primary Types of Materials

(i) Opaque Materials: All radiation absorbed over first few microns of the surface ex. wood, bark, metals

(ii) Semi-transparent Materials: Transparency can be wavelength dependent

ex. glass & water transparent in visible spectrum but opaque in IR

↳ Greenhouse effect: Visible light through glass, gets absorbed and IR released that cannot pass through the glass, temperature ↑

## Blackbody emissive power

$\left\{\frac{W}{m^2}\right\}$ $E_b = \sigma T_s^4$ $\sigma = 5.67 \times 10^{-8} \dfrac{W}{m^2 K^4}$

↑ temp in Kelvin!

Stefan-Boltzmann constant

## Wavelength Dependence (+ temperature)

• Different amounts of radiation given off at different wavelengths

$\Rightarrow$ Spectral blackbody emissive power

$E_{b,\lambda}(\lambda, T) = \dfrac{C_1}{\lambda\left[\exp\left(\frac{C_2}{\lambda T}\right) - 1\right]}$

where $\begin{cases} C_1 = 3.74 \times 10^8 \ \frac{W \mu m^2}{m^2} = 2\pi h C_0^2 \\ C_2 = 1.44 \times 10^4 \ \mu m \cdot K = \frac{h C_0}{k} \end{cases}$

for wavelengths in microns (μm)

## Wien's Displacement Law

$(\lambda T)_{max\ power} = 2897.8 \ \mu m \cdot K$

## Total Emission (usually most important)

$E_b(T) = \int_0^\infty E_{b,\lambda} \, d\lambda = \sigma T_s^4$

## Radiative Properties

(i) Emissivity: Real bodies emit a fraction of the blackbody emissive power

$\varepsilon(T) = \dfrac{E(T)}{E_b(T)}$ ~ Actual emissive power / ~ Ideal (blackbody) emissive power

• Most Metals: $\varepsilon \in (0.85, 0.95)$

$\Rightarrow$ Emissive Power: $E_{emitted} = \varepsilon \sigma T_s^4$ $\left\{\frac{W}{m^2}\right\}$

• Properties for incident radiation (irradiation)

(ii) Absorptivity ($\alpha$)

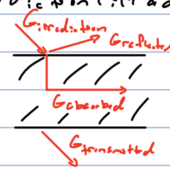
$\alpha = \dfrac{G_{absorbed}}{G}$

(iii) Reflectivity ($\rho$)

$\rho = \dfrac{G_{reflected}}{G}$

(iv) Transmissivity ($\tau$)

$\tau = \dfrac{G_{transmitted}}{G}$




### Energy Balance

$G = \alpha G + \rho G + \tau G$

$\Rightarrow 1 = \alpha + \rho + \tau$

• If opaque ($\tau = 0$)

$\alpha + \rho = 1$

$1 \ Pa = 1 \ \dfrac{kg}{m \cdot s^2} = \dfrac{J}{m^3}$