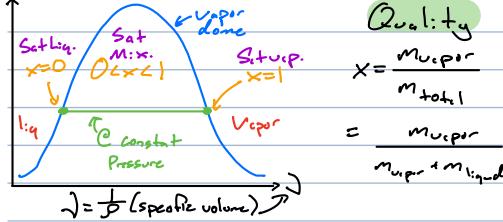


Modes of Energy Storage @ Microscale

- Structure @ Microscale defines macrostate of energy
- | | | |
|---|---------------------------------|---------------------|
| Sensible Modes Related to activity & motion | (i) Translational Motion (KE) | (ii) Phase Changes |
| | (iii) Vibrational Motion | (iv) Chemical Bonds |
| | (vii) Rotational Motion | (vi) Nuclear Bonds |
| | Latent modes Structural changes | |
| | (viii) Electron Translation | |

Energy can cross boundary as: (i) Heat (ii) Work (iii) Mass
Mass Flow Rate: $\dot{m} = \rho V_{avg} A_c$

Phase Change Fluids (Tables)



Phase Regions

- (i) Compressed Liquid $\rightarrow @ \text{given } P: T < T_{sat}$
- (ii) Saturated mixture $(O_{excl}) \rightarrow T = T_{sat} \& P = P_{sat}$
- (iii) Saturated Vapor $(x=1) \rightarrow T = T_{sat} \& P = P_{sat}$
- (iv) Saturated Liquid ($x=0$) $\rightarrow T = T_{sat} \& P = P_{sat}$
- (v) Superheated Vapor $\rightarrow T > T_{sat} \& P < P_{sat}$

No quality in compressed liquid or ST regions

Closed Systems



$$\text{1st Law: } (Q_{in} - Q_{out}) + (W_{in} - W_{out}) = U_2 - U_1 \quad \text{or} \quad \Delta U = Q - W$$

Can be an isobaric or isothermal process

Q_{in} : Heat Transfer into system

Q_{out} : Heat Transfer out of system

W_{in} : Work done on the system

W_{out} : Work done by the system

Boundary Work (Generally)

$$W_{piston} = \int_1^2 P dV = P \Delta V = P_m \Delta V = W_{out}$$

\hookrightarrow True for isobaric systems

$$W_{piston} = W_{out} \rightarrow W_{out} > 0 \text{ if volume } \uparrow$$

Polytropic Processes

$$PV^n = \text{Constant} \Rightarrow P_1 V_1^n = P_2 V_2^n \quad \& \quad W_{piston} = \frac{P_2 V_2 - P_1 V_1}{1-n}$$

Ideal Gas EoS

Law: $PV = nRT$ \hookrightarrow Universal Gas Constant

EoS: $PV = mRT$ \hookrightarrow Particular Gas Constant

$$P = RT \quad R = \frac{\bar{R}}{M_w}$$

Specific Volume \rightarrow

$$\bar{V} = \frac{1}{\rho} = \frac{V}{m} \Rightarrow m = \frac{V}{\bar{V}}$$

$$m = \frac{V}{\bar{V}} \quad \& \quad V = m \bar{V}$$

Finding D_u \hookrightarrow Treat air as ideal in this course

1) Ideal Gases

- Internal energy only a function of temperature
- Use specific heat value

C_v : Specific Heat @ Constant Vol.

$$C_v = \frac{du}{dt} = C_v \text{ (constant)} \quad \Delta u = C_v (T_2 - T_1)$$

C_p : Specific heat @ constant Press.

\hookrightarrow See 'Open Systems'

2) True Liquids / Solids

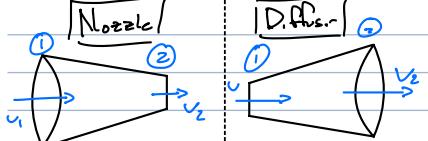
- $C_v = C_p = C$

$$\Delta u = u_2 - u_1 = \int_1^2 c_v dt = C_{avg}(T_2 - T_1)$$

$$\Delta u = C \Delta T \quad \& \quad \Delta u = m \Delta T$$

Steady Flow Devices

1) Nozzles / Diffusers



$$V \uparrow \text{ as } A \downarrow, P_2 < P_1 \quad V \downarrow \text{ as } A \uparrow, P_2 > P_1$$

$$Exchanges enthalpy for KE \\ O = \dot{m} [(h_2 - h_1) + \frac{1}{2} (V_2^2 - V_1^2)]$$

$$Assume: \Delta PE = 0, adiabatic, W = 0 \text{ (Pisopic system)}$$

Second Law of Thermodynamics

- Quantitatively: (i) Processes proceed spontaneously in a particular direction

(ii) Energy has quality, not just quantity

(iii) Sets limits on theoretical Performance

Modes of Heat Transfer

- (i) Conduction: Energy Transfers from higher to lower energetic matter due to direct interaction
- (ii) Convection: Enhanced conduction due to fluid motion
- (iii) Radiation: Energy transfer via electromagnetic waves

First Law on a Total Basis

$$(Q_{in} - Q_{out}) + (W_{in} - W_{out}) + (E_{mass,in} - E_{mass,out}) = (U_2 - U_1) + \Delta KE + \Delta PE$$

Phase Change Fluids (Tables)

Phase Regions

- (i) Compressed Liquid $\rightarrow @ \text{given } P: T < T_{sat}$
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Boundary Work (Generally)

$$W_{piston} = \int_1^2 P dV = P \Delta V = P_m \Delta V = W_{out}$$

\hookrightarrow True for isobaric systems

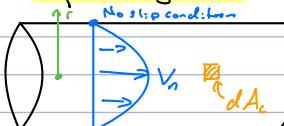
$$W_{piston} = W_{out} \rightarrow W_{out} > 0 \text{ if volume } \uparrow$$

Example Constants (i) & (ii) must be true for: ideal gas

$$(i) n=0; \text{constant } P \text{ isobaric} \quad (ii) \gamma = \frac{C_p}{C_v} \text{ is constant } S \text{ isentropic}$$

$$(iii) n=1; \text{constant } T \text{ isothermal} \quad (iv) n \rightarrow \infty; \text{constant } V \text{ isochoric}$$

Open Systems



Expressions \rightarrow avg velocity

$$\dot{m} = \rho V_{avg} A_c \quad \text{velocity profile}$$

$$\text{reality: } \dot{m} = \int A_c \rho V dA_c$$

$$V_{avg} = \frac{1}{A_c} \int V dA_c$$

$$\dot{m} = \frac{\dot{m}}{A_c} = V_{avg} A_c$$

Elements

$$(i) Kinetic Energy (\frac{1}{2} V_{avg}^2)$$

$$(ii) Internal energy (u)$$

$$(iii) Potential Energy (gz)$$

$$(iv) Flow Work Energy (PV)$$

Focus on Steady Flow Devices

mass (i) $m_{in} = m_{out} \rightarrow$ zero accumulation

energy (ii) $E_{in} = E_{out}$

1st Law:

$$(Q_{in} - Q_{out}) + (W_{in} - W_{out}) + (E_{mass,in} - E_{mass,out}) = 0$$

For steady operations

$$\Rightarrow (Q_{in} - Q_{out}) + (W_{in} - W_{out}) = \dot{m} [h_2 - h_1 + \frac{1}{2} (V_2^2 - V_1^2) + g(z_2 - z_1)]$$

$$h = u + PV \rightarrow E_{mass} = \dot{m} (h + \frac{1}{2} V^2 + gz)$$

$$\text{Calculating } \Delta h \quad h = u + PV \rightarrow E_{mass} = \dot{m} (h + \frac{1}{2} V^2 + gz)$$

1) Ideal Gases

$$C_p = \frac{dh}{dT} \quad @ \text{constant } P \text{ pressure}$$

$$\Delta h = \int_1^2 C_p dT$$

$$= C_p (T_2 - T_1)$$

2) True Liquids/Solids

$$C_p = C_v = C$$

$$\Delta h = C_{avg} \Delta T$$

$$= C_{avg} (T_2 - T_1)$$

3) Phase Change Fluids

$$C_p = C_v = C$$

$$\Delta h = h_f + x(h_g - h_f)$$

$$= x h_g + (1-x) h_f$$

Heat Exchangers



Transfer heat from hot to cold fluid

Assume: $T_{in,1} = T_{out,2}$

$\cdot T_{out,1} = 0$

$\cdot \Delta PE = 0$

$\cdot \Delta KE = 0$

Energy Analysis based on boundary color

$$* \quad Q_{out} = \dot{m}_H (h_2 - h_1)$$

$$* \quad Q_{in} = \dot{m}_C (h_2 - h_1)$$

$$* \quad Q_{heat} = \dot{m}_H (h_2 - h_1) + \dot{m}_C (h_2 - h_1)$$

$$* \quad Q_{out} = \dot{m}_H (h_2 - h_1) = \dot{m}_H (h_2 - h_1)$$

Heat Engines Converts heat (less valuable) into work (more valuable)

4V Elements

Examples: Steam Power Plant, Car Engine

(i) Receives heat from high Temp res. T_H

(ii) Rejects heat to low temp res. T_L

(iii) Converts a portion of the heat into work

(iv) Operates as a cycle

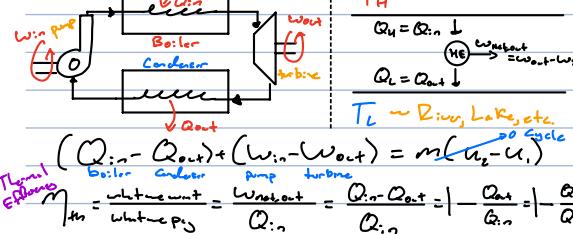
T_H

$O \downarrow \text{in} \quad W \uparrow \text{out}$

$O \downarrow \text{out} \quad O \uparrow \text{in}$

T_L

Rankine Cycle Power plant as a heat engine



Carnot Principles Carnot cycles are ideal cycles/conditions

(i) Efficiencies of an irreversible (real) cycle is always less than a reversible cycle

(ii) Efficiency of all reversible engines acting between T_H and T_L is the same

$\Rightarrow \left(\frac{Q_H}{Q_L} \right)_{rev} = \left(\frac{T_H}{T_L} \right)_{ideal} \rightarrow \eta_{Carnot} = 1 - \frac{T_L}{T_H} \leftarrow \text{must be absolute from } \eta_{th} = 1 - \frac{Q_L}{Q_H}$

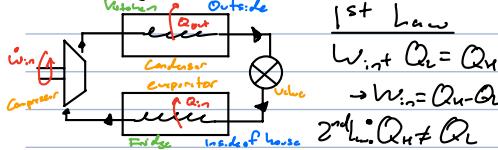
Carnot Heat Engine

- (i) Isothermal expansion @ T_H (iii) Isothermal compression @ T_L
- (ii) Adiabatic expansion: $T_H \rightarrow T_L$ (iv) Adiabatic compression

$$CoP_{Carnot} = \frac{1}{\frac{T_L}{T_H} - 1}$$

$$CoP_{HP, Carnot} = \frac{1}{1 - \frac{T_L}{T_H}}$$

Refrigeration cycle valid for fridges, ACs



$$CoP_R = \frac{Q_{in}}{W_{in}} = \frac{Q_L}{W_{in}} = \frac{Q_L}{Q_H - Q_L} = \frac{1}{\frac{Q_H}{Q_L} - 1}, \quad CoP_R = [2, 6] \text{ usually}$$

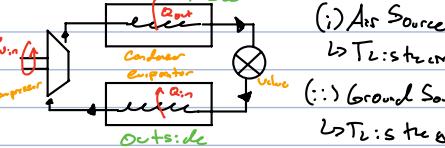
T_H

$$\begin{aligned} Q_H &\uparrow Q_H = Q_{in} \\ W_{in} &\rightarrow R \end{aligned}$$

T_L

$$\begin{aligned} Q_L &\uparrow Q_L = Q_{in} \\ W_{in} &\rightarrow \end{aligned}$$

Heat Pump Two Types



T_H

$$\begin{aligned} Q_H &\uparrow Q_H = Q_{out} \\ W_{in} &\rightarrow HP \end{aligned}$$

T_L

$$\begin{aligned} Q_L &\uparrow Q_L = Q_{in} \\ W_{in} &\rightarrow \end{aligned}$$

Entropy

- (i) Entropy change as a measure of energy quality/degradation
- (ii) Entropy as a measure of molecular system disorder \leftarrow more microstates \rightarrow more entropy

For an Internally Reversible Process

$$dS = \left(\frac{dQ}{T} \right)_{\text{ideal}} \quad dS = \frac{ds}{T} \text{ mass specific}$$

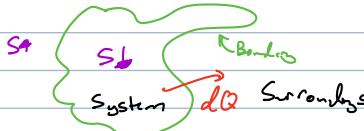
Total Entropy Change

$$\Delta S_{\text{total}} = \Delta s_{\text{sys}} + \Delta s_{\text{sur}}$$

(i) $\Delta s_{\text{tot}} > 0 \Rightarrow \text{real/irreversible}$

(ii) $\Delta s_{\text{tot}} = 0 \Rightarrow \text{ideal/reversible}$

(iii) $\Delta s_{\text{tot}} < 0 \Rightarrow \text{impossible!}$



For a Real Process

$$dS > \frac{dQ}{T} \Rightarrow \Delta s = \int_1^2 \frac{dQ}{T} \text{ Not Q and T relationship}$$

$$\Delta s_{\text{gen}} = \int_1^2 \left(\frac{dQ}{T} \right)_{\text{ideal}} + S \leftarrow \text{generated through irreversibilities}$$

Calculating Entropy Change

$$\begin{aligned} \text{Assume } T_1 &= T_0 \quad \text{valid for surroundings} \\ \Delta s_{\text{sur}} &= \int_1^2 \frac{dQ}{T} = \int_1^2 \frac{dQ}{T_0} = \frac{Q_2}{T_0} \leftarrow \text{Transfer from 1 to 2} \end{aligned}$$

(ii) More Generally

$$\begin{aligned} dQ - dW &= dU \quad \text{dude} \\ \text{Assume } dQ &= \text{inflow} \quad \text{dude} \quad \text{dude} \quad \text{dude} \\ dQ - dW &= dU \quad \text{dude} \quad \text{dude} \quad \text{dude} \quad \text{dude} \\ T ds - P dV &= dU \quad \text{Gibbs Equation} \end{aligned}$$

Evaluating Δs

1) Ideal Gases

$$\Delta s_{\text{sys}} = C_{V,\text{gas}} \ln \left(\frac{T_2}{T_1} \right) + R \ln \left(\frac{V_2}{V_1} \right)$$

$$\Delta s_{\text{sys}} = C_{P,\text{gas}} \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{P_2}{P_1} \right)$$

1) Isentropic Processes $\Delta s_{\text{sys}} = 0 \rightarrow s_2 = s_1$

2) $T \propto c$ Liquids/Solids

2) $T \propto c$ Liquids/Solids

$$\text{Incompressible: } dV \approx 0 \quad C_p = C_V = C \quad dP \approx 0$$

$$\Delta s_{\text{sys}} = C \ln \left(\frac{T_2}{T_1} \right)$$

3) Phase Change Fluids

3) Phase Change Fluids

$$\Delta s_{\text{sys}} = C \ln \left(\frac{T_2}{T_1} \right) = 0$$

$$C \ln \left(\frac{T_2}{T_1} \right) = 0 \quad T_2 = T_1 \text{ take true}$$

Solids & liquids \leftarrow incompressible liquids

require isentropic process to be isothermal

3) Phase Change Fluids

$$s_2 = s_1$$

Use tables and interpolate

Beware of false assumptions in question!

Isentropic Efficiency

(i) Turbines $\eta_{isentropic} = \frac{\text{actual}}{\text{isentropic}} = \frac{h_1 - h_2A}{h_1 - h_2S}$

Diff Enthalpies $h_{2A} = \text{isentropic}$

$h_{2S} = \text{actual}$

(ii) Compressors $\eta_c = \frac{\text{isentropic}}{\text{actual}} = \frac{h_{2S} - h_1}{h_{2A} - h_1}$

Diff Enthalpies $h_{2A} = \text{isentropic}$

$h_{2S} = \text{actual}$

Watch for h_{2A} vs. h_{2S} & η

1) Ideal Gases

$$\frac{T_2s}{T_1} = \left(\frac{V_1}{V_2} \right)^{k-1} \quad R = C_p - C_v$$

$$\frac{T_2s}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} \quad k = \frac{C_p}{C_v}$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^k$$

All 3 valid
only for an
ideal gas in an
isentropic process

1) Isentropic Processes $\Delta s_{\text{sys}} = 0 \rightarrow s_2 = s_1$

2) $T \propto c$ Liquids/Solids

$$\frac{T_2s}{T_1} = \left(\frac{V_1}{V_2} \right)^{k-1}$$

$$\frac{V_2}{V_1} = \left(\frac{T_2}{T_1} \right)^{\frac{1}{k-1}}$$

$$\frac{P_2}{P_1} = \left(\frac{T_1}{T_2} \right)^{\frac{1}{k-1}}$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^{\frac{1}{k-1}}$$

$$\frac{V_2}{V_1} = \left(\frac{P_2}{P_1} \right)^{\frac{k}{k-1}}$$

$$\frac{P_2}{P_1} = \left(\frac{T_1}{T_2} \right)^{\frac{k}{k-1}}$$

$$\frac{T_2s}{T_1} = \left(\frac{P_1}{P_2} \right)^{\frac{k}{k-1}}$$

$$\frac{P_2}{P_1} = \left(\frac{T_1}{T_2} \right)^{\frac{1}{k}}$$

$$\frac{V_2}{V_1} = \left(\frac{T_1}{T_2} \right)^{\frac{1}{k}}$$

$$\frac{P_2}{P_1} = \left(\frac{T_1}{T_2} \right)^{\frac{k}{k-1}}$$

$$\frac{V_2}{V_1} = \left(\frac{P_1}{P_2} \right)^{\frac{1}{k}}$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^{\frac{k-1}{k}}$$

External Flow

- Flow over bodies \Rightarrow aerodynamic forces (C_L , C_D , F_L , F_D)
- Two primary mechanisms:
 - (i) Friction: $T = \mu \frac{dy}{dx}$

\hookrightarrow Shear stress over surface area \Rightarrow frictional force

(ii) Pressure difference: Reduced pressure make

\hookrightarrow pressure differences over surface \Rightarrow pressure force

• External Flow: $Re_{\text{crit}} = S \times 10^5$ less than trees laminar

Note: For a tree, velocity $\uparrow \Rightarrow C_D$ as leaves move in direction of wind and add to drag (pink dot evolution)

Aerodynamic Force Balances

(i) Take-off (C_L) velocity for an airplane

$$V_{\text{to}} = \sqrt{\frac{2W}{\rho g_2 A}} \quad \text{where } W = \text{mass} \text{ and } C_L \text{ is lift coefficient}$$

(ii) Terminal (V_t) velocity splittings, suspensions

$$V_t = \sqrt{\frac{2V_0}{C_D \cdot A}} \cdot \frac{P_s - P_f}{\rho f} \quad \text{where } f \text{ face solid and fluid respectively}$$

Lift Force

• Lift coefficient depends on

(i) Shape

(ii) Angle of attack (α)

\hookrightarrow angle between chord & horizontal

• Still: Lift Coefficient increases with angle of attack up to still point

\hookrightarrow Typically occurs around 15°

• Effect of flaps: increase lift coefficient at the expense of drag

Drag and Lift

• Effects that influence aerodynamic forces

(i) Fluid Properties (P_{air})

(ii) Object Properties / Shape

a) Streamlined

b) Blunt or Buffet

(iii) Flow Regime (Laminar vs. Turbulent)

• F_L : Lift force - Acts perpendicular to free stream

• F_D : Drag force - Acts along free stream

• Got C_D values from Tables based on (i) Shape and (ii) Reynolds number

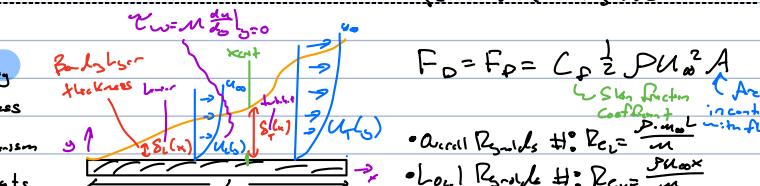
Drag Force

(i) Skin friction drag

\rightarrow Due to shear stress

\rightarrow Dominant mechanism

for streamlined objects



Definitions / Expressions

$$\bullet C_D = \frac{F_D}{\frac{1}{2} \rho U_\infty^2 A}$$

$$\Rightarrow C_D \frac{1}{2} \rho U_\infty^2 A$$

$$\bullet C_L = \frac{F_L}{\frac{1}{2} \rho U_\infty^2 A}$$

$$\Rightarrow F_L = C_L \frac{1}{2} \rho U_\infty^2 A$$

C_D, C_L

can cancel

Appropriate Area for C_D , C_L

(i) Blunt Front Area

(ii) Streamlined body

c) Cylinder: $A_F = \pi D L$

Plat Form Area (A_p)

d) Sphere: $A_F = \frac{\pi}{4} D^2$ $A_p = b \cdot c$

Span Chord length

• Overall Reynolds #: $Re_{\text{crit}} = \frac{D_{\text{chord}} \cdot U_\infty}{\nu}$

• Local Reynolds #: $Re_x = \frac{U_\infty x}{\nu}$

Describes boundary layer

• Flow over a flat plate can be... (To get C_D value)

(i) Entry Linear

$$C_{f,\text{linear}} = \frac{1.33}{Re_2}$$

(ii) Prandtl Turbulent

$$C_{f,\text{turbulent}} = \frac{0.074}{Re_2^{1/4}}$$

(iii) Combination of Laminar & Turbulent + BL

$$\Rightarrow C_D = \frac{0.074}{Re_2^{1/4}} - 1742^2 / Re_2$$

• Form Drag (Pressure Drag)

\rightarrow Due to flow blocking / Separation

\rightarrow Dominant mechanism for bluff bodies

• Decrease wake size also reduces form drag at the expense of friction drag and goal is not reduced total drag

Basic slightly streamlined no separation

• Form pressure drag \uparrow due to \uparrow Re.

• Form drag due to friction drag \uparrow , overall \downarrow

1. [Chapter 10] For flow over a plate, the variation of velocity with vertical distance y from the plate is given as $u(y) = ay + by^2$ where a and b are constants. Obtain a relation for the wall shear stress in terms of a , b , and μ .

$$u(y) = ay + by^2$$

$$\tau = \mu \frac{du}{dy}$$

↑ Shear Stress

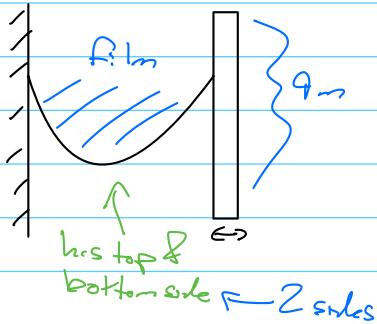
$$\rightarrow \tau = \mu \frac{d}{dy} (ay + by^2)$$

$$\tau = \mu a + 2\mu b y$$

$$y=0 \text{ at wall} \therefore \tau_w = \mu a + 2\mu b(0) = \mu a$$

$\tau_w = \mu a$

2. [Chapter 10] The surface tension of a liquid is to be measured using a liquid film suspended on a U-shaped wire frame with a 9 cm long movable side. If the force needed to move the wire is 0.045 N, determine the surface tension of this liquid in air.



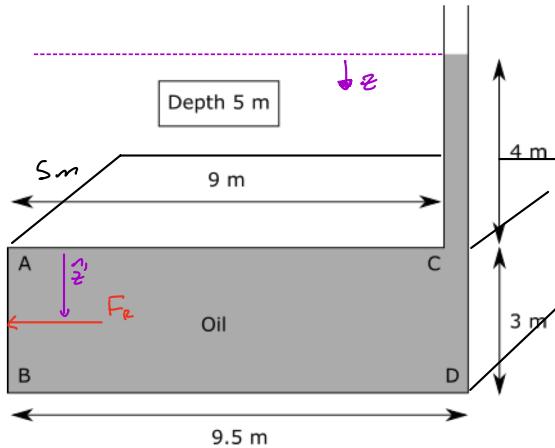
$$F = 2\sigma L$$

Force applied to move curve
film has 2 surfaces
surface tension
Length of wire (m)

$$\sigma = \frac{F}{2L} = \frac{0.045 \text{ N}}{2(0.09 \text{ m})} = 0.25 \text{ N/m}$$

$$\sigma = 0.25 \text{ N/m}$$

3. [Chapter 11] The following $3 \text{ m} \times 9.5 \text{ m} \times 5 \text{ m}$ tank shown is filled with oil of SG=0.8. Specific gravity (SG) is the ratio of the fluid density to the density of water. Depth here is the depth into the page.



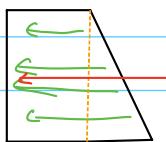
- (a) the magnitude and the location of the line of action of the resultant force acting on surface AB.
- (b) The pressure force acting on surface BD.
- (c) Will the force acting on surface BD equal the weight of oil in the tank? Find a relation.

$$SG_{0:1} = \frac{\rho_{0:1}}{\rho_{H_2O}} = 0.8, \rho_{H_2O} = 997.048 \frac{\text{kg}}{\text{m}^3}$$

$$0.8 \rho_{H_2O} = \rho_{0:1}$$

$$\rightarrow \rho_{0:1} = 797.638 \frac{\text{kg}}{\text{m}^3}$$

a)



$$F_R = F_{R,T} + F_{R,R}$$

$$F_{R,T} = \frac{1}{2} \rho g h (h_w)$$

$$= \frac{1}{2} (797.638)(4.81)(3)(15)$$

$$= 176059 \text{ N}$$

$$F_R = 645549 \text{ N}$$

$$z' = 5.64 \text{ m}$$

$$F_{R,R} = \rho g h_i A$$

Unidirectional
Position des freie
Flüssigkeit
oberfläche

$$= (797.638)(0.81)(4)(15)$$

$$= 469440 \text{ N}$$

$$F_R z' = \frac{2}{3} h F_{R,T} + \frac{1}{3} h F_{R,R}$$

$$\rightarrow z' = 1.64 \text{ m}$$

z' should satisfy $z=0$

$$z' = z' + 4 \text{ m} = 5.64 \text{ m}$$

b) $F_R = \rho g h_i A$

$h_i = 7 \text{ m}$ from free-surface

$$A = 9.5 \times 5 \text{ m} = 47.5 \text{ m}^2$$

$$F_R = 797.638 * 9.81 * 7 * 47.5 \text{ m}$$

$$= 2.6 \times 10^6 \text{ N}$$

$$F_R = 2.6 \text{ MN}$$

c) $W_{0:1} = mg, m = \rho_{0:1} \times V$

$$Volume = (Top \text{ } K) + (Top \text{ } B:+)$$

$$= (9.5 \times 3 \times 5) + (0.5 \times 4 \times 5)$$

$$= 152.5 \text{ m}^3$$

$$\rightarrow W_{0:1} = 797.638 \frac{\text{kg}}{\text{m}^3} (152.5 \text{ m}^3) (9.81 \frac{\text{m}}{\text{s}^2})$$

$$= 1.19 \times 10^6 \text{ N}$$

$$W_{0:1} = 1.19 \text{ MN}$$

$$\frac{BD_{F_R}}{W_{0:1}} = \frac{2.6 \text{ MN}}{1.19 \text{ MN}} = 2.18 \text{ suggests } \boxed{F_R = 2.18 W_{0:1}}$$

Trevor Swan

$$1. Re = \frac{\text{inertial}}{\text{viscous}}$$

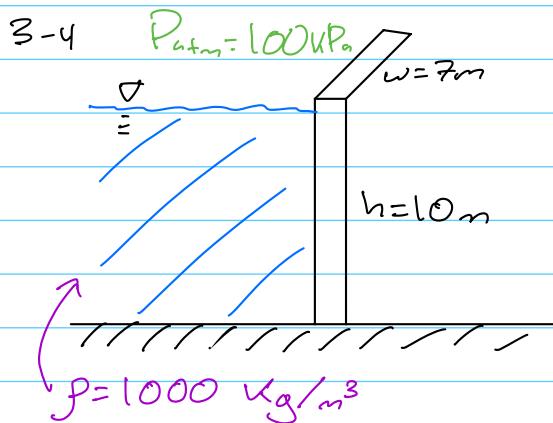
ECHE225: QUIZ #6 // 10.17.24

$$2. u(h=0) = 0 \text{ no-slip}$$

$$u(h=A) = V_0$$

$u(h=C) > 0$, but $< V_0$, increases from 0 to A

3-4



$$3. P_{\text{gage}} = P_{\text{absolute}} - P_{\text{atm}} \quad P_{\text{atm}} = 100 \text{ kPa}$$

$$P_{\text{absolute}} = P_{\text{gage}} + P_{\text{atm}} \quad P_{\text{gage}} = \rho gh$$

$$4. P_{\text{absolute}}(h=5) = 100 + S \rho g$$

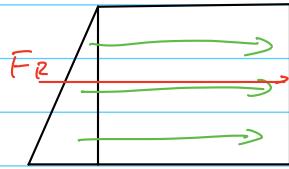
$$= 100 \text{ kPa} + S_m \left(1000 \frac{\text{kg}}{\text{m}^3} \right) (9.81 \frac{\text{m}}{\text{s}^2})$$

$$= 100 + 49050 \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$$

$$1 \text{ N} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \rightarrow \text{our answer is in } \frac{\text{N}}{\text{m}}, 1 \text{ Pa} = 1 \frac{\text{N}}{\text{m}}$$

$$= 100 + 49.050 = 149.05 \text{ kPa}$$

4. Resultant Force



height is distance from free surface to 10m

$$F_{R,T} = \frac{1}{2} \rho g h (h w) \leftarrow P_{\text{abs}} \cdot A$$

$$= \frac{1}{2} (1000 \frac{\text{kg}}{\text{m}^3}) (9.81 \frac{\text{m}}{\text{s}^2}) (10 \text{ m}) (10 \times 7 \text{ m}^2)$$

$$= 3433,500 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} = \text{N}$$

$$F_{R,R} = \rho g h (h w)$$

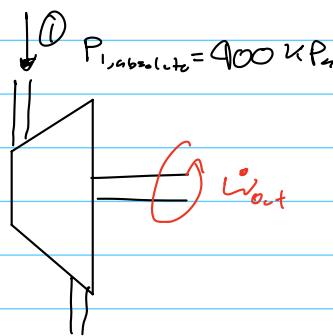
$$= (1000 \frac{\text{kg}}{\text{m}^3}) (9.81 \frac{\text{m}}{\text{s}^2}) (10 \text{ m}) (10 \times 7 \text{ m}^2)$$

$$= 6,867,000 \text{ N}$$

$$F_R = F_{R,T} + F_{R,R} = 10,300,500 \text{ N}$$

$$F_R = 10,300 \text{ kN}$$

1. [Chapter 12] In a hydroelectric power plant, water enters the turbine nozzles at 900 kPa absolute with a low velocity. If the nozzle outlets are exposed to atmospheric pressure of 100 kPa, determine the maximum velocity to which water can be accelerated by the nozzles before striking the turbine blades.



$$P_1 + \frac{1}{2} \rho V_1^2 + \rho g z_1 = P_2 + \frac{1}{2} \rho V_2^2 + \rho g z_2$$

$P_1 = P_{\text{absolute}}$

$P_2 = P_{\text{atm}}$

$$P_{1,\text{absolute}} = P_{\text{atm}} + \frac{1}{2} \rho V_2^2$$

$$V_2^2 = \frac{2}{\rho} (P_{1,\text{absolute}} - P_{\text{atm}})$$

$$P_{1,\text{absolute}} = 900,000 \text{ Pa} \quad \& \quad P_{\text{atm}} = 100,000 \text{ Pa}$$

$$P_a = \frac{k_g}{m \cdot s^2}$$

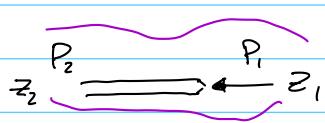
$$1 \text{ kPa} = 1000 \text{ Pa}$$

$$V_2 = \sqrt{\frac{2}{497 \frac{\text{kg}}{\text{m}^3}} (900,000 \text{ Pa} - 100,000 \text{ Pa})}$$

$$= 40.06 \text{ m/s}$$

$$V_2 = 40.06 \text{ m/s}$$

2. [Chapter 12] A Pitot-static probe is used to measure the speed of an aircraft flying at 4000 m. Assume that the density of the atmosphere at that height is 0.82 kg/m^3 . If the differential pressure reading is 3.5 kPa, determine the speed of the aircraft.



$$P_1 + \frac{1}{2} \rho V_1^2 + \rho g z_1 = P_2 + \frac{1}{2} \rho V_2^2 + \rho g z_2 \quad z_1 = z_2 = 4000$$

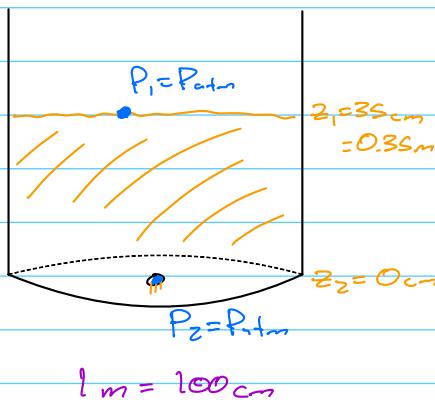
$$P_1 - P_2 = 3500 \text{ Pa}$$

$$\begin{aligned} P_1 - P_2 &= \frac{1}{2} \rho V_2^2 \\ \rightarrow V_2^2 &= \frac{2}{\rho} (P_1 - P_2) \end{aligned}$$

$$V_2 = \sqrt{\frac{2}{0.82} (3500 \text{ Pa})} = 92.4 \text{ m/s}$$

$V_2 = 92.4 \text{ m/s}$

3. [Chapter 12] While traveling on a dirt road, the bottom of a car hits a sharp rock and a small hole develops at the bottom of its gas tank. If the height of the gasoline in the tank is 35 cm, determine the initial velocity of the gasoline at the hole. Discuss how the velocity will change with time and how the flow will be affected if the lid of the tank is closed tightly.



$$P_1 + \frac{1}{2} \rho V_1^2 + \rho g z_1 = P_2 + \frac{1}{2} \rho V_2^2 + \rho g z_2$$

Surface v is negligible

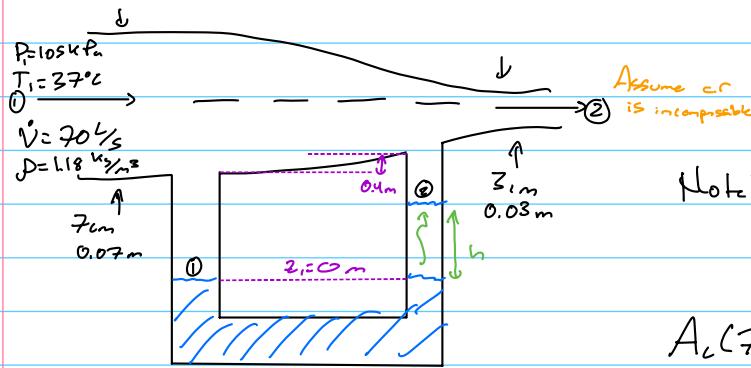
$$\rho z_1 = \frac{1}{2} V_2^2$$

$$V_2 = \sqrt{2(9.81 \frac{m}{s^2})(0.35 m)} = 2.62$$

$$V_2 = 2.62 \text{ m/s}$$

- As z_1 decreases, $\rho g z_1$ will also decrease
- If tank is sealed tightly, then P_1 is not necessarily $Patm$, so $P_1 + \rho g z_1 = \frac{1}{2} \rho V_2^2 \rightarrow V_2 = \sqrt{\frac{2}{\rho}(P_1 + \rho g z_1)}$, so velocity will most likely be dependent both to tank pressure and oil density.

4. [Chapter 12] Air at 105 kPa and 37°C flows upward through a 7-cm-diameter inclined duct at a rate of 70 L/s. The duct diameter is then reduced to 3 cm through a reducer. The pressure change across the reducer is measured by a water manometer. The elevation difference between the two points on the pipe where the two arms of the manometer are attached is 0.4 m. Determine the differential height between the fluid levels of the two arms of the manometer.



- $P_{1a} = P_{1w} = P_1$ } As P_a is isobaric,
- $P_{2a} = P_{2w} = P_2$ } Column pressure is equivalent +

Note: Steady state $\Rightarrow \dot{m}_1 = \dot{m}_2$

Incompressible $\Rightarrow \dot{V}_1 = \dot{V}_2 = \dot{V} = 70 \text{ L/s}$

$$\Rightarrow \dot{V} = \dot{V}_{avg} A_c \Rightarrow V_{avg} = \frac{\dot{V}}{A_c}$$

$$A_c(7\text{cm}) = \pi r^2 = \frac{\pi}{4}(D)^2 = \frac{\pi}{4}(0.07\text{m})^2 = 0.0038\text{m}^2$$

$$A_c(3\text{cm}) = \frac{\pi}{4}(0.03\text{m})^2 = 0.00071\text{m}^2$$

$\{air\}$ $P_1 + \frac{1}{2} \rho_a V_1^2 + \rho_a g z_1 = P_2 + \frac{1}{2} \rho_a V_2^2 + \rho_a g z_2 \Rightarrow P_1 - P_2 = \frac{1}{2} \rho_a (V_2^2 - V_1^2)$

$\{water\}$ $P_1 + \frac{1}{2} \rho_w V_1^2 + \rho_w g z_1 = P_2 + \frac{1}{2} \rho_w V_2^2 + \rho_w g z_2 \Rightarrow P_1 - P_2 = \rho_w g h$

$\{Eqn. 1\}$ $\frac{1}{2} \rho_a (V_2^2 - V_1^2) = \rho_w g h \Rightarrow h = \frac{\rho_a (V_2^2 - V_1^2)}{2 \rho_w g}$

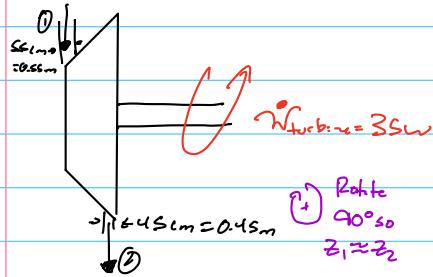
$$\left\{ \begin{array}{l} \rho_a = 1.18 \frac{\text{kg}}{\text{m}^3} \\ \rho_w = 997 \frac{\text{kg}}{\text{m}^3} \end{array} \right.$$

$\{Vol. 1\}$ $V_1 = \frac{\dot{V}}{A_c} = \frac{70 \text{ L/s} \times \frac{1 \text{ m}^3}{1000 \text{ L}}}{0.0038 \text{ m}^2} = 18.42 \text{ m/s}$
 $1 \text{ L} = 0.001 \text{ m}^3$

$$V_2 = \frac{\dot{V}}{A_c} = \frac{70 \text{ L/s} \times \frac{1 \text{ m}^3}{1000 \text{ L}}}{0.00071 \text{ m}^2} = 98.6 \text{ m/s}$$

$$h = 57 \text{ cm}$$

$\{Solve!\}$ $h = \frac{1.18 \frac{\text{kg}}{\text{m}^3} ((98.6 \text{ m/s})^2 - (18.42 \text{ m/s})^2)}{2(997 \frac{\text{kg}}{\text{m}^3})(9.81 \text{ m/s}^2)} = 0.566 \text{ m} = 56.6 \text{ cm}$



$$P_1 - P_2 = 125 \text{ kPa}$$

$$\dot{V}_1 = 2.5 \text{ m}^3/\text{s}$$

$$\text{Incompressible: } V_1 = V_2 = \dot{V}$$

$$\rho_w = 997 \text{ kg/m}^3, \alpha = 1$$

Turbulent

$$h_L = 3.25 \text{ m}$$

$$\frac{P_1}{\rho g} + \alpha \frac{V_1^2}{2g} + z_1 + h_{pump, \text{in}} = \frac{P_2}{\rho g} + \alpha \frac{V_2^2}{2g} + z_2 + h_{turbine, \text{out}} + h_L$$

$$\begin{aligned} h_{turbine, \text{out}} &= \frac{P_1}{\rho g} - \frac{P_2}{\rho g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_L \\ &= \frac{P_1 - P_2}{\rho g} + \frac{\dot{V}_1^2 - \dot{V}_2^2}{2g} - h_L \end{aligned}$$

$$\boxed{\text{velocities}} \quad \dot{V} = V_{avg} A_c$$

$$V_1 = \frac{\dot{V}_1}{\frac{\pi}{4}(D)^2} = \frac{2.5 \text{ m}^3/\text{s}}{\frac{\pi}{4}(0.55 \text{ m})^2} = 10.52 \text{ m/s}$$

$$V_2 = \frac{\dot{V}_2}{\frac{\pi}{4}(D)^2} = \frac{2.5 \text{ m}^3/\text{s}}{\frac{\pi}{4}(0.45 \text{ m})^2} = 15.72 \text{ m/s}$$

$$2. h_{turbine, \text{out}} = \frac{125,000 \text{ Pa}}{997 \text{ kg/m}^3 (9.81 \text{ m/s}^2)} + \frac{(10.52 \text{ m/s})^2 - (15.72 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} - 3.25 \text{ m} = \boxed{2.58 \text{ m}}$$

$$4. \eta_t = \frac{W_{turbine}}{W_{turbine, \text{out}}}, h_{turbine, \text{out}} = \frac{W_{turbine, \text{out}}}{\dot{m} g}, W_{turbine} = 35 \text{ kW}$$

$$\dot{m} = \dot{V} \rho = 2.5 \text{ m}^3/\text{s} (997 \text{ kg/m}^3) = 2492.5 \text{ kg/s}$$

Σw^2

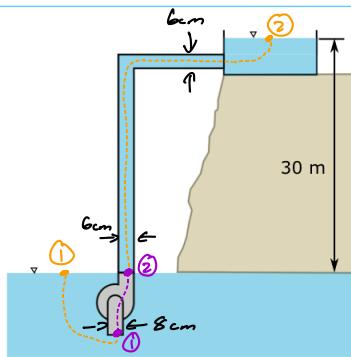
$$W_{turbine, \text{out}} = \dot{m} g h_{turbine, \text{out}} = (2492.5 \text{ kg/s})(9.81 \text{ m/s}^2)(2.58 \text{ m}) = 63084.68 \frac{\text{kg} \cdot \text{m}^2}{\text{s}^3}$$

$$= 63.084 \text{ kW}$$

$$\eta_t = \frac{35 \text{ kW}}{63.084 \text{ kW}} = 0.5548$$

$$\boxed{\eta_t = 55.48\%}$$

1. [Chapter 12] Underground water is to be pumped by a 78 percent efficient 5-kW submerged pump to a pool whose free surface is 30 m above the underground water level. The diameter of the pipe is 8 cm on the intake side and 6 cm on the discharge side. Determine (a) the maximum flow rate of water and (b) the pressure difference across the pump. Assume the elevation difference between the pump inlet and outlet and the effect of the kinetic energy correction factors to be negligible.



Given

F: d

- $\eta_{\text{pump}} = 78\%$
- $\Delta z = z_2 - z_1 = 30 \text{ m}$
- $d = 1 \text{ m}, \rho = 997 \text{ kg/m}^3$
- $\dot{w}_{\text{pump}} = 5 \text{ kW}$
- $A_{c2} = \frac{\pi}{4}(0.08 \text{ m})^2 = 0.005024 \text{ m}^2$
- $A_{c1} = \frac{\pi}{4}(0.06 \text{ m})^2 = 0.00283 \text{ m}^2$

a) m

b) $\Delta P = P_2 - P_1$

$$a) \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 + h_{\text{pump},u} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_{\text{turbine},u} + h_L \quad \begin{matrix} z_2 - z_1 = 30 \text{ m} \\ \text{pump} \end{matrix} \quad \begin{matrix} \text{p: p: } \text{level negligible} \\ \text{compared to position} \end{matrix}$$

$h_{\text{pump},u} = 30 \text{ m}$

$\dot{w}_{\text{pump},u} = 30 \text{ m}$

$\dot{w}_{\text{pump}} = \frac{\dot{w}_{\text{pump},u}}{\eta} = \frac{30}{0.78} = 39.1 \text{ m/s}$

$\dot{m} = \frac{\dot{w}_{\text{pump},u}}{30 \cdot 9.81} = \frac{39.1}{30 \cdot 9.81} = 1.325 \text{ kg/s}$

$\dot{m} = 1.325 \text{ kg/s}$

$$\eta_{\text{pump}} = \frac{\dot{w}_{\text{pump}}}{\dot{w}_{\text{pump}}} \Rightarrow \dot{w}_{\text{pump}} = 0.78 \text{ (5 kW)} = 3.9 \text{ kW} = 3900 \text{ W}$$

$$b) \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 + h_{\text{pump},u} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_{\text{turbine},u} + h_L \quad \begin{matrix} z_2 - z_1 = 30 \text{ m} \\ \text{negligible pipes are so close} \end{matrix}$$

$$\frac{1}{\rho g} (P_2 - P_1) = \frac{1}{2g} (V_1^2 - V_2^2) + h_{\text{pump},u} \quad V = \frac{\dot{m}}{\rho A_{c2}} \quad \left| V_1 = \frac{1.325}{997 A_{c2}} = 2.65797 \text{ m/s} \right.$$

$$\Delta P = P_2 - P_1 = \frac{\rho}{2} (V_1^2 - V_2^2) + \rho g h_{\text{pump},u}$$

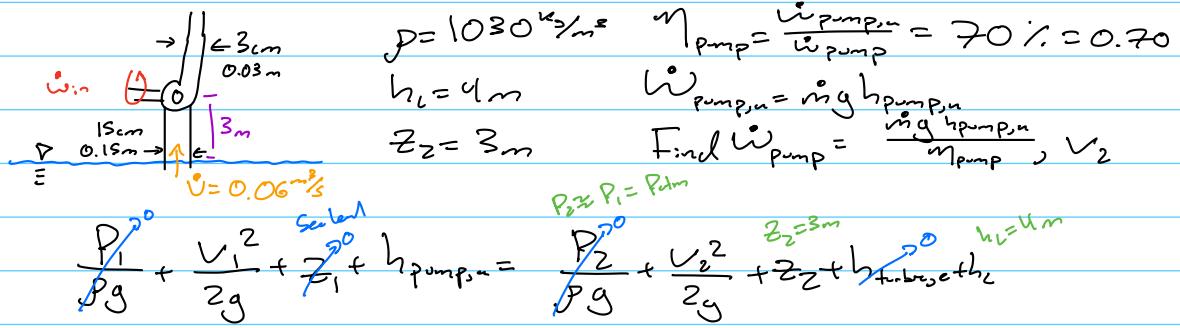
$$V_2 = \frac{1.325}{997 \cdot 0.00283} = 4.69607 \text{ m/s}$$

$$\Delta P = \frac{997}{2} ((2.65797)^2 - (4.69607)^2) + (997)(4.69607)(30)$$

$$= 285432 \text{ Pa}$$

$\Delta P = 286 \text{ kPa}$

2. [Chapter 12] A fireboat is to fight fires at coastal areas by drawing seawater with density 1030 kg/m^3 through a 15-cm-diameter pipe at a rate of $0.06 \text{ m}^3/\text{s}$ and discharging it through a nozzle with an exit diameter of 3 cm. The total irreversible head loss of the system is 4 m, and the position of the nozzle is 3 m above sea level. For a pump efficiency of 70 percent, determine the required shaft power input to the pump and the water discharge velocity.



$$h_{\text{pump},n} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + Z_2 + h_2 = \frac{1}{2g} (V_2^2 - V_1^2) + 3 \text{ m} + 4 \text{ m}$$

$$V_1 = \frac{\dot{V}}{A_{c_1}} = \frac{0.06 \text{ m}^3/\text{s}}{\frac{\pi}{4}(0.15 \text{ m})^2} = 3.40 \text{ m/s} \quad h_{\text{pump},n} = \frac{1}{2(9.81)} (84.68^2 - 3.40^2) + 7 \text{ m}$$

$$V_2 = \frac{\dot{V}}{A_{c_2}} = \frac{0.06 \text{ m}^3/\text{s}}{\frac{\pi}{4}(0.03 \text{ m})^2} = 84.88 \text{ m/s}$$

$$\rightarrow \dot{V}_{\text{pump}} = (61.8 \text{ m/s})(4.81 \text{ m/s})(373.618 \text{ m})$$

$$\dot{m} = \rho \dot{V}$$

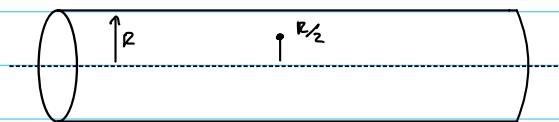
$$\dot{m} = (1030 \text{ kg/m}^3)(0.06 \text{ m}^3/\text{s}) \\ = 61.8 \text{ kg/s}$$

$$= 323584 \text{ Pa}$$

$$0.70$$

$\dot{V}_{\text{pump}} = 324 \text{ m/s}$
$V_2 = 84.88 \text{ m/s}$

3. [Chapter 14] In fully developed laminar flow in a circular pipe the velocity at $R/2$ (midway between the wall surface and the centerline) is measured to be 13 m/s. Determine the velocity at the center of the pipe.



$$u(r=R/2) = 13 \text{ m/s}$$

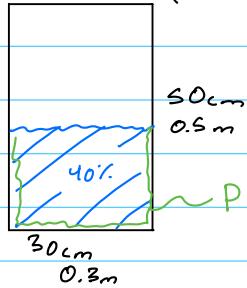
Find $u(R=0)$ ← maximum velocity
↑ center of pipe

$$\left. \begin{aligned} u(r) &= u_{\max} \left(1 - \frac{r^2}{R^2}\right) \\ u(r=R/2) &= u_{\max} \left(1 - \frac{(R/2)^2}{R^2}\right) \end{aligned} \right\} 13 \text{ m/s} = u_{\max} \left(1 - \frac{R^2/4}{R^2}\right)$$

$$u_{\max} = \frac{13 \text{ m/s}}{3/4} = \frac{13 \cdot 4}{8} \text{ m/s} = \frac{52}{8} \text{ m/s} \approx 17.33 \text{ m/s}$$

$u_{\max} = 17.33 \text{ m/s}$

1. This question is not related to this week's homework



$$\begin{aligned} A_c &= 0.3m \cdot 0.4(0.5m) \\ &= 0.06 m^2 \end{aligned}$$

$$\begin{aligned} P &= (0.3) + 2(0.2m) \\ &= 0.7 \quad \text{wetted perimeter} \end{aligned}$$

• Steady-state, laminar flow assumed

$$\bullet SG = 0.88, V_{avg} = 2 \text{ m/s}$$

$$R_{c,ext} = 2300 \text{ (constant)}$$

$$R_c = \frac{\rho V_{avg} D_h}{M}$$

$$\rightarrow M = \frac{\rho V_{avg} D_h}{R_c}$$

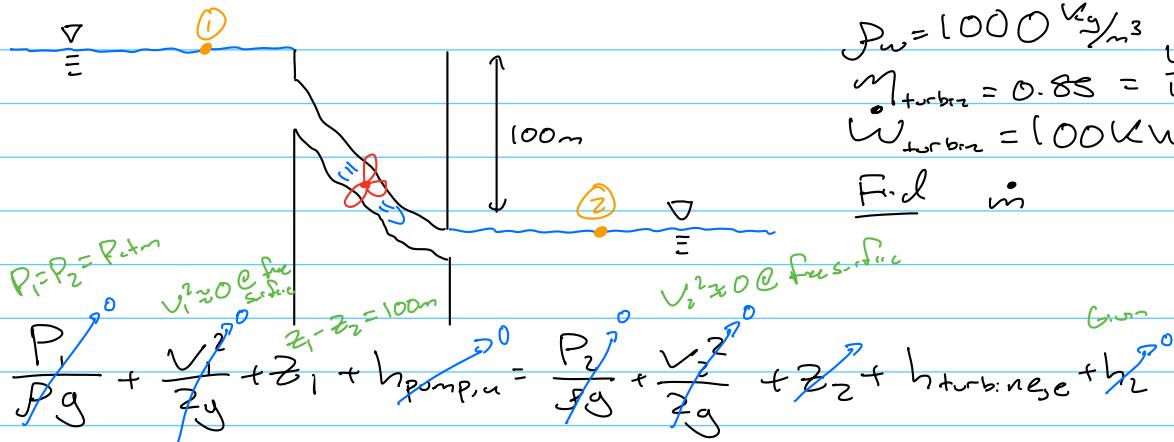
$$D_h = \frac{4 A_c}{P} \quad \rho = SG \cdot \rho_{water}$$

$$\begin{aligned} \rho &= 0.85 \cdot 1000 \frac{\text{kg}}{\text{m}^3} = 850 \frac{\text{kg}}{\text{m}^3} \\ D_h &= \frac{4(0.06 \text{ m}^2)}{0.7 \text{ m}} = 0.3429 \text{ m} \end{aligned}$$

$$\begin{aligned} M &= \frac{850 \frac{\text{kg}}{\text{m}^3} \cdot 2 \text{ m/s} \cdot 0.3429 \text{ m}}{2300} \\ &= 0.2534 \frac{\text{kg}}{\text{m/s}} \end{aligned}$$

$$M = 0.25 \frac{\text{kg}}{\text{m/s}}$$

2.



$$\begin{aligned} \rho_w &= 1000 \frac{\text{kg}}{\text{m}^3} \\ \eta_{turbine} &= 0.85 = \frac{\omega_{turbine}}{\omega_{turbine,c}} \\ \dot{W}_{turbine,c} &= 100 \text{ kW} \end{aligned}$$

Find m

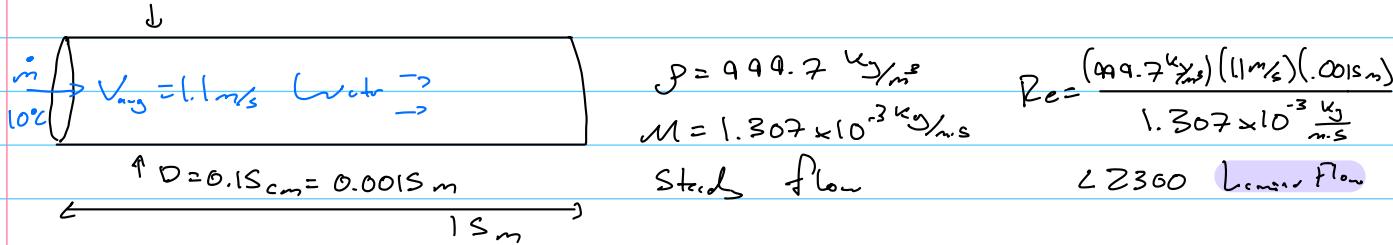
$$h_{turbine,c} = 100 \text{ m}$$

$$\dot{W}_{turbine,c} = \frac{\dot{W}_{turbine}}{\eta_{turbine}} = \frac{100 \text{ kW}}{0.85} = 117.647 \text{ kW} = 117,647 \text{ W} \quad \left\{ \frac{\text{kg} \cdot \text{m}^2}{\text{s}^3} \right\}$$

$$h_{turbine,c} = \frac{\dot{W}_{turbine,c}}{mg} \rightarrow m = \frac{\dot{W}_{turbine,c}}{g \cdot h_{turbine,c}} = \frac{117,647 \text{ W}}{(9.81 \text{ m/s}^2)(100 \text{ m})} = 119.93 \frac{\text{kg}}{\text{s}}$$

$$\boxed{m = 119.93 \frac{\text{kg}}{\text{s}}}$$

1. [Chapter 14] Water at 10°C ($\rho = 999.7 \text{ kg/m}^3$, $\mu = 1.307 \times 10^{-3} \text{ kg/m}\cdot\text{s}$) is flowing steadily in a 0.15-cm-diameter, 15-m-long pipe at an average velocity of 1.1 m/s. Determine:
- pressure drop
 - head loss
 - pumping power required to overcome this pressure drop.



a) Flow is laminar

$$\Delta P_L = \frac{32 \mu L V_{\text{avg}}}{D^2} = \frac{32 (1.307 \times 10^{-3} \frac{\text{kg}}{\text{m}\cdot\text{s}})(15 \text{ m})(1.1 \text{ m/s})}{(0.0015 \text{ m})^2} = 306709 \text{ Pa}$$

$$\Delta P_L = 306.7 \text{ kPa}$$

b) $h_L = \frac{\Delta P_L}{\rho g} \Rightarrow h_L = \frac{306709.33 \frac{\text{kg}}{\text{m}\cdot\text{s}^2}}{(999.7 \frac{\text{kg}}{\text{m}^3})(9.81 \text{ m/s}^2)} = 31.27$

derived from bernoulli:

$$h_c = 31.3 \text{ m}$$

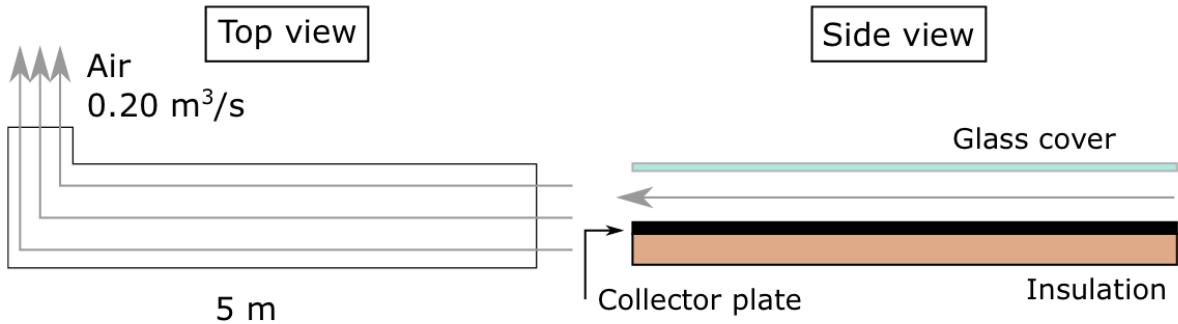
c) $\frac{W_{\text{pump}}}{m} = h_L \Rightarrow W_{\text{pump}} = (31.27 \text{ m})(9.81 \text{ m/s}^2) = (31.27 \text{ m})(0.00194 \frac{\text{kg}}{\text{s}})(9.81 \frac{\text{m}}{\text{s}^2})$

$$\begin{aligned} m &= \rho V_{\text{avg}} A_C \\ &= 999.7 \frac{\text{kg}}{\text{m}^3} \cdot 1.1 \frac{\text{m}}{\text{s}} \cdot \frac{\pi}{4} (0.0015 \text{ m})^2 \\ &= 0.00194 \frac{\text{kg}}{\text{s}} \end{aligned}$$

$$= 0.5951 \frac{\text{kg}}{\text{s}^3} \quad \text{Solve}$$

$$W_{\text{pump}} = 0.595 \text{ W}$$

2. [Chapter 14] Consider an air solar collector that is 1 m wide and 5 m long and has a constant spacing of 3 cm between the glass cover and the collector plate. Air flows at an average temperature of 45°C at a rate of 0.20 m³/s through the 1-m-wide edge of the collector along the 5-m-long passageway. Disregarding the entrance and roughness effects and the 90° bend, determine the pressure drop in the collector.



$$Re = \frac{\rho V_{avg} D}{\mu} = \frac{(1.205 \frac{\text{kg}}{\text{m}\cdot\text{s}})(6.67 \frac{\text{m}}{\text{s}})(0.05 \text{ m})}{(1.82 \times 10^{-5} \frac{\text{N}\cdot\text{s}}{\text{m}\cdot\text{s}})} = 25746 \dots > 2300 \quad T_w, b-l-t-t!$$

$$\dot{V} = \frac{\dot{m}}{\rho} \rightarrow \dot{V} = V_{avg} A_c \Rightarrow V_{avg} = \frac{\dot{V}}{A_c} = \frac{0.20 \frac{\text{m}^3}{\text{s}}}{(1\text{m})(0.03\text{m})} = 6.67 \frac{\text{m}}{\text{s}}, \beta = 1.205 \frac{\text{kg}}{\text{m}\cdot\text{s}}$$

$$D_h = \frac{u A_c}{\dot{m}} = \frac{q (1\text{m} \cdot 0.03\text{m})}{2(1\text{m}) + 2(0.03\text{m})} = 0.0583 \text{ m}$$

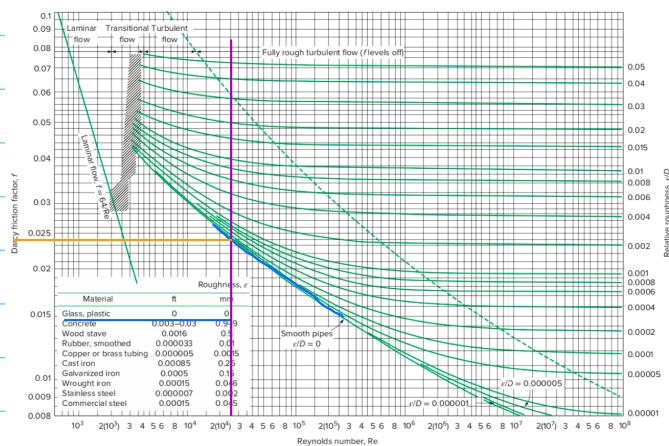


FIGURE A-27
The Moody chart for the friction factor for fully developed flow in circular pipes for use in the head loss relation $\Delta P_L = f \frac{L}{D} \frac{\rho V^2}{2}$. Friction factors in the turbulent flow are evaluated from the Colebrook equation $\frac{1}{\sqrt{f}} = -2 \log_{10} \left(\frac{e/D}{3.7} + \frac{2.51}{Re \sqrt{f}} \right)$.

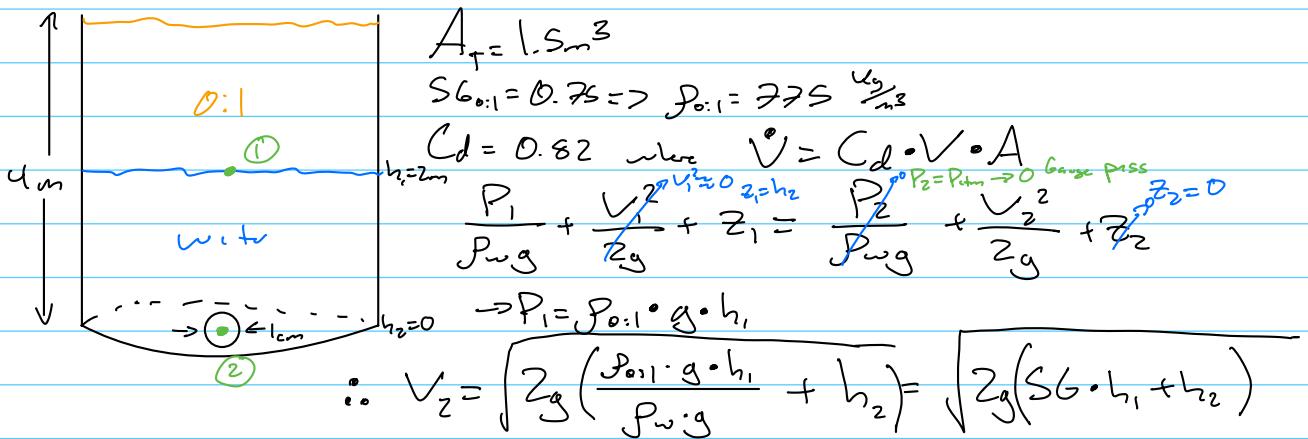
$$f = 0.024, \Delta P_2 = f \frac{L}{D} \cdot \frac{1}{2} \rho u_{avg}^2$$

$$\Delta P_2 = 0.024 \left(\frac{5\text{m}}{0.0583\text{m}} \right) \cdot \left(\frac{1}{2} \cdot 1.205 \frac{\text{kg}}{\text{m}\cdot\text{s}} (6.67 \frac{\text{m}}{\text{s}})^2 \right) = 55.2 \frac{\text{kg}}{\text{m}^2}$$

$$\Delta P_2 = 55.2 P_2$$

Reasonably close
considering rounding!!

- what to
know*
3. [Chapter 14] A 4-m high cylindrical tank having a cross-sectional area $A_T = 1.5 \text{ m}^2$ is filled with equal volumes of water and oil. The specific gravity of the oil is 0.75. A 1 cm-diameter hole at the bottom of the tank is opened and water starts to flow out. If the discharge coefficient of the hole is $C_d = 0.82$, determine how long it will take for the water in the tank to completely drain. Assume that the tank is open to the atmosphere. The discharge coefficient represents a fraction of the expected volumetric flow rate through the opening: $\dot{V} = C_d V A$.



$$\dot{V} = C_d \cdot V \cdot A \quad A_2 = \frac{\pi}{4} (0.01)^2$$

$$\rightarrow \dot{V}_2 = C_d \cdot V_2 \cdot A_2 \quad = 7.85 \times 10^{-5} \text{ m}^3/\text{s}$$

$$\therefore \dot{V}_2 = C_d \cdot \sqrt{2g (SG \cdot h_1 + h_2)} \cdot 7.85 \times 10^{-5} \text{ m}^2 \quad \text{also } \dot{V}_2 = -A_{T_{\text{out}}} \frac{dh_2}{dt}$$

$$\Rightarrow -A_T \frac{dh_2}{dt} = (7.85 \times 10^{-5} \text{ m}^2) C_d \sqrt{2g (SG \cdot h_1 + h_2)}$$

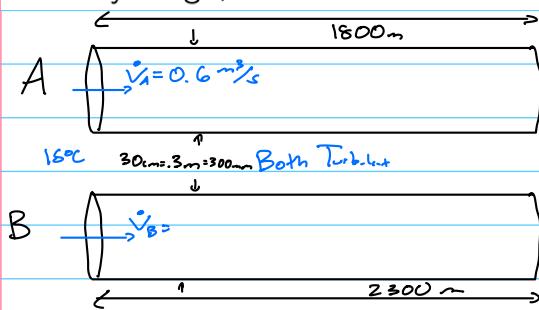
$$\text{so} \quad \int_{h_2}^0 \frac{dh_2}{\sqrt{2g (SG \cdot h_1 + h_2)}} = - \frac{(7.85 \times 10^{-5}) C_d}{A_T} \int_0^t dt$$

$$t = - \frac{A_T}{(7.85 \times 10^{-5}) C_d} \int_{h_2}^0 \frac{dh_2}{\sqrt{2g (SG \cdot h_1 + h_2)}}$$

$$= - \frac{1.5 \text{ m}^2}{(7.85 \times 10^{-5} \text{ m}^2)(0.82)} \cdot \int_2^0 \frac{dh_2}{\sqrt{2(0.81)(0.75 \cdot 2 \text{ m} + h_2)}} = 6707.94 \text{ seconds}$$

$$t = 1.89 \text{ hours}$$

4. [Chapter 14] A certain part of the cast iron piping of a water distribution system involves a parallel section. Both parallel pipes have a diameter of 30 cm and the flow is fully turbulent. One of the branches (pipe A) is 1800 m long while the other branch (pipe B) is 2300 m long. If the flow rate through pipe A is $0.6 \text{ m}^3/\text{s}$, determine the flow rate through pipe B. Disregard minor loss and assume the water temperature to be 15°C . Show that the flow is fully rough, and thus the friction factor is independent of Reynolds number.



$$\text{For Parallel pipes:}$$

$$\dot{V}_{\text{Tot}} = \dot{V}_A + \dot{V}_B$$

$$\dot{V} = \dot{V}_{\text{Avg}} \cdot A_c$$

$$\Delta P_A = \Delta P_B$$

$$\text{Assume } \rho = 1000 \frac{\text{kg}}{\text{m}^3}$$

We will assume fully rough, so $f_1 = f_2$

Ep is const.

$$\text{Assumption: } f \cdot \frac{L_A}{D} \cdot \frac{1}{2} \rho V_{A,\text{avg}}^2 = f \cdot \frac{L_B}{D} \cdot \frac{1}{2} \rho V_{B,\text{avg}}^2, \text{ where } V_{A,\text{avg}} = \frac{\dot{V}_A}{A_c} \text{ and } V_{B,\text{avg}} = \frac{\dot{V}_B}{A_c}$$

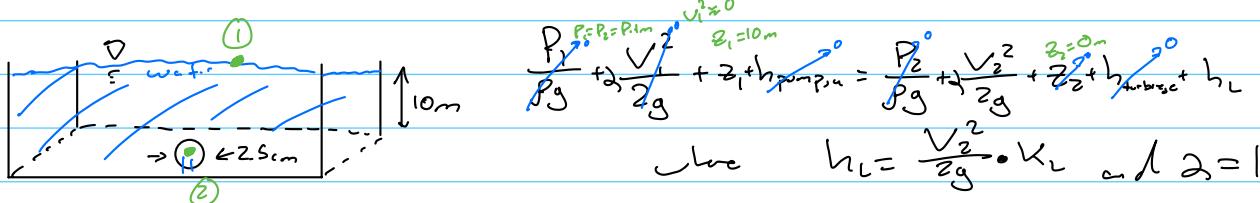
$$L_A \cdot V_{A,\text{avg}}^2 = L_B \cdot V_{B,\text{avg}}^2$$

$$L_A \left(\frac{V_A}{A_c} \right)^2 = L_B \left(\frac{V_B}{A_c} \right)^2$$

$$\boxed{\dot{V}_B = 0.531 \frac{\text{m}^3}{\text{s}}}$$

$$L_A \left(\frac{V_A^2}{A_c^2} \right) = L_B \left(\frac{V_B^2}{A_c^2} \right) \Rightarrow \dot{V}_B = \dot{V}_A \sqrt{\frac{L_A}{L_B}} = 0.6 \frac{\text{m}^3}{\text{s}} \sqrt{\frac{1800\text{m}}{2300\text{m}}} = 0.5308$$

5. [Chapter 14] Water is to be withdrawn from a 10-m-high reservoir by drilling a 2.5-cm-diameter hole at the bottom surface. Disregarding the effect of the kinetic energy correction factor, determine the flow rate of water through the hole if (a) the entrance of the hole is well-rounded and (b) the entrance is sharp-edged.



$$z_1 = \frac{V_2^2}{2g} + K_L \frac{V_2^2}{2g} = V_2^2 \left(\frac{1}{2g} + \frac{K_L}{2g} \right)$$

$$\Rightarrow V_2^2 = z_1 \left(\frac{1}{2g} + \frac{K_L}{2g} \right)^{-1} \quad \Rightarrow \quad V_2 = \sqrt{10 \left(\frac{1}{2(9.81 \text{ m/s}^2)} + \frac{K_L}{2(9.81 \text{ m/s}^2)} \right)^{-1}}$$

a) Well-rounded; $K_L = 0.03$

$$V_2 = \sqrt{10 \left(\frac{1}{2(9.81 \text{ m/s}^2)} + \frac{0.03}{2(9.81 \text{ m/s}^2)} \right)^{-1}} = 13.8016 \text{ m/s}$$

$$\dot{V}_2 = \frac{V_2}{A_c} = (13.8016 \text{ m/s}) \left(\frac{\pi}{4} (0.025 \text{ m})^2 \right) = 0.006775 \text{ m}^3/\text{s}$$

b) Sharp-edged; $K_L = 0.50$

$$V_2 = \sqrt{10 \left(\frac{1}{2(9.81 \text{ m/s}^2)} + \frac{0.50}{2(9.81 \text{ m/s}^2)} \right)^{-1}} = 11.4368 \text{ m/s}$$

$$\dot{V}_2 = (11.4368 \text{ m/s}) \left(\frac{\pi}{4} (0.025 \text{ m})^2 \right) = 0.005614 \text{ m}^3/\text{s}$$

a) $\dot{V}_2 = 6.72 \times 10^{-3} \text{ m}^3/\text{s}$ and b) $5.61 \times 10^{-3} \text{ m}^3/\text{s}$

1.

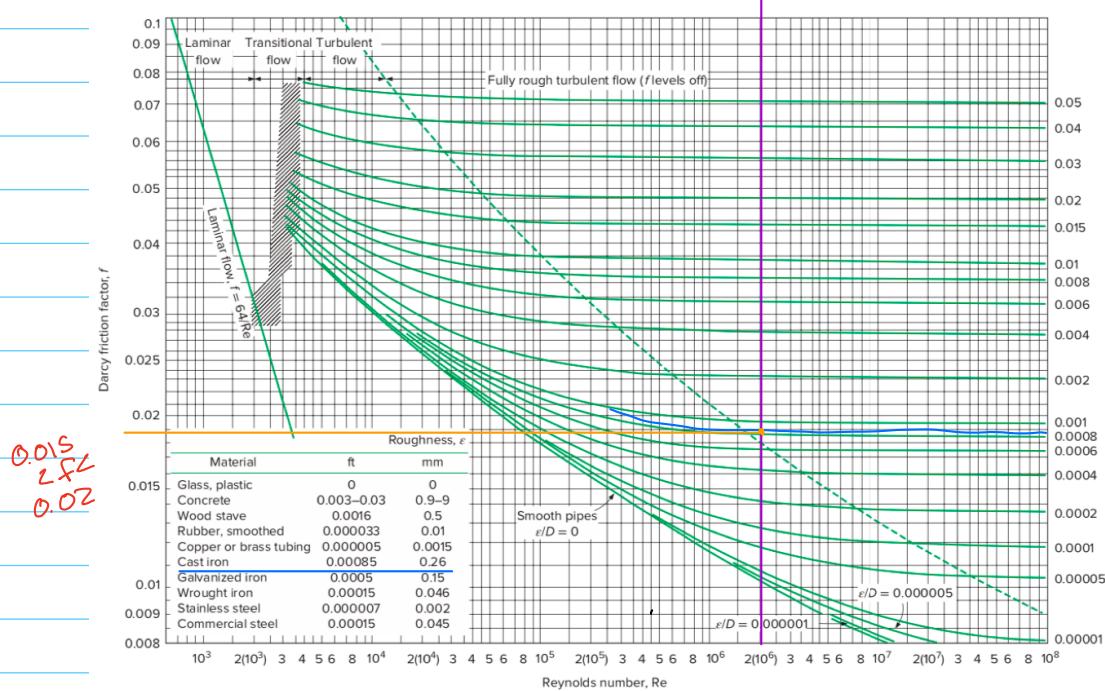


FIGURE A-27

The Moody chart for the friction factor for fully developed flow in circular pipes for use in the head loss relation $\Delta P_L = f \frac{L}{D} \frac{\rho V^2}{2}$. Friction factors in the turbulent flow are evaluated from the Colebrook equation $\frac{1}{\sqrt{f}} = -2 \log_{10} \left(\frac{e/D}{3.7} + \frac{2.51}{Re \sqrt{f}} \right)$.

$$\begin{aligned} \text{Given: } & D_{\text{outer}} = 850 \text{ mm} \\ & \sqrt{V_{\text{max}}^2} = 0.00062 \text{ m}^2/\text{s} \\ & \text{Roughness: } e = 0.008 \text{ m} \\ & \text{Length: } L = 40 \text{ m} \\ & \text{Head loss: } z_2 - z_1 = 8 \text{ mm} \end{aligned}$$

$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$

$V_2 = \sqrt{2g z_2} = \sqrt{2(9.81 \text{ m/s}^2)(0.008 \text{ m})} = 8.8589 \text{ m/s}$

$$\Delta P_2 = \frac{32 \mu L V_{\text{avg}}}{D^2} = \frac{32 (0.527 \text{ kg/m.s})(40 \text{ m})(8.86 \text{ m/s})}{(0.008 \text{ m})^2} = 93384.400 \text{ Pa}$$

$$\boxed{\Delta P_2 = 93384.400 \text{ kPa}}$$

$$\boxed{V_2 = 8.8589 \text{ m/s}}$$

$$Re = \frac{\rho V_{\text{avg}} D}{\mu} = \frac{850 \text{ kg/m.s} \cdot (8.8589 \text{ m/s})(0.008 \text{ m})}{0.527 \text{ kg/m.s}} = 114.3084$$

$$\boxed{Re = 114.3084}$$

Cast iron

$$\epsilon = 0.26 \text{ mm}$$

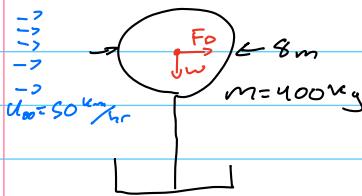
Diameter

$$D_{\text{inner}} = 300 \text{ mm}$$

$$\frac{\epsilon}{D} = 0.00087$$

$$Re_c = 2 \times 10^6$$

1. [Chapter 15] An 8-m-diameter hot air balloon that has a total mass of 400 kg is standing still in air on a windless day. The balloon suddenly is subjected to a 50 km/hr wind. Determine the initial acceleration of the balloon in the horizontal direction.



$$\sum F_x = m a_x \Rightarrow a_x = \frac{F_D}{m}$$

$$F_D = C_D \frac{1}{2} \rho U_\infty^2 A$$

$$\rho_{air} = 1.205 \frac{\text{kg}}{\text{m}^3} \quad \text{and} \quad m = 1.82 \times 10^{-5} \frac{\text{kg}}{\text{m}^3}$$

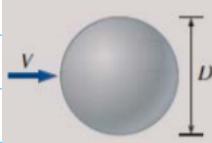
$$A = \pi r^2 = \frac{\pi}{4} D^2 = 16\pi \text{ m}^2 = 50.266 \text{ m}^2$$

$$U_\infty = 50 \frac{\text{km}}{\text{hr}} = 13.89 \frac{\text{m}}{\text{s}}$$

$$C_D = ? \quad \text{Tables!}$$

C_D Tables!

Sphere, $A = \pi D^2 / 4$



Laminar:
 $Re \leq 2 \times 10^5$
 $C_D = 0.5$
Turbulent:
 $Re \geq 2 \times 10^6$
 $C_D = 0.2$

See Fig. 11-36 for C_D vs. Re for smooth and rough spheres.

$$Re = \frac{\rho U_\infty D}{\mu} = \frac{(1.205 \frac{\text{kg}}{\text{m}^3})(13.89 \frac{\text{m}}{\text{s}})(8 \text{ m})}{(1.82 \times 10^{-5} \frac{\text{kg}}{\text{m} \cdot \text{s}})} = 7.36 \times 10^6 \Rightarrow \text{Turbulent}$$

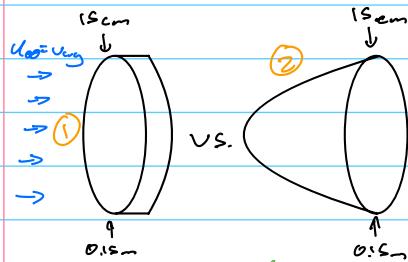
$$\therefore U_{sc} \quad C_D = 0.2$$

$$a_x = \frac{1}{m} F_D = \frac{1}{400 \text{ kg}} \cdot (0.2 \cdot \frac{1}{2} \cdot 1.205 \frac{\text{kg}}{\text{m}^3} \cdot (13.89 \frac{\text{m}}{\text{s}})^2 \cdot 50.266 \text{ m}^2)$$

$$= 2.92 \frac{\text{m}}{\text{s}^2}$$

$$a_x = 2.92 \frac{\text{m}}{\text{s}^2}$$

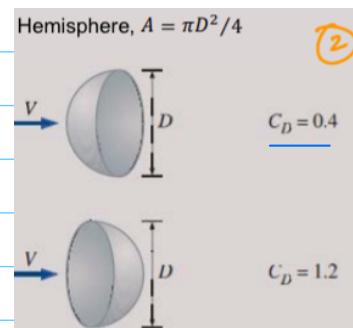
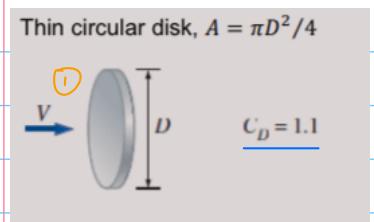
2. [Chapter 15] To reduce the drag coefficient and thus improve the fuel efficiency of cars, the design of side rearview mirrors has changed dramatically in recent decades from a simple circular plate to a streamlined shape. Determine the amount of fuel and money saved per year as a result of replacing a 15-cm-diameter flat mirror by one with a hemispherical back. Assume the car is driven 25,000 km a year at an average speed of 95 km/hr. Take the density and price of gasoline to be 0.75 kg/L and \$0.90/L, respectively; the heating value of gasoline to be 44,000 kJ/kg and the overall efficiency of the engine to be 30 percent.



$$F_D = \frac{1}{2} C_D \rho U_\infty^2 A$$

$$\text{where } A = \frac{\pi}{4} (0.15m)^2 = 0.0176715 \text{ m}^2$$

$$R_{dp} = \frac{\rho U_\infty D}{M} = \frac{(1.205 \frac{\text{kg}}{\text{m}^3})(26.39 \frac{\text{m}}{\text{s}})(0.15m)}{1.82 \times 10^{-5} \frac{\text{kg}}{\text{m} \cdot \text{s}}} = 2.62 \times 10^5$$



$$\textcircled{1} F_D = \frac{1}{2} \cdot 1.1 \cdot 1.205 \frac{\text{kg}}{\text{m}^3} (26.39 \frac{\text{m}}{\text{s}})^2 A \text{ m}^2 \\ = 8.156 \frac{\text{kg} \cdot \text{m}}{\text{s}^2}$$

$$\textcircled{2} F_D = \frac{1}{2} \cdot 0.4 \cdot 1.205 \frac{\text{kg}}{\text{m}^3} (26.39 \frac{\text{m}}{\text{s}})^2 A \text{ m}^2 \\ = 2.466 \frac{\text{kg} \cdot \text{m}}{\text{s}^2}$$

$$E_{drag} = F_D \cdot \text{Distance} \quad \left\{ \frac{\text{kg} \cdot \text{m}^2}{\text{m}} \right\}$$

$$\textcircled{1} E_{drag} = 8.156 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \cdot 25000000 \frac{\text{m}}{\text{year}} \\ = 2.04 \times 10^8 \frac{\text{J}}{\text{year}} = 2.04 \times 10^8 \frac{\text{kg} \cdot \text{m}^2}{\text{year}}$$

$$\textcircled{2} E_{drag} = 2.466 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \cdot 25000000 \frac{\text{m}}{\text{year}} \\ = 7.415 \times 10^7 \frac{\text{J}}{\text{year}} = 7.415 \times 10^7 \frac{\text{kg} \cdot \text{m}^2}{\text{year}}$$

$$E_{fuel} = \frac{E_{drag}}{0.30}$$

$$\textcircled{1} E_{fuel} = 6.8 \times 10^8 \frac{\text{kg} \cdot \text{m}^2}{\text{year}}$$

$$\textcircled{2} E_{fuel} = 2.47 \times 10^8 \frac{\text{kg} \cdot \text{m}^2}{\text{year}}$$

Fuel required and money required

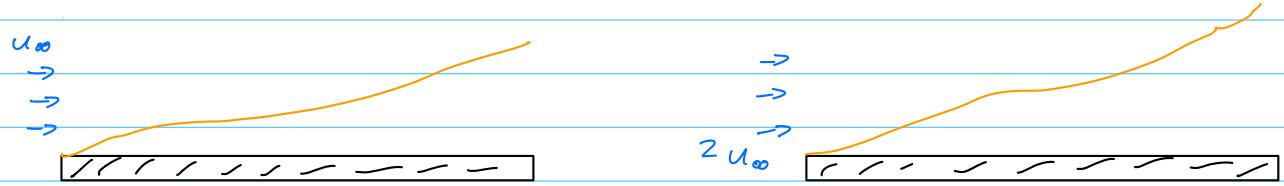
$$\textcircled{1} F_{ul} = \frac{6.8 \times 10^8 \frac{\text{kg} \cdot \text{m}^2}{\text{year}}}{(44,000 \frac{\text{kJ}}{\text{kg}})(0.75 \frac{\text{kg}}{\text{l}})} = 20.6 \frac{\text{l}}{\text{year}} \text{ so } \$18.54/\text{year}$$

$$\textcircled{2} F_{ul} = \frac{2.47 \times 10^8 \frac{\text{kg} \cdot \text{m}^2}{\text{year}}}{(44,000 \frac{\text{kJ}}{\text{kg}})(0.75 \frac{\text{kg}}{\text{l}})} = 7.48 \frac{\text{l}}{\text{year}} \text{ so } \$6.73/\text{year}$$

$$\text{Fuel} = \frac{E_{drag}}{\text{Material} \cdot \rho_{gas}} \quad \left\{ \frac{\text{L}}{\text{year}} \right\}$$

Saving
\$13.12/year
\$11.81/year

3. [Chapter 15] Consider laminar flow of a fluid over a flat plate. Now the freestream velocity is doubled. Determine the change in the drag force on the plate. Assume that the flow remains laminar.



$$F_D = C_f \frac{1}{2} \rho u_\infty^2 A$$

$$\frac{F_{D_2}}{F_{D_1}} = \frac{C_{f_2} \frac{1}{2} \rho (2u_\infty)^2 A}{C_{f_1} \frac{1}{2} \rho (u_\infty)^2 A} = \frac{4 C_{f_2}}{C_{f_1}}$$

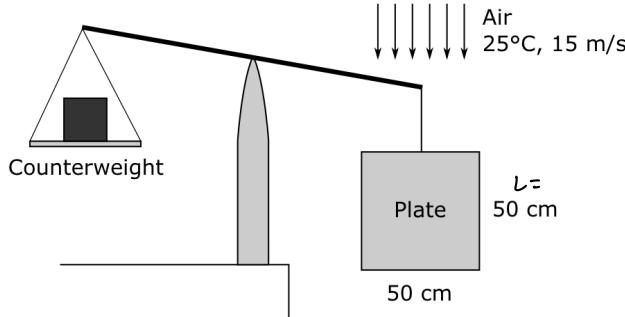
$$C_{f_2} = \frac{4 C_{f_1}}{\text{Re}_2^{1/2}} = F_{D_2} = \frac{4 \left(\frac{1}{\text{Re}_{D_1}^{1/2}} \right)}{\frac{1}{\text{Re}_2^{1/2}}} F_{D_1} = 4 \left(\frac{\text{Re}_{D_1}^{2/3}}{\text{Re}_2^{2/3}} \right) F_{D_1} = 4 \left(\frac{\frac{\partial u_\infty L}{\partial x}}{\frac{\partial^2 u_\infty L}{\partial x^2}} \right)^{1/2} F_{D_1}$$

$$\text{Re}_2 = \frac{\rho u_\infty L}{\mu}$$

$$= 4 \left(\frac{1}{2} \right)^{1/2} F_{D_1}$$

$$F_{D_2} = 2.83 F_{D_1}$$

4. [Chapter 15] The weight of a thin flat plate 50 cm x 50 cm in size is balanced by a counterweight that has a mass of 2 kg. Now a fan is turned on and air at 1 atm and 25°C flows downward over both surfaces of the plate (front and back in the sketch) with a freestream velocity of 15 m/s. Determine the mass of the counterweight that needs to be added in order to balance the plate in this case.



$$\rho_{air} = 1.184 \frac{kg}{m^3}$$

$$M_{air} = 1.84 \times 10^{-3} \frac{kg}{m \cdot s}$$

$$U_\infty = 15 \text{ m/s}$$

$$Re_L = \frac{(1.184 \frac{kg}{m^3})(15 \frac{m}{s})(0.50 \text{ m})}{1.84 \times 10^{-3} \frac{kg}{m \cdot s}} = 4.83 \times 10^5$$

< \$5 \times 10^5 \Rightarrow \text{Laminar}\$

$$F_D = C_f \frac{1}{2} \rho U_\infty^2 A \quad \text{where } C_f = \frac{1.33}{Re_L^{1/2}}$$

$$C_f = 1.33 / (4.83 \times 10^5)^{1/2} = 0.00192$$

$$2 \text{ sides of the plate so } C_f = 0.00384$$

$$F_D = 0.00384 \cdot \frac{1}{2} \cdot 1.184 \frac{kg}{m^3} \cdot (15 \frac{m}{s})^2 \cdot (0.50 \text{ m})^2 = 0.1279 \text{ N}$$

$$W = mg \Rightarrow m = \frac{W}{g} = \frac{0.1279 \text{ N}}{9.81 \frac{m}{s^2}} = 0.013 \text{ kg}$$

$$m = 13 \text{ g}$$

5. [Chapter 15] A jumbo jet airplane has a mass of about 400,000 kg when fully loaded with over 400 passengers and takes off at a speed of 280 km/hr. Determine the takeoff speed when the airplane has 100 empty seats assuming that each passenger with luggage is 150 kg and the wing and flap settings are maintained the same.

$$V_{\text{take-off}} = \sqrt{\frac{2W}{\rho C_L A}}$$

$$400,000 \text{ kg} \Rightarrow V_{\text{to}} = 280 \frac{\text{km}}{\text{hr}}$$

$$\frac{V_{\text{to},2}}{V_{\text{to},1}} = \frac{\sqrt{\frac{2W_2}{\rho C_L A}}}{\sqrt{\frac{2W_1}{\rho C_L A}}} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{m_2}{m_1 g}}$$

$$V_{\text{to},2} = 274.7 \frac{\text{km}}{\text{hr}}$$

$$V_{\text{to},2} = V_{\text{to},1} \sqrt{\frac{m_2}{m_1}} \quad \text{so } V_{\text{to},2} = 280 \frac{\text{km}}{\text{hr}} \sqrt{\frac{m_2}{400,000 \text{ kg}}}$$

For 100 less passengers, 15000 kg less

$$\Rightarrow m_2 = 385,000 \text{ kg}$$

$$V_{\text{to},2} = 280 \frac{\text{km}}{\text{hr}} \sqrt{\frac{385,000 \text{ kg}}{400,000 \text{ kg}}} = 274.7 \frac{\text{km}}{\text{hr}}$$

Exam 2 Notes



Fluid Mechanics

- Study of stationary and moving fluids (states & dynamics)

- Fluids: Captures both liquids & gases

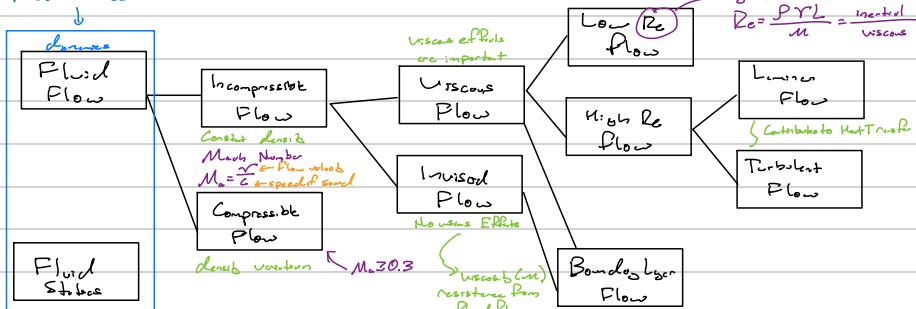
- Applications

- 1) Aerodynamics: Flow \Rightarrow Forces (lift, drag)
 - 2) Hydrodynamics - ships, subs, etc.
 - 3) Fluid Power - hydraulics, pneumatics
 - 4) Fluid Handling - Pipe Flows
 - 5) Hydrostatics
- 6) Biofluid Flows
 - 7) Geophysical Flows - atmosphere, circulates ocean currents
 - 8) Astrophysics / Motion of cosmic objects & galaxies
 - 9) Shock Waves
 - 10) Compressible flow phenomena

- Extreme range of length and time scales

\rightarrow split up entire space of applications into various regimes

Fluid Mechanics



Governing equations

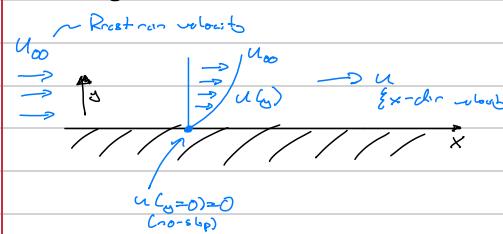
- Can be quite complex

Navier-Stokes equation: $\rho \frac{D\mathbf{u}}{Dt} = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + \rho g_x$ Fay Fmax is not covered in this course

\hookrightarrow nonlinear, partial differential equation

- We will rely on empirical relations, empirical geometries for engineering applications

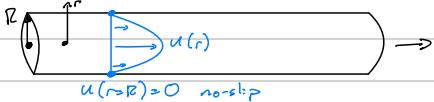
Nomenclature



- No-slip condition: Fluid adjacent to a solid surface takes the velocity of that surface

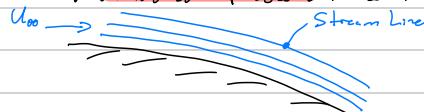
- External Flow: Surface in an infinite flow field (can move far away from boundary)

- Internal Flow: Flow is fully bounded by a solid surface

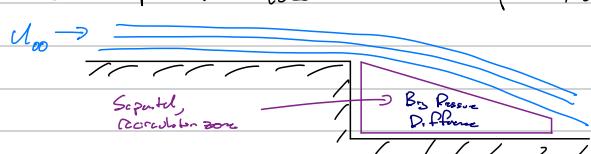


Flow Separation

a) Attached Flow: Flow follows the shape of a body



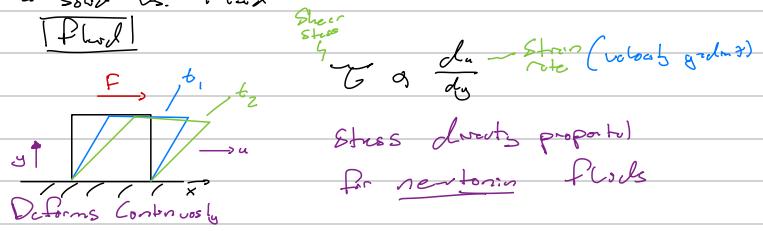
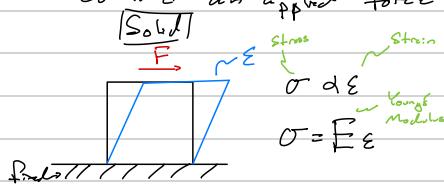
b) Separated Flow: Flow separates/doesn't follow surface due to abrupt geometry change



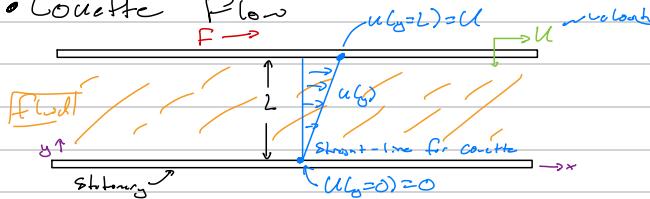
Viscoelasticity

- Resistance to flow, molecular friction

Consider an applied force on a solid vs. fluid



- Couette Flow



$$\text{Force} \Rightarrow \text{Shear Stress on the Fluid: } \tau = \frac{F}{A}$$

$$\text{Force} \Rightarrow \text{Upper plate velocity } U$$

$$\tau \propto \frac{dU}{dy}$$

Newtonian Fluids: Common fluids like water, air, oil, etc.

Non-newtonian Fluids viscosity changes based on force

- To find the force

$$\tau = \tau A = \mu \frac{dU}{dy} A \quad \text{Couette Flow}$$

For all newtonian fluids

- Introduce proportionality constant for newtonian fluids

$$\tau = \mu \frac{dU}{dy} \quad \begin{matrix} \text{Dynamic Viscosity} \\ \text{Also known as } \eta = \mu \end{matrix}$$

- More Generally



Surface Tension

- Comparative attractions between similar and dissimilar molecules

- Some effects of surface tension

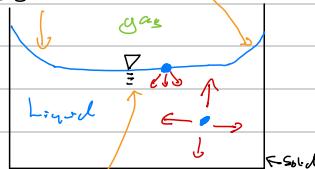
(i) Spherical Droplets

↳ minimum energy state

Pendant Droplet

(ii) Wetting Behavior
(v) Capillary Rises

(iii) Meniscus



(iv) Free Surface

- Requires some energy to increase free surface area

$$\text{area} \Rightarrow \text{surface tension } (\sigma)$$

$$\sigma \left[\frac{\text{energy}}{\text{area}} = \frac{\text{force} \times \text{distance}}{\text{area}} = \frac{\text{force}}{\text{length}} \right] \quad \begin{matrix} \text{Energy} \\ \text{in mJ} \end{matrix}$$

Surface Area of a Sphere is $4\pi r^2$

- Consider a spherical droplet



As fluid is injected, surface area is created
↳ molecules move from the interior to the surface \rightarrow takes some amount of energy

- Energy stored in the droplet

$$dE = \sigma dA_s = \sigma \cdot 8\pi r dr$$

$$\left. \begin{array}{l} A_s = 4\pi r^2 \\ dA_s = 8\pi r dr \end{array} \right\}$$

- Work required to increase A_s

$$dW = F dr = (P_i - P_o) A_s dr = (P_i - P_o) 4\pi r^2 dr$$

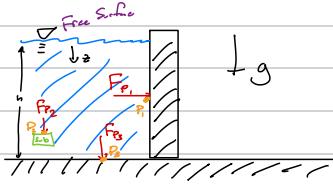
- Equate dE and dW

$$dE = \sigma \cancel{8\pi r^2 dr} = 4\cancel{\pi r^2 dr} (P_i - P_o) = dW \Rightarrow (P_i - P_o) = \frac{2\sigma}{r}$$

Hydrostatics

Hydrostatics Def. Impact of pressure on submerged surfaces

Applications: Dams, Tanks



Pressure

(i) Pressure acts isotropically

(acts the same in all directions)

(ii) Pressure leads to a pressure force

$$\text{Pressure} \left[\frac{\text{force}}{\text{area}} \right] \rightarrow F_{\text{pressure}} = \int P dA$$

(iii) Pressure force on a submerged object (like tank) acts normal to the surface, and always acts inward

(iv) Pressure varies with depth according to the hydrostatic pressure variation

$$\text{net pressure force} \rightarrow \frac{dP}{dz} = \rho g \quad \begin{array}{l} \text{density} \\ \text{constant} \end{array}$$

Accel due to gravit

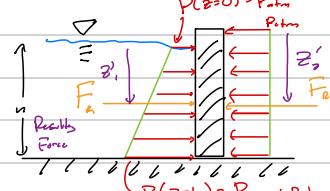
$$\Rightarrow P(z) = \int \rho g dz = P(z=0) + \rho g z$$

Assume const. ρ

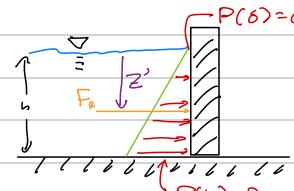
• Convenient to write things in gauge pressure:

$$\text{Gauge Pressure} = P_{\text{absolute}} - P_{\text{atm}}$$

Absolute Pressure



Gauge Pressure



Goals (i) Determine the resultant force (F_R)

(ii) Determine where this single force acts (line of action)

1) Resultant Force

$$dA = w dz$$

$$F_R = \int_A P(z) dA = \int_0^h P(z) w dz$$

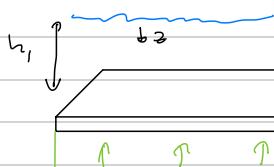
a) Linear Pressure Distribution

$$F_R = w \int_0^h \rho g z dz = w \left[\frac{1}{2} \rho g z^2 \right]_0^h = \frac{1}{2} \rho g h^2 w = \frac{1}{2} \rho g h (h_w)$$

Average Pressure

$$\checkmark \text{Area}$$

b) Uniform Pressure



$$F_R = \rho g h \cdot A$$

Buoyant Force

2) Line of Action

Location needs to lead to the same moment as the force distribution

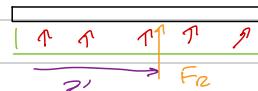
a) Linear Pressure Distribution

$$z' = \frac{2}{3} h$$

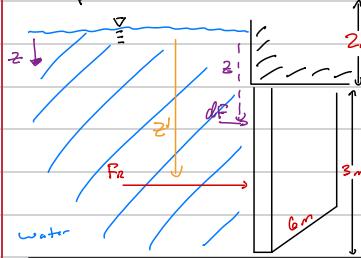


b) Uniform Pressure

$$z' = \text{half way across}$$



Example 11-4s



• Find

1) Resultant Force (F_r)

2) where F_r acts

2 Approaches possible Approaches

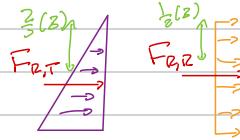
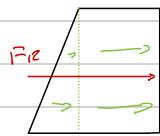
1) Integral Approach

$$F_r = \int P(z) dA = w \int_2^5 \rho g z dz = \frac{\rho g w}{2} (5^2 - 2^2) = 618,030 \text{ N} \rightarrow [F_r = 618 \text{ kN}]$$

$$F_r z' = \int_2^5 \rho g z^2 dz = \rho g w \int_2^5 z^2 dw = \frac{\rho g w}{3} (5^3 - 2^3) = 2,245,540 \text{ N}\cdot\text{m}$$

$$618,030 \text{ N} z' = 2,245,540 \text{ N}\cdot\text{m} \rightarrow [z' = 3.71 \text{ m}]$$

2) Decomposition



$$F_{r,T} = P_{avg} \cdot A \quad \text{Area} \\ = \frac{1}{2} \rho g (3)(5)(6) \\ = 265 \text{ kN}$$

$$F_{r,B} = P_A \cdot A \\ = 2 \rho g (3)(6) \\ = 363 \text{ kN}$$

$$\Rightarrow F_r = F_{r,T} + F_{r,B} = 618 \text{ kN}$$

$$[F_r = 618 \text{ kN}]$$

Take the moment about the base $\Rightarrow \hat{z}'$

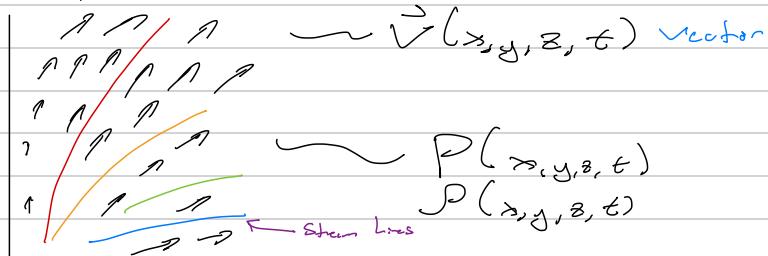
$$F_r \hat{z}' = F_{r,T} \frac{2}{3}(3) + F_{r,B} \frac{1}{3}(3) \Rightarrow \hat{z}' = 1.71 \text{ m}$$

$$[\hat{z}' = 3.71 \text{ m}]$$

Shift to the free surface: $z' = 2m + \frac{1}{3}(3) = 3.71 \text{ m}$

Fluid Dynamics & The Bernoulli Equation

Flow Fluid



Stream line \rightarrow Line formed from the tangent to trajectory at any point

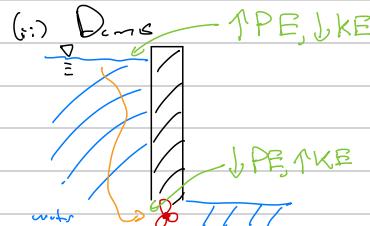
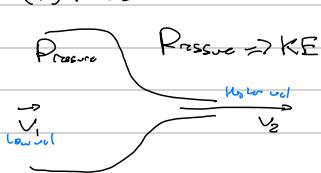
- For steady flows $[\vec{V}(x,y,z)]$ all fluid particles passing through a certain point follow the same streamlines

Bernoulli Equation

- Provides a relationship between pressure, elevation, and velocity
 - Conservation of mechanical energy

Applications

(i) Nozzle



Derivation

- Conservation of Linear Momentum applied to a fluid element

1) Acceleration of a fluid element

$$v_s = f(s, t) \quad \text{in general velocity is a function of two variables}$$

$$v_s = \frac{ds}{dt}?$$

- Total change in v_s

multidimentional $\rightarrow dv_s = \frac{\partial v_s}{\partial s} ds + \frac{\partial v_s}{\partial t} dt$

- Divide through by dt

$$a = \frac{dv_s}{dt} = \frac{\partial v_s}{\partial s} \cdot \frac{ds}{dt} + \frac{\partial v_s}{\partial t}$$

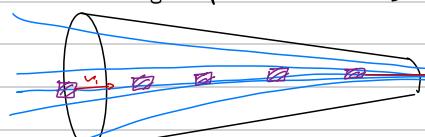
Change in velocity
at a point with respect
to time

$$\frac{\partial v_s}{\partial t} = 0 \quad \text{implies by 'steady-flow'}$$

$$\text{Sub back in } v_s = \frac{ds}{dt}$$

$$\therefore a = v_s \frac{\partial v_s}{\partial s}$$

Physical picture of $v_s \frac{ds}{dt}$



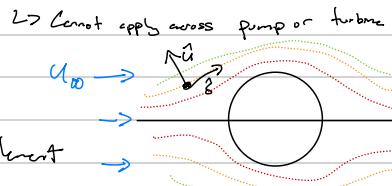
→ Acceleration due to movement through

a velocity field despite steady flow

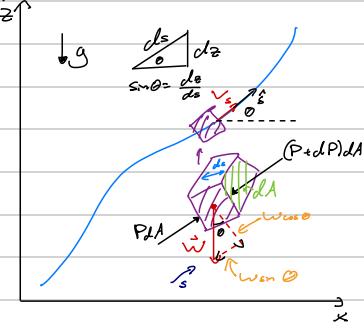
• N2L along stream line $\rightarrow \sum F_s = m a_s$

Assumptions

- (i) Steady flow - no transient effects
- (ii) Inviscid - viscous effects negligible
- (iii) Incompressible (ρ constant)
- (iv) Flow along a stream line
- (v) No heat transfer
- (vi) No work interactions



2) Forces



Forces

- 1) Weight; $W = mg$; $m = \rho dV = \rho dA ds$
- 2) Pressure Force; Normal force due to pressure

$$\sum F_s = -W \sin \theta + P dA - (P + dP) dA$$

$$\sum F_s = -dP dA - mg \sin \theta = m a_s = m v_s \frac{dv_s}{ds}$$

$$\Rightarrow -dP - P g \frac{ds}{ds} = \rho v_s dv_s$$

$$= \frac{1}{2} \rho d(v_s^2)$$

$$\Rightarrow dP + Pg ds + \frac{1}{2} \rho d(v_s^2) = 0$$

$$\frac{dP}{P} + g ds + \frac{1}{2} d(v_s^2) = 0$$

• Integrate Expression

$$\int \frac{dP}{P} + g z + \frac{1}{2} v_s^2 = \text{Constant} \quad \text{Valid for compressible or incompressible flow}$$

→ For constant density

$$\frac{P}{\rho} + \frac{1}{2} v_s^2 + g z = \text{Constant}$$

Starts point for Bernoulli:

Equation along a stream

• Bernoulli Equation + continuity applied between two points:

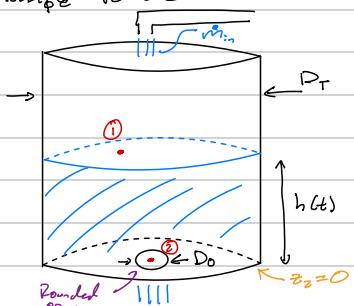
$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g z_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g z_2$$



(Continuity) Bernoulli Equations

- Assumptions for this form
- ① Steady
 - ② Incompressible
 - ③ No heat transfer
 - ④ Flow along a stream
 - ⑤ No frictional losses

Example 12-22



$P_1 = P_{atm}$

$v_1^2 = 0$ due to free surface

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g z_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g z_2$$

$P_1 = P_{atm}$ so $P_1 = P_2$

$$\Rightarrow v_1^2 = \frac{\pi D_I^2}{4} = v_2^2 = \frac{\pi D_o^2}{4}$$

so $(\frac{D_o}{D_I})^2$ used

At steady-state conditions:

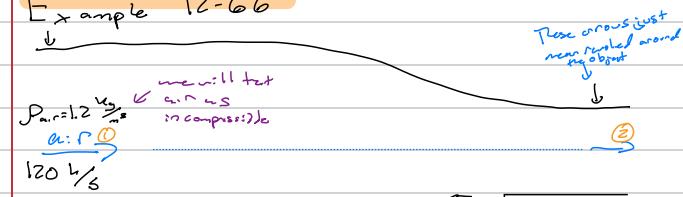
$$\therefore h_{max} = \frac{1}{2g} \left(\frac{v_{max}^2}{\pi D_o^2} \right)$$

$$\dot{m}_{in} = \dot{m}_{out} = \rho V_2 A_o = \rho \sqrt{2gh_{max}} \frac{\pi D_o^2}{4}$$

- Find h_{max} (equilibrium fill level)
in terms of \dot{m}_{in} , D_o , P_{atm} , etc.

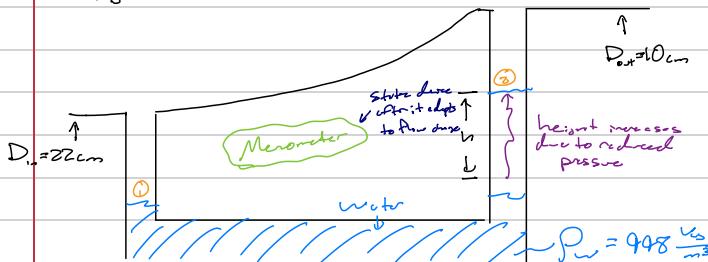
P_2 also is P_{atm} because it is an open hole into the atmosphere

Example 12-66



Problem

- Velocity should increase due to conservation of mass
- Pressure should decrease; velocity needs to go up to increase from somewhere, and this is due to pressure



Find: The height 'h' that the water goes up when wind goes through the tunnel

Wind goes through tunnel creates a low pressure zone at end of nozzle so transition acts 'suck up'

$$\text{Bernoulli Eq: } P_1 + \frac{1}{2} \rho V_1^2 + \rho g Z_1 = P_2 + \frac{1}{2} \rho V_2^2 + \rho g Z_2 \quad \text{air}$$

$$P_{1w} + \frac{1}{2} \rho V_{1w}^2 + \rho g Z_{1w} = P_{2w} + \frac{1}{2} \rho V_{2w}^2 + \rho g Z_{2w} \quad \text{water}$$

$$\begin{cases} P_1 = P_{1w} = P_1 \\ P_2 = P_{2w} = P_2 \end{cases}$$

Ans ρ is low, can assume column pressurization

$$\rightarrow \text{Air: } P_1 - P_2 = \frac{1}{2} \rho_a (V_2^2 - V_1^2) \stackrel{\text{eqn to}}{\Rightarrow} h = \frac{1}{2} \frac{\rho_a}{\rho_wg} (V_2^2 - V_1^2)$$

$$\rightarrow \text{Water: } P_1 - P_2 = \rho_w g h$$

$$\text{Find what's } m_1 = m_2 \text{ Steady-state} \quad m = \rho V_A \text{ and } \dot{m} = \rho V_A A = \frac{\dot{m}}{A}$$

\rightarrow If incompressible, we can say $V_1 = V_2$

$$\therefore V_1 = V_2 = V = 120 \frac{\text{ft}}{\text{s}} = 0.12 \frac{\text{m}}{\text{s}}$$

$$\Rightarrow V_1 = \frac{V}{A_1} = \frac{V}{\frac{\pi D_1^2}{4}} = 3.16 \frac{\text{m}}{\text{s}} \quad \text{and} \quad V_2 = \frac{V}{A_2} = \frac{V}{\frac{\pi D_2^2}{4}} = 15.8 \frac{\text{m}}{\text{s}} \quad \left. \right\} h = 1.37 \text{ cm}$$

Energy Form of the Bernoulli Equation

General Energy Form of the Bernoulli Equation

Goal: Start with energy conservation to relax some of the assumptions

- Rot based on system of the first law of thermodynamics

$$(Q_{in} - Q_{out}) + (w_{in} - w_{out}) = m \left[(h_2 - h_1) + \frac{1}{2} (V_2^2 - V_1^2) + g(z_2 - z_1) \right]$$

(i) Define b_m

$$\begin{cases} q = \dot{Q}/m \\ w = \dot{W}/m \end{cases} \quad \begin{cases} \text{if } q \neq 0 \\ \text{if } w \neq 0 \end{cases}, \quad q_{net,in} = q_{in} - q_{out} \Rightarrow q_{net,in} + w_{net,in} = (h_2 - h_1) + \frac{1}{2} (V_2^2 - V_1^2) + g(z_2 - z_1)$$

(ii) Use definition of h : $\begin{cases} h = u + p \end{cases} = u + \frac{P}{\rho}$... $h_1 = u_1 + \frac{P_1}{\rho_1}$
 $h_2 = u_2 + \frac{P_2}{\rho_2}$

$$\Rightarrow \frac{P_1}{\rho_1} + \frac{1}{2} V_1^2 + g z_1 + w_{net,in} = \frac{P_2}{\rho_2} + \frac{1}{2} V_2^2 + g z_2 + u_2 - u_1 + q_{net,in}$$

mechanical energy in to added mechanical energy
turbine (from pump/turbine)

mechanical energy out of turbine

internal energy and heat transfer

- For ideal scenarios (no irreversibilities)

$$u_2 = u_1 + q_{net,in} \Rightarrow u_2 - u_1 - q_{net,in} = 0$$

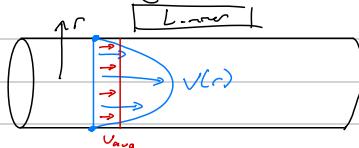
- For nonideal scenarios (fluid friction present)

$$u_2 - u_1 > q_{net,in} \Rightarrow u_2 - u_1 = q_{net,in} + e_{losses}$$

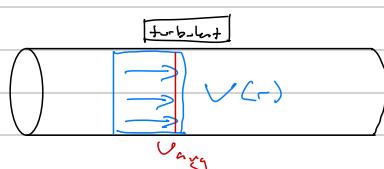
$$(u_2 - u_1 - q_{net,in}) = e_{losses}$$

$$\Rightarrow \frac{P_1}{\rho_1} + \frac{1}{2} V_1^2 + g z_1 + w_{pump} = \frac{P_2}{\rho_2} + \frac{1}{2} V_2^2 + g z_2 + w_{turbine} + e_{losses}$$

Reiner Energy correction factor $\times E_{total} = \int_{A_0} \frac{1}{2} V(r) dA_0 \Rightarrow KE \text{ correction factor } (\lambda)$



• For laminar flow: $\lambda = 2$



Most pipe flows are turbulent

• For turbulent flow: $\lambda = 1.03 \approx 1$

$$\frac{P_1}{\rho_1} + \frac{1}{2} \lambda V_1^2 + g z_1 + w_{pump} = \frac{P_2}{\rho_2} + \frac{1}{2} \lambda V_2^2 + g z_2 + w_{turbine} + e_{losses}$$

Energy Equation in terms of power λ is omitted for case of pump but should be multiplied by the KE term
 \rightarrow rot basis ($\times m$) \rightarrow Actual \rightarrow Actual work

$$m \left[\frac{P_1}{\rho_1} + \frac{1}{2} V_1^2 + g z_1 \right] + \dot{w}_{pump} = m \left[\frac{P_2}{\rho_2} + \frac{1}{2} V_2^2 + g z_2 \right] + \dot{w}_{turbine} + \dot{E}_{losses}$$

$$\rightarrow \dot{E}_{losses} = \dot{E}_{losses, pump} + \dot{E}_{losses, turb} + \dot{E}_{losses, piping}$$

- For Pump

$$\dot{w}_{pump,u} = \dot{w}_{pump} - \dot{E}_{losses, pump} \Rightarrow \eta_{pump} = \frac{\dot{w}_{pump,u}}{\dot{w}_{pump}}$$

useful
Actual

(isentropic: $\eta = 1$)

- For Turbine

$$\dot{w}_{turbine} = \dot{w}_{turbine} + \dot{E}_{losses, turb} \Rightarrow \eta_{turbine} = \frac{\dot{w}_{turbine}}{\dot{w}_{turbine}}$$

Contractible
Actual

$$P + \text{Volumetric Flowrate} = \frac{\text{Velocity}}{\text{Cross Sec Area}}$$

$$\Rightarrow m \left[\frac{P_1}{\rho_1} + \frac{1}{2} V_1^2 + g Z_1 \right] + \dot{W}_{\text{pump,u}} = m \left[\frac{P_2}{\rho_2} + \frac{1}{2} V_2^2 + g Z_2 \right] + \dot{W}_{\text{turbine}} + \dot{E}_{\text{losses, piping}}$$

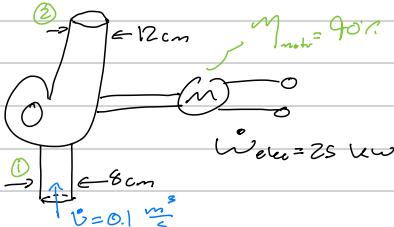
Unit Bernoulli Equation in terms of head & usual static point

→ head has dimensions of length, Diminish by $\frac{mg}{g}$

$$h_{\text{pump,u}} = \frac{\dot{W}_{\text{pump,u}}}{mg}, h_{\text{turbine}} = \frac{\dot{W}_{\text{turbine}}}{mg}, h_L = \frac{\dot{E}_{\text{losses,piping}}}{mg} \leftarrow \text{Frictional head loss}$$

$$\Rightarrow \frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + Z_1 + h_{\text{pump,u}} = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + Z_2 + h_{\text{turbine}} + h_L \quad \begin{array}{l} \text{Irrecoverable head loss between (1) and (2)} \\ \text{due to fluid friction} \end{array}$$

Example 12-47



Given

- $Q = 0.1 \text{ m}^3/\text{s}$
- Pressure rise: 250 kPa $\leftarrow \Delta P = P_2 - P_1$, here
- KB correction factor: $\lambda = 1.05$

Find: Pump efficiency

$$\eta_{\text{pump}} = \frac{\dot{W}_{\text{pump,u}}}{\dot{W}_{\text{pump}}} \quad \dot{W}_{\text{pump}} = \eta_{\text{motor}} \times \dot{W}_{\text{elec}}$$

$$\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + Z_1 + h_{\text{pump,u}} = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + Z_2 + h_{\text{turbine}} + h_L \quad \begin{array}{l} \text{outlet and inlet are closed, } s=0 \\ \text{there are not enough piping to consider losses due to piping} \end{array}$$

$$h_{\text{pump,u}} = \frac{P_2 - P_1}{\rho g} + \frac{V_2^2 - V_1^2}{2g}$$

$$\text{Volume} = \frac{C}{A_C} \quad V_1 = \frac{C}{A_C} = \frac{C}{\frac{\pi}{4} D^2} = 0.89 \frac{m^3}{s}$$

$$\text{Incompressible} \rightarrow V_1 = V_2 = C \quad V_2 = \frac{C}{A_{C_2}} = \frac{C}{\frac{\pi}{4} (\frac{D}{2})^2} = 8.81 \frac{m^3}{s}$$

$$\rightarrow h_{\text{pump,u}} = \frac{\Delta P}{\rho g} + \frac{V_2^2 - V_1^2}{2g} = \frac{250000 \text{ Pa}}{860 \times 9.81} + 1.05 \frac{8.81^2 - 0.89^2}{2 \times 9.81}^2$$

$$= 29.6 - 17 = 12.6 \text{ m}$$

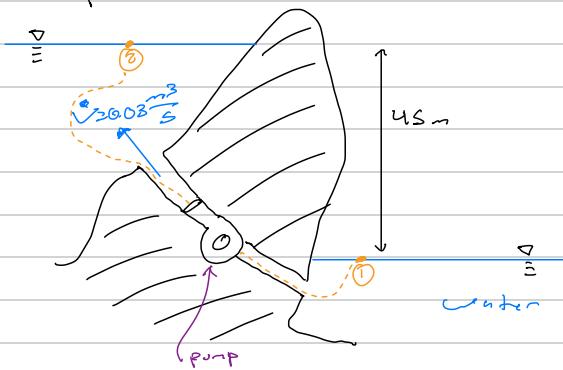
useless pump
pump to get pressure
watch your units

$$\dot{W}_{\text{pump,u}} = \dot{m} g h_{\text{pump,u}} = \rho \dot{V} g h_{\text{pump,u}} = 10.66 \text{ kW} = 10.66 \text{ kW}$$

$$\eta_{\text{pump}} = \frac{\dot{W}_{\text{pump,u}}}{\dot{W}_{\text{pump}}} = \frac{10.66 \text{ kW}}{\eta_{\text{motor}} \dot{W}_{\text{elec}}} = \frac{10.66 \text{ kW}}{22.5 \text{ kW}} = 47.4\%$$

$$\boxed{\eta_{\text{pump}} = 47.4\%}$$

Example 12-60



Pump delivers 20kW of useful power

Find

- (i) Irreversible Head loss (h_L)
- (ii) Lost mechanical power

1) Frictional Head Loss

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 + h_{pump,u} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_{friction} + h_L$$

$$\Rightarrow h_L = h_{pump,u} - \Delta Z$$

velocities small relative to
Z changes at free surface

$$\Delta Z = Z_1 - Z_2 = 45m$$

$$h_{pump,u} = \frac{\dot{E}_{pump,u}}{\dot{m}g} = \frac{20kW}{\rho g} = 67.96m$$

$$h_L = h_{pump,u} - \Delta Z = 67.96 - 45 = 23m$$

With pump working you
could nose plow 67.96m

$$h_L = 23m$$

2) Lost Mechanical Power

$$\dot{E}_{losses, pipe} = h_L \dot{m}g = 67.96 \times 56.2 \times 9.81 = 36.5kW$$

$$\dot{E}_{losses, pipe} = 6.76kW$$

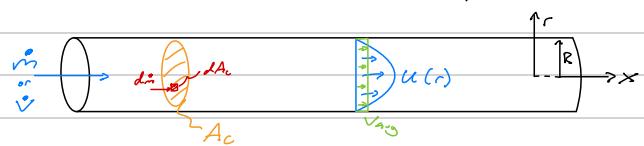
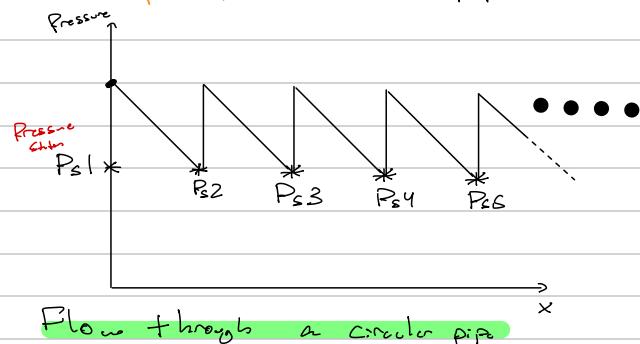
$$h_L = \frac{\dot{E}_{losses, pipe}}{\dot{m}g}$$

Internal Flow

Internal Flow

Goal: Calculate frictional head loss (h_f) based on velocity, viscosity etc.

Example: Trans-Alaska Pipeline ~800 mi



• Mass flow rate:

$$\text{Overall} \quad \dot{m} = \rho V_{\text{Avg}} A_c$$

Local

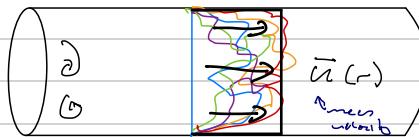
$$\dot{m} = \int_{A_c} P u(r) dA_c$$

• Equate \dot{m} expressions: $\int P u(r) dA_c = \rho V_{\text{Avg}} A_c$

→ Assume incompressible and circular pipe: $P_1 = P_2 = P$, $A_c = \pi R^2$ & $dA_c = 2\pi r dr$
 $\Rightarrow V_{\text{Avg}} = \frac{2}{R^2} \int_0^R u(r) r dr$

Laminar

Turbulent



• Smooth, orderly flow

• Low velocity, high viscosity

→ Flow regime based on Reynolds Number (Re)

• High mixing chaotic flow

• High velocity, low viscosity

$$\text{dimensionless } Re = \frac{\rho V_{\text{Avg}} D}{\mu}$$

• Ratio of inertial (momentum) effects to viscous effects

• Transition from laminar to turbulent based on a critical Reynolds number

$$Re_{\text{crit}} = 2300 \text{ for internal flow}$$

$$Re_{\text{crit}} = 5 \times 10^5 \text{ for external flow}$$

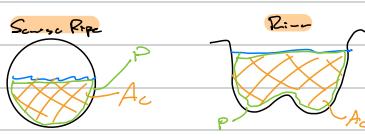
• For non-circular cross sections, use hydrodynamic diameter D_h

$$D_h = \frac{4 A_c}{P}$$

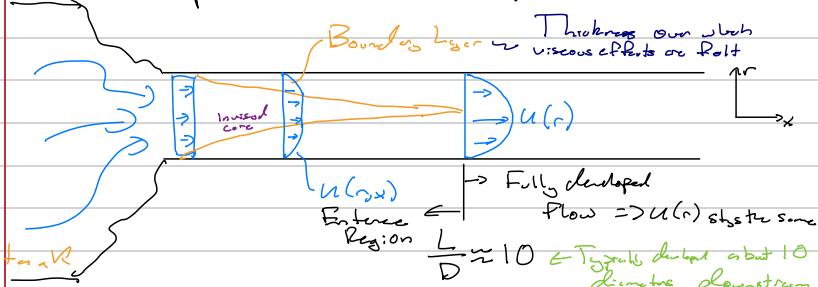
Flow cross section
Area
Perimeter



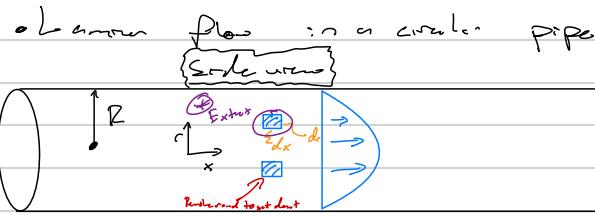
or



• Developing vs. Fully Developed Flow



Dominators



{ end view }

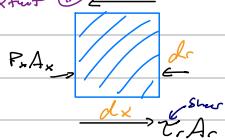


Assumptions

- Steady, Laminar ($R_e \leq 300$) flow
- Assume incompressible flow and constant properties
- Straight, circular pipe
- Fully developed

Approach: Apply Newton's 2nd Law ($\sum F_x = m_{\text{max}}$) to a short slip fluid element

$$\text{Extent } \textcircled{1} \text{ Traversed} \Rightarrow \sum F_x = 0$$



Forces

(i) Friction/shear stress

(ii) Pressure

Sum forces

$$(P_Ax) - (P_Ax)_{\text{ext}} + (r\tau_s)_{\text{ext}} - (r\tau_s)_{\text{ext}} = 0$$

• Reverse sign due to τ_s

$$r \frac{dP}{dx} - P_Ax + (r\tau_s)_{\text{ext}} - (r\tau_s)_{\text{ext}} = 0 \quad \text{Newton's 2nd Law}$$

$$\tau_s = -M \frac{du}{dr}$$

$$\rightarrow \text{Take Limit as } dr, dx \rightarrow 0 \Rightarrow r \frac{dP}{dx} + \frac{d(r\tau_s)}{dr} = 0$$

$$\Rightarrow r \frac{dP}{dx} = M \frac{du}{dr} \quad (\text{assumed } M \text{ constant})$$

$$\text{Only terms on } \frac{dP}{dx} = \frac{M}{r} \frac{d}{dr} \left(\frac{du}{dr} \right) \quad (\text{only depends on } r)$$

$$g(x) = f(r) \Rightarrow \text{to both } g(x) = f(r) = \text{constant must be true}$$

$$\Rightarrow \frac{d}{dr} \left(r \frac{du}{dr} \right) = \frac{M}{r} \frac{dP}{dx} \quad (\text{constant})$$

• Integrate twice:

$$u(r) = \frac{r^2}{4M} \frac{dP}{dx} + C_1 \ln r + C_2 \quad (\text{constant of integration})$$

• Need 2 boundary conditions to solve for C_1, C_2

1) No-slip condition: $u(r=R) = 0$

2) Symmetric: $\frac{du}{dr}(r=0) = 0$ comes from flow being fully developed & laminar

$$\Rightarrow u(r) = \frac{R^2}{4M} \frac{dP}{dx} \left(1 - \frac{r^2}{R^2} \right)$$

$\frac{dP}{dx} \neq 0$ not be too small
to push water upstream (i.e. flow)

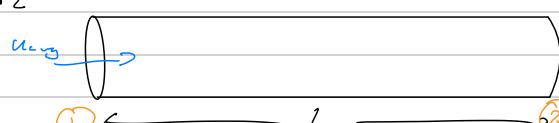
$$\bullet \text{Average velocity: } U_{\text{avg}} = \frac{2}{R^2} \int_0^R u(r) r dr = - \frac{R^2}{8M} \frac{dP}{dx} \quad \text{For laminar flow: } U_{\text{avg}} = \frac{1}{2} U_{\text{max}}$$

• Remove to isolate $\frac{dP}{dx} \equiv \Delta P_L$

$$\frac{dP}{dx} = - \frac{8M U_{\text{avg}} L}{R^2}$$

$$\Rightarrow \Delta P_L = \frac{8M U_{\text{avg}} L}{R^2}$$

$$= \frac{32M U_{\text{avg}} L}{D^2}$$



$$\leftarrow \frac{dP}{dx} = \frac{P_2 - P_1}{L}$$

$\Delta P_L = P_1 - P_2$
pressure loss

$$\dot{W}_{\text{pump}} = \dot{V} \Delta P_L$$

Incompressible assumed

• Pumping power to overcome frictional losses

- To generalize results, define the Darcy Friction Factor (f)

$$\Delta P_L = f \frac{L}{D} \frac{1}{2} \rho u_{avg}^2$$

dynamical pressure

$$\frac{\Delta P_L}{\frac{1}{2} \rho u_{avg}^2} = f \frac{L}{D}$$

Einstein ends up being dimensionless

- Go from $\Delta P_L \rightarrow h_L$

$$h_L = \frac{\Delta P_L}{\rho g} = f \cdot \frac{L}{D} \cdot \frac{u_{avg}^2}{2g}$$

- Fiction factor for laminar flow in a circle $\frac{f D}{R_p}$

pipe: $\Delta P_L = \frac{32 \mu L u_{avg}}{D^2}$

want to look like this
dimensionless
 $= \frac{L}{D} \frac{1}{2} \rho u_{avg}^2 \left(\frac{64 \mu}{R_p D} \right) \frac{1}{Re}$ Reynolds #

$$\Rightarrow f = \frac{64}{Re} \quad \text{only based on const (64) and reynolds number w/ respect to pipe diameter (} R_p \text{)}$$

For turbulent flow

- Empirical rather than analytical (based on exponents)

Experiments show: $f = f(R_c, \frac{\epsilon}{D})$
 \rightarrow Surface roughness



Curve fit experimental data \rightarrow Colebrook equation

$$\frac{1}{\sqrt{f}} = -2.0 \log_{10} \left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re \sqrt{f}} \right)$$

\rightarrow Implicit equation for $f \rightarrow$ 1 to solve!

- Approximate explicit form \rightarrow Hazen-Williams Equations

$$\frac{1}{\sqrt{f}} \approx -1.8 \log_{10} \left[\left(\frac{\epsilon/D}{3.7} \right)^{1.11} + \frac{6.9}{Re} \right] \quad \text{2% error introduced}$$

- Or, plot colebrook equation plotted as Moody Diagram

(i) Re on x-axis Calculate ϵ/D , find intersection of Re and ϵ/D on curves

(ii) ϵ/D on secondary y-axis

Result comes from reading across to primary vertical axis \rightarrow now you have f !

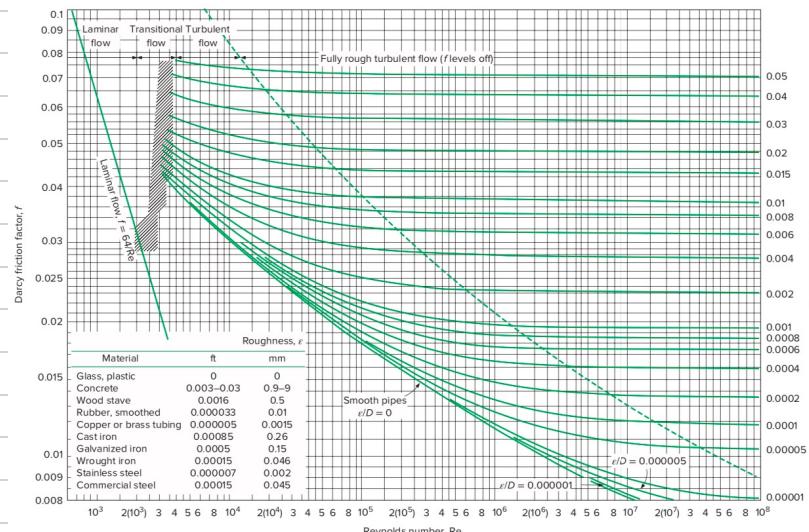
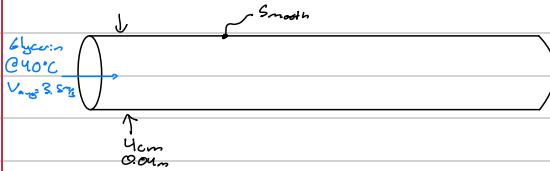


FIGURE A-27

The Moody chart for the friction factor for fully developed flow in circular pipes for use in the head loss relation $\Delta P_L = f \frac{L}{D} \frac{\rho V^2}{2}$. Friction factors in the turbulent flow are evaluated from the Colebrook equation $\frac{1}{\sqrt{f}} = -2 \log_{10} \left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re \sqrt{f}} \right)$.

Example 14-48



Find pressure loss per 10m of length

$$\text{Liquor } @ 40^\circ\text{C} \quad \rho = 1282 \text{ kg/m}^3 \\ \mu = 0.27 \text{ kg/m.s}$$

$$\text{For water } M_{\text{water}} = 0.000018 \frac{\text{kg}}{\text{m.s}} \\ \text{For air } M_{\text{air}} = 0.001 \frac{\text{kg}}{\text{m.s}}$$

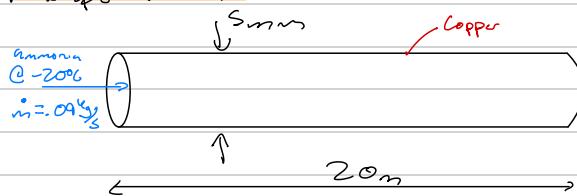
$$\Delta P_L = f \frac{L}{D} \frac{V_{\text{avg}}^2}{2} \quad \text{Unknown}$$

$$Re_D = \frac{\rho V_{\text{avg}} D}{\mu} = 649.2 \times 2300 \Rightarrow \text{Laminar flow}$$

$$\therefore f = \frac{64}{Re_D} = \frac{64}{649.2} = 0.099$$

$$\therefore \Delta P_L = 189,000 \text{ Pa per 10m}$$

Example 14-51



Find ΔP_L , h_L , h_{pump}

$$\text{Ammonia } @ -20^\circ\text{C} \quad \rho = 665.1 \frac{\text{kg}}{\text{m}^3} \\ \text{Turb } A-17 \quad \mu = 2.361 \times 10^{-4} \frac{\text{kg}}{\text{m.s}}$$

$$Re_D = \frac{\rho V_{\text{avg}} D}{\mu} \quad \dot{m} = \rho V_{\text{avg}} A_c \Rightarrow V_{\text{avg}} = \frac{\dot{m}}{\rho A_c} = 6.9 \text{ m/s} \Rightarrow Re_D = 97,100 \Rightarrow \text{Turbulent flow}$$

$$\Delta P_L = f \frac{L}{D} \frac{V_{\text{avg}}^2}{2}$$

Go to Moody Diagram

$$Re_D = \frac{\dot{m}}{\mu} \quad \frac{\epsilon}{D} = \frac{0.0015 \text{ mm}}{3 \text{ mm}} = .0005$$

$$\therefore \Delta P_L = 12.86 \text{ kPa}$$

$$h_L = \frac{\Delta P_L}{\rho g} = 1.89 \text{ m}$$

$$h_{\text{pump}} = \dot{V} \Delta P_L = \dot{m} g h_L = 167 \text{ W}$$

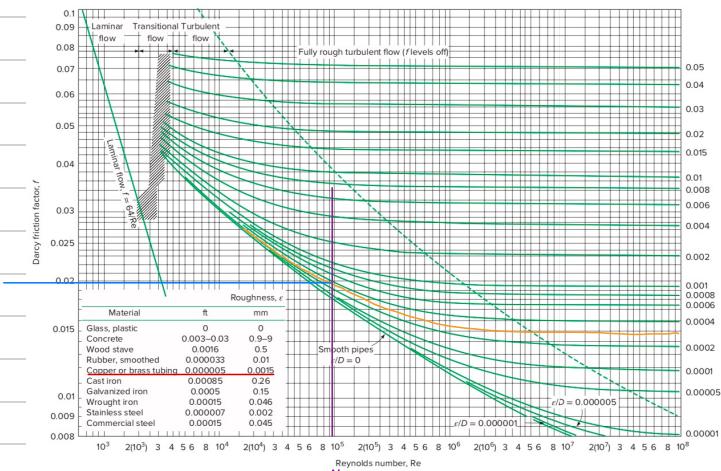


FIGURE A-27 $Re = 97,000 = 4.7 \times 10^4$

The Moody chart for the friction factor for fully developed flow in circular pipes for use in the head loss relation $\Delta P_L = f \frac{L}{D} \frac{\rho V^2}{2}$. Friction factors in the turbulent flow are evaluated from the Colebrook equation $\frac{1}{f} = -2 \log_{10} \left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re \sqrt{f}} \right)$.

$f = \frac{1}{\sqrt{f}}$

Minor Losses

- piping losses from other components
 - (i) Elbows, Tees
 - (ii) Valves
 - (iii) Pipe transitions (reducing or expanding)
 - (iv) Entrance/exit to/from piping

Modeling them

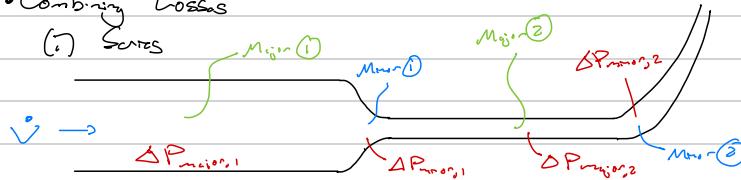
$$h_{L, \text{minor}} = K_L \cdot \frac{V_{AO}^2}{2g}$$

↑

Component specific minor loss coefficient K_L tables give scenarios with K_L values for some ΔP s (KE constant factor)

Combining Losses

(i) Series

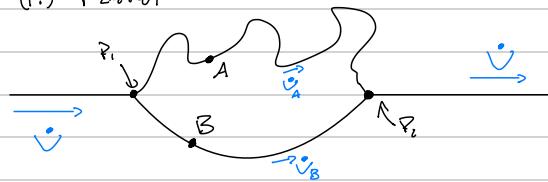


a) V constant (incompressible)

b) ΔP_s added

$$\Rightarrow h_{z,\text{tot}} = \sum h_{L,\text{major}} + \sum h_{L,\text{minor}}$$

(ii) Parallel



a) $V = V_A + V_B$

b) $\Delta P_A = \Delta P_B$

$$\Rightarrow h_{L,A} = h_{L,B}$$

External Flow

External Flow

- Flow on bodies \Rightarrow aerodynamic forces (bP_{drag})

\rightarrow Two Primary Mechanisms

(i) Friction $T = \mu \frac{du}{dy}$ \rightarrow loss in the boundary layer

(ii) Pressure Differences $P_1 + \frac{1}{2} \rho V_1^2 = P_2 + \frac{1}{2} \rho V_2^2$

Wake $P_b \approx P_\infty$

• Shear stress over surface area \Rightarrow friction force

• Pressure Differences on surface \Rightarrow pressure force

$Re_{crit} = 2300$ for internal flow

$Re_{crit} = 5 \times 10^5$ for external flow

$2D$ vs. $3D$ flows

• 2D analysis valid if $\frac{L}{D}$ is large

Defining Forces & moments

• Defn. Lift & Drag Coefficients

- Drag Coeff.

Dimensionless $C_D = \frac{F_D}{\frac{1}{2} \rho U_\infty^2 A} \Rightarrow F_D = C_D \frac{1}{2} \rho U_\infty^2 A$

- Lift Coeff.

$C_L = \frac{F_L}{\frac{1}{2} \rho U_\infty^2 A}$

• Effects that influence aerodynamic forces

- Fluid Properties (ρ, μ)
- Object Properties / Shape
 - Streamlined
 - Blunt or bluff body
- Flow Regime (Laminar vs. Turbulent)

• Magnitude of C_D

$P_2 = P_{\text{atm}} = P_\infty + \frac{1}{2} \rho U_\infty^2$

$(P_2 - P_\infty) A \Rightarrow C_D \approx 1$

$\frac{1}{2} \rho U_\infty^2 A$

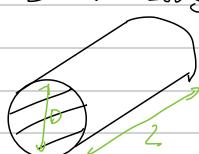
• C_D can be greater than 1

Appropriate area for C_D, C_L

- Depends on shape type

- Blunt body-frontal area (A_p)
- Streamlined body \rightarrow planform area (A_p)

$U_\infty \rightarrow$

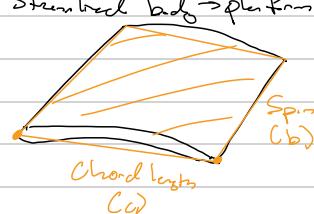


• For a cylinder

$$A_p = DL$$

• For a sphere

$$A_p = \frac{\pi D^2}{4}$$



$$A_p = b_c$$

- Get C_D from table

→ C_D depends on

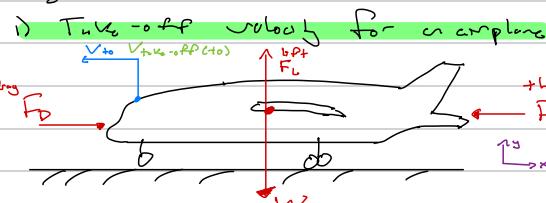
i) Shape

ii) Reynolds number → Check restrictions on tables

For a free wake $\Delta L \uparrow \Rightarrow C_D \downarrow$. Laws turn \rightarrow flow decelerates

Laws reduce drag at lower wind speeds

Aerodynamic Force Balances



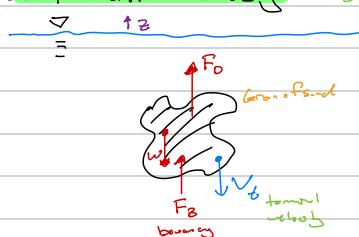
At force equilibrium

$$\sum F_x = 0 \Rightarrow F_t = F_D = C_D \frac{1}{2} \rho V_{turb}^2 A$$

$$\sum F_y = 0 \Rightarrow W = F_L = C_L \frac{1}{2} \rho V_{turb}^2 A$$

$$V_{turb} = \sqrt{\frac{2W}{\rho C_L A}}$$

2) Terminal Velocity - sedimentation, suspensions



$$\begin{cases} F_D = C_D \frac{1}{2} \rho_f V_tilde_turb^2 A \\ W = mg \quad \text{and} \quad \rho_f V_tilde_turb^2 A = \rho_s V_g \\ F_B = W_{displaced} = m_f g = \rho_f V_g \end{cases}$$

$$\begin{aligned} \text{Sum forces: } & \sum F_z = 0 \\ C_D \frac{1}{2} \rho_f V_tilde_turb^2 A + \rho_f V_g - \rho_s V_g &= 0 \\ V_tilde_turb &= \sqrt{\frac{2(V_g - \rho_s)}{\rho_f C_D A}} \end{aligned}$$

For parachute: $C_D = 1.3$

For spherical grains: $C_D = \frac{24}{Re} (Re < 1)$

Drag Force

Two main types:

i) Skin friction drag

→ Due to shear stress

→ Dominant mechanism for streamlined objects

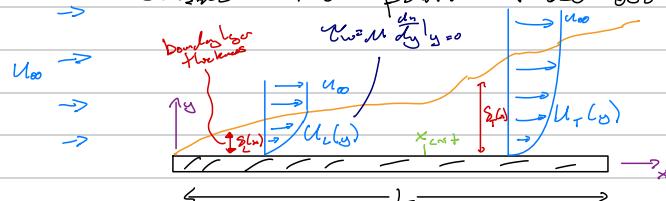
ii) Form drag (pressure drag)

→ Due to flow blocking/separation

→ Dominant mechanism for bluff bodies

Skin Friction Drag

Consider the parallel flow over a flat plate



Overall Reynolds number

$$Re_x = \frac{\rho U_{infty} L}{\mu}$$

Local Reynolds number

$$Re_x = \frac{\rho U_{infty} x}{\mu}$$

$$Re_{crit} = S \times 10^5$$

\hookrightarrow transition for x_{crit}

Regions: $Re < Re_{crit} \Rightarrow$ Laminar boundary layer
 $Re > Re_{crit} \Rightarrow$ Turbulent boundary layer

$$F_D = F_F = C_D \frac{1}{2} \rho U_{infty}^2 A$$

{Skin friction coef.

• Friction Coefficient β

→ Laminar BL ($Re_x < S \times 10^5$)

$$\frac{\delta(x)}{x} = \frac{4.91}{Re_x^{1/2}}$$

Boundary Layer

• Friction Coefficient β

→ Turbulent BL

• Friction Coefficient β

• Friction Coefficient β

$$\frac{\delta(x)}{x} = \frac{0.38}{Re_x^{1/8}}$$

$$C_{F,x} = \frac{0.089}{Re_x^{1/8}}$$

• Integrate to get a total front. coefficient: $C_F = \frac{1}{2} \int_0^L C_{F,x} dx$

• Flow over a flat plate can be:

(i) Entry Laminar

$$C_{F,laminar} = \frac{1.33}{Re_x^{1/2}}$$

(ii) Primary Turbulent

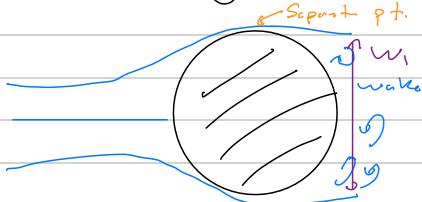
$$C_{F,turbulent} = \frac{0.074}{Re_x^{1/8}}$$

(iii) Combination of Laminar & Turbulent BL

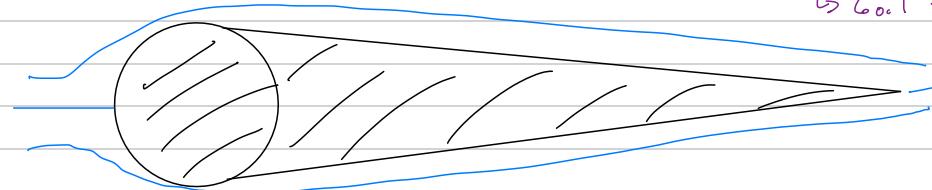
$$C_F = \frac{1}{2} \left(\int_0^{x_{crit}} C_{F,x,laminar} dx + \int_{x_{crit}}^L C_{F,x,turbulent} dx \right)$$

$$\Rightarrow C_F = \frac{0.074}{Re_x^{1/8}} - \frac{1.742}{Re_x}$$

Streamlining - Design to reduce drag



- ↑ Form / pressure drag



- ↓ Form Drag but ↑ Friction drag
- Goal: s is not drag ↓

no
Separate

- Form drag ↓
- Friction drag ↑
- Overall Drag ↓

Lift Force



$$\text{Result: } F_L = C_L \frac{1}{2} \rho U_\infty^2 A_p \quad A_p = b c \text{ Frontage}$$

- Lift coeff (C_L) depends on

(i.) Shape

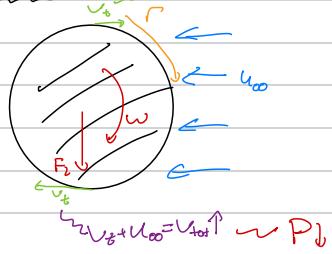
(ii.) Angle of attack (α)

↳ Angle between chord and horizontal



$$V_t - U_\infty = V_{tot} \downarrow \sim P_1$$

B.II

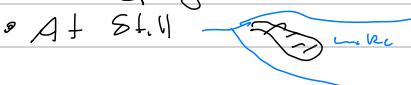


$$V_t + U_\infty = V_{tot} \uparrow \sim P_D$$

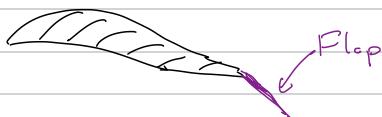
Stall

- Lift Coeff increases with angle of attack
- ↳ up to stall point

• Stall typically occurs around 18°

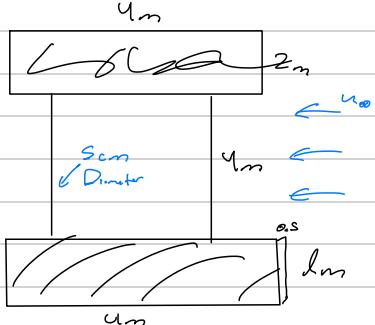


Effect of Flaps



- Flaps increase lift coeff at the expense of drag

Example IS-88



$$M_\infty = 150 \frac{m}{s} = 41.6 \text{ m/s}$$

Air $\rho = 1.2 \text{ kg/m}^3$

$$M = 1.73 \times 10^{-8} \frac{m}{s}$$

End: (i) Drift tension

(ii) Drift posts

$$Re = \frac{\rho u_\infty D}{\mu} = 1.2 \times 10^7$$

(i) Drag on ship

$$F_D = C_D \frac{1}{2} \rho U_\infty^2 A$$

$$C_D \Rightarrow 1.5-4 \text{ Tables} \xrightarrow{\text{Revised}} C_D = 1.10 + 0.02 \left(\frac{2}{D} + \frac{D}{L} \right) = 1.15$$

$$F_{D,\text{ship}} = 10,382 \text{ N}$$

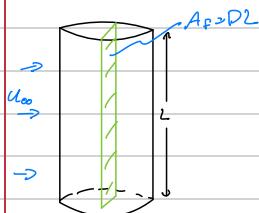
(ii) Drag on Roots/Posts

a) Look at $\frac{L}{D}$

b) Check Tables In case out of the

c) Solo

$$F_{\text{pole}} = C_{D,\text{pole}} \frac{1}{2} \rho U_\infty^2 A_p$$



Check Reynolds Number:

$$Re_p = \frac{\rho U_\infty D}{\mu} = (1.73 \times 10^{-8}) D \Rightarrow L \text{ mm} \text{ but } > 10^4$$

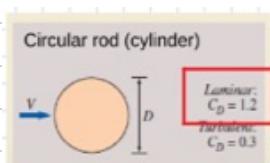
Two possibilities for C_D

a) 2-D approx

$$C_D \approx 1.2$$

b) Finite Cylinder

$$\frac{L}{D} = \frac{u_m}{U_\infty} = 80 \Rightarrow C_{D,\text{pole}} \approx 1.2$$



Finite cylinder, vertical, $A = LD$	
L/D	C_D
1	0.6
2	0.7
5	0.8
10	0.9
40	1.0
∞	1.2

Values are for laminar flow ($Re \leq 2 \times 10^5$)

$$\Rightarrow F_{\text{pole}} = 271 \text{ N}$$

$$2 \text{ supports} \rightarrow \text{multiplied} \quad F_{\text{pole}} \times 2 \Rightarrow F_{\text{pole}} = 2 \times 271 = 542 \text{ N}$$