

Reminders

- HW 3B is be due on Monday, October 7

Office Hours:

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Day	Time	Location	Personnel
Monday	4 – 5PM	AW Smith 105	Shruti
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Learning Objectives

- After the next two classes students should be able to:
 - Convert between the volumetric flowrate and mass flowrate for incompressible fluids
 - Look up physical properties in Table B.1
 - Specific gravity, molecular weight, critical T and P
 - Define an equation of state
 - State the assumptions of the ideal gas law
 - State when it is appropriate to apply the ideal gas law



Equations of state

- An **equation of state** is a function that relates the molar quantity and volume of a gas to temperature and pressure
 - What are n , P and V ? → process variables!
- They are empirically derived!



René Magritte, 1928-29. Currently on display at Los Angeles County Museum of Art, USA



“The famous pipe. How people reproached me for it! And yet, could you stuff my pipe? No, it’s just a representation, is it not? So if I had written on my picture ‘This is a pipe’, I’d have been lying!”

René Magritte



CASE WESTERN RESERVE
UNIVERSITY

Christine Duval, 2024. Currently on display in AW Smith Building.

$$PV = nRT$$

The gas is not ideal.

“Oh, ideal gas law. How often it is used in chemical engineering. And yet, is any gas truly ideal? No, the ideal gas law is how we choose to represent it. So if I tell the class, ‘The gas is ideal’, I’d have been lying!”

Christine Duval

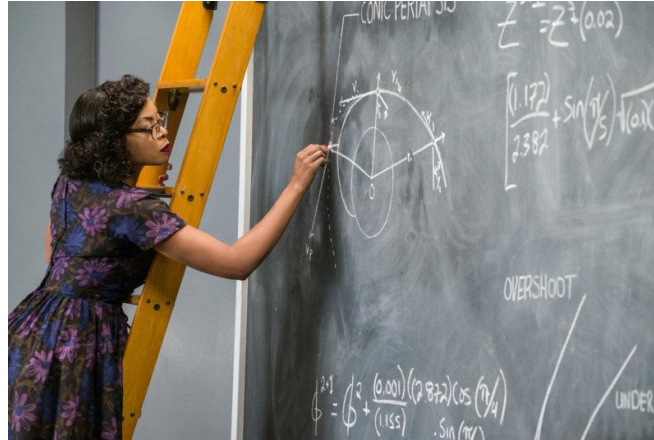


CASE WESTERN RESERVE
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Thermodynamics: A play in 3 acts



Collect a lot of data on process variables (P, V, and T) in a specific range of conditions for many chemical species



Use known properties and math to create equations that fit all of the data – call them “equations of state”



Use the models and 2 of the process variables to predict the third process variable



And the
MORAL
of the
STORY
is...

No gas is ideal...gases may
behave ideally in a certain
set of conditions.

Equations of state are just
models that do a good job
of describing what we
observe.



Ideal Gas Law

$$PV = nRT$$

- Assumptions
 - Large number of molecules in random motion
 - Volume of individual molecules \ll volume of gas
 - No intermolecular forces b/w molecules
 - Perfectly elastic collisions
 - Temperature of a gas depends kinetic energy of the molecules
- Conditions for which the ideal gas law applies:
 - ▣ “High” temperature (above 0 Celsius)
 - ▣ “Low” pressure (around 1 atm)



How do we know if a gas behaves ideally?

1. Calculate $\frac{RT}{P} = \hat{V}$

2. Assess. We can use the ideal gas law when...

$$\hat{V} = \frac{RT}{P} > 5 \frac{L}{mol} \text{ for diatomic gases}$$

$$\hat{V} = \frac{RT}{P} > 20 \frac{L}{mol} \text{ for other gases}$$



You may remember from chemistry class...

Table 5.2-1 Standard Conditions for Gases

System	T_s	P_s	V_s	n_s
SI	273 K	1 atm	0.022415 m ³	1 mol
CGS	273 K	1 atm	22.415 L	1 mol
American Engineering	492°R	1 atm	359.05 ft ³	1 lb-mole

- This is only true at standard conditions for 1 mol of gas. The 22.415 L/mol is not a conversion factor
- We must use equations of state to relate V and moles for any other T and P



Equation of state

Equations of state are mathematical functions that relate molar volume, temperature, and pressure. (\hat{V} , T, P)

- If the ideal gas law does not apply...
 - ▣ **Virial EOS**
 - ▣ **Van der Waals EOS**
 - ▣ **Compressibility Factor EOS**
 - ▣ Soave-Redlich-Kwong (SRK) EOS
 - ▣ Many, many others

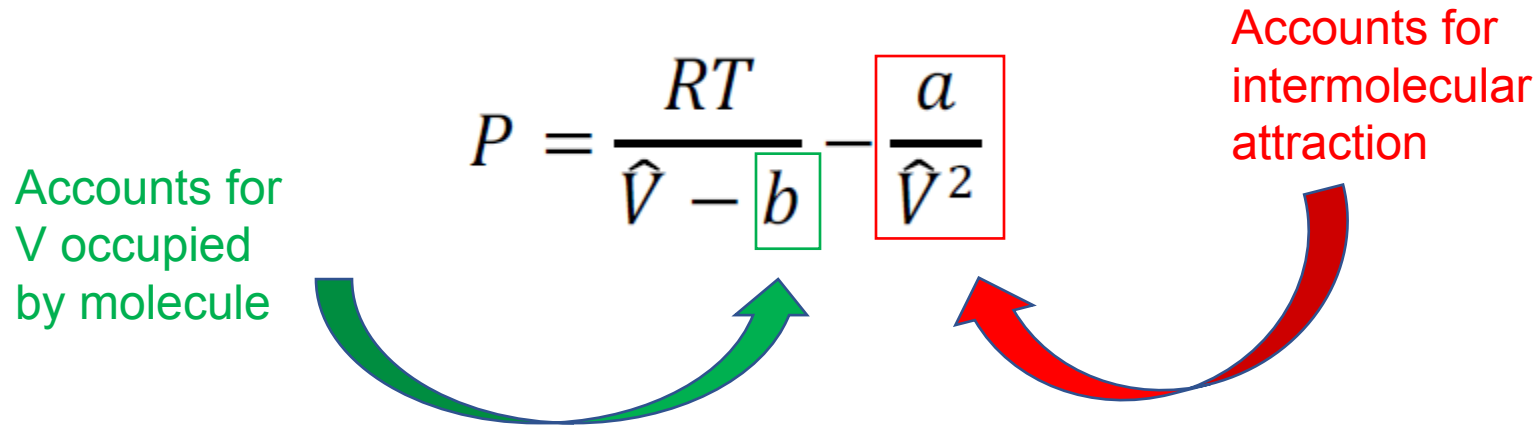


Van der Waals EOS

$$P = \frac{RT}{\hat{V} - b} - \frac{a}{\hat{V}^2}$$

Accounts for V occupied by molecule

Accounts for intermolecular attraction



Useful when these assumptions don't apply:

- Volume of molecules \ll volume of gas
- No intermolecular forces between molecules



Van der Waals EOS

$$P = \frac{RT}{\hat{V} - b} - \frac{a}{\hat{V}^2}$$

How do we find a and b?

$$a = \frac{27R^2T_c^2}{64P_c} \quad \text{Look it up!}$$
$$b = \frac{RT_c}{8P_c} \quad \text{Look it up!}$$



Compressibility Factor EOS

$$P\hat{V} = zRT$$

How do we find z ?

$$z = f(\text{Tr}, \text{Pr})$$

$$T_r = \frac{T}{T_c} \text{ Look it up!}$$

$$P_r = \frac{P}{P_c} \text{ Look it up!}$$



We can get more complicated...

$$P = \frac{RT}{\hat{V} - b} - \frac{\alpha a}{\hat{V}(\hat{V} + b)}$$

Soave-Redlich-Kwong

$$a = 0.42747 \frac{R^2 T_c^2}{P_c} \quad b = 0.08664 \frac{RT_c}{P_c}$$

$$\alpha = \left[1 + (0.48508 + 1.55171\omega + 0.1561\omega^2) \left(1 - \sqrt{\frac{T}{T_c}} \right) \right]^2$$



Why do almost all EOS reference critical T and P?

- **T_c and P_c:** Highest temperature and corresponding pressure at which species can coexist in two phases (liquid and vapor)
- **Law of corresponding states:** Certain physical properties of a gas strongly depend on the proximity of the gas to its critical state
- What does the **T_r** and **P_r** tell us?



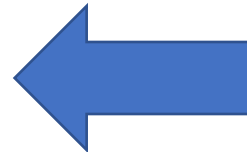
Law of corresponding states

- Certain physical properties of a gas strongly depend on the proximity of the gas to its critical state



The whole point

- We want to choose an EOS that works for our system
- What do we mean by “works”?
 - Allows us to predict properties (T,V,P, composition) reliably



This is not a pipe, but it's a great representation of one.

Equations of state are often *empirical* To determine the fitting parameters (a,b,z,etc...) we need to first **look up P_c and T_c** .



Unit 4: Single phase systems and equations of state

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$$P_r = \frac{P}{P_c} \text{ Look it up!}$$



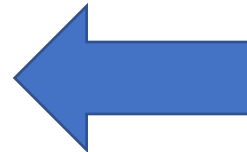
Why do almost all EOS reference critical T and P?

- **T_c and P_c:** Highest temperature and corresponding pressure at which species can coexist in two phases (liquid and vapor)
- **Law of corresponding states:** Certain physical properties of a gas strongly depend on the proximity of the gas to its critical state
- Reduced temperature and pressure tell us how close we are to the critical state of the gas.



The whole point

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- What do we mean by “works”?
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