Modes of Energy Storage @ Microscale

Structure @ Microscale defines macrostate of energy

Sensible Modes: Related to activity & motion & closely to temp.
- (i) Translational Motion (KE)
- (ii) Vibrational Motion
- (iii) Rotational Motion
- (iv) Electron Translation

Latent modes / Structural changes:
- (v) Phase Changes
- (vi) Chemical Bonds
- (vii) Nuclear Bonds

Energy can cross boundary as: (i) Heat (ii) Work (iii) Mass

Mass Flow Rate: $\dot{m} = \rho V_{Avg} A_c$

Modes of Heat Transfer

(i) **Conduction:** Energy Transfers from higher to lower energetic matter due to direct interaction

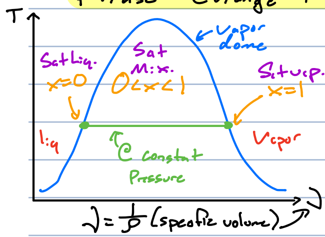
(ii) **Convection:** Enhanced conduction due to fluid motion

(iii) **Radiation:** Energy transfer via electromagnetic waves

First Law on a Total Basis

$$(Q_{in} - Q_{out}) + (W_{in} - W_{out}) + (E_{mass_{in}} - E_{mass_{out}}) = (U_2 - U_1) + \Delta KE + \Delta PE$$

Phase Change Fluids (Tables)



Quality

$$x = \frac{m_{vapor}}{m_{total}} = \frac{m_{vapor}}{m_{vapor} + m_{liquid}}$$

Phase Regions

(i) Compressed Liquid
- → @ given P: $T < T_{sat}$
- → @ given T: $P > P_{sat}$

(ii) Saturated Liquid (x=0)
- → $T = T_{sat}$ & $P = P_{sat}$

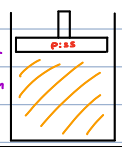
(iii) Saturated mixture (0<x<1)
- → $T = T_{sat}$ & $P = P_{sat}$

(iv) Saturated Vapor (x=1)
- → $T = T_{sat}$ & $P = P_{sat}$

(v) Superheated Vapor → $T > T_{sat}$ & $P < P_{sat}$

No quality in compressed liquid or SH regions

Closed Systems



1^{st} Law: $\underbrace{(Q_{in} - Q_{out}) + (W_{in} - W_{out})}_{\text{Can be on rate basis too}} = U_2 - U_1$ {J or kJ}

$= \underset{\text{System mass}}{m}\,\underset{\text{Mass specific internal energy}}{(u_2 - u_1)}$ {kJ/kg}

Q_{in}: Heat Transfer into system

Q_{out}: Heat Transfer out of system

W_{in}: Work done on the system

W_{out}: Work done by the system

Boundary Work (generally)

$$W_{piston} = \int_1^2 P\,dV = P\Delta V = P_m \Delta \nu = W_{out}$$

True for isobaric situations

$W_{piston} = W_{out} \rightarrow W_{out} > 0$ if volume ↑

Polytropic Processes

$PV^n = \text{Constant}$ (n: polytropic constant) $\Rightarrow P_1 V_1^n = P_2 V_2^n$ & $W_{piston} = \frac{P_2 V_2 - P_1 V_1}{1-n}$

Example Constants — (ii) & (iii) must be true for ideal gas
- (i) $n = 0$; constant P: isobaric
- (ii) $n = 1$; constant T: isothermal
- (iii) $\gamma = \frac{c_p}{c_v}$; constant S: isentropic
- (iv) $n \rightarrow \infty$; constant V: isochoric

Ideal Gas EoS

Law: $PV = n\bar{R}T$ ($\bar{R}$: Universal Gas Constant)

EoS: $PV = mRT$ (R: Particular Gas Constant)

$P\nu = RT$

$P = \rho RT$

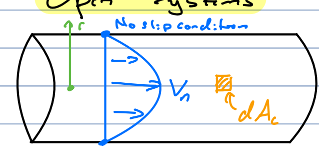
$R = \frac{\bar{R}}{MW}$

Specific Volume ν

$$\nu = \frac{1}{\rho} = \frac{V}{m} \Rightarrow m = \frac{V}{\nu}$$

$$\dot{m} = \frac{\dot{V}}{\nu} \;\&\; V = m\nu$$

Open Systems



- 0 velocity @ the wall
- Increased velocity @ center

Expressions

$\dot{m} = \rho V_{Avg} A_c$ (avg velocity)

locally $\dot{m} = \int_{A_c} \rho V_n \, dA_c$ (velocity profile)

$V_{Avg} = \frac{1}{A_c}\int V_n \, dA_c$

volume flow rate $\dot{V} = \frac{\dot{m}}{\rho} = V_{Avg} A_c$

E_{mass} elements
- (i) Kinetic Energy ($\frac{1}{2}V_{Avg}^2$)
- (ii) Internal energy (u)
- (iii) Potential Energy (gz)
- (iv) Flow Work Energy ($P\nu$)

Focus on Steady Flow Devices
- mass (i) $\dot{m}_{in} = \dot{m}_{out}$
- energy (ii) $\dot{E}_{in} = \dot{E}_{out}$

zero accumulation

1^{st} Law: $(\dot{Q}_{in} - \dot{Q}_{out}) + (\dot{W}_{in} - \dot{W}_{out}) + (\dot{E}_{mass_{in}} - \dot{E}_{mass_{out}}) = 0$

For steady open systems

$$\Rightarrow (\dot{Q}_{in} - \dot{Q}_{out}) + (\dot{W}_{in} - \dot{W}_{out}) = \dot{m}\left[(h_2 - h_1) + \tfrac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1)\right]$$

Finding Δu

Treat air as ideal in this course

1) Ideal Gases
- Internal energy only a function of temperature
- Use specific heat value

c_v: Specific Heat @ constant Vol.

$$c_v = \frac{du}{dT} \Rightarrow \Delta u = c_{v_{Avg}}(T_2 - T_1)$$ (Typically constant)

c_p: Specific heat @ ~~constant Press.~~

↳ See 'Open Systems'

2) True Liquids / Solids
- $c_v = c_p = c$

$\Delta u = u_2 - u_1 = \int_1^2 c\,dT = c_{Avg}(T_2 - T_1)$

$\underset{\text{Mass specific}}{\Delta u = c\Delta T}$ & $\underset{\text{total basis}}{\Delta U = mc\Delta T}$

3) Phase Change Fluids

(i) For a saturated mixture

$$u = x u_g + (1-x) u_f$$

Let ϕ be the property, so generally:

Both f & g found in the table properties

$\phi = \phi_f + x(\phi_g - \phi_f)$ — Called interpolation

$\phi = x\phi_g + (1-x)\phi_f$

(ii) For compressed liquids
- u & ν much stronger functions of temp than Pressure

Shortcut: Evaluate compressed liquid (CL) properties based on saturated liquid value. e.g. $u_{CL} = u_f(T)$

↳ Note that $u_{CL} \neq u_g(T)$

Calculating Δh

$h \equiv u + P\nu \rightarrow E_{mass} = m(h + \frac{1}{2}V^2 + gz)$

1) Ideal Gases

$c_p = \left.\frac{dh}{dT}\right|_P$ (@ constant pressure)

$\Delta h = \int_1^2 c_p\,dT = c_p(T_2 - T_1)$

2) True Liquids/Solids

$c_p = c_v = c$

$\Delta h = c_{Avg}\Delta T = c_{Avg}(T_2 - T_1)$

3) Phase Change Fluids

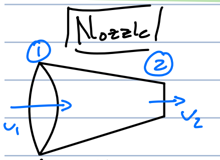
Go to tables

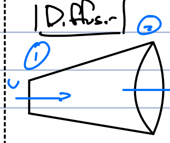
$h = h_f + x(h_g - h_f)$

$h = x h_g + (1-x) h_f$

Steady Flow Devices

1) Nozzles / Diffusers





V↑ as A↓, $P_2 < P_1$ | V↓ as A↑, $P_2 > P_1$

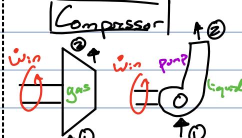
Exchanges enthalpy for KE

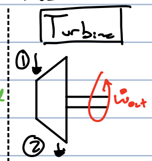
$0 = \dot{m}\left[(h_2 - h_1) + \frac{1}{2}(V_2^2 - V_1^2)\right]$ Relates Δh & KE

Assume: $\Delta PE \approx 0$, adiabatic, $W \approx 0$ (Passive system)

2) Compressors / Turbines

Fans also compressors, minimal P↑





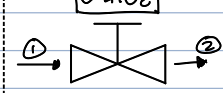
P↑, T↑ / P↑, min T↑ | Extracts fluid power

Relates Δh and Work

$\dot{W}_{in} - \dot{W}_{out} = \dot{m}(h_2 - h_1)$

Assume: $\Delta PE \approx$, $\dot{Q}$ small, $\Delta KE \approx 0$ by design

3) Throttles



$P_1 > P_2$

$\Delta h = 0$

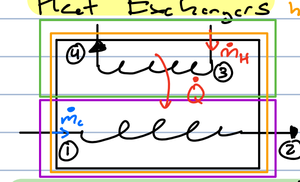
$h_1 = h_2$

Assume: $\Delta PE, \Delta KE \approx 0$

$Q_{net} \approx 0$, $W_{net} \approx 0$

Heat Exchangers

Transfer heat from hotter → colder fluid



Assume:
- $\dot{W}_{net} = 0$
- $\Delta PE \approx 0$
- $\Delta KE \approx 0$

Energy Analysis based on boundary color

* $\dot{Q}_{out} = \dot{m}_H(h_4 - h_3)$

* $\dot{Q}_{in} = \dot{m}_c(h_2 - h_1)$

* $Q_{in} = Q_{out}$: $\dot{Q}_{Net} = \dot{m}_H(h_4 - h_3) + \dot{m}_c(h_2 - h_1)$

$\dot{m}_c(h_2 - h_1) = \dot{m}_H(h_3 - h_4)$

Second Law of Thermodynamics

Quantified by (i) Heat Engine (ii) Entropy

(i) Processes proceed spontaneously in a particular direction

(ii) Energy has quality, not just quantity

(iii) Sets limits on theoretical Performance

Heat Engines

Converts heat (less valuable) into work (more valuable)

4 Key Elements

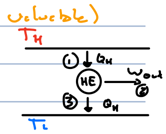
Examples: Steam Power Plant, Car Engine

(i) Receive Heat from high Temp res. T_H

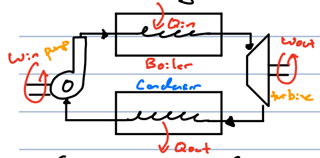
(ii) Convert a portion of the heat into work

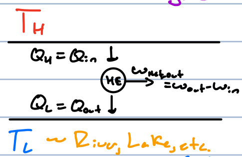
(iii) Rejects heat to low temp res. T_L

(iv) Operates as a cycle



Rankine Cycle — Power plant as a heat engine





$$\underbrace{(Q_{in}}_{\text{boiler}} - \underbrace{Q_{out})}_{\text{condenser}} + (\underbrace{W_{in}}_{\text{pump}} - \underbrace{W_{out}}_{\text{turbine}}) = m(\cancel{u_2 - u_1}) \quad \xrightarrow{} 0 \text{ Cycle}$$

Thermal Efficiency

$$\eta_{th} = \frac{\text{what we want}}{\text{what we pay}} = \frac{W_{net,out}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{Q_L}{Q_H}$$

Carnot Principles — Carnot cycles are ideal cycles/conditions

(i) Efficiencies of an irreversible (real) cycle is always less than a reversible cycle

(ii) Efficiency of all reversible engines acting between T_H and T_L is the same

$$\Rightarrow \left(\frac{Q_H}{Q_L}\right)_{rev} = \left(\frac{T_H}{T_L}\right)_{absolute} \longrightarrow \eta_{th,carnot} = 1 - \frac{T_L}{T_H} \leftarrow \text{must be absolute} \quad \text{from } \eta_{th} = 1 - \frac{Q_L}{Q_H} \text{ general}$$

Carnot Heat Engine

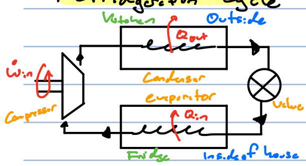
(i) Isothermal expansion @ T_H (iii) Isothermal compression @ T_L

(ii) Adiabatic expansion: $T_H \to T_L$ (iv) Adiabatic compression

$$CoP_{R,carnot} = \frac{1}{\frac{T_H}{T_L} - 1}$$

$$CoP_{HP,carnot} = \frac{1}{1 - \frac{T_L}{T_H}}$$

Refrigeration cycle — valid for: fridges, ACs

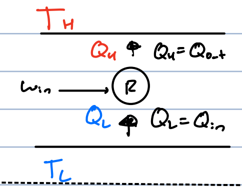


1st Law

$W_{in} + Q_L = Q_H$

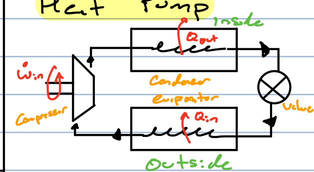
$\rightarrow W_{in} = Q_H - Q_L$

2nd Law: $Q_H \neq Q_L$



$$CoP_R = \frac{Q_{in}}{W_{in}} = \frac{Q_L}{W_{in}} = \frac{Q_L}{Q_H - Q_L} = \frac{1}{\frac{Q_H}{Q_L} - 1}, \quad CoP_R = [2, 6] \text{ usually}$$

Heat Pump



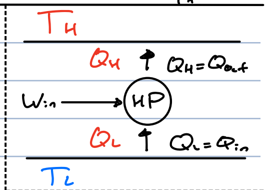
Two Types

(i) Air Source

↳ T_L is the air

(ii) Ground Source

↳ T_L is the ground



$$CoP_{HP} = \frac{Q_{out}}{W_{in}} = \frac{Q_H}{W_{in}} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - \frac{Q_L}{Q_H}}, \quad CoP_{HP} = [1, 2] \text{ usually}$$

Entropy

(i) Entropy change as a measure of energy quality/degradation

(ii) Entropy as a measure of molecular system disorder ← increases as heat is added

For Cycles / Processes — irreversibility ⇒ entropy generation

- Ideal cycles/processes → reversible
- Real cycles/processes → irreversible

Assume surr. are isolated... No heat/work out of surr.

For an Internally Reversible Process

$$dS = \left(\frac{dQ}{T}\right)_{ideal} \leftarrow \text{absolute!} \qquad ds = \frac{dS}{m} \text{ mass specific}$$

For a Real Process

$$dS > \frac{dQ}{T} \overset{\text{Integrate}}{\Rightarrow} \Delta s = \int_1^2 \frac{dQ}{T} \quad \text{Need } Q \text{ and } T \text{ relationship}$$

Δs can be +/− depending on the sign of dQ

Total Entropy Change — always nonnegative

$$\Delta s_{total} = \Delta s_{sys} + \Delta s_{surr}$$

(i) $\Delta s_{tot} > 0 \Rightarrow$ real (irreversible)

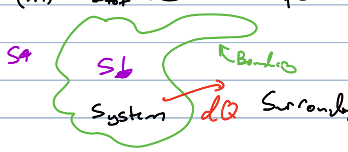
(ii) $\Delta s_{tot} = 0 \Rightarrow$ ideal (reversible)

(iii) $\Delta s_{tot} < 0 \Rightarrow$ impossible!

$$\Delta s_{sys} = \int_1^2 \left(\frac{dQ}{T}\right)_{ideal} + S_{generated} \leftarrow \text{generated through irreversibilities}$$

$$\Delta s_{sys} = \int_1^2 \left(\frac{dQ}{T}\right)_{sys,ideal} + \int_1^2 \left(\frac{dQ}{T}\right)_{surr,ideal} + S_{generated}$$

In the Rankine Cycle, Boiler and Compressor are both isobaric (operate @ Constant Pressure)



Calculating Entropy Change

(i) Isothermal Process — valid for surroundings

Assume fixed T_0

$$\Delta s_{surr} = \int_1^2 \frac{dQ}{T} = \int_1^2 \frac{dQ}{T_0} = \frac{Q_2}{T_0} \leftarrow \text{Transfer from } 1 \to 2$$

(ii) More Generally

$$\left.\begin{aligned} &\underset{(Q_{in}-Q_{out})}{dQ} - \underset{(W_{in}-W_{out})}{dW} = \underset{\text{Change in energy}}{dU} \\ &dS = \frac{dQ}{T} \quad T\,dS - \underset{\text{boundary work}}{P\,dV} = dU \end{aligned}\right\} \underset{\text{by mass}}{\overset{\text{divide}}{\Longrightarrow}} T\,ds = du + P\,dv \overset{h = u + Pv}{\Longrightarrow} T\,ds = dh - v\,dP$$

Gibbs Equation

Evaluating Δs

1) Ideal Gases

$$\Delta s_{sys} = C_{v,Avg} \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right)$$

enthalpy form

$$\Delta s_{sys} = C_{p,Avg} \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$$

2) True Liquids/Solids

Incompressible: $dv \approx 0$, $dP \approx 0$

$C_p = C_v = c$

$$\Delta s_{sys} = c \ln\left(\frac{T_2}{T_1}\right)$$

3) Phase Change Fluids

$$\Delta s_{sys} = s_2 - s_1$$

Use tables for s_2, s_1

↳ interpolate w/ quality

Isentropic Efficiency

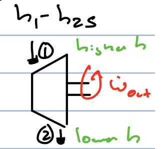
(i) Turbines — isentropic turbine produces most power

$$\eta_t = \frac{\text{actual}}{\text{isentropic}} = \frac{h_1 - h_{2A}}{h_1 - h_{2s}}$$

Diff Enthalpies

h_{2s} = isentropic

h_{2A} = actual



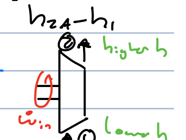
(ii) Compressors — isentropic compressor requires least power

$$\eta_c = \frac{\text{isentropic}}{\text{actual}} = \frac{h_{2s} - h_1}{h_{2A} - h_1}$$

Diff Enthalpies

h_{2s} = isentropic

h_{2A} = actual



Watch for h_{2A} vs. h_{2s} & ηs

Isentropic Processes $\Delta s_{sys} = 0 \longrightarrow s_2 = s_1$

1) Ideal Gases

$$\frac{T_{2s}}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1} \qquad R = c_p - c_v \qquad k = \frac{c_p}{c_v}$$

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

$$\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^{k}$$

All 3 valid only for an ideal gas in an isentropic process

2) True Liquids/Solids

$$\Delta s_{sys} = c \ln\left(\frac{T_2}{T_1}\right) = 0$$

$$c \ln\left(\frac{T_2}{T_1}\right) = 0 \qquad T_2 = T_1 \text{ to be true}$$

Solids & incompressible liquids require isentropic process to be isothermal

3) Phase Change Fluids

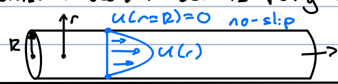
$$s_2 = s_1$$

Use tables and interpolate

Beware of false assumptions in questions!

Nomenclature

- No-slip condition: Fluid adjacent to a solid surface takes the velocity of that surface
- External Flow: Surface in an infinite flow fluid
- Internal Flow: Flow is fully bounded by a solid surface

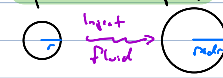


- Boundary Layer (BL): Thickness over which viscous effects are felt

Surface Tension

- Comparative attraction between similar and dissimilar molecules
- Some Effects of Surface Tension
  (i) Spherical Droplets (ii) Capillary Rise
  (iii) Meniscus
  (iv) Free Surface
  (v) Wetting Behavior

Spherical Droplet



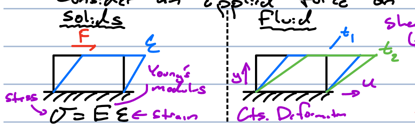
$\left[\sigma = \frac{\text{energy}}{\text{area}} = \frac{\text{force}}{\text{length}}\right]$ $\left\{\frac{N}{m}\right\}$ Energy required to ↑ surface area

$dE = \sigma\, dA_s;\ dA_s = 8\pi r\, dr$
$dW = (P_i - P_0)4\pi r^2 dr$
$\Big\} (P_i - P_0) = \frac{2\sigma}{r}$

Viscosity

Resistance to flow, molecular friction

→ Consider an applied force on a solid vs. fluid

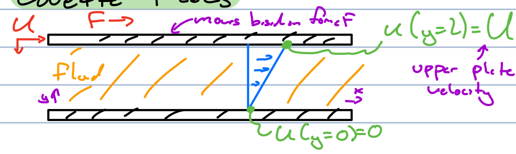


$\sigma = E\varepsilon$

Shear stress $\tau \propto \frac{du}{dy}$ directly proportional for newtonian fluids (Eliminates air, oil, etc.)

- Dynamic Viscosity $\equiv \mu = \nu\rho$ (kinematic viscosity)(density)
- Non-newtonian fluid viscosity changes based on force

Couette Flow



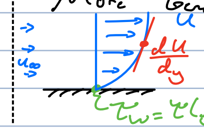
- Shear stress on the fluid: $\tau = F/A$
  - ↳ For general fluids: $F = \tau \cdot A$

Finding the Force

① $F = \tau A = \mu \frac{du}{dy} A$ all

② $F = \tau A = \mu \frac{U}{L} A$ Couette

More Generally



Shear Flow

$\tau_w = \tau(y=0)$ wall shear stress

Hydro Statics

Impact of pressure on engineered surfaces

Applications: Dams, Tanks

Pressure $\left[\frac{\text{force}}{\text{area}}\right]$

Pressure: (i) Acts isotropically (Same in all directions)
(ii) Leads to a pressure force: $F_{\text{pressure}} = \int P\, dA$
(iii) Pressure force on a submerged object (like a dam) always acts normal to the surface and inward
(iv) Varies with depth according to hydrostatic pressure variation

$$\frac{dP}{dz} = \rho g \Rightarrow P(z) = \int \rho g\, dz = P(z=0) + \rho g z$$

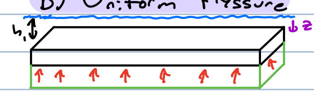
Resultant Force

$F_R = \int_A P(z)\, dA = \int_0^h P(z)\, w\, dz$

a) Linear Pressure Distribution

$F_R = w\int_0^h \rho g z\, dz$
$= (\frac{1}{2}\rho g h)(h w)$
average pressure, area

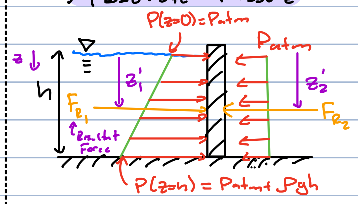
b) Uniform Pressure



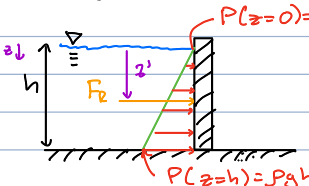
$F_R = \rho g h_1 A$

Gauge Pressure: $P_{\text{gauge}} = P_{\text{absolute}} - P_{\text{atm}}$

Absolute Pressure



Gauge Pressure

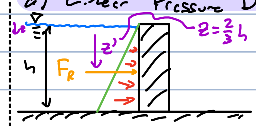


- Two methods: Decomposition & Integration

Line of Action

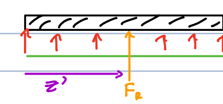
Location needs to lead to the same moment as the force distribution

a) Linear Pressure Distribution



$z' = \frac{2}{3}h$

b) Uniform Pressure



z is halfway across

Fluid Dynamics

- Stream Lines: Line formed tangent to the velocity at every point
- For steady flows, all fluid particles passing through a center point follow the same streamlines

Bernoulli Equation

Standard Form

- Provides a relationship between pressure, elevation, and velocity

Applications: Nozzles, Dams, etc.

Assumptions: (i) Steady Flow - no transient effects
(ii) Inviscid - viscous effects negligible
(iii) Incompressible (ρ constant)
(iv) Flow along a streamline
(v) No heat transfer
(vi) No work interactions may be considered for this form

Relationship: $\frac{P}{\rho} + \frac{1}{2}V^2 + gz = \text{constant}$ → Called the conventional form

$\Rightarrow P_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2$

Energy form of the Bernoulli Equation

$$\frac{P_1}{\rho} + \frac{1}{2}\alpha_1 V_1^2 + gz_1 + w_{\text{pump}} = \frac{P_2}{\rho} + \frac{1}{2}\alpha_2 V_2^2 + gz_2 + w_{\text{turbine}} + e_{\text{loss}}$$

↳ Kinetic energy correction factor (α): laminar $\alpha = 2$ and turbulent $\alpha \approx 1.05 \approx 1$

$\dot{E}_{\text{loss}} = \dot{E}_{\text{loss,pump}} + \dot{E}_{\text{loss,turbine}} + \dot{E}_{\text{loss,piping}}$

mass based "power" ↳ $\dot{W}_{\text{pump},u} = \dot{W}_{\text{pump}} - \dot{E}_{\text{loss,pump}}$ (useful) $\Rightarrow \eta_{\text{pump}} = \frac{\dot{W}_{\text{pump},u}}{\dot{W}_{\text{pump}}}$

$\dot{W}_{\text{turbine},e} = \dot{W}_{\text{turbine}} + \dot{E}_{\text{loss,turbine}}$ (extractable, actual) $\eta_{\text{turbine}} = \frac{\dot{W}_{\text{turbine}}}{\dot{W}_{\text{turbine},e}}$

Also, $\dot{W}_{\text{pump}} = \eta_{\text{motor}} \cdot \dot{W}_{\text{elec}}$

Bernoulli Equation in Terms of Head (h)

divide by $\dot{m}g$

$h_{\text{pump},u} = \frac{\dot{W}_{\text{pump},u}}{\dot{m}g}$, $h_{\text{turbine},e} = \frac{\dot{W}_{\text{turbine}}}{\dot{m}g}$, $h_L = \frac{\dot{E}_{\text{loss,piping}}}{\dot{m}g}$

$$\Rightarrow \frac{P_1}{\rho g} + \frac{\alpha_1 V_1^2}{2g} + z_1 + h_{\text{pump},u} = \frac{P_2}{\rho g} + \frac{\alpha V_2^2}{2g} + z_2 + h_{\text{turbine},e} + h_L$$ ← piping

h_L is the irreversible head loss between ① and ② due to fluid friction → The diff. states

Internal Flow

Laminar Flow

- Smooth, orderly flow
- Low velocity, high viscosity

Turbulent Flow

- High mixing, chaotic flow
- High velocity, low viscosity

⇒ Flow Regime based on the Reynolds number

$Re = \frac{\rho V_{avg} D}{\mu}$ ← Characteristic length; ← dynamic viscosity

- Ratio of inertial to viscous effects
- Re_{crit} is transition from laminar to turbulent flow

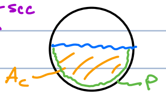
Internal Flow: $Re_{crit} = 2300$

Non-circular Cross Sections

- Use hydraulic diameter for D

$D_h = \frac{4A_c}{P}$ ← cross-sec area; ← wetted perimeter

Aqueducts, rivers, or sewage pipes



Developed Flow

- Entrance region is before flow is fully developed, typically $L/D \approx 10$
- BL present in entrance region

$$u(r) = \boxed{-\frac{R^2}{4\mu}\cdot\frac{dP}{dx}}\left(1 - \frac{r^2}{R^2}\right)$$
maximum velocity

$\frac{dP}{dx} < 0$ must be true to push fluid in +direction (+u flow)

Let $\frac{dP}{dx} \equiv \Delta P_L = P_1 - P_2$

- Average velocity: $u_{avg} = -\frac{R^2}{8\mu}\cdot\frac{dP}{dx}$ For laminar flow $u_{avg} = \frac{1}{2}u_{max}$

Laminar $\Rightarrow \Delta P_L = \frac{8\mu u_{avg} L}{R^2} = \frac{32\mu u_{avg} L}{D^2}$

- Pumping power to overcome frictional losses: $\dot{W}_{\text{pump},L} = \dot{V}\Delta P_L$

Generalize with Darcy Friction Factor (f)

$\Delta P_L = f\frac{L}{D}\frac{1}{2}\rho u_{avg}^2$; $h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{u_{avg}^2}{2g}$

(i) Laminar Flow

$\Delta P_L = \frac{L}{D}\cdot\frac{1}{2}\rho u_{Avg}^2\left(64\frac{\mu}{\rho u_{avg} D}\right) \Rightarrow f = \frac{64}{Re_D}$

(ii) Turbulent flow: Use moody tables or...

① $\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$ Colebrook

② $\frac{1}{\sqrt{f}} \approx -1.8\log_{10}\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right]$ Haaland (2% error)

$\frac{\varepsilon}{D} = \frac{\text{roughness}}{\text{diameter}}$

Minor Losses

(i) Elbows, Tees (ii) Piping Transitions
(iii) Valves (iv) Entrance/Exit

$\Rightarrow h_L = K_L \cdot \frac{V_{avg}^2}{2g}$ K_L from tables

Combining Losses

(i) Series a) $\dot{V}$ const (ii) Parallel a) $\dot{V} = \dot{V}_a + \dot{V}_b$
b) $h_L = \sum h_{L,major} + \sum h_{L,minor}$ b) $\Delta P_A = \Delta P_B \Rightarrow h_{L_A} = h_{L_B}$

External Flow

- Flow over bodies => aerodynamic forces (lift, drag)
- Two primary mechanisms:

(i) Friction: $\tau = \mu \frac{du}{dy}$

↳ Shear stress over surface area => frictional force

(ii) Pressure Differences: Reduced pressure in wake

↳ pressure differences over surface => pressure force

- External Flow: $Re_{crit} = 5 \times 10^5$ less than this is laminar

Note: For a tree, velocity ↑ => C_D↓ as leaves move in direction of wind and fold to reduce drag (part of evolution)

Aerodynamic Force Balances

(i) Take-off (to) velocity for an airplane

$$V_{to} = \sqrt{\frac{2W}{\rho C_L A}}$$

where $W = mg$ and C_L is lift coefficient

(ii) Terminal (t) Velocity

sedimentation, suspensions

$$V_t = \sqrt{\frac{2Vg}{C_D \cdot A} \cdot \frac{\rho_S - \rho_F}{\rho_F}}$$

where s, f are solid and fluid respectively

Lift Force

- Lift coefficient depends on
  (i) Shape
  (ii) Angle of attack (α)
  ↳ angle between chord & horizontal
- Stall: Lift Coefficient increases with angle of attack up to stall point
  ↳ Typically Occurs around 15°
- Effect of flaps: Increase lift coefficient at the expense of drag

Drag and Lift

- Effects that influence aerodynamic forces

(i) Fluid Properties (ρ, μ)

(ii) Object Properties / Shape
  a) Streamlined
  b) Blunt or Bluffed

(iii) Flow Regime (Laminar vs. Turbulent)

- F_L: Lift force - Acts perpendicular to free stream
- F_D: Drag force - Acts along free stream

Definitions / Expressions

- $C_D = \frac{F_D}{\frac{1}{2}\rho u_\infty^2 A}$ => $C_D \frac{1}{2}\rho u_\infty^2 A$

C_D and C_L can exceed 1

- $C_L = \frac{F_L}{\frac{1}{2}\rho u_\infty^2 A}$ => $F_L = C_L \frac{1}{2}\rho u_\infty^2 A$

Appropriate Area for C_D, C_L

(i) Blunt Frontal Area
  a) Cylinder $A_F = DL$
  b) Sphere $A_F = \frac{\pi}{4} D^2$

(ii) Streamlined body — Planform Area (A_p)

$A_p = b \cdot c$ (span b, chord length c)

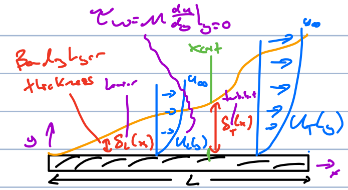
- Get C_D values from Tables based on (i) Shape and (ii) Reynolds number

Drag Force

(i) Skin friction drag

→ Due to shear stress

→ Dominant mechanism for streamlined objects



$$F_D = F_f = C_f \frac{1}{2}\rho u_\infty^2 A$$

↳ Skin friction coefficient; A: Area in contact with fluid

- Overall Reynolds #: $Re_L = \frac{\rho u_\infty L}{\mu}$
- Local Reynolds #: $Re_x = \frac{\rho u_\infty x}{\mu}$

- Critical Reynolds #: $Re_{crit} = 5 \times 10^5$

$$\begin{cases} Re_x < Re_{crit} \Rightarrow \text{Laminar} \\ Re_x > Re_{crit} \Rightarrow \text{Turbulent} \end{cases}$$

Describes boundary layer

- Flow over a flat plate can be... (To get C_f value)

(i) Entirely Laminar: $C_{f,laminar} = 1.33 / \sqrt{Re_L}$

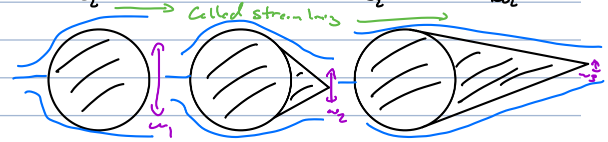
(ii) Primarily Turbulent: $C_{f,turbulent} = 0.074 / Re_L^{1/5}$

(iii) Combination of Laminar & Turbulent BL: => $C_f = 0.074 / Re_L^{1/5} - 1742 / Re_L$

(ii) Form Drag (Pressure Drag)

→ Due to flow blocking / Separation

→ Dominant mechanism for bluff bodies

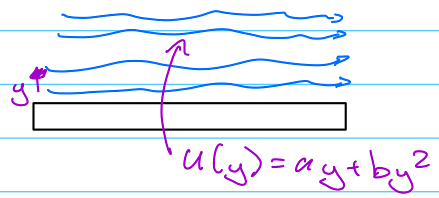


Basic — Slightly streamlined — no Separation

- Decreasing wake size decreases form drag at the expense of friction drag and goal is net reduced total drag

- Basic: ↑ form/pressure drag
- Slightly streamlined: ↓ form, ↑ friction
- No separation: form drag ↓↓, friction drag ↑↑, overall ↓

1. [Chapter 10] For flow over a plate, the variation of velocity with vertical distance y from the plate is given as $u(y) = ay + by^2$ where a and b are constants. Obtain a relation for the wall shear stress in terms of a, b, and μ.



$$\tau = \mu \frac{du}{dy}$$

Shear Stress

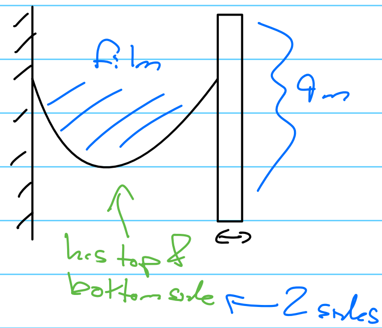
$$\boxed{\tau_w = \mu a}$$

$$\rightarrow \tau = \mu \frac{d}{dy}(ay + by^2)$$

$$\tau = \mu a + 2\mu b y$$

$y = 0$ @ wall: $\tau_w = \mu a + 2\mu b(0) = \mu a$

2. [Chapter 10] The surface tension of a liquid is to be measured using a liquid film suspended on a U-shaped wire frame with a 9 cm long movable side. If the force needed to move the wire is 0.045 N, determine the surface tension of this liquid in air.



$$F = 2\sigma L$$

Force applied to move wire

film has 2 surfaces

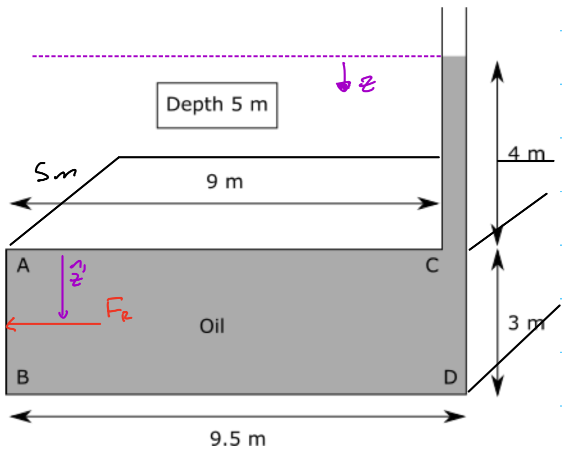
surface tension

Length of wire (m)

$$\boxed{\sigma = 0.25\ N/m}$$

$$\sigma = \frac{F}{2L} = \frac{0.045\ N}{2(0.09\ m)} = 0.25\ N/m$$

3. [Chapter 11] The following 3 m x 9.5 m x 5 m tank shown is filled with oil of SG=0.8. Specific gravity (SG) is the ratio of the fluid density to the density of water. Depth here is the depth into the page.



(a) the magnitude and the location of the line of action of the resultant force acting on surface AB.

(b) The pressure force acting on surface BD.

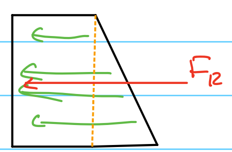
(c) Will the force acting on surface BD equal the weight of oil in the tank? Find a relation.

$SG_{oil} = \frac{\rho_{oil}}{\rho_{H_2O}} = 0.8$, $\rho_{H_2O} = 997.048 \frac{kg}{m^3}$

$0.8\,\rho_{H_2O} = \rho_{oil}$

$\rightarrow \rho_{oil} = 797.638\ kg/m^3$

a)



$F_{R,T} = \underbrace{\tfrac{1}{2}}_{\text{Linear distribution}} \underbrace{\rho g h}_{\text{Avg. Pressure}} \underbrace{(h w)}_{\text{Area}}$

$= \frac{1}{2}(797.638)(9.81)(3)(15)$

$= 176059\ N$

$F_R = F_{R,T} + F_{R,R}$

$F_{R,R} = \rho g h_1 A$

Uniform Distribution; dis from surface; Area; hw

$= (797.638)(9.81)(4)(15)$

$= 469440\ N$

$\boxed{F_R = 645499\ N \quad z' = 5.64\ m}$

$F_R \hat{z}' = \frac{2}{3} h F_{R,T} + \frac{1}{2} h F_{R,R}$

$= \frac{2}{3}\cdot 3 F_{R,T} + \frac{1}{2}\cdot 3 F_{R,R}$

$\rightarrow \hat{z}' = 1.64\ m$

z' should start at z=0

$z' = \hat{z}' + 4m = 5.64\ m$

b) $F_R = \rho g h_1 A$

$h_1 = 7m$ from free surface

$A = 9.5m \times 5m = 47.5 m^2$

$F_R = 797.638 * 9.81 * 7 * 47.5m$

$= 2.6 \times 10^6\ N$

$\boxed{F_R = 2.6\ MN}$

c) $W_{oil} = mg$, $m = \rho_{oil} \times V$

Volume = (Tank) + (Top Bit)

$= (9.5 \times 3 \times 5) + (0.5 \times 4 \times 5)$

$= 152.5\ m^3$

$\rightarrow W_{oil} = 797.638 \frac{kg}{m^3}(152.5 m^3)(9.81 \frac{m}{s^2})$

$= 1.19 \times 10^6\ N$

$\boxed{W_{oil} = 1.19\ MN}$

$\frac{BD_{F_R}}{W_{oil}} = \frac{2.6\ MN}{1.19\ MN} = 2.18$ suggests $\boxed{F_R = 2.18\, W_{oil}}$

Trevor Swan **ECHE225: QUIZ #6 // 10.17.24**

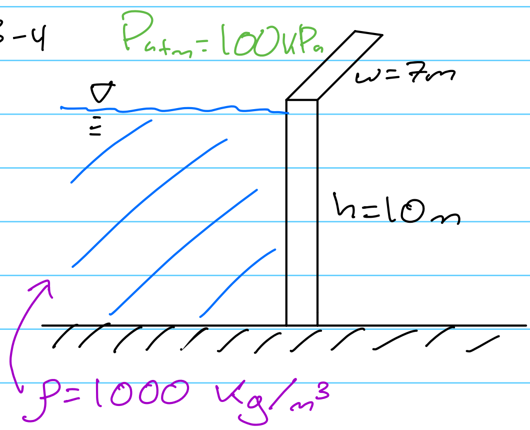
1. $Re = \frac{\text{inertial}}{\text{viscous}}$

2. $u(h=0) = 0$ no-slip

$u(h=A) = V_0$

$u(h=C) > 0$, but $< V_0$, increases from 0 to A

3-4



3. $P_{gage} = P_{absolute} - P_{atm}$

$P_{absolute} = P_{gage} + P_{atm}$

$\left.\right\}$ $P_{atm} = 100kPa$, $P_{gage} = \rho g h$

$\therefore P_{absolute}(h=5) = 100 + 5\rho g$

$= 100\ kPa + 5m\left(1000\frac{kg}{m^3}\right)\left(9.81\frac{m}{s^2}\right)$

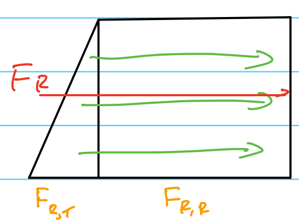
$= 100 + 49050\ \frac{kg}{m \cdot s^2}$

$1\ N = \frac{kg \cdot m}{s^2}$ → our answer is in $\frac{N}{m}$, $1\ Pa = 1\ N/m$

$= 100 + 49.050 = \boxed{149.05\ kPa}$

4. Resultant Force

height is distance from free surface $h = 10m$



$F_{R,T} = \frac{1}{2}\rho g h(hw)$ ← $P_{avg} \cdot A$

$= \frac{1}{2}\left(1000\frac{kg}{m^3}\right)\left(9.81\ \frac{m}{s^2}\right)(10m)(10 \times 7m^2)$

$= 3{,}433{,}500\ \frac{kg \cdot m}{s^2} = N$

$F_{R,R} = \rho g h(hw)$

$= \left(1000\frac{kg}{m^3}\right)\left(9.81\frac{m}{s^2}\right)(10m)(10 \times 7m^2)$

$= 6{,}867{,}000\ N$

$F_R = F_{R,T} + F_{R,R} = 10{,}300{,}500\ N$

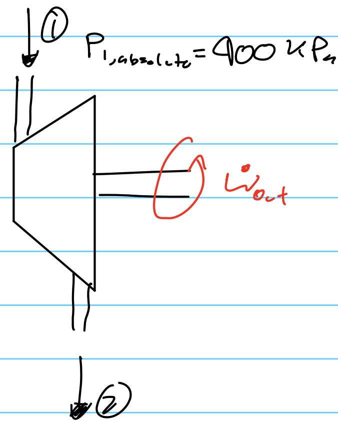
$\boxed{\cancel{F_R = 10{,}300\ kN}}$

$F_R = w\int_0^h \rho g z = \frac{1}{2}\rho g h^2 w$

$= \frac{1}{2}\ 1000(9.81)(10)^2(7) = 3433500$

$= \boxed{3434\ kN}$

1. [Chapter 12] In a hydroelectric power plant, water enters the turbine nozzles at 900 kPa absolute with a low velocity. If the nozzle outlets are exposed to atmospheric pressure of 100 kPa, determine the maximum velocity to which water can be accelerated by the nozzles before striking the turbine blades.



$V_1^2 \approx 0$ $\quad z_1 \approx z_2$

$$\underbrace{P_1}_{P_1 = P_{1,absolute}} + \cancel{\tfrac{1}{2}\rho V_1^2}^{\,0} + \cancel{\rho g z_1} = \underbrace{P_2}_{P_2 = P_{atm}} + \tfrac{1}{2}\rho V_2^2 + \cancel{\rho g z_2}^{\,0}$$

$$P_{1,absolute} = P_{atm} + \tfrac{1}{2}\rho V_2^2 \qquad \rho = 997\ \tfrac{kg}{m^3}$$

$$V_2^2 = \frac{2}{\rho}\left(P_{1,absolute} - P_{atm}\right)$$

$$P_{1,absolute} = 900{,}000\ Pa \ \&\ P_{atm} = 100{,}000\ Pa$$

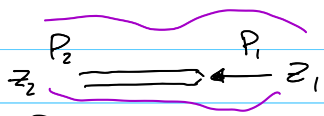
$$Pa = \frac{kg}{m \cdot s^2}$$

$$1\ kPa = 1000\ Pa$$

$$V_2 = \sqrt{\frac{2}{997\ kg/m^3}\left(900{,}000\ Pa - 100{,}000\ Pa\right)}$$

$$= 40.06\ m/s \qquad \boxed{V_2 = 40.06\ m/s}$$

2. [Chapter 12] A Pitot-static probe is used to measure the speed of an aircraft flying at 4000 m. Assume that the density of the atmosphere at that height is 0.82 kg/m^3. If the differential pressure reading is 3.5 kPa, determine the speed of the aircraft.



$$P_1 + \frac{1}{2}\rho \cancel{V_1^2}^{\,0} + \cancel{\rho g z_1}^{\,0} = P_2 + \frac{1}{2}\rho V_2^2 + \cancel{\rho g z_2}^{\,0} \qquad z_1 = z_2 = 4000\,\text{m}$$

$v_1 \approx 0$ @ entrance

$P_1 - P_2 = 3500\ \text{Pa}$

$$P_1 - P_2 = \frac{1}{2}\rho V_2^2$$

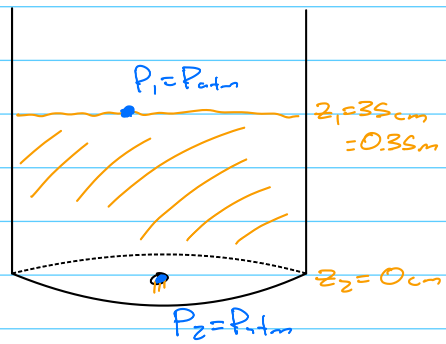
$$\rightarrow V_2^2 = \frac{2}{\rho}(P_1 - P_2)$$

$$V_2 = \sqrt{\frac{2}{0.82\,\text{kg/m}^3}(3500\,\text{Pa})}$$

$$= 92.4\ \text{m/s}$$

$$\boxed{V_2 = 92.4\ \text{m/s}}$$

3. [Chapter 12] While traveling on a dirt road, the bottom of a car hits a sharp rock and a small hole develops at the bottom of its gas tank. If the height of the gasoline in the tank is 35 cm, determine the initial velocity of the gasoline at the hole. Discuss how the velocity will change with time and how the flow will be affected it the lid of the tank is closed tightly.



$1\,m = 100\,cm$

$P_1 = P_2$

$$\cancel{P_1} + \frac{1}{2}\cancel{\rho}\cancel{v_1^2}^{\approx 0} + \cancel{\rho} g z_1 = \cancel{P_2} + \frac{1}{2}\cancel{\rho} v_2^2 + \cancel{\rho g z_2}^{0}$$

surface v is negligible

$$g z_1 = \frac{1}{2} v_2^2$$

$$v_2 = \sqrt{2(9.81\,m/s^2)(0.35\,m)}$$

$$= 2.62$$

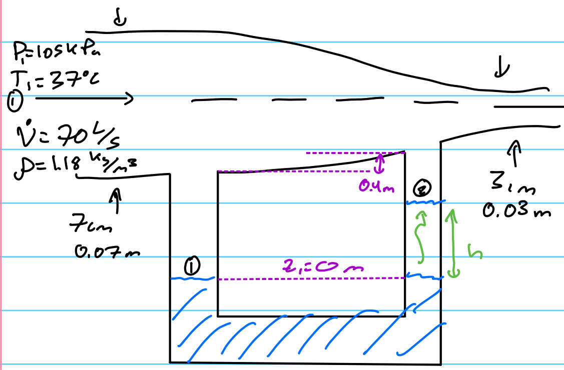
$$\boxed{v_2 = 2.62\,m/s}$$

- As z_1 decreases $\sqrt{2 g z_1}$ will also decrease
- If Tank is sealed tightly, then P_1 is not necessarily P_{atm}, So

$$P_1 + \rho g z_1 = \frac{1}{2}\rho v_2^2 \rightarrow v_2 = \sqrt{\frac{2}{\rho}(P_1 + \rho g z)},$$ so velocity will

most likely be dependent on both the tank pressure and oil density.

4. [Chapter 12] Air at 105 kPa and 37°C flows upward through a 7-cm-diameter inclined duct at a rate of 70 L/s. The duct diameter is then reduced to 3 cm through a reducer. The pressure change across the reducer is measured by a water manometer. The elevation difference between the two points on the pipe where the two arms of the manometer are attached is 0.4 m. Determine the differential height between the fluid levels of the two arms of the manometer.



Assume air is incompressible

- $P_{1a} = P_{1w} = P_1$
- $P_{2a} = P_{2w} = P_2$

As ρ_a is so low, column pressure is equivalent

Note: Steady State $\Rightarrow \dot{m}_1 = \dot{m}_2$

Incompressible $\Rightarrow \dot{V}_1 = \dot{V}_2 = \dot{V} = 70\ L/s$

$\hookrightarrow \dot{V} = \underset{\text{velocity}}{V_{Avg}} A_c \Rightarrow V_{Avg} = \frac{\dot{V}}{A_c}$

$A_c(7_{cm}) = \pi r^2 = \frac{\pi}{4}(D)^2 = \frac{\pi}{4}(0.07m)^2 = 0.0038\,m^2$

$A_c(3_{cm}) = \frac{\pi}{4}(0.03m)^2 = 0.00071\,m^2$

$A_{c_1} = 0.0038\,m^2,\ A_{c_2} = 0.00071\,m^2$

$\{air\}$ $P_1 + \frac{1}{2}\rho_a V_1^2 + \cancel{\rho_a g z_1} = P_2 + \frac{1}{2}\rho_a V_2^2 + \cancel{\rho_a g z_1} \Rightarrow P_1 - P_2 = \frac{1}{2}\rho_a (V_2^2 - V_1^2)$ ($z_1 = z_2 = 0$)

$\{water\}$ $P_1 + \cancel{\frac{1}{2}\rho_w V_1^2} + \cancel{\rho_w g z_1} = P_2 + \cancel{\frac{1}{2}\rho_w V_2^2} + \rho_w g z_2 \Rightarrow P_1 - P_2 = \rho_w g h$ ($V_1^2 \approx 0$, $z_1 = 0$, $V_2^2 \approx 0$, $z_2 = h$)

$\boxed{Equate}$ $\frac{1}{2}\rho_a (V_2^2 - V_1^2) = \rho_w g h \Rightarrow h = \frac{\rho_a (V_2^2 - V_1^2)}{2\rho_w g}$

$\rho_a = 1.18\ kg/m^3$
$\rho_w = 997\ kg/m^3$

$\boxed{velocities}$ $V_1 = \frac{\dot{V}}{A_{c_1}} = \frac{70\ L/s \times \frac{1\,m^3}{1000\,L}}{0.0038\ m^2} = 18.42\ m/s$

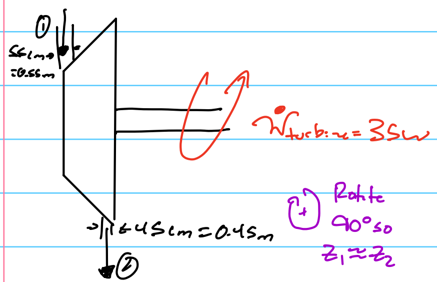
$1L = 0.001\,m^3$

$V_2 = \frac{\dot{V}}{A_{c_2}} = \frac{70\ L/s \times \frac{1\,m^3}{1000\,L}}{0.00071\ m^2} = 98.6\ m/s$

$\boxed{h = 57\,cm}$

$\boxed{Solve!}$ $h = \frac{1.18\ kg/m^3 \left((98.6\ m/s)^2 - (18.42\ m/s)^2\right)}{2(997\ kg/m^3)(9.81\ m/s)} = 0.566\ m = 57\,cm$

Trevor Swan **ECHE225: QUIZ #7 // 10.24.24**



$P_1 - P_2 = 125\ kPa$

$\dot{V}_1 = 2.5\ m^3/s$

Incompressible: $\dot{V}_1 = \dot{V}_2 = \dot{V}$

$\rho_w = 997\ kg/m^3,\ \alpha = 1$ ← Turbulent

2. Want to know: $h_{turbine,e}$

4. Want to know: efficiency η_t

$h_L = 3.25m$

$$\frac{P_1}{\rho g} + \alpha \frac{V_1^2}{2g} + \cancel{z_1}^{\,z_1 \approx z_2} + \cancel{h_{pump,u}}^{\,\text{no pump}} = \frac{P_2}{\rho g} + \alpha \frac{V_2^2}{2g} + \cancel{z_2} + h_{turbine,e} + h_L$$

$$h_{turbine,e} = \frac{P_1}{\rho g} - \frac{P_2}{\rho g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_L$$

$$= \frac{P_1 - P_2}{\rho g} + \frac{V_1^2 - V_2^2}{2g} - h_L$$

velocities: $\dot{V} = v_{Avg} A_c$

$$v_1 = \frac{\dot{V}_1}{\frac{\pi}{4}(D)^2} = \frac{2.5\ m^3/s}{\frac{\pi}{4}(0.55m)^2} = 10.52\ m/s$$

$$V_2 = \frac{\dot{V}_2}{\frac{\pi}{4}(D)^2} = \frac{2.5\ m^3/s}{\frac{\pi}{4}(0.45m)^2} = 15.72\ m/s$$

2. $$h_{turbine,e} = \frac{125{,}000 Pa}{997\ kg/m^3 (9.81 m/s^2)} + \frac{(10.52 m/s)^2 - (15.72 m/s)^2}{2(9.81 m/s^2)} - 3.25m = \boxed{2.58\ m}$$

4. $$\eta_t = \frac{\dot{W}_{turbine}}{\dot{W}_{turbine,e}},\quad h_{turbine,e} = \frac{\dot{W}_{turbine,e}}{\dot{m} g},\quad \dot{W}_{turbine} = 35kW$$

$$\dot{m} = \dot{V}\rho = 2.5\ m^3/s\,(997\ kg/m^3) = 2492.5\ kg/s$$

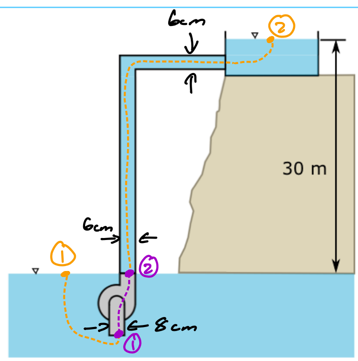
{W}

$$\dot{W}_{turbine,e} = \dot{m} g h_{turbine,e} = (2492.5\ kg/s)(9.81 m/s^2)(2.58\ m) = 63084.68\ \frac{kg \cdot m^2}{s^3}$$

$$= 63.084\ kW$$

$$\eta_t = \frac{35kW}{63.084kW} = 0.5548 \qquad \boxed{\eta_t = 55.48\%}$$

1. [Chapter 12] Underground water is to be pumped by a 78 percent efficienct 5-kW submerged pump to a pool whose free surface is 30 m above the underground water level. The diameter of the pipe is 8 cm on the intake side and 6 cm on the discharge side. Determine (a) the maximum flow rate of water and (b) the pressure difference across the pump. Assume the elevation difference between the pump inlet and outlet and the effect of the kinetic energy correction factors to be negligible.



Given

- $\eta_{pump} = 78\%$
- $\Delta z = z_2 - z_1 = 30\,m$
- $\alpha \approx 1$, $\rho = 997\, kg/m^3$
- $\dot{W}_{pump} = 5\,kW$
- $A_{c2} = \frac{\pi}{4}(0.08m)^2 = 0.005\, m^2$
- $A_{c1} = \frac{\pi}{4}(0.06m)^2 = 0.00283\, m^2$

Find

a) $\dot{m}$

b) $\Delta P = P_2 - P_1$

a) $$\cancel{\frac{P_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + \cancel{z_1} + h_{pump,u} = \cancel{\frac{P_2}{\rho g}} + \cancel{\frac{V_2^2}{2g}} + z_2 + \cancel{h_{turbine,e}} + \cancel{h_L}$$

$z_2 - z_1 = 30m$ pump piping head negligible compared to position

$$h_{pump,u} = 30m$$

$$\frac{\dot{W}_{pump,u}}{\dot{m} g} = 30m$$

$$\eta_{pump} = \frac{\dot{W}_{pump,u}}{\dot{W}_{pump}} \Rightarrow \dot{W}_{pump} = 0.78(5\,kW) = 3.9\,kW = 3900\,W$$

$$\rightarrow \dot{m} = \frac{\dot{W}_{pump,u}}{30m \cdot 9.81 m/s^2} = \frac{3900\,W}{30 \cdot 9.81} = 13.25\; kg/s$$

$$\boxed{\dot{m} = 13.25\; kg/s}$$

b) $z_2 \approx z_1$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1} + h_{pump,u} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{z_2} + \cancel{h_{turbine,e}} + \cancel{h_L}$$

negligible as pipes are so close

$$\frac{1}{\rho g}(P_2 - P_1) = \frac{1}{2g}(V_1^2 - V_2^2) + h_{pump,u}$$

$$V = \frac{\dot{m}}{\rho A_c} \quad \Big| \quad V_1 = \frac{13.25}{997 A_{c2}} = 2.65797\, m/s$$

$$V_2 = \frac{13.25}{997 kg/m^3\, A_{c1}} = 4.69607\, m/s$$

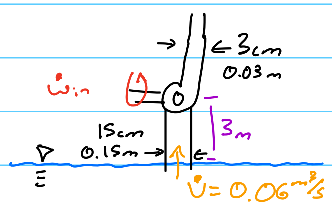
$$\Delta P = P_2 - P_1 = \frac{\rho}{2}(V_1^2 - V_2^2) + \rho g h_{pump,u}$$

$$\Delta P = \frac{997\, kg/m^3}{2}\left((2.66 m/s)^2 - (4.70 m/s)^2\right) + (997\, kg/m^3)(9.81 m/s^2)(30m)$$

$$= 285\,932\; Pa$$

$$\boxed{\Delta P = 286\; kPa}$$

2. [Chapter 12] A fireboat is to fight fires at coastal areas by drawing seawater with density 1030 kg/m^3 through a 15-cm-diameter pipe at a rate of 0.06 m^3/s and discharging it through a nozzle with an exit diamter of 3 cm. The total irreversible head loss of the system is 4 m, and the position of the nozzle is 3 m above sea level. For a pump efficiency of 70 percent, determine the required shaft power input to the pump and the water discharge velocity.



$\rho = 1030\ kg/m^3$ $\quad \eta_{pump} = \dfrac{\dot{W}_{pump,u}}{\dot{W}_{pump}} = 70\% = 0.70$

$h_L = 4m$ $\quad \dot{W}_{pump,u} = \dot{m} g h_{pump,u}$

$z_2 = 3m$ $\quad$ Find $\dot{W}_{pump} = \dfrac{\dot{m} g h_{pump,u}}{\eta_{pump}}$, V_2

$P_2 \approx P_1 = P_{atm}$ $\quad$ Sea level $\quad z_2 = 3m$ $\quad h_L = 4m$

$$\cancel{\frac{P_1}{\rho g}}^{0} + \frac{V_1^2}{2g} + \cancel{z_1}^{0} + h_{pump,u} = \cancel{\frac{P_2}{\rho g}}^{0} + \frac{V_2^2}{2g} + z_2 + \cancel{h_{turbine,e}}^{0} + h_L$$

$$h_{pump,u} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + z_2 + h_L = \frac{1}{2g}\left(V_2^2 - V_1^2\right) + 3m + 4m$$

$$V_1 = \frac{\dot{V}}{A_{c_1}} = \frac{0.06\ m^3/s}{\frac{\pi}{4}(0.15m)^2} = 3.40\ m/s \qquad h_{pump,u} = \frac{1}{2(9.81)}\left(84.88^2 - 3.40^2\right) + 7m$$

$$= 373.618\ m$$

$$V_2 = \frac{\dot{V}}{A_{c_2}} = \frac{0.06\ m^3/s}{\frac{\pi}{4}(0.03m)^2} = \underline{84.88\ m/s}$$

$$\rightarrow \dot{W}_{pump} = \frac{(61.8\ kg/s)(9.81\ m/s^2)(373.618m)}{0.70}$$

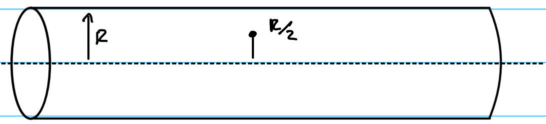
$$\dot{m} = \rho \dot{V}$$

$$\dot{m} = (1030\ kg/m^3)(0.06\ m^3/s) \qquad = 323584\ Pa$$

$$= 61.8\ kg/s$$

$$\boxed{\begin{aligned} \dot{W}_{pump} &= 324\ kPa \\ V_2 &= 84.88\ m/s \end{aligned}}$$

3. [Chapter 14] In fully developed laminar flow in a circular pipe the velocity at R/2 (midway between the wall surface and the centerline) is measured to be 13 m/s. Determine the velocity at the center of the pipe.



$u(r = R/2) = 13\,m/s$

<u>Find</u> $u(R=0)$ ← maximum velocity

↑ center of pipe

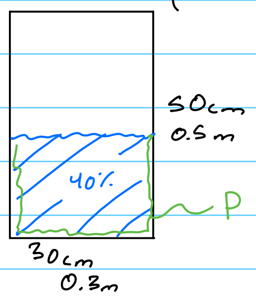
$$\left. u(r) = u_{max}\left(1 - \frac{r^2}{R^2}\right)\right|_{r=R/2} \qquad 13\,m/s = u_{max}\left(1 - \frac{R^2/4}{R^2}\right)$$

$$u(r = R/2) = u_{max}\left(1 - \frac{(R/2)^2}{R^2}\right) \qquad u_{max} = \frac{13\,m/s}{3/4} = \frac{13 \cdot 4}{3}\,m/s = \frac{52}{3}\,m/s \approx 17.33\,m/s$$

$$\boxed{u_{max} = 17.33\,m/s}$$

Trevor Swan **ECHE225: QUIZ #8 // 10.31.24**

1. This question is not related to this week's homework



- $A_c = 0.3m \cdot 0.4(.5m)$
  $= 0.06\ m^2$ ($0.4 \times 0.5 = 0.2$)
- $P = (0.3) + 2(0.2m)$
  $= 0.7$ wetted perimeter
- Steady-state, laminar flow desired
- $SG = 0.85$, $V_{Avg} = 2\ m/s$

$Re_{crit} = 2300$ (laminar flow)

$Re = \dfrac{\rho V_{Avg} D_h}{\mu}$

$\rightarrow \mu = \dfrac{\rho V_{Avg} D_h}{Re}$

$D_h = \dfrac{4 A_c}{P}$ $\rho = SG \cdot \rho_{water}$

$\rho = 0.85 \cdot 1000\ kg/m^3 = 850\ kg/m^3$

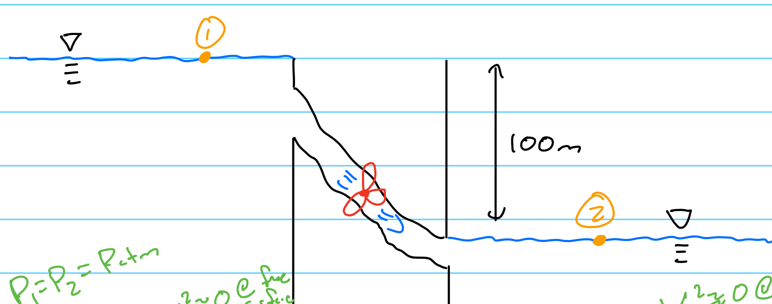
$D_h = \dfrac{4(0.06\ m^2)}{0.7m} = 0.3429m$

$\mu = \dfrac{850\ kg/m^3 \cdot 2m/s \cdot 0.3429m}{2300}$

$= 0.2534\ \dfrac{kg}{m \cdot s}$

$$\boxed{\mu = 0.25\ \frac{kg}{m \cdot s}}$$

2.



$\rho_w = 1000\ kg/m^3$

$\eta_{turbine} = 0.85 = \dfrac{\dot{W}_{turbine}}{\dot{W}_{turbine,e}}$

$\dot{W}_{turbine} = 100kW$

Find $\dot{m}$

$P_1 = P_2 = P_{atm}$ $V_1^2 \approx 0$ @ free surface $z_1 - z_2 = 100m$ $V_2^2 \approx 0$ @ free surface Given

$$\cancel{\frac{P_1}{\rho g}}^{0} + \cancel{\frac{V_1^2}{2g}}^{0} + z_1 + \cancel{h_{pump,u}}^{0} = \cancel{\frac{P_2}{\rho g}}^{0} + \cancel{\frac{V_2^2}{2g}}^{0} + \cancel{z_2} + h_{turbine,e} + \cancel{h_L}^{0}$$

$h_{turbine,e} = 100\ m$

$\dot{W}_{turbine,e} = \dfrac{\dot{W}_{turbine}}{\eta_{turbine}} = \dfrac{100kW}{0.85} = 117.647\ kW = 117{,}647W$ $\left\{ \dfrac{kg \cdot m^2}{s^3} \right\}$

$h_{turbine,e} = \dfrac{\dot{W}_{turbine,e}}{\dot{m} g} \rightarrow \dot{m} = \dfrac{\dot{W}_{turbine,e}}{g \cdot h_{turbine}} = \dfrac{117{,}647\ W}{(9.81 m/s^2)(100m)} = 119.93\ kg/s$

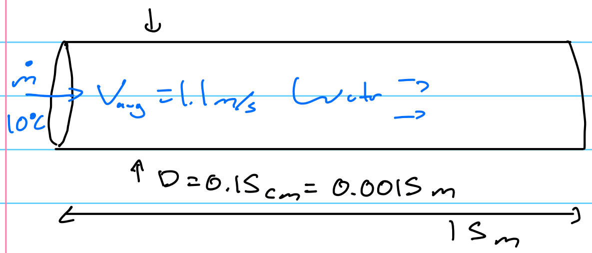
$$\boxed{\dot{m} = 119.93\ kg/s}$$

1. [Chapter 14] Water at 10°C ($\rho = 999.7$ kg/m^3, $\mu = 1.307\times10^{-3}$ kg/m·s) is flowing steadily in a 0.15-cm-diameter, 15-m-long pipe at an average velocity of 1.1 m/s. Determine:

   (a) pressure drop

   (b) head loss

   (c) pumping power required to overcome this pressure drop.



$\rho = 999.7$ kg/m^3

$\mu = 1.307\times10^{-3}$ kg/m·s

Steady flow

$$Re = \frac{(999.7\,\text{kg/m}^3)(1.1\,\text{m/s})(.0015\,\text{m})}{1.307\times10^{-3}\,\frac{\text{kg}}{\text{m}\cdot\text{s}}} < 2300$$ Laminar Flow

a) Flow is Laminar

$$\Delta P_L = \frac{32 \mu L u_{avg}}{D^2} = \frac{32(1.307\times10^{-3}\,\frac{\text{kg}}{\text{m}\cdot\text{s}})(15\,\text{m})(1.1\,\text{m/s})}{(0.0015\,\text{m})^2} = 306709\text{ Pa}$$

$$\boxed{\Delta P_L = 306.7\text{ kPa}}$$

b) $h_L = \frac{\Delta P_L}{\rho g}$ $\Rightarrow$ $h_L = \frac{306709.33\,\frac{\text{kg}}{\text{m}\cdot\text{s}^2}}{(999.7\,\text{kg/m}^3)(9.81\,\text{m/s}^2)} = 31.27$ $\boxed{h_L = 31.3\text{ m}}$

derived from bernoulli:

c) $\frac{\dot{W}_{pump}}{\dot{m} g} = h_L \Rightarrow \dot{W}_{pump} = (31.27\,\text{m})\dot{m}(9.81\,\text{m/s}^2) = (31.27\,\text{m})(0.00194\,\text{kg/s})(9.81\,\text{m/s}^2)$

$\dot{m} = \rho V_{avg} A_c$

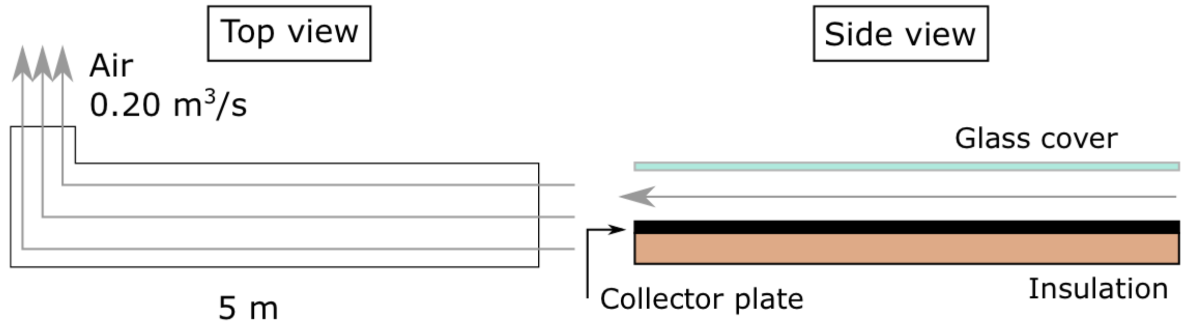
$= 999.7\,\frac{\text{kg}}{\text{m}^3}\cdot 1.1\,\text{m/s}\cdot\frac{\pi}{4}(0.0015\,\text{m})^2$

$= 0.00194$ kg/s

$= 0.5951\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}^3}$ {W}

$$\boxed{\dot{W}_{pump} = 0.595\text{ W}}$$

2. [Chapter 14] Consider an air solar collector that is 1 m wide and 5 m long and has a constant spacing of 3 cm between the glass cover and the collector plate. Air flows at an average temperature of 45°C at a rate of 0.20 m^3/s through the 1-m-wide edge of the collector along the 5-m-long passageway. Disregarding the entrance and roughness effects and the 90° bend, determine the pressure drop in the collector.



$$Re = \frac{\rho V_{avg} D_h}{\mu} = \frac{(1.205\ \frac{kg}{m^3})(6.67\ m/s)(0.0583\ m)}{(1.82\times10^{-5}\ \frac{kg}{m\cdot s})} = 25746... > 2300 \quad \text{Turbulent!}$$

$$\dot{V} = \frac{\dot{m}}{\rho} \rightarrow \dot{V} = V_{avg} A_c \Rightarrow V_{avg} = \frac{\dot{V}}{A_c} = \frac{0.20\ m^3/s}{(1\,m)(.03\,m)} = 6.67\ m/s, \quad \rho = 1.205\ kg/m^3$$

$$D_h = \frac{4A_c}{P} = \frac{4(1\,m\cdot .03\,m)}{2(1\,m)+2(.03\,m)} = 0.0583\ m$$

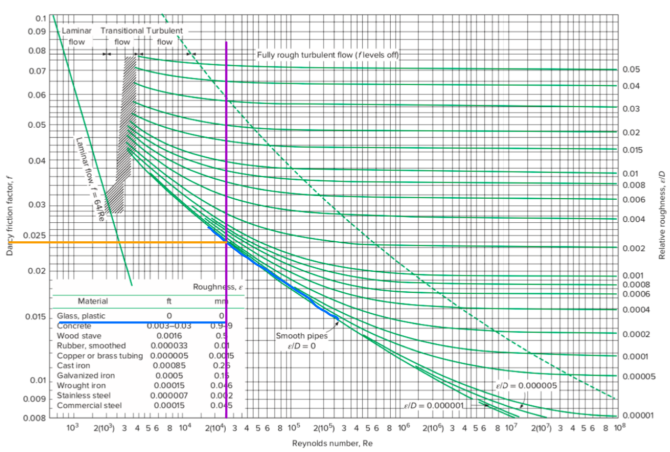


FIGURE A–27
The Moody chart for the friction factor for fully developed flow in circular pipes for use in the head loss relation $\Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2}$. Friction factors in the turbulent flow are evaluated from the Colebrook equation $\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{Re\sqrt{f}}\right)$.

$$f = 0.024, \quad \Delta P_L = f\frac{L}{D}\cdot\frac{1}{2}\rho U_{avg}^2$$

$$\Delta P_L = 0.024\left(\frac{5\,m}{.0583\,m}\right)\cdot\left(\frac{1}{2}\cdot 1.205\frac{kg}{m^3}\cdot(6.67\ m/s)^2\right)$$

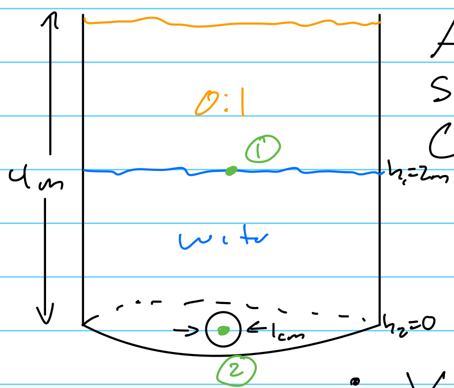
$$= 55.2\ \frac{kg}{m\cdot s^2}$$

$$\boxed{\Delta P_L = 55.2\ Pa}$$

Reasonably close considering rounding!

what the fuck

3. [Chapter 14] A 4-m high cylindrical tank having a cross-sectional area $A_T = 1.5$ m^2 is filled with equal volumes of water and oil. The specific gravity of the oil is 0.75. A 1 cm-diameter hole at the bottom of the tank is opened and water starts to flow out. If the discharge coefficient of the hole is $C_d = 0.82$, determine how long it will take for the water in the tank to completely drain. Assume that the tank is open to the atmoshere. The discharge coefficient represents a fraction of the expected volumetric flow rate through the opening: $\dot{V} = C_d V A$.



$A_T = 1.5 m^3$

$SG_{oil} = 0.75 \Rightarrow \rho_{oil} = 775 \ kg/m^3$

$C_d = 0.82$ where $\dot{V} = C_d \cdot V \cdot A$

$$\frac{P_1}{\rho_w g} + \frac{\cancel{V_1^2}}{2g} + z_1 = \frac{\cancel{P_2}}{\rho_w g} + \frac{V_2^2}{2g} + \cancel{z_2}$$

$V_1^2 \approx 0$ $\quad z_1 = h_2$ $\quad P_2 = P_{atm} \to 0$ Gauge press $\quad z_2 = 0$

$\Rightarrow P_1 = \rho_{oil} \cdot g \cdot h_1$

$$\therefore V_2 = \sqrt{2g\left(\frac{\rho_{oil} \cdot g \cdot h_1}{\rho_w \cdot g} + h_2\right)} = \sqrt{2g(SG \cdot h_1 + h_2)}$$

$\dot{V} = C_d V \cdot A$ $\qquad A_2 = \frac{\pi}{4}(.01)^2$

$\Rightarrow \dot{V}_2 = C_d \cdot V_2 \cdot A_2$ $\qquad = 7.85 \times 10^{-5} m^2$

$\therefore \dot{V}_2 = C_d \cdot \sqrt{2g(SG \cdot h_1 + h_2)} \cdot 7.85 \times 10^{-5} m^2$ $\quad$ also $\dot{V}_2 = -A_{Tank} \frac{dh_2}{dt}$

$$\Rightarrow -A_T \frac{dh_2}{dt} = (7.85 \times 10^{-5} m^2) C_d \sqrt{2g(SG \cdot h_1 + h_2)}$$

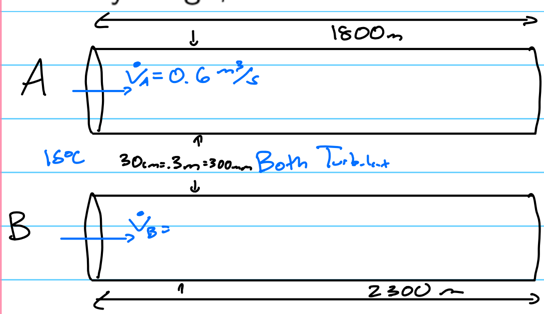
$$\text{so} \int_{h_2}^{0} \frac{dh_2}{\sqrt{2g(SG \cdot h_1 + h_2)}} = -\frac{(7.85 \times 10^{-5}) C_d}{A_T} \int_0^t dt$$

$$t = -\frac{A_T}{(7.85 \times 10^{-5}) C_d} \int_{h_2}^{0} \frac{dh_2}{\sqrt{2g(SG \cdot h_1 + h_2)}}$$

$$= -\frac{1.5 m^2}{(7.85 \times 10^{-5} m^2)(0.82)} \cdot \int_2^0 \frac{dh_2}{\sqrt{2(9.81)(0.75 \cdot 2m + h_2)}} = 6797.94 \text{ seconds}$$

$$\boxed{t = 1.89 \text{ hours}}$$

4. [Chapter 14] A certain part of the cast iron piping of a water distribution system involves a parallel section. Both parallel pipes have a diameter of 30 cm and the flow is fully turbulent. One of the branches (pipe A) is 1800 m long while the other branch (pipe B) is 2300 m long. If the flow rate through pipe A is 0.6 m^3/s, determine the flow rate through pipe B. Disregard minor loss and assume the water temperature to be 15°C. Show that the flow is fully rough, and thus the friction factor is independent of Reynolds number.



For parallel pipes:

$\dot{V}_{Tot} = \dot{V}_A + \dot{V}_B$

&

$\Delta P_A = \Delta P_B$

$\dot{V} = V_{Avg} \cdot A_c$

Assume $\rho = 1000\ kg/m^3$

We will assume fully rough, so $f_1 = f_2$

ε/D is constant

Assumption

Same fluid

$$\cancel{f} \cdot \frac{L_A}{\cancel{D}} \cdot \frac{1}{\cancel{2}}\cancel{\rho} V_{A,avg}^2 = \cancel{f} \cdot \frac{L_B}{\cancel{D}} \cdot \frac{1}{\cancel{2}}\cancel{\rho} V_{B,avg}^2 \quad \text{, where } V_{A,avg} = \frac{\dot{V}_A}{A_c} \text{ and } V_{B,avg} = \frac{\dot{V}_B}{A_c}$$

Same diameter

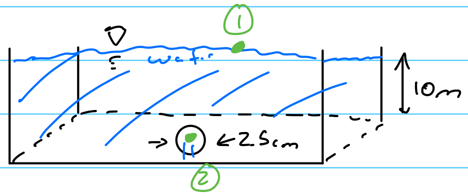
$$L_A \cdot V_{A,avg}^2 = L_B \cdot V_{B,avg}^2$$

$$L_A \left(\frac{\dot{V}_A}{A_c}\right)^2 = L_B \left(\frac{\dot{V}_B}{A_c}\right)^2$$

$$\boxed{\dot{V}_B = 0.531\ m^3/s}$$

$$L_A \left(\frac{\dot{V}_A^2}{\cancel{A_c^2}}\right) = L_B \left(\frac{\dot{V}_B^2}{\cancel{A_c^2}}\right) \Rightarrow \dot{V}_B = \dot{V}_A \sqrt{\frac{L_A}{L_B}} = 0.6\ m^3/s \sqrt{\frac{1800m}{2300m}} = 0.5308$$

5. [Chapter 14] Water is to be withdrawn from a 10-m-high reservoir by drilling a 2.5-cm-diameter hole at the bottom surface. Disregarding the effect of the kinetic energy correction factor, determine the flow rate of water through the hole if (a) the entrance of the hole is well-rounded and (b) the entrance is sharp-edged.



$P_1 = P_2 = P_{atm}$, $V_1^2 \approx 0$, $z_1 = 10m$, $z_2 = 0m$

$$\cancel{\frac{P_1}{\rho g}} + \alpha_1 \cancel{\frac{V_1^2}{2g}} + z_1 + \cancel{h_{pump,u}}^{\,0} = \cancel{\frac{P_2}{\rho g}} + \alpha_2\frac{V_2^2}{2g} + \cancel{z_2} + \cancel{h_{turbine,e}}^{\,0} + h_L$$

where $h_L = \frac{V_2^2}{2g}\cdot K_L$ and $\alpha = 1$

$$z_1 = \frac{V_2^2}{2g} + K_L\frac{V_2^2}{2g} = V_2^2\left(\frac{1}{2g} + \frac{K_L}{2g}\right)$$

$$\Rightarrow V_2^2 = z_1\left(\frac{1}{2g} + \frac{K_L}{2g}\right)^{-1} \rightsquigarrow V_2 = \sqrt{10m\left(\frac{1}{2(9.81 m/s^2)} + \frac{K_L}{2(9.81 m/s^2)}\right)^{-1}}$$

a) Well-rounded; $K_L = 0.03$

$$V_2 = \sqrt{10m\left(\frac{1}{2(9.81 m/s^2)} + \frac{0.03}{2(9.81 m/s^2)}\right)^{-1}} = 13.8016\ m/s$$

$$\dot{V}_2 = \frac{V_2}{A_c} = (13.8016\ m/s)\left(\frac{\pi}{4}(.025m)^2\right) = 0.006775\ m^3/s$$

b) Sharp-edged; $K_L = 0.50$

$$V_2 = \sqrt{10m\left(\frac{1}{2(9.81 m/s^2)} + \frac{0.50}{2(9.81 m/s^2)}\right)^{-1}} = 11.4368\ m/s$$

$$\dot{V}_2 = (11.4368\ m/s)\left(\frac{\pi}{4}(.025m)^2\right) = 0.005614\ m^3/s$$

$$\boxed{a)\ \dot{V}_2 = 6.77\times10^{-3}\ m^3/s \quad \text{and} \quad b)\ 5.61\times10^{-3}\ m^3/s}$$

Trevor Swan **ECHE225: QUIZ #9 // 11.07.24**

1.

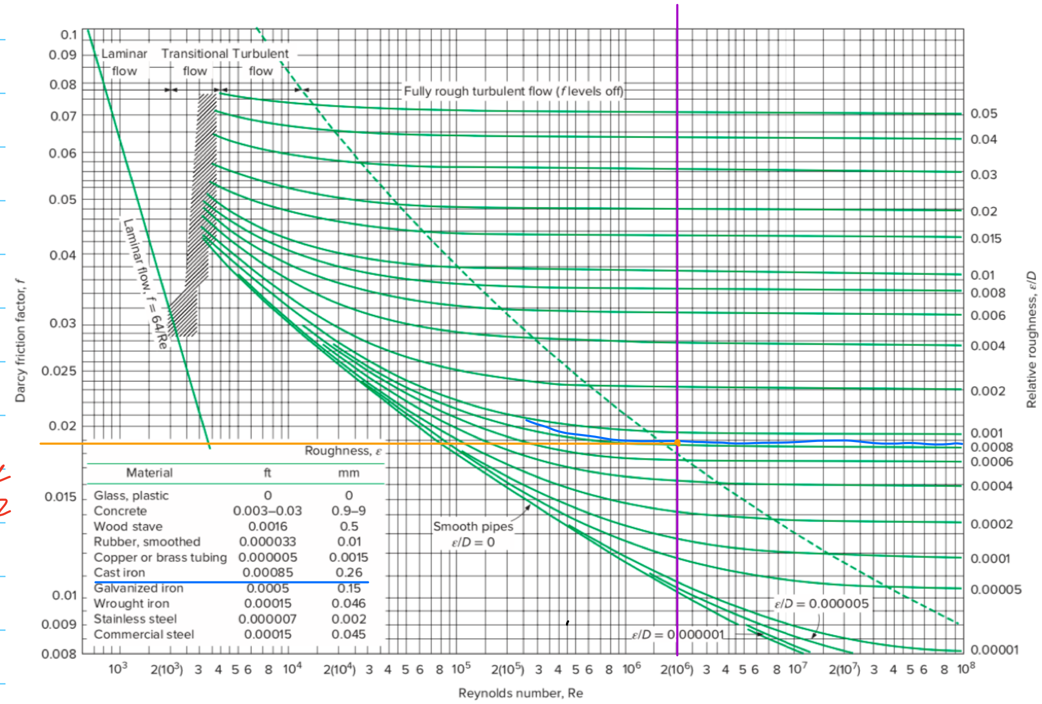


FIGURE A–27

The Moody chart for the friction factor for fully developed flow in circular pipes for use in the head loss relation $\Delta P_L = f \frac{L}{D}\frac{\rho V^2}{2}$. Friction factors in the turbulent flow are evaluated from the Colebrook equation $\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$.

Cast Iron

$\varepsilon = 0.26\,mm$

Diameter

$30\,cm = 300\,mm$

$\varepsilon/D = 0.00087$

$Re = 2\times10^6$

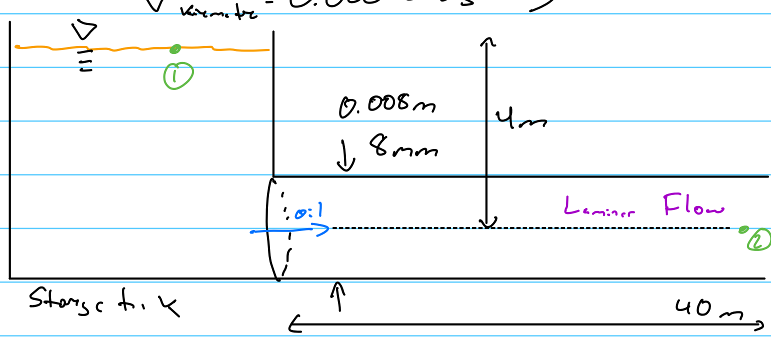
$0.015 < f < 0.02$

Correction on next page

~~2.~~ Oil: $\rho_{oil} = 850\,kg/m^3$

$\nu_{kinematic} = 0.00062\,m^2/s$

$\mu_{Dynamic} = \nu_{kinematic}\cdot\rho = 0.00062\,m^2/s \cdot 850\,kg/m^3 = 0.527\,\frac{kg}{m\cdot s}$



$P_1 = P_2 = P_{hydro}$

$\cancel{\frac{P_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + z_1 = \cancel{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + \cancel{z_2}$ $\quad z_1 = 4m$, free surface

$V_2 = \sqrt{2 g z_1}$

$= \sqrt{2(9.81\,m/s^2)(4m)}$

$= 8.8589\,m/s$

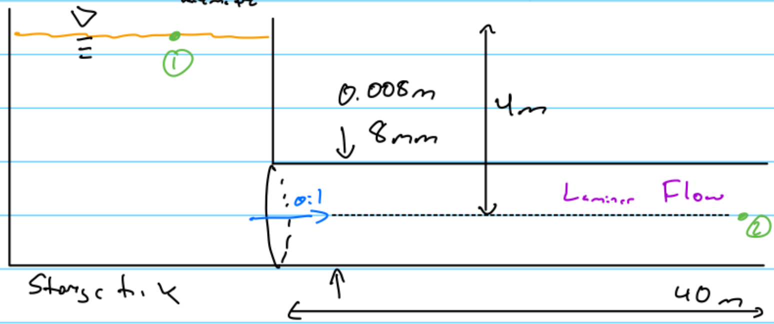
$\Delta P_2 = \frac{32\mu L u_{avg}}{D^2} = \frac{32\,(0.527\,\frac{kg}{m\cdot s})(40m)(8.86\,m/s)}{(0.008m)^2} = 93384400\,\frac{kg}{m\cdot s^2}$

$\boxed{\Delta P_2 = 93384.400\ kPa}$ $\quad \boxed{V_2 = 8.8589\,m/s}$

$Re = \frac{\rho u_{avg} D}{\mu} = \frac{850\,kg/m^3\cdot(8.8589\,m/s)(0.008m)}{0.527\,kg/m\cdot s} = 114.3084$

$\boxed{Re = 114.3084}$

3. Oil: $\rho_{oil} = 850\ kg/m^3$, $\nu_{kinematic} = 0.00062\ m^2/s$ $\Big\}$ $\mu_{Dynamic} = \nu_{kinematic} \cdot \rho = 0.00062\ m^2/s \cdot 850\ kg/m^3 = 0.527\ \frac{kg}{m\cdot s}$



$P_1 = P_2 = P_{hydro}$

$$\cancel{\frac{P_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}}^{\,0} + z_1 = \cancel{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + \cancel{z_2}^{\,0}$$

$z_1 = 4m$, free surface

$$V_2 = \sqrt{2 g z_1} = \sqrt{2(9.81\ m/s^2)(4m)} = 8.8589\ m/s$$

$$\Delta P_2 = \frac{32 \mu L u_{avg}}{D^2} = \frac{32\,(0.527\ \frac{kg}{m\cdot s})(40m)(8.86\ m/s)}{(0.008m)^2} = 93384400\ \frac{kg}{m\cdot s^2}$$

$\boxed{\Delta P_2 = 93384.400\ kPa}$ $\boxed{V_2 = 8.8589\ m/s}$

$$Re = \frac{\rho u_{avg} D}{\mu} = \frac{850\ kg/m^3 \cdot (8.8589\ m/s)(0.008m)}{0.527\ kg/m\cdot s} = 114.3084$$

$\boxed{Re = 114.3084}$

$$P_{1\,gauge} = \rho g h = (850\ kg/m^3)(9.81\ m/s^2)(4m) = 33354\ \underbrace{\frac{kg\cdot m}{s^2}}_{\{N\}} = 33.354\ kN/m^2$$

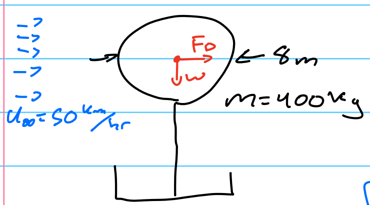
$$\Delta P = P_1 - P_2 = P_1 - P_{atm} = P_{gauge} = 33.354\ kPa$$

$$\dot{V} = \frac{\Delta P \pi D^4}{128 \mu L} = \frac{(33354\ \frac{kg\cdot m}{s^2\cdot m^2})\pi(0.008m)^4}{128(.527\ \frac{kg}{m\cdot s})(40m)} = 1.59\times10^{-7}\ \frac{\frac{kg\cdot m^3}{s^2}}{kg/s} = 1.59\times10^{-7}\ m^3/s$$

$$V_{avg} = \frac{\dot{V}}{A_c} = \frac{1.59\times10^{-7}\ m^3/s}{\frac{\pi}{4}(0.008m)^2} = 3.164\times10^{-3}\ m/s$$

$$Re = \frac{\rho V_{avg} D}{\mu} = \frac{(850\ kg/m^3)(3.164\times10^{-5}\ m/s)(0.008m)}{0.527\ kg/m\cdot s} = \underbrace{0.0408}_{\ll 2300} \Rightarrow \boxed{\text{Laminar Flow!}}$$

1. [Chapter 15] An 8-m-diameter hot air ballooon that has a total mass of 400 kg is standing still in air on a windless day. The balloon suddenly is subjected to a 50 km/hr wind. Determine the initial acceleration of the balloon in the horizonal direction.



$\Sigma F_x = m a_x \Rightarrow a_x = \frac{F_D}{m}$

$F_D = C_D \frac{1}{2} \rho u_\infty^2 A$

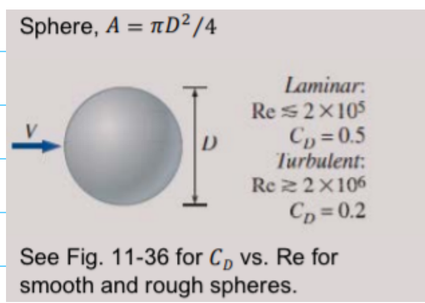
$\rho_{air} = 1.205\ kg/m^3$ and $\mu = 1.82 \times 10^{-5}\ \frac{kg}{m \cdot s}$

frontal area $A = \pi r^2 = \frac{\pi}{4} D^2 = 16\pi\ m^2 = 50.266\ m^2$

$u_\infty = 50\ km/hr = 13.89\ m/s$

$C_D = ?$ Tables!

C_D Tables!



$Re = \frac{\rho u_\infty D}{\mu} = \frac{(1.205\ \frac{kg}{m^3})(13.89\ m/s)(8\ m)}{(1.82 \times 10^{-5}\ \frac{kg}{m \cdot s})}$

$= 7.36 \times 10^6 \Rightarrow$ Turbulent

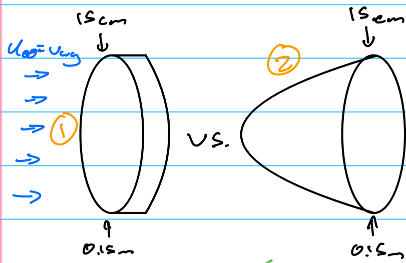
$\therefore$ Use $C_D = 0.2$

$a_x = \frac{1}{m} F_D = \frac{1}{400kg} \cdot \left(0.2 \cdot \frac{1}{2} \cdot 1.205\ \frac{kg}{m^3} \cdot (13.89\ m/s)^2 \cdot 50.266\ m^2\right)$

$= 2.92\ m/s^2$

$\boxed{a_x = 2.92\ m/s^2}$

2. [Chapter 15] To reduce the drag coefficient and thus improve the fuel efficiency of cars, the design of side rearview mirrors has changed dramatically in recent decades from a simple circular plate to a streamlined shape. Determine the amount of fuel and money saved per year as a result of replacing a 15-cm-diameter flat mirror by one with a hemispherical back. Assume the car is driven 25,000 km a year at an average speed of 95 km/hr. Take the density and price of gasoline to be 0.75 kg/L and $0.90/L, respectively; the heating value of gasoline to be 44,000 kJ/kg and the overall efficiency of the engine to be 30 percent.



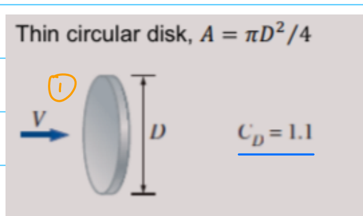
- 25000 km/year driven
- $V_{avg} = 95$ km/hr $= 26.39$ m/s
- $\rho_{gas} = 0.75$ kg/L, $0.90/L
- Heating value of gas: 44,000 kJ/kg
- $\eta_{engine} = 30\%$

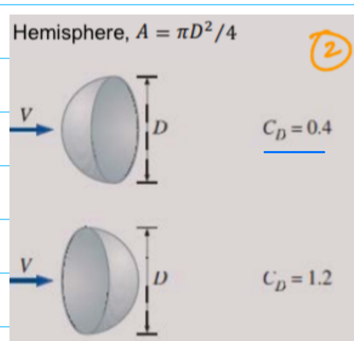
Air
$\rho = 1.205$ kg/m³
$\mu = 1.82\times10^{-5}$ kg/m·s

$$F_D = \tfrac{1}{2} C_D \rho u_\infty^2 A$$

where $A = \frac{\pi}{4}(0.15\,m)^2 = 0.017671S\ m^2$

$$Re_D = \frac{\rho u_\infty D}{\mu} = \frac{(1.205\ kg/m^3)(26.39\ m/s)(0.15\,m)}{1.82\times10^{-5}\ \frac{kg}{m\cdot s}} = 2.62\times10^5$$





① $F_D = \frac{1}{2}\cdot 1.1 \cdot 1.205\ \frac{kg}{m^3}\cdot(26.39\ m/s)^2 \cdot A\ m^2$
$= 8.156\ \frac{kg\cdot m}{s}$

② $F_D = \frac{1}{2}\cdot 0.4 \cdot 1.205\ \frac{kg}{m^3}\cdot(26.39\ m/s)^2 \cdot A\ m^2$
$= 2.966\ \frac{kg\cdot m}{s^2}$

$E_{drag} = F_D \cdot \text{Distance}$ $\left\{\frac{kJ \text{ or } J}{time}\right\}$

① $E_{drag} = 8.156\ \frac{kg\cdot m}{s}\cdot 25000000\ \frac{m}{year}$
$= 2.04\times10^8\ \frac{J}{year} = 2.04\times10^5\ \frac{kJ}{year}$

② $E_{drag} = 2966\ \frac{kg\cdot m}{s}\cdot 25000000\ \frac{m}{year}$
$= 7.415\times10^7\ \frac{J}{year} = 7.42\times10^4\ \frac{kJ}{year}$

$E_{required} = \dfrac{E_{drag}}{0.30}$

① $E_{required} = 6.8\times10^5\ \frac{kJ}{year}$

② $E_{required} = 2.47\times10^5\ \frac{kJ}{year}$

$\text{Fuel} = \dfrac{E_{required}}{\text{Heating value}\cdot\rho_{gas}}$ $\left\{\frac{L}{year}\right\}$

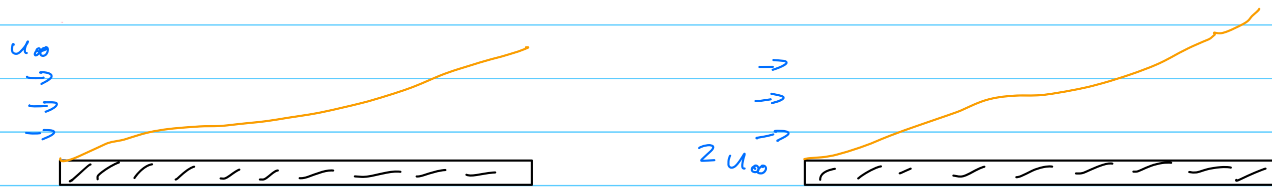
Fuel required and money required

① $\text{Fuel} = \dfrac{6.8\times10^5\ \frac{kJ}{year}}{(44{,}000\ \frac{kJ}{kg})(0.75\ \frac{kg}{L})} = 20.6\ \frac{L}{year}$ so $18.54/year

② $\text{Fuel} = \dfrac{2.47\times10^5\ \frac{kJ}{year}}{(44{,}000\ \frac{kJ}{kg})(0.75\ \frac{kg}{L})} = 7.48\ \frac{L}{year}$ so $6.73/year

Saving
≈ $13.12/year
≈ $11.81/year

3. [Chapter 15] Consider laminar flow of a fluid over a flat plate. Now the freestream velocity is doubled. Determine the change in the drag force on the plate. Assume that the flow remains laminar.



$$F_D = C_f \tfrac{1}{2}\rho u_\infty^2 A$$

$$\frac{F_{D_2}}{F_{D_1}} = \frac{C_{f_2} \cancel{\tfrac{1}{2}\rho} (2\cancel{u_\infty})^2 \cancel{A}}{C_{f_1} \cancel{\tfrac{1}{2}\rho} (\cancel{u_\infty})^2 \cancel{A}} = \frac{4C_{f_2}}{C_{f_1}}$$

$$F_{D_2} = \frac{4C_{f_2}}{C_{f_1}} \cdot F_{D_1} = F_{D_2} = \frac{4\left(\cancel{1.33}/Re_{L_2}^{1/2}\right)}{\cancel{1.33}/Re_{L_1}^{1/2}} F_{D_1} = 4\left(\frac{Re_{L_1}^{1/2}}{Re_{L_2}^{1/2}}\right) F_{D_1} = 4\left(\frac{\frac{\cancel{\rho u_\infty L}}{\cancel{\mu}}}{\frac{\cancel{\rho} 2\cancel{u_\infty L}}{\cancel{\mu}}}\right)^{1/2} F_{D_1}$$

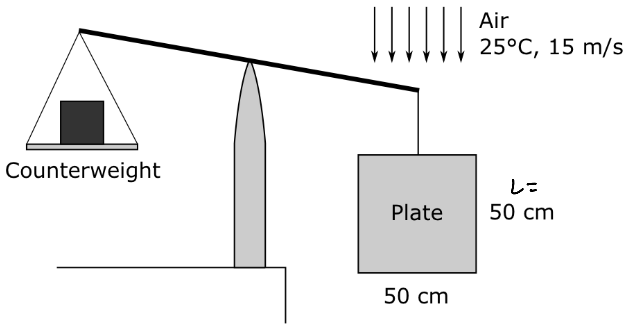
$$C_f = \frac{1.33}{Re_{L}^{1/2}}$$

$$Re_L = \frac{\rho u_\infty L}{\mu}$$

$$= 4\left(\frac{1}{2}\right)^{1/2} F_{D_1}$$

$$\boxed{F_{D_2} = 2.83 F_{D_1}}$$

4. [Chapter 15] The weight of a thin flat plate 50 cm x 50 cm in size is balanced by a counterweight that has a mass of 2 kg. Now a fan is turned on and air at 1 atm and 25°C flows downward over both surfaces of the plate (front and back in the sketch) with a freestream velocity of 15 m/s. Determine the mass of the counterweight that needs to be added in order to balance the plate in this case.



$\rho_{air} = 1.184 \ kg/m^3$

$\mu_{air} = 1.84 \times 10^{-5} \ \frac{kg}{m \cdot s}$

$u_\infty = 15 \ m/s$

$$Re_L = \frac{(1.184 \ kg/m^3)(15 \ m/s)(0.50 \ m)}{1.84 \times 10^{-5} \ \frac{kg}{m \cdot s}} = 4.83 \times 10^5$$

$< 5 \times 10^5 \Rightarrow$ Laminar

$F_D = C_f \frac{1}{2} \rho u_\infty^2 A$ where $C_f = \frac{1.33}{Re_L^{1/2}}$

$C_f = 1.33/(4.82 \times 10^5)^{1/2} = 0.00192$

2 sides of the plate so $C_f = 0.00384$

$F_D = 0.00384 \cdot \frac{1}{2} \cdot 1.184 \frac{kg}{m^3} \cdot (15 \ m/s)^2 \cdot (0.50 \ m)^2 = 0.1279 \ N$

$\vec{W} = mg \Rightarrow m = \frac{\vec{W}}{g} = \frac{0.1279 \ N}{9.81 \ m/s^2} = 0.013 \ kg$

$\boxed{m = 13 g}$

5. [Chapter 15] A jumbo jet airplane has a mass of about 400,000 kg when fully loaded with over 400 passengers and takes off at a speed of 280 km/hr. Determine the takeoff speed when the airplane has 100 empty seats assuming that each passenger with luggage is 150 kg and the wing and flap settings are maintained the same.

$$V_{take\text{-}off} = \sqrt{\frac{2W}{\rho C_L A}}$$

$400{,}000$ kg $\Rightarrow V_{to} = 280 \frac{km}{hr}$

$$\frac{V_{to,2}}{V_{to,1}} = \frac{\sqrt{2W_2/\cancel{\rho C_L A}}}{\sqrt{2W_1/\cancel{\rho C_L A}}} = \sqrt{\frac{m_2 \cancel{g}}{m_1 \cancel{g}}} = \sqrt{\frac{m_2}{m_1}}$$

$$\boxed{V_{to,2} = 274.7 \frac{km}{hr}}$$

$$V_{to,2} = V_{to,1}\sqrt{\frac{m_2}{m_1}} \quad \text{so} \quad V_{to,2} = 280 \,{}^{km}/_{hr} \sqrt{m_2/400{,}000\,kg}$$

For 100 less passengers, 15,000 kg less

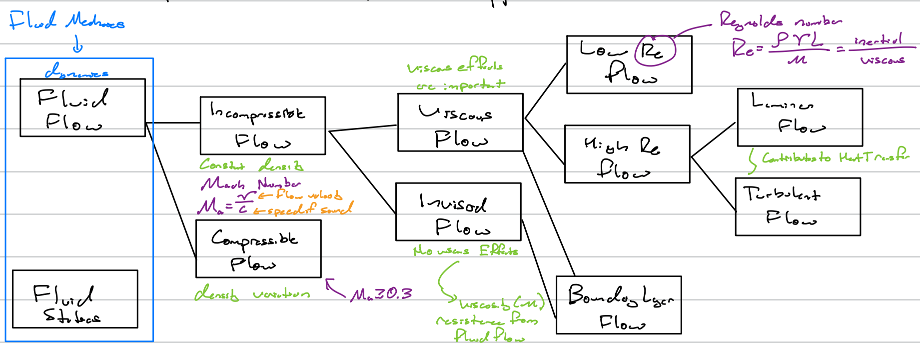
$\Rightarrow m_2 = 385{,}000$ kg

$$V_{to,2} = 280 \,{}^{km}/_{hr} \sqrt{\frac{385{,}000\,kg}{400{,}000\,kg}} = 274.7 \frac{km}{hr}$$

Exam 2 Notes

Fluid Mechanics

- Study of stationary and moving fluids (statics & dynamics)
- Fluids: Captures both liquids & gases
- Applications
  1) Aerodynamics: Flow => forces (lift, drag)
  2) Hydrodynamics – ships, subs, etc.
  3) Fluid Power – hydraulics, pneumatics (lpt)
  4) Fluid Handling – Pipe Flows
  5) Hydrostatics
  6) Bio fluid Flows
  7) Geophysical Flows – atmospheric circulation, ocean currents
  8) Astrophysics / Motion of cosmic objects & galaxies
  9) Shock Waves ← compressible flow phenomenon
- Extreme range of length and time scales
  → split up entire space of applications into various regimes



Governing equations

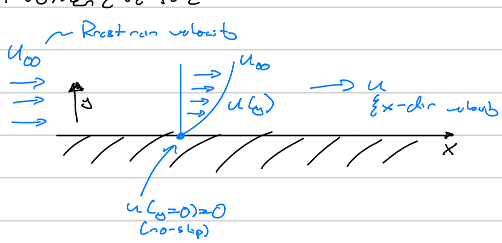
- Can be quite complex

Navier-Stokes Equation (x-dir only):
$$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \rho g_x$$
F_{ey} $F = ma$ ↳ not covered in this course

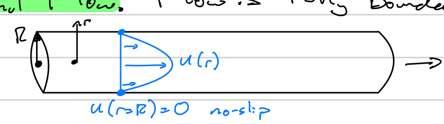
↳ non-linear, partial differential equations

- We will rely on empirical relations, simplified geometries for engineering applications

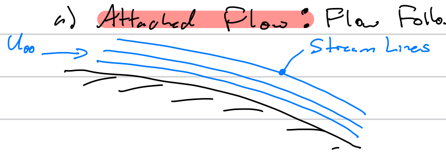
Nomenclature



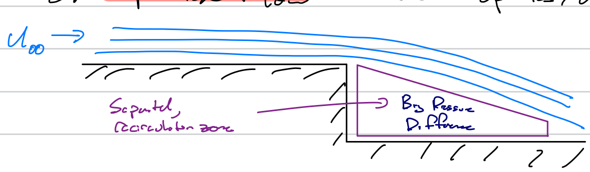
- No-slip condition: Fluid adjacent to a solid surface takes the velocity of that surface
- External Flow: Surface in an infinite flow field (can move far enough away from it and not see it)
- Internal Flow: Flow is fully bounded by a solid surface



- Flow Separation
  a) Attached Flow: Flow follows the shape of a body



  b) Separated Flow: Flow separates / doesn't follow surface due to abrupt geometry flow

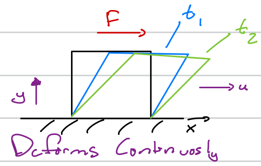


Viscosity

- Resistance to flow, molecular friction
  - → Consider an applied force on a solid vs. fluid

Stress $\sigma \propto \varepsilon$ Strain

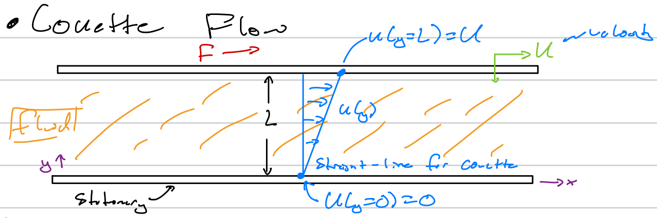
$\sigma = E\varepsilon$ ← Young's Modulus



Shear Stress: $\tau \propto \frac{du}{dy}$ ← Strain rate (velocity gradient)

Stress directly proportional for newtonian fluids

- Couette Flow



- Force => Shear Stress on the fluid: $\tau = \frac{F}{A}$
- Force => upper plate velocity U

$$\tau \propto \frac{dU}{dy}$$

Newtonian Fluids: Common fluids like water, air, oil, etc.

Non-newtonian fluids viscosity changes based on force

- Introduce proportionality constant for newtonian fluids

$$\tau = \mu \frac{dU}{dy}$$

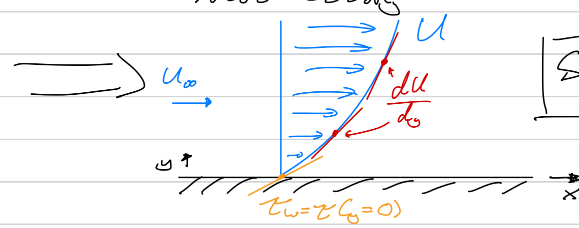
μ: Dynamic viscosity. Also Kinematic viscosity: $\nu = \frac{\mu}{\rho}$

- To find the force

$$\tau = \tau A = \mu \frac{dU}{dy} A = \mu \frac{U}{L} A$$

For all newtonian fluids; Couette Flow

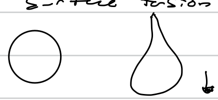
- More Generally



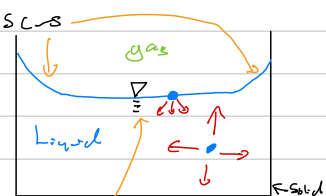
$\boxed{\text{Shear Flow}}$

Surface Tension

- Competing Attractions between similar and dissimilar molecules
- Some effects of surface tension
  - (i) Spherical Droplets:
    ↳ minimum energy state



  - (iv) Wetting Behavior
  - (v) Capillary Rise
  - (ii) Meniscus



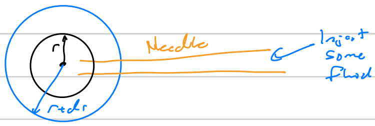
  - (iii) Free Surface

Surface Area of a Sphere is $4\pi r^2$

- Requires some energy to increase the surface area => surface tension (σ)

$$\sigma \left[\frac{\text{energy}}{\text{area}} = \frac{\text{force} \times \text{distance}}{\text{area}} = \frac{\text{force}}{\text{length}}\right] \left\{\frac{N}{m}\right\}$$

- Consider a spherical droplet



- As fluid is injected, surface area is created
  ↳ molecules moved from the interior to the surface ⇝ takes some amount of energy

$$A_s = 4\pi r^2$$
$$dA_s = 8\pi r\,dr$$

- Energy stored in the droplet

$$dE = \sigma\, dA_s = \sigma\, 8\pi r\,dr$$

- Work required to increase A_s

$$dW = F\,dr = (P_i - P_o) A_s\,dr = (P_i - P_o)\,4\pi r^2\,dr$$

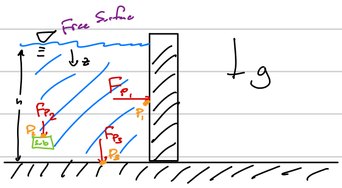
- Equate dE and dW

$$dE = \sigma\, \overset{2}{\cancel{8}}\cancel{\pi}\cancel{r}\cancel{dr} = 4\cancel{\pi} r^{\cancel{2}}\cancel{dr}(P_i - P_o) = dW \implies (P_i - P_o) = \frac{2\sigma}{r}$$

Hydrostatics

Hydrostatics Def: Impact of pressure on engineered surfaces

Applications: Dams, Tanks



Pressure

(i) Pressure acts isotropically (acts the same in all directions)

(ii) Pressure leads to a pressure force

Pressure $\left[\frac{\text{force}}{\text{area}}\right] \longrightarrow F_{pressure} = \int P\,dA$

(iii) Pressure force on a submerged object (like the dam) acts normal to the surface, and always acts inward!

(iv) Pressure varies with depth according to the hydrostatic pressure variation

$$\underbrace{\frac{dP}{dz}}_{\text{net pressure force}} = \boxed{\rho g}$$

(density, weight, Accel due to gravity)

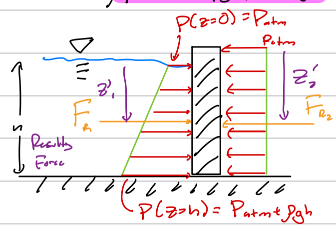
$$\Rightarrow p(z) = \int \rho g\,dz = p(z=0) + \rho g z$$

Assume constant ρ

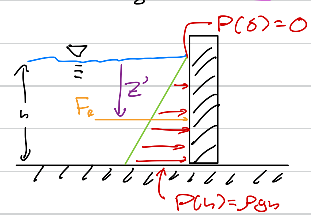
• Convenient to write things in gauge pressure:

$$P_{gauge} = P_{absolute} - P_{atm}$$

Absolute Pressure



Gauge Pressure



Goals (i) Determine the resultant force (F_R)

(ii) Determine where this single force acts (line of action)

1) Resultant Force

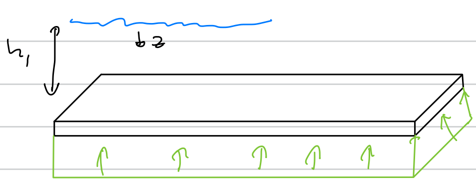
$dA = w\,dz$

$$F_R = \int_A p(z)\,dA = \int_0^h p(z)\,w\,dz$$

a) Linear Pressure Distribution

$$F_R = w\int_0^h \rho g z\,dz = w\left[\tfrac{1}{2}\rho g z^2\right]_0^h = \tfrac{1}{2}\rho g h^2 w = \underbrace{\boxed{\tfrac{1}{2}\rho g h}}_{\text{Average Pressure}}\underbrace{\boxed{(hw)}}_{\text{Area}}$$

b) Uniform Pressure



$$F_R = \rho g h_1 A$$

g

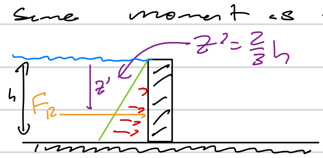
Buoyant Force

2) Line of Action

Location need to lead to the same moment as the force distribution

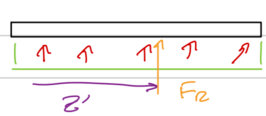
a) Linear Pressure Distribution

$$z' = \tfrac{2}{3}h$$

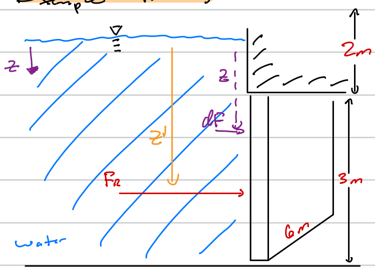


b) Uniform Pressure

z' is half way across



Example 11-45



- Find
  1) Resultant Force (F_R)
  2) Where F_R acts

2 Approaches possible Approaches

1) Integral Approach

$$F_R = \int P(z)\,dA = w\int_2^5 \rho g z\,dz = \frac{w\rho g}{2}(5^2-2^2) = 618{,}030\ \text{N} \rightarrow \boxed{F_R = 618\ \text{kN}}$$

Gate down to 5
Gate is 2m deep

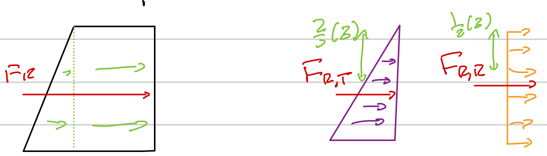
lever arm

$$F_R z' = \int_2^5 \rho g z\, z\, w\, dz = \rho g w \int_2^5 z^2\,dw = \frac{\rho g w}{3}(5^3-2^3) = 2{,}245{,}540\ \text{N·m}$$

dF

$$618{,}030\ \text{N}\ z' = 2{,}245{,}540\ \text{N·m} \rightarrow \boxed{z' = 3.71\text{m}}$$

2) Decomposition



$F_{R,T} = P_{avg} \cdot A$ Area
$= \frac{1}{2}\rho g(3)(3)(6)$
$= 265$ kN

$F_{R,R} = P \quad A$
$= 2\rho g \quad (3)(6)$
$= 353$ kN

$\Rightarrow F_R = F_{R,T} + F_{R,R} = 618$ kN $\quad \boxed{F_R = 618\text{kN}}$

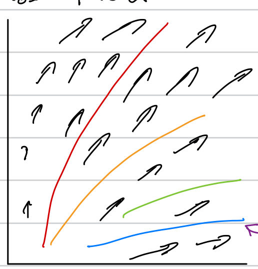
Take the moment about the hinge $\Rightarrow \hat{z}'$

$$F_R \hat{z}' = F_{R,T}\frac{2}{3}(3) + F_{R,R}\frac{1}{2}(3) \Rightarrow \hat{z}' = 1.71\text{m} \qquad \boxed{z' = 3.71\text{m}}$$

Shift to the free surface: $z' = 2\text{m} + \hat{z}' = 3.71\text{m}$

Fluid Dynamics & The Bernoulli Equation

Flow Fluid



$\sim \vec{V}(x, y, z, t)$ vector

$\sim p(x, y, z, t)$

$\rho(x, y, z, t)$

Stream lines → Line formed from the tangent to the velocity at every point

- For Steady Flows $[\vec{v}(x, y, z)]$ all fluid particles passing through a certain point follow the same stream lines

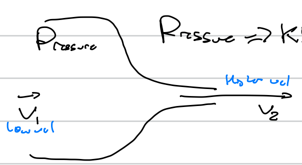
Bernoulli Equation

→ Provides a relationship between pressure, elevation, and velocity

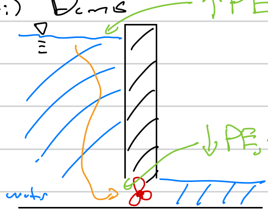
- Conservation of mechanical energy

Applications

(i) Nozzle



(ii) Dams



Assumptions

(i) Steady flow - no transient effects

(ii) Inviscid - viscous effects negligible

(iii) Incompressible (ρ constant)

(iv) Flow along a stream line

(v) No heat transfer

(vi) No work interactions

↳ Cannot apply across pump or turbine

Derivation

- Conservation of Linear Momentum applied to a fluid element

1) Acceleration of a fluid element

$V_s = f(s, t)$ in general ← velocity is a function of two variables

$V_s = \frac{ds}{dt}$?

- Total change in V_s

multivariable derivative → $dV_s = \frac{\partial V_s}{\partial s} ds + \frac{\partial V_s}{\partial t} dt$

- Divide through by dt

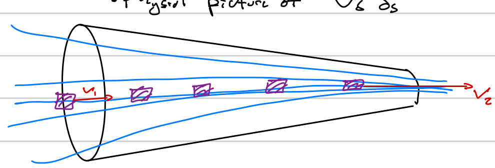
$a = \frac{dV_s}{dt} = \frac{\partial V_s}{\partial s} \cdot \frac{ds}{dt} + \frac{\partial V_s}{\partial t}$

← change in velocity at a point with respect to time

- $\frac{\partial V_s}{\partial t} = 0$ implied by 'steady-flow'
- Sub back in $V_s = \frac{ds}{dt}$

$\therefore a = V_s \frac{\partial V_s}{\partial s}$

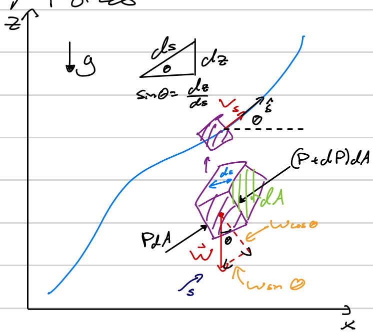
- Physical picture of $V_s \frac{\partial V_s}{\partial s}$



→ Acceleration due to movement through a velocity field despite steady flow

- N2L along stream line : $\sum F_s = m a_s$

2) Forces



Forces

1) Weight; $W = mg$; $m = \rho dV = \rho dA ds$
2) Pressure Force; Normal force due to pressure

$$\sum F_s = -W\sin\theta + \cancel{PdA} - (\cancel{P} + dP)dA$$

$$\sum F_s = -dPdA - mg\sin\theta = ma_s = mV_s\frac{dv_s}{ds}$$

$\frac{\partial v_s}{\partial s} = \frac{dv_s}{ds}$ since $V_s = f(s)$

$$-dP\cancel{dA} - \rho\,\cancel{dA}\,\cancel{ds}\,g\frac{dz}{\cancel{ds}} = \rho\,\cancel{dA}\,\cancel{ds}\,V_s\frac{dv_s}{\cancel{ds}}$$

$$\Rightarrow -dP - \rho g dz = \rho V_s dv_s = \tfrac{1}{2}\rho d(V_s^2)$$

$$\Rightarrow dP + \rho g dz + \tfrac{1}{2}\rho d(V_s^2) = 0$$

$$\frac{dP}{\rho} + g dz + \frac{1}{2}d(V_s^2) = 0$$

- Integrate Expression

$$\int\frac{dP}{\rho} + gz + \frac{1}{2}V_s^2 = \text{Constant}$$

Valid for compressible or incompressible Flow

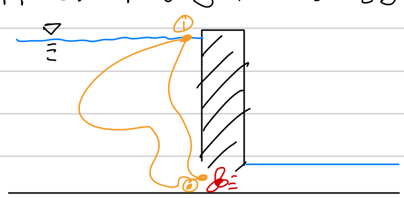
→ For constant density

$$\frac{P}{\rho} + \frac{1}{2}V_s^2 + gz = \text{Constant}$$

Starting point for Bernoulli Equation along a streamline

- Bernoulli Equation typically applied between two points:

$$P_1 + \tfrac{1}{2}\rho V_1^2 + \rho g z_1 = P_2 + \tfrac{1}{2}\rho V_2^2 + \rho g z_2$$

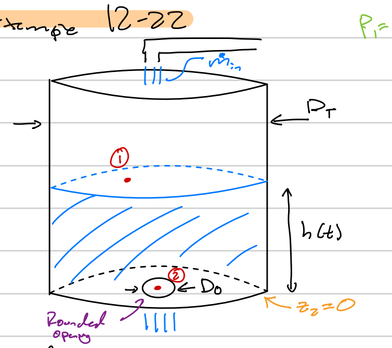


(Conventional) Bernoulli Equation

$$P + \tfrac{1}{2}\rho V^2 + \rho g z = \text{constant}$$

Assumptions for this form:
① Steady
② Incompressible
③ Inviscid Flow
④ Flow along a streamline
⑤ No heat or work interactions

Example 12-22



- Find h_{max} (equilibrium fill level) in terms of $\dot{m}_{in}$, D_0, ρ, etc.

P_2 also is P_{atm} because it is an open hole into the atmosphere

$P_1 = P_{atm}$

$V_1^2 \approx 0$ due to large area diff

$$\cancel{P_1} + \tfrac{1}{2}\cancel{\rho V_1^2} + \rho g z_1 = \cancel{P_2} + \tfrac{1}{2}\rho V_2^2 + \cancel{\rho g z_2}$$

$z_1 = h$, $z_2 = 0$, $P_2 = P_{atm}$ ∴ $P_1 = P_2$

mass: $\rho V_1 A_1 = \rho V_2 A_2$ ← conservation of mass

$$\Rightarrow V_1\frac{\pi D_T^2}{4} = V_2\frac{\pi D_0^2}{4}$$

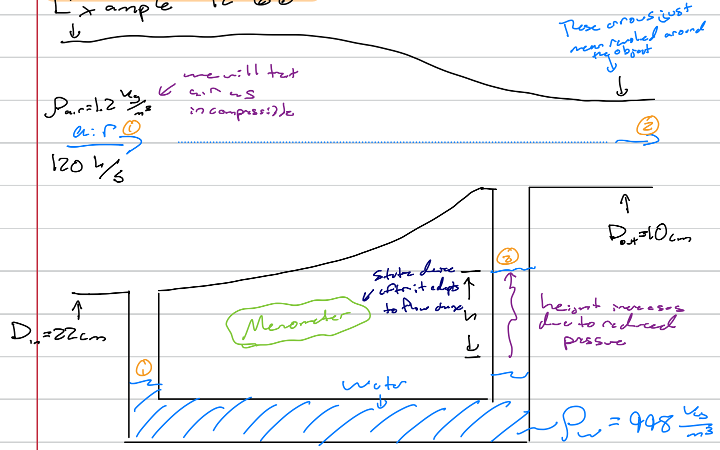
← Diameters given so $\left(\frac{D_0}{2}\right)^2$ used

$$\Rightarrow V_2 = \sqrt{2gh}$$

- At steady-state conditions: $\dot{m}_{in} = \dot{m}_{out} = \rho V_2 A_0 = \rho\sqrt{2gh_{max}}\,\frac{\pi D_0^2}{4}$

$$\therefore\ \boxed{h_{max} = \frac{1}{2g}\left(\frac{4\dot{m}_{in}}{\pi D_0^2\rho}\right)^2}$$

Example 12-66



Predict

- Velocity should increase due to conservation of mass
- Pressure should decrease; velocity needs to get its increased from somewhere, and that's the pressure

Find: The height 'h' that the water goes up when wind goes through the tunnel

↳ wind goes through tunnel creates a low pressure zone at end of nozzle so the water gets 'sucked' up

Bernoulli EQ: $P_{1a} + \frac{1}{2}\rho_a V_{1a}^2 + \cancel{\rho_a g z_{1a}} = P_{2a} + \frac{1}{2}\rho_a V_{2a}^2 + \cancel{\rho_a g z_{2a}}$ {air} $\quad z_{1a} = z_{2a}$

$P_{1w} + \cancel{\frac{1}{2}\rho_w V_{1w}^2} + \cancel{\rho_w g z_{1w}} = P_{2w} + \cancel{\frac{1}{2}\rho_w V_{2w}^2} + \rho_w g z_{2w}$ {water}

no velocity change

$z_{2w} - z_{1w} = h$

Assume: $\begin{cases} P_{1a} = P_{1w} = P_1 \\ P_{2a} = P_{2w} = P_2 \end{cases}$

Air's ρ is low, can assume column pressure is the same

→ Air: $P_1 - P_2 = \frac{1}{2}\rho_a (V_2^2 - V_1^2)$ $\quad \overset{\text{equate expressions}}{\Longrightarrow} \quad h = \frac{1}{2}\frac{\rho_a}{\rho_w g}(V_2^2 - V_1^2)$

we still need velocities

→ Water: $P_1 - P_2 = \rho_w g h$

Find velocities: $\dot{m}_1 = \dot{m}_2$ Steady-state $\quad \dot{m} = \rho V_{avg} A$ and $\dot{V} = V_{avg} A = \frac{\dot{m}}{\rho}$

→ If incompressible, we can say that $\dot{V}_1 = \dot{V}_2$

$\therefore \dot{V}_1 = \dot{V}_2 = \dot{V} = 120\ L/s = 0.12\ m^3/s$

$\Rightarrow V_1 = \frac{\dot{V}}{A_1} = \frac{\dot{V}}{\frac{\pi D_1^2}{4}} = 3.16\ m/s$ and $V_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{\frac{\pi D_2^2}{4}} = 15.3\ m/s$ $\Bigg\}$ $\boxed{h = 1.37\ cm}$

Energy Form of the Bernoulli Equation

General Energy Form of the Bernoulli Equation

Goal: Start with energy conservation to relax some of the assumptions

- Rate based open system of the first law of thermodynamics

$$(\dot{Q}_{in} - \dot{Q}_{out}) + (\dot{W}_{in} - \dot{W}_{out}) = \dot{m}\left[(h_2-h_1) + \tfrac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1)\right]$$

(i) Divide by $\dot{m}$ $\begin{cases} q = \dot{Q}/\dot{m} \\ w = \dot{W}/\dot{m} \end{cases}$ $\left\{\frac{kJ}{kg}\right\}$, $q_{net,in} = q_{in} - q_{out}$, $w_{net,in} = w_{in} - w_{out}$ $\Rightarrow q_{net,in} + w_{net,in} = (h_2 - h_1) + \tfrac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1)$

(ii) Use definition of h: $\left\{ h \equiv u + pv = u + \frac{P}{\rho} \right.$ ... $h_1 = u_1 + \frac{P_1}{\rho_1}$, $h_2 = u_2 + \frac{P_2}{\rho_2}$

$$\Rightarrow \underbrace{\boxed{\frac{P_1}{\rho_1} + \frac{1}{2}V_1^2 + gz_1}}_{\text{mechanical energy in to the system}} + \underbrace{\boxed{w_{net,in}}}_{\substack{\text{added mechanical energy} \\ (\dot{W}_{pump} - \dot{W}_{turbine})}} = \underbrace{\boxed{\frac{P_2}{\rho_2} + \frac{1}{2}V_2^2 + gz_2}}_{\text{mechanical energy out of the system}} + \underbrace{\boxed{u_2 - u_1 + q_{net,in}}}_{\text{internal energy and heat transfer}}$$

- For ideal scenarios (no irriversibilities)

$$u_2 = u_1 + q_{net,in} \Rightarrow u_2 - u_1 - q_{net,in} = 0$$

- For nonideal scenarios (fluid friction present)

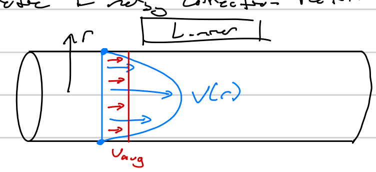
$$u_2 - u_1 > q_{net,in} \Rightarrow u_2 - u_1 = \overset{+/-}{q_{net,in}} + \overset{\text{Only positive}}{e_{losses}}$$

$$(u_2 - u_1 - q_{net,in}) = e_{losses}$$

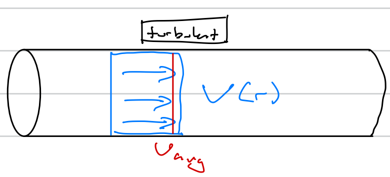
$$\Rightarrow \frac{P_1}{\rho_1} + \frac{1}{2}V_1^2 + gz_1 + w_{pump} = \frac{P_2}{\rho_2} + \frac{1}{2}V_2^2 + gz_2 + w_{turbine} + e_{losses}$$

Kinetic Energy correction factor

$KE_{total} = \int_{A_c} \frac{1}{2} V(r)\, dA_c \Rightarrow$ KE correction factor (α)



- For laminar flow: $\alpha = 2$



Most pipe flows are turbulent

- For turbulent flow: $\alpha = 1.03 \cong 1$

$$\therefore \frac{P_1}{\rho_1} + \frac{1}{2}\alpha_1 V_1^2 + gz_1 + w_{pump} = \frac{P_2}{\rho_2} + \frac{1}{2}\alpha_2 V_2^2 + gz_2 + w_{turbine} + e_{losses}$$

Energy Equation in terms of power

α is omitted for ease of writing but should be multiplied by the KE term

→ rate basis ($\times \dot{m}$)

Actual work   Actual work

$$\dot{m}\left[\frac{P_1}{\rho_1} + \frac{1}{2}V_1^2 + gz_1\right] + \dot{W}_{pump} = \dot{m}\left[\frac{P_2}{\rho_2} + \frac{1}{2}V_2^2 + gz_2\right] + \dot{W}_{turbine} + \dot{E}_{losses}$$

$$\rightarrow \dot{E}_{losses} = \dot{E}_{losses,pump} + \dot{E}_{losses,turbine} + \dot{E}_{losses,piping}$$

- For Pump

$$\underbrace{\dot{W}_{pump,u}}_{\substack{\text{useful} \\ \text{(isentropic ideal)}}} = \underbrace{\dot{W}_{pump}}_{\text{Actual}} - \dot{E}_{losses,pump} \Rightarrow \eta_{pump} = \frac{\dot{W}_{pump,u}}{\dot{W}_{pump}}$$

- For Turbine

$$\underbrace{\dot{W}_{turbine,e}}_{\text{extractable}} = \underbrace{\dot{W}_{turbine}}_{\text{Actual}} + \dot{E}_{losses,turbine} \Rightarrow \eta_{turbine} = \frac{\dot{W}_{turbine}}{\dot{W}_{turbine,e}}$$

1 pt $\frac{\text{Volumetric Flowrate}}{\text{Cross Sec Area}} = \text{Velocity}$

$$\Rightarrow \dot{m}\left[\frac{P_1}{\rho_1} + \frac{1}{2}V_1^2 + gz_1\right] + \dot{W}_{pump,u} = \dot{m}\left[\frac{P_2}{\rho_2} + \frac{1}{2}V_2^2 + gz_2\right] + \dot{W}_{turbine,e} + \dot{E}_{losses,piping}$$

Writing Bernoulli Equation in terms of head ← usual starting point

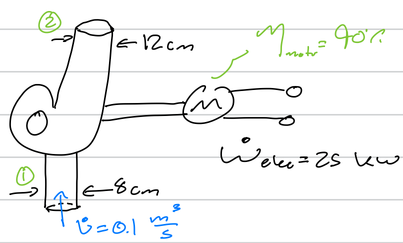
→ head has dimensions of length; Divide by $\dot{m}g$

$$\underset{\text{head}}{h_{pump,u}} = \frac{\dot{W}_{pump,u}}{\dot{m}g}, \quad \underset{\text{head}}{h_{turbine,e}} = \frac{\dot{W}_{turbine}}{\dot{m}g}, \quad \underset{\text{head}}{h_L} = \frac{\dot{E}_{losses,piping}}{\dot{m}g} \leftarrow \text{frictional head loss}$$

$$\Rightarrow \frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 + h_{pump,u} = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2 + h_{turbine,e} + h_L$$

Irreversible head loss between ① and ② due to fluid friction

Example 12-47



Given

- 0.1: $\rho = 860\ kg/m^3$
- Pressure rise: 250 kPa ← $\Delta P = P_2 - P_1$ here
- KE correction factor: $\alpha = 1.05$

Find: Pump efficiency

$$\eta_{pump} = \frac{\dot{W}_{pump,u}}{\dot{W}_{pump}} \quad \dot{W}_{pump} = \eta_{motor} \times \dot{W}_{elec}$$

$$\frac{P_1}{\rho_1 g} + \alpha\frac{V_1^2}{2g} + \cancel{z_1} + h_{pump,u} = \frac{P_2}{\rho_2 g} + \alpha\frac{V_2^2}{2g} + \cancel{z_2} + \cancel{h_{turbine,e}}^{\,0} + \cancel{h_L}^{\,0}$$

$z_1 \approx z_2$

outlet and inlet are close, so there's not enough piping to consider losses due to piping

$$h_{pump,u} = \frac{P_2 - P_1}{\rho g} + \alpha\frac{V_2^2 - V_1^2}{2g}$$

$$\text{velocity} = \frac{\dot{V}}{A_c} \qquad V_1 = \frac{\dot{V}}{A_{c_1}} = \frac{\dot{V}}{\frac{\pi}{4}D^2} = 19.89\ m/s$$

$$\text{incompressible} \rightarrow \dot{V}_1 = \dot{V}_2 = \dot{V} \qquad V_2 = \frac{\dot{V}}{A_{c_2}} = \frac{\dot{V}}{\frac{\pi}{4}D^2} = 8.84\ m/s$$

$$\rightarrow h_{pump,u} = \frac{\Delta P}{\rho g} + \alpha\frac{V_2^2 - V_1^2}{2g} = \frac{\overset{\{N/m^2\}}{250{,}000\ Pa}}{\underset{\{N/m^3\}}{860 \times 9.81}} + 1.05\,\frac{8.84^2 - 19.89^2}{2 \times 9.81}$$

$$= 29.6 \quad \underset{\text{use less pump power to get pressure rise}}{-17} \quad = 12.6\ m$$

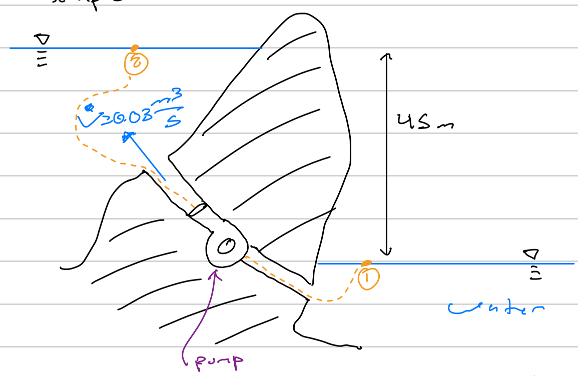
Watch your units

$$\dot{W}_{pump,u} = \dot{m}g\,h_{pump,u} = \rho\dot{V}g\,h_{pump,u} = 10{,}660\ W = 10.66\ kW$$

$$\eta_{pump} = \frac{\dot{W}_{pump,u}}{\dot{W}_{pump}} = \frac{10.66\ kW}{\eta_{motor}\cdot\dot{W}_{elec}} = \frac{10.66\ kW}{22.5\ kW} = 47.4\%$$

$$\boxed{\eta_{pump} = 47.4\%}$$

Example 12-60



• Pump delivers 20 kW of useful power

Find

(i) Irreversible head loss (h_L)

(ii) Lost mechanical power

1) Frictional Head Loss

velocities small relative to z changes at free surface

$P_1 = P_2 = P_{atm}$, $V_1^2 \approx 0$, $V_2^2 \approx 0$, $\Delta z = z_2 - z_1 = 45 m$

$$\frac{\cancel{P_1}}{\rho g} + \frac{\cancel{V_1^2}}{2g} + z_1 + h_{pump,u} = \frac{\cancel{P_2}}{\rho g} + \frac{\cancel{V_2^2}}{2g} + z_2 + \cancel{h_{turbine,e}} + h_L$$

$$\Rightarrow h_L = h_{pump,u} - \Delta z$$

with $\dot W_{pump,u}$ work, you could raise fluid 67.96 m

$$h_{pump,u} = \frac{\dot W_{pump,u}}{\dot m g} = \frac{20 kW}{\rho \dot V g} = 67.96 m$$

$$h_L = h_{pump,u} - \Delta z = 68 - 48 = 23 m$$

$$\boxed{h_L = 23 m}$$

2) Lost Mechanical Power

$$\dot E_{losses, piping} = h_L \dot m g = 6756.5 W$$

$$\boxed{\dot E_{loss, piping} = 6.76 kW}$$

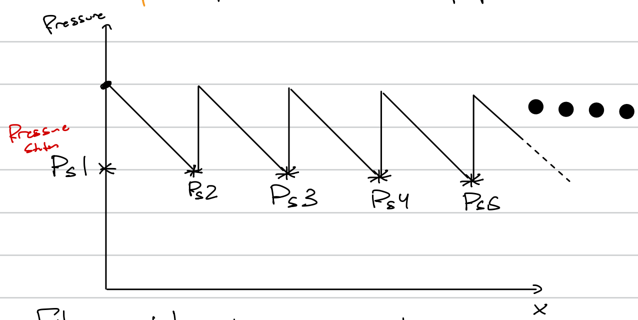
from $h_L = \frac{\dot E_{loss, pipe}}{\dot m g}$

Internal Flow

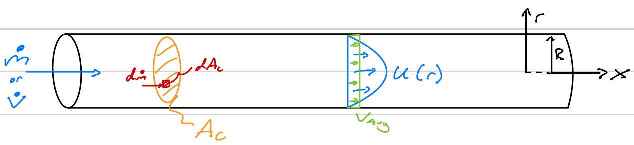
Internal Flow

Goal: Calculate frictional head loss (h_L) based on velocity, viscosity, etc.

Example: Trans-Alaska Pipeline ~800 mi



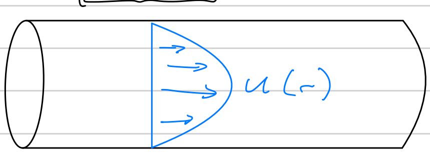
Flow through a circular pipe



- Mass flow rate:
  - Overall: $\dot{m} = \rho V_{Avg} A_c$
  - Local: $\dot{m} = \int_{A_c} \rho u(r)\, dA_c$ ← $d\dot{m}$

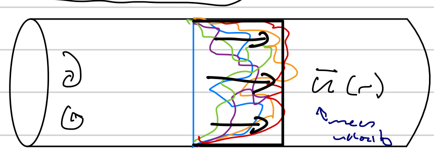
- Equate $\dot{m}$ expressions: $\int \rho u(r) dA_c = \rho V_{Avg} A_c$
  - → Assume incompressible and circular pipe: $\rho_1 = \rho_2 = \rho$, $A_c = \pi R^2$ & $dA_c = 2\pi r\, dr$
  - $\Rightarrow V_{Avg} = \frac{2}{R^2}\int_0^R u(r)\, r\, dr$

Laminar



- Smooth, orderly flow
- Low velocity, high viscosity

Turbulent



- High mixing chaotic flow
- High velocity, low viscosity

→ Flow regime based on the Reynold's Number (Re)

$$Re = \frac{\rho V_{Avg} D}{\mu}$$

dimensionless; density; Avg velocity; Characteristic length; Dynamic viscosity

- Ratio of inertial (momentum) effects to viscous effects
- Transition from laminar to turbulent based on a critical reynold's number

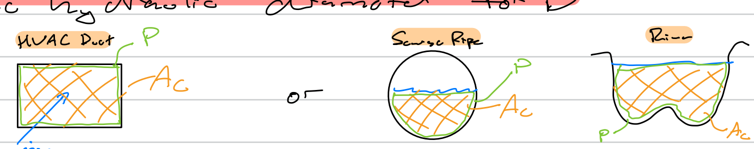
$Re_{crit} = 2300$ for internal flow

$Re_{crit} = 5 \times 10^5$ for external flow

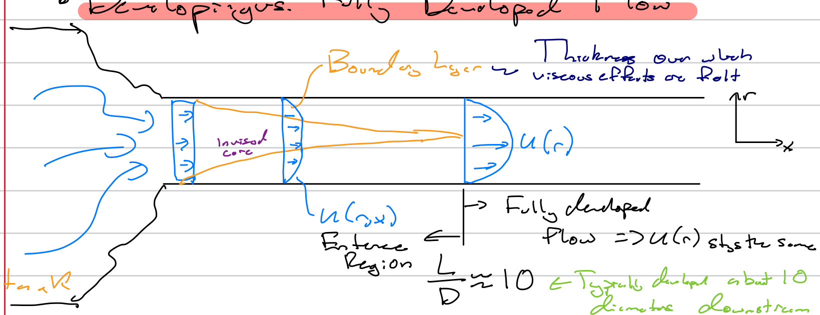
- For non-circular cross sections, use hydraulic diameter for D

$$D_h = \frac{4 A_c}{P}$$

A_c ← flow cross sec. area; P ← wetted perimeter



- Developing vs. Fully Developed Flow



$\frac{L}{D} \approx 10$ ← Typically developed about 10 diameters downstream

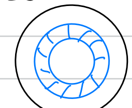
Derivation

- Laminar flow in a circular pipe

Side view

Steady: $\frac{\partial u}{\partial t} = 0$, $u\frac{\partial u}{\partial x} = 0$ ← from assumption 4

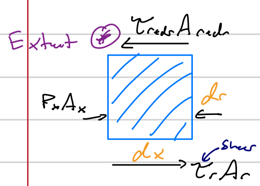
End view



Assumptions
(i) Steady, Laminar ($Re \leq 2300$) flow
(ii) Assume incompressible flow and constant properties
(iii) Straight, circular pipe
(iv) Fully developed

Approach: Apply Newton's 2nd Law ($\sum F_x = ma_x$) to a donut shaped fluid element

$$\Rightarrow \sum F_x = 0$$



Forces
(i) Friction / Shear stress
(ii) Pressure

Sum forces

$$(P\,2\pi r\,dr) - (P\,2\pi r\,dr)_{x+dx} + (\tau\,2\pi r\,dx)_r - (\tau\,2\pi r\,dx)_{r+dr} = 0$$

- Rearrange, divide by $2\pi r\,dr\,dx$

$$r\,\frac{P_{x+dx} - P_x}{dx} + \frac{(r\tau)_{r+dr} - (r\tau)_r}{dr} = 0$$

Newtonian fluid: $\tau = -\mu\frac{du}{dr}$

→ Take limit as $dr, dx \to 0$: $r\frac{dP}{dx} + \frac{d(r\tau)}{dr} = 0$

$$\Rightarrow r\frac{dP}{dx} = \mu\frac{d}{dr}\left(r\frac{du}{dr}\right)$$ assumed μ constant

Only depends on x $\boxed{\frac{dP}{dx}} = \boxed{\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)}$ only depends on r

$g(x) = f(r) \Rightarrow$ to be true $g(x) = f(r) =$ constant must be true

$$\Rightarrow \frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{r}{\mu}\frac{dP}{dx}$$ ← constant

- Integrate twice:

$$u(r) = \frac{r^2}{4\mu}\frac{dP}{dx} + c_1 \ln r + c_2$$ Constants of integration

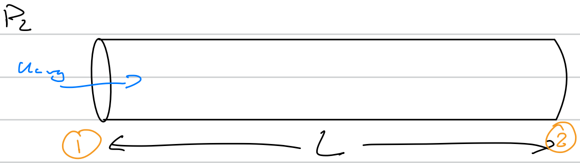
- Need 2 boundary conditions to solve for c_1, c_2
  1) No-slip condition: $u(r=R) = 0$
  2) Symmetric: $\frac{du}{dr}(r=0) = 0$ ← comes from flow being fully developed & laminar

$$\Rightarrow u(r) = \boxed{-\frac{R^2}{4\mu}\frac{dP}{dx}}\left(1 - \frac{r^2}{R^2}\right)$$ Maximum velocity

$\frac{dP}{dx} < 0$ must be true to push water positive direction (+u flow)

- Average velocity: $u_{avg} = \frac{2}{R^2}\int_0^R u(r)\,r\,dr = -\frac{R^2}{8\mu}\frac{dP}{dx}$ ← For laminar flow: $u_{avg} = \frac{1}{2}u_{max}$
- Rewrite to isolate $\frac{dP}{dx} \equiv \Delta P_L$

$$\frac{dP}{dx} = \frac{-8\mu u_{avg}}{R^2}$$



$\frac{dP}{dx} = \frac{P_2 - P_1}{L}$ so $\Delta P_L = P_1 - P_2$ Pressure loss

$$\Rightarrow \Delta P_L = \frac{8\mu u_{avg} L}{R^2}$$
$$= \frac{32\mu u_{avg} L}{D^2}$$

- Pumping power to overcome frictional losses

$$\dot{W}_{pump_L} = \dot{V}\Delta P_L$$ incompressible assumed

- To generalize results, define the Darcy friction factor (f)

$$\Delta P_L = f\,\frac{L}{D}\cdot\boxed{\frac{1}{2}\rho u_{Avg}^2} \text{ — dynamic pressure}$$

$$\frac{\Delta P_L}{\frac{1}{2}\rho u_{Avg}^2} = f\frac{L}{D}$$ Equation ends up being dimensionless

- Go from $\Delta P_L \to h_L$

$$h_L = \frac{\Delta P_L}{\rho g} = f\cdot\frac{L}{D}\cdot\frac{u_{avg}^2}{2g}$$

- Friction factor for laminar flow in a circular pipe

pipe: $$\Delta P_L = \frac{32\mu L u_{Avg}}{D^2}$$

want it to look like this

$$\left\{\Delta P_L = f\,\frac{L}{D}\,\frac{1}{2}\rho u_{Avg}^2\right\}$$

$$= \left(\frac{L}{D}\right)\left(\frac{1}{2}\rho u_{Avg}^2\right)\left(64\,\frac{\mu}{\rho u_{avg} D}\right) \text{ — } \frac{1}{Re_D} \text{ Reynolds \#}$$

dimensionless

$$\Rightarrow f = \frac{64}{Re_D}$$ only based on const (64) and reynolds number w/ respect to pipe diameter (Re_D)

- For turbulent flow
  - Empirical rather than analytical (based on experiments)
  - Experiments show: $f = f\left(Re, \frac{\varepsilon}{D}\right)$ — surface roughness

→ Surface roughness

ε — surface roughness

- Curve fit experimental data → Colebrook equation

$$\frac{1}{\sqrt{f}} = -2.0\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

↳ Implicit equation for f → hard to solve!

- Approximate explicit form → Haaland Equation

$$\frac{1}{\sqrt{f}} \approx -1.8\log_{10}\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right]$$ ~ 2% error introduced

- Or, plot colebrook equation plotted as 'Moody Diagram'
  (i) Re on x-axis
  (ii) ε/D on secondary y-axis

Calculate ε/D, find intersection of Re and roughness curve

Result comes from reading across to primary vertical axis → now you have f!

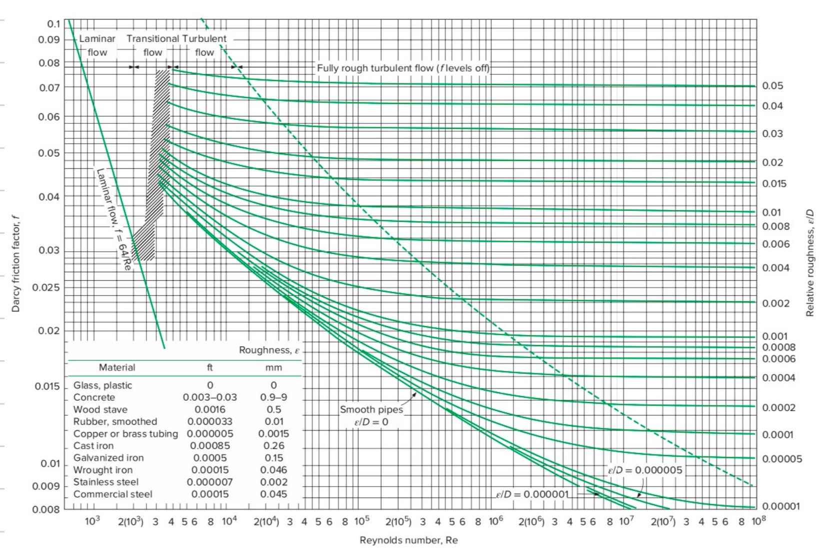
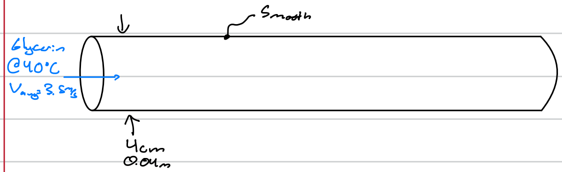


FIGURE A–27

The Moody chart for the friction factor for fully developed flow in circular pipes for use in the head loss relation $\Delta P_L = f\,\frac{L}{D}\,\frac{\rho V^2}{2}$. Friction factors in the turbulent flow are evaluated from the Colebrook equation $\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$.

Example 14-48



Find pressure loss per 10m of length

Glycerin @ 40°C $\begin{cases} \rho = 1282 \ kg/m^3 \\ \mu = 0.27 \ kg/m \cdot s \end{cases}$

For water, $\mu_{air} = 0.000018 \ \frac{kg}{m \cdot s}$
& air for ref. $\mu_{water} = 0.001 \ \frac{kg}{m \cdot s}$

$$\Delta P_L = \boxed{f} \frac{L}{D} \frac{1}{2} \rho V_{avg}^2$$

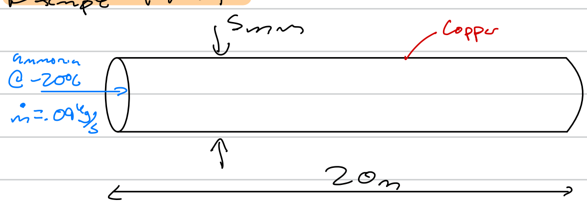
Unknown

$$Re_D = \frac{\rho V_{avg} D}{\mu} = 644.2 \ 2300 \Rightarrow \text{Laminar flow}$$

dimensionless

$$\therefore f = \frac{64}{Re_D} = \frac{64}{644.2} = 0.099 \qquad \therefore \boxed{\Delta P_L = 189{,}000 \text{ Pa per } 10m}$$

Example 14-51



Find ΔP_L, h_L, $\dot{W}_{pump,L}$

Ammonia @ -20°C $\begin{cases} \rho = 665.1 \ \frac{kg}{m^3} \\ \mu = 2.361 \times 10^{-4} \ \frac{kg}{m \cdot s} \end{cases}$

Table A-17

$$Re_D = \frac{\rho V_{avg} D}{\mu} \quad \Bigg| \quad \dot{m} = \rho V_{avg} A_c \Rightarrow V_{avg} = \frac{\dot{m}}{\rho A_c} = 6.9 \ m/s \Rightarrow Re_D = 97{,}100 \Rightarrow \text{Turbulent Flow}$$

$$\Delta P_L = f \frac{L}{D} \frac{1}{2} \rho V_{avg}^2$$

Go to Moody Diagram

$$\left.\begin{matrix} Re_D \\ \varepsilon/D \end{matrix}\right\} \xRightarrow{\text{moody}} f = 0.02$$

$$\frac{\varepsilon}{D} = \frac{0.0015mm}{5mm} = .0003$$

$$\therefore \Delta P_L = 12.36 \ kPa$$

$$h_L = \frac{\Delta P_L}{\rho g} = 1.89 \ m$$

$$\dot{W}_{pump} = \dot{V} \Delta P_L = \dot{m} g h_L = 167 W$$

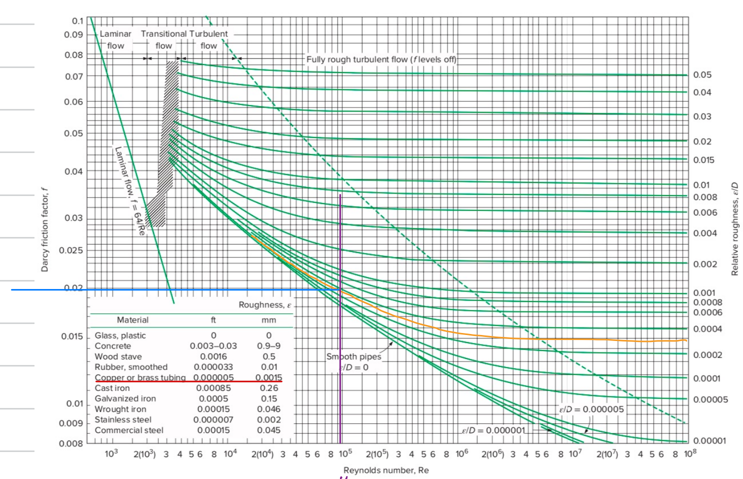


FIGURE A-27 $Re = 97{,}000 = 9.7 \times 10^4$

The Moody chart for the friction factor for fully developed flow in circular pipes for use in the head loss relation $\Delta P_L = f \frac{L}{D} \frac{\rho V^2}{2}$. Friction factors in the turbulent flow are evaluated from the Colebrook equation $\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$.

$$\rho = \frac{m}{V} \qquad V = \frac{m}{\rho}$$

Minor Losses

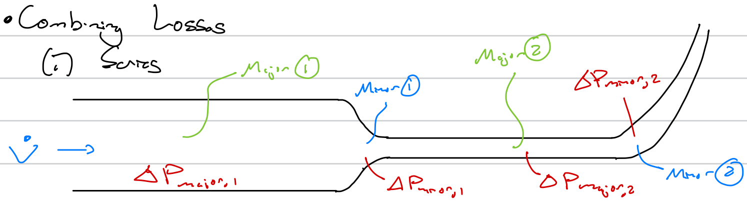
- Piping losses from other components
  - (i) Elbows, Tees
  - (ii) Valves
  - (iii) Piping Transitions (reducing or expanding)
  - (iv) Entrance/exit to/from piping

Modeling them

$$h_{L,minor} = K_L \cdot \frac{V_{avg}^2}{2g}$$

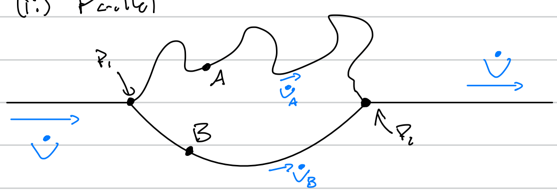
component specific minor loss coefficient → Tables give scenarios with K_L values, for some α appears (KE correction factor)

- Combining Losses
  - (i) Series



a) $\dot{V}$ constant (incompressible)
b) ΔP_s added
$\Rightarrow h_{L,tot} = \sum h_{L,major} + \sum h_{L,minor}$

  - (ii) Parallel

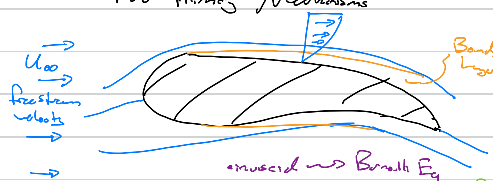


a) $\dot{V} = \dot{V}_A + \dot{V}_B$
b) $\Delta P_A = \Delta P_B$
$\Rightarrow h_{L,A} = h_{L,B}$

External Flow

External Flow

- Flow over bodies => aerodynamic forces (lift, drag)
  - → Two Primary Mechanisms

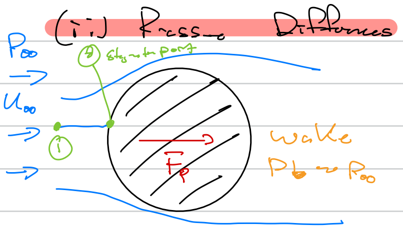


(i) Friction $\tau = \mu \boxed{\frac{du}{dy}}$ ⇝ large in the boundary layer (could be low)

Boundary layer ⇝ Viscous effects important

- Shear stress over surface area => frictional forces

(ii) Pressure Differences



$$P_1 + \underbrace{\tfrac{1}{2}\rho V_1^2}_{\text{Dynamic pressure}} = P_2 + \cancel{\tfrac{1}{2}\rho V_2^2}^{\,0}$$

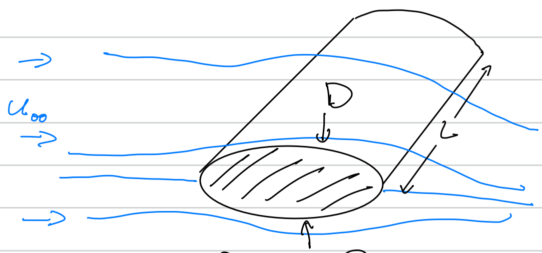
(P_∞, U_∞; ↑ Stagnation pressure)

- Pressure Differences over surface => pressure force

$Re_{crit} = 2300$ for internal flow

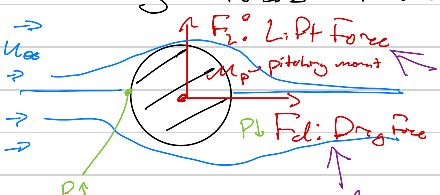
$Re_{crit} = 5\times10^5$ for external flow

2-D vs. 3-D Flows



- 2-D analysis valid if $\frac{L}{D}$ is large

Defining Forces & moments



↖ Acts perpendicular to freestream by definition

↑ Acts along freestream

- Effects that influence aerodynamic forces
  - (i) Fluid Properties (ρ, μ)
  - (ii) Object Properties / Shape
    - a) Streamlined (U_∞ →)
    - b) Blunt or bluffed (U_∞ →)
  - (iii) Flow Regime (Laminar vs. Turbulent)

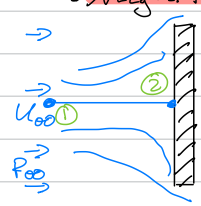
- Define Lift & Drag Coefficients
  1) Drag Coefficient

     dimensionless $C_D = \dfrac{F_D}{\frac{1}{2}\rho U_\infty^2 A} \Rightarrow F_D = C_D \tfrac{1}{2}\rho U_\infty^2 A$

  2) Lift Coefficient

     $C_L = \dfrac{F_L}{\frac{1}{2}\rho U_\infty^2 A}$

- Magnitude of C_D



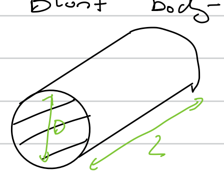
$P_2 = P_{stag} = P_\infty + \frac{1}{2}\rho U_\infty^2$

$\dfrac{(P_2 - P_\infty)A}{\frac{1}{2}\rho U_\infty^2 A} \Rightarrow C_D \approx 1$

- C_D can be greater than 1!!

- Appropriate area for C_D, C_L
  - Depends on shape type
    - i) Blunt body – frontal area (A_f)
    - 2) Streamlined body → planform area (A_p)

U_∞ →

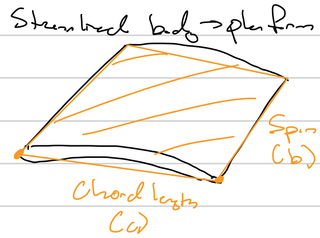


- for a cylinder

  $A_f = DL$

- for a sphere

  $A_f = \dfrac{\pi D^2}{4}$



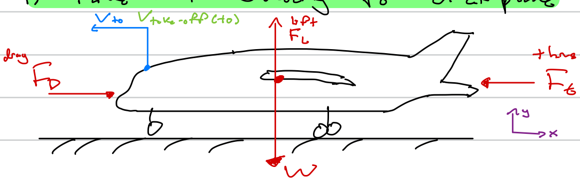
$A_p = bc$

• Get C_D from tables
→ C_D depends on
i) Shape
2) Reynolds number → Check restrictions on tables

For a tree, velocity ↑ ⟹ C_D ↓. Leaves turn into flow direction, so leaves reduce drag at high wind speeds

Aerodynamic Force Balances

1) Take-off velocity for an airplane



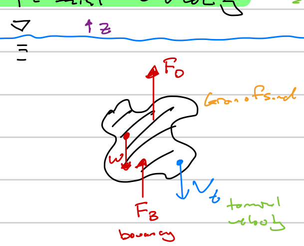
• At force equilibrium

$$\sum F_x = 0 \Rightarrow F_t = F_D = C_D \tfrac{1}{2}\rho V_{to}^2 A$$

$$\sum F_y = 0 \Rightarrow W = F_L = C_L \tfrac{1}{2}\rho V_{to}^2 A$$

$$V_{to} = \sqrt{\frac{2W}{\rho C_L A}}$$

2) Terminal Velocity – sedimentation, suspensions



$$\begin{cases} F_D = C_D \tfrac{1}{2}\rho_f V_t^2 A \\ W = m_s g = \rho_s V g \\ F_B = W_{f,displaced} = m_f g = \rho_f V g \end{cases}$$

sand, fluid, volume

archimedes principle

• Sum forces : $\sum F_z = 0$

$$C_D \tfrac{1}{2}\rho_f V_t^2 A + \rho_f V g - \rho_s V g = 0$$

$$V_t = \sqrt{\frac{2Vg}{C_D A} \cdot \frac{\rho_s - \rho_f}{\rho_f}}$$

For parachute: $C_D = 1.3$

For spherical grain of sand: $C_D = \frac{24}{Re}$ $(Re < 1)$

Drag Force

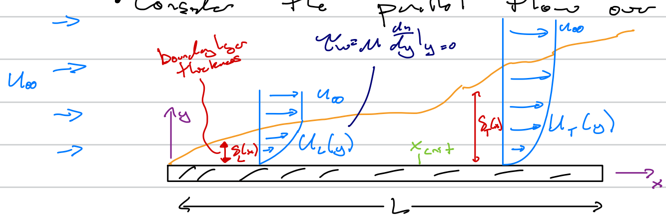
• Two main types:

1) Skin friction drag
→ Due to shear stress
→ Dominant mechanism for streamlined objects

2) Form drag (pressure drag)
→ Due to flow blockage/separation
→ Dominant mechanism for bluff bodies

Skin Friction Drag

• Consider the parallel flow over a flat plate



$$F_D = F_f = C_f \tfrac{1}{2}\rho U_\infty^2 A$$

A: area in contact with fluid

C_f: skin friction coef.

• Overall Reynolds number

$$Re_L = \frac{\rho U_\infty L}{\mu}$$

• Local Reynolds number

$$Re_x = \frac{\rho U_\infty x}{\mu}$$

• Critical Reynolds #

$$Re_{crit} \simeq 5 \times 10^5$$

↳ Can solve for x_{crit}

describes boundary layer
• $Re_x < Re_{crit} \Rightarrow$ Laminar
• $Re_x > Re_{crit} \Rightarrow$ Turbulent

• Friction Coefficient
Boundary layer

→ Laminar BL $(Re_x < 5\times10^5)$

$$\frac{\delta(x)}{x} = \frac{4.91}{Re_x^{1/2}} \quad \Big| \quad C_{f,x} = \frac{0.664}{Re_x^{1/2}}$$

→ Turbulent BL

$$\frac{\delta(x)}{x} = \frac{0.38}{Re_x^{1/5}} \quad \Big| \quad C_{f,x} = \frac{0.059}{Re_x^{1/5}}$$

• Integrate to get a total friction coefficient: $C_f = \frac{1}{L}\int_0^L C_{f,x}\,dx$

• Flow over a flat plate can be:

(i) Entirely Laminar

$$C_{f,laminar} = \frac{1.33}{Re_L^{1/2}}$$

(ii) Primarily Turbulent

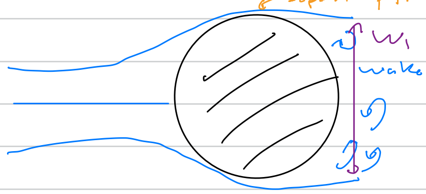
$$C_{f,turbulent} = \frac{0.074}{Re_L^{1/5}}$$

(iii) Combination of Laminar & Turbulent BL

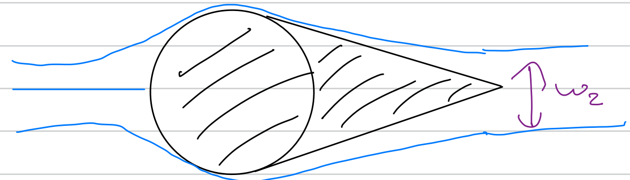
$$C_f = \frac{1}{L}\left(\int_0^{x_{crit}} C_{f,x,laminar}\,dx + \int_{x_{crit}}^{L} C_{f,x,turbulent}\,dx\right)$$

$$\Rightarrow C_f = \frac{0.074}{Re_L^{1/5}} - \frac{1742}{Re_L}$$

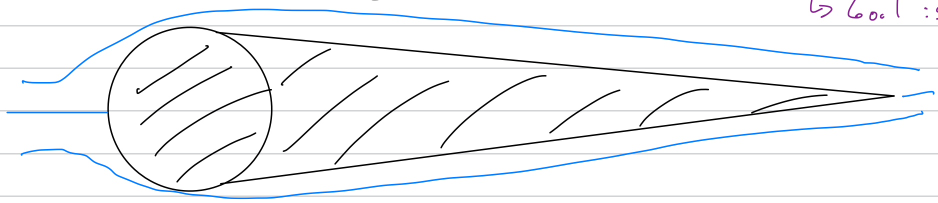
Streamlining – Design to reduce drag



- ↑ form / pressure drag



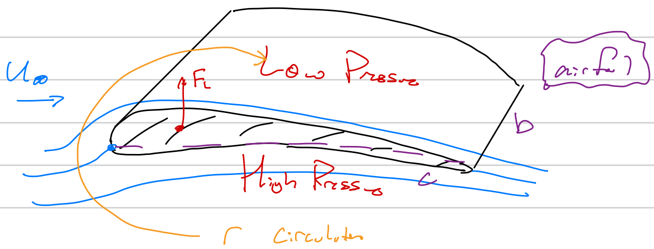
- ↓↓ Form Drag but ↑ Friction drag
  ↳ Goal is net drag ↓

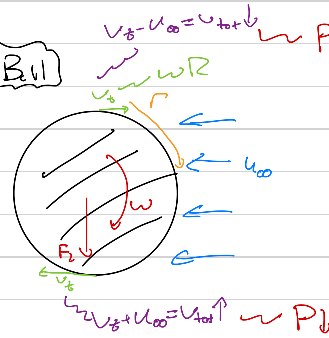


no Separation

- Form drag ↓↓
- Friction drag ↑↑
- Overall Drag ↓

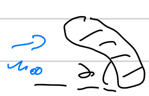
Lift Force





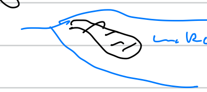
Recall: $F_L = C_L \frac{1}{2} \rho u_\infty^2 A_p$ $A_p = bc$ for wings

- Lift coef (C_L) depends on
  (i) Shape
  (ii) Angle of attack (α)
  ↳ Angle between chord and horizontal

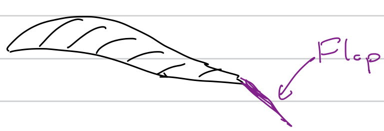


Stall

- Lift Coef increases with angle of attack
  ↳ up to stall point
- Stall typically occurs around 15°
- At Stall

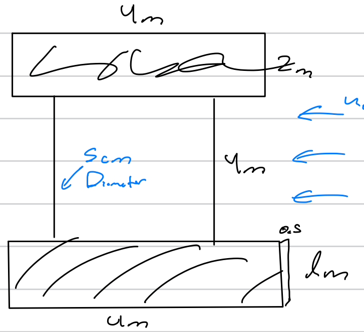


Effect of Flaps



- Flaps increase lift coef at the expense of drag

Example 15-88



$u_\infty = 150 \ \frac{km}{hr} = 41.6 \ m/s$

Air $\rho = 1.3 \ kg/m^3$

$\mu = 1.73 \times 10^{-5} \ \frac{kg}{m \cdot s}$

Find: (i) Drag on the sign
(ii) Drag on posts

$$Re = \frac{\rho u_\infty L}{\mu} = 1.25 \times 10^7 \quad (L = 4m)$$

(i) Drag on sign

$$F_D = C_D \tfrac{1}{2} \rho u_\infty^2 A \quad (A = 4m \times 2m)$$

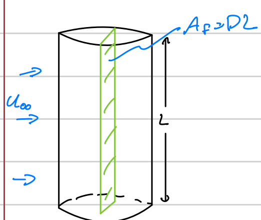
$C_D \Rightarrow$ 15-4 Tables (rectangular plate) $\Rightarrow$ Recalled $C_D = 1.10 + 0.02\left(\frac{2}{D} + \frac{D}{2}\right) = 1.15$

$$\boxed{F_{D,sign} = 10{,}382 N}$$

(ii) Drag on Rods/Posts

a) Look at L/D
b) Check Tables — 1m comes out of the
c) Solve

$$F_{pole} = C_{D,pole} \tfrac{1}{2} \rho u_\infty^2 A_f \quad (A_f = DL)$$



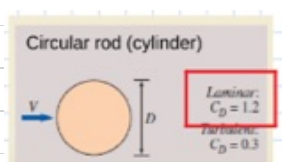
• Check Reynolds Number:

$$Re_D = \frac{\rho u_\infty D}{\mu} = 1.57 \times 10^5 \Rightarrow \text{Laminar but} > 10^4$$

• Two possibilities for C_D

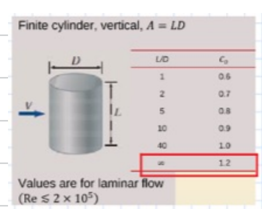
a) 2-D approx

$C_D = 1.2$



b) Finite Cylinder

$\frac{L}{D} = \frac{4m}{5/100 \ m} = 80 \Rightarrow C_D = 1.2$



$\Rightarrow F_{pole} = 271 N$

• 2 supports → multiply F_{pole} by 2: $F_{poles} = 2 * 271 = 542 N$