

Homework 3

Due 2/6/25

Reading Chapter 4 of the textbook will help you answer these questions. In lectures 5-6 and part of 7, you will have seen relevant background and examples which should help you solve these problems as well. Unless otherwise noted, write out by hand (e.g. not Minitab) to solve. If it asked for you to perform a problem in Minitab, please include your output in the answer. Remember to follow the step-by-step procedure for hypothesis testing outlined in class.

Also remember the proper sketches for hypothesis testing should have:

- 1) The type of reference distribution (e.g., normal dist., t-distribution among others), draw the shape and write the defining parameters (e.g., μ , σ among others)
- 2) Show where the test statistic lies on your sketch
- 3) For solving using fixed significance level method – show the acceptance region, rejection region, alpha, as well as the critical value
- 4) For p-value method, shade in the p-value and report that value

- 1) PVC pipe is manufactured with a population mean diameter of 1.01 inch and a standard deviation of 0.00300 inch. Find the probability that a random sample of $n = 9$ sections of pipe will have a sample mean diameter greater than 1.009 inch and less than 1.012 inch. Report to 0.000 accuracy. Remember to provide a sketch of this probability problem as part of your answer.
- 2) Output from a software package follows (Z is the test statistic) – Report to 0.000 accuracy.

One-Sample Z:

Test of $\mu = 20$ vs > 20

The assumed standard deviation = 0.75

| Variable | N | Mean | SE Mean | Z | P |
|----------|----|--------|---------|---|---|
| x | 10 | 19.889 | 0.237 | ? | ? |

- a) Fill in the missing items (Z: the test statistic and P: the p-value). Calculate by hand. Make a sketch of the reference distribution (with its parameters defined), labeling the test statistic and the p-value.
 - b) Provide a Minitab output which has these items (perform the same test that was in this chart)
 - c) Is this a 1-sided or 2-sided test?
 - d) Use the table in your book appendix, and the data in this table to construct a 95% 2-sided confidence interval on the mean. Comment on what the 95% CI really means (we are 95% confident that ...? And what does 95% confident mean?).
 - e) What would the P-value be if the alternative hypothesis was $H_1: \mu \neq 20$? Calculate by hand, and provide a sketch of the reference distribution with the proper labels to illustrate your answer.
- 3) Medical researchers have developed a new artificial heart constructed of primarily titanium and plastic. The heart will last and operate almost indefinitely once it is implanted into a patient's

body, but the battery pack needs to be recharged every four hours. A random sample of 50 battery packs is selected and subjected to a life test. The average life of the samples is 4.05 hours. Assume that battery life is normally distributed, with a standard deviation of 0.20 hours.

- a. Is there evidence to support the claim that the mean battery life of the population exceeds 4 hours? Remember to provide a sketch to illustrate your answer. Use $\alpha=0.05$
 - b. Construct a 1-sided, lower bound confidence interval on the mean with $\alpha=0.05$ by hand, and explain what it represents. (Report this with to an accuracy of 0.00)
 - c. Check these answers in Minitab and report the outputs.
- 4) Do problem 4.5 (a-b) in your book by hand (4.7 in the 7th edition), and then show a Minitab output for this test as part c (no need to do the part c in the book). Comment on what the p-value means in part b (i.e., what is the definition of a p-value). Assume 3 significant digits for reporting. Provide a sketch with the proper labels to illustrate your answer.
- 5) Explain the difference between type I and type II error, and their relationship to the convention that a rejection of the null hypothesis is considered a strong conclusion, but not rejecting is considered weak. Use the one sample z-test to explain, and drawings.