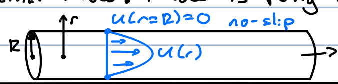
Nomenclature

- No-slip condition: Fluid adjacent to a solid surface takes the velocity of that surface
- External Flow: Surface in an infinite flow fluid
- Internal Flow: Flow is fully bounded by a solid surface



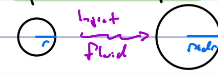
- Boundary layer (BL): Thickness over which viscous effects are felt

Surface Tension

- Comparative attraction between similar and dissimilar molecules
- Some Effects of Surface Tension

(i) Spherical Droplets (v) Capillary Rise
(ii) Meniscus
(iii) Free Surface
(iv) Wetting Behavior

Spherical Droplet



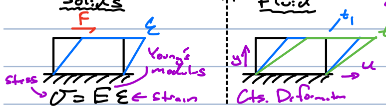
$\left[\sigma = \frac{\text{energy}}{\text{area}} = \frac{\text{force}}{\text{length}}\right] \left\{\frac{N}{m}\right\}$ Energy required to ↑ surface area

$dE = \sigma\, dA_s;\ dA_s = 8\pi r\, dr$, $dW = (P_i - P_o)4\pi r^2 dr$ $\Big\}\ (P_i - P_o) = \frac{2\sigma}{r}$

Viscosity

Resistance to flow, molecular friction

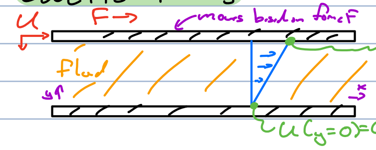
→ Consider an applied force on a solid vs. fluid



Shear stress $\tau \propto \frac{du}{dy}$ — directly proportional for newtonian fluids (Ex: water, air, oil, etc.)

- Dynamic Viscosity $\equiv \mu = \nu\rho$ (kinematic viscosity)(density)
- Non-newtonian fluid viscosity changes based on force

Couette Flow



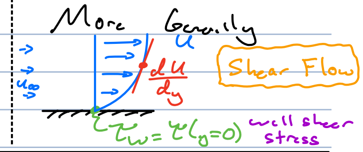
- Shear stress on the fluid: $\tau = F/A$
  - ↳ For general fluids: $F = \tau \cdot A$

Finding the Force

① $F = \tau A = \mu \frac{du}{dy} A$ all

② $F = \tau A = \mu \frac{U}{L} A$ Couette

More Generally



Hydro Statics

Impact of pressure on engineered surfaces

Applications: Dams, Tanks

Pressure $\left[\frac{\text{force}}{\text{area}}\right]$

Pressure: (i) Acts isotropically (same in all directions)

(ii) Leads to a pressure force: $F_{pressure} = \int P\, dA$

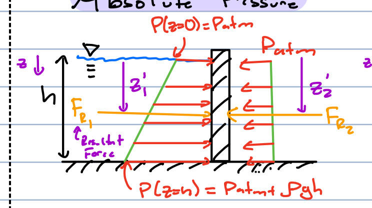
(iii) Pressure force on a submerged object (like a dam) always acts normal to the surface and inward

(iv) Varies with depth according to hydrostatic pressure variation

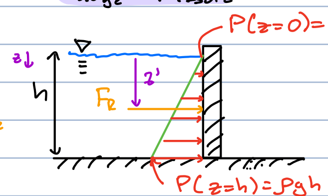
$$\frac{dP}{dz} = \rho g \Rightarrow P(z) = \int \rho g\, dz = P(z=0) + \rho g z$$

Gauge Pressure: $P_{gauge} = P_{absolute} - P_{atm}$

Absolute Pressure



Gauge Pressure



- Two methods: Decomposition & Integration

Line of Action

Location needs to lead to the same moment as the force distribution

Resultant Force

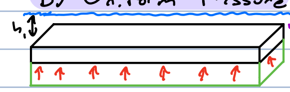
$F_R = \int_A P(z)\, dA = \int_0^h P(z)\, w\, dz$

a) Linear Pressure Distribution

$$F_R = w\int_0^h \rho g z\, dz = \left(\tfrac{1}{2}\rho g h\right)(hw)$$

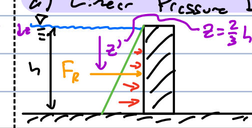
(average pressure)(area)

b) Uniform Pressure



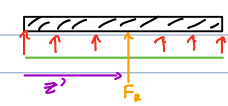
$F_R = \rho g h_1 A$

a) Linear Pressure Distribution



$z' = \frac{2}{3}h$

b) Uniform Pressure



z is halfway across

Fluid Dynamics

- Stream Lines: Line formed tangent to the velocity at every point
- For steady flows, all fluid particles passing through a center point follow the same stream lines

Bernoulli Equation

Standard Form

- Provides a relationship between pressure, elevation, and velocity

Applications: Nozzles, Dams, etc.

Assumptions: (i) Steady Flow - no transient effects

(ii) Inviscid - viscous effects negligible

(iii) Incompressible (ρ constant)

(iv) Flow along a stream line

(v) No heat transfer

(vi) No work interactions may be considered for this form

Relationship: $\frac{P}{\rho} + \frac{1}{2}V^2 + gz = \text{constant}$ → Called the conventional form

$$\Rightarrow P_1 + \tfrac{1}{2}\rho V_1^2 + \rho g z_1 = P_2 + \tfrac{1}{2}\rho V_2^2 + \rho g z_2$$

Energy form of the Bernoulli Equation

$$\frac{P_1}{\rho} + \frac{1}{2}\alpha_1 V_1^2 + gz_1 + w_{pump} = \frac{P_2}{\rho_2} + \frac{1}{2}\alpha_2 V_2^2 + g z_2 + w_{turbine} + e_{losses}$$

↳ Kinetic energy correction factor (α): laminer $\alpha = 2$ and turbulent $\alpha \approx 1.05 \approx 1$

$\dot{E}_{losses} = \dot{E}_{losses,pump} + \dot{E}_{losses,turbine} + \dot{E}_{losses,piping}$

mass base "power" ↳ $\begin{cases} \dot{W}_{pump,u} = \dot{W}_{pump} - \dot{E}_{losses,pump} \\ \dot{W}_{turbine,e} = \dot{W}_{turbine} + \dot{E}_{losses,turbine} \end{cases}$ (useful; extractable; actual) $\Rightarrow$ $\eta_{pump} = \frac{\dot{W}_{pump,u}}{\dot{W}_{pump}}$, $\eta_{turbine} = \frac{\dot{W}_{turbine,e}}{\dot{W}_{turbine,e}}$

Also, $\dot{W}_{pump} = \eta_{motor} \cdot \dot{W}_{elec}$

Bernoulli Equation in Terms of Head (h)

divide by $\dot{m}g$

$h_{pump,u} = \frac{\dot{W}_{pump,u}}{\dot{m}g}$, $h_{turbine,e} = \frac{\dot{W}_{turbine}}{\dot{m}g}$, $h_L = \frac{\dot{E}_{losses,piping}}{\dot{m}g}$

$$\Rightarrow \frac{P_1}{\rho_1 g} + \frac{\alpha_1 V_1^2}{2g} + z_1 + h_{pump,u} = \frac{P_2}{\rho_2 g} + \frac{\alpha V_2^2}{2g} + z_2 + h_{turbine,e} + h_L$$ ← piping

h_L is the irreversible head loss between ① and ② due to fluid friction → The diff. states

Internal Flow

Laminar Flow

- Smooth, orderly flow
- Low velocity, high viscosity

Turbulent Flow

- High mixing, chaotic flow
- High velocity, low viscosity

⇒ Flow Regime based on the Reynolds number

$Re = \frac{\rho V_{avg} D}{\mu}$ ← characteristic length; μ ← dynamic viscosity

- Ratio of inertial to viscous effects
- Re_{crit} is transition from laminar to turbulent flow

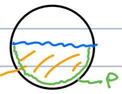
Internal Flow: $Re_{crit} = 2300$

Non-circular Cross Sections

- Use hydraulic diameter D_h

$D_h = \frac{4A_c}{P}$ ← cross-sec area; P ← wetted perimeter

Ducts, rivers, or sewage pipes



Developed Flow

- Entrance region is before flow is fully developed, typically $L/D \approx 10$
- BL present in entrance region

$$u(r) = \boxed{-\frac{R^2}{4\mu}\cdot\frac{dP}{dx}}\left(1 - \frac{r^2}{R^2}\right)$$

maximum velocity

$\frac{dP}{dx} < 0$ must be true to push fluid in + direction (+u flow)

Let $\frac{dP}{dx} \equiv \Delta P_L = P_1 - P_2$

- Average velocity: $u_{avg} = -\frac{R^2}{8\mu}\cdot\frac{dP}{dx}$ For laminar flow $u_{avg} = \frac{1}{2}u_{max}$

$\Rightarrow \Delta P_L = \frac{8\mu u_{avg} L}{R^2} = \frac{32\mu u_{avg} L}{D^2}$ (Laminar)

- Pumping power to overcome frictional losses: $\dot{W}_{pump_L} = \dot{V}\Delta P_L$

Generalize with Darcy Friction Factor (f)

$\Delta P_L = f\frac{L}{D}\frac{1}{2}\rho u_{avg}^2;\ h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{u_{avg}^2}{2g}$

(i) Laminar Flow

$\Delta P_L = \frac{L}{D}\cdot\frac{1}{2}\rho u_{avg}^2\left(64\frac{\mu}{\rho u_{avg} D}\right) \Rightarrow f = \frac{64}{Re_D}$

(ii) Turbulent flow

Use moody tables or...

① $\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$ Colebrook

② $\frac{1}{\sqrt{f}} \approx -1.8\log_{10}\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right]$ Haaland (2% error)

$\frac{\varepsilon}{D} = \frac{\text{roughness}}{\text{diameter}}$

Minor Losses

(i) Elbows, Tees (iii) Piping Transitions
(ii) Valves (iv) Entrance/Exit

$\Rightarrow h_L = K_L \cdot \frac{V_{avg}^2}{2g}$ K_L from tables

Combining Losses

(i) Series a) $\dot{V}$ const (ii) Parallel a) $\dot{V} = \dot{V}_a + \dot{V}_b$

b) $h_L = \Sigma h_{L,major} + \Sigma h_{L,minor}$ b) $\Delta P_A = \Delta P_B = h_{L_A} = h_{L_B}$

External Flow

- Flow over bodies => aerodynamic forces (lift, drag)
- Two primary mechanisms:

(i) Friction: $\tau = \mu \frac{du}{dy}$

↳ Sheer stress over surface area => frictional force

(ii) Pressure Differences: Reduced pressure in wake

↳ pressure differences over surface => pressure force

- External Flow: $Re_{crit} = 5 \times 10^5$ less than this is laminar

Note: For a tree, velocity ↑ => C_D↓ as leaves move in direction of wind and fold to reduce drag (part of evolution)

Aerodynamic Force Balances

(i) Take-off (to) velocity for an airplane

$$V_{to} = \sqrt{\frac{2W}{\rho C_L A}}$$

where $W = mg$ and C_L is lift coefficient

(ii) Terminal (t) Velocity

sedimentation, suspensions

$$V_t = \sqrt{\frac{2 V g}{C_D \cdot A} \cdot \frac{\rho_s - \rho_f}{\rho_f}}$$

where s, f are solid and fluid respectively

Lift Force

- Lift coefficient depends on

(i) Shape

(ii) Angle of attack (α)

↳ angle between chord & horizontal

- Stall: Lift Coefficient increases with angle of attack up to stall point

↳ Typically Occurs around 15°

- Effect of flaps: Increase lift coefficient at the expense of drag

Drag and Lift

- Effects that influence aerodynamic forces

(i) Fluid Properties (ρ, μ)

(ii) Object Properties / Shape

a) Streamlined

b) Blunt or Bluffed

(iii) Flow Regime (Laminar vs. Turbulent)

- F_L: Lift force - Acts perpendicular to free stream
- F_D: Drag force - Acts along free stream

Definitions / Expressions

- $C_D = \frac{F_D}{\frac{1}{2}\rho u_\infty^2 A}$

$\Rightarrow C_D \frac{1}{2} \rho u_\infty^2 A$

- $C_L = \frac{F_L}{\frac{1}{2}\rho u_\infty^2 A}$

$\Rightarrow F_L = C_L \frac{1}{2} \rho u_\infty^2 A$

C_D and C_L can exceed 1

Appropriate Area for C_D, C_L

(i) Blunt Frontal Area

a) Cylinder $A_F = DL$

b) Sphere $A_F = \frac{\pi}{4} D^2$

(ii) Streamlined body

Planform Area (A_p)

$A_p = b \cdot c$ (span b, chord length c)

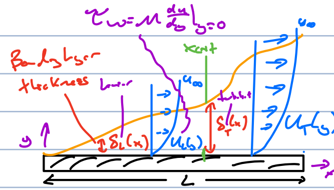
- Get C_D values from Tables based on (i) Shape and (ii) Reynolds number

Drag Force

(i) Skin friction drag

→ Due to shear stress

→ Dominant mechanism for streamlined objects



$$F_D = F_P = C_f \frac{1}{2} \rho u_\infty^2 A$$

↳ Skin friction coefficient; A: Area in contact with fluid

- Overall Reynolds #: $Re_L = \frac{\rho u_\infty L}{\mu}$
- Local Reynolds #: $Re_x = \frac{\rho u_\infty x}{\mu}$

- Critical Reynolds #: $Re_{crit} = 5 \times 10^5$

$$\begin{cases} Re_x < Re_{crit} \Rightarrow \text{Laminar} \\ Re_x > Re_{crit} \Rightarrow \text{Turbulent} \end{cases}$$

Describes boundary layer

- Flow over a flat plate can be... (To get C_f value)

(i) Entirely Laminar

$C_{f,laminar} = 1.33 / \sqrt{Re_L}$

(ii) Primarily Turbulent

$C_{f,turbulent} = 0.074 / Re_L^{1/5}$

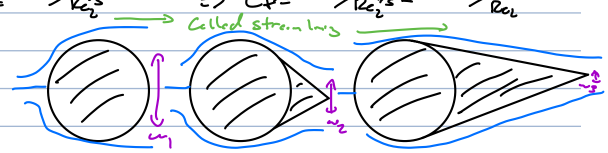
(iii) Combination of Laminar & Turbulent BL

$\Rightarrow C_F = 0.074 / Re_L^{1/5} - 1742 / Re_L$

(ii) Form Drag (Pressure Drag)

→ Due to flow blocking / Separation

→ Dominant mechanism for bluff bodies



- Decreasing wake size decreases form drag at the expense of friction drag and goal is net reduced total drag

Basic
- ↑ form/pressure drag

slightly streamlined
- ↓ form, ↑ friction

no separation
- form drag ↓↓, friction drag ↑↑, overall ↓