



Exam 1 Notes



Micro vs. Macroscale

Thought Exercise

- Refrigerator is closed, insulated room
• Fridge Turned on at t=0



1) Closed Fridge up same down
a) Fridge Temperature? \uparrow - \downarrow Inside cools

b) Room Temperature? \uparrow - \downarrow Room warms
Room heats due to cooling process

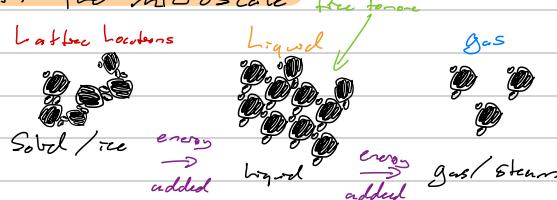
2) Open Fridge

a) Average Room Temp? \uparrow - \downarrow no Fridge, room cools
energy needed to

Two perspectives on energy

- intuition
1) Microscale: Behavior of atoms, molecules, electrons, etc.
2) Macroscale: Pumps, turbines, power plants, etc.

At the microscale



Takeaway

Structure @ Macroscale
dictates Macro state
of energy

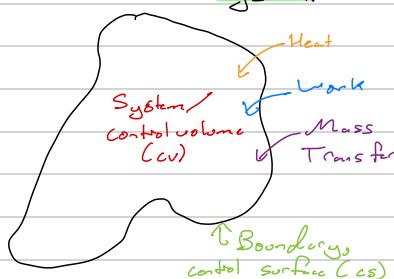
→ Modes of energy storage @ the microscale

- 1) Translational Motion (KE)
 - 2) Vibrational Energy
 - 3) Rotational Motion
 - 4) Electron Translation
 - 5) Phase Changes
 - 6) Chemical Bonds
 - 7) Nuclear Bonds
- 1-4: Sensible modes. Related to activity/motion. Related closely to temperature
- Sum up \rightarrow Changes to internal energy
- U or u
- | | |
|----------------|---------------------|
| Capital U | Total Energy |
| Lower-case u | kJ/kg |
| mass basis | J/kg , ... |

5-7: Latent modes. Tend to structural changes

→ Modeling Energy @ the macroscale

Define a system:

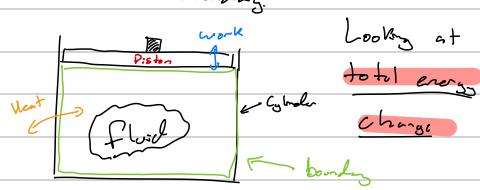


Energy can cross boundary in 3 ways:

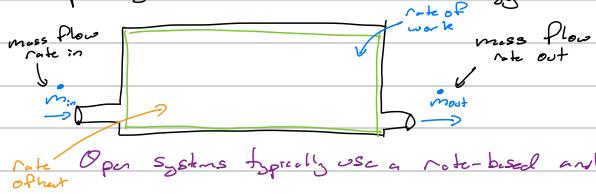
- 1) Heat
- 2) Work
- 3) Mass Transfer

Open vs. Closed Systems

Closed System: No mass transfer across boundary.



Open System: All forms of energy transfer are possible



$$\dot{m} = \rho V_{avg} A_C$$

Mass flow rate (\dot{m}):
 \dot{m} = density ρ \times average velocity V_{avg} \times cross-section area A_C

Application: Wind Turbine



If we opened up the system to the ground it would make a big difference bc the unaffected wind cancels itself out

First Law of Thermodynamics

→ **Conservation of Energy:** Energy cannot be created nor destroyed

→ Can also write total energy as a mass-specific basis:

$$\frac{E}{m} = e = u + \frac{1}{2} m v^2 + g z$$

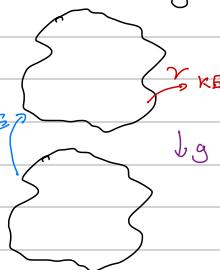
↓ KE ↓ PE ↓ elevation

- 1) Energy stored within a system
- 2) Energy transfer across the boundary

① Stored Energy

$$E = PE + KE + U$$

↓ KE ↓ PE ↓ Internal Energy (IE)



② Energy crossing Boundary

- a) Heat Transfer (Q) - energy transfer due to a temperature difference
- b) Work (w) - rate of heat transfer \dot{Q} : Total Heat Transfer over some time t

$$\dot{Q} = \text{rate of heat transfer} \leftarrow \dot{E}_{KE}, w = \frac{\dot{Q}}{t}$$

Modes of Heat Transfer

- 1) **Conduction:** energy transfer from higher to lower energetic matter due to direct interaction
- 2) **Convection:** enhanced conduction due to fluid motion
- 3) **Radiation:** energy transfer via electromagnetic waves
 ↗ Physics definition: $w = Fd$; force acting over a distance

Work (w): energy transfer w/o a temperature difference

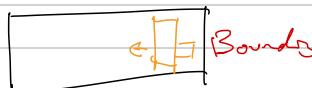
w : Total work over some time \dot{w}

\dot{w} : Rate of work \dot{E}_{W3} (watts)

→ Different Types of Work

1) Mechanical Work:

- a) Moving Boundary - piston
- b) Shaft work



2) Electrical Work:

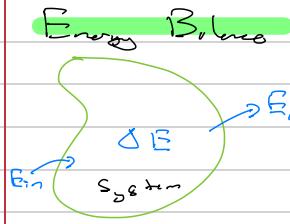


Mass Flow Energy

$E_{mass} = \text{Total Energy}$, transferred due to mass transfer over some time \dot{E}_{W3}

\dot{E}_{mass} : rate Basis \dot{E}_{W3}

First Law of Thermodynamics



$$E_{in} - E_{out} = \Delta E \quad \text{Heat + work}$$

$$\cdot E_{in} = Q_{in} + W_{in} + E_{mass,in}$$

$$\cdot E_{out} = Q_{out} + W_{out} + E_{mass,out}$$

$$\cdot \Delta E = E_{final} - E_{initial}$$

$$\hookrightarrow \Delta E = (U_2 - U_1) + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1)$$

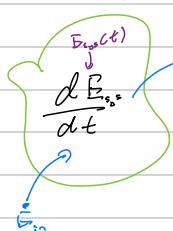
Starting point for Law on total Basis

This is an altered derivation of stored Energy formula

Combining Equations for unitary Law

$$(Q_{in} - Q_{out}) + (W_{in} - W_{out}) + (E_{mass,in} - E_{mass,out}) = (U_2 - U_1) + \frac{1}{2}(m_2 V_2^2 - m_1 V_1^2) + g(m_2 z_2 - m_1 z_1)$$

→ On a Rate Basis



$$E_{in} - E_{out} = \frac{dE}{dt}$$

Starting point for Law on total Basis

Pure Substances

Properties of Pure Substances

- No mixtures (except air)

Goal: Calculate some properties given others

→ Given the thermodynamic state of a substance

→ Specify two independent state variables

Three primary types of substances

1) Ideal Gases (or close to/really)

- Blastic Collisions
- No Intermolecular Forces
- Simple, Low density gases

Approach: Use equations

2) True Liquids (Incompressible Liquids)

→ No Phase change

Approach: Use (different) equations

3) Phase Change Fluids (Sticky things)

Approach: Use tables + Props

→ or software

Examples of Ideal Gases

- air
- N_2

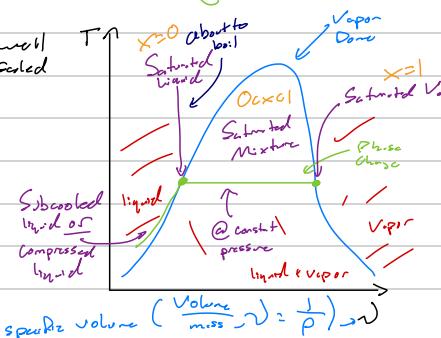
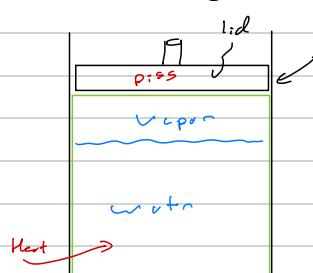
Examples of True Liquids

- water
- oil

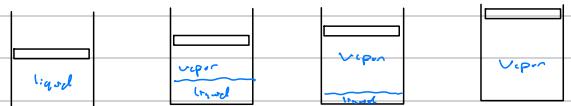
Examples of Phase Change Liquids

- water/steam
- refrigerants

Phase Change Process Boiling @ constant pressure



• Phase change quantified by the quality: $X = \frac{m_{\text{vapor}}}{m_{\text{total}}} = \frac{m_{\text{vapor}}}{m_{\text{vapor}} + m_{\text{liquid}}}$



Regions

1) Compressed Liquid Region

At a given pressure: $T < T_{sat}$

At a given temperature: $P > P_{sat}$ \leftarrow compressed liquid

2) Saturated Liquid

$$\text{Ques} \rightarrow X = 0, T = T_{sat} \neq P = P_{sat}$$

3) Saturated mixture

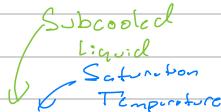
$$0 < X < 1, T = T_{sat} \neq P = P_{sat}$$

4) Saturated Vapor

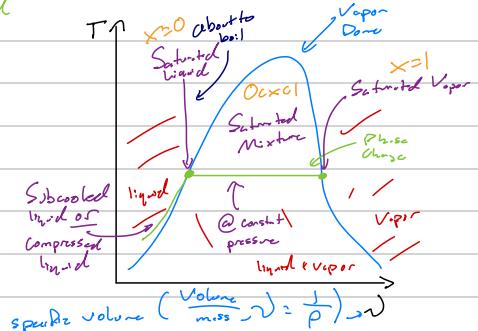
$$X = 1, T = T_{sat} \neq P = P_{sat}$$

5) Superheated vapor

$$T > T_{sat}, P < P_{sat}$$



Revolving



Working with Steam Tables

a) Saturated water (mixture)

(i) Table A4 (by temperature)

(ii) Table A5 (by Pressure)

b) Superheated vapor \rightarrow Table A6

c) Compressed liquid \rightarrow Table A7

Steps for Property Lookup

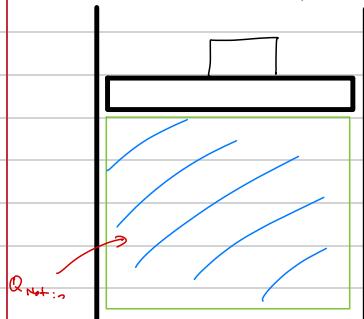
- 1) Start with Saturated mixture table
- 2) Identify the phase (mixture, vapor, or liquid)
- 3) Go to appropriate table

Example: Given water @ 90°C , 50 kPa \rightarrow Determine phase Outlined in Textbook

$$E_{\text{meas}} = m_e e_{\text{meas}} = m_e \left(\frac{P}{P_0} + \frac{V^2}{2} + gz \right)$$

Closed Systems

- Consider a piston-cylinder system



1st Law

$$(Q_{in} - Q_{out}) + (W_{in} - W_{out}) = U_2 - U_1 \quad \text{or } \sum KJ \quad \text{or } \sum \frac{KJ}{kg}$$

System Mass $\rightarrow m(U_2 - U_1)$

Mass specific internal energy $\rightarrow \frac{\sum KJ}{\sum kg}$

$$Q_{net,in} - W_{net,out}$$

$$\Rightarrow Q_{net,in} - W_{net,out} = m(U_2 - U_1)$$

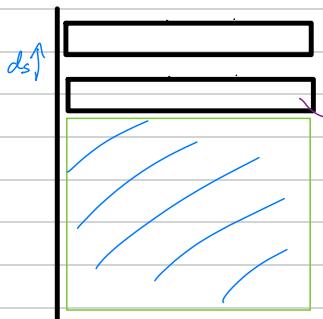
Q_{in} : Heat transfer into the system

W_{in} : Work done on the system

Q_{out} : Heat transfer out of the system

W_{out} : Work done by the system

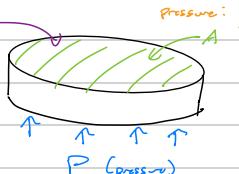
Calculating boundary work



①
②

$$dW = F_{ext} ds$$

$$= P(A_{ext}) ds$$



Integrate between ① to ②

$$\text{Pressure: } \frac{F_{ext}}{A_{ext}}, \text{ area} \rightarrow \text{force}$$

$$W_{piston} = \int_1^2 P dV = W_{out}$$

Done by the system
Positive when volume increases

• If pressure is constant (isobaric process)

$$\rightarrow W_{piston} = P \int_1^2 dV = P(V_2 - V_1) = P\Delta V$$

Writing 1st law is
most common place to start

• Find Heat Added from ① → ②

$$1^{\text{st}} \text{ Law: } (Q_{in} - Q_{out}) + (W_{in} - W_{out}) = m(U_2 - U_1)$$

Heat added only

$$\text{Work: } W_{piston} = P(V_2 - V_1) = P_m(V_2 - V_1)$$

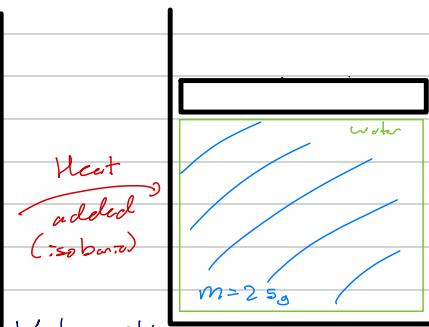
↑ Specific volume

① Go to sat vapor table @ 300 kPa
Sat $V_{\text{apor}}(V_1) = 0.60582 \frac{m^3}{kg}$
Sat $V_{\text{apor}}(U_1) = 2543.2 \frac{kg}{m^3}$
Sat Temp = 133°; 200 > 133 ∴ Superheated

Example: Closed System



①



②

$$P_1 = 300 \text{ kPa} \quad T_2 = 200^\circ\text{C}, P_2 = P_1 = 300 \text{ kPa}$$

Saturated vapor \Leftrightarrow Treat as phase change fluid

② Go to superheated table @ 0.3 mPa

Positive is
 w_{out}

$$V_2 = 0.716 \frac{m^3}{kg}$$

$$U_2 = 2681.0 \frac{KJ}{kg}$$

$$\text{Calculations: } W_{piston} = P_m(V_2 - V_1) = 0.83 \text{ KJ}$$

$$Q_{in} = m(U_2 - U_1) + w_{out} = 3.5 \text{ KJ}$$

Polytropic Processes

Boundary work for a polytropic process

$$\rightarrow \text{Polytropic process: } PV^n = \text{Constant} = C \Rightarrow P = CV^{-n}$$

$$P_1 V_1^n = P_2 V_2^n$$

• Some Examples

<u>n</u>	<u>constant</u>
0	pressure \rightarrow isobaric
1	temperature \rightarrow isothermal
$\gamma = \frac{C_p}{C_v}$	entropy \rightarrow isentropic
$\rightarrow \infty$	volume \rightarrow isochoric

Gamma = ratio of specific heats

Example: Piston-Cylinder with Nitrogen Entertations: deg's



Want to use $\frac{P_2 V_2 - P_1 V_1}{1-n}$

$$P_1 V_1^n = P_2 V_2^n$$

$$V_2^n = \frac{P_1 V_1^n}{P_2}$$

$$\ln(V_2^n) = \ln\left(\frac{P_1 V_1^n}{P_2}\right)$$

$$\ln(V_2) = \frac{1}{n} \cdot \ln\left(\frac{P_1 V_1^n}{P_2}\right)$$

$$V_2 = e^{\left(\frac{1}{n} \cdot \ln\left(\frac{P_1 V_1^n}{P_2}\right)\right)} = 0.08636 \text{ m}^3$$

Back to Work

$$\begin{aligned} W_{\text{poly}} &= \int_{V_1}^{V_2} P \, dV \\ &= \int_{V_1}^{V_2} C V^{-n} \, dV \\ &= C \left[\frac{1}{1-n} V^{1-n} \right]_{V_1}^{V_2} \\ &= C \left(\frac{V_2^{1-n} - V_1^{1-n}}{1-n} \right) \\ &= C \left(\frac{\frac{P_2}{C} \cdot V_2 - \frac{P_1}{C} \cdot V_1}{1-n} \right) \\ \Rightarrow W_{\text{poly}} &= \frac{P_2 V_2 - P_1 V_1}{1-n} \end{aligned}$$

$$\begin{aligned} V_1 &= 0.07 \text{ m}^3 & V_2 &=? \\ P_1 &= 130 \text{ kPa} & P_2 &= 100 \text{ kPa} \\ T_1 &= 120^\circ \text{C} & T_2 &= 100^\circ \text{C} \end{aligned}$$

Find work done by Process

Ideal Gas Law / Eq of State

$$PV = nRT$$

$$\text{For ECHE225: } PV = mRT$$

\hookrightarrow Specific Ratio to a particular Gas

$$\rightarrow P = PRT \text{ or } P_f = RT$$

$$PV = mRT$$

$$mR = \frac{PV}{T}$$

$$\rightarrow \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$V_2 = \frac{T_2 P_1 V_1}{P_2 T_1} = 0.0866 \text{ m}^3$$

Positive Number so this is work out

Finding Δu

3.) Phase change fluids

a) For a saturated mixture

$$u = X_u g + (1-X) u_f$$

or

$$u = u_f + X(u_g - u_f)$$

$$v = v_g + (1-X)v_f$$

or

$$v = v_f + X(v_g - v_f)$$

$h = \dots$ Same for all

$s = \dots$ other fluid values

b) For compressed liquids

u, v much stronger functions of temperature than pressure

Shortcut/Estimate: Use pure compressed liquid properties at the saturation

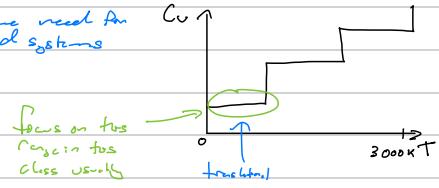
$$u_{cl} = u_f(T)$$

\uparrow comp. \uparrow \uparrow sat. liquid \uparrow \uparrow on temperature.

\rightarrow It is incorrect to say $u_{cl} = u_f(T)$

1) Ideal Gases

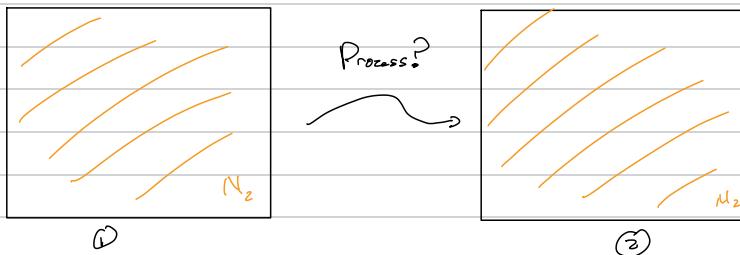
- Internal energy is only a function of temperature
- Use specific heat value - energy required to increase temperature by 1K (per unit mass) $\sum \frac{C_V}{M \cdot K}$
- ↳ Two types:
 - C_V : Specific heat at constant volume
 - ↳ directly relates to changes in internal energy
 - what we need for closed systems
 - $C_V = \frac{du}{dT} \rightarrow du = C_V(T) dT$
 - $\Delta u = u_2 - u_1 = \int_1^2 C_V(T) dT$
 - C_V is typically constant: $u_2 - u_1 = C_V, \text{Avg} (T_2 - T_1)$
- C_p : Specific heat at constant pressure
 - ↳ directly relates to enthalpy
 - open systems will discuss later



2) True Liquids/Solids

$$\begin{cases} \hookrightarrow C_V = C_p = C \\ \Delta u = u_2 - u_1 = \int_1^2 C(T) dT = C_{\text{Avg}} (T_2 - T_1) \quad \text{definition} \\ \Delta u = C \Delta T \quad \text{mass specific} \\ \Delta u = m c \Delta T \quad \text{total basis} \end{cases}$$

Example: Rigid Tank filled with N_2



$$P_1 = 100 \text{ kPa}$$

$$T_1 = 25^\circ\text{C} = 298.15 \text{ K}$$

$$P_2 = 300 \text{ kPa}$$

Rigid Tank = Constant volume
Treat N_2 as an ideal gas
Process is unknown

Steps

- 1) Write 1st law and simplify
- 2) Set eqn. 1 to ideal gas
- 3) Divide T_2 with $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$
- 4) Collect constants and solve

Since it's ideal

$$1^{\text{st}} \text{ Law: } (Q_{in} - Q_{out}) + (W_{in} - W_{out}) = m(u_2 - u_1) = m C_V (T_2 - T_1)$$

ideal G.s E.S: $PV = mRT \leftarrow T$ must be in Kelvin to work correctly

$$V_1 = V_2 \quad \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2} \rightarrow T_2 = \frac{T_1 P_2}{P_1} = 3T_1 = 894 \text{ K}$$

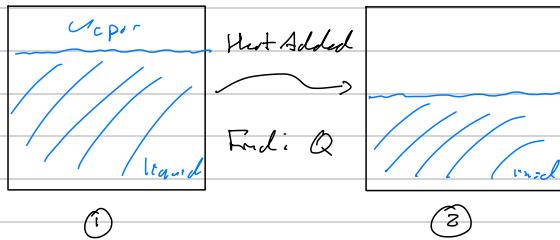
Table lookup

$T \text{ (K)}$	$C_V \left(\frac{KJ}{kg \cdot K} \right)$
298 \approx 300	0.743
894 \approx 900	0.849

$$\left. \begin{array}{l} \\ \end{array} \right\} C_{\text{V, Avg}} = \frac{0.743 + 0.849}{2} =$$

$$Q_{in} = m C_V \Delta T \Rightarrow q_{in} = \frac{Q_{in}}{m} \rightarrow 475 \text{ kJ/kg}$$

Example S-30: 10L water in a rigid tank



$$T_1 = 100^\circ\text{C}$$

$$x_1 = 0.123$$

$$T_2 = 150^\circ\text{C}$$

$$v_2 = v_1 = 0.207 \frac{\text{m}^3}{\text{kg}}$$

$$\text{1st Law: } (Q_{in} - Q_{out}) + (W_{in} - W_{out}) = m(u_2 - u_1)$$

$$v_1 = x_1 v_g + (1-x_1) v_f = 0.207 \frac{\text{m}^3}{\text{kg}}$$

$$u_1 = x_1 u_{g,1} + (1-x_1) u_{f,1} = 678.8 \frac{\text{kJ}}{\text{kg}}$$

plus
water

$$\therefore T_2 = 150^\circ\text{C}$$

$$v_2 = 0.207 \frac{\text{m}^3}{\text{kg}}$$

Go to temperature table @ 150°C
if $v < v_f \rightarrow$ compressed liquid
if $v > v_g \rightarrow$ superheated vapor
if $v_f < v < v_g \rightarrow$ saturated mixture

Volume remains the same $\Rightarrow v_2 = v_1 = v$
Mass remains the same

From Sat Water Table @ 100°C

$$v_1 = 1.622 \frac{\text{m}^3}{\text{kg}}$$

$$u_{g,1} = 2506 \frac{\text{kJ}}{\text{kg}}$$

$$v_{f,1} = 0.00104 \frac{\text{m}^3}{\text{kg}}$$

$$u_{f,1} = 419.06 \frac{\text{kJ}}{\text{kg}}$$

From Sat Water Table @ 150°C

$$v_{f,2} = 0.001091 \frac{\text{m}^3}{\text{kg}}$$

$$u_{g,2} = 0.3924 \frac{\text{kJ}}{\text{kg}}$$

$$v_{g,2} = 631.66 \frac{\text{m}^3}{\text{kg}}$$

$$u_{g,2} = 2559.1 \frac{\text{kJ}}{\text{kg}}$$

$$v_2 = v_{f,2} + x_2(v_{g,2} - v_{f,2}) = 0.207 \frac{\text{m}^3}{\text{kg}}$$

$$x_2 = \frac{0.207 - v_{f,2}}{v_{g,2} - v_{f,2}} = 0.525$$

Find mixture internal energy value

$$u_2 = u_{g,2} x_2 + (1-x_2) u_{f,2} = 1643.6 \frac{\text{kJ}}{\text{kg}}$$

Back to 1st Law: $Q_{in} = m(u_2 - u_1) \rightarrow 10\text{L} \rightarrow m^3 = 0.01$

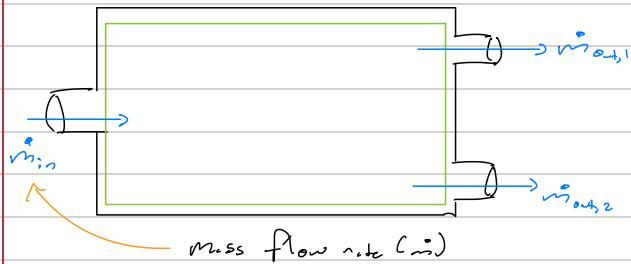
- Find mass? $m = \rho V = \frac{m}{V} = \frac{0.01}{0.207} = 0.048 \text{ kg}$

$$\therefore Q_{in} = 0.048(u_2 - u_1) = \boxed{4.8 \text{ kJ}}$$

$$J = \frac{V}{m} \quad P = \frac{m}{V}$$

$$V = m J$$

Open Systems



- mass crossing in area per unit time

density ρ avg velocity V_{avg} cross sectional area

- Simple Expression: $\dot{m} = \rho V_{avg} A_c$



- No velocity @ the wall
- Linearized velocity at center of pipe

velocity profile

- Locally: $\dot{m} = \int_{A_c} \rho v_n dA_c$

- Equate \dot{m} expressions
Flow: incompressible

$$\rho V_{avg} A_c = \int_{A_c} \rho V_n dA_c$$

$$= \rho \int_{A_c} V_n dA_c$$

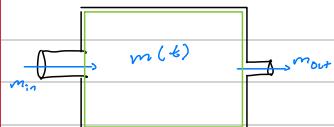
Volumetric Flow Rate

$$\dot{V} = \frac{\dot{m}}{\rho} = V_{Avg} A_c \quad (\text{in } \frac{\text{m}^3}{\text{s}})$$

$$\Rightarrow V_{Avg} = \frac{1}{A_c} \int V_n dA_c$$

Conservation Laws

1) Mass



$$\dot{m}_{in} - \dot{m}_{out} = \Delta m \quad (\text{total b.s.s.})$$

$$\dot{m}_{in} - \dot{m}_{out} = \frac{dm}{dt} \quad \begin{matrix} \text{Rate of mass} \\ \text{accumulation} \end{matrix} \quad \left\{ \frac{kg}{s} \right\}$$

Extend to multiple inlets/outlets: $\sum_i \dot{m}_{in,i} - \sum_j \dot{m}_{out,j} = \frac{dm}{dt}$

2) Energy (1st Law Thermodynamics)

$$E_{in} - E_{out} = \Delta E_{sys} \quad (\text{total b.s.s.})$$

$$\dot{E}_{in} - \dot{E}_{out} = \frac{dE_{sys}}{dt} \quad \begin{matrix} \text{Focus for open systems} \\ \text{(control b.s.s.)} \end{matrix}$$

Mass transfer heat work rates of

will focus on steady flow analysis $\Rightarrow \frac{d}{dt} = 0$, in and out balance P & En laws
↳ steady flow devices

\Rightarrow No accumulation of mass or energy inside our system

• For Steady Equations

1) Mass: $\dot{m}_{in} = \dot{m}_{out}$ (or $\dot{m} = 0$)

2) Energy: $\dot{E}_{in} = \dot{E}_{out}$

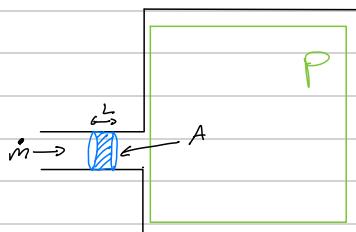
Corresponds to steady, normal operations

1st Law for Steady Open Systems

$$(\dot{Q}_{in} - \dot{Q}_{out}) + (\dot{W}_{in} - \dot{W}_{out}) + (\dot{E}_{mass,in} - \dot{E}_{mass,out}) = 0$$

net rate heat transfer
 net rate work transfer
 net rate mass transfer

Flow Work Energy



$$\begin{aligned} W_{Flow} &= PL \\ &= PA\bar{L} \quad \leftarrow \text{Flow Fluid is with zero force} \\ &= PV \end{aligned}$$

- \dot{E}_{mass} elements
 - (i) Kinetic Energy $\left(\frac{1}{2}\dot{V}_{avg}^2\right)$
 - (ii) Internal Energy (U)
 - (iii) Potential Energy (gz)
 - (iv) Flow Work Energy (PV)

$$\frac{W_{Flow}}{m} = m_{Flow} = \frac{PV}{m} = P$$

$$\dot{V} = \frac{V}{m}$$

Energy Terms Brought together

$$\dot{E}_{mass} = m \underbrace{(U + PV)}_{\text{enthalpy } (h)} + \frac{1}{2} \dot{V}_{avg}^2 + gz$$

- Introduce the enthalpy $h = U + PV$:
 $h = U + PV$ (also total $H = U + PV$)
- $\Rightarrow \dot{E}_{mass} = m (h + \frac{1}{2} \dot{V}_{avg}^2 + gz)$

Conservation of Energy (single inlet, single outlet)

$$(\dot{Q}_{in} - \dot{Q}_{out}) + (\dot{W}_{in} - \dot{W}_{out}) + m [(h_{in} - h_{out}) + \frac{1}{2} (V_{in}^2 - V_{out}^2) + g(z_{in} - z_{out})] = 0$$

- (i) Move Flow Energy to RHS, define
- (ii) Let $out = 2, in = 1$

$$\Rightarrow \dot{Q}_{out} + \dot{W}_{out} = m [h_2 - h_1 + \frac{1}{2} (V_2^2 - V_1^2) + g(z_2 - z_1)]$$

Enthalpy Calculating Δh @ constant pressure

$$\text{D) Ideal Gas: } C_p = \frac{dh}{dT} \Big|_P$$

$$\Rightarrow \Delta h = \int_1^2 C_p dT$$

2) True Liquids: $C_p = C_v = C$

$$\Delta h = C_{avg} \Delta T$$

3) Phase Change Fluids

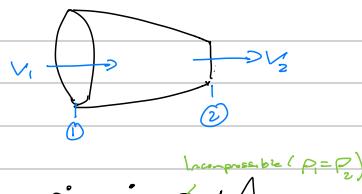
- Tables with missing values

$$\begin{aligned} h &= h_f + x(h_g - h_f) \quad \text{Interpolation} \\ h &= xh_g + (1-x)h_f \end{aligned}$$

Steady Flow Devices

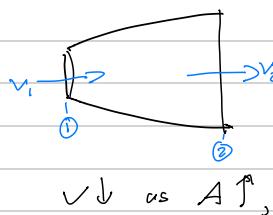
1) Nozzles / Diffusers

Nozzle



$$\text{mass: } \dot{m}_1 = \rho_1 V_1 A_1 \\ = P_2 V_2 A_2 = \dot{m}_2 \\ \Rightarrow V_1 \uparrow \text{ as } A \downarrow, P_2 < P_1$$

Diffuser



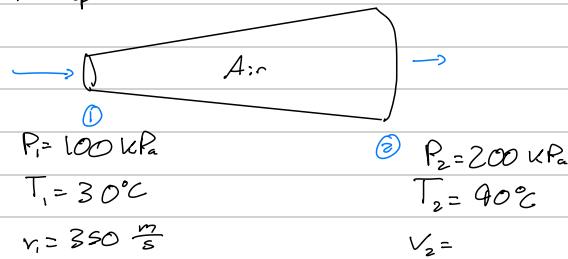
$$V \downarrow \text{ as } A \uparrow, P_2 > P_1$$

$$\text{Energy Analysis: } Q_{\text{Net,in}} + W_{\text{Net,in}} = \dot{m} [h_2 - h_1 + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1)]$$

Assumptions: $\Delta PE \approx 0$ ($z_2 \approx z_1$)
 $\dot{Q}_{\text{net,in}} = 0$ (adiabatic)
 $W_{\text{net,in}} \approx 0$ (positive s_{gen})

• Exchanges enthalpy for kinetic energy

Example 6-26



Find: V_2 , something about the areas?

$$\Rightarrow V_2 = \sqrt{V_1^2 - 2c_p \Delta T} \quad \text{where } c_p = 1.007 \frac{\text{kJ}}{\text{kgK}}$$

Substituting values: $V_2 = \sqrt{40.7^2 - 2 \times 1.007 \times 1000 \times (40 - 30)} = 40.7 \frac{\text{m}}{\text{s}}$

$$\text{Energy: } 0 = \dot{m} [(h_2 - h_1) + \frac{1}{2}(V_2^2 - V_1^2)]$$

$$\dot{m}(h_2 - h_1) = \dot{m}(\frac{1}{2}(V_2^2 - V_1^2))$$

$$2(h_2 - h_1) = V_2^2 - V_1^2$$

$$V_2 = \sqrt{V_1^2 - 2(h_2 - h_1)}$$

Need to calculate Δh

$$\text{Ideal gas: } \Delta h = c_p A_v \Delta T$$

$$\frac{m^2}{s^2} = \frac{J}{kg}$$

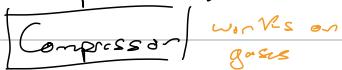
$$\text{mass: } \dot{m}_1 = \dot{m}_2 \\ P_1 V_1 A_1 = P_2 V_2 A_2 \rightarrow \frac{A_2}{A_1} = \frac{P_1 V_1}{P_2 V_2} \quad \text{If } P \text{ is constant} \rightarrow \frac{A_2}{A_1} = \frac{V_1}{V_2} = 6.6$$

$$\rightarrow \text{Use ideal gas EoS: } P = \frac{P}{RT}, P_1 = \frac{P_1}{RT_1} \text{ and } P_2 = \frac{P_2}{RT_2}$$

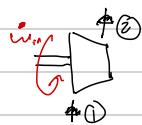
Don't make bad assumptions

$$\frac{P_1 V_1}{P_2 V_2} = \frac{V_1}{V_2} \left(\frac{P_1 R T_2}{R T_1 P_2} \right) = 8.6 \quad \text{Area ratio should be 5.1}$$

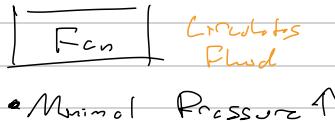
2) Compressors / Turbines



- As $P \uparrow$, so does T_{mp}



- As $P \uparrow$, only slight $T_{\text{mp}} \uparrow$



- Minimal Pressure \uparrow

Example 6-42

Conditions provided for adiabatic steam turbine



$$\begin{aligned} P_1 &= 10 \text{ MPa} \\ T_1 &= 500^\circ\text{C} \end{aligned}$$

$$w_{\text{out}} = \dot{m} (h_1 - h_2)$$

$$\left. \begin{array}{l} P_1 \\ T_1 \end{array} \right\} \begin{array}{l} \text{Steam} \\ \text{Turbine} \end{array} \xrightarrow{\text{SH}} h_1 = 3328 \frac{\text{kJ}}{\text{kg}}$$

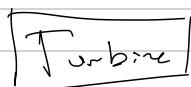
$$\left. \begin{array}{l} P_2 \\ \text{low } P_a \end{array} \right\} \begin{array}{l} \text{Subcooled} \\ \text{Water} \end{array} \xrightarrow{} h_g = 191.82 \frac{\text{kJ}}{\text{kg}}$$

$$h_f = 2583.9 \frac{\text{kJ}}{\text{kg}}$$

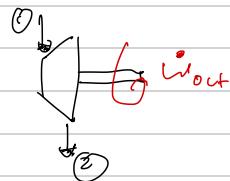
$$h_2 = h_g + x(h_f - h_g) = 2344.2 \frac{\text{kJ}}{\text{kg}}$$

- Find w_{out} for $\dot{m} = 2 \frac{\text{kg}}{\text{s}}$

$$w_{\text{out}} = 2 \frac{\text{kg}}{\text{s}} (3328 - 2344.2) \frac{\text{kJ}}{\text{kg}} = 2061 \text{ kW} = 2.06 \text{ MW}$$



- Extracts fluid power to produce shaft work



Energy Analysis

$$\dot{Q}_{\text{net,in}} + \dot{W}_{\text{net,in}} = \dot{m} [h_{\text{in}} - h_{\text{out}} + \frac{1}{2}(v_{\text{2}}^2 - v_{\text{1}}^2) + g(z_{\text{2}} - z_{\text{1}})]$$

Assumptions: $\Delta PE \approx 0$

Reduces friction \dot{Q} typically small

$\Delta KE \approx 0$ desired result

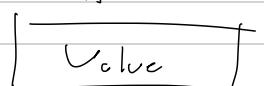
KE converted to Enthalpy P_v via losses devices

Focuses on \dot{Q} vs \dot{W}

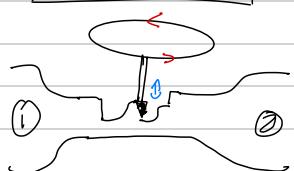
$$\dot{W}_{\text{net,in}} = \dot{m} (h_{\text{2}} - h_{\text{1}})$$

$$\dot{W}_{\text{in}} - \dot{W}_{\text{out}} = \dot{m} (h_{\text{2}} - h_{\text{1}})$$

3) Throttles \rightarrow Valve



Controls Flow, Control Pressure



$$Q_{\text{Net}} + \dot{W}_{\text{Net}} = m [(h_2 - h_1) + \frac{1}{2} (v_2^2 - v_1^2) + g (z_2 - z_1)]$$

$$\text{Assumptions: } \Delta P_E \approx 0$$

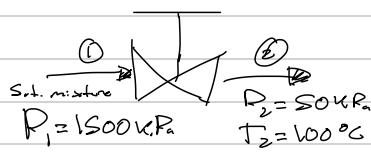
$$\Delta K_E \approx 0$$

$$\dot{Q}_{\text{Net}} \approx 0$$

$$\dot{W}_{\text{Net}} \approx 0$$

$$\text{enthalpy conserved: } h_1 = h_2, P_1 > P_2$$

Example: Steam flowing through a valve



$$P_1 = 1500 \text{ kPa}, T_1 = 100^\circ \text{C}$$

$$P_2 = 500 \text{ kPa}, T_2 = 100^\circ \text{C}$$

$$\text{1st law: } h_1 = h_2$$

$$P_2 \underset{\substack{\text{Steam} \\ \text{Turb}}}{\gtrsim} \underset{\text{Tables}}{\gtrsim} h_2 = 2682.4 \frac{\text{kJ}}{\text{kg}} = h_1$$

$$h_1 = 2684.2 \underset{\substack{\text{Steam mix} \\ P_1 = 1500 \text{ kPa}}}{\gtrsim} h_1 = h_g + x(h_f - h_g) \rightarrow x = \frac{h_1 - h_g}{h_f - h_g}$$

$$h_f = 844.85 \frac{\text{kJ}}{\text{kg}}$$

$$h_g = 2791.0 \frac{\text{kJ}}{\text{kg}}$$

$$\Rightarrow x = 0.49$$

Recap

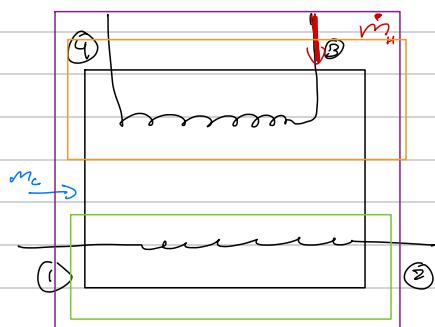
$$(i) \text{ Nozzles / Diffusers: } (h_2 - h_1) + \frac{1}{2} (v_2^2 - v_1^2) = 0$$

$$(ii) \text{ Compressors / Turbines: } \dot{W}_{\text{Net}} = m (h_2 - h_1)$$

$$(iii) \text{ Turbines: } h_1 = h_2$$

Heat Exchangers

Transfers heat from hotter fluid to colder fluid



$$\text{Energy Analysis: } (Q_{in} - Q_{out}) + (h_1 - h_2) = m [(h_2 - h_1) + \frac{1}{2} (v_2^2 - v_1^2) + g (z_2 - z_1)]$$

$$\text{Assumptions: } \dot{W}_{\text{Net}} = 0, \Delta P_E = 0$$

$$\Delta K_E \approx 0$$

If Boundary is Purple

$$Q_{\text{out}} = m_H (h_2 - h_1) + m_C (h_2 - h_1)$$

If Boundary is Green

$$Q_{in} = m_C (h_2 - h_1)$$

$$\left. \begin{array}{l} Q_{in} = Q_{out} \\ m_C (h_2 - h_1) = m_H (h_2 - h_1) \end{array} \right\}$$

If Boundary is Orange

$$Q_{out} = m_H (h_2 - h_1)$$

Second Law of Thermodynamics

1st Law: Accounting of Energy

Implications of the 2nd Law

- (i) Processes proceed spontaneously (naturally) in a particular direction
- (ii) Energy has quality, not just quantity
↳ 2nd Law ↳ 1st Law
- (iii) Sets limits on theoretical performance
- (iv) Helps determine degree of chemical reaction completion

Ways to Quantify the 2nd Law

- (i) Heat Engine \rightarrow energy conversion
- (ii) Entropy - measure of disorder

Heat Engines: Device that converts heat into work

Example: Steam Power Plant, Car Engine,

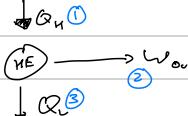
- Consists of 4 key elements

(i) Receives heat from a high-Temp Reservoir at T_H

↳ Q_{in}

(ii) Converts a portion of the heat \rightarrow work

T_H

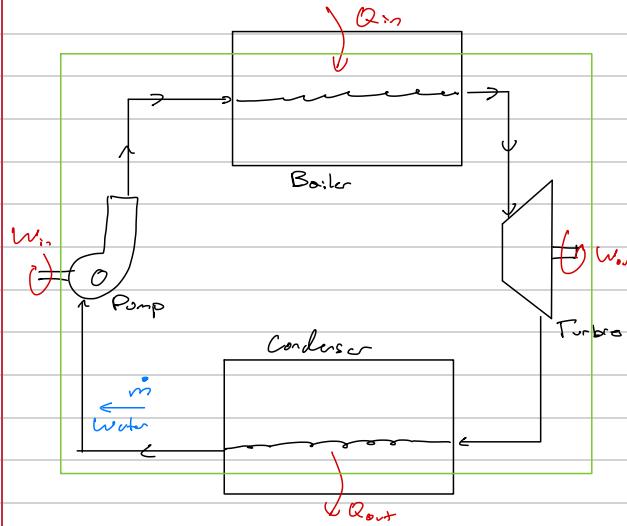


(iii) Rejects heat to a low-temp reservoir at T_L

T_L

(iv) Operates as a cycle

Power Plant as a heat engine (Rankine Cycle)



1st Law Analysis

$$(Q_{in} - Q_{out}) + (W_{in} - W_{out}) = m(c_2 - c_1)$$

boiler condenser pump turbine

Opener a cycle

$T_H \sim$ Combustion Temperature

$$Q_H = Q_{in}$$

$$Q_L = Q_{out}$$

$T_L \sim$ River, Ocean, Lake Etc...

- Define Thermal Efficiency

$$\eta_{th} = \frac{W_{net,out}}{W_{net,in} + W_{in}} = \frac{W_{net,out}}{Q_{in}} \Rightarrow \eta_{th} = \frac{W_{net,out}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

2nd Law

Typical Thermal Efficiencies

- Car Engine (spark ignition): $\eta_{th} = 25-30\%$

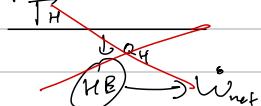
- Diesel Engine: $\eta_{th} \approx 40\%$

- Combined Cycle Power Plant (gas turbine + steam turbine): $\eta_{th} \approx 60\%$

Two Statements of the 2nd Law

1) Kelvin-Planck Statement

a) Impossible to generate work from a single reservoir



In other words, we must reject some heat

b) $\eta_{th} \neq 1$; Thermal Efficiency is never 100%!

→ Define the ideal heat engine

↪ Carnot Cycle

• Form ideal heat engine from ideal processes

Ideal Process → Reversible Process: A process that can be reversed without impact on the surroundings

→ Idealizations on an actual process

• Pump - reversible compression: Requires minimum work

• Turbine - reversible expansion: Produces maximum work

→ Some irreversibilities

(i) Chemical Reactions

(ii) Heat Transfer through a finite temperature difference

(iii) Inelastic deformations

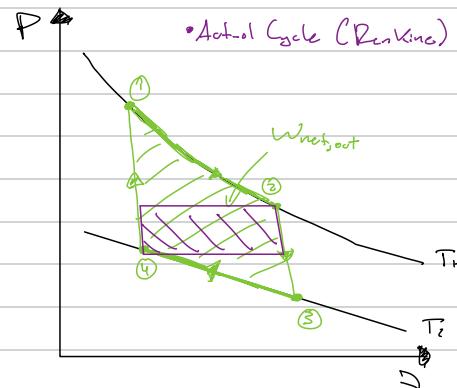
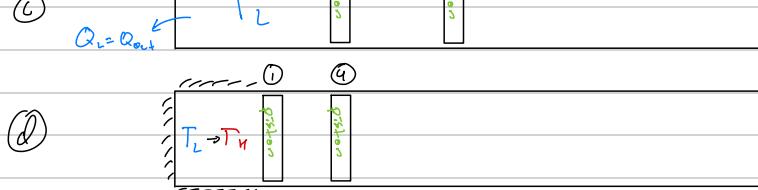
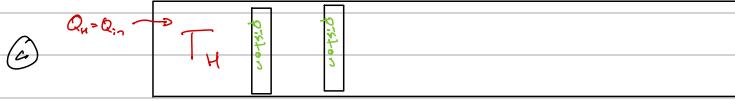
(iv) Friction (or losses)

(v) Mixing of dissimilar fluids

(vi) Electrical Resistance

→ Carnot Heat Engine (Ideal Heat Engine)

① → ②



$$W_{piston} = \int P dV$$

Heat and Work in Cycle

a) Isothermal expansion @ T_H

b) Adiabatic expansion: $T_H \rightarrow T_L$

Cooling

Pump

Work

c) Isothermal compression @ T_L

d) Adiabatic compression

Area under or above curve

Carnot Principles

- 1) Efficiency of an irreversible Carnot cycle is always less than a reversible cycle operating between T_H and T_L
- 2) Efficiency of all reversible engines acting between T_H and T_L is the same

- Introduce the Thermodynamic (absolute) temperature

$$\left(\frac{Q_H}{Q_L} \right)_{rev} = \left(\frac{T_H}{T_L} \right)_{absolute}$$

→ For a heat engine (general):

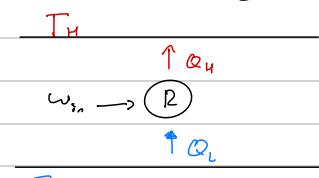
$$\eta_{th,gen} = \frac{w_{net,out}}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

→ For a Carnot heat Engine:

$$\eta_{th,Carnot} = 1 - \frac{T_L}{T_H} \quad \text{3 must be absolute}$$

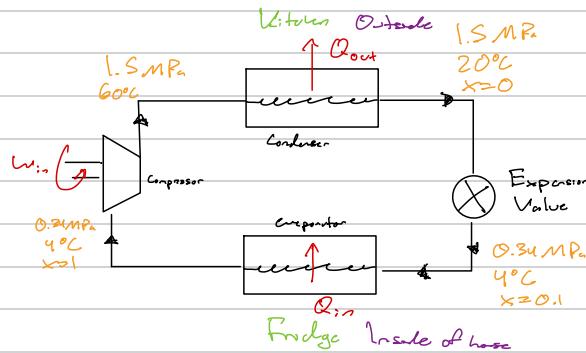
Refrigeration Cycles & Heat Pumps

Refrigeration Cycle



Example values in orange

onto cycle to the right



Valid for:

- Refrigerators
- Air conditioners

1st Law

$$w_{in} + Q_L = Q_H$$

$$(w_{in} = Q_H - Q_L)$$

Coefficient of Performance (COP)

$$COP_R = \frac{Q_{in}}{w_{in}} = \frac{Q_L}{w_{in}} = \frac{Q_L}{Q_H - Q_L} = \frac{1}{\frac{Q_H}{Q_L} - 1} \Rightarrow COP_R > 1$$

Can also be used for heat engines
1st Law Relationship

abides by 2nd Law so $Q_H \neq Q_L$

Typical Values: $COP_R = [2, 6]$ ← Higher COP_R is better, less work needed

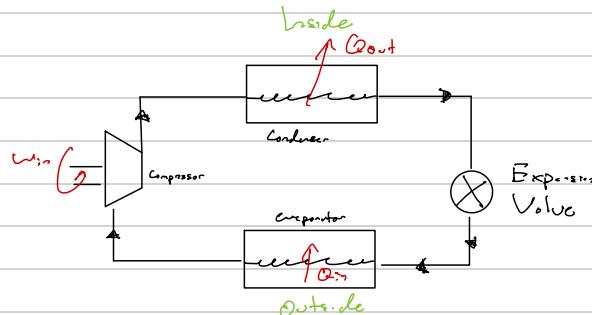
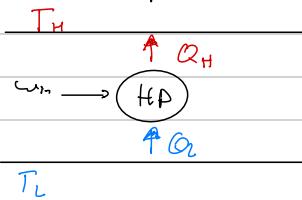
→ Generally: $COP_R = \frac{1}{\frac{Q_H}{Q_L} - 1}$

Apply Thermodynamic temp. Scale

→ Ideal (Carnot) refrigerator: $COP_{R,ideal} = \frac{1}{\frac{T_H}{T_L} - 1}$ ← COP_R horizontally based on source temps

Note that $COP_{R,ideal} > COP_R$ generally

Heat Pump



Coefficient of Performance

$$COP_{HP} = \frac{Q_{out}}{w_{in}} = \frac{Q_H}{w_{in}} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - \frac{Q_L}{Q_H}}$$

Typical Values: $COP_{HP} = [1, 2]$

→ Ideal Heat Pump: $COP_{HP,ideal} = \frac{1}{1 - \frac{T_L}{T_H}}$

Two Types of Heat Pumps

1) Air Source

↳ T_L is the air

2) Ground Source

↳ T_L is the ground

Example 2 - S3

- Heat Pump Cycle
- R134a enters condenser at 800 kPa and 35°C
- at $\dot{m} = 0.018 \frac{\text{kg}}{\text{s}}$ and leaves at 800 kPa as a saturated liquid.
- Compressor consumes 1.2 kW of power

Find (i) COP_{HP}

- (ii) Rate of heat absorption from the outside air

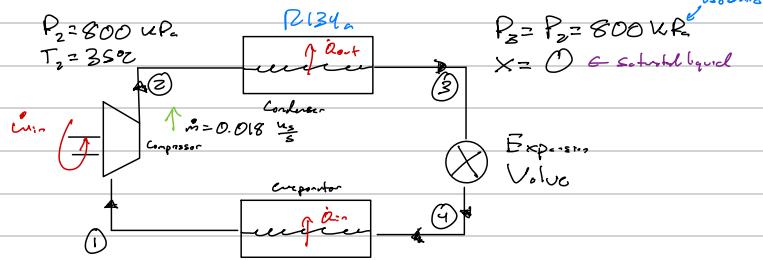
$$\text{COP}_{\text{HP}} = \frac{\dot{Q}_{\text{out}}}{\dot{w}_{\text{in}}} = ?$$

$$\text{COP}_{\text{HP}} = \frac{3.16}{1.2} = 2.64$$

$$\boxed{\text{COP}_{\text{HP}} = 2.64}$$

$$\dot{Q}_{\text{in}} = \dot{Q}_{\text{out}} + \dot{w}_{\text{in}} = 3.16 \text{ kW} - 1.2 \text{ kW} = 1.96 \text{ kW}$$

$$\boxed{\dot{Q}_{\text{in}} = 1.96 \text{ kW}}$$



$$W_{\text{in}} \text{ Given: } W_{\text{in}} = 1.2 \text{ kW}$$

1st Law, open system

$$\dot{Q}_{\text{out}} = \dot{m}(h_2 - h_3) = \dot{Q}_H$$

Tables: (i)

$$\begin{aligned} T_2 &= 35^\circ\text{C} \\ P_2 &= 800 \text{ kPa} \end{aligned} \quad \left. \begin{array}{l} \xrightarrow{\text{SH}} h_{310} = \dots \\ h_{400} = \dots \end{array} \right. \xrightarrow{\substack{\text{Interpolate} \\ \text{from tables}}} h_2 = 271.2 \frac{\text{kJ}}{\text{kg}}$$

$$\begin{aligned} \text{From tables} \\ \text{R134a tables} \\ \text{in open system} \\ A = V \\ x = 0 \\ P_3 = 800 \text{ kPa} \end{aligned} \quad \left. \begin{array}{l} \xrightarrow{\text{SH}} h_f = h_3 = 24.48 \\ h_{400} = \dots \end{array} \right. \xrightarrow{\substack{\text{Shaded} \\ \text{Liquid}}} h_3 = 24.48 \frac{\text{kJ}}{\text{kg}}$$

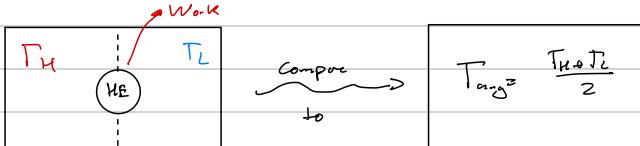
$$\rightarrow \dot{Q}_H = \dot{m}(h_2 - h_3) = 3.16 \text{ kW} = \dot{Q}_{\text{out}}$$

Entropy

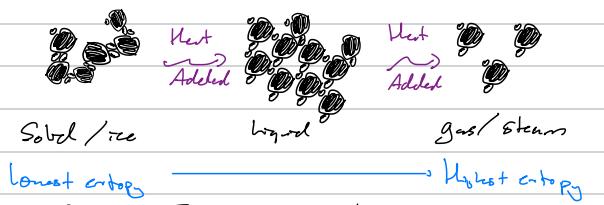
- For cycles / processes
 - Ideal cycles/processes \rightarrow reversible
 - Real cycles/processes \rightarrow irreversible
- Irreversibility \Rightarrow entropy generation

Want to think about entropy

(i) Entropy change as a measure of energy quality/ degradation



(ii) Entropy as a measure of molecular system disorder



- Define Entropy change based on heat transfer for an initially reversible process

$$dS \equiv \left(\frac{dQ}{T} \right)_{\text{ideal}}$$

↓ Total change $\left\{ \frac{dQ}{T} \right\}$
in entropy $\left\{ \frac{dQ}{T} \right\}$

Make sure you're using absolute temperature!

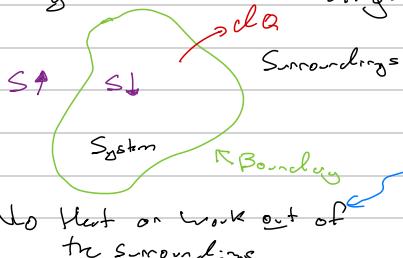
\hookrightarrow Mass specific basis: $dS \equiv \frac{dS}{m} \equiv \left\{ \frac{dQ}{mT} \right\}$

For a real process: $dS > \frac{dQ}{T}$

- Integrate to get total entropy change for a process

$$\Delta S = \int_1^2 \frac{dQ}{T} \quad \text{would need to know how } Q \text{ & } T \text{ are related}$$

- System in its surroundings



$\rightarrow dS$ can be positive or negative depending on the sign of dQ

\rightarrow Assume surroundings are isolated

No heat or work out of the surroundings

- Total Entropy change is always positive

$$\Delta S_{\text{tot}} = \Delta S_{\text{sys}} + \Delta S_{\text{surroundings}}$$

Three Possibilities

(i) $\Delta S_{\text{tot}} > 0 \Rightarrow$ real process (impossible)

(ii) $\Delta S_{\text{tot}} = 0 \Rightarrow$ ideal process (reversible)

(iii) $\Delta S_{\text{tot}} < 0 \Rightarrow$ impossible!

- For a real system:

$$\Delta S_{\text{sys}} = \int_1^2 \left(\frac{dQ}{T} \right)_{\text{ideal}} + S_{\text{generated}}$$

$$\Delta S_{\text{tot}} = \int_1^2 \left(\frac{dQ}{T} \right)_{\text{sys, real}} + \int_1^2 \left(\frac{dQ}{T} \right)_{\text{sur, ideal}} + S_{\text{generated}}$$

entropy generated through irreversibilities

Calculating Entropy Change

1) Isothermal Process (valid for surroundings)

Assume fixed temperature T_0

$$\Delta S_{\text{sur}} = \int_{1,}^2 \frac{dQ}{T} \geq \int_{1,}^2 \frac{dG}{T_0} = -\frac{Q_2}{T_0}$$

Heat Transfer from 1 to 2

2) More Generally

$$dQ - dw = du$$

$(P_{\text{in}} - P_{\text{out}})$ $(w = -PdV)$ \downarrow Change in energy

$$dS \frac{dQ}{T} + T dS - P dV = du$$

\downarrow Divide by mass: \downarrow lower case vars for division by mass

$$TdS = du + PdV$$

Gibbs Equations

Gibbs Equations - enthalpy form

$$TdS = du + PdV$$

• Recall: $h = u + Pv$

$$\begin{aligned} \rightarrow dh &\equiv du + d(PV) \\ &= du + PdV + VdP \\ \Rightarrow dh &= du - PdV - VdP \end{aligned}$$

$$TdS = dh - PdV - VdP$$

$$TdS = dh - VdP$$

• Evaluating ΔS

1) Phase change fluids

→ Use tables for s_1, s_2

$$\Delta S_{\text{sys}} = s_2 - s_1$$

2) Ideal Gases

$$TdS = du + PdV$$

$$\rightarrow \text{Ideal Gas EoS: } PV = RT \Rightarrow P = \frac{RT}{V}$$

$$TdS = du + \frac{RTdV}{V}$$

$$\rightarrow \text{(ideal gas) Internal Energy } \rightarrow dU = C_V dT$$

$$TdS = C_V dT + \frac{RTdV}{V}$$

$$ds = C_V \frac{dT}{T} + R \frac{dV}{V}$$

→ Integrate and assume C_V is a constant

$$\Delta S_{\text{sys}} = C_V, \text{Avg} \ln \left(\frac{T_2}{T_1} \right) + R \ln \left(\frac{V_2}{V_1} \right)$$

If we started with enthalpy form:

$$\Delta S_{\text{sys}} = C_p, \text{Avg} \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{P_2}{P_1} \right)$$

Equally valid, choose baselines of your quantities

3) True (Incompressible) liquids/solids

$$TdS = du + PdV \xrightarrow{\text{Incompressible}} dV \approx 0$$

$$TdS = dh - VdP \xrightarrow{\text{Incompressible}} dP \approx 0$$

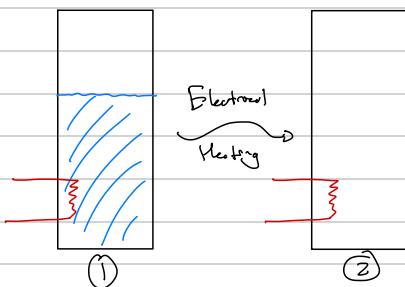
$$\rightarrow TdS = C_V dT \quad \left\{ \begin{array}{l} C_V = C_P = C \\ \text{for these substances} \end{array} \right.$$

$$\rightarrow TdS = C dT$$

$$\rightarrow \text{Integrate: } \Delta S_{\text{sys}} = C \ln \left(\frac{T_2}{T_1} \right)$$

Exams will be more approachable based instead of derivative based

Example 8-30) Un-insulated tank $K \approx 3 \text{ kg/m}$



$$P_1 = 200 \text{ kPa}$$

$\frac{3}{4}$ of mass is liquid
↓

All vapor (no superheat)

$$\gamma_2 = \gamma_1$$

$$\text{Find: } \Delta s_{\text{sys}}$$

Stat at tables:

$$\sum \frac{\delta Q}{T}$$

$$Q_{\text{add}} \quad \text{at } 1^\circ \quad x_1 = \frac{1 - \gamma_{\text{sat}}}{1 + \gamma_{\text{sat}}} = 0.25$$

$$P_1 \quad \left. \begin{array}{l} \xrightarrow{\text{Sut.}} S_f = 1.8302 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \\ \xrightarrow{\text{mix}} S_g = 7.127 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \end{array} \right\} \quad S_i = x_1 S_{\text{sat}} (1 - x_1) S_p = 2.9294 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

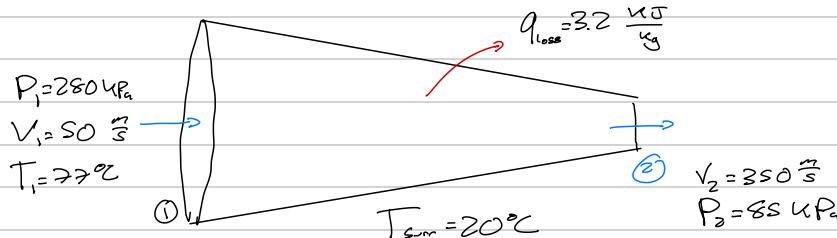
$$\left. \begin{array}{l} \xrightarrow{\text{Sut.}} \gamma_f = 0.001061 \frac{\text{m}^3}{\text{kg}} \\ \xrightarrow{\text{mix}} \gamma_g = 0.88578 \frac{\text{m}^3}{\text{kg}} \end{array} \right\} \quad \gamma_i = x_1 \gamma_g + (1 - x_1) \gamma_f = 0.222 \frac{\text{m}^3}{\text{kg}}$$

$$Q_{\text{add}} \quad \text{at } 2^\circ \quad x_2 = 1, \text{ all v.p.} \quad \checkmark \text{ Inert prop.}$$

$$\gamma_2 = 0.222 \frac{\text{m}^3}{\text{kg}} \quad \left. \begin{array}{l} \xrightarrow{\text{Sut.}} S_i = S_g = 6.63 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \\ x_2 = 1 \end{array} \right\} \quad \text{Look for } 0.222 \frac{\text{m}^3}{\text{kg}} \text{ on Res. or Temp tables}$$

$$\rightarrow \Delta s_{\text{sys}} = m(S_2 - S_1) = +11.1 \frac{\text{kJ}}{\text{kg}} \quad \checkmark \text{ added heat, entropy should increase}$$

Example 8-81) Entropy change for air in a nozzle



$$\text{Find: } \text{a) } T_2$$

$$\text{b) } \Delta s_{\text{total}}$$

$$\sum \frac{\delta Q}{T}$$

$$\text{1st Law: } (Q_{\text{in}} - Q_{\text{out}}) + (C_v \ln \frac{T_2}{T_1}) = m [(h_2 - h_1) + \frac{1}{2} (v_2^2 - v_1^2)]$$

$$\text{Dissipation: } -q_{\text{loss}} = (h_2 - h_1) + \frac{1}{2} (v_2^2 - v_1^2)$$

$$\text{Treat as Ideal: } -q_{\text{loss}} = c_{\text{p,avg}} (T_2 - T_1) + \frac{1}{2} (v_2^2 - v_1^2) \quad \text{Table: } c_{\text{p,avg}} = 1005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$\text{Plug in: } T_2 = 24.11^\circ\text{C}$$

For total entropy change: $\Delta s_{\text{tot}} = \Delta s_{\text{sys}} + \Delta s_{\text{sur}}$

System given pressure, use enthalpy form

$$\Delta s_{\text{sys}} = C_{\text{p,avg}} \ln \left(\frac{T_2}{T_1} \right) - R_{\text{air}} \ln \left(\frac{P_2}{P_1} \right) \xrightarrow{\text{Plug}} \Delta s_{\text{sys}} = 0.18 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$\downarrow 1005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \quad \text{absolute} \quad R_{\text{air}} = 0.2870 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

Surroundings treat surroundings as isothermal

$$\Delta s_{\text{sur}} = \frac{Q}{T_0} \quad \left| \begin{array}{l} \text{use} \\ \text{constant w.r.t. surroundings} \end{array} \right. \quad \Delta s_{\text{sur}} = \frac{+3.2}{20 + 273.15} \frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 0.01 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

Total

$$\Delta s_{\text{tot}} = 0.18 + 0.01 = 0.19 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \Rightarrow \text{Real since } \Delta s_{\text{tot}} > 0$$

$$\boxed{\Delta s_{\text{tot}} = 0.19 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}}$$

1. Isentropic Processes

- $\Delta s_{isent} = 0$

$$\Rightarrow \Delta s_{isent} = s_2 - s_1 = 0 \Rightarrow s_2 = s_1$$

- By substance type

(i) Phase Change fluids \rightarrow tables

$$\Rightarrow s_2 = s_1$$

(ii) Ideal Gases

$$\Delta s_{isent} = c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{V_2}{V_1}\right) \xrightarrow{\text{Due to being isentropic}}$$

$$\Rightarrow \ln\left(\frac{T_2}{T_1}\right) = -\frac{R}{c_v} \ln\left(\frac{V_2}{V_1}\right) \rightarrow \ln\left(\frac{T_2}{T_1}\right) = \ln\left[\left(\frac{V_1}{V_2}\right)^{R/c_v}\right]$$

- Ratio of specific heats: $K = \frac{C_p}{C_v}$

- For ideal gases: $R = C_p - C_v$

$$\therefore \ln\left(\frac{T_2}{T_1}\right) = \ln\left[\left(\frac{V_1}{V_2}\right)^{K-1}\right] \Rightarrow \frac{T_{2s}}{T_1} = \left(\frac{V_1}{V_2}\right)^{K-1}$$

This is ok true for an ideal gas in an isentropic process

Take T_{2s} as output of process

- Start with enthalpy for ...

$$\Delta s_{isent} = c_{p,avg} \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right) \xrightarrow{\text{isentropic process}}$$

Similar algebra

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}}$$

Derived similarly with $K = \frac{C_p}{C_v}$ & $\frac{R}{c_v} = K-1$

- Combine specific volume expression with enthalpy one...

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^K$$

Also valid only for ideal gases in an isentropic process

Caution: watch out for assumptions when problem-solving

- True Liquids/Solids

$$\Delta s_{isent} = C \ln\left(\frac{T_2}{T_1}\right) \xrightarrow{\text{isentropic}}$$

$$C \ln\left(\frac{T_2}{T_1}\right) = 0$$

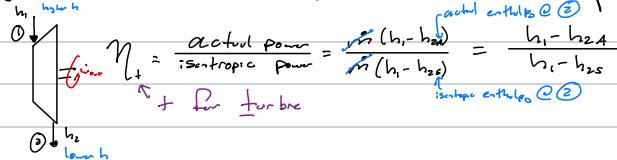
For solids/incompressible liquids, process must be isothermal to be isentropic

$\leftarrow T_2 = T_1$ to be true!

- Use isentropic processes to define process/component efficiency

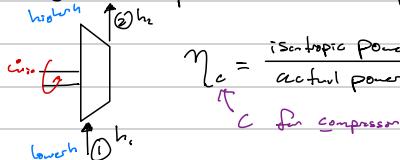
\Rightarrow Isentropic Efficiency

(i) Turbines - isentropic turbine will produce the most power (largest possible fraction)



$$\eta_t = \frac{h_1 - h_{2s}}{h_1 - h_2}$$

(ii) Compressors - isentropic compressor will require the least power



$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1}$$

Example 8-118) Adiabatic CO_2 Compressor

State 1: $P_1 = 100 \text{ kPa}$, $T_1 = 300 \text{ K}$, $m = 1.8 \frac{\text{kg}}{\text{s}}$

State 2: $P_2 = 600 \text{ kPa}$, $T_2 = 450 \text{ K}$

$\eta_c = \frac{h_{2s} - h_1}{h_{2A} - h_1} = \frac{C_{PAg} (T_{2s} - T_1)}{C_{PAg} (T_{2A} - T_1)}$

$T_{2s} = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{\kappa-1}{\kappa}}$ $\xrightarrow[\text{CO}_2]{\text{Tables}} \kappa = 1.2635$

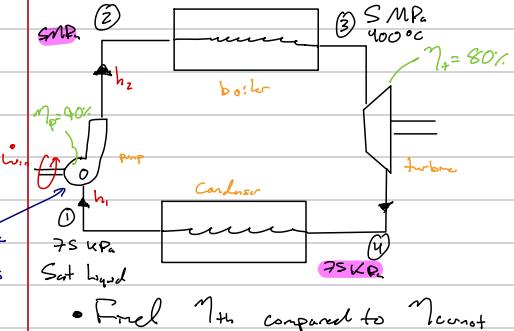
$\Rightarrow T_{2s} = 434.2 \text{ K}$

$\therefore \eta_c = \frac{(434.2 \text{ K} - 300 \text{ K})}{(450 \text{ K} - 300 \text{ K})} = 0.895$

$\eta_c = 89.5\%$

Review Lecture

Example Steam Power Plant (Rankine cycle) ← 4 components



$$\eta_{th} = 1 - \frac{\dot{Q}_i}{\dot{Q}_{in}} = \frac{\dot{W}_{net}}{\dot{Q}_{in}}$$

$$\eta_{carnot} = 1 - \frac{T_L}{T_H}$$

State ①

$$P_1 = 75 \text{ kPa} \xrightarrow{s_{1t}} T_1 = 91.76^\circ\text{C}, s_p = s_1 = 1.2132 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$x_1 = 0 \quad h_p = h_1 = 384.44 \frac{\text{kJ}}{\text{kg}}$$

Note: $\dot{W}_{pump} = \dot{V}_{in} = \dot{m}(h_{2t} - h_1)$

Boiler & compressor are both isobaric (constant P)

Looks@ both case - State ②S: $s_2 = s_1 = 1.2132 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

Interpolation not on exam 3

For actual pump:

$$\eta_p = \frac{h_{2A} - h_1}{h_{2t} - h_1} = 0.40$$

$$\Rightarrow h_{2A} = 390.74 \frac{\text{kJ}}{\text{kg}} \text{ vs. } h_{2t} = 390.107 \frac{\text{kJ}}{\text{kg}}$$

$$\dot{W}_{pump} = \dot{m}(h_{2A} - h_1) = \dot{m}(6.3) \text{ kW}$$

State ②

$$P_2 = 3 \text{ MPa} \xrightarrow{s_{2t}} s_2 < s_{2c} < s_g \xrightarrow{\text{comp.}} h_{2s} = 390.107 \frac{\text{kJ}}{\text{kg}}$$

$$s_{20} < s_2 < s_{100}$$

State ③

$$T_3 = 400^\circ\text{C} \xrightarrow{s_{3t}} h_3 = 3146.2 \frac{\text{kJ}}{\text{kg}}$$

$$P_3 = 3 \text{ MPa} \xrightarrow{s_{3s}} s_3 = 6.6483 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

Isentropic expansion from ③ → ④s

$$P_4 = 75 \text{ kPa} \xrightarrow{s_{4t}} x_{4s} = 0.871 \quad \text{from } s_{4s}$$

$$s_4 = s_3 = 6.6483 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \xrightarrow{\text{mix}} h_{4s} = 2382.8 \frac{\text{kJ}}{\text{kg}}$$

$$\text{For actual turbine } \eta_t = \frac{h_3 - h_{4a}}{h_{3t} - h_{4s}} = 0.80$$

$$\Rightarrow h_{4a} = 2523.6 \frac{\text{kJ}}{\text{kg}}$$

$$\dot{W}_{turbine} = \dot{W}_{out} = \dot{m}(h_3 - h_{4a}) = \dot{m}(663) \text{ kW}$$

Boiler heat addition:

$$\dot{Q}_{in} = \dot{m}(h_2 - h_{2a}) = \dot{m}(2806) \text{ kW}$$

Thermal Efficiency (η_{th})

$$\eta_{th} = \frac{\dot{W}_{net} + \dot{W}_{pump}}{\dot{Q}_{in}}$$

$$= \frac{\dot{m}(663 - 6.8)}{\dot{m}(2806)} = 23.4\%$$

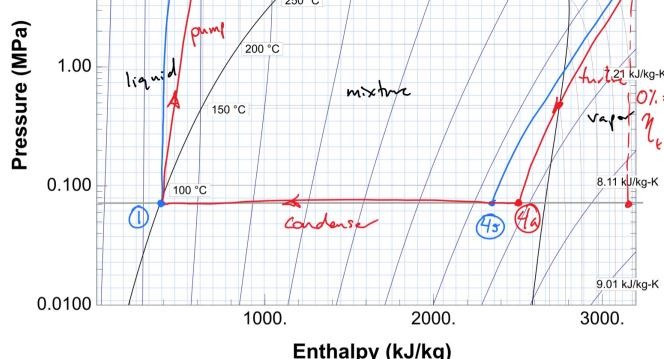
Carnot Efficiency (η_{carnot})

$$\eta_{carnot} = 1 - \frac{T_L}{T_H}$$

← temp into pump ($T_{sat} @ 75 \text{ kPa}$)

← temp into turbine

$$= 1 - \frac{41.8 + 273.15}{400 + 273.15} = 45.8\%$$



Exam 2 Notes



Fluid Mechanics

- Study of stationary and moving fluids (statics & dynamics)

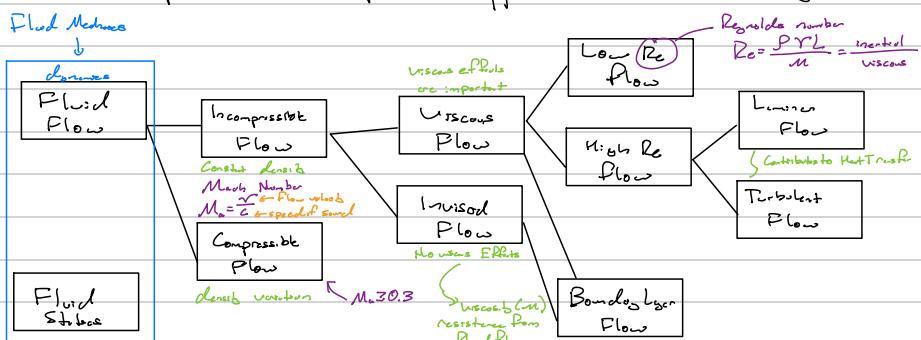
- Fluids: Captures both liquids & gases

- Applications

- 1) Aerodynamics: Flow \Rightarrow forces (lift, drag)
 - 2) Hydrodynamics - ships, subs, etc.
 - 3) Fluid Power - hydraulics, pneumatics
 - 4) Fluid Handling - Pipe Flows
 - 5) Hydrostatics
 - 6) Bio-fluid Flows
 - 7) Geophysical Flows - atmospheric circulation, ocean currents
 - 8) Astrophysics / Motion of cosmic objects & galaxies
 - 9) Shock Waves
- Compressible flow phenomenon

- Extreme range of length and time scales

→ split up entire space of applications into various regimes



General equations

- Can be quite complex

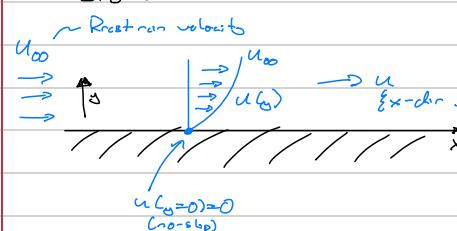
Navier-Stokes equation: $\rho \frac{D\mathbf{u}}{Dt} = -\frac{\partial \mathbf{P}}{\partial x} + \mu \left(\frac{\partial^2 \mathbf{u}}{\partial x^2} + \frac{\partial^2 \mathbf{u}}{\partial y^2} + \frac{\partial^2 \mathbf{u}}{\partial z^2} \right) + \rho g_x$

Fay Fmax is not covering this case

↳ nonlinear, partial differential equations

- We will rely on empirical relations, empirical geometries for engineering applications

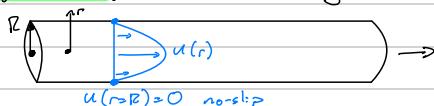
Numerical flow



- No-slip condition: Fluid adjacent to a solid surface takes the velocity of that surface

- External Flow: Surface in an infinite flow field (common for wind tunnel)

- Internal Flow: Flow is fully bounded by a solid surface

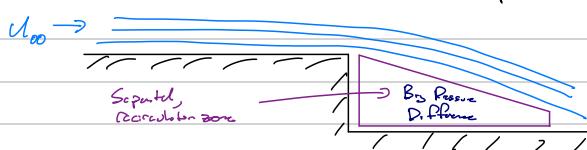


Flow Separation

a) **Attached Flow:** Flow follows the shape of a body



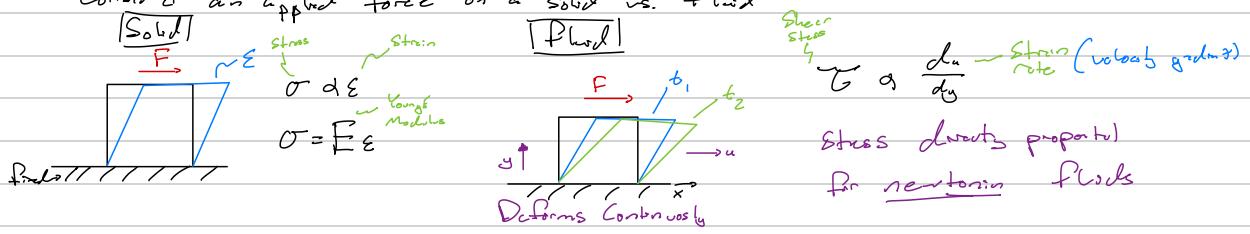
b) **Separated Flow:** Flow separates / doesn't follow surface due to abrupt geometry. Flow



Viscosity

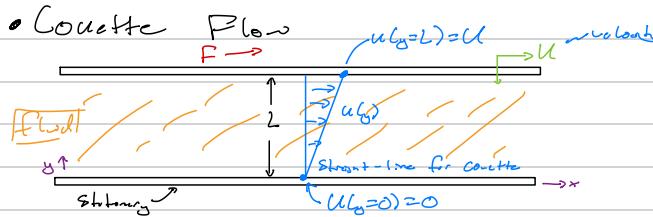
- Resistance to flow, molecular friction

Consider an applied force on a solid vs. fluid



$$\tau \propto \frac{du}{dy} \quad \text{Strain rate (velocity gradient)}$$

Stress directly proportional
for Newtonian fluids



- Force \Rightarrow Shear Stress on the fluid: $\tau = \frac{F}{A}$
 - Force \Rightarrow Upper plate velocity U
- $$\tau \propto \frac{dU}{dy}$$

Newtonian Fluids: Common fluids like water, air, oil, etc.

Non-newtonian Fluids viscosity changes based on force

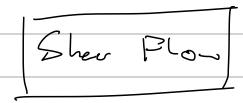
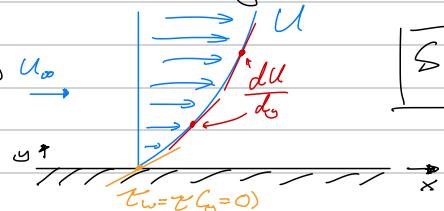
- To find the force

$$\tau = \tau A = \mu \frac{dU}{dy} \quad A = \mu \frac{U}{L} \quad \text{For all Newton Fluids}$$

- Introduce proportionality constant for Newtonian Fluids

$$\tau = \mu \frac{dU}{dy} \quad \text{Dynamic Viscosity} \quad \eta = \mu$$

- More Generally



Surface Tension

- Comparative attractions between similar and dissimilar molecules

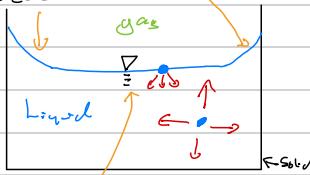
- Some effects of surface tension

(i) Spherical Droplets
 \hookrightarrow minimum energy state



(ii) Wetting Behavior
(v) Capillary Rise

(iii) Meniscus



Surface Area of a Sphere is $4\pi r^2$

(iv) Free Surface

- Requires some work to increase free surface area
- area \Rightarrow surface tension (σ)

$$\sigma \left[\frac{\text{energy}}{\text{area}} = \frac{\text{Force} \times \text{distance}}{\text{area}} = \frac{\text{Force}}{\text{length}} \right] \quad \text{SI Unit: } \text{N/m}$$

- Consider a spherical droplet



As fluid is injected, surface area is created
↳ molecules move from the interior to the surface \rightarrow takes some amount of energy

- Energy stored in the droplet

$$dE = \sigma dA_s = \sigma 8\pi r dr$$

$$\left. \begin{aligned} A_s &= 4\pi r^2 \\ dA_s &= 8\pi r dr \end{aligned} \right\}$$

- Work required to increase A_s

$$dW = F dr = (P_i - P_o) A_s dr = (P_i - P_o) 4\pi r^2 dr$$

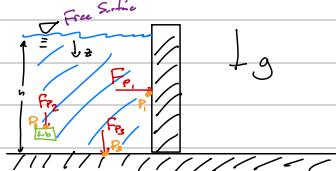
- Equate dE and dW

$$dE = \sigma 8\pi r dr = 4\pi r^2 dr (P_i - P_o) = dW \Rightarrow (P_i - P_o) = \frac{2\sigma}{r}$$

Hydrostatics

Hydrostatics Def. Impact of pressure on submerged surfaces

Applications: Dams, Tanks



Pressure

(i) Pressure acts isotropically

(acts the same in all directions)

(ii) Pressure leads to a pressure force

$$\text{Pressure } \left[\frac{\text{Force}}{\text{area}} \right] \rightarrow F_{\text{pressure}} = \int P dA$$

(iii) Pressure force on a submerged object (like tank) acts normal to its surface, and always acts inward

(iv) Pressure varies with depth according to the hydrostatic pressure variation

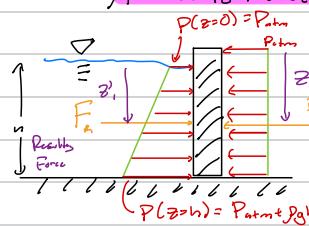
$$\frac{dP}{dz} = \rho g \quad \begin{array}{l} \text{Net pressure} \\ \text{Due to depth} \\ \text{Accel due to gravit}\end{array}$$

$$\Rightarrow P(z) = \int \rho g dz = P(z=0) + \rho g z \quad \begin{array}{l} \text{Assume const } P \\ \text{At } z=0\end{array}$$

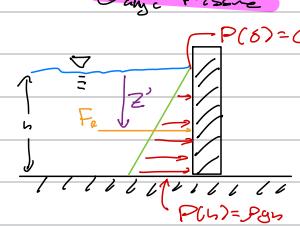
• Convenient to write things in gauge pressure:

$$P_{\text{gauge}} = P_{\text{absolute}} - P_{\text{atm}}$$

Absolute Pressure



Gauge Pressure



Goals (i) Determine the resultant force (F_R)

(ii) Determine where this single force acts (line of action)

1) **Resultant Force** $dA = w dz$

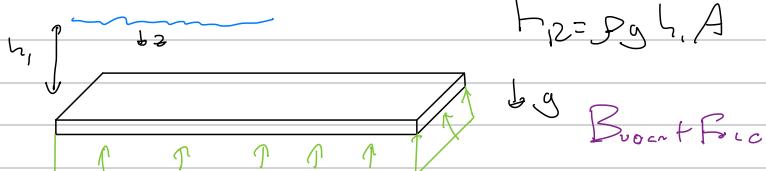
$$F_R = \int_A P(z) dA = \int_0^h P(z) w dz$$

a) Linear Pressure Distribution

$$F_R = w \int_0^h \rho g z dz = w \left[\frac{1}{2} \rho g z^2 \right]_0^h = \frac{1}{2} \rho g h^2 w = \boxed{\frac{1}{2} \rho g h (h_w)}$$

Average Pressure \downarrow
Area \downarrow

b) Uniform Pressure



$$F_R = P g h \cdot A$$

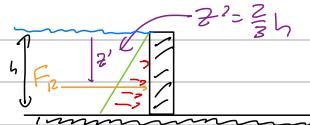
Buoyant Force

2) Line of Action

Location needs to lead to the same moment as the force distribution

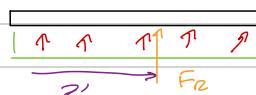
a) Linear Pressure Distribution

$$z' = \frac{2}{3} h$$

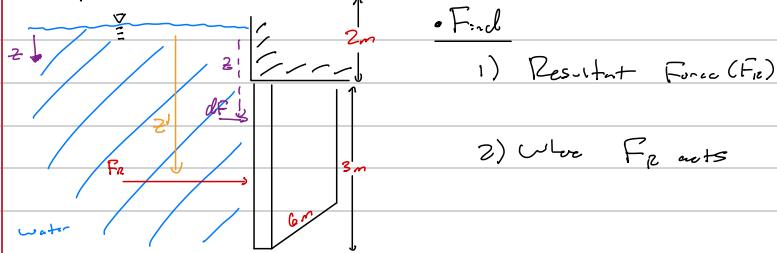


b) Uniform Pressure

z' is half way across



Example 11-4s



• $F_{R,z}$

1) Resultant Force (F_R)

2) Where F_R acts

2 Approaches possible Approaches

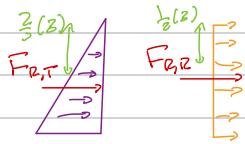
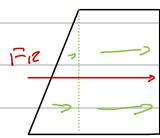
1) Integral Approach

$$F_R = \int P(z) dA = w \int_0^S \rho g z \, dz = \frac{\rho g S}{2} (S^2 - 2^2) = 618,030 \text{ N} \rightarrow F_R = 618 \text{ kN}$$

$$F_R z' = \int_0^S \rho g z^2 \, dz = \rho g w \int_0^S z^2 \, dz = \frac{\rho g w}{3} (S^3 - 2^3) = 2,248,540 \text{ N.m}$$

$$618,030 \text{ N} \cdot z' = 2,248,540 \text{ N.m} \rightarrow z' = 3.71 \text{ m}$$

2) Decomposition



$$F_{R,T} = P_{avg} \cdot A \quad \text{Area} \\ = \frac{1}{2} \rho g (S)(\bar{z})(6) \\ = 265 \text{ kN}$$

$$\Rightarrow F_R = F_{R,T} + F_{R,R} = 618 \text{ kN}$$

$$F_{R,R} = P \cdot A \\ = 2 \rho g (S)(6) \\ = 363 \text{ kN}$$

$$F_R = 618 \text{ kN}$$

Take the moment about the base $\Rightarrow z'$

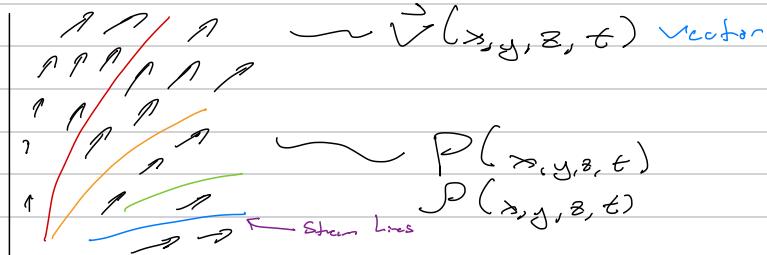
$$F_R z' = F_{R,T} \frac{2}{3}(S) + F_{R,R} \frac{1}{2}(S) \Rightarrow z' = 1.71 \text{ m}$$

$$z' = 3.71 \text{ m}$$

Shift to the free surface: $z' = 2m + \frac{1}{2}(S) = 3.71 \text{ m}$

Fluid Dynamics & The Bernoulli Equation

Flow Fluid



Stream lines \rightarrow Line formed from the tangent to trajectory at any point

• For steady flows $[V(x, y, z)]$ all fluid particles passing through a certain point follow the same stream lines

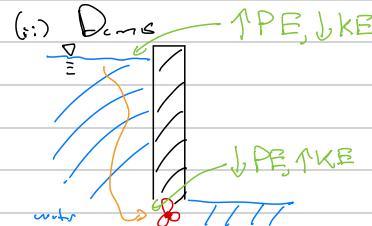
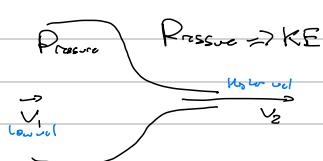
Bernoulli Equation

\rightarrow Provides a relationship between pressure, elevation, and velocity

- Conservation of mechanical energy

Applications

(i) Nozzle



Derivation

• Conservation of Linear Momentum applied to a fluid element

i) Acceleration of a fluid element

$$V_s = f(s, t) \quad \text{in general} \quad \text{velocity is a function of two variables}$$

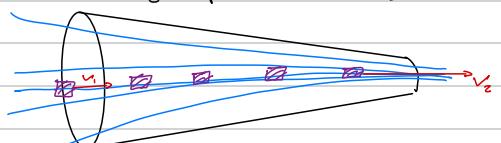
$$V_s = \frac{ds}{dt} ?$$

- Total change in V_s
- multidimensional $\Rightarrow dV_s = \frac{\partial V_s}{\partial s} ds + \frac{\partial V_s}{\partial t} dt$
- Divide through by dt
- $$\alpha = \frac{dV_s}{dt} = \frac{\partial V_s}{\partial s} \frac{ds}{dt} + \frac{\partial V_s}{\partial t}$$

- $\frac{\partial V_s}{\partial t} = 0$ implied by "steady-flow"
- Sub back in $V_s = \frac{ds}{dt}$
- $\alpha = V_s \frac{\partial V_s}{\partial s}$

Change in velocity
at a point with respect
to time

Physical picture of $V_s \frac{ds}{dt}$



\rightarrow Acceleration due to movement through
a velocity field despite steady flow

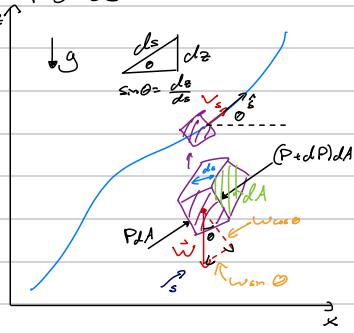
$$\bullet \text{NBL along stream line} : \sum F_s = m \alpha_s$$

Assumptions

- (i) Steady flow - no transient effects
- (ii) Inviscid - viscous effects negligible
- (iii) Incompressible (ρ const.)
- (iv) Flow along a stream line
- (v) No heat transfer
- (vi) No work interactions

\rightarrow Cannot apply across pump or turbine

2) Forces



Forces

- 1) Weight; $W = mg$; $m = \rho dV = \rho dA dz$
- 2) Pressure Force; Normal force due to pressure

$$\sum F_x = -W \sin \theta + P dA - (P + dP) dA$$

$$\sum F_x = -dP dA - mg \sin \theta = m a_s = m v_s \frac{dv_s}{ds} - dP dA - \rho dA dz g \frac{dz}{ds} = \rho dA dz v_s \frac{dv_s}{ds}$$

$$\Rightarrow -dP - \rho g dz = \rho v_s dv_s \\ = \frac{1}{2} \rho d(v_s^2)$$

$$\Rightarrow dP + \rho g dz + \frac{1}{2} \rho d(v_s^2) = 0$$

$$\frac{dP}{\rho} + g dz + \frac{1}{2} d(v_s^2) = 0$$

$$\frac{dv_s}{ds} = \frac{dA}{ds}$$

since $v_s = f(s)$

• Integrate Expression

$$\int \frac{dP}{\rho} + g z + \frac{1}{2} V_s^2 = \text{Constant} \quad \text{Valid for compressible or incompressible flow}$$

→ For constant density

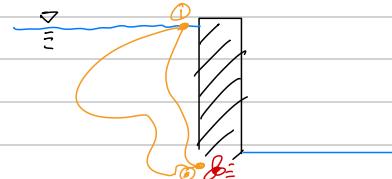
$$\frac{P}{\rho} + \frac{1}{2} V_s^2 + g z = \text{Constant}$$

Start point for Bernoulli:

Equation along a stream to

• Bernoulli: Equation + principle applied between two points:

$$P_1 + \frac{1}{2} \rho V_1^2 + \rho g z_1 = P_2 + \frac{1}{2} \rho V_2^2 + \rho g z_2$$



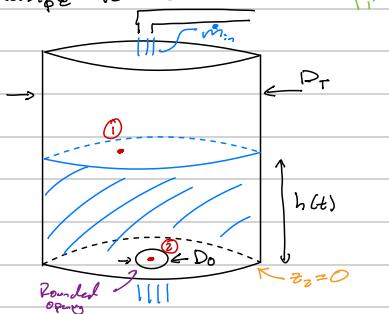
(Continental) Bernoulli Equations

$$P + \frac{1}{2} \rho V^2 + \rho g z = \text{constant}$$

Assumptions for this form

- ① Steady
- ② Flow along a stream
- ③ Incompressible
- ④ No heat transfer
- ⑤ Laminar flow

Example 12-22



$V_2 = 0$ due to free surface

$$P_1 + \frac{1}{2} \rho V_1^2 + \rho g z_1 = P_2 + \frac{1}{2} \rho V_2^2 + \rho g z_2$$

$P_1 = P_{atm}$ so $P_1 = P_2$

constant \rightarrow

in mass

$$\rho V_1 A_1 = \rho V_2 A_2$$

$$\Rightarrow V_1 \frac{\pi D_1^2}{4} = V_2 \frac{\pi D_2^2}{4}$$

Directly given
so $(\frac{D_2}{D_1})^2$ used

$$V_2 = \sqrt{2gh}$$

• At steady-state conditions:

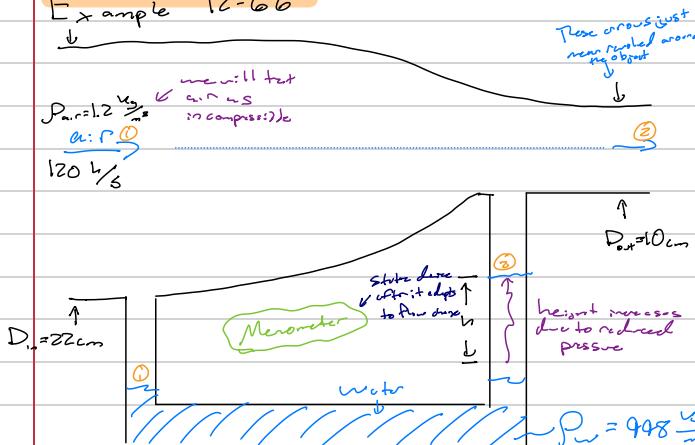
$$\therefore h_{max} = \frac{1}{2g} \left(\frac{4m_{in}}{\pi D_2^2 \rho} \right)^2$$

$$m_{in} = m_{out} = \rho V_2 A_o = \rho \sqrt{2gh_{max}} \frac{\pi D_o^2}{4}$$

- Find h_{max} (equilibrium fill level)
in terms of m_{in} , D_o , P_{atm} .

P_2 also is P_{atm} because it is an open hole into the atmosphere

Example 12-66



Problem

- Velocity should increase due to conservation of mass
- Pressure should decrease; velocity needs to go up; its increased from some place, and that's the pressure

Find: The height 'h' that the water goes up

when wind goes through tunnel

Wind goes through tunnel creates a low pressure zone at end of nozzle so transition has 'suck up'

$$\text{Bernoulli Eq: } P_1 + \frac{1}{2} \rho V_1^2 + \rho g Z_1 = P_2 + \frac{1}{2} \rho V_2^2 + \rho g Z_2 \quad \begin{cases} \text{air} \\ \text{water} \end{cases}$$

$$P_{1w} + \frac{1}{2} \rho V_{1w}^2 + \rho g Z_{1w} = P_{2w} + \frac{1}{2} \rho V_{2w}^2 + \rho g Z_{2w} \quad \begin{cases} \text{water} \end{cases}$$

$$\rightarrow \text{Air: } P_1 - P_2 = \frac{1}{2} \rho_a (V_2^2 - V_1^2) \quad \begin{matrix} \text{express} \\ \text{as} \end{matrix} \quad h = \frac{1}{2} \frac{\rho_a}{\rho_w g} (V_2^2 - V_1^2) \quad \begin{matrix} \text{use still} \\ \text{measured} \end{matrix}$$

$$\rightarrow \text{Water: } P_1 - P_2 = \rho_w g h$$

$$\begin{cases} P_{1a} = P_{1w} = P_1 \\ P_{2a} = P_{2w} = P_2 \end{cases}$$

Ans ρ is low, can assume column pressure = same

$$\text{Find what's } \dot{m}_1 = \dot{m}_2 \quad \text{Steady-state} \quad \dot{m} = \rho V_{avg} A \quad \text{and } \dot{c} = V_{avg} A = \frac{\dot{m}}{\rho}$$

$$\rightarrow \text{If incompressible, we can say } t_1 + V_1 = t_2 + V_2$$

$$\therefore V_1 = V_2 = \dot{V} = 120 \frac{\text{m}}{\text{s}} = 0.12 \frac{\text{m}^3}{\text{s}}$$

$$\Rightarrow V_1 = \frac{\dot{V}}{A_1} = \frac{\dot{V}}{\frac{\pi D_1^2}{4}} = 3.16 \frac{\text{m}^3}{\text{s}} \quad \text{and} \quad V_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{\frac{\pi D_2^2}{4}} = 15.8 \frac{\text{m}^3}{\text{s}} \quad \left. \right\} h = 1.37 \text{ cm}$$

Energy Form of the Bernoulli Equation

General Energy Form of the Bernoulli Equation

Goal: Start with energy conservation to relax some of the assumptions

- Rate based open system of the first law of thermodynamics

$$(Q_{in} - Q_{out}) + (w_{in} - w_{out}) = m \left[(h_2 - h_1) + \frac{1}{2} (V_2^2 - V_1^2) + g(z_2 - z_1) \right]$$

(i) Define basis $\begin{cases} q = \dot{Q}/m \\ w = \dot{W}/m \end{cases}$ $\Rightarrow q_{net,in} = q_m - q_{out}$ $\Rightarrow q_{net,in} + w_{net,in} = (h_2 - h_1) + \frac{1}{2} (V_2^2 - V_1^2) + g(z_2 - z_1)$

(ii) Use definition of h : $\begin{cases} h = u + p \\ h_1 = u_1 + \frac{P_1}{\rho_1} \\ h_2 = u_2 + \frac{P_2}{\rho_2} \end{cases}$

$$\Rightarrow \frac{P_1}{\rho_1} + \frac{1}{2} V_1^2 + g z_1 + w_{net,in} = \frac{P_2}{\rho_2} + \frac{1}{2} V_2^2 + g z_2 + (u_2 - u_1 + q_{net,in})$$

mechanical energy is to added mechanical energy
turbine (from pump/turbine)

mechanical energy is to added mechanical energy
of the system

internal energy and heat transfer

- For ideal scenarios (no resistances)

$$u_2 = u_1 + q_{net,in} \Rightarrow u_2 - u_1 - q_{net,in} = 0$$

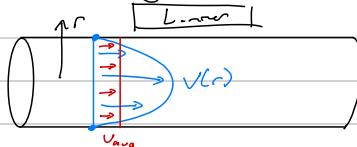
- For nonideal scenarios (fluid friction present)

$$u_2 - u_1 > q_{net,in} \Rightarrow u_2 - u_1 = q_{net,in} + e_{losses}$$

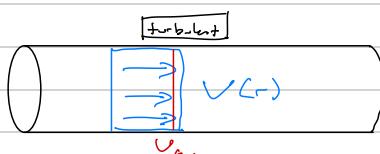
$$(u_2 - u_1 - q_{net,in}) = e_{losses}$$

$$\Rightarrow \frac{P_1}{\rho_1} + \frac{1}{2} V_1^2 + g z_1 + w_{pump} = \frac{P_2}{\rho_2} + \frac{1}{2} V_2^2 + g z_2 + w_{turbine} + e_{losses}$$

Kinetic Energy correction factor $KE_{total} = \int_{A_0} \frac{1}{2} V(r)^2 dA_0 \Rightarrow KE \text{ correction factor } (\lambda)$



• For laminar flow: $\lambda = 2$



• For turbulent flow: $\lambda = 1.03 \approx 1$

$$\therefore \frac{P_1}{\rho_1} + \frac{1}{2} \lambda V_1^2 + g z_1 + w_{pump} = \frac{P_2}{\rho_2} + \frac{1}{2} \lambda V_2^2 + g z_2 + w_{turbine} + e_{losses}$$

Energy Equation in terms of power λ is omitted for clarity but should be multiplied by the KE_{total}
 \rightarrow rate basis ($\times m$) \rightarrow Actual work

$$\therefore \dot{m} \left[\frac{P_1}{\rho_1} + \frac{1}{2} V_1^2 + g z_1 \right] + \dot{w}_{pump} = \dot{m} \left[\frac{P_2}{\rho_2} + \frac{1}{2} V_2^2 + g z_2 \right] + \dot{w}_{turbine} + \dot{e}_{losses}$$

$$\therefore \dot{E}_{losses} = \dot{E}_{losses,pump} + \dot{E}_{losses,turb} + \dot{E}_{losses,piping}$$

- For Pump

$$\dot{w}_{pump,n} = \dot{w}_{pump} - \dot{E}_{losses,pump} \Rightarrow \eta_{pump} = \frac{\dot{w}_{pump,n}}{\dot{w}_{pump}}$$

Actual
Work

(isentropic: $\lambda=1$)

- For Turbine

$$\dot{w}_{turbine} = \dot{w}_{turbine} + \dot{E}_{losses,turb} \Rightarrow \eta_{turbine} = \frac{\dot{w}_{turbine}}{\dot{w}_{turbine}}$$

Actual
Work

$$\text{Vol. Flowrate} = \frac{\dot{V}_P}{\text{Cross Sec. Area}}$$

$$\Rightarrow \dot{m} \left[\frac{P_1}{\rho_1} + \frac{1}{2} V_1^2 + g Z_1 \right] + \dot{W}_{pump,u} = \dot{m} \left[\frac{P_2}{\rho_2} + \frac{1}{2} V_2^2 + g Z_2 \right] + \dot{W}_{turbine} + \dot{E}_{losses,piping}$$

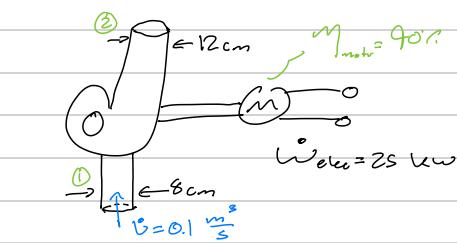
Writing Bernoulli Equation in terms of head & usual static point

→ head has dimensions of length, Divide by $\frac{g}{mg}$

$$h_{pump,u} = \frac{\dot{W}_{pump,u}}{mg}, h_{turbine} = \frac{\dot{W}_{turbine}}{mg}, h_2 = \frac{\dot{E}_{losses,piping}}{mg} \leftarrow \text{frictional head loss}$$

$$\Rightarrow \frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + Z_1 + h_{pump,u} = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + Z_2 + h_{turbine} + h_2 \leftarrow \begin{array}{l} \text{Inviscid head loss but near (1) and (2)} \\ \text{due to fluid friction} \end{array}$$

Example 12-47



Given

- $\eta_{motor} = 90\%$
- $\dot{W}_{elc} = 25 \text{ kW}$
- $P_1 = 860 \text{ kPa}$
- Pressure rise: $250 \text{ kPa} \leftarrow \Delta P = P_2 - P_1$, here
- KE correction factor: $\lambda = 1.05$

Fnd: Pump efficiency

$$\eta_{pump} = \frac{\dot{W}_{pump,u}}{\dot{W}_{pump}} \leftarrow \dot{W}_{pump} = \eta_{motor} \times \dot{W}_{elc}$$

$$\frac{P_1}{\rho_1 g} + 2 \frac{V_1^2}{2g} + Z_1 + h_{pump,u} = \frac{P_2}{\rho_2 g} + 2 \frac{V_2^2}{2g} + Z_2 + h_{turbine} + h_2 \leftarrow \begin{array}{l} \text{outlet and inlet are closed, so} \\ \text{there is not enough piping to consider} \\ \text{losses due to piping} \end{array}$$

$$h_{pump,u} = \frac{P_2 - P_1}{\rho g} + 2 \frac{V_2^2 - V_1^2}{2g}$$

$$V_{outlet} = \frac{C}{A_2} \quad V_1 = \frac{C}{A_1} = \frac{C}{\frac{\pi}{4} D_1^2} = 0.89 \frac{m}{s}$$

$$\text{incompressible} \rightarrow V_1 = V_2 = V \quad V_2 = \frac{C}{A_2} = \frac{C}{\frac{\pi}{4} D_2^2} = 8.84 \frac{m}{s}$$

$$\rightarrow h_{pump,u} = \frac{\Delta P}{\rho g} + 2 \frac{V_2^2 - V_1^2}{2g} = \frac{250,000 \text{ Pa}}{1000 \text{ kg/m}^3 \times 9.81} + 1.05 \frac{8.84^2 - 0.89^2}{2 \times 9.81}$$

$$= 29.6 \rightarrow 17 = 12.6 \text{ m}$$

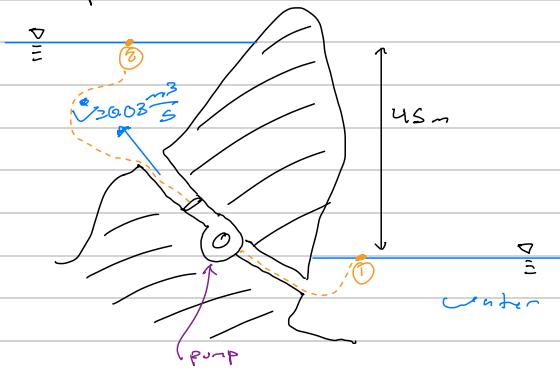
use less pump power to get pressure Watch your units

$$\dot{W}_{pump,u} = \dot{m} g h_{pump,u} = \rho V g h_{pump,u} = 10.66 \text{ kW} = 10.66 \text{ kW}$$

$$\eta_{pump} = \frac{\dot{W}_{pump,u}}{\dot{W}_{pump}} \rightarrow \frac{10.66 \text{ kW}}{\eta_{motor} \cdot \dot{W}_{elc}} = \frac{10.66 \text{ kW}}{22.8 \text{ kW}} = 47.4\%$$

$$\boxed{\eta_{pump} = 47.4\%}$$

Example 12-60



Pump delivers 20kW of useful power

Find

- (i) Irreversible head loss (h_L)
- (ii) Lost mechanical power

1) Frictional Head Loss

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 + h_{pump,u} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_{pump,u} + h_L$$

velocities small relative
z changes at free surface

$$\Delta z = z_2 - z_1 = 45 \text{ m}$$

$$\Rightarrow h_L = h_{pump,u} - \Delta z$$

$$h_{pump,u} = \frac{\dot{E}_{pump,u}}{\dot{m} g} = \frac{20 \text{ kW}}{\rho \dot{V} g} = 67.96 \text{ m}$$

$$h_L = h_{pump,u} - \Delta z = 67.96 - 45 = 23 \text{ m}$$

$$h_L = 23 \text{ m}$$

With pumping work you could raise fluid 67.96 m

2) Lost Mechanical Power

$$\dot{E}_{losses, pipe} = h_L \dot{m} g = 67.96 \text{ kW}$$

$$\dot{E}_{losses, pipe} = 6.76 \text{ kW}$$

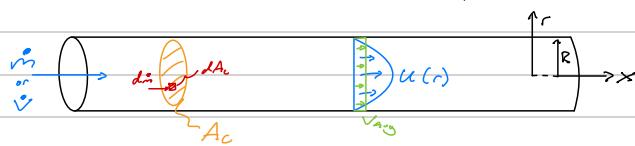
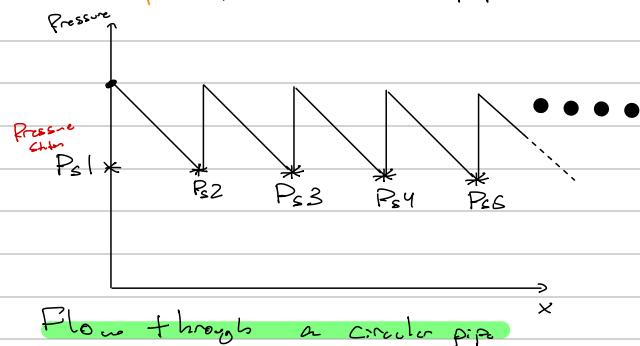
$$h_L = \frac{\dot{E}_{losses, pipe}}{\dot{m} g}$$

Internal Flow

Internal Flow

Goal: Calculate frictional head loss (h_f) based on velocity, viscosity etc.

Example: Trans-Alaska Pipeline ~800mi



• Mass Flux rate:

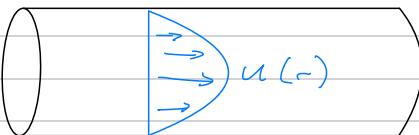
$$\begin{aligned} \text{Overall} \quad \dot{m} &= \rho V_{avg} A_c \\ \text{Local} \quad \dot{m} &= \int_{A_c} \rho u(r) dA_c \end{aligned}$$

• Equate \dot{m} expressions: $\int_{A_c} \rho u(r) dA_c = \rho V_{avg} A_c$

$$\rightarrow \text{Assume incompressible and circular pipe: } \rho_1 = \rho_2 = \rho, A_c = \pi R^2 \text{ and } dA_c = 2\pi r dr$$

$$\Rightarrow V_{avg} = \frac{2}{R^2} \int_0^R u(r) r dr$$

Laminar



Turbulent



• Smooth, orderly flow

• Low velocity, high viscosity

\rightarrow Flow regime based on Reynolds Number (Re)

• High mixing chaotic flow

• High velocity, low viscosity

$$\text{dimensionless } Re = \frac{\rho V_{avg} D_{hyd}}{\mu}$$

Ansatz: D_{hyd} = hydraulic diameter

• Ratio of inertial (momentum) effects to viscous effects

• Transition from laminar to turbulent based on a critical Reynolds number

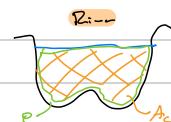
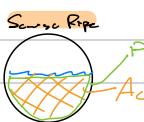
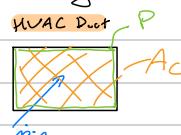
$$Re_{crit} = 2300 \text{ for internal flow}$$

$$Re_{crit} = 5 \times 10^5 \text{ for external flow}$$

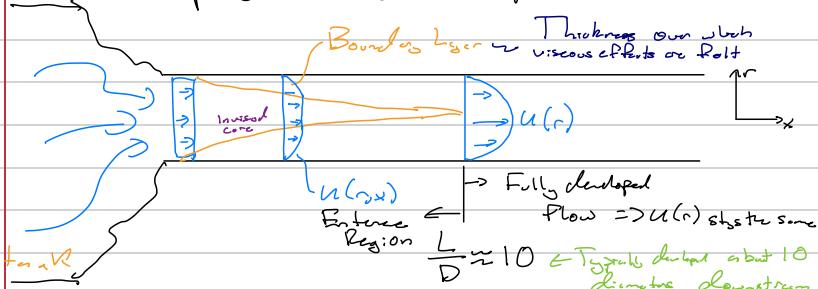
e For non-circular cross sections, use hydrodynamic diameter for D

$$D_h = \frac{4 A_c}{P}$$

Flow crosssec. over wetted perim.

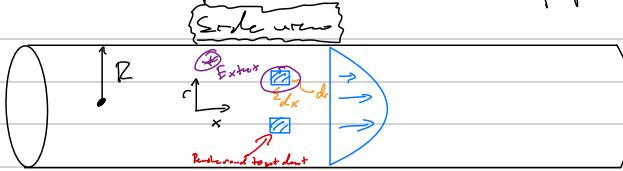


• Developing Fully Developed Flow



Dominos

• Laminar flow in a circular pipe



end view

$$\frac{\partial u}{\partial r} = 0 \quad \frac{\partial u}{\partial x} = 0$$

From assumption 4

Assumptions

- (i) Steady, linear ($R_e < 300$) flow
- (ii) Assume incompressible flow and constant properties
- (iii) Straight, circular pipe
- (iv) Fully developed

Approach: Apply Newton's 2nd Law ($\sum F_x = m a$) to a short element

$$\text{Extent } \Delta x \quad \text{Width } \Delta r \Rightarrow \sum F_x = 0$$

Forces

- (i) Friction / Shear stress
- (ii) Pressure

Sum Forces

$$(P_2 \pi r^2 \Delta x) - (P_1 \pi r^2 \Delta x) + (\tau 2\pi r \Delta x)_{\text{ext}} - (\tau 2\pi r \Delta x)_{\text{int}} = 0$$

• Rearrange divide by $2\pi r \Delta x$

$$\frac{P_{\text{ext}} - P_{\text{int}}}{\Delta x} + \left(\frac{\tau}{r} \right)_{\text{ext}} - \left(\frac{\tau}{r} \right)_{\text{int}} = 0 \quad \text{Newton's 2nd Law}$$

$$\tau = -M \frac{du}{dr}$$

$$\rightarrow \text{Take limit as } \Delta x \rightarrow 0: r \frac{dP}{dx} + \frac{d(\tau)}{dr} = 0$$

$$\Rightarrow r \frac{dP}{dx} = M \frac{d}{dr} \left(r \frac{du}{dr} \right) \quad \text{assumed } M \text{ constant}$$

$$\text{only depends on } r \quad \frac{dP}{dx} = \frac{M}{r} \frac{d}{dr} \left(\frac{du}{dr} \right)$$

$$g(x) = f(r) \Rightarrow \text{to both } g(x) = f(r) = \text{constant must be true}$$

$$\Rightarrow \frac{d}{dr} \left(r \frac{du}{dr} \right) = \frac{f}{M} \frac{dP}{dx} \quad \text{constant}$$

• Integrate twice:

$$u(r) = \frac{r^2}{4M} \frac{dP}{dx} + C_1 \ln r + C_2$$

Constant of integration

• Need 2 boundary conditions to solve for C_1, C_2

1) No-slip condition: $u(r=R) = 0$

2) Symmetric: $\frac{du}{dr}(r=0) = 0$ ← comes from flowing fully developed & laminar

$$\Rightarrow u(r) = \frac{R^2}{4M} \frac{dP}{dx} \left(1 - \frac{r^2}{R^2} \right)$$

$\frac{dP}{dx} \neq 0$ must be true
to push water upstream (in flow)

For laminar flow:
 $u_{\text{avg}} = \frac{1}{2} u_{\text{max}}$

• Average velocity: $u_{\text{avg}} = \frac{2}{R^2} \int_0^R u(r) r dr = - \frac{R^2}{8M} \frac{dP}{dx}$

• Rewrite to isolate $\frac{dP}{dx} \equiv \Delta P_L$

$$\frac{dP}{dx} = - \frac{8M u_{\text{avg}}}{R^2}$$

$$\Rightarrow \Delta P_L = \frac{8Mu_{\text{avg}}L}{R^2}$$

$$= \frac{32Mu_{\text{avg}}L}{D^2}$$

$$\dot{W}_{\text{pump}} = \rho \Delta P_L$$

Incompressible assumed

• Pumping power to overcome frictional losses

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L}$$

$$\Rightarrow \Delta P_L = P_1 - P_2$$

pressure loss

- To generalize results, define the Darcy Friction Factor (f)

$$\Delta P_L = f \frac{L}{D} \frac{1}{2} \rho u_{avg}^2$$

dynamical pressure

$$\frac{\Delta P_L}{\frac{1}{2} \rho u_{avg}^2} = f \frac{L}{D}$$

Euclidean ends up being dimensionless

- Go from $\Delta P_L \rightarrow h_L$

$$h_L = \frac{\Delta P_L}{\rho g} = f \cdot \frac{L}{D} \cdot \frac{u_{avg}^2}{2g}$$

- Friction factor for laminar flow in a circular pipe

$$f: p_c - p_i = \frac{32 \mu L}{D^2}$$

want to look like this

$$= \frac{L}{D} \frac{1}{2} \rho u_{avg}^2 \left(64 \frac{\mu}{\rho u_{avg} D} \right) \frac{1}{Re_p}$$

Reynolds #

$$\Rightarrow f = \frac{64}{Re_p}$$

only based on const (64) and Reynolds number w/ respect to pipe diameter (Re_p)

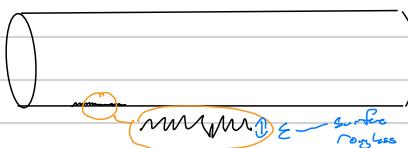
- For turbulent flow

- Empirical rather than analytical (based on experiments)

$$\text{Experiments show } f = f(Re, \frac{\epsilon}{D})$$

Surface roughness

\rightarrow Surface roughness



Curve fit experimental data \rightarrow Colebrook equation

$$\frac{1}{\sqrt{f}} = -2.0 \log_{10} \left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re \sqrt{f}} \right)$$

\rightarrow Implicit equation for $f \rightarrow$ 1 to solve!

- Approximate explicit form \rightarrow Hazen and Equations

$$\frac{1}{\sqrt{f}} \approx -1.8 \log_{10} \left[\left(\frac{\epsilon/D}{3.7} \right)^{1.11} + \frac{6.9}{Re} \right]$$

2% error introduced

- Or, plot colebrook equation plotted as Moody Diagram

(i) Re on x-axis Calculate ϵ/D , find intersection of Re and roughness line

(ii) ϵ/D on secondary y-axis

Result comes from reading across to primary vertical axis \rightarrow now you have f !

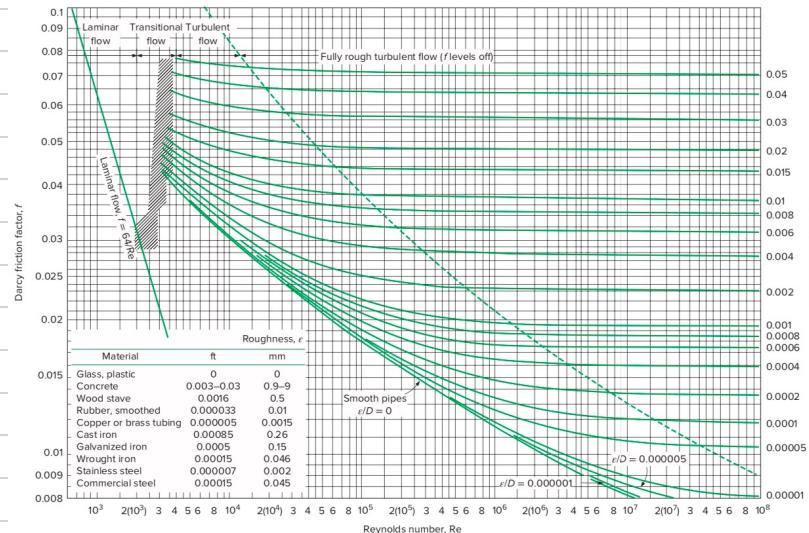
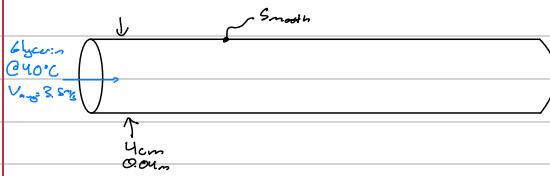


FIGURE A-27

The Moody chart for the friction factor for fully developed flow in circular pipes for use in the head loss relation $\Delta P_L = f \frac{L}{D} \frac{\rho V^2}{2}$. Friction factors in the turbulent flow are evaluated from the Colebrook equation $\frac{1}{\sqrt{f}} = -2 \log_{10} \left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re \sqrt{f}} \right)$.

Example 14-48



Find pressure loss per 10 m of length

$$\text{Glycerin } 40^\circ C \quad \begin{cases} \rho = 1282 \frac{\text{kg}}{\text{m}^3} \\ \mu = 0.0018 \frac{\text{kg}}{\text{m}\cdot\text{s}} \end{cases}$$

$$\text{For water: } \mu_{\text{water}} = 0.00018 \frac{\text{kg}}{\text{m}\cdot\text{s}} \quad \text{So for water: } \mu_{\text{water}} = 0.001 \frac{\text{kg}}{\text{m}\cdot\text{s}}$$

$$\Delta P_L = f \frac{L}{D} \frac{V_{avg}^2}{2}$$

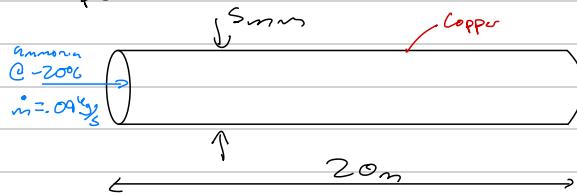
Unknown

$$Re_D = \frac{\rho V_{avg} D}{\mu} = 649.2 \times 2300 \Rightarrow \text{Laminar flow}$$

$$\therefore f = \frac{64}{Re_D^2} = \frac{64}{649.2^2} = 0.099$$

$$\Delta P_L = 189,000 \text{ Pa per 10m}$$

Example 14-51



Find ΔP_L , h_L , $W_{pump,L}$

$$\text{Ammonia } @ -20^\circ C \quad \begin{cases} \rho = 662.1 \frac{\text{kg}}{\text{m}^3} \\ \mu = 2.361 \times 10^{-4} \frac{\text{kg}}{\text{m}\cdot\text{s}} \end{cases}$$

$$Re_D = \frac{\rho V_{avg} D}{\mu}$$

$$v_i = \rho V_{avg} A_c \Rightarrow V_{avg} = \frac{v_i}{A_c} = 6.9 \text{ m/s} \Rightarrow Re_D = 97,100 \Rightarrow \text{Turbulent Flow}$$

$$\Delta P_L = f \frac{L}{D} \frac{V_{avg}^2}{2}$$

Go to Moody Diagram

$$Re_D \quad \left\{ \begin{array}{l} \text{moody} \\ \frac{\epsilon}{D} \end{array} \right\} \Rightarrow f = 0.02$$

$\frac{\epsilon = 0.0015 \text{ mm}}{D = 10 \text{ mm}} = .0003$

$$\therefore \Delta P_L = 1236 \text{ kPa}$$

$$h_L = \frac{\Delta P_L}{\rho g} = 189 \text{ m}$$

$$W_{pump} = \dot{V} \Delta P_L = \dot{m} g h_L = 167 \text{ W}$$

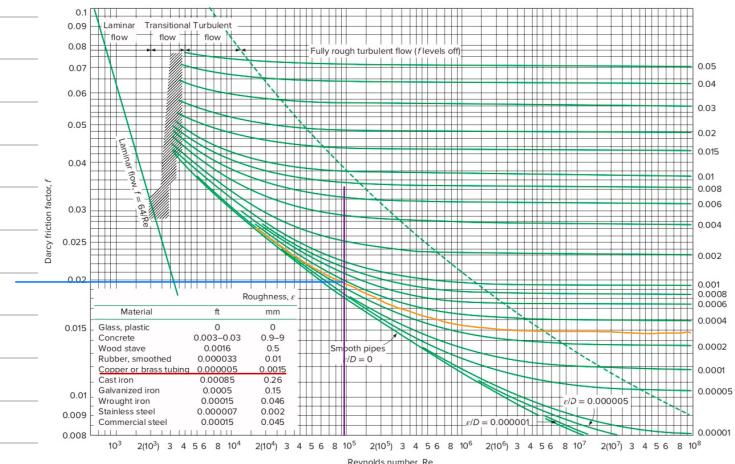


FIGURE A-27 $Re = 97,000 = 4.7 \times 10^4$

The Moody chart for the friction factor for fully developed flow in circular pipes for use in the head loss relation $\Delta P_L = f \frac{L}{D} \frac{\rho V^2}{2}$. Friction factors in the turbulent flow are evaluated from the Colebrook equation: $\frac{1}{f} = -2 \log_{10} \left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re \sqrt{f}} \right)$



Minor Losses

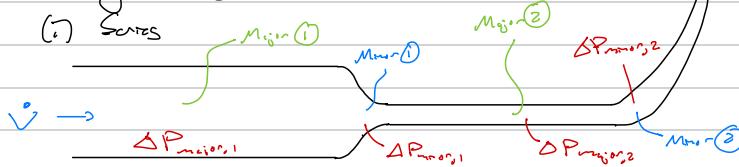
- piping losses from other components
 - (i) Elbows, Tees
 - (ii) Valves
 - (iii) piping transitions (reducing or expanding)
 - (iv) entrance/exit to/from piping

Modeling them

$$h_{L, \text{minor}} = K_L \cdot \frac{V_{avg}^2}{2g}$$

Component specific minor loss coefficient K_L tables give scenarios with V_{avg} for cone & appr. (KE correction factor)

• Combining Losses

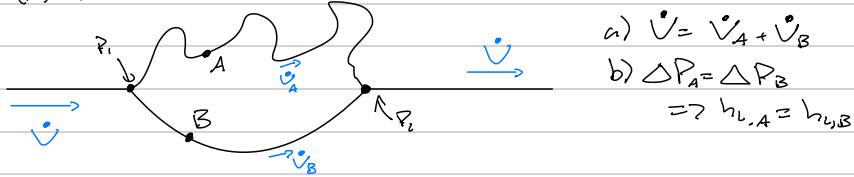


a) \dot{V} constant (incompressible)

b) ΔP_s added

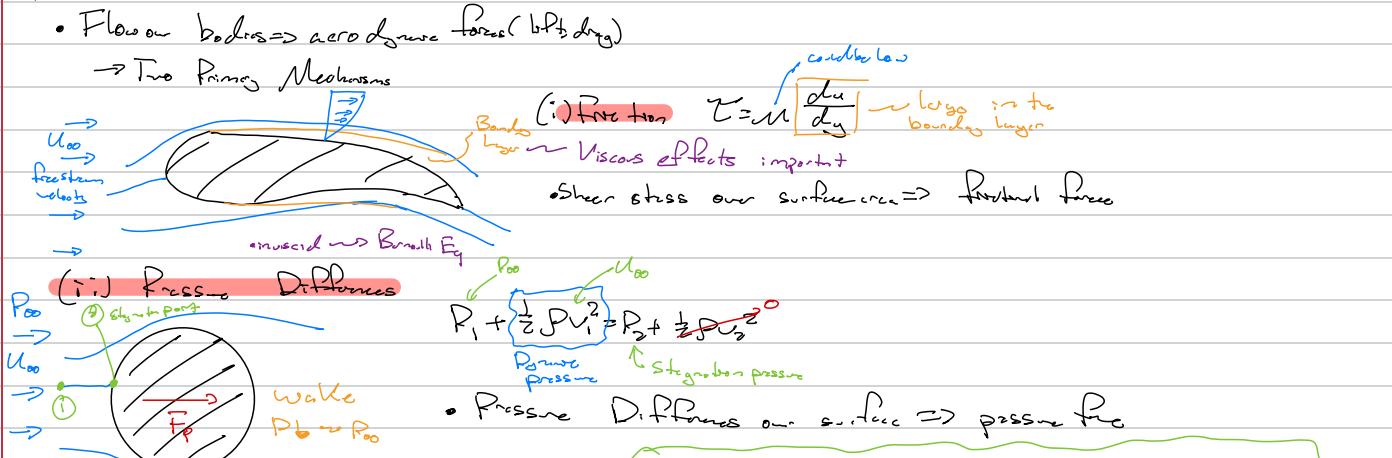
$$\Rightarrow h_{L, \text{tot}} = \sum h_{L, \text{major}} + \sum h_{L, \text{minor}}$$

(ii) Parallel

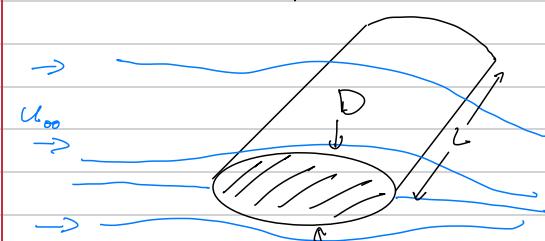


External Flow

External Flow

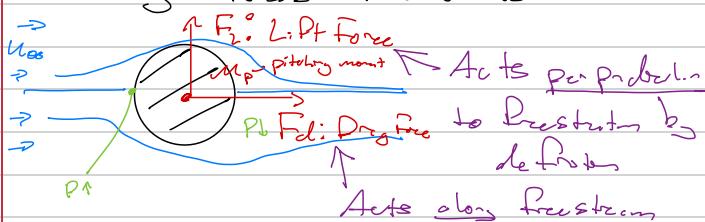


2D vs. 3-D Flows



• 2-D analysis valid if $\frac{L}{D}$ is large

Defining Forces & moments



• Defining Lift & Drag Coefficients

i) Drag Coefficient

$$\text{Dimensionless } C_D = \frac{F_D}{\frac{1}{2} \rho U_\infty^2 A} \Rightarrow F_D = C_D \frac{1}{2} \rho U_\infty^2 A$$

ii) Lift Coefficient

$$C_L = \frac{F_L}{\frac{1}{2} \rho U_\infty^2 A}$$

• Effects that influence aerodynamic forces

(i) Fluid Properties (ρ, μ)

(ii) Object Properties / Shape

a) Streamlined

b) Blunt or bluff

(iii) Flow Regime (Laminar vs. Turbulent)

• Magnitude of C_D

$$P_2 = P_{\text{stagn}} = P_\infty + \frac{1}{2} \rho U_\infty^2$$

$$(P_2 - P_\infty) A \Rightarrow C_D \approx \frac{1}{2} \rho U_\infty^2 A$$

• C_D can be greater than 1.0

• Appropriate area for C_D, C_L

• Depends on shape type

i) Blunt body-frontal area (A_F)

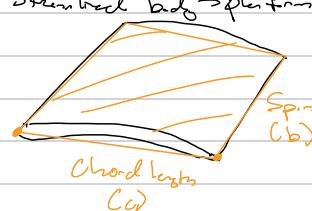


• For a cylinder

$$A_F = \pi D^2$$

• For a sphere

$$A_F = \frac{\pi D^2}{4}$$



$$A_F = b c$$

- Get C_D from table

→ C_D depends on

i) Shape

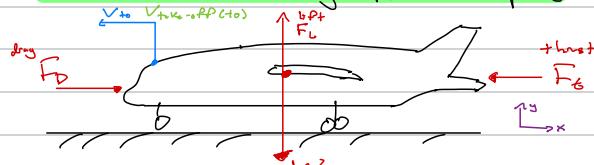
ii) Reynolds number → check restrictions on turbines

For a free velocity $\uparrow \Rightarrow C_D \downarrow$. Laws from \rightarrow flow direction, so

Laws reduce drag at higher wind speeds

Aerodynamic Force Balances

- i) Turb - off velocity for an airplane



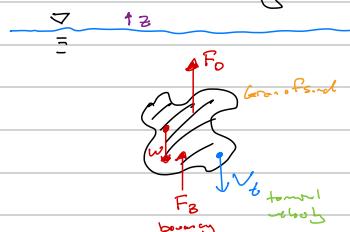
At force equilibrium

$$\sum F_x = 0 \Rightarrow F_T = F_D = C_D \frac{1}{2} \rho V_{turb}^2 A$$

$$\sum F_y = 0 \Rightarrow W = F_L = C_L \frac{1}{2} \rho V_{turb}^2 A$$

$$V_{turb} = \sqrt{\frac{2W}{\rho C_L A}}$$

- ii) Terminal Velocity - sedimentation, suspensions



$$\begin{cases} F_D = C_D \frac{1}{2} \rho_D V_turb^2 A \\ \text{and } \text{displaced volume} \\ W = m g = \rho_s V g \\ F_B = W_{\text{displaced}} = m_p g = \rho_f V g \end{cases}$$

$$\bullet \text{Sum Forces: } \sum F_z = 0 \\ C_D \frac{1}{2} \rho_D V_turb^2 A + D_f V g - \rho_s V g = 0$$

$$V_turb = \sqrt{\frac{2Vg}{C_D A} \cdot \frac{\rho_s - \rho_f}{\rho_D}}$$

For parachute: $C_D = 1.3$

For spherical granofls: $C_D = \frac{24}{Re} (Re < 1)$

Drag Force

- Two main types:

- i) Skin friction drag

→ Due to shear stress

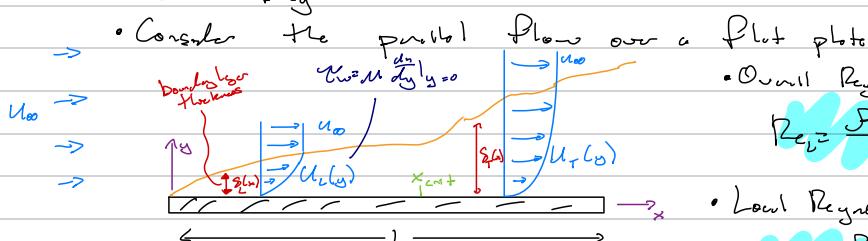
→ Dominant mechanism for streamlined objects

- ii) Form drag (pressure drag)

→ Due to flow blocking/separation

→ Dominant mechanism for bluff bodies

Skin Friction Drag



$$F_D = F_f = C_D \frac{1}{2} \rho U_{infty}^2 A \quad \text{area in contact with fluid}$$

• Skin friction coeff.

Boundary Layer

• Friction Coefficient $\delta(x)$

→ Laminar BL ($Re_x < 5 \times 10^5$)

$$\delta(x) = \frac{4.91}{Re_x^{1/4}}$$

$$C_{f,lmn} = \frac{0.664}{Re_x^{1/4}}$$

• Low Reynolds number

$$Re_x = \frac{\rho U_{infty} x}{\mu}$$

• Critical Reynolds num.

$$Re_{crit} = S \times 10^5$$

↳ Criterion for x_{crit}

• $Re_x < Re_{crit} \Rightarrow$ laminar boundary layer

• $Re_x > Re_{crit} \Rightarrow$ turbulent

$$\bullet \text{Integrate to get total friction coeff: } C_f = \frac{1}{2} \int_0^L C_{f,lmn} dx$$

• Flow over a flat plate can be:

$$(i) \text{ Entry Laminar: } C_{f,lmn} = \frac{1.33}{Re_x^{1/4}}$$

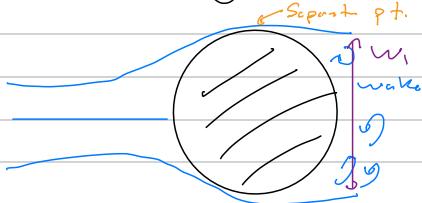
$$(ii) \text{ Parabolic Turbulent: } C_{f,turbulent} = \frac{0.074}{Re_x^{1/4}}$$

(iii) Combination of Laminar & Turbulent BL

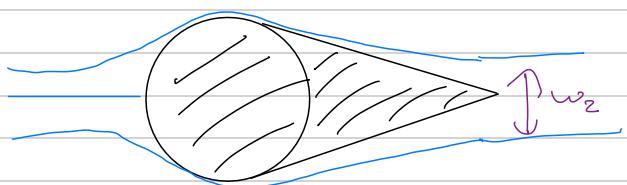
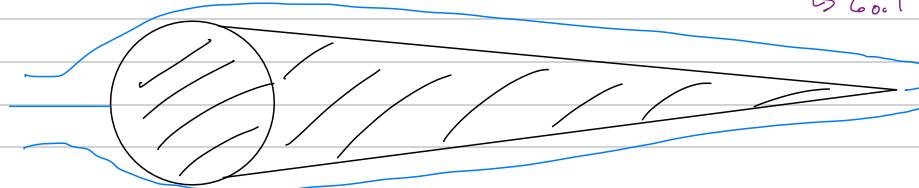
$$C_f = \frac{1}{2} \left(\int_0^{x_{crit}} C_{f,lmn} dx + \int_{x_{crit}}^L C_{f,turbulent} dx \right)$$

$$\Rightarrow C_f = \frac{0.074}{Re_x^{1/4}} - \frac{1.742}{Re_x}$$

Streamlining - Design to reduce drag



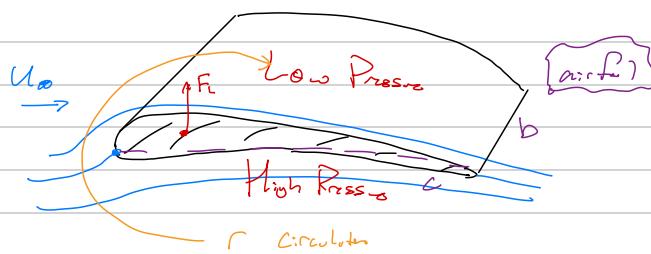
- ↑ Form / pressure drag



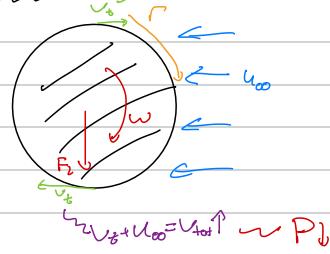
- ↓ Form Drag but ↑ Penetration drag
↳ C.D. is not drag ↓

- no Separation
- Form drag ↓
- Front drag ↑↑
- Overall drag ↓

Lift Force



$$V_t - U_\infty = U_{\text{tot}} \text{ in } P_1$$



$$U_t + U_\infty = U_{\text{tot}} \uparrow \text{ in } P_1$$

Result: $F_L = C_L \frac{1}{2} \rho U_\infty^2 A_p$ $A_p = b c$ for wings

- Lift coefficient (C_L) depends on

(i.) Shape

(ii.) Angle of attack (α)

↳ Angle between chord and horizontal

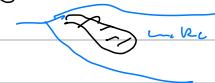


Stall

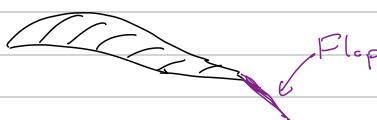
- Lift Coef increases with angle of attack
↳ up to stall point

• Stall typically occurs around 18°

• At stall

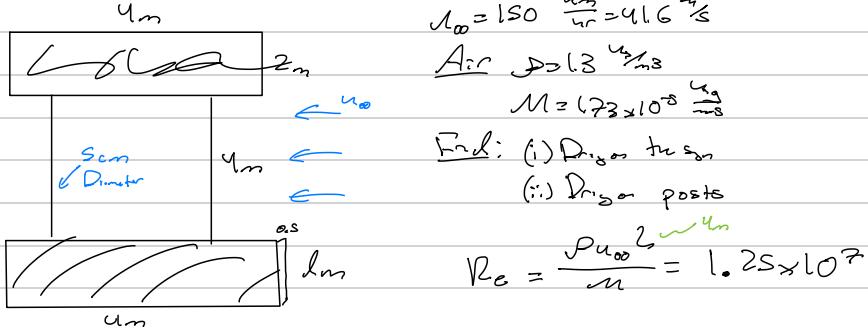


Effect of Flaps



- Flaps increase lift coefficient at the expense of drag

Example IS-88



(i) Drag on stem

$$F_D = C_D \frac{1}{2} \rho U_{\infty}^2 A$$

$$C_D \Rightarrow 1.5 - 4 \text{ To be used} \quad \text{rectangular plate} \Rightarrow C_D = 1.10 + 0.02 \left(\frac{L}{D} + \frac{P}{D} \right) = 1.15$$

$$F_{D,\text{stem}} = 100382 \text{ N}$$

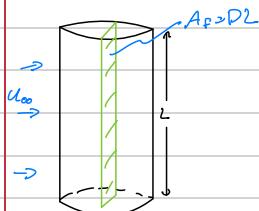
(ii) Drag on Roots/Posts

a) Look at $\frac{L}{D}$

b) Check tables L comes out of the

c) Solve

$$F_{\text{post}} = C_{D,\text{post}} \frac{1}{2} \rho U_{\infty}^2 A_p$$



Check Reynolds Number:

$$Re_p = \frac{\rho U_{\infty} D}{\mu} = 1.73 \times 10^8 \Rightarrow L \text{ mm but } > 10^4$$

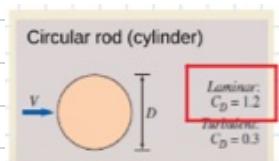
Two possibilities for C_D

a) 2-D approx

$$C_D \approx 1.2$$

b) Finite Cylinders

$$\frac{L}{D} = \frac{u_m}{S_{\infty} D} = 80 \Rightarrow C_D \approx 1.2$$



Finite cylinder, vertical, $A = LD$	
L/D	C_D
1	0.6
2	0.7
5	0.8
10	0.9
40	1.0
\approx	1.2

Values are for laminar flow ($Re \leq 2 \times 10^5$)

$$\Rightarrow F_{\text{post}} = 2 \times 1 \text{ N}$$

$$2 \text{ supports} \rightarrow \text{multiplied} \quad F_{\text{post}} \cdot 2 \Rightarrow F_{\text{total}} = 2 \times 2 \text{ N} = 4 \text{ N}$$

Exam 3 Notes



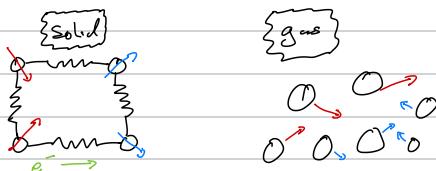
Introduction to Heat Transfer

Heat Transfer

- Energy Transfer through a temperature difference

- Three modes of heat transfer

(i) **Conduction** - Energy transfer due to the direct interaction between particles

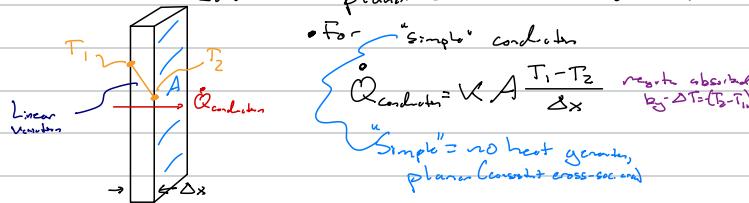


→ Conduction is described by Fourier's Law

$$Q_{\text{Conduction}} = -k A \frac{dT}{dx} \quad \begin{array}{l} \text{Heat Flux } \frac{dQ}{dx} \\ \text{not total} \end{array}$$

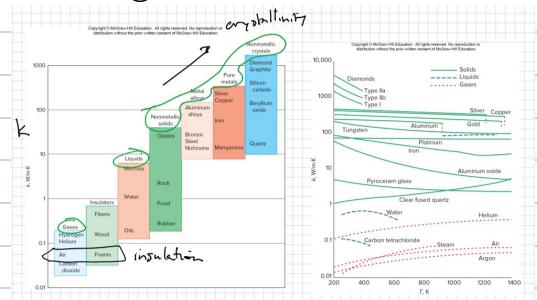
\downarrow
 (W/m²) Thermal Conductivity
 \downarrow
 Cross-sectional area

→ Consider a planar wall (1-D conduction)



• Typical thermal conductors with

gas < solids < metals



→ For Transient Heat Transfer

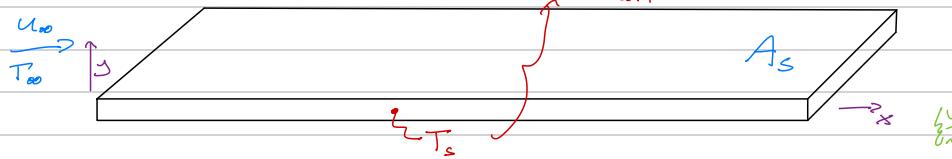
- Additional parameter is important

$$\frac{Q}{A} = \frac{k}{x} \cdot \frac{T_1 - T_2}{x} = \frac{\rho \cdot c}{x} \cdot \frac{x}{x} \cdot \frac{T_1 - T_2}{x}$$

thermal conductivity
 $\frac{Q}{A}$
 $\frac{T_1 - T_2}{x}$
 density
 specific heat capacity

(ii) **Convection** - Enhanced conduction through bulk fluid motion

↳ moving fluid → large temperature gradient



→ Convection described by Newton's law of Cooling:

$$Q_{\text{Convection}} = h \cdot A_s (T_s - T_\infty)$$

→ Heat transfer coefficient

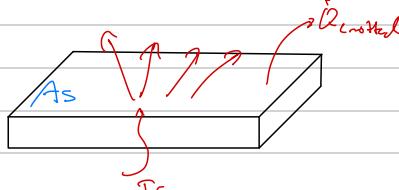
↳ proportional to effect of flow: $h \propto U_{\text{infty}}$

(iii) **Radiation** - energy transfer due to electromagnetic waves

→ Does not require a medium

→ All matter at non-zero temperature emits radiation

→ Radiated radiation described by the Stefan-Boltzmann Law:



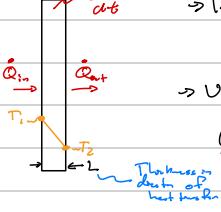
$$Q_{\text{radiated}} = \epsilon \sigma A_s T^4$$

emissivity
 Stefan-Boltzmann
 constant $\sigma = 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \text{K}^4}$

Conduction

Thermal Resistance Network

Consider Planar Wall


$$\Rightarrow \text{1st Law: } \dot{Q}_{in} - \dot{Q}_{out} = \frac{dE}{dt}$$

Steady-state

$$\Rightarrow \dot{Q}_{in} = \dot{Q}_{out} = \dot{Q}_{\text{conduction, wall}}$$

Use Fourier's Law

$$\dot{Q}_{\text{conduction}} = kA_c \frac{T_1 - T_2}{L}$$

Thickness
depth of
metallization

$\rightarrow D_c \text{ for thermal Resistance}$

$$\dot{Q}_{\text{conduction}} = \frac{\Delta T}{R_{\text{cond.}}}$$

Planar

→ Use Fourier's Law again...

$$\Rightarrow R_{\text{cond}} = \frac{L}{kA}$$