ENGR 225: Spring 2024

Homework #9: External flow

Due: April 5

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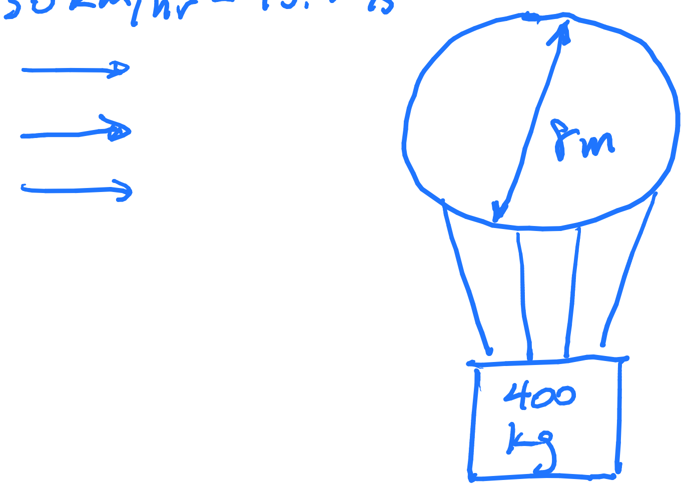
1. [Chapter 15] An 8-m-diameter hot air ballooon that has a total mass of 400 kg is standing still in air on a windless day. The balloon suddenly is subjected to a 50 km/hr wind. Determine the initial acceleration of the balloon in the horizonal direction.

$U_\infty = 50\,km/hr = 13.9\,m/s$



Air @ 20°C

$\rho = 1.204\,\frac{kg}{m^3}$

$\mu = 1.825\times10^{-5}\,\frac{kg}{m\cdot s}$

- Drag force acting on the balloon:

$$F_D = C_D\,\tfrac{1}{2}\rho U_\infty^2 A$$

Frontal area: $\frac{\pi D^2}{4}$

- Model balloon as a sphere $\Rightarrow C_D$
  - Need Reynolds number to determine C_D

$$Re = \frac{\rho U_\infty D}{\mu} = \frac{(1.204)(13.9)(8)}{1.825\times10^{-5}} = 7.3\times10^6$$

$\Rightarrow$ turbulent

From Table 15-2 $(Re \gtrsim 2\times10^6) \Rightarrow C_D = 0.2$

$$F_D = (0.2)\tfrac{1}{2}(1.204)(13.9)^2\frac{\pi(8)^2}{4} = 1167\,N$$

- To get initial acceleration: $F = ma$

$$a = \frac{F_D}{m} = \frac{1167}{400} = 2.9\,\frac{m}{s^2}$$

2. [Chapter 15] To reduce the drag coefficient and thus improve the fuel efficiency of cars, the design of side rearview mirrors has changed dramatically in recent decades from a simple circular plate to a streamlined shape. Determine the amount of fuel and money saved per year as a result of replacing a 15-cm-diameter flat mirror by one with a hemispherical back. Assume the car is driven 25,000 km a year at an average speed of 95 km/hr. Take the density and price of gasoline to be 0.75 kg/L and \$0.90/L, respectively; the heating value of gasoline to be 44,000 kJ/kg and the overall efficiency of the engine to be 30 percent.

$U_\infty = 95\ \text{km/hr} = 26.4\ \text{m/s}$

$D = 15\ \text{cm} = 0.15\ \text{m}$

Air @ 20°C $\Rightarrow \begin{cases} \rho = 1.2\ \frac{kg}{m^3} \\ \mu = 1.83\times10^{-5}\ \frac{kg}{m\cdot s} \end{cases}$

$$Re = \frac{\rho U_\infty D}{\mu} = 2.6\times10^5$$

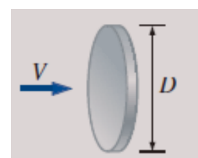
- Drag force

Frontal area: $\frac{\pi D^2}{4}$

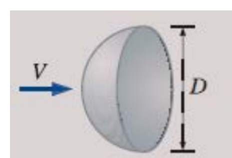
$$F_D = C_D \frac{1}{2}\rho U_\infty^2 A$$

- C_D values from Table 15-2:

Thin circular disk : $C_D = 1.1$



Hemisphere : $C_D = 0.4$



- Energy lost to drag (per year):

$$E_{drag} = (F_D)\left(\frac{\text{distance traveled}}{\text{year}}\right) \quad \left\{\frac{kJ}{yr}\right\}$$

- Fuel energy required to overcome drag:

$$E_{required} = \frac{E_{drag}}{\eta_{engine}} \quad \left\{\frac{kJ}{yr}\right\}$$

30%

- Fuel required $= \dfrac{E_{required}}{\text{heating value}} \quad \left\{\frac{kg}{yr}\right\}$

- Fuel required $= \dfrac{E_{required}}{(\text{heating value})(\rho_{fuel})} = \left\{\frac{L}{yr}\right\}$

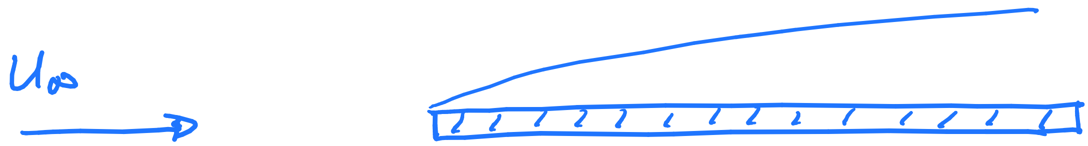
Mirror type	F_D (N)	E_{drag} $\left(\frac{kJ}{yr}\right)$	$E_{required}$ $\left(\frac{kJ}{yr}\right)$
Thin Disk	8.12	2.03×10^5	6.77×10^5
Hemisphere	2.95	7.38×10^4	2.46×10^5

\$0.90/L

Mirror type	Fuel required $\left(\frac{L}{yr}\right)$	Cost $\left(\frac{\$}{yr}\right)$
Thin Disk	20.5	18.46
Hemisphere	7.46	6.71

Savings: 13.1 L/yr $11.75/yr

3. [Chapter 15] Consider laminar flow of a fluid over a flat plate. Now the freestream velocity is doubled. Determine the change in the drag force on the plate. Assume that the flow remains laminar.



- Drag force on a flat plate:

$$F_D = C_f \frac{1}{2} \rho U_\infty^2 A$$

- If $U_{\infty 2} = 2 U_{\infty 1}$ find change in drag force

$$\frac{F_{D2}}{F_{D1}} = \frac{C_{f2} \cancel{\frac{1}{2}} \cancel{\rho} U_{\infty 2}^2 \cancel{A}}{C_{f1} \cancel{\frac{1}{2}} \cancel{\rho} U_{\infty 1}^2 \cancel{A}} = \frac{C_{f2} (2 \cancel{U_{\infty 1}})^2}{C_{f2} \cancel{U_{\infty 1}^2}} = \frac{4 C_{f2}}{C_{f1}}$$

- For laminar flow: $C_f = \dfrac{1.33}{Re_L^{1/2}}$

with $Re_L = \dfrac{\rho U_\infty L}{\mu}$

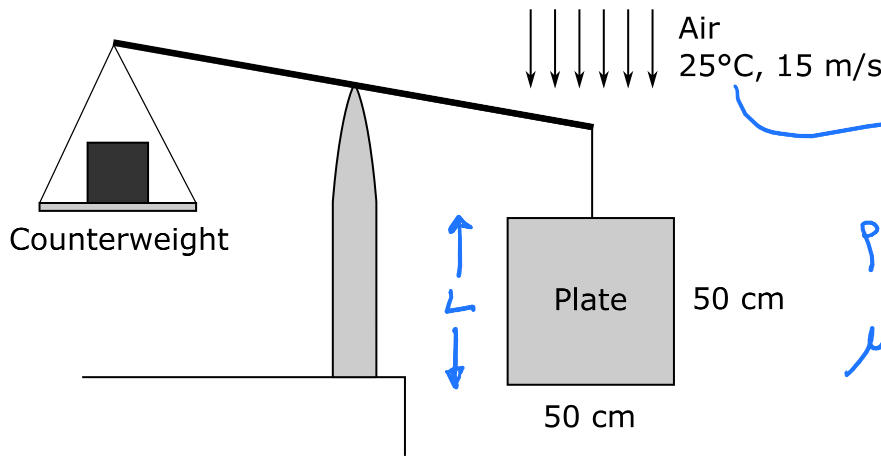
$$\frac{F_{D2}}{F_{D1}} = 4 \frac{C_{f2}}{C_{f1}} = 4 \frac{\cancel{1.33} / Re_{L2}^{1/2}}{\cancel{1.33} / Re_{L1}^{1/2}} = 4 \frac{Re_{L1}^{1/2}}{Re_{L2}^{1/2}}$$

$$= 4 \frac{(\cancel{\rho} U_{\infty 1} \cancel{L} / \cancel{\mu})^{1/2}}{(\cancel{\rho} U_{\infty 2} \cancel{L} / \cancel{\mu})^{1/2}} = 4 \frac{U_{\infty 1}^{1/2}}{U_{\infty 2}^{1/2}} = 4 \frac{\cancel{U_{\infty 1}^{1/2}}}{(2 \cancel{U_{\infty 1}})^{1/2}}$$

$$= \frac{4}{\sqrt{2}} = 2.83$$

$$\hookrightarrow F_{D2} = 2.83 F_{D1}$$

4. [Chapter 15] The weight of a thin flat plate 50 cm x 50 cm in size is balanced by a counterweight that has a mass of 2 kg. Now a fan is turned on and air at 1 atm and 25°C flows downward over both surfaces of the plate (front and back in the sketch) with a freestream velocity of 15 m/s. Determine the mass of the counterweight that needs to be added in order to balance the plate in this case.



$\rho = 1.184 \ kg/m^3$

$\mu = 1.85 \times 10^{-5} \ \frac{kg}{m \cdot s}$

- Need to calculate the drag force on the plate

$$F_D = C_f \tfrac{1}{2} \rho U_\infty^2 A$$

- To get C_f, first calculate Re to determine flow regime

laminar $< 5\times10^5$

$$Re_L = \frac{\rho U_\infty L}{\mu} = \frac{(1.184)(15)(50/100)}{1.85\times10^{-5}} = 4.8\times10^5$$

- For laminar flow over flat plate:

$$C_f = \frac{1.33}{Re_L^{1/2}} = 0.001919$$

- Drag force: $F_D = C_f \tfrac{1}{2}\rho U_\infty^2 A$

Both sides

$$= (0.0019)\tfrac{1}{2}(1.18)(15)^2\left(\tfrac{50}{100}\right)\left(\tfrac{50}{100}\right)(2)$$

$$= 0.1278 \ N$$

- Additional counterweight:

$$W = mg = F_D \Rightarrow m = \frac{F_D}{g} = \frac{0.1278}{9.81} = 0.013 \ kg = 13 \ g$$

5. [Chapter 15] A jumbo jet airplane has a mass of about 400,000 kg when fully loaded with over 400 passengers and takes off at a speed of 280 km/hr. Determine the takeoff speed when the airplane has 100 empty seats assuming that each passenger with luggage is 150 kg and the wing and flap settings are maintained the same.

- At liftoff weight = lift force:

$$W = F_L = C_L \tfrac{1}{2} \rho v^2 A$$

$$\Longrightarrow v = \sqrt{\frac{2W}{\rho C_L A}}$$

- Comparing the take-off speed under two weight scenarios:

$$\frac{v_2}{v_1} = \frac{\sqrt{\frac{\cancel{2}W_1}{\cancel{\rho}\cancel{C_L}\cancel{A}}}}{\sqrt{\frac{\cancel{2}W_2}{\cancel{\rho}\cancel{C_L}\cancel{A}}}}$$

ρ, C_L, A the same for both cases

$$= \sqrt{\frac{W_1}{W_2}} = \sqrt{\frac{m_1 \cancel{g}}{m_2 \cancel{g}}} = \sqrt{\frac{m_2}{m_1}}$$

$$v_2 = v_1 \sqrt{\frac{m_1}{m_2}} \qquad v_1 = 280 \frac{km}{hr} \qquad m_1 = 400{,}000 \text{ kg}$$

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1 passenger w/ luggage = 150 kg

100 passengers = (100)(150) = 15,000 kg

$m_2 = m_1 - 15{,}000 = 385{,}000$ kg

$$v_2 = 280 \sqrt{\frac{385}{400}} = 275 \frac{km}{hr}$$

Answers

1. 2.9 m/s^2
2. 13.1 L/year, $11.75/year
3. 2.8x increase
4. 13 g
5. 275 km/hr