Modes of Energy Storage @ Microscale

Structure @ Microscale defines macrostate of energy

Sensible Modes: Related to activity & motion & closely to temp.
- (i) Translational Motion (KE)
- (ii) Vibrational Motion
- (iii) Rotational Motion
- (iv) Electron Translation

Latent modes / Structural changes:
- (v) Phase Changes
- (vi) Chemical Bonds
- (vii) Nuclear Bonds

Energy can cross boundary as: (i) Heat (ii) Work (iii) Mass

Mass Flow Rate: $\dot{m} = \rho \mathcal{V}_{Avg} A_c$

Modes of Heat Transfer

(i) **Conduction:** Energy Transfers from higher to lower energetic matter due to direct interaction

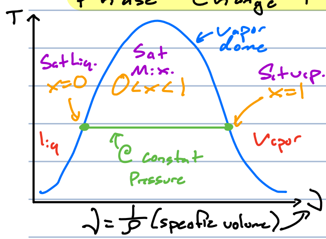
(ii) **Convection:** Enhanced conduction due to fluid motion

(iii) **Radiation:** Energy transfer via electromagnetic waves

First Law on a Total Basis

$$(Q_{in} - Q_{out}) + (W_{in} - W_{out}) + (E_{mass_{in}} - E_{mass_{out}}) = (U_2 - U_1) + \Delta KE + \Delta PE$$

Phase Change Fluids (Tables)



Quality

$$x = \frac{m_{vapor}}{m_{total}} = \frac{m_{vapor}}{m_{vapor} + m_{liquid}}$$

Phase Regions

(i) Compressed Liquid
- → @ given P: $T < T_{sat}$
- → @ given T: $P > P_{sat}$

(ii) Saturated Liquid ($x=0$)
- → $T = T_{sat}$ & $P = P_{sat}$

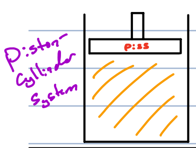
(iii) Saturated mixture ($0<x<1$)
- → $T = T_{sat}$ & $P = P_{sat}$

(iv) Saturated Vapor ($x=1$)
- → $T = T_{sat}$ & $P = P_{sat}$

(v) Superheated Vapor → $T > T_{sat}$ & $P < P_{sat}$

No quality in compressed liquid or SH regions

Closed Systems



1^{st} Law: $\underbrace{(Q_{in} - Q_{out}) + (W_{in} - W_{out})}_{\text{Can be on rate basis too}} = U_2 - U_1$ {J or kJ}

$= \underbrace{m}_{\text{System mass}} \underbrace{(u_2 - u_1)}_{\text{Mass specific internal energy}}$ {kJ/kg}

Q_{in}: Heat Transfer into system W_{in}: Work done on the system

Q_{out}: Heat Transfer out of system W_{out}: Work done by the system

Boundary Work (generally)

$$\mathcal{W}_{piston} = \int_1^2 P\,dV = P\Delta V = P_m \Delta v = \mathcal{W}_{out}$$

True for isobaric situations

$\mathcal{W}_{piston} = \mathcal{W}_{out} \rightarrow \mathcal{W}_{out} > 0$ if volume ↑

Polytropic Processes

$$PV^n = \text{Constant} \Rightarrow P_1 V_1^n = P_2 V_2^n \;\&\; \mathcal{W}_{piston} = \frac{P_2 V_2 - P_1 V_1}{1-n}$$

(n ← polytropic constant)

Example Constants

(iii) & (iii) must be true for ideal gas

- (i) $n=0$; constant P isobaric
- (ii) $n=1$; constant T isothermal
- (iii) $\gamma = \frac{c_p}{c_v}$; constant S isentropic
- (iv) $n \rightarrow \infty$; constant V isochoric

Ideal Gas EoS

Law: $PV = n\bar{R}T$ ← Universal Gas Constant

EoS: $PV = mRT$ ← Particular Gas Constant

$Pv = RT$

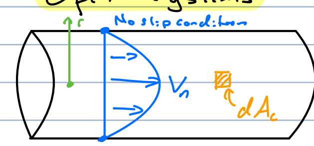
$P = \rho RT$ $R = \frac{\bar{R}}{MW}$

Specific Volume v

$$v = \frac{1}{\rho} = \frac{V}{m} \Rightarrow m = \frac{V}{v}$$

$$\dot{m} = \frac{\dot{V}}{v} \;\&\; V = mv$$

Open Systems



- 0 velocity @ the wall
- Increased velocity @ center

Focus on Steady Flow Devices

- mass (i) $\dot{m}_{in} = \dot{m}_{out}$
- energy (ii) $\dot{E}_{in} = \dot{E}_{out}$

zero accumulation

Expressions

$\dot{m} = \rho \mathcal{V}_{Avg} A_c$ (avg velocity)

locally $\dot{m} = \int_{A_c} \rho V_n \, dA_c$ (velocity profile)

$V_{Avg} = \frac{1}{A_c}\int V_n \, dA_c$

volume flow rate $\dot{V} = \frac{\dot{m}}{\rho} = V_{Avg} A_c$

$\dot{E}_{mass}$ elements

- (i) Kinetic Energy ($\frac{1}{2}V_{Avg}^2$)
- (ii) Internal energy (u)
- (iii) Potential Energy (gz)
- (iv) Flow Work Energy (Pv)

1^{st} Law: $(\dot{Q}_{in} - \dot{Q}_{out}) + (\dot{W}_{in} - \dot{W}_{out}) + (\dot{E}_{mass_{in}} - \dot{E}_{mass_{out}}) = 0$

For steady open systems

$$\Rightarrow (\dot{Q}_{in} - \dot{Q}_{out}) + (\dot{W}_{in} - \dot{W}_{out}) = \dot{m}\left[(h_2 - h_1) + \tfrac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1)\right]$$

Finding Δu

Treat air as ideal in this course

1) Ideal Gases

- Internal energy only a function of temperature
- Use specific heat value

c_v: Specific Heat @ constant Vol.

$$c_v = \frac{du}{dT} \Rightarrow \Delta u = c_{v_{Avg}}(T_2 - T_1)$$ (Typically constant)

c_p: Specific heat @ constant Press.

↳ See 'Open Systems'

2) True Liquids / Solids

- $c_v = c_p = c$

$\Delta u = u_2 - u_1 = \int_1^2 c\,dT = c_{Avg}(T_2 - T_1)$

$\underbrace{\Delta u = c\Delta T}_{\text{mass specific}}$ & $\underbrace{\Delta U = mc\Delta T}_{\text{total basis}}$

3) Phase Change Fluids

(i) For a saturated mixture

$$u = x u_g + (1-x) u_f$$

Let Φ be the property, so generally:

Both Equally valid for all table properties; called interpolation

$$\Phi = \Phi_f + x(\Phi_g - \Phi_f)$$
$$\Phi = x\Phi_g + (1-x)\Phi_f$$

(ii) For compressed liquids
- u & v much stronger functions of temp than Pressure

Shortcut: Evaluate compressed liquid (CL) properties based on saturated liquid value. eg. $u_{CL} = u_f(T)$

↳ Note that $u_{CL} \neq u_g(T)$

Calculating Δh

$h \equiv u + Pv \rightarrow E_{mass} = m(h + \frac{1}{2}V^2 + gz)$

1) Ideal Gases

$c_p = \left.\frac{dh}{dT}\right|_P$ ← @ constant pressure

$\Delta h = \int_1^2 c_p\,dT = c_p(T_2 - T_1)$

2) True Liquids/Solids

$c_p = c_v = c$

$\Delta h = c_{Avg}\Delta T = c_{Avg}(T_2 - T_1)$

3) Phase Change Fluids

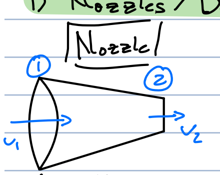
Go to tables ⇓

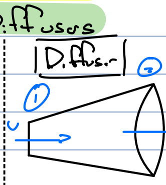
$h = h_f + x(h_g - h_f)$

$h = xh_g + (1-x)h_f$

Steady Flow Devices

1) Nozzles / Diffusers





V↑ as A↓, $P_2 < P_1$ | V↓ as A↑, $P_2 > P_1$

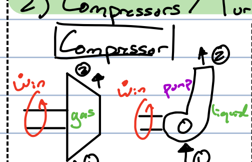
Exchanges enthalpy for KE

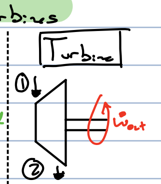
$$0 = \dot{m}\left[(h_2 - h_1) + \tfrac{1}{2}(V_2^2 - V_1^2)\right]$$ Relates Δh & KE

Assume: $\Delta PE \approx 0$, adiabatic, $W \approx 0$ ← Passive system

2) Compressors / Turbines

Fans also compressors, minimal P↑





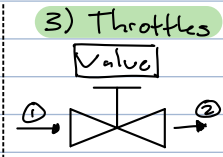
P↑, T↑ / P↑, $_{min}$ T↑ | Extracts fluid power

Relates Δh and Work

$$\dot{W}_{in} - \dot{W}_{out} = \dot{m}(h_2 - h_1)$$

Assume: $\Delta PE \approx$, $\dot{Q}$ small, $\Delta KE \approx 0$ by design

3) Throttles



$P_1 > P_2$

$\Delta h = 0$

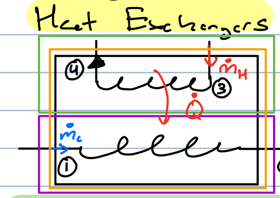
$h_1 = h_2$

Assume: $\Delta PE, \Delta KE \approx 0$

$\dot{Q}_{net} \approx 0$, $\dot{W}_{net} \approx 0$

Heat Exchangers

Transfer heat from hotter → colder fluid



Assume:
- $\mathcal{W}_{net} = 0$
- $\Delta PE \approx 0$
- $\Delta KE \approx 0$

Energy Analysis based on boundary color

- ✳ $\dot{Q}_{out} = \dot{m}_H(h_4 - h_3)$
- ✳ $\dot{Q}_{in} = \dot{m}_c(h_2 - h_1)$
- ✳ $\dot{Q}_{Net} = \dot{m}_H(h_4 - h_3) + \dot{m}_c(h_2 - h_1)$

$Q_{in} = Q_{out}$ ⟹ $\dot{m}_c(h_2 - h_1) = \dot{m}_H(h_3 - h_4)$

Second Law of Thermodynamics

Quantified by (i) Heat Engine (ii) Entropy

- (i) Processes proceed spontaneously in a particular direction
- (ii) Energy has quality, not just quantity
- (iii) Sets limits on theoretical Performance

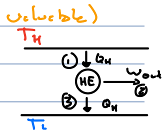
Heat Engines

Converts heat (less valuable) into work (more valuable)

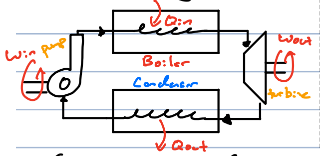
4 Key Elements

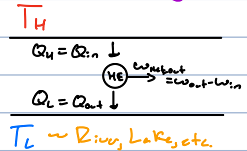
Examples: Steam Power Plant, Car Engine

- (i) Receive Heat from high Temp res. T_H
- (ii) Convert a portion of the heat into work
- (iii) Rejects heat to low temp res. T_L
- (iv) Operates as a cycle



Rankine Cycle
Power plant as a heat engine





$$(\underset{\text{boiler}}{Q_{in}} - \underset{\text{condenser}}{Q_{out}}) + (\underset{\text{pump}}{W_{in}} - \underset{\text{turbine}}{W_{out}}) = m\cancel{(u_2 - u_1)}$$ $\xrightarrow{}$ 0 cycle

Thermal Efficiency
$$\eta_{th} = \frac{\text{what we want}}{\text{what we pay}} = \frac{W_{net,out}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{Q_L}{Q_H}$$

Carnot Principles
Carnot cycles are ideal cycles/conditions

(i) Efficiencies of an irreversible (real) cycle is always less than a reversible cycle

(ii) Efficiency of all reversible engines acting between T_H and T_L is the same

$$\Rightarrow \left(\frac{Q_H}{Q_L}\right)_{rev} = \left(\frac{T_H}{T_L}\right)_{absolute} \longrightarrow \eta_{th,carnot} = 1 - \frac{T_L}{T_H} \leftarrow \text{must be absolute}$$ from general $\eta_{th} = 1 - \frac{Q_L}{Q_H}$

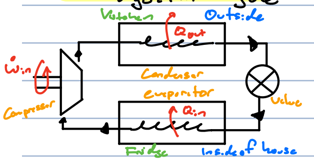
Carnot Heat Engine
(i) Isothermal expansion @ T_H (iii) Isothermal compression @ T_L

(ii) Adiabatic expansion: $T_H \to T_L$ (iv) Adiabatic compression

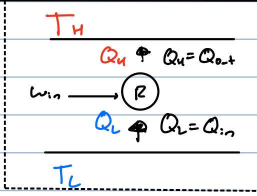
$$CoP_{R,carnot} = \frac{1}{\frac{T_H}{T_L} - 1}$$

$$CoP_{HP,carnot} = \frac{1}{1 - \frac{T_L}{T_H}}$$

Refrigeration cycle
valid for: fridges, ACs

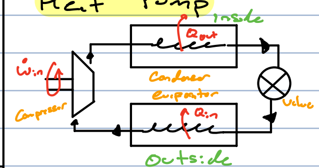


1st Law
$W_{in} + Q_L = Q_H$
$\rightarrow W_{in} = Q_H - Q_L$
2nd Law: $Q_H \neq Q_L$



$$CoP_R = \frac{Q_{in}}{W_{in}} = \frac{Q_L}{W_{in}} = \frac{Q_L}{Q_H - Q_L} = \frac{1}{\frac{Q_H}{Q_L} - 1}, \quad CoP_R = [2, 6] \text{ usually}$$

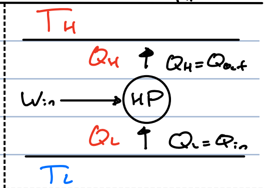
Heat Pump



Two Types

(i) Air Source
↳ T_L is the air

(ii) Ground Source
↳ T_L is the ground



$$CoP_{HP} = \frac{Q_{out}}{W_{in}} = \frac{Q_H}{W_{in}} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - \frac{Q_L}{Q_H}}, \quad CoP_{HP} = [1, 2] \text{ usually}$$

Entropy
(i) Entropy change as a measure of energy quality/degradation

(ii) Entropy as a measure of molecular system disorder ← increases as heat is added

For Cycles/Processes
irreversibility ⇒ entropy generation

- Ideal cycles/processes → reversible
- Real cycles/processes → irreversible

Assume surr. are isolated... No heat/work out of surr

For an Internally Reversible Process
$$dS = \left(\frac{dQ}{T}\right)_{ideal} \leftarrow \text{absolute!} \quad \Big| \quad ds = \frac{dS}{m} \text{ mass specific}$$

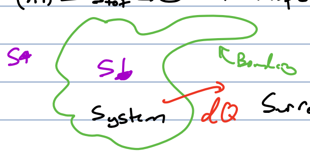
Total Entropy Change
always nonnegative

$$\Delta s_{total} = \Delta s_{sys} + \Delta s_{surr}$$

(i) $\Delta s_{tot} > 0 \Rightarrow$ real (irreversible)

(ii) $\Delta s_{tot} = 0 \Rightarrow$ ideal (reversible)

(iii) $\Delta s_{tot} < 0 \Rightarrow$ impossible!



For a Real Process
$$ds > \frac{dQ}{T} \overset{\text{Integrate}}{\Longrightarrow} \Delta s = \int_1^2 \frac{dQ}{T}$$ Need Q and T relationship

Δs can be +/- depending on the sign of dQ

$$\Delta s_{sys} = \int_1^2 \left(\frac{dQ}{T}\right)_{ideal} + S_{generated} \leftarrow \text{generated through irreversibilities}$$

$$\Delta s_{sys} = \int_1^2 \left(\frac{dQ}{T}\right)_{sys,ideal} + \int_1^2 \left(\frac{dQ}{T}\right)_{surr,ideal} + S_{generated}$$

In the Rankine Cycle, Boiler and Compressor are both isobaric (operate @ Constant Pressure)

Calculating Entropy Change
(i) Isothermal Process
valid for surroundings

Assume fixed T_0

$$\Delta s_{surr} = \int_1^2 \frac{dQ}{T} = \int_1^2 \frac{dQ}{T_0} = \frac{Q_2}{T_0} \leftarrow \text{Transfer from } 1 \to 2$$

(ii) More Generally
$$\underset{(Q_{in}-Q_{out})}{dQ} - \underset{(W_{in}-W_{out})}{dW} = \underset{\text{Change in energy}}{dU}$$ divide by mass $\Longrightarrow T ds = du + P dv \overset{h = u + Pv}{\Longrightarrow} T ds = dh - v dP$

$dS = \frac{dQ}{T}$; boundary work

$$T dS - P dV = dU$$

Gibbs Equation

Evaluating Δs
1) Ideal Gases
$$\Delta s_{sys} = c_{v,Avg} \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right)$$

enthalpy form

$$\Delta s_{sys} = c_{p,Avg} \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$$

2) True Liquids/Solids
Incompressible: $dv \approx 0$, $dP \approx 0$

$c_p = c_v = c$

$$\Delta s_{sys} = c \ln\left(\frac{T_2}{T_1}\right)$$

3) Phase Change Fluids
$$\Delta s_{sys} = s_2 - s_1$$

Use tables for s_2, s_1
↳ interpolate w/ quality

Isentropic Processes
$\Delta s_{sys} = 0 \longrightarrow s_2 = s_1$

1) Ideal Gases
$$\frac{T_{2s}}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1} \qquad R = c_p - c_v \qquad k = \frac{c_p}{c_v}$$

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

$$\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^{k}$$

All 3 valid only for an ideal gas in an isentropic process

2) True Liquids/Solids
$$\Delta s_{sys} = c \ln\left(\frac{T_2}{T_1}\right) = 0$$

$$c \ln\left(\frac{T_2}{T_1}\right) = 0 \quad T_2 = T_1 \text{ to be true}$$

Solids & incompressible liquids require isentropic process to be isothermal

3) Phase Change Fluids
$$s_2 = s_1$$

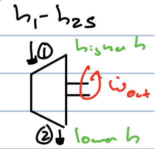
Use tables and interpolate

Beware of false assumptions in questions!

Isentropic Efficiency
(i) Turbines
isentropic turbine produces most power

$$\eta_t = \frac{\text{actual}}{\text{isentropic}} = \frac{h_1 - h_{2A}}{h_1 - h_{2s}}$$

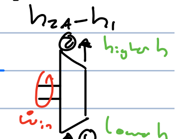
Diff Enthalpies
h_{2s} = isentropic
h_{2A} = actual



(ii) Compressors
isentropic compressor requires least power

$$\eta_c = \frac{\text{isentropic}}{\text{actual}} = \frac{h_{2s} - h_1}{h_{2A} - h_1}$$

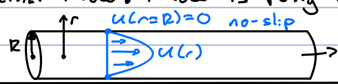
Diff Enthalpies
h_{2s} = isentropic
h_{2A} = actual



Watch for h_{2A} vs. h_{2s} & ηs

Nomenclature

- No-slip condition: Fluid adjacent to a solid surface takes the velocity of that surface
- External Flow: Surface in an infinite flow fluid
- Internal Flow: Flow is fully bounded by a solid surface

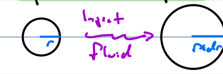


- Boundary layer (BL): Thickness over which viscous effects are felt

Surface Tension

- Comparative attraction between similar and dissimilar molecules
- Some Effects of Surface Tension
  - (i) Spherical Droplets (v) Capillary Rise
  - (ii) Meniscus
  - (iii) Free Surface
  - (iv) Wetting Behavior

Spherical Droplet



$\left[\sigma = \frac{\text{energy}}{\text{area}} = \frac{\text{force}}{\text{length}}\right]$ $\left\{\frac{N}{m}\right\}$ Energy required to ↑ surface area

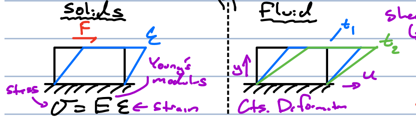
$\left\{ dE = \sigma\, dA_s;\ dA_s = 8\pi r\, dr \atop dW = (P_i - P_o)4\pi r^2 dr \right\} (P_i - P_o) = \frac{2\sigma}{r}$

Viscosity

Resistance to flow, molecular friction

→ Consider an applied force on a solid vs. fluid

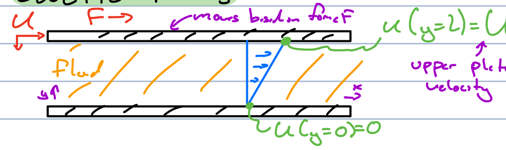
Solids | Fluid



Shear stress $\tau \propto \frac{du}{dy}$ — directly proportional for newtonian fluids (Ex: water, air, oil, etc.)

- Dynamic Viscosity $\equiv \mu = \nu\rho$ (kinematic viscosity)(density)
- Non-newtonian fluid viscosity changes based on force

Couette Flow



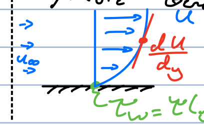
- Shear stress on the fluid: $\tau = F/A$
  - ↳ For general fluids: $F = \tau \cdot A$

Finding the Force

① $F = \tau A = \mu \frac{du}{dy} A$ all

② $F = \tau A = \mu \frac{U}{L} A$ Couette

More Generally



Hydro Statics

Impact of pressure on engineered surfaces

Applications: Dams, Tanks

Pressure [force/area]

Pressure: (i) Acts isotropically (Same in all directions)

(ii) Leads to a pressure force: $F_{pressure} = \int P\, dA$

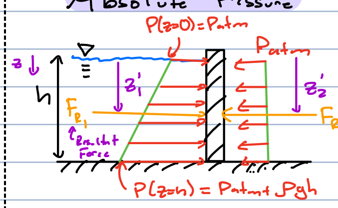
(iii) Pressure force on a submerged object (like a dam) always acts normal to the surface and inward

(iv) Varies with depth according to hydrostatic pressure variation

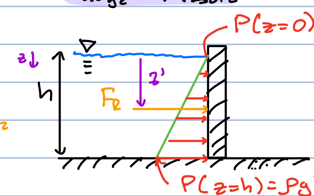
$$\frac{dP}{dz} = \rho g \Rightarrow P(z) = \int \rho g\, dz = P(z=0) + \rho g z$$

Gauge Pressure: $P_{gauge} = P_{absolute} - P_{atm}$

Absolute Pressure



Gauge Pressure



- Two methods: Decomposition & Integration

Resultant Force

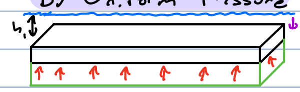
$F_R = \int_A P(z)\, dA = \int_0^h P(z)\, w\, dz$

a) Linear Pressure Distribution

$F_R = w\int_0^h \rho g z\, dz$

$= (\frac{1}{2}\rho g h)(h w)$ — gauge pressure, area

b) Uniform Pressure

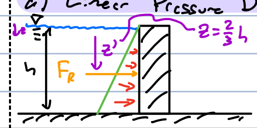


$F_R = \rho g h_1 A$

Line of Action

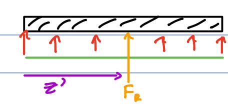
Location needs to lead to the same moment as the force distribution

a) Linear Pressure Distribution



$z' = \frac{2}{3}h$

b) Uniform Pressure



z is halfway across

Fluid Dynamics

- Stream Lines: Line formed tangent to the velocity at every point
- For steady flows, all fluid particles passing through a center point follow the same stream lines

Bernoulli Equation

Standard Form

- Provides a relationship between pressure, elevation, and velocity

Applications: Nozzles, Dams, etc.

Assumptions: (i) Steady Flow - no transient effects

(ii) Inviscid - viscous effects negligible

(iii) Incompressible (ρ constant)

(iv) Flow along a stream line

(v) No heat transfer

(vi) No work interactions may be considered for this form

Relationship: $\frac{P}{\rho} + \frac{1}{2}V^2 + gz = \text{constant}$ → Called the conventional form

$\Rightarrow P_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2$

Energy form of the Bernoulli Equation

$$\frac{P_1}{\rho} + \frac{1}{2}\alpha_1 V_1^2 + gz_1 + w_{pump} = \frac{P_2}{\rho_2} + \frac{1}{2}\alpha_2 V_2^2 + g z_2 + w_{turbine} + e_{losses}$$

↳ Kinetic energy correction factor (α): laminar $\alpha = 2$ and turbulent $\alpha \approx 1.05 \approx 1$

$\dot{E}_{losses} = \dot{E}_{losses,pump} + \dot{E}_{losses,turbine} + \dot{E}_{losses,piping}$

mass based "power" ↳ $\left\{ \dot{w}_{pump,u} = \dot{w}_{pump} - \dot{E}_{losses,pump} \atop \dot{w}_{turbine,e} = \dot{w}_{turbine} + \dot{E}_{losses,turbine} \right.$ (useful; extractable; actual) $\Rightarrow \eta_{pump} = \frac{\dot{w}_{pump,u}}{\dot{w}_{pump}}$, $\eta_{turbine} = \frac{\dot{w}_{turbine,e}}{\dot{w}_{turbine,e}}$

Also, $\dot{w}_{pump} = \eta_{motor} \cdot \dot{w}_{elec}$

Bernoulli Equation in Terms of Head (h)

divide by $\dot{m}g$

$h_{pump,u} = \frac{\dot{w}_{pump,u}}{\dot{m}g}$, $h_{turbine,e} = \frac{\dot{w}_{turbine}}{\dot{m}g}$, $h_L = \frac{\dot{E}_{losses,piping}}{\dot{m}g}$

$$\Rightarrow \frac{P_1}{\rho_1 g} + \frac{\alpha_1 V_1^2}{2g} + z_1 + h_{pump,u} = \frac{P_2}{\rho_2 g} + \frac{\alpha V_2^2}{2g} + z_2 + h_{turbine,e} + h_L \leftarrow \text{piping}$$

h_L is the irreversible head loss between ① and ② due to fluid friction → The diff. states

Internal Flow

Laminar Flow

- Smooth, orderly flow
- Low velocity, high viscosity

Turbulent Flow

- High mixing, chaotic flow
- High velocity, low viscosity

⇒ Flow Regime based on the Reynolds number

$Re = \frac{\rho V_{avg} D}{\mu}$ ← Characteristic length; ← dynamic viscosity

- Ratio of inertial to viscous effects
- Re_{crit} is transition from laminar to turbulent flow

Internal Flow: $Re_{crit} = 2300$

Non-circular Cross Sections

- Use hydraulic diameter D

$D_h = \frac{4A_c}{P}$ ← cross-sec area; ← wetted perimeter

Aqueducts, rivers, or sewage pipes

Developed Flow

- Entrance region is before flow is fully developed, typically $L/D \approx 10$
- BL present in entrance region

$u(r) = \boxed{-\frac{R^2}{4\mu}\cdot\frac{dP}{dx}}\left(1 - \frac{r^2}{R^2}\right)$ maximum velocity

$\frac{dP}{dx} < 0$ must be true to push fluid in +direction (+u flow)

Let $\frac{dP}{dx} \equiv \Delta P_L = P_1 - P_2$

- Average velocity: $u_{avg} = -\frac{R^2}{8\mu}\cdot\frac{dP}{dx}$ For laminar flow $u_{avg} = \frac{1}{2}u_{max}$

$\Rightarrow \Delta P_L = \frac{8\mu u_{avg} L}{R^2} = \frac{32\mu u_{avg} L}{D^2}$ (Laminar)

- Pumping power to overcome frictional losses: $\dot{w}_{pump,L} = \dot{V}\Delta P_L$

Generalize with Darcy Friction Factor (f)

$\Delta P_L = f\frac{L}{D}\frac{1}{2}\rho u_{avg}^2$; $h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{u_{avg}^2}{2g}$

(i) Laminar Flow

$\Delta P_L = \frac{L}{D}\cdot\frac{1}{2}\rho u_{avg}^2\left(64\frac{\mu}{\rho u_{avg} D}\right) \Rightarrow f = \frac{64}{Re_D}$

(ii) Turbulent flow Use moody tables or...

① $\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$ Colebrook $\frac{\varepsilon}{D} = \frac{\text{roughness}}{\text{diameter}}$

② $\frac{1}{\sqrt{f}} \approx -1.8\log_{10}\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right]$ Haaland (2% error)

Minor Losses

(i) Elbows, Tees (ii) Piping Transitions

(iii) Valves (iv) Entrance/Exit

$\Rightarrow h_L = K_L\cdot\frac{V_{avg}^2}{2g}$ K_L from tables

Combining Losses

(i) Series a) $\dot{V}$ const (ii) Parallel a) $\dot{V} = \dot{V}_A + \dot{V}_B$

b) $h_L = \sum h_{L,major} + \sum h_{L,minor}$ b) $\Delta P_A = \Delta P_B \Rightarrow h_{L,A} = h_{L,B}$

External Flow

- Flow over bodies => aerodynamic forces (lift, drag)
- Two primary mechanisms:

(i) Friction: $\tau = \mu \frac{du}{dy}$

↳ Shear stress over surface area => frictional force

(ii) Pressure Differences: Reduced pressure in wake

↳ pressure differences over surface => pressure force

- External Flow: $Re_{crit} = 5 \times 10^5$ less than this is laminar

Note: For a tree, velocity ↑ => C_D↓ as leaves move in direction of wind and fold to reduce drag (perk of evolution)

Aerodynamic Force Balances

(i) Take-off (to) velocity for an airplane

$$V_{to} = \sqrt{\frac{2W}{\rho C_L A}}$$ where $W = mg$ and C_L is lift coefficient

(ii) Terminal (t) Velocity

sedimentation, suspensions

$$V_t = \sqrt{\frac{2Vg}{C_D \cdot A} \cdot \frac{\rho_s - \rho_f}{\rho_f}}$$ where s, f are solid and fluid respectively

Lift Force

- Lift coefficient depends on
  - (i) Shape
  - (ii) Angle of attack (α)
    - ↳ angle between chord & horizontal
- Stall: Lift Coefficient increases with angle of attack up to stall point
  - ↳ Typically Occurs around 15°
- Effect of flaps: Increase lift coefficient at the expense of drag

Drag and Lift

- Effects that influence aerodynamic forces

(i) Fluid Properties (ρ, μ)

(ii) Object Properties / Shape

a) Streamlined

b) Blunt or Bluffed

(iii) Flow Regime (Laminar vs. Turbulent)

- F_L: Lift force - Acts perpendicular to free stream
- F_D: Drag force - Acts along free stream

Definitions / Expressions

- $C_D = \frac{F_D}{\frac{1}{2}\rho u_\infty^2 A}$
  $\Rightarrow F_D = C_D \frac{1}{2}\rho u_\infty^2 A$
- $C_L = \frac{F_L}{\frac{1}{2}\rho u_\infty^2 A}$
  $\Rightarrow F_L = C_L \frac{1}{2}\rho u_\infty^2 A$

C_D and C_L can exceed 1

Appropriate Area for C_D, C_L

(i) Blunt Frontal Area

a) Cylinder $A_F = DL$

b) Sphere $A_F = \frac{\pi}{4} D^2$

(ii) Streamlined body

Planform Area (A_p)

$A_p = b \cdot c$ (b: span, c: chord length)

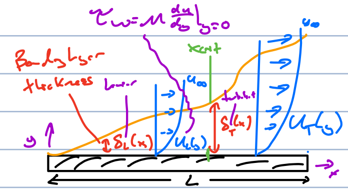
- Get C_D values from Tables based on (i) Shape and (ii) Reynolds number

Drag Force

(i) Skin friction drag

→ Due to shear stress

→ Dominant mechanism for streamlined objects



$$F_D = F_f = C_f \frac{1}{2}\rho u_\infty^2 A$$

↳ C_f: Skin friction coefficient; A: Area in contact with fluid

- Overall Reynolds #: $Re_L = \frac{\rho u_\infty L}{\mu}$
- Local Reynolds #: $Re_x = \frac{\rho u_\infty x}{\mu}$

- Critical Reynolds #: $Re_{crit} = 5 \times 10^5$

$$\begin{cases} Re_x < Re_{crit} \Rightarrow \text{Laminar} \\ Re_x > Re_{crit} \Rightarrow \text{Turbulent} \end{cases}$$ Describes boundary layer

- Flow over a flat plate can be... (To get C_f value)

(i) Entirely Laminar: $C_{f,laminar} = 1.33/\sqrt{Re_L}$

(ii) Primarily Turbulent: $C_{f,turbulent} = 0.074/Re_L^{1/5}$

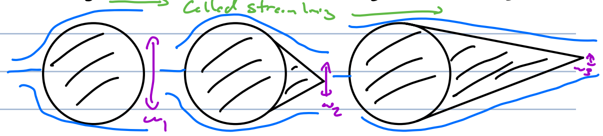
(iii) Combination of Laminar & Turbulent BL: $\Rightarrow C_f = 0.074/Re_L^{1/5} - 1742/Re_L$

(ii) Form Drag (Pressure Drag)

→ Due to flow blocking / Separation

→ Dominant mechanism for bluff bodies

Called streamlining



- Decreasing wake size decreases form drag at the expense of friction drag and goal is net reduced total drag

Basic: • ↑ form/pressure drag

slightly streamlined: • ↓ form, ↑ friction

no separation: • form drag ↓↓, friction drag ↑↑, overall ↓

Heat Transfer

- Energy Transfer through a temperature difference

(i) Conduction – Energy Transfer due to the direct interaction of particles

Simple Conduction: $\dot{Q}_{cond} = kA_c \frac{T_1 - T_2}{\Delta x}$

↳ From Fourier's Law: $\dot{Q}_{cond} = -kA_c \frac{dT}{dx}$

- Gasses << Solids due to crystallinity

(ii) Convection: Enhanced conduction through bulk fluid motion

↳ moving fluid → large temperature gradient

Described by: $\dot{Q}_{conv} = h \cdot A_s (T_s - T_\infty)$

↳ From Newton's Law of Cooling

Heat Transfer Coefficient: $h \uparrow$ as $u_\infty \uparrow$

(iii) Radiation: Energy transfer due to electromagnetic waves

- Does not require a medium and all matter at a non-zero temperature emit radiation.

Described by: $\dot{Q}_{emitted} = \varepsilon \sigma A_s T_s^4$ (emissivity; surface area; absolute surface temp)

↳ Stefan-Boltzmann Eq

where σ = Stefan-Boltzmann constant = $5.67 \times 10^{-8} \frac{W}{m^2K^4}$

Thermal Resistance Network

(i) Conduction

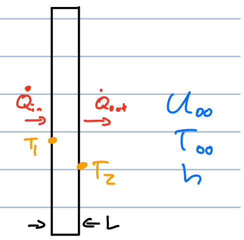
$\dot{Q}_{cond} = kA_c \frac{(T_1 - T_2)}{L} = kA_c \frac{\Delta T}{L}$

$\dot{Q}_{cond,wall} = \frac{\Delta T}{R_{cond}} \rightsquigarrow R_{cond} = \frac{L}{kA}$

(ii) Convection

$\dot{Q}_{conv} = hA_s(T_2 - T_\infty) = hA_s \Delta T$

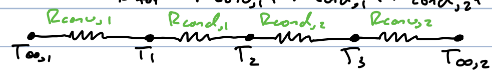
$\dot{Q}_{conv} = \frac{\Delta T}{R_{conv}} \rightsquigarrow R_{conv} = \frac{1}{hA_s}$



Series Resistances

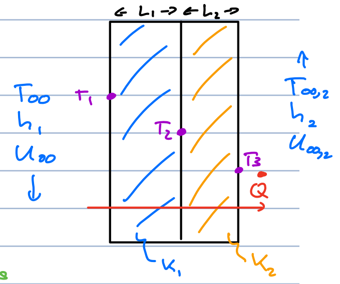
- $\dot{Q}$ is constant ⟹ $\Delta T = \dot{Q} R_{tot}$

↳ $R_{tot} = R_{conv,1} + R_{cond,1} + R_{cond,2} + R_{conv,2}$



$\dot{Q} = \frac{T_{first} - T_{last}}{R_{tot}} = \frac{T_{\infty 1} - T_1}{R_{conv,1}} = \frac{T_1 - T_3}{R_{cond,1} + R_{cond,3}} = \dots$

Resistances must correspond with temperatures

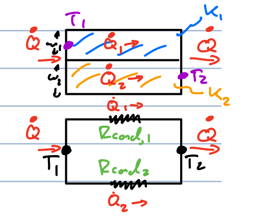


Resistances in Parallel

- ΔT is constant, $\dot{Q}$s added individually $\dot{Q}_{tot} = \Sigma \dot{Q}_i$

⟹ $\dot{Q} = \frac{\Delta T}{R}$

↳ $\frac{1}{R_{tot}} = \frac{1}{R_{cond,1}} + \frac{1}{R_{cond,2}} + \frac{1}{R_{cond,3}} + \dots$

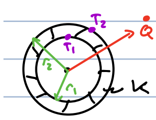


- For some problems like —ww—[ ]—ww—, collapse parallel resistances to get R_{middle}, add to the ones in series!!

Conduction in Radial Systems

(i) Cylindrical Coordinates

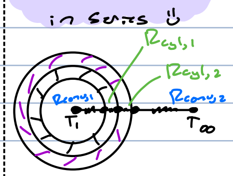
Still Fourier's Law:



$\dot{Q}_r = \frac{2\pi L k}{\ln\left(\frac{r_2}{r_1}\right)}(T_1 - T_2)$

$R_{cyl} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi L k}$

Radial can also be in series!!



(ii) Spherical Coordinates ($A_c = 4\pi r^2$) ← still Fourier's

$\dot{Q} = \frac{4\pi r_1 r_2 k}{r_2 - r_1}(T_1 - T_2)$ $R_{sph} = \frac{r_2 - r_1}{4\pi r_1 r_2 k}$

Finned Heat Transfer

↳ Still uses Fourier's Law to Represent $\dot{Q}_{conduction}$

$\dot{Q}_{cond} = -kA_c \frac{dT}{dx}$

⟹ $0 = \frac{d}{dx}\left(kA_c \frac{dT}{dx}\right) - hP[T(x) - T_\infty]$

Applications

(i) Electronics Cooling

(ii) Heat Exchangers

→ Car Radiator

→ HVAC Systems

(iii) Extreme ... (Tanks, etc.)

Two Different Boundary Condition Assumptions

(i) Assume infinitely long fin tip

a) $T(x \to \infty) \to T_\infty \Rightarrow \theta(x \to \infty) \to 0$

b) $T(x=0) \to T_b \Rightarrow \theta(x=0) \to \theta_b$

$\theta(x) = T(x) - T_\infty$

$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-mx}$ for $m = \sqrt{\frac{hP}{kA_c}}$

$T(x) = (e^{-mx})(T_b - T_\infty) + T_\infty$

$\dot{Q} = \sqrt{hPkA_c}\,(T_b - T_\infty)$

P is the Perimeter

(ii) Assume Adiabatic Fin Tip, Finite length

a) $\dot{Q}\big|_{x=L} = 0 = -kA_c \frac{dT}{dx}\big|_{x=L} \Rightarrow \frac{d\theta}{dx}\big|_{x=L} = 0$

b) $\theta(0) = \theta_b$

$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh[m(L-x)]}{\cosh(mL)}$ Hyperbolic cosine and tangent

$\dot{Q} = \dot{Q}\big|_{x=0} = \sqrt{hPkA_c}\,(T_b - T_\infty)\tanh(mL)$

$T(x) = \frac{\cosh[m(L-x)]}{\cosh(mL)}(T_b - T_\infty) + T_\infty$; $m = \sqrt{\frac{hP}{kA_c}}$

where...

$\cosh(x) = \frac{1 + e^{-2x}}{2e^{-x}}$ and $\tanh(x) = \frac{e^{2x} + 1}{e^{2x} - 1}$

Transient Heat Transfer

Lumped System Analysis: Assume temp varies only with time $T = T(t)$

$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$ where $b = \frac{hA_s}{\rho V c} = \frac{h}{\rho L_c \cdot c}$ & assumption holds

(c = specific heat; L_c = characteristic length)

$\dot{Q}(t) = hA_s[T_\infty - T(t)]$ decreases with time

$Q_{total} = mc[T(t_{final}) - T_i]$ total heat transfer from $t=0$ to $t=f$

$Q_{max} = mc[T_\infty - T_i]$ maximum possible energy change

- Assumption holds for Biot # ≤ 0.1 ⤳ completely uniform temperature and can use Lumped System Analysis (LSA)

Biot # = $Bi = \frac{\text{Convection @ surface}}{\text{Conduction within}} = \frac{hL_c}{k}$ where $L_c = \frac{V}{A_s}$ (volume / surface area)

↳ If only given thickness (t) of object, assume $t \ll$ width (w)

$= L_c = \frac{V}{A_s} = \frac{Lwt}{2Lw + 2Lt} = \frac{wt}{2(w+t)} \approx \frac{t}{2}$ can use in 'b' $b = \frac{h}{\rho \cdot L_c \cdot c}$

Convection and External Flow

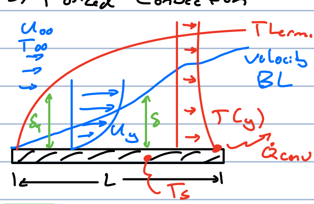
1) Natural / Free Convection

- Flow is induced by temp gradients

2) Forced Convection → Imposed Velocity

Recall $Re_L = \frac{\rho u_\infty L}{\mu}$

Characteristic diameter: $Re_{D_h} = \frac{\rho u_\infty D}{\mu}$, $D_h = \frac{4A_c}{P}$



Boundary Layer

- Define BL thickness

$u_{BL} = 0.99\,u_\infty = y = \delta(x)$

$T_{BL} = T_s + 0.99(T_\infty - T_s) = y = \delta_T(x)$

Flux: $q''_{conv} = h\Delta T = \frac{\dot{Q}_{conv}}{A_s} \Rightarrow \dot{Q}_{conv} = q''_{conv} \cdot A_s$

$q''_{cond} = k_{fluid} \frac{\Delta T}{L} = \frac{\dot{Q}_{cond}}{A_s} \Rightarrow \dot{Q}_{cond} = q''_{cond} \cdot A_s$

Prandtl Number

$Pr = \frac{\text{viscosity}}{\text{diffusivity}} = \frac{\nu}{\alpha}$ (dimless; kinematic viscosity; thermal diffusivity) Tables!

Typical Values

(i) $Pr < 1$ ⟹ liquid metals

(ii) $Pr \approx 1$ ⟹ gases (air)

(iii) $Pr > 1$ ⟹ oils, water

Nusselt Number

$Nu_L = \frac{Lh}{k_{fluid}}$ | $Nu_x = \frac{xh_x}{k_{fluid}}$

(i) Local

a) Laminar: $Nu_x = 0.332 \cdot Re_x^{1/2} \cdot Pr^{1/3}$

b) Turbulent: $Nu_x = 0.0296 \cdot Re_x^{4/5} \cdot Pr^{1/3}$

(ii) Average Note: If $Re > 5 \times 10^5$, use T&L unless stated to be fully turbulent!

a) Laminar: $Nu_L = 0.664 \cdot Re_L^{1/2} \cdot Pr^{1/3}$

b) Turbulent: $Nu_L = 0.037 \cdot Re_L^{4/5} \cdot Pr^{1/3}$

c) L & T: $Nu_L = (0.037 Re_L^{4/5} - 871) \cdot Pr^{1/3}$

Convection & Internal Flow

Energy Contained in the Fluid as:

entities:
- (i) Internal Energy
- (ii) Kinetic (KE) & Potential (PE) Energy
- (iii) Flow Work Energy

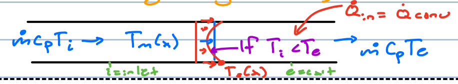
• For true liquids: $\Delta h = c_p \Delta T$

Energy Analysis

$\dot{Q}_{in} = h A_s [T_s(x) - T_m(x)]$ — $\dot{Q}_{in}$ due to convection

$q''_{in} = h_x [T_s(x) - T_m(x)]$ — $\dot{Q}_{in}$ on a flux basis (per unit area)

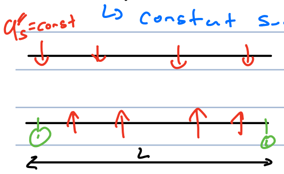
↳ generally constant, but can be local



Consider Two Possible Cases

(i) q''_s constant (along x) → const $\dot{Q}$, like solar radiation

↳ constant surface heat flux



$\dot{Q}_{in} = q''_s A_s = \dot{m} c_p (T_e - T_i)$

$T_e = T_i + \frac{q''_s A_s}{\dot{m} c_p} = T_i + \frac{q''_s P L}{\dot{m} c_p}$

$T_m(x) = \frac{q''_s P x}{\dot{m} c_p}$

What about T_s

Newton's Law of Cooling

$q''_s = h[T_s(x) - T_m(x)]$ ← constant if fully developed

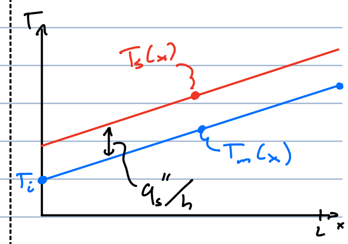
$\Rightarrow T_s(x) = T_m(x) + \frac{q''_s}{h}$

Determining h

- For Laminar Pipe
- Circular Pipe
- q''_s constant

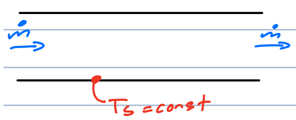
$Nu_D = \frac{hD}{k_{fluid}} = \frac{48}{11} \approx 4.36$

$\Rightarrow h = \frac{48}{11} \cdot \frac{k_{fluid}}{D}$



(ii) T_s constant (along x) → like ducts, submerged pipes

↳ constant surface temperature



$\dot{Q}_{conv} = h A_s [T_s - T_m(x)]$

$\dot{Q}_{in} = h A_s \Delta T_{lm} = \dot{m} c_p (T_e - T_i)$

↳ $T_s - T_m(x) = \Delta T_{avg} \approx \Delta T_{lm} = \frac{\Delta T_e - \Delta T_i}{\ln\left(\frac{\Delta T_e}{\Delta T_i}\right)}$

Use log mean diff bc T_m changes with position x.

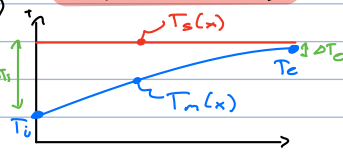
$T_e = T_s - (T_s - T_i) \cdot \left(\exp\left[-\frac{hPL}{\dot{m} c_p}\right]\right)$

$T_m(x) = T_s - (T_s - T_i) \cdot \left(\exp\left[-\frac{hPx}{\dot{m} c_p}\right]\right)$

h for constant T_s, laminar flow, circular pipe

$Nu_D = 3.66$

$\Rightarrow h = 3.66 \cdot \frac{k_{fluid}}{D}$



Laminar Flow for other cross-sections

$D_H = \frac{4A_c}{P}$

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TABLE 19–3

Nusselt number and friction factor for fully developed laminar flow in tubes of various cross sections ($D_h = 4A_c/p$, $Re = V_{avg} D_h/\nu$, and $Nu = hD_h/k$)

Tube Geometry	a/b or θ°	Nusselt Number T_s = Const.	Nusselt Number $\dot{q}_s$ = Const.
Circle	—	3.66	4.36
Rectangle	a/b		
	1	2.98	3.61
	2	3.39	4.12
	3	3.96	4.79
	4	4.44	5.33
	6	5.14	6.05
	8	5.60	6.49
	∞	7.54	8.24
Ellipse	a/b		
	1	3.66	4.36
	2	3.74	4.56
	4	3.79	4.88
	8	3.72	5.09
	16	3.65	5.18
Isosceles Triangle	θ		
	10°	1.61	2.45
	30°	2.26	2.91
	60°	2.47	3.11
	90°	2.34	2.98
	120°	2.00	2.68

Turbulent Flow ($Re_D > 2300$)

Dittus-Boelter Equation

$Nu_D = 0.023 \cdot Re_D^{4/5} \cdot Pr^n$

where: $n = 0.4$ if fluid is heated; $n = 0.3$ if fluid is cooled

Thermal Radiation

- Radiation Requires no medium
- All objects at non-zero (absolute) temperature emit radiation
- Amount of radiation proportional to temperature
- Thermal Radiation is a subset of the full EM spectrum (microns (μm): 0.1μm < λ < 100μm)
- Effects that affect radiative transfer
  - (i) Temperature
  - (ii) Geometry, direction
  - (iii) Wavelength
  - (iv) Material Properties

Blackbody Radiation

- Perfect emitter and absorber
  - a) Emits maximum radiation at a given T
  - b) Absorbs all incident radiation
  - c) Diffuse/uniform radiations

Examples: Sun @ 5800K, Snow & white paint in IR

Can be temperature or wavelength specific

Two Primary Types of Materials

(i) Opaque Materials: All radiation absorbed over first few microns of the surface ex. wood, bark, metals

(ii) Semi-transparent Materials: Transparency can be wavelength dependent

ex. glass & water transparent in visible spectrum but opaque in IR

↳ Green house effect: Visible light through glass, gets absorbed and IR released that cannot pass through the glass, temperature ↑

Blackbody emissive power

$E_b = \sigma T_s^4$ $\{\frac{W}{m^2}\}$ $\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$

← temp in Kelvin!

Stefan-Boltzmann constant

Wavelength Dependence (+ temperature)

- Different amounts of radiation given off at different wavelengths

$\Rightarrow$ Spectral blackbody emissive power

$E_{b,\lambda}(\lambda, T) = \frac{C_1}{\lambda[\exp(\frac{C_2}{\lambda T}) - 1]}$

where $C_1 = 3.74 \times 10^8 \frac{W \mu m^4}{m^2} = 2\pi \hbar c_0^2$

$C_2 = 1.44 \times 10^4 \mu m \cdot K = \frac{\hbar c_0}{k}$

for wavelengths in microns (μm)

Wien's Displacement Law

$(\lambda T)_{max\ power} = 2897.8\ \mu m \cdot K$

Total Emission (usually most important)

$E_b(T) = \int_0^\infty E_{b,\lambda}\, d\lambda = \sigma T_s^4$

Radiative Properties

(i) Emissivity: Real bodies emit a fraction of the blackbody emissive power

$\varepsilon(T) = \frac{E(T)}{E_b(T)}$ ~ Actual emissive power / ~ Ideal (blackbody) emissive power

• Most Metals: $\varepsilon \in (0.85, 0.95)$

$\Rightarrow$ Emissive Power: $E_{emitted} = \varepsilon \sigma T_s^4$ $\{\frac{W}{m^2}\}$

• Properties for incident radiation (irradiation)

(ii) Absorptivity (α)

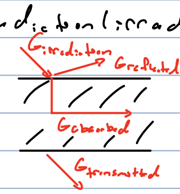
$\alpha = \frac{G_{absorbed}}{G}$

(iii) Reflectivity (ρ)

$\rho = \frac{G_{reflected}}{G}$

(iv) Transmissivity (τ)

$\tau = \frac{G_{transmitted}}{G}$



Energy Balance

$G = \alpha G + \rho G + \tau G$

$\Rightarrow 1 = \alpha + \rho + \tau$

• If opaque ($\tau = 0$)

$\alpha + \rho = 1$

$1\ Pa = 1 \frac{kg}{m \cdot s^2} = \frac{J}{m^3}$

ECHE 225: Fall 2024

Homework #11: Heat transfer, steady-state conduction

Due: November 21

1. [Chapter 17] Consider a person standing in a room at 20°C with an exposed surface area of 1.5 m^2. The deep body temperature of the human body is 37°C and the thermal conductivity of human tissue is about 0.3 W/m·K. The body is losing heat a rate of 180 W by natural convection and radiation to the surroundings. Taking the body temprature 0.5 cm beneath the skin to be 37°C, determine the skin temperature of the person.

2. [Chapter 17] Consider a 1.2-m-high and 2-m-wide double-pane window consisting of two 3-mm-thick layers of glass (k=0.78 W/m·K) separated by a 12-mm-wide stagnant air space (k=0.026 W/m·K). Determine the steady rate of heat transfer through this double-pane window and the temperature of its inner surface for a day during which the room is maintained at 22°C while the temperature of the outdoors is -8°C. Take the convection heat transfer coefficients on the inner and outer surfaces to be $h_i = 15$ W/m^2K and $h_o = 30$ W/m^2K and disregard any heat transfer by radiation.

3. [Chapter 17] Clothing made of several thin layers of fabric with trapped air in between, often called ski clothing, is commonly used in cold climates because it is light, fashionable, and a very effective thermal insulator. So it is no surprise that such clothing has largely replaced thick and heavy old-fashioned coats. Consider a jacket of four layers of 0.1-mm-thick synthetic fabric (k=0.13 W/m·K) and 2 mm thick air (k=0.026 W/m·K) between each of the layers. Assuming the inner surface temperature of the jacket to be 30°C and the surface area to be 1.25 m^2, determine the rate of heat loss through the jacket when the temperature of the outdoors is 4°C. What would your response be if the jacket is made of a single layer of 0.4-mm-thick sythetic fabric? What should be the thickness of wool fabric (k=0.035 W/m·K) if the person is to achieve the same level of thermal comfort wearing a thick wool coat instead of a 4-layer ski jacket?

4. [Chapter 17] A typical section of a building wall extends in and out of the page and is repeated in the vertical direction. The wall support members are made of steel (k=50 W/m·K). The support members are 8 cm (t_{23}) x 0.5 cm (L_B). The remainder of the inner wall space is filled with insulation (k=0.03 W/m·K) and measures 8 cm (t_{23}) x 60 cm (L_A). The inner wall is made of gypsum board (k=0.5 W/m·K) that is 1 cm thick (t_{12}) and the outer wall is made of brick (k=1.0 W/m·K) that is 10 cm thick (t_{34}). What is the average heat flux through this wall when $T_1 = 20°$C and $T_4 = 35°$C?

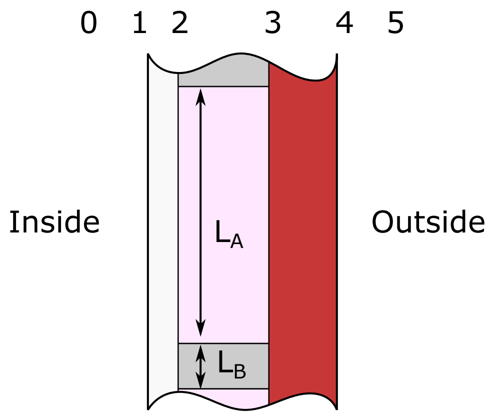
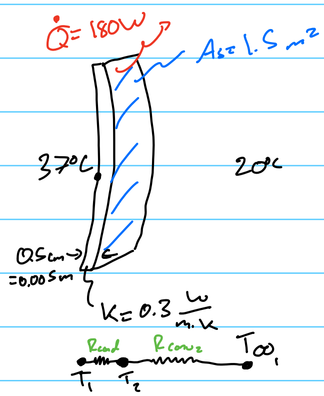


Figure 1: Schematic for Problem 4

5. [Chapter 17] A 9-mm-diameter spherical ball at 50°C is covered by a 1-mm-thick plastic insulation (k=0.13 W/m·K). The ball is exposed to a medium at 15°C with a combined convection and radiation heat transfer coefficient of 20 W/m^2·K. Determine if the plastic insulation on the ball will increase or decrease heat transfer from the ball.

1. [Chapter 17] Consider a person standing in a room at 20°C with an exposed surface area of 1.5 m^2. The deep body temperature of the human body is 37°C and the thermal conductivity of human tissue is about 0.3 W/m·K. The body is losing heat a rate of 180 W by natural convection and radiation to the surroundings. Taking the body temprature 0.5 cm beneath the skin to be 37°C, determine the skin temperature of the person.



$$\dot{Q} = \frac{\Delta T}{R} \Rightarrow \Delta T = \dot{Q}R$$

$$\boxed{T_2 = 35°C}$$

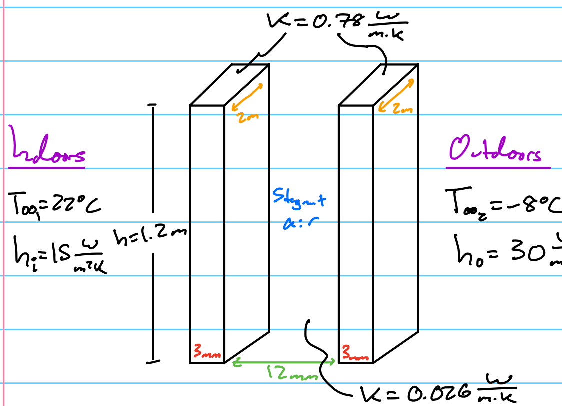
$$T_1 - T_2 = \dot{Q} \cdot \frac{L}{kA_s}$$

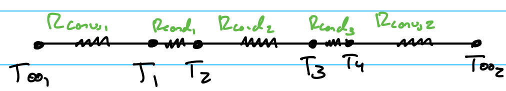
$$T_2 = T_1 - \dot{Q} \cdot \frac{L}{kA_s}$$

$$= \left[(37°C + 273.15) - 180W \left(\frac{0.005\,m}{0.3\,\frac{W}{mK} \cdot 1.5\,m^2} \right) \right] - 273.15$$

$$= 35°C$$

2. [Chapter 17] Consider a 1.2-m-high and 2-m-wide double-pane window consisting of two 3-mm-thick layers of glass (k=0.78 W/m·K) separated by a 12-mm-wide stagnant air space (k=0.026 W/m·K). Determine the steady rate of heat transfer through this double-pane window and the temperature of its inner surface for a day during which the room is maintained at 22°C while the temperature of the outdoors is -8°C. Take the convection heat transfer coefficients on the inner and outer surfaces to be $h_i = 15$ W/m^{2}K and $h_o = 30$ W/m^{2}K and disregard any heat transfer by radiation.





Find: $\dot{Q}$ and T_1

$$\dot{Q} = \frac{T_{\infty_1} - T_{\infty_2}}{R_{conv,1} + 2 \cdot R_{cond,1} + R_{cond,2} + R_{conv,2}} \leftarrow R_{tot}$$

$$R_{conv,1} = \frac{1}{hA_s} = \left(15 \frac{W}{m^2K} \cdot (2m \times 1.2m)\right)^{-1} = 0.02778 \frac{K}{W}$$

$$R_{cond,1} = \frac{L}{kA_s} = 0.003\,m \left(0.78 \frac{W}{mK} \cdot (2m \times 1.2m)\right)^{-1}$$

same as $R_{cond,3}$

$$= 0.001603 \frac{K}{W}$$

$$R_{cond,2} = \frac{L}{kA_s} = 0.012\,m \left(0.026 \frac{W}{m \cdot K} \cdot (2m \times 1.2m)\right)^{-1}$$

$$= 0.1923 \frac{K}{W}$$

$$R_{conv,2} = \frac{1}{hA_s} = \left(30 \frac{W}{m^2K} \cdot (2m \times 1.2m)\right)^{-1} = 0.01389 \frac{K}{W}$$

$$R_{tot} = 0.23776 \frac{K}{W}$$

$$\dot{Q} = \frac{22°C - -8°C}{0.237176 \frac{K}{W}} = 126.5W$$

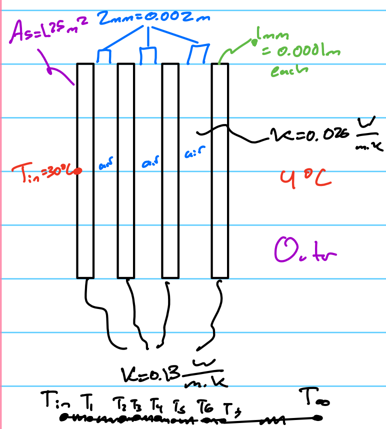
$$\dot{Q} = \frac{T_{\infty_1} - T_1}{R_{conv,1}} \Rightarrow T_1 = T_{\infty_1} - \dot{Q} R_{conv,1}$$

$$= 22°C - \left[126.5W \left(0.02778 \frac{K}{W}\right)\right]$$

$$= 18.5°C$$

$$\boxed{\begin{array}{l} \dot{Q} = 126.5W \\ T_1 = 18.5°C \end{array}}$$

3. [Chapter 17] Clothing made of several thin layers of fabric with trapped air in between, often called ski clothing, is commonly used in cold climates because it is light, fashionable, and a very effective thermal insulator. So it is no surprise that such clothing has largely replaced thick and heavy old-fashioned coats. Consider a jacket of four layers of 0.1-mm-thick synthetic fabric (k=0.13 W/m·K) and 2 mm thick air (k=0.026 W/m·K) between each of the layers. Assuming the inner surface temperature of the jacket to be 30°C and the surface area to be 1.25 m^2, determine the rate of heat loss through the jacket when the temperature of the outdoors is 4°C. What would your response be if the jacket is made of a single layer of 0.4-mm-thick sythetic fabric? What should be the thickness of wool fabric (k=0.035 W/m·K) if the person is to achieve the same level of thermal comfort wearing a thick wool coat instead of a 4-layer ski jacket?



$$R_{cond,fabric} = \frac{L}{kA_s} = \frac{0.0001\,m}{(0.13\,\frac{W}{m\cdot K})(1.25\,m^2)} = 0.000615\,\frac{K}{W}$$

$$R_{cond,air} = \frac{L}{kA_s} = \frac{0.002\,m}{(0.026\,\frac{W}{m\cdot K})(1.25\,m^2)} = 0.06154\,\frac{K}{W}$$

$$R_{tot} = 4R_{cond,fib} + 3R_{cond,air}$$

$$= 0.1871\,\frac{K}{W}$$

$$\dot{Q} = \frac{30°C - 4°C}{0.1871\,\frac{K}{W}} = 138.46\,W$$

$$\boxed{a)\ \dot{Q} = 138.46\,W}$$

Remove air gaps: $R_{tot} = 4R_{cond,fabr}$

$$= 0.00246\,\frac{K}{W}$$

$$\dot{Q} = \frac{30°C - 4°C}{0.00246\,\frac{K}{W}} = 10569.1\,W$$

$$\boxed{b)\ \dot{Q} = 10569.1}$$

$$138.46 = \frac{26°C}{\frac{L}{(0.035\,\frac{W}{m\cdot K})(1.25\,m^2)}} = \frac{26°C\,(0.035\,\frac{W}{m\cdot K})(1.25\,m^2)}{L}$$

$$L = \frac{26°C\,(0.035\,\frac{W}{m\cdot K})(1.25\,m^2)}{138.46\,W} = 0.00819\,m$$

$$\boxed{c)\ L = 8.19\,mm}$$

4. [Chapter 17] A typical section of a building wall extends in and out of the page and is repeated in the vertical direction. The wall support members are made of steel (k=50 W/m·K). The support members are 8 cm (t_{23}) x 0.5 cm (L_B). The remainder of the inner wall space is filled with insulation (k=0.03 W/m·K) and measures 8 cm (t_{23}) x 60 cm (L_A). The inner wall is made of gypsum board (k=0.5 W/m·K) that is 1 cm thick (t_{12}) and the outer wall is made of brick (k=1.0 W/m·K) that is 10 cm thick (t_{34}). What is the average heat flux through this wall when $T_1 = 20$°C and $T_4 = 35$°C?

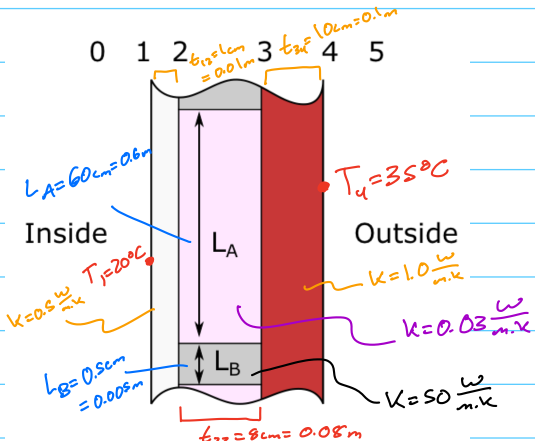
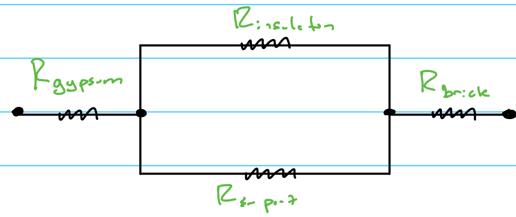


Figure 1: Schematic for Problem 4



$$R_{gypsum} = \frac{0.01\,m}{0.5\,\frac{W}{m\cdot K}\cdot A_{gypsum}} = 3.31\,\frac{K}{W}$$

$$R_{insulation} = \frac{0.08\,m}{0.03\,\frac{W}{m\cdot K}\cdot A_{insulation}} = 55.56\,\frac{K}{W}$$

$$R_{support} = \frac{0.08\,m}{50\,\frac{W}{m\cdot K}\cdot A_{support}} = 4.0\,\frac{K}{W}$$

$$R_{brick} = \frac{0.1\,m}{1.0\,\frac{W}{m\cdot K}\cdot A_{brick}} = 1.68\,\frac{K}{W}$$

$$R_{tot} = 3.31 + \left(\frac{1}{55.56} + \frac{1}{4.0}\right)^{-1} + 1.65 = 8.69\,\frac{K}{W}$$

$$A_{total} = lw = (0.01m + 0.08m + 0.1m)(0.6m + 0.005m) = 0.11495\,m^2$$

Find: $\dot{Q} = \frac{T_1 - T_4}{R_{tot}} = \frac{15°C}{R_{total}}$

q is heat flux => $\{\frac{W}{m^2}\}$

→ T in °C (basically K)

∴ R_{total} must be in $\frac{m^2 \cdot K}{W}$

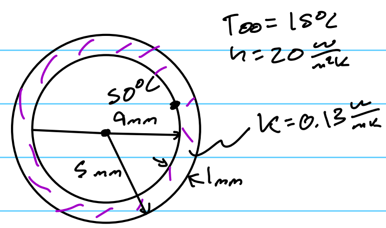
usually its in $\frac{K}{W}$, remove Area from equation?

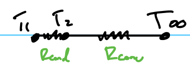
$$\dot{Q} = \frac{15°C}{8.69\,\frac{K}{W}} = 1.73\,W$$

$$q = \frac{1.73\,W}{0.11495\,m^2} = 15.02\,\frac{W}{m^2}$$

$\cancel{\boxed{q = 15.02\,\frac{W}{m^2}}}$ $q = 50\,\frac{W}{m^2}$? how

5. [Chapter 17] A 9-mm-diameter spherical ball at 50°C is covered by a 1-mm-thick plastic insulation (k=0.13 W/m·K). The ball is exposed to a medium at 15°C with a combined convection and radiation heat transfer coefficient of 20 W/m^2·K. Determine if the plastic insulation on the ball will increase or decrease heat transfer from the ball.





$$\dot{Q}_{insulation} = \frac{50°C - 15°C}{R_{cond,ins} + R_{conv,both}} \qquad \dot{Q}_{no\ insulation} = \frac{50°C - 15°C}{R_{conv,sph}}$$

$$R_{conv,sph} = \frac{1}{hA} = \frac{1}{20 \frac{W}{m^2K}(4\pi(.0045m)^2)} = 196.488 \frac{K}{W}$$

$$R_{conv,both} = \frac{1}{hA} = \frac{1}{20 \frac{W}{m^2K}(4\pi(.005m)^2)} = 159.16 \frac{K}{W}$$

$$R_{cond,ins} = \frac{0.005m - 0.0045m}{4\pi(.005m \cdot .0045m)0.13\frac{W}{mK}} = 13.603 \frac{K}{W}$$

$$\Rightarrow \dot{Q}_{insulation} = \frac{35°C}{159.16\frac{K}{W} + 13.603\frac{K}{W}} = 0.2026\ W$$

$$\Rightarrow \dot{Q}_{no\ insulation} = \frac{35°C}{196.488\frac{K}{W}} = 0.178\ W$$

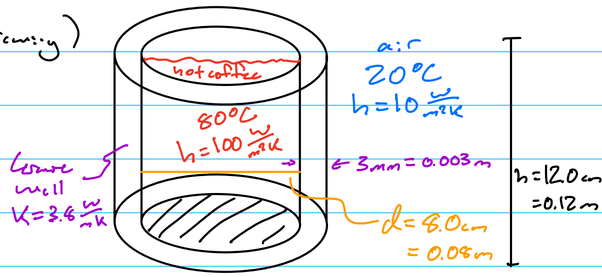
$\boxed{\text{Insulation increases heat Transfer here}}$

Answers

1. 35°C
2. 126 W, 18.5 °C
3. 139 W, 10600 W, 8.18 mm
4. 50 W/m^2
5. Insulation will increase heat transfer (0.18 W without insulation, 0.22 W with insulation)

Trevor Swan **ECHE225: QUIZ #11 // 11.21.24**

Drawing)



Inner wall area $= \pi d h = \pi(0.08m)(0.12m)$
$= 0.0302 \ m^2$

Outer wall area $= \pi d h = \pi(0.08m + 2(0.003)m)(0.12m)$
$= 0.0324 \ m^2$

1. Find $\dot{Q} = \frac{\Delta T}{R_{tot}}$

$R_{coffee} = \frac{1}{hA_s} = \frac{1}{100 \frac{W}{m^2K}(.0302 m^2)} = 0.331 \frac{K}{W}$

radius $= \frac{d}{2}$

$\frac{rad._1}{rad._2} = \frac{\frac{d_1}{2}}{\frac{d_2}{2}} = \frac{d_1}{d_2}$

$R_{cyl} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi h K} = \frac{\ln\left(\frac{d_2}{d_1}\right)}{2\pi h K} = \frac{\ln\left(\frac{0.08m + 2(.003m)}{0.08m}\right)}{2\pi(0.12m)3.8\frac{W}{mK}} = 0.0252 \frac{K}{W}$

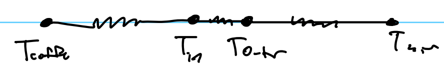
$R_{air} = \frac{1}{hA_s} = \frac{1}{10\frac{W}{m^2K}(.0324m^2)} = 3.086 \frac{K}{W}$

$R_{tot} = R_{coffee} + R_{cyl} + R_{air} = 0.331 + 0.0252 + 3.086 = 3.4422 \frac{K}{W}$

$\dot{Q} = \frac{80-20 \ K}{3.4422 \frac{K}{W}} = \boxed{17.43 \ W}$

2. Find $q = \frac{\dot{Q}}{A_s} = \frac{17.43W}{0.0324m^2} = 537.96 \frac{W}{m^2}$ $\boxed{q = 537.96 \frac{W}{m^2}}$

3. Find T_{outer}



$\dot{Q} = \frac{T_{coffee} - T_{outer}}{R_{coffee} + R_{cyl}}$

$T_{coffee} - T_{outer} = \dot{Q}(R_{coffee} + R_{cyl})$

$T_{outer} = T_{coffee} - \dot{Q}(R_{coffee} + R_{cyl})$

$T_{outer} = 80°C - 17.43W(0.331\frac{K}{W} + 0.0252\frac{K}{W})$
$= 80°C - 6.209 \ K$

$\boxed{T_{outer} = 73.791 \ °C}$

CASE WESTERN RESERVE UNIVERSITY
Case School of Engineering

ECHE 225: Fall 2024

Homework #12: Fins, transient heat transfer, forced convection

Due: December 5

1. [Chapter 17] Consider a very long rectangular fin attached to a flat surface such that the temperature at the end of the fin is essentially that of the surrounding air (i.e. 25°C) The width and thickness of the fin are 5 cm and 1 mm, respectively, and its thermal conductivity is 230 W/m·K. The temperature at the base of the fin is 50°C and the heat transfer coefficient is 30 W/m^2·K. Estimate the fin temperature at a distance of 5 cm from the base and the rate of heat loss from the entire fin.

2. [Chapter 17] A 10 mm diameter and 50 cm long aluminum fin (k=237 W/m·K) is attached to a surface. If the heat transfer coefficient is 15 W/m^2·K, determine the percent error in the rate of heat transfer from the fin when the infinitely long assumption is used instead of the adiabatic fin tip assumption.

3. [Chapter 18] Obtain a relation for the time required for a lumped system to reach the average temperature ($\frac{T_i+T_\infty}{2}$) where T_i is the initial temperature and T_∞ is the temperature of the surroundings.

4. [Chapter 18] Metal plates (k=200 W/m·K, ρ=3000 kg/m^3, c$_p$=900 J/kg·K) with a thickness of 1 cm are being heated in an oven for 2 minutes. Air in the oven is maintained at 800°C with a convection heat transfer coefficient of 200 W/m^2·K. If the initial temperature of the plate is 20°C, determine the temperature of the plates when they are removed from the oven.

5. [Chapter 18] Carbon steel balls (ρ=8000 kg/m^3, k=60 W/m·K, c$_p$=0.5 kJ/kg·K) 10 mm in diameter are annealed by heating them first to 900°C in a furnace and then allowing them to cool slowly to 100°C in ambient air at 35°C. If the average heat transfer coefficient is 75 W/m^2·K, determine how long the annealing process will take. If 2000 balls are to be annealed per hour, determine the total rate of heat transfer from the balls to the ambient air.

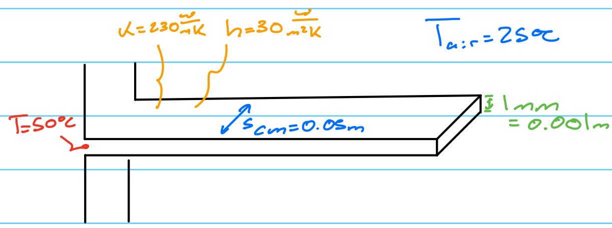
6. [Chapter 19] Hot carbon dioxide exhaust gas at 1 atm is being cooled by flat plates. The gas at 220°C flows in parallel over the upper and lower surfaces of a 1-m long flat plate at a velocity of 5 m/s. If the flat plate surface temperature is maintained at 80°C, determine:
   (a) the local convection heat transfer coefficient at 0.5 m from the leading edge
   (b) the average convection heat transfer coefficient over the entire plate
   (c) the total heat flux to the plate.

7. An array of power transistors, dissipating 6 W of power each, are to be cooled by mounting them on a 50 cm x 50 cm square aluminum plate and blowing air at 35°C over the plate with a fan at a velocity of 2 m/s. The average temperature of the plate is not to exceed 65°C. Assuming the heat transfer from the back side of the plate to be negligible and disregarding radiation, determine the maximum number of transistors that can be placed on this plate.

8. [Chapter 19] Combustion gases (calculate properties based on air) passing through a 5-cm internal diameter circular tube are used to vaporize waste water at atmospheric pressure. Hot gases enter the tube at 115 kPa and 250°C at a mean velocity of 3 m/s and leave at 150°C. If the average heat transfer coefficient is 120 W/m^2·K and the inner surface temperature of the tube is 110°C determine (a) the tube length and (b) the rate of evaporation of the water.

9. [Chapter 19] Consider a 12-m-long smooth rectangular tube (with sides $a = 50$ mm and $b = 25$ mm) that is maintained at a constant surface temperature. Liquid water enters the tube at 20°C with a mass flow rate of 0.01 kg/s. Determine the tube surface temperature necessary to heat the water to the desired outlet temperature of 80°C.

1. [Chapter 17] Consider a very long rectangular fin attached to a flat surface such that the temperature at the end of the fin is essentially that of the surrounding air (i.e. 25°C) The width and thickness of the fin are 5 cm and 1 mm, respectively, and its thermal conductivity is 230 W/m·K. The temperature at the base of the fin is 50°C and the heat transfer coefficient is 30 W/m^2·K. Estimate the fin temperature at a distance of 5 cm from the base and the rate of heat loss from the entire fin.



Assume infinitely long fin

Find $\begin{cases} \text{Temp at } x = 5\,cm = 0.05\,m \\ \dot{Q} \text{ from entire film} \end{cases}$

$$\boxed{\begin{aligned} T(0.05\,m) &= 36.06\,°C \\ \dot{Q} &= 4.69\,W \end{aligned}}$$

$T(x) = (e^{-mx})(T_b - T_\infty) + T_\infty$, $m = \sqrt{\dfrac{hP}{kA_c}}$

$\dot{Q} = \sqrt{hPkA_c}\,(T_b - T_\infty)$

$P = 2(0.05\,m) + 2(0.001\,m) = 0.102\,m$

$A_c = (0.05\,m)(0.001\,m) = 0.00005\,m^2$

$$m = \sqrt{\frac{(30\,\frac{W}{m^2K})(0.102\,m)}{(230\,\frac{W}{mK})(0.00005\,m^2)}} = 16.3122\,\frac{1}{m}$$

$T(0.05\,m) = (e^{-16.3122\,\frac{1}{m}(0.05\,m)})(50°C - 25°C) + 25°C$

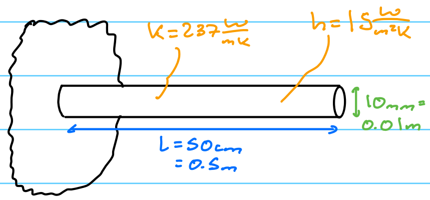
$= 36.06\,°C$

$\dot{Q} = \sqrt{30\,\frac{W}{m^2K}(0.102\,m)(230\,\frac{W}{mK})(0.00005\,m^2)}\,(50-25)$

$= 4.69\,W$

1.3%

2. [Chapter 17] A 10 mm diameter and 50 cm long aluminum fin (k=237 W/m·K) is attached to a surface. If the heat transfer coefficient is 15 W/m^2·K, determine the percent error in the rate of heat transfer from the fin when the infinitely long assumption is used instead of the adiabatic fin tip assumption.



$P = \pi d = \pi(0.01m) = 0.03142\ m$

$A_c = \pi\left(\frac{D}{2}\right)^2 = \frac{\pi}{4}(0.01m)^2 = 0.0000785\ m^2$

Infinitely Long $\dot{Q} = \sqrt{hPkA_c}\,(T_b - T_\infty)$

$\dot{Q}_{IL} = \sqrt{15\frac{W}{m^2K}(0.03142m)(237\frac{W}{mK})(0.0000785m^2)}\,(T_b - T_\infty)$

$= 0.09364\frac{W}{K}(T_b - T_\infty)$

Adiabatic Tip $\dot{Q} = \sqrt{hPkA_c}\,(T_b - T_\infty)\tanh(mL)$

$\sqrt{\frac{hP}{kA_c}}$ $m = \sqrt{\frac{(15\frac{W}{m^2K})(0.03142m)}{(237\frac{W}{mK})(0.0000785m^2)}} = 5.03315$

$\tanh(mL) = \tanh(5.03315\frac{1}{m}\cdot 0.5m) = 0.987048$

$\dot{Q}_{AT} = 0.09364\frac{W}{K}(T_b - T_\infty)(0.987048)$

$\%\text{error} = \frac{(\cancel{0.09364\frac{W}{K}(T_b-T_\infty)}) - (\cancel{0.09364\frac{W}{K}(T_b-T_\infty)}(0.987048))}{\cancel{0.09364\frac{W}{K}(T_b-T_\infty)}} = 0.01296$

$\boxed{\%\ \text{error} = 1.3\%}$ *Note this is the same as $1 - \tanh(mL)$*

3. [Chapter 18] Obtain a relation for the time required for a lumped system to reach the average temperature $(\frac{T_i+T_\infty}{2})$ where T_i is the initial temperature and T_∞ is the temperature of the surroundings.

$$\frac{T(t)-T_\infty}{T_i-T_\infty} = e^{-bt} \quad \text{where} \quad b = \frac{hA_s}{\rho V c}$$

$$\frac{\frac{T_i+T_\infty}{2}-T_\infty}{T_i-T_\infty} = e^{-bt}$$

$$\frac{\frac{T_i}{2}-\frac{T_\infty}{2}}{T_i-T_\infty} = e^{-bt}$$

$$\frac{\cancel{(T_i-T_\infty)}(\frac{1}{2})}{\cancel{T_i-T_\infty}} = e^{-bt}$$

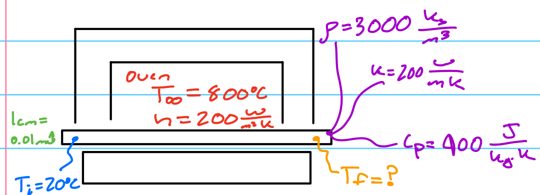
$$\frac{1}{2} = e^{-bt}$$

$$\ln(\tfrac{1}{2}) = -bt$$

$$t = -\frac{\ln(1/2)}{b}$$

$$\boxed{t = \frac{0.693}{b}\ s}$$

4. [Chapter 18] Metal plates (k=200 W/m·K, ρ=3000 kg/m^3, c_p=900 J/kg·K) with a thickness of 1 cm are being heated in an oven for 2 minutes. Air in the oven is maintained at 800°C with a convection heat transfer coefficient of 200 W/m^2·K. If the initial temperature of the plate is 20°C, determine the temperature of the plates when they are removed from the oven.



$t = 2\,\text{minutes} = 120\,s$

$$\frac{T(t)-T_\infty}{T_i-T_\infty} = e^{-bt}, \quad b = \frac{hA_s}{\rho V c} = \frac{h}{\rho L_c c}$$

$$\boxed{T(2\,min) = 667.935°C}$$

Assume $t \ll w$

$$Bi = \frac{hL_c}{k}; \quad L_c = \frac{V}{A_s} = \frac{\cancel{L}wt}{2\cancel{L}w + 2\cancel{L}t} = \frac{wt}{2(w+t)} \approx \frac{t}{2} = \frac{0.01m}{2} = 0.005m$$

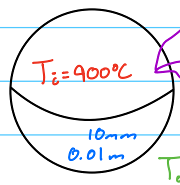
$$\hookrightarrow Bi\ \# = \frac{(200\,\frac{W}{m^2K})(0.005m)}{200\,\frac{W}{mK}} = 0.005$$

$$T(t) = T_\infty + e^{-bt}(T_i - T_\infty)$$

$$b = \frac{200\,\frac{W}{m^2K}}{(3000\,\frac{kg}{m^3})(0.005m)(900\,\frac{J}{kg\cdot K})} = 0.0148148\,\frac{1}{s}$$

$$T(120s) = 800°C + e^{-0.0148\frac{1}{s}(120\,s)}(20°C - 800°C)$$
$$= 667.935°C$$

5. [Chapter 18] Carbon steel balls (ρ=8000 kg/m^3, k=60 W/m·K, c_p=0.5 kJ/kg·K) 10 mm in diameter are annealed by heating them first to 900°C in a furnace and then allowing them to cool slowly to 100°C in ambient air at 35°C. If the average heat transfer coefficient is 75 W/m^2·K, determine how long the annealing process will take. If 2000 balls are to be annealed per hour, determine the total rate of heat transfer from the balls to the ambient air.



$\rho = 8000 \frac{kg}{m^3}$

$k = 60 \frac{W}{mK}$

$c_p = 0.5 \frac{kJ}{kg\cdot K} = 500 \frac{J}{kg\cdot K}$

$T_\infty = 35°C$

$h = 75 \frac{W}{m^2K}$

$$A_s = 4\pi r^2 = 4\pi\left(\frac{0.01m}{2}\right)^2 = 3.142\times10^{-4}\ m^2$$

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{0.01m}{2}\right)^2 = 5.236\times10^{-7}\ m^3$$

$$Bi = \frac{hL_c}{k} = \frac{hV}{kA_s} = \frac{(75 \frac{W}{m^2K})(5.236\times10^{-7} m^3)}{(60 \frac{W}{mK})(3.142\times10^{-4} m^2)}$$

$$= 0.00208 \checkmark$$

$$b = \frac{hA_s}{\rho V c} = \frac{(75 \frac{W}{m^2K})(3.142\times10^{-4} m^2)}{(8000 \frac{kg}{m^3})(5.236\times10^{-7} m^3)(500 \frac{J}{kg\cdot K})}$$

$$= 0.01125 \ \frac{1}{s}$$

a) $T(t) = T_\infty + e^{-bt}(T_i - T_\infty)$

$$100°C = 35°C + e^{-0.01125t}(900°C - 35°C)$$

$$0.0751 = e^{-0.01125t}$$

$$t = \frac{1}{-0.01125}\ln(0.0751) = 230.128 \text{ seconds}$$

$$230.128\ sec \times \frac{1\ min}{60\ sec} = \boxed{3.84\ min}$$

b) $Q = \overbrace{\rho V}^{m} c_p [T_i - T_f]$

$\dot{Q}_{total} = n \cdot \dot{Q} \cdot \frac{1}{\Delta t}$

$$\dot{Q}_{total} = 2000 \cdot \dot{Q} \cdot \frac{1}{3600s}$$

$$Q = 8000 \frac{kg}{m^3}(5.236\times10^{-7} m^3)(500 \frac{J}{kgK})(900-100)$$

$$= 1675.52\ J$$

$$\dot{Q}_{total} = 2000 \cdot 1675.52\ J \cdot \frac{1}{3600s}$$

$$= 930.844\ W$$

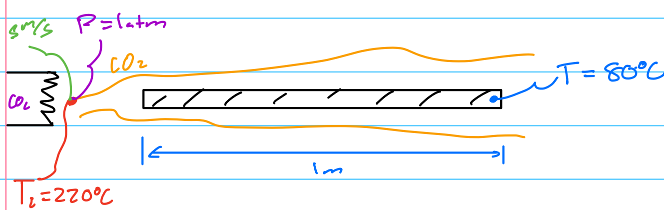
$$\boxed{930.844W}$$

6. [Chapter 19] Hot carbon dioxide exhaust gas at 1 atm is being cooled by flat plates. The gas at 220°C flows in parallel over the upper and lower surfaces of a 1-m long flat plate at a velocity of 5 m/s. If the flat plate surface temperature is maintained at 80°C, determine:

(a) the local convection heat transfer coefficient at 0.5 m from the leading edge

(b) the average convection heat transfer coefficient over the entire plate

(c) the total heat flux to the plate.



CO_2 @ 1 atm, Film Temp (A-23)

$T_f = \frac{T_s + T_\infty}{2} = \frac{80°C + 220°C}{2} = 150°C$

$\rho = 1.2675\ \frac{kg}{m^3}$

$c_p = 957.4\ \frac{J}{kg \cdot K}$

$K = 0.02652\ \frac{W}{m \cdot K}$

$\mu = 2.063 \times 10^{-5}\ \frac{kg}{m \cdot s}$

$Pr = 0.7445$

a) $h_x = \frac{Nu_x K_{fluid}}{x}$, at $x = 0.5\,m$

$$Re_x = \frac{\rho u_\infty x}{\mu} = \frac{(1.2675\ \frac{kg}{m^3})(5\ \frac{m}{s})(0.5\,m)}{2.063 \times 10^{-5}\ \frac{kg}{m \cdot s}} = 153599 = 1.5 \times 10^5 < 5 \times 10^5 \text{ Laminar}$$

$$Nu_x = 0.332 \cdot Re_x^{1/2} \cdot Pr^{1/3} = 0.332 \cdot 153599^{1/2} \cdot 0.7445^{1/3} = 117.929$$

$$h_x = \frac{117.929\,(0.02652\ \frac{W}{mK})}{0.5\,m} = 6.25495\ \frac{W}{m^2K}$$

$$\boxed{h_x = 6.25\ \frac{W}{m^2K}}$$

b) $$Re_L = \frac{\rho u_\infty L}{\mu} = \frac{(1.2675\ \frac{kg}{m^3})(5\ \frac{m}{s})(1\,m)}{2.063 \times 10^{-5}\ \frac{kg}{m \cdot s}} = 307198 = 3.07 \times 10^5 \text{ Laminar}$$

$$Nu_L = 0.664 \cdot Re_L^{1/2} \cdot Pr^{1/3} = 0.664 \cdot 307198^{1/2} \cdot 0.7445^{1/3} = 333.554$$

$$h = \frac{333.554\,(0.02652\ \frac{W}{mK})}{1\,m} = 8.84585\ \frac{W}{m^2K}$$

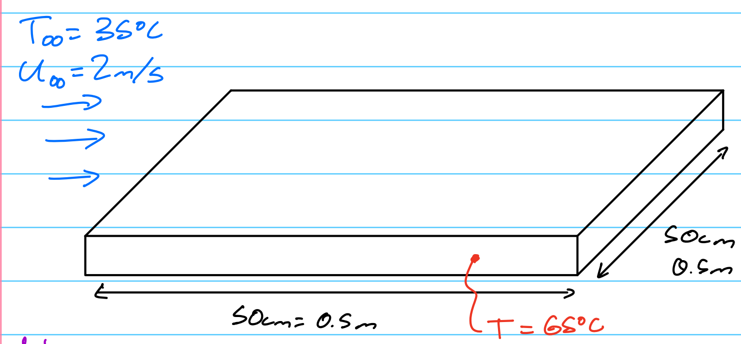
$$\boxed{h = 8.85\ \frac{W}{m^2K}}$$

c) $q''_{conv} = h \Delta T$

$$q''_{conv} = (8.85\ \tfrac{W}{m^2K})(220°C - 80°C) = 1239\ \tfrac{W}{m^2}$$

$$\boxed{q''_{conv} = 1239\ \frac{W}{m^2}}$$

7. An array of power transistors, dissipating 6 W of power each, are to be cooled by mounting them on a 50 cm x 50 cm square aluminum plate and blowing air at 35°C over the plate with a fan at a velocity of 2 m/s. The average temperature of the plate is not to exceed 65°C. Assuming the heat transfer from the back side of the plate to be negligible and disregarding radiation, determine the maximum number of transistors that can be placed on this plate.



Air @ 1atm, Film Temp (A-)

$$T_f = \frac{35°C + 65°C}{2} = 50°C$$

$\rho = 1.092 \frac{kg}{m^3}$

$k = 0.02735 \frac{W}{mK}$

$\mu = 1.963 \times 10^{-5} \frac{kg}{m \cdot s}$

$Pr = 0.7228$

$$\frac{Lh}{k_{fluid}} = Nu_L$$

$$Re_L = \frac{(1.092 \frac{kg}{m^3})(2 \frac{m}{s})(0.5 m)}{1.963 \times 10^{-5} \frac{kg}{m \cdot s}} = 55629.1 = 5.56 \times 10^4 < 5 \times 10^5 \text{ laminar}$$

$$Nu_L = 0.664 \cdot Re_L^{1/2} \cdot Pr^{1/3} = 0.664 \cdot 55629.1^{1/2} \cdot 0.7228^{1/3} = 140.548$$

$$h = \frac{Nu_L \cdot k_{fluid}}{L} = \frac{140.548 (0.02735 \frac{W}{mK})}{0.5 m} = 7.68798 \frac{W}{m^2 K}$$

$$q''_{conv} = \frac{\dot{Q}_{conv}}{A_s} \Rightarrow A_s = (0.5 m)^2 = 0.25 m^2$$

$$q''_{conv} = h \Delta T = (7.68798 \frac{W}{m^2 K})(65°C - 35°C) = 230.639 \frac{W}{m^2}$$

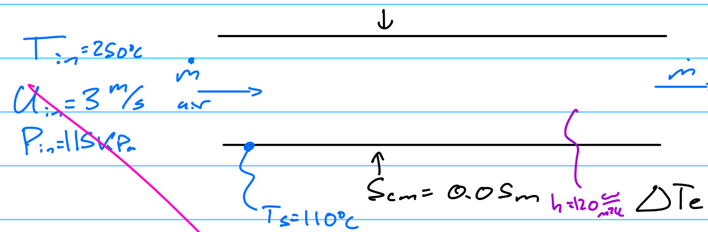
$$\dot{Q}_{conv} = \frac{230.639 \frac{W}{m^2}}{0.25 m^2} = 57.6598 W$$

$$\text{Total Transistors: } \frac{57.6598 W}{6 W} = 9.60997$$

$\boxed{\text{Can have 9 total}}$

Incorrect, most likely incorrect table values?

8. [Chapter 19] Combustion gases (calculate properties based on air) passing through a 5-cm internal diameter circular tube are used to vaporize waste water at atmospheric pressure. Hot gases enter the tube at 115 kPa and 250°C at a mean velocity of 3 m/s and leave at 150°C. If the average heat transfer coefficient is 120 W/m²·K and the inner surface temperature of the tube is 110°C determine (a) the tube length and (b) the rate of evaporation of the water.



$\Delta T_e = 150 - 110 = 40$

$\Delta T_i = 250 - 110 = 140$

Air @ ~1 atm

200°C: $R = 287\ J/kgK$

$C_p = 1023\ \frac{J}{kgK}$

$K = 0.03779\ \frac{W}{mK}$

$\mu = 2.577 \times 10^{-5}\ \frac{kg}{ms}$

$Pr = 0.6974$

a) $T_m = \frac{T_i + T_e}{2} = \frac{250 + 150}{2} = 200°C$

$\Delta T_{lm} = \frac{\Delta T_e - \Delta T_i}{\ln\left(\frac{\Delta T_e}{\Delta T_i}\right)} = \frac{40 - 140}{\ln\left(\frac{40}{140}\right)} = 79.824°C$

$PV = mRT$

$\rho = \frac{P}{R\,T_m} = \frac{115{,}000\ Pa}{(287 \frac{J}{kgK})(200°C)} = 0.84687\ \frac{kg}{m^3}$

$A_c = \frac{\pi}{4}(0.05m)^2 = 0.001964\ m^2$

$\dot{m} = \rho V_{avg} A_c = (0.84687 \frac{kg}{m^3})(3 \frac{m}{s})(0.001964\ m^2) = 0.00499\ \frac{kg}{s}$

$\dot{Q} = hPL\Delta T_{lm} = \dot{m} C_p (T_e - T_i)$

$\dot{Q} = \dot{m} C_p (T_e - T_i)$

$= (0.00499 \frac{kg}{s})(1023 \frac{J}{kgK})(-100)$

$= -510.477\ W$

$L = \frac{\dot{Q}}{hP\Delta T_{lm}}$

$= \frac{510.477\ W}{(120 \frac{W}{m^2K})(\pi\, 0.05m)(79.824)} = 0.339$

$\boxed{L = 33.9\ cm}$ should be 30.7 cm

See next page

b) $\dot{Q} = \dot{m}_w h_{vapor}$

Table A-3

$h_{fg} = 2257\ \frac{kJ}{kg}$

$\dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{0.510477\ kW}{2257 \frac{kJ}{kg}} = 0.000226175\ \frac{kg}{s} \times \frac{3600s}{1hr}$

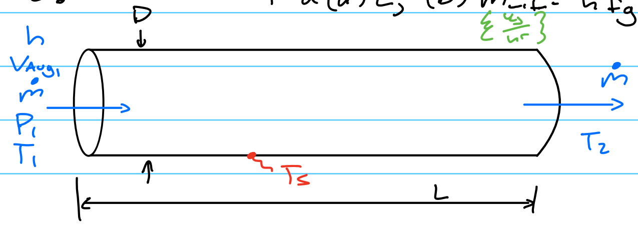
$\boxed{\dot{m} = 0.8469\ kg/hr}$ Should be 0.736 kg/hr

Homework 12 Corrections

8. Find (a) L, (b) $\dot{m}_{water} = \dfrac{\dot{Q}}{h_{fg}}$ $\left\{\frac{kg}{hr}\right\}$

rate of evaporation

$$T_{film} = \frac{250°C + 150°C}{2} = 200°C$$



Air @ $T_{film} = 200°C$, 115 kPa

$R = 0.287 \frac{kJ}{kgK}$

$c_p = 1023 \frac{J}{kgK}$

$k = 0.03779 \frac{W}{mK}$

$\mu = 2.577 \times 10^{-5} \frac{kg}{m \cdot s}$

$Pr = 0.6674$

$PV = m\bar{R}T$ 1 kPa $= \frac{kJ}{m^3}$ Use entrance temp to get density

$$\rho = \frac{P}{\bar{R}T} = \frac{115 \text{ kPa}}{0.287 \frac{kJ}{kgK} \cdot (250+273.15)} = 0.76593 \frac{kg}{m^3}$$

$$\dot{m} = \rho V_{avg} A_c = \left(0.76593 \frac{kg}{m^3}\right)\left(3 \, m/s\right)\left(\pi\left(\frac{0.05\,m}{2}\right)^2\right)$$
$$= 0.0045 \, kg/s$$

$$\dot{Q} = \dot{m} c_p (T_e - T_i)$$
$$= (0.0045 \, kg/s)\left(1023 \frac{J}{kgK}\right)(250-150)$$
$$= 460.35 \, W$$

a) $\dot{Q} = h A_s \Delta T_{lm}$

$$\dot{Q} = h P L \Delta T_{lm}$$

$$L = \frac{\dot{Q}}{h P \Delta T_{lm}} = \frac{460.35 \, W}{\left(120 \frac{W}{m^2K}\right)(0.15708\,m)(78.82°C)} = 0.3098 \, m$$

$h = 120 \frac{W}{m^2 K}$

$$\boxed{L = 30.98 \, cm}$$

$P = \pi(0.05\,m) = 0.15708 \, m$

$$\Delta T_{lm} = \frac{\Delta T_e - \Delta T_i}{\ln\left(\frac{\Delta T_e}{\Delta T_i}\right)} = \frac{(150-110)-(250-110)}{\ln\left(\frac{(150-110)}{(250-110)}\right)} = 78.82°C$$

b) $\dot{m}_{water} = \dfrac{\dot{Q}}{h_{fg}}$

$$\boxed{\dot{m}_{water} = 0.734 \, kg/hr}$$

$h_{fg,water} = 2257 \frac{kJ}{kg}$

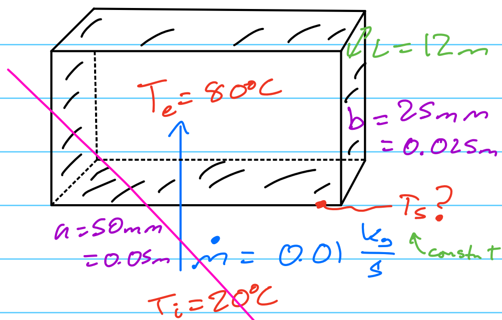
$$\dot{m}_{water} = \frac{0.46035 \, kW}{2257 \frac{kJ}{kg}} = 0.000203965 \, kg/s \times 3600$$

Incorrect, most likely incorrect table values?

9. [Chapter 19] Consider a 12-m-long smooth rectangular tube (with sides $a = 50$ mm and $b = 25$ mm) that is maintained at a constant surface temperature. Liquid water enters the tube at 20°C with a mass flow rate of 0.01 kg/s. Determine the tube surface temperature necessary to heat the water to the desired outlet temperature of 80°C.

Average Temp

$T_m = \frac{T_e + T_i}{2} = 50°C$



$$T_e = T_s - (T_s - T_i)e^{-\frac{hA_s}{\dot{m}c_p}}$$

$$T_e = T_s - T_s e^{-\frac{hA_s}{\dot{m}c_p}} - T_i \cdot e^{-\frac{hA_s}{\dot{m}c_p}}$$

$$-T_s + T_s e^{-\frac{hA_s}{\dot{m}c_p}} = -T_e - T_i \cdot e^{-\frac{hA_s}{\dot{m}c_p}}$$

$$T_s(1 - e^{-\frac{hA_s}{\dot{m}c_p}}) = T_e + T_i \cdot e^{-\frac{hA_s}{\dot{m}c_p}}$$

$$T_s = \frac{T_e + T_i \cdot e^{-\frac{hA_s}{\dot{m}c_p}}}{1 - e^{-\frac{hA_s}{\dot{m}c_p}}}$$

$$T_s = \frac{T_e + T_i \cdot e^{-\frac{hA_s}{\dot{m}c_p}}}{1 - e^{-\frac{hA_s}{\dot{m}c_p}}}$$

$$Re_D = \frac{\rho V_{avg} D_h}{\mu}$$

$\dot{m} = 0.01 \frac{kg}{s}$

$A_s = 2(12m \cdot 0.05m) + 2(12m \cdot 0.025m) = 1.8 \ m^2$

$c_p = 4181 \ J/kg{\cdot}K$, $k = 0.644 \frac{W}{m{\cdot}K}$ Table A-15 @ 50°C

$\rho = 988.1 \frac{kg}{m^3}$

$\mu = 0.547 \times 10^{-3} \frac{kg}{m{\cdot}s}$

$D_h = \frac{4A_c}{P} = \frac{4(0.05m \cdot 0.025m)}{2(0.05m + 0.025m)} = 0.0333 \ m$

$V_{avg} = \frac{\dot{m}}{\rho A_c} = \frac{0.01}{988.1 \cdot (0.05 \cdot 0.025)} = 0.00810 \ m/s$

$$Re_D = \frac{(988.1 \frac{kg}{m^3})(0.00810 \frac{m}{s})(0.0333m)}{(0.547 \times 10^{-3} \frac{kg}{m{\cdot}s})} = 487.24 << 2300 \text{ Laminar!}$$

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TABLE 19–3

Nusselt number and friction factor for fully developed laminar flow in tubes of various cross sections ($D_h = 4A_c/p$, $Re = V_{avg}D_h/\nu$, and $Nu = hD_h/k$)

$D_H = \frac{4A_c}{P}$

Tube Geometry	a/b or $\theta°$	Nusselt Number T_s = Const.	Nusselt Number $\dot{q}_s$ = Const.
Circle	—	3.66	4.36
Rectangle	a/b		
	1	2.98	3.61
	2	3.39	4.12
	3	3.96	4.79
	4	4.44	5.33
	6	5.14	6.05
	8	5.60	6.49
	∞	7.54	8.24
Ellipse	a/b		
	1	3.66	4.36
	2	3.74	4.56
	4	3.79	4.88
	8	3.72	5.09
	16	3.65	5.18
Isosceles Triangle	θ		
	10°	1.61	2.45
	30°	2.26	2.91
	60°	2.47	3.11
	90°	2.34	2.98
	120°	2.00	2.68

$3.39 = \frac{hD_h}{k}$ See next page

$\frac{3.39k}{D_h} = h \Rightarrow h = \frac{3.39(0.644 \frac{W}{m{\cdot}K})}{0.0333 \ m} = 65.56 \frac{W}{m^2K}$

$\Rightarrow e^{-\frac{hA_s}{\dot{m}c_p}} = e^{\left(-\frac{65.56 \frac{W}{m^2K} \cdot 1.8m^2}{0.01 \frac{kg}{s} \cdot 4181 \frac{J}{kg{\cdot}K}}\right)} = 0.0595$

Rectangle

$\left\{ \begin{array}{l} a = 50mm \\ b = 25mm \end{array} \right. \Rightarrow \frac{a}{b} = 2$

Ts Constant

$Nu = 3.39$

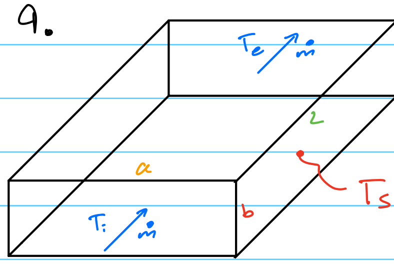
$$T_s = \frac{80°C + 20°C(0.0595)}{1 - 0.0595} = 86.3°C$$

$$\boxed{T_s = 86.3°C}$$

Should be 83.8 °C

Homework 12 Corrections

9.



Water: $\dot{m} = 0.01\ \frac{kg}{s}$ @ 20°C

$\hookrightarrow T_i = 20°C$ and $T_e = 80°C$

$T_{avg} = \frac{20°C + 80°C}{2} = 50°C \rightarrow$ Table

$c_p = 4181\ J/kg \cdot K$

$\rho = 988.1\ \frac{kg}{m^3}$

$\mu = 0.547 \times 10^{-3}\ \frac{kg}{m \cdot s}$

$k = 0.644\ \frac{W}{mK}$

$V_{avg} = \frac{\dot{m}}{\rho A_c} = \frac{0.01\ \frac{kg}{s}}{(988.1\ \frac{kg}{m^3})(0.00125 m^2)}$

$= 0.0081\ m/s$

Tube: Find T_s, assume const T_s

$a = 50mm = 0.05m$

$b = 25mm = 0.025m$ $\Big\} \frac{a}{b} = 2$

$L = 12\ m$

$A_s = 2L(a+b) = 1.8 m^2$

$A_c = ab = 0.00125 m^2$

$P = 2(a+b) = 0.15 m$

$D_h = \frac{4A_c}{P} = \frac{4(0.00125 m^2)}{0.15m}$

$= 0.0333\ m$

$Re_{D_h} = \frac{(988.1\ \frac{kg}{m^3})(0.0081\ \frac{m}{s})(0.0333 m)}{0.547 \times 10^{-3}\ \frac{kg}{m \cdot s}} = 487.24 < 2300$ laminar!

$\frac{a}{b} = 2 \rightarrow Nu = 3.39$ from 19.3 table

$Nu_{D_h} = \frac{h D_h}{k} \rightarrow h = \frac{Nu_{D_h} k}{D_h}$

$$h = \frac{(3.39)(0.664\ \frac{W}{mK})}{(0.0333 m)} = 67.59\ \frac{W}{m^2 K}$$

$$T_e = T_s - (T_s - T_i) \cdot \underbrace{\exp\left[-\frac{hPL}{\dot{m}\, c_p}\right]}_{0.0544824}$$

$80°C = T_s - (T_s - 20°C)\, 0.0544824$

$80°C = T_s - 0.0544824 T_s + 1.08965$

$80°C = 0.945518 T_s + 1.08965$

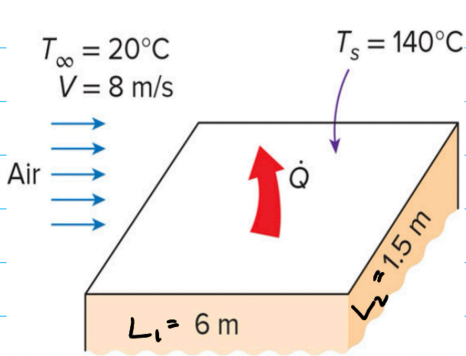
$T_s = 83.4°C$

Answers

1. 36°C, 4.7 W
2. 1.3%
3. 0.693/b
4. 668°C
5. 3.8 minutes, 931 W
6. (a) 6.25 $W/m^2 \cdot K$, (b) 8.85 $W/m^2 \cdot K$, (c) 1238 W/m^2
7. 9
8. 30.7 cm, 0.736 kg/hr
9. 83.8°C

Trevor Swan **ECHE225: QUIZ #12 // 12.04.24**

$P_{atm} = 83.4$ kPa ← From elevation = 1610m



Given

$T_\infty = 20°C$

$U_\infty = 8.00\ m/s$

$T_s = 140°C$

$T_{film} = T_f = \frac{140+20}{2} = 80°C$

$L_1 = 6m$

$L_2 = 1.5m$

Given Properties of air @ 1atm, T_f

$\rho = 0.9994\ \frac{kg}{m^3}$

$k = 0.02953\ \frac{W}{mK}$

$\mu = 2.096 \times 10^{-5}\ \frac{kg}{m \cdot s}$

$P_r = 0.7154$

→ Assume Ideal Gas

$P_{atm} = 101.325$ kPa

$P_{atm} = 83.4$ kPa

2. $PV = m\bar{R}T$

$P_1 = \rho_1 \bar{R} T$

$\frac{P_1}{\rho_1} = \frac{P_2}{\rho_2}$ → $\rho_2 = \frac{P_2 \rho_1}{P_1} = \frac{(83.4\ kPa)(0.994\ \frac{kg}{m^3})}{101.325\ kPa} = 0.823\ \frac{kg}{m^3}$

$\boxed{\rho_2 = 0.823\ \frac{kg}{m^3}}$

3. Rate of heat transfer (kW) along $L_1 = 6m$

$Re_{L} = \frac{\rho U_\infty L}{\mu} = \frac{(0.823\ \frac{kg}{m^3})(8\ m/s)(6m)}{2.096 \times 10^{-5}\ \frac{kg}{m \cdot s}} = 1884732.8 = 18.84 \times 10^5 > 5 \times 10^5$ turbulent

These were looking for mixed BL Nu

$Nu_L = 0.037 \cdot (18.84 \times 10^5)^{4/5} \cdot (0.7154)^{1/3} = 3465.63$

$h = \frac{Nu_L\ k_{fluid}}{L} = \frac{(3465.63)(0.02953\ \frac{W}{mK})}{6m} = 17.057\ \frac{W}{m^2K}$

$\underbrace{\dot{Q}_{conv} = \frac{h \Delta T}{A_s}}_{\text{should be } \dot{Q}_{conv} = h\Delta T \cdot A_s} = \frac{(17.057\ \frac{W}{m^2K})(120°C)}{(6m \cdot 1.5m)} = 227.42\ W$ $\boxed{\dot{Q}_{conv} = 0.23\ kW}$

4. Rate of heat transfer (kW) along $L_2 = 1.5m$

$Re_{L} = \frac{(0.823\ \frac{kg}{m^3})(8\ m/s)(1.5m)}{2.096 \times 10^{-5}\ \frac{kg}{m \cdot s}} = 471183 = 4.7 \times 10^5 < 5 \times 10^5$ laminar

$Nu_L = 0.664(4.7 \times 10^5)^{1/2}(0.7154)^{1/3} = 407.13$

$h = \frac{Nu_L\ k_{fluid}}{L} = \frac{(407.13)(0.02953\ \frac{W}{mK})}{1.5m} = 8.015\ \frac{W}{m^2K}$

$\underbrace{\dot{Q}_{conv} = \frac{h \Delta T}{A_s}}_{\text{should be } \dot{Q}_{conv} = h\Delta T \cdot A_s} = \frac{(8.015\ \frac{W}{m^2K})(120°C)}{(6m \cdot 1.5m)} = 106.867\ W$ $\boxed{\dot{Q}_{conv} = 0.11\ kW}$

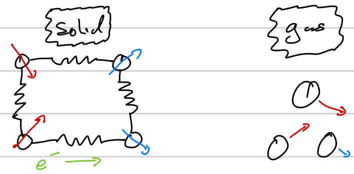
Exam 3 Notes

Introduction to Heat Transfer

Heat Transfer

- Energy Transfer through a temperature difference
- Three modes of heat transfer

(i) Conduction - Energy transfer due to the direct interaction between particles



→ Conduction is described by Fourier's Law

$$\dot{Q}_{conduction} = -kA\frac{dT}{dx}$$

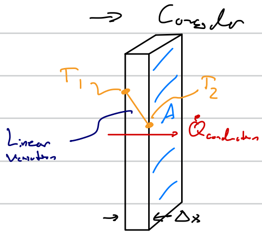
Heat flows from hot to cold

temperature gradient in x-dir

$\{W/m\cdot K\}$ thermal conductivity

cross-sectional area

→ Consider a plane wall (1-D conduction)



- For "simple" conduction

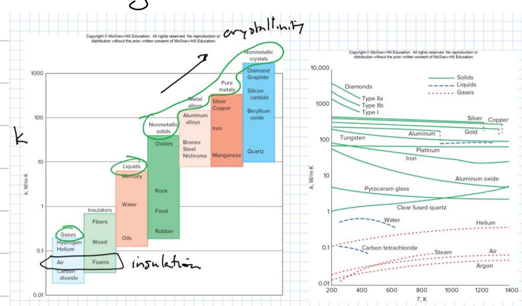
$$\dot{Q}_{conduction} = kA\frac{T_1-T_2}{\Delta x}$$

negative absorbed by $\Delta T = (T_2 - T_1)$

"simple" = no heat generation, planar (constant cross-sec. area)

- Typical thermal conductivity values

gas << solids ← crystallinity



→ For Transient Heat Transfer

- Additional parameter is important

$$\alpha = \frac{k}{\rho \cdot c}$$

$\{m^2/s\}$ thermal conductivity

Stored energy

density

specific heat

(ii) Convection - Enhanced conduction through bulk fluid motion

↳ moving fluid → large temperature gradient

$\{W/m^2\cdot K\}$ Convective heat transfer coefficient

Surface area in contact with flow

→ Convection described by Newton's Law of Cooling: $\dot{Q}_{convection} = h\cdot A_s(T_s - T_\infty)$

→ Heat transfer coefficient

h captures the effect of flow: h↑ as u_∞↑

(iii) Radiation - energy transfer due to electromagnetic waves

→ Does not require a medium

→ All matter at non-zero temperature emits radiation

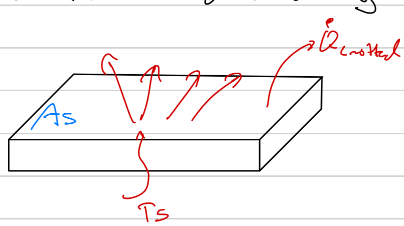
→ Emitted radiation described by the Stefan-Boltzmann Law: $\dot{Q}_{emitted} = \varepsilon\sigma A_s T_s^4$

emissivity

Surface area

Surface temperature

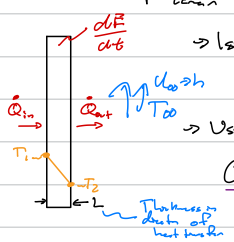
Stefan-Boltzmann Constant $\sigma = 5.67 \times 10^{-8}\ \frac{W}{m^2K^4}$



Conduction

Thermal Resistance Network

- Conduction Plane Wall



→ 1st Law: $\dot{Q}_{in} - \dot{Q}_{out} = \frac{dE}{dt}$ steady-state

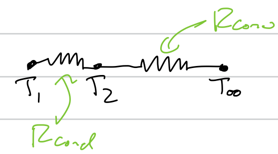
⟹ $\dot{Q}_{in} = \dot{Q}_{out} = \dot{Q}_{conduction,wall}$

→ Using Fourier's Law

$$\dot{Q}_{cond,wall} = kA_c \frac{T_1 - T_2}{L}$$ ($T_1 - T_2 = \Delta T$)

→ Define thermal Resistance

$$\dot{Q}_{cond,wall} = \frac{\Delta T}{R_{cond.}}$$



→ Using Fourier's Law again...

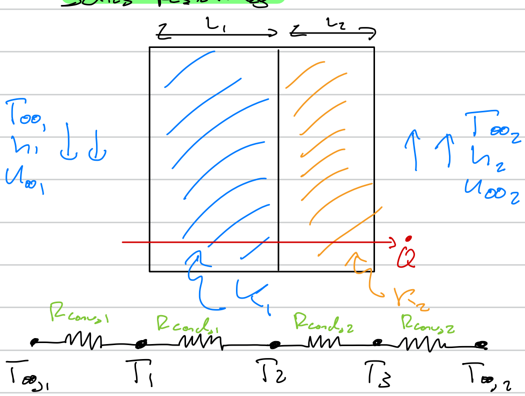
Plane wall

$$\Rightarrow R_{cond} = \frac{L}{kA}$$

- Convective Resistance: $\dot{Q}_{conv} = hA_s(T_2 - T_\infty)$ ⟹ $R_{convection} = \frac{1}{hA_s}$

More Complexity:

Series Resistance



- For thermal resistances in series:
  - (i) $\dot{Q}$ same for all
  - (ii) Individual ΔTs sum to total ΔT

$$\dot{Q} = \frac{\Delta T}{R}; \quad \Delta T = \dot{Q}R$$

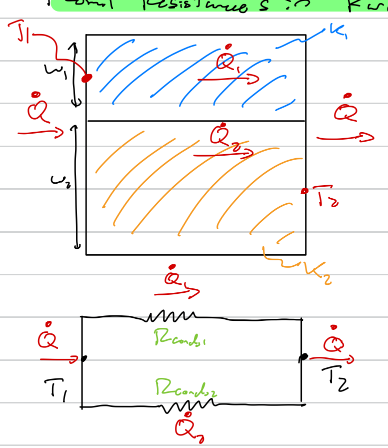
$$\Rightarrow R_{tot} = R_{conv,1} + R_{cond,1} + R_{cond,2} + R_{conv,2}$$

$$\dot{Q} = \frac{T_{first} - T_{last}}{R_{tot}}$$

$$= \frac{T_{\infty,1} - T_1}{R_{conv,1}} = \frac{T_1 - T_3}{R_{cond,1} + R_{cond,2}} = \dots$$

make sure you use corresponding resistance

Thermal Resistances in Parallel

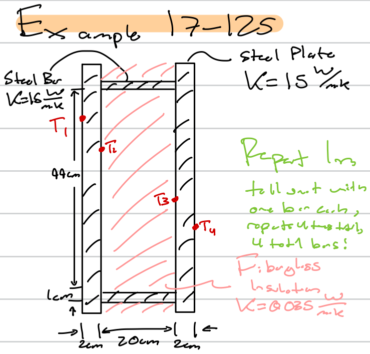


- For thermal Resistances in parallel
  - (i) Individual $\dot{Q}$s add to $\dot{Q} = \dot{Q}_1 + \dot{Q}_2$
  - (ii) ΔTs the same for all

$$\dot{Q} = \frac{\Delta T}{R}$$

$$\frac{1}{R_{tot}} = \frac{1}{R_{cond_1}} + \frac{1}{R_{cond_2}} + \dots$$

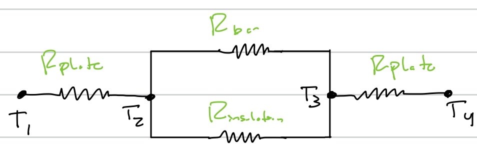
Example 17-125



Repeat 1 m tall unit with one bar each, repeated 4 total bars!

Fiberglass insulation $k = 0.035\,\frac{W}{mK}$

- wall is 4 m tall, 6 m deep
- Find (i) $\dot{Q}$ for $\Delta T = 22°C$
  - (ii) Can the effect of the bars be ignored?



$$\dot{Q} = \frac{\Delta T_{tot}}{R_{tot}} = \frac{T_1 - T_4}{R_{tot}}$$ ($T_1 - T_4 = 22°C$) Need to find R_{tot}

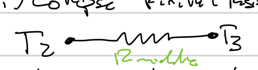
$$R_{plate} = \frac{L_p}{k_p A_p} = \frac{2/100\ m}{15\frac{W}{m \cdot K} \cdot (1\ m)(6\ m)} = 0.000222\ \frac{K}{W}$$

L_p: dimension in direction of heat transfer; 1 m tall repeat unit; 6 m deep

$$R_{bar} = \frac{20/100\ m}{15\frac{W}{m \cdot K}(\frac{1}{100}m)(6\ m)} = 0.222\ \frac{K}{W}$$

$$R_{insulation} = \frac{20/100\ m}{(.035\frac{W}{m \cdot K})(\frac{99}{100}m)(6\ m)} = 0.962\ \frac{K}{W}$$

(i) Collapse Parallel Resistance



$$\frac{1}{R_{middle}} = \frac{1}{R_{bar}} + \frac{1}{R_{insulation}}$$

$$\Rightarrow R_{middle} = 0.1805\ \frac{K}{W}$$

(ii) Calculate R_{tot}

$$R_{tot} = 2R_{plate} + R_{middle} = 0.1809\ \frac{K}{W}$$

(iii) Calculate Unit Cell $\dot{Q}$

$$\dot{Q}_{unit\ cell} = \frac{\Delta T_{tot}}{R_{tot}} = \frac{22°C}{.1809\frac{K}{W}} = 121.5\ W$$

Change so bars not included

(iv) Calculate $\dot{Q}_{tot}$

$$\dot{Q}_{tot} = 4\dot{Q}_{uc} = 486\ W$$

Example 17-125 (continued)

R_{plate} — R_{middle} — R_{plate}

T_1 — T_2 — T_3 — T_4

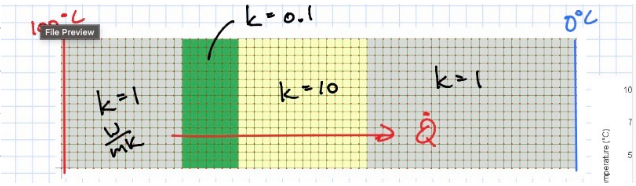
$\dot{Q}_{wc} = \frac{T_1 - T_2}{R_{plate}} \Rightarrow \Delta T_{plate} = 0.027°C$

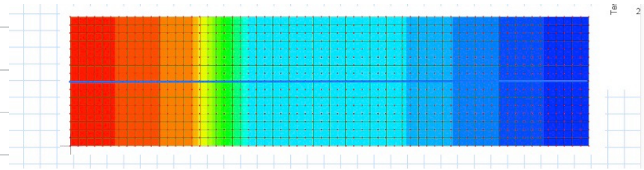
$\dot{Q} = \frac{T_2 - T_3}{R_{middle}} \Rightarrow \Delta T_{middle} = 21.98°C$

(i) $\dot{Q}_{bar} = \frac{T_2 - T_3}{R_{bar}} = 98.8W$ & (ii) $\dot{Q}_{insulation} = \frac{T_2 - T_3}{R_{insulation}} = 22.8W$

Bar takes up 1% of area but transfers 81% of the heat through the middle => thermal bridge.

- Visualize with Finite - Element Model



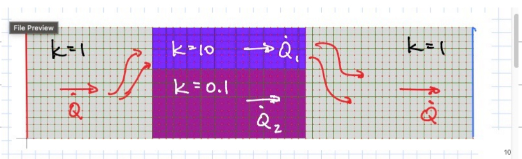


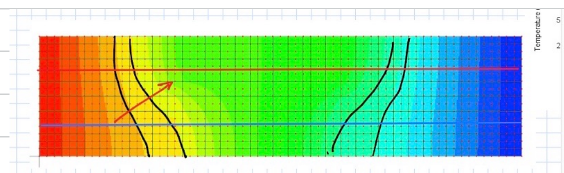
- Resistances in Parallel

a) $\dot{Q}$s add

b) ΔT the same

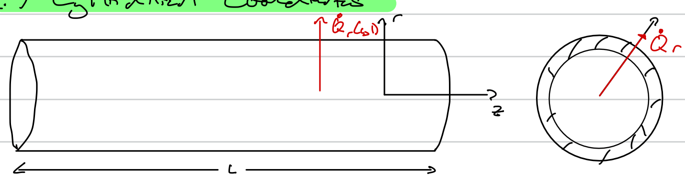
$\Rightarrow \frac{1}{R_{tot}} = \sum_i \frac{1}{R_i}$





Conduction in Radial Systems

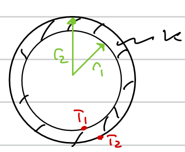
(i) Cylindrical Coordinates



- Conduction (still) governed by Fourier's Law

$$\dot{Q}_r = -kA_c \frac{dT}{dr} \text{ ; if still steady} \longrightarrow \dot{Q} \text{ constant}$$

$$\Rightarrow \dot{Q}_r = -k\, 2\pi r L \frac{dT}{dr}$$



$$\int_{r_1}^{r_2} \frac{\dot{Q}}{2\pi L} \frac{dr}{r} = -k \int_{T_1}^{T_2} dT$$

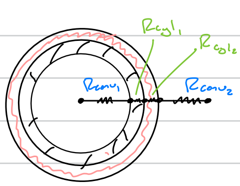
$$\frac{\dot{Q}}{2\pi L} \ln\left(\frac{r_2}{r_1}\right) = k(T_1 - T_2) \Rightarrow \dot{Q}_r = \frac{\Delta T}{R_{cyl}} = \underbrace{\boxed{\frac{2\pi L k}{\ln(r_2/r_1)}}}_{} (T_1 - T_2) \text{ ; } R_{cyl} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi L k}$$

$\frac{1}{R_{cyl}}$

(ii) Spherical Coordinates ($A_c = 4\pi r^2$)

$$\Rightarrow R_{sph} = \frac{r_2 - r_1}{4\pi\, r_1 r_2 k}$$

- Radial thermal Resistances can also be in series

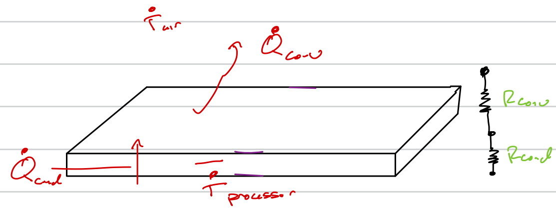


Finned Heat Transfer

Applications

(i) Electronics cooling
(ii) Heat Exchangers
→ Car Radiator
→ Condenser, Evaporator of HVAC systems
(iii) Extremities
→ Ears
→ Toes, Fingers
→ Nose

• Consider simple resistance network



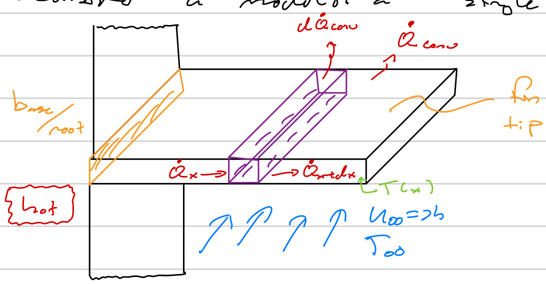
Thickness

$$R_{cond} = \frac{t}{kA_c} = \frac{10^{-4}}{A_c}$$

$$R_{conv} = \frac{1}{hA_s} = \frac{10^{-1}}{A_c}$$

1000×

• Consider a model of a simple fin



• Energy balance on a differential element

$$\dot{Q}_{in} - \dot{Q}_{out} = \cancel{\frac{dE}{dt}}^{0} \text{ steady}$$

$$\dot{Q}_x - (\dot{Q}_{x+dx} + d\dot{Q}_{conv}) = 0$$

$$d\dot{Q}_{conv} = h\,dA_s[T(x) - T_\infty]$$

$$= h\underbrace{P\Delta x}_{dA_s}[T(x) - T_\infty]$$

Perimeter

$$\frac{\dot{Q}_{x+dx} - \dot{Q}_x}{\Delta x} + hP[T(x) - T_\infty] = 0$$

• As $\Delta x \to 0$: $\frac{d\dot{Q}_{cond}}{dx} + hP[T(x) - T_\infty] = 0$

• Represent $\dot{Q}_{conduction}$ using Fourier's Law: $\dot{Q}_{cond} = -kA_c\frac{dT}{dx}$

$$\Rightarrow \frac{d}{dx}\left(kA_c\frac{dT}{dx}\right) - hP[T(x) - T_\infty] = 0$$

• For constant k, A_c:

$$\frac{d^2T}{dx^2} - \underbrace{\boxed{\frac{hP}{kA_c}}}_{\text{all constants, let } m^2}[T(x) - T_\infty] = 0$$

• Introduce a change of variables: $\theta(x) = T(x) - T_\infty \Rightarrow \frac{d^2\theta}{dx^2} - m^2\theta(x) = 0$

{ 2nd order, ODE
{ Homogeneous, const. coefficients

• Solution to ODE: combination of exponentials

$$\theta(x) = c_1e^{mx} + c_2e^{-mx}$$

← need to find c_1, c_2 from boundary conditions

Boundary Conditions

i) Assume infinitely-long fins

a) $T(x \to \infty) \to T_\infty \Rightarrow \theta(x \to \infty) \to 0 \Rightarrow c_1 = 0$

b) $T(x=0) \to T_b \Rightarrow \theta(x=0) = T_b - T_\infty = \theta_b$

$$\Rightarrow \frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-mx} \quad \text{where} \quad m = \sqrt{\frac{hP}{kA_c}} \Rightarrow T(x) = (e^{-mx})(T_b - T_\infty) + T_\infty$$

$$\dot{Q} = \sqrt{hPkA_c}\,(T_b - T_\infty)$$

• Heat dissipated by the fin:

$$\dot{Q}\Big|_{x=0} = -K A_c \frac{dT}{dx}\Big|_{x=0} = \sqrt{hPKA_c}\,(T_b - T_\infty)$$

• Consider a second boundary condition case:

2) Adiabatic fin tip, finite length L

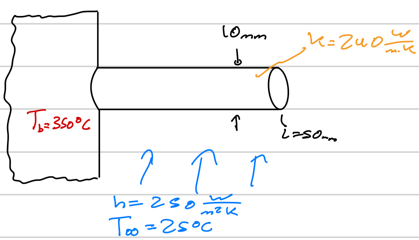
a) $\dot{Q}\big|_{x=L} = 0 = -K A_c \frac{dT}{dx}\big|_{x=L} \Rightarrow \frac{d\theta}{dx}\big|_{x=L} = 0$

b) $\theta(0) = \theta_b$

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh[m(L-x)]}{\cosh(mL)}$$

$$\dot{Q}\big|_{x=0} = \sqrt{hPKA_c}\,(T_b - T_\infty)\tanh(mL)$$

Example 17-127



1) Find $\dot{Q}$ for:

a) Infinite fin assumption

b) Adiabatic fin tip

2) Plot temperature profiles

↳ From plot infinite fin assumption is not great as the end tip temp is not the expected 25°C

a) Infinite fin: $\dot{Q} = \sqrt{hPKA_c}\,(T_b - T_\infty)$

$$\left.\begin{aligned} P &= \pi D \\ A_c &= \frac{\pi D^2}{4} \end{aligned}\right\} \quad \dot{Q} = 125W$$

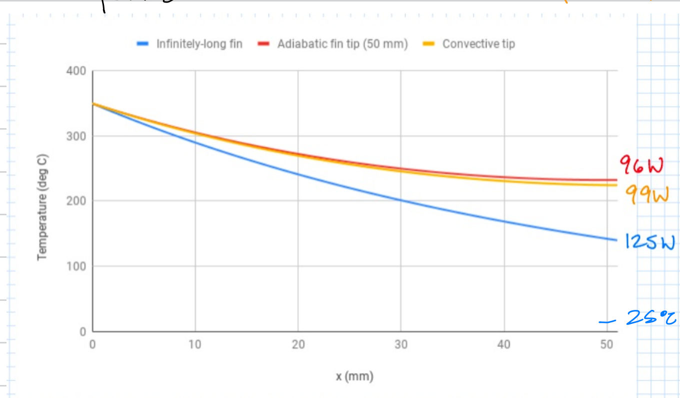
b) Adiabatic fin: $\dot{Q} = \boxed{\sqrt{hPKA_c}\,(T_b - T_\infty)}\tanh(mL)$

tanh always less than 1

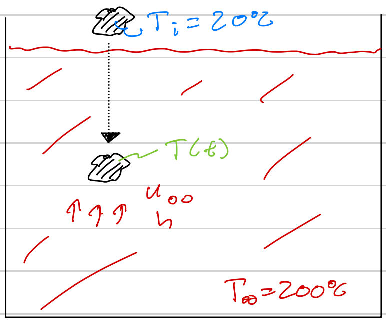
$$m = \sqrt{\frac{hP}{KA_c}} \quad \Big\} \quad \dot{Q} = 96.3W$$

2) Plot Temperature over time

Convective tip = best/most realistic assumption



Transient Heat Transfer

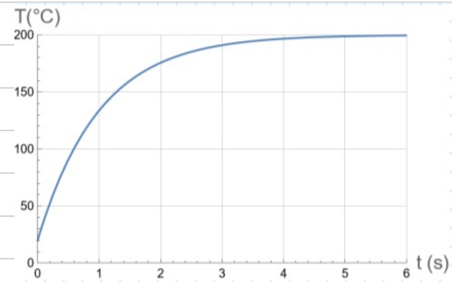


Lumped System Analysis

→ Assume temperature only varies with time: $T = T(t)$ ← Temperature as a function of time

- Factors that influence this assumption
  - (i) Size, shape (geometric attributes)
  - (ii) Material properties ~ k
  - (iii) Environment Properties ~ h

Analysis



↳ Start with energy balance (1st Law)

$$\underbrace{\dot{Q}_{in}}_{\text{Convection}} - \cancel{\dot{Q}_{out}} = \frac{d\boxed{E}}{dt} \quad E \sim \underbrace{U}_{\text{Internal Energy}} + \cancel{KE} + \cancel{PE}$$

$$\Rightarrow h A_s [T_\infty - T(t)] = \frac{dU}{dt} \overset{\text{bc solid obj}}{=} \frac{d}{dt}(\rho \underset{\text{volume}}{V} \underset{\text{specific heat}}{c} T)$$

$$h A_s [T_\infty - T(t)] = \rho V c \frac{dT}{dt}$$

$$\frac{dT}{T - T_\infty} = -\frac{hA_s}{\rho V c} dt \rightsquigarrow \text{Integrate from } t=0 \text{ to } t=t \quad \int_{T_i}^{T} \frac{dT}{T-T_\infty} = -\frac{hA_s}{\rho V c} \int_0^t dt$$

$$\ln(T - T_\infty)\Big|_{T_i}^{T} = -\frac{hA_s}{\rho V c} t \Big|_0^t$$

$$\therefore \ln\left[\frac{T - T_\infty}{T_i - T_\infty}\right] = -\underbrace{\boxed{\frac{hA_s}{\rho V c}}}_{b} t \Rightarrow \frac{T - T_\infty}{T_i - T_\infty} = e^{-bt} \quad \text{where } b = \frac{hA_s}{\rho V c} \text{ \& assumptions hold}$$

- Rate of heat transfer to our object

$$\dot{Q}(t) = h A_s [T_\infty - T(t)] \quad \text{will decrease with time}$$

- Maximum possible energy change

$$\dot{Q}_{max} = mc[T_\infty - T_i]$$

- Total heat transfer between $t=0$ and t_f

$$\dot{Q}_{total} = mc[T(t_{final}) - T_i]$$

Valid conditions for lumped system analysis

- Define a dimensionless quantity
  → Conduction within the body vs. convection to the body

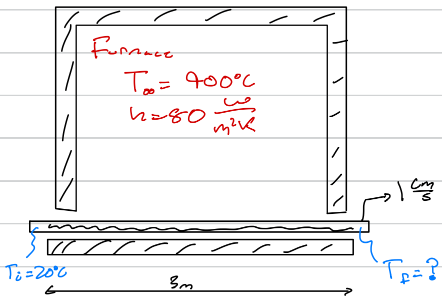
$$\frac{\text{Convection @ surface}}{\text{Conduction within}} = \frac{h A_s \cancel{\Delta T}}{k A_c \frac{\cancel{\Delta T}}{L_c}} = \frac{h L_c}{k} = Bi \text{ aka Biot \#} \quad \text{where } L_c = \frac{V}{A_s} \begin{array}{l} \sim \text{Volume of object} \\ \sim \text{surface area of object} \end{array}$$

(Newton's law of cooling → Convection; Fourier's Law → Conduction)

- Lumped System Analysis valid for small Bi #
  - $Bi \to 0 \Rightarrow$ completely uniform temperature
  - Calculations show that lumped system analysis (LSA) valid for $Bi < 0.1$

In this class, if you're $Bi \# > 0.1$, you probably did something wrong ☺

Example 18-18

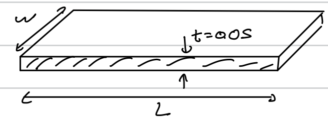


Stainless Steel Object:

- $k = 21 \frac{W}{mK}$
- $\rho = 8900 \frac{kg}{m^3}$
- $C_p = 570 \frac{J}{kgK}$
- 5mm Thick

Find object exit temperature T_f

- Lumped system analysis valid



$$B_i = \frac{hL_c}{k} \; ; \; L_c = \frac{V}{A_s} = \frac{\cancel{L}wt}{2\cancel{L}w + 2\cancel{L}t}$$

$$= \frac{wt}{2(w+t)} \quad \text{assume } t \ll w$$

$$= \frac{t}{2} = 2.5 \, mm$$

$$\Rightarrow B_i = \frac{80\left(\frac{2.5}{1000}\right)}{21} = 0.00952 \quad \text{valid ✓}$$

$$\therefore \frac{\overset{T_f}{T} - \overset{900°C}{T_\infty}}{\underset{20°C}{T_i} - \underset{900°C}{T_\infty}} = e^{-bt} \qquad b = \frac{h A_s}{\rho V c} \quad \frac{A_s}{V} = \frac{1}{L_c}$$

- Calculate b

$$b = \frac{h}{\rho L_c c} = 0.00702 \, \frac{1}{s}$$

- 3m long @ $1 \frac{cm}{s}$

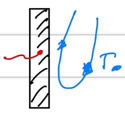
$$\frac{300 \, cm}{1 \frac{cm}{s}} = 300s \text{ total}$$

$$\Rightarrow \boxed{T_f = T(t = 300s) = 793°C}$$

Convection & External Flow

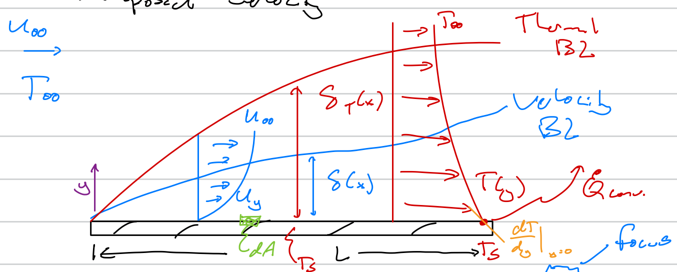
1) Natural / Free Convection

- Flow is induced by temperature gradients



2) Forced Convection

→ Imposed velocity



- Define BL thickness

$$u_{BL} = 0.99\, u_\infty \Rightarrow y = \delta$$

$$T_{BL} = T_s + 0.99(T_\infty - T_s) \Rightarrow y = \delta_T$$

- For Convection: $\dot{Q}_{convection} = \{h\} A_s [T_s - T_\infty]$
- Apply a surface energy balance

$$\dot{Q}_{conduction} = \dot{Q}_{convection}$$

local value, varies w/ x

$$-k_{fluid}\, dA \frac{dT}{dy}\Big|_{y=0} = h_x\, dA (T_s - T_\infty)$$

$h \uparrow$ as u_∞

- solve for h_x: $h_x(x) = \dfrac{-k_{fluid} \boxed{\frac{dT}{dy}\big|_{y=0}}}{T_s(x) - T_\infty}$

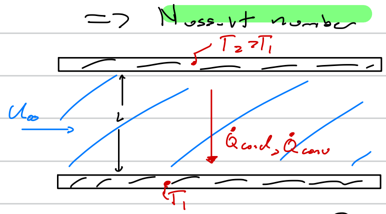
- Generally more interested in overall effect

→ Average heat transfer coefficient

$$h_{avg} = \frac{1}{L}\int_0^L h_x\, dx$$

- We will use correlations (mostly empirical)

⇒ Nusselt number



- Driving Potential

$$\Delta T = T_2 - T_1$$

→ write conduction & convection on a flux basis

$$q''_{conv} = h\Delta T = \frac{\dot{Q}_{conv}}{A_s}$$

$$q''_{cond} = k_{fluid}\frac{\Delta T}{L}$$

Take the ratios

$$\frac{q''_{conv}}{q''_{cond}} = \frac{h\,\Delta T}{k_{fluid}\frac{\Delta T}{L}} = \frac{L h}{k_{fluid}} = Nu_L$$

$$\frac{x\, h_x}{k_{fluid}} = Nu_x$$

{ $Nu = 1 \Rightarrow$ recover cond. limit; $\uparrow Nu \Rightarrow$ strong conv. enhancement }

- Reynolds # will be important

↳ Need another parameter ⇝ describe relative size of the two BLs (thermal vs. velocity BL)

1) Viscosity affects the velocity BL
2) Thermal Diffusivity affects the thermal BL

⇒ Second Parameter: Prandtl #

diffusion of momentum

$$Pr = \frac{\text{viscosity}}{\text{diffusivity}} = \frac{\nu}{\alpha}$$

ν ~ kinematic viscosity $\nu = \frac{\mu}{\rho}$ $\left\{\frac{m^2}{s}\right\}$

α ~ thermal diffusivity $\alpha = \frac{k}{\rho c}$ $\left\{\frac{m^2}{s}\right\}$

↳ diffusion of heat

→ Typical values:

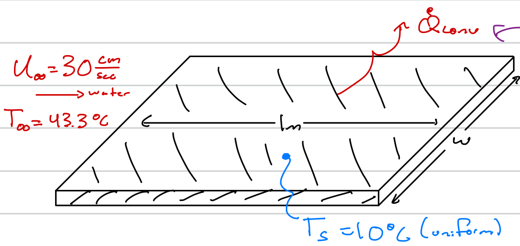
1) $Pr \approx 1$ ⇒ gases (air)
2) $Pr > 1$ ⇒ oils, water
3) $Pr < 1$ ⇒ liquid metals

Nussel Number (Continued)

- Correlations structure: $Nu = f(Re, Pr) \Rightarrow h$
- Correlations (constant surface temperature T_s), flat plate
  1) Local
     a) Laminar: $Nu_x = \frac{h_x \cdot x}{k_{fluid}} = 0.332 \cdot Re_x^{1/2} \cdot Pr^{1/3}$
     b) Turbulent: $Nu_x = 0.0296 \cdot Re_x^{4/5} \cdot Pr^{1/3}$
  2) Average
     also denoted $\bar{h}$ or h
     a) Laminar: $Nu = \frac{h_{avg} L}{k_{fluid}} = 0.664 \cdot Re_L^{1/2} \cdot Pr^{1/3}$
     most likely
     b) Turbulent: $Nu = 0.037 \cdot Re^{4/5} \cdot Pr^{1/3}$
     c) Laminar + Turbulent: $Nu = (0.037 Re^{4/5} - 871) \cdot Pr^{1/3}$

Example 19-16

- Flow of hot water over a cold flat plate (consider only top surface)



$\dot{Q}_{conv} = h A_s (T_s - T_\infty)$

$= h L w (T_s - T_\infty)$

$\frac{\dot{Q}_{conv}}{w} = hL(T_s - T_\infty)$

Even though $\dot{Q}$ is into plate its ok since we'll just get a negative $\dot{Q}_{conv}$

- Evaluate Properties @ film temp.

$T_f = \frac{T_s + T_\infty}{2} = 26.6\bar{6}°C$

film temp avg.

- From table A-15:

$\rho = 996.7 \frac{kg}{m^3}$

$\mu = 8.6 \times 10^{-4} \frac{kg}{m \cdot s}$

$Pr = 5.9$

$k = 0.61 \frac{W}{m \cdot K}$

- Find rate of heat transfer per unit width

- Calculate Re_L

$Re_L = \frac{\rho U_\infty L}{\mu} = 3.5 \times 10^5 \Rightarrow$ Laminar BL over entire plate $\Rightarrow$

- Laminar Correlation

$Nu = 0.664 Re_L^{1/2} Pr^{1/3} = 707.4$

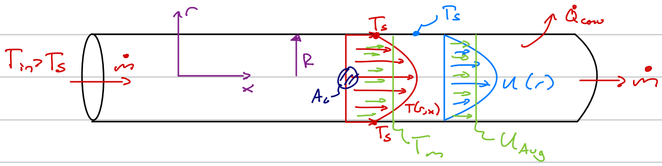
- Calculate h from $Nu = \frac{hL}{k_{fluid}}$

$\Rightarrow h = \frac{Nu \, k_{fluid}}{L} = 431.3 \frac{W}{m^2 K}$

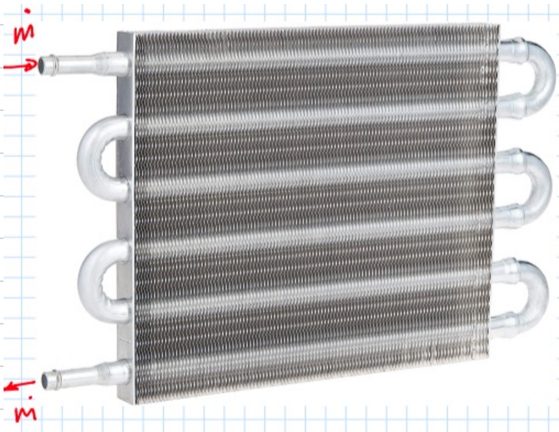
- Solve: $\frac{\dot{Q}_{conv}}{w} = hL(T_s - T_\infty)$

$= \boxed{-14{,}360 \frac{W}{m}}$

Convection & Internal Flow



Can be applied to heat exchangers, for example



- Energy contained in the fluid as:
  - (i) Internal Energy
  - (ii) Kinetic Energy, KE
  - Potential Energy, PE
  - (iii) Flow Work Energy

  ⟶ enthalpy

- For true liquids: $\Delta h = c_p \Delta T$

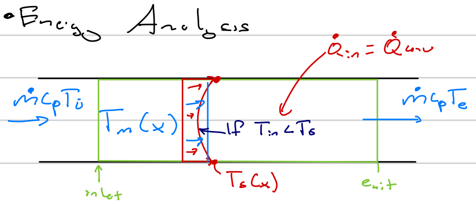
- Mean temperature - track the energy flowing through a cross-section of the pipe

$$\dot{E}_{fluid} = \dot{m} c_p T_m = \int_{A_c} c_p T(r) \underbrace{\rho u(r) dA_c}_{d\dot{m}} \quad \leftarrow \text{general description}$$

→ For a circular pipe:

$$T_m = \frac{1}{\dot{m} c_p} \int_0^R \rho c_p T(r) u(r) \underbrace{2\pi r dr}_{dA_c} = \frac{2}{u_{avg} R^2} \int_0^R T(r) u(r) r dr$$

- Energy Analysis



→ 1st Law, open system

$$\dot{Q}_{in} - \cancel{\dot{Q}_{out}}^{\,0} = \dot{m}(h_e - h_i) \quad \text{(net in)}$$

- $\dot{Q}_{in}$ due to convection

$$\dot{Q}_{in} = h A_s [T_s(x) - T_m(x)]$$

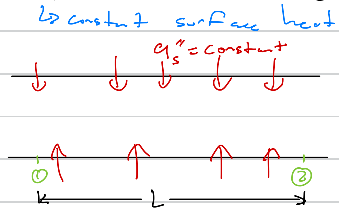
- $\dot{Q}_{in}$ on a flux basis (per unit area)

$$q''_{in} = h_x [T_s(x) - T_m(x)]$$

[generally constant but can be local]

- Consider two possible cases
  1) q''_s constant (along x) ⟶ constant $\dot{Q}$, solar radiation

     ⟶ constant surface heat flux



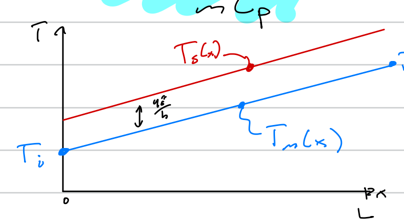
$$\dot{Q}_{in} = \underbrace{q''_s}_{constant} A_s = \dot{m} c_p (T_e - T_i)$$

- Solve for exit Temperature

$$T_e = T_i + \frac{q''_s A_s}{\dot{m} c_p} = T_i + \frac{q''_s P L}{\dot{m} c_p}$$

- Or generally

$$T_m(x) = \frac{q''_s P x}{\dot{m} c_p}$$



- What about T_s?

  → Newton's Law of Cooling: $q''_s = h[T_s(x) - T_m(x)]$

  [constant if thermally fully developed]

$$\Rightarrow T_s(x) = T_m(x) + \frac{q''_s}{h}$$

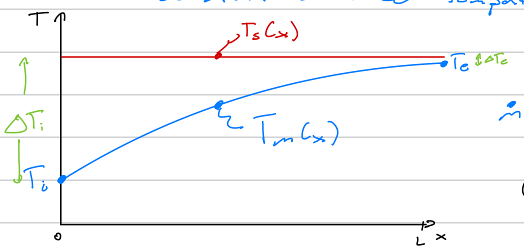
- Determining h for laminar flow in a circular pipe

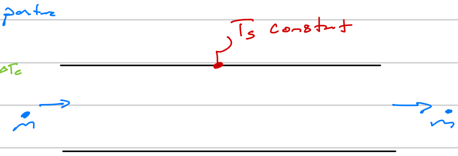
  → For q''_s constant ⟹ $h = \frac{48}{11} \frac{k_{fluid}}{D}$

$$\Rightarrow Nu_D = \frac{hD}{k_{fluid}} = \frac{48}{11} \approx 4.36$$

2) T_s constant (along x) ⟿ ducts, submerged pipes

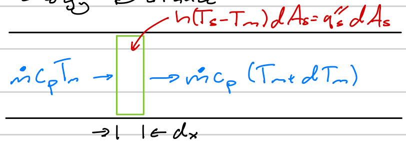
↳ Constant surface temperature





$$\dot{Q}_{conv} = hA_s[T_s - T_m(x)]$$

• Energy Balance



$$\cancel{\dot{m}c_pT_m} - \dot{m}c_p(\cancel{T_m} + dT_m) + q_s''\, dA_s = 0$$

$$\dot{m}c_p\, dT_m = h(T_s - T_m)\,dA_s = h(T_s - T_m)P\,dx$$

→ Integrate from inlet to the exit (i) to (e)

$$\ln\left(\frac{T_s - T_e}{T_s - T_i}\right) = -\frac{h\,\underbrace{PL}_{A_s}}{\dot{m}c_p}$$

$$\Rightarrow T_e = T_s - (T_s - T_i)e^{-\frac{hPL}{\dot{m}c_p}}$$

• Or Generally:

$$T_m(x) = T_s - (T_s - T_i)e^{-\frac{hPx}{\dot{m}c_p}}$$

• h for constant T_s, laminar flow, circular pipe ⇒ $Nu_D = 3.66$

$$\dot{Q}_{in} = hA_s\underbrace{[T_s - T_m(x)]}_{\Delta T_{avg}?} = \dot{m}c_p(T_e - T_i)$$

$$\Delta T_{avg} = \Delta T_{lm} = \frac{\Delta T_e - \Delta T_i}{\ln\left(\frac{\Delta T_e}{\Delta T_i}\right)}$$

log-mean (lm) temperature diff

Laminar Flow for other cross sections

TABLE 19–3

Nusselt number and friction factor for fully developed laminar flow in tubes of various cross sections ($D_h = 4A_c/p$, $Re = V_{avg}D_h/\nu$, and $Nu = hD_h/k$)

$$D_H = \frac{4A_c}{P}$$

Tube Geometry	a/b or θ°	Nusselt Number T_s = Const.	Nusselt Number $\dot{q}_s$ = Const.
Circle	—	3.66	4.36
Rectangle	a/b		
	1	2.98	3.61
	2	3.39	4.12
	3	3.96	4.79
	4	4.44	5.33
	6	5.14	6.05
	8	5.60	6.49
	∞	7.54	8.24
Ellipse	a/b		
	1	3.66	4.36
	2	3.74	4.56
	4	3.79	4.88
	8	3.72	5.09
	16	3.65	5.18
Isosceles Triangle	θ		
	10°	1.61	2.45
	30°	2.26	2.91
	60°	2.47	3.11
	90°	2.34	2.98
	120°	2.00	2.68

Turbulent Flow ($Re_D > 2300$)

• Empirical correlations:

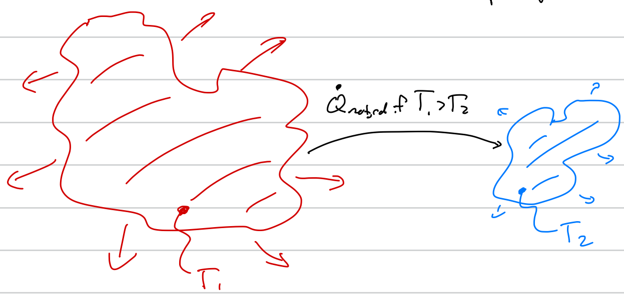
1) Dittus-Boelter Equation

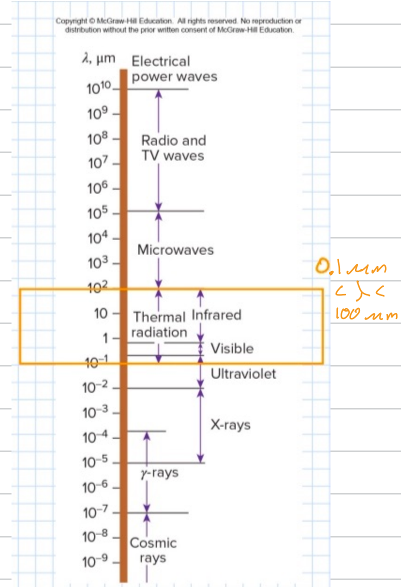
$$Nu_D = 0.023 \cdot Re_D^{4/5} \cdot Pr^n$$

where $\begin{cases} n = 0.4 & \text{: if fluid is heated} \\ n = 0.3 & \text{: if fluid is cooled} \end{cases}$

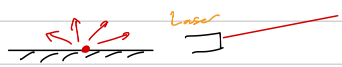
Thermal Radiation

- Radiation requires no medium
- All objects at non-zero (absolute) temperature emit radiation
- Amount of emitted radiation proportional to temperature





- Thermal Radiation is a subset of the full electromagnetic spectrum
- Effects that affect radiative transfer
  (i) Temperature
  (ii) Geometry, direction



  (iii) Wavelength
  (iv) Material Properties

Blackbody Radiation

- Perfect emitter and absorber
  a) Emits maximum radiation at a given T
  b) Absorbs all incident radiation
  c) Diffuse/uniform radiations

Examples:
(i) Sun - behaves like a blackbody at 5800K   temp specific
(ii) Snow
(iii) White paint
} behave as black bodies in infrared   wavelength specific

- Blackbody emissive power

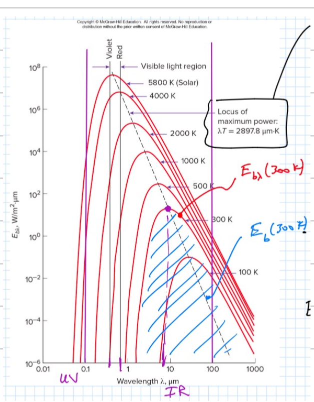
$$E_b = \sigma T_s^4$$ where σ is Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$

$\{\frac{W}{m^2}\}$ Temperature needs to be absolute ~ Kelvin!!!

- Wavelength dependence (+ temperature)
  - Different amounts of radiation given off at different wavelengths
  ⇒ Spectral blackbody emissive power

$$E_{b,\lambda}(\lambda, T) = \frac{C_1}{\lambda^5 \left[\exp\left(\frac{C_2}{\lambda T}\right) - 1\right]}$$

where: $C_1 = 3.74 \times 10^8 \frac{W \mu m^2}{m^2} = 2\pi h C_0^2$ for wavelengths in microns (μm)
$C_2 = 1.44 \times 10^4 \ \mu m \cdot K = \frac{hC_0}{k}$



Wien's Displacement Law

$$(\lambda T)_{max\,power} = 2897.8 \ \mu m \cdot K$$

Example: Room temperature (298K)

$$\lambda = \frac{2897.8}{298} = 9.72 \ \mu m$$

- Total emission usually most important

$$E_b(T) = \int_0^\infty E_{b,\lambda} \, d\lambda = \sigma T_s^4$$

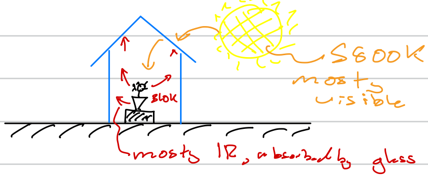
• Two primary types of materials:

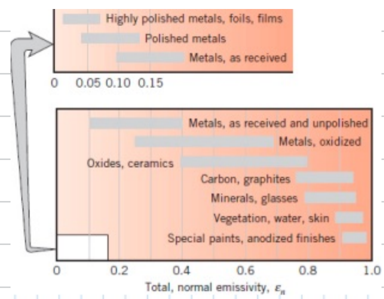
(i) Opaque Materials

- All radiation absorbed over the first few microns of the surface
- Examples: wood, bricks, metals

(ii) Semi-transparent materials

- Transparency can be wavelength dependent
- Examples: Glass, water transparent in visible spectrum but opaque in the infrared (IR)
- Greenhouse Effect





Radiative Properties

(i) Emissivity (ε)

- Real bodies emit a fraction of the blackbody emissive power

$$\varepsilon(T) = \frac{E(T)}{E_b(T)}$$

$E(T)$ — Actual emissive power

$E_b(T)$ — Ideal (blackbody) emissive power

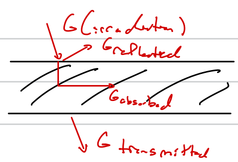
- Most materials $\varepsilon \in (0.85, 0.95)$

$\Rightarrow$ Emissive Power: $E_{emitted} = \varepsilon \sigma T_s^4 \; \{\frac{W}{m^2}\}$

• Properties for incident radiation (irradiation)

(ii) Absorptivity (α)

$$\alpha = \frac{G_{absorbed}}{G}$$



(iii) Reflectivity (ρ)

$$\rho = \frac{G_{reflected}}{G}$$

(iv) Transmissivity (τ)

$$\tau = \frac{G_{transmitted}}{G}$$

Energy Balance on this material

$$G = \alpha G + \rho G + \tau G$$

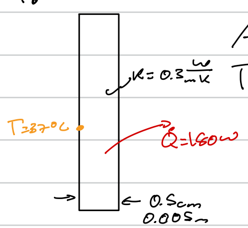
$$1 = \alpha + \rho + \tau$$

- If opaque ($\tau = 0$)

$$\alpha + \rho = 1$$

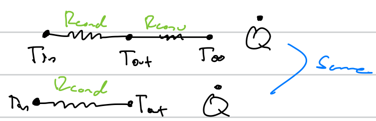
Homework 11 Problems

1.



$A_s = 1.5\ m^2$

$T_\infty = 20°C$

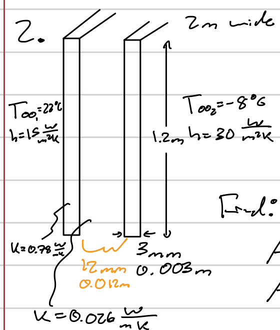


$\dot{Q} = kA_s \frac{(T_1 - T_2)}{L}$

$180W = (0.3 \frac{W}{mK})(1.5 m^2)\left(\frac{37°C - T_2}{0.005m}\right)$

$T_2 = 35°C$

2.



$R_{conv,1} = \frac{1}{hA_s} = \frac{1}{15 \frac{W}{m^2K}(2.4 m^2)} = 0.0278 \frac{K}{W}$

$R_{cond,glass} = 2 \cdot \frac{L}{kA_c} = 2 \cdot \frac{0.003m}{0.78 \frac{W}{mK}(2.4m^2)} = 0.00321 \frac{K}{W}$

$R_{cond,air} = \frac{L}{kA_c} = \frac{0.012m}{0.026 \frac{W}{mK}(2.4m^2)} = 0.1923 \frac{K}{W}$

$R_{conv,2} = \frac{1}{hA_s} = \frac{1}{30 \frac{W}{m^2K}(2.4m^2)} = 0.01389 \frac{K}{W}$

Find: $\dot{Q}, T_1$

$R_{tot} = 0.2372 \frac{K}{W}$

$A_s = 2m \times 1.2m = 2.4m^2$

$A_c = A_s$

$\dot{Q} = \frac{\Delta T}{R}$

$\dot{Q} = (30°C)(0.2372 \frac{°C}{W})^{-1}$

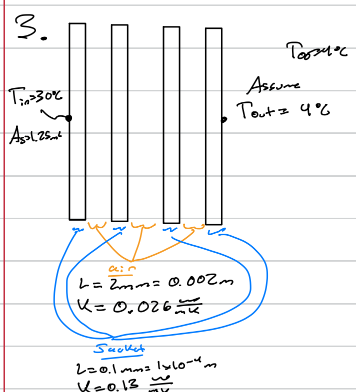
$= 126.476$

$\dot{Q} = 126.476$

$\dot{Q} = \frac{T_{\infty,1} - T_1}{R_{cond,1}} = 2(R_{cond,1}\dot{Q} - T_{\infty,1}) = T_1$

$T_1 = 18.48°C$

3.



$R_{cond,air} = \frac{0.002m}{(0.026 \frac{W}{mK})(1.25m^2)} = 0.0615 \frac{K}{W}$

$R_{cond,jacket} = \frac{1\times10^{-4}m}{(0.13 \frac{W}{mK})(1.25m^2)} = 0.000615 \frac{K}{W}$

$R_{tot} = 3R_{cond,air} + 4R_{cond,jacket} = 0.187 \frac{K}{W}$

$\dot{Q} = \frac{26°C}{0.187 \frac{K}{W}} = 139.037W$

$R_{cond,fabric} = \frac{4\times10^{-4}m}{(0.13 \frac{W}{mK})(1.25m^2)} = 0.00246 \frac{K}{W}$

$\dot{Q} = \frac{26°C}{0.002462 \frac{K}{W}} = 10560.5 W$

$139.037W = (0.035 \frac{W}{mK})(1.25m^2)\frac{26°C}{L}$

$L = 0.00818m = 8.18mm$

4.

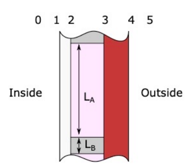


Figure 1: Schematic for Problem 4

$R_{inside} = \frac{0.01m}{(0.5 \frac{W}{mK})(0.00605m^2)} = 3.306 \frac{K}{W}$

$R_{outside} = \frac{0.1m}{(1.0 \frac{W}{mK})(0.0605m^2)} = 1.653 \frac{K}{W}$

$R_{bar} = \frac{0.08m}{(50 \frac{W}{mK})(4\times10^{-4}m^2)} = 4 \frac{K}{W}$

$R_{insul} = \frac{0.08m}{(0.03 \frac{W}{mK})(0.048m^2)} = 55.56 \frac{K}{W}$

$\frac{1}{R_{middle}} = \frac{1}{R_{bar}} + \frac{1}{R_{insul}}$

$\frac{1}{R_{middle}} = 0.2679$

$R_{middle} = 3.73 \frac{K}{W}$

$R_{tot} = 8.689 \frac{K}{W}$

$L = L_A + L_B = 0.605m$

$A_{12} = (0.605m)(0.01m) = 0.00605m^2$

$A_{34} = (0.605m)(0.10m) = 0.0605m^2$

$A_{bar} = 4\times10^{-4} m^2$

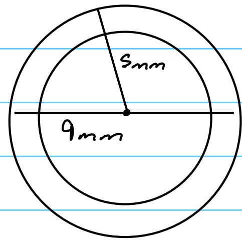
$A_{insul} = 0.048 m^2$

$A_{tot} = 0.1149 5m^2$

$\dot{Q} = \frac{35°C - 20°C}{8.689 \frac{°C}{W}} = 1.7263$

$\dot{q} = \frac{\dot{Q}}{A_{tot}} = \frac{1.7263W}{0.11493m^2} = 15.02 \frac{W}{m^2}$

5.



$$\dot{Q}_{no\ insulation} = hA_s \Delta T = 20\,\frac{W}{m^2K}\left(4\pi\left(\frac{0.009m}{2}\right)^2\right)(35°C) = 0.178\ W$$

$$\dot{Q}_{insulated} = \frac{35°C}{R_{conv,1} + R_{cond,1}} = 0.2026\,W$$

$$R_{conv,1} = \frac{1}{hA_s} = \frac{1}{\left(20\,\frac{W}{m^2K}\right)\left(4\pi\left(\frac{0.01m}{2}\right)^2\right)} = 159.155\,\frac{K}{W}$$

$$R_{cond,1} = \frac{4\pi r_1 r_2 k}{r_2 - r_1} = \left(\frac{4\pi(0.005m)(0.0045m)(0.13\,\frac{W}{mK})}{0.005m - 0.0045m}\right)^{-1} = 13.603\,\frac{K}{W}$$

Insulation will cause heat transfer to increase bc