ECHE313 Homework 5 - Due 02/20/25 — Trevor Swan (tcs94)

1.

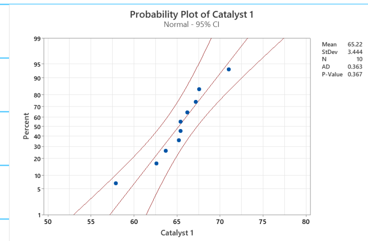
Catalyst 1	Catalyst 2
57.9	66.4
66.2	71.7
65.4	70.3
65.4	69.3
65.2	64.8
62.6	69.6
67.6	68.6
63.7	69.4
67.2	65.3
71.0	68.8

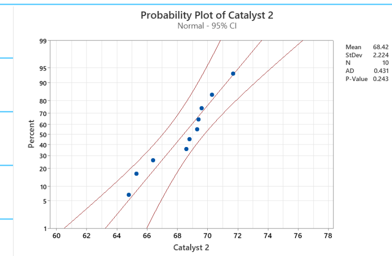
Population Std $= 3\ g/L = \sigma_1 = \sigma_2$

a)

Statistics

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
Catalyst 1	10	0	65.22	1.08922	3.44442	57.9	63.425	65.4	67.3	71
Catalyst 2	10	0	68.42	0.703294	2.22401	64.8	66.125	69.05	69.775	71.7





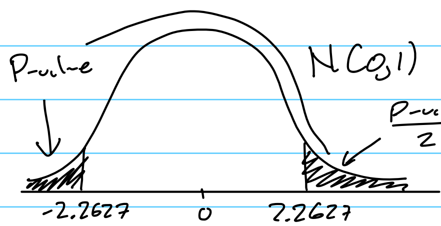
p-value > 0.05 for both catalysts' normality, so insufficient evidence to reject normality.

b) Inference on mean concentrations of two normally distributed populations w/ known variance

→ Two-sample Z $\quad H_0: \mu_1 = \mu_2 \quad H_1: \mu_1 \neq \mu_2$

$$Z_0 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n}}} = \frac{68.42 - 65.22}{\sqrt{\frac{3^2}{10} + \frac{3^2}{10}}} = 2.2627$$

Reject if: P-value $< \alpha = 0.05$



$$\text{P-value} = 2(1 - P\{Z_0 < 2.2627\})$$
$$= 2(1 - 0.98809) = 0.02382$$

P-value $= 0.022$ $\alpha = 0.05$ ⇒ Reject H_0. We have sufficient evidence to reject the claim that the two catalysts yield the same concentrations. We accept that the catalyst choice impacts mean active concentrations.

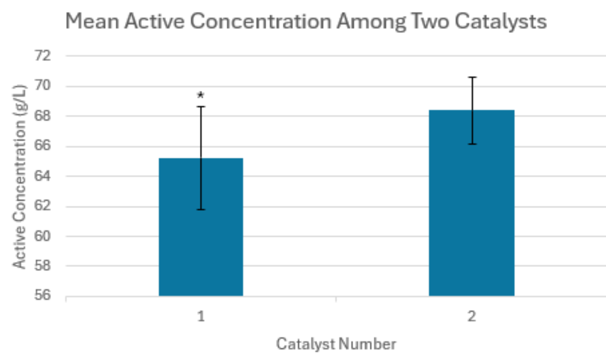
c) $\bar{x}_1 - \bar{x}_2 - Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \leq \mu_1 - \mu_2 \leq \bar{x}_1 - \bar{x}_2 + Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

$$\text{Lower} = 68.42 - 65.22 - 2.2627\sqrt{\frac{3^2}{10} + \frac{3^2}{10}} = 0.165$$
$$\text{Upper} = 68.42 - 65.22 + 2.2627\sqrt{\frac{3^2}{10} + \frac{3^2}{10}} = 6.23$$

$0.165 \leq \mu_1 - \mu_2 \leq 6.23$

In repeated sampling, this method will produce intervals that capture the true mean 95% of the time. As our interval does not cross 0, it supports our conclusion that the mean active concentration depends on catalyst choice.

d)



Mean active concentration of liquid laundry detergent based on two different catalyst types. Data are represented as the mean ± standard deviation with n = 1. (*) represents $p < 0.05$ compared to the second catalyst's performance, using a two-sided, t-sample z-test in Minitab and by-hand with known population standard deviation of 3 g/L. We have sufficient evidence to reject the claim that the two catalysts yield the same concentrations, and accept that the concentration is dependent on catalyst choice.

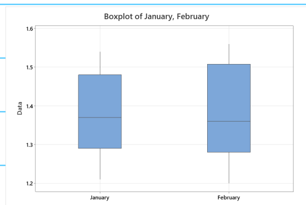
2.

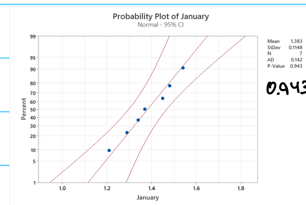
January	February
1.45	1.54
1.37	1.41
1.21	1.56
1.54	1.37
1.48	1.20
1.29	1.31
1.34	1.27
	1.35

Problem 4.11, data btwn months, not techs

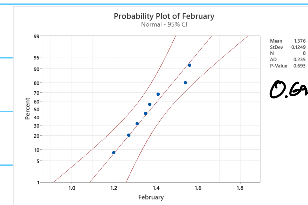
Statistics

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
January	7	0	1.38286	0.0434091	0.114850	1.21	1.29	1.37	1.48	1.54
February	8	0	1.37625	0.0441563	0.124893	1.2	1.28	1.36	1.5075	1.56





0.943



0.693

Distributed evenly and all about the same so assuming equivalence is reasonable

p-values both > 0.05, so we do not have evidence to reject that the data are normally distributed

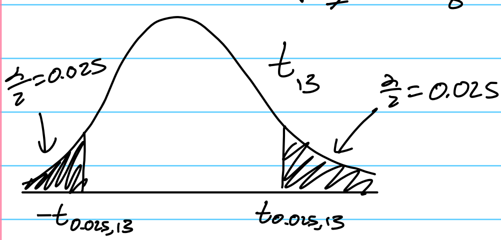
a) Inference about mean surface finish, unknown variance, assume equal variance

$H_0: \mu_1 = \mu_2 \quad H_a: \mu_1 \neq \mu_2$

$$S_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(7-1)0.114850^2 - (8-1).124893^2}{7+8-2}} = 0.1204$$

$$\rightarrow T_0 = \frac{1.38286 - 1.37625}{0.1204\sqrt{\frac{1}{7}+\frac{1}{8}}} = 0.106$$

DoF: $7+8-2=13$



Reject if $\begin{cases} T_0 \geq t_{0.025,13} \\ T_0 \leq -t_{0.025,13} \end{cases}$

$t_{0.025,13} = 2.160$

$\Rightarrow$ Fail to Reject $\quad 0.106 < |2.160|$

Fail to reject Ho. We have insufficient evidence to claim that the mean surface finish varies between months of January and February

b) Based on our findings, I would conclude that the mean surface finishes independent of month, so work can be conducted on either Jan or Feb. If we rejected Ho, I would prevent work in the 'defect' month to prevent errors in products' surface finish.

c) $\bar{x}_1 - \bar{x}_2 - t_{\frac{\alpha}{2}, n_1+n_2-2}\, s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_2 \le \bar{x}_1 - \bar{x}_2 + t_{\frac{\alpha}{2}, n_1+n_2-2}\, s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

$$\left.\begin{aligned} \text{Lower} &= 1.38286 - 1.37625 - 2.160\left(.1204\sqrt{\tfrac{1}{7}+\tfrac{1}{8}}\right) = -.128 \\ \text{Upper} &= 1.38286 - 1.37625 + 2.160\left(.1204\sqrt{\tfrac{1}{7}+\tfrac{1}{8}}\right) = .141 \end{aligned}\right\} -.128 \le \mu_1 - \mu_2 \le .141$$

In repeated sampling, this method will produce intervals that capture the true mean difference in surface finish 95% of the time. The interval crosses 0, so it supports our conclusion to fail to reject that the months are the same.

d) P-value $= 0.917 > \alpha = 0.05$, so fail to reject H_0. Conclusion is the same as stated in part (b), yay!

Test

Null hypothesis $H_0: \mu_1 - \mu_2 = 0$
Alternative hypothesis $H_1: \mu_1 - \mu_2 \neq 0$

T-Value	DF	P-Value
0.11	13	0.917

3.

Two-Sample T-Test and CI

Sample	N	Mean	StDev	SE Mean
1	15	54.73	2.13	0.55
2	20	58.64	5.28	1.2

Difference = mu (1) - mu (2)
Estimate for difference: -3.91
95% upper bound for difference: ?
T-test of difference = 0 (vs <): T-value = -3.00 P-value = ? DF = ?

a) Two-Sample T-test

One-sided test:

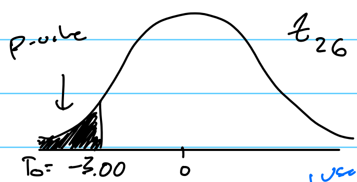
$H_0: \mu_1 - \mu_2 = 0$

$H_a: \mu_1 - \mu_2 > 0$

$\frac{S_2^2}{S_1^2} = \frac{2.13^2}{5.28^2} = 6.14$ Cannot use Pooled!

$$DF = \nu = \frac{\left(\frac{2.13^2}{15} + \frac{5.28^2}{20}\right)^2}{\frac{\left(\frac{2.13^2}{15}\right)^2}{15-1} + \frac{\left(\frac{5.28^2}{20}\right)^2}{20-1}} = 26.45 \rightarrow DF = 26$$

Upper = −1.69



P-value = $P\{t_{26} > 3.00\}$

btwn $(\overset{3.067}{0.0025}, \overset{2.779}{0.005})$

P-value $\in (0.0025, 0.005)$

use t dist.

$$Upper = \bar{x}_1 - \bar{x}_2 + t_{0.05,26}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = 54.73 - 58.64 + 1.706\sqrt{\frac{2.13^2}{15} + \frac{5.28^2}{20}} = -1.69$$

b) $\alpha = 0.05$ → done above, p-value $\in (0.0025, 0.005) < 0.05 = \alpha$

$\alpha = 0.01$ → p-value $\in (0.0025, 0.005) < 0.01 = \alpha$

In both cases, p-value < α, so we reject H_0. We have sufficient evidence to reject the claim that the mean difference is 0. We accept that it is less than 0 for both choices of α.

c) If $H_a: \mu_1 \neq \mu_2$, then p-value $\in 2(0.0025, 0.005)$

p-value $\in (0.005, 0.01)$

If $\alpha = 0.05$, the upper bound for our new interval is still less than alpha ($0.01 < 0.05$), so we reject H_0 still. Our conclusion will be the exact same as found in (b).

4.

Inspector	Micrometer Caliper	Vernier Caliper	Difference
1	0.150	0.151	−0.001
2	0.151	0.150	0.001
3	0.151	0.151	0.000
4	0.152	0.150	0.002
5	0.151	0.151	0.000
6	0.150	0.151	−0.001
7	0.151	0.153	−0.002
8	0.153	0.155	−0.002
9	0.152	0.154	−0.002
10	0.151	0.151	0.000
11	0.151	0.150	0.001
12	0.151	0.152	−0.001

Statistics

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median
Micrometer Caliper	12	0	0.151167	0.0002410	0.0008348	0.15	0.151	0.151
Vernier Caliper	12	0	0.151583	0.0004680	0.0016214	0.15	0.15025	0.151
Caliper Difference	12	0	-0.0004167	0.0003786	0.0013114	-0.002	-0.00175	-0.0005

Variable	Q3	Maximum
Micrometer Caliper	0.15175	0.153
Vernier Caliper	0.15275	0.155
Caliper Difference	0.00075	0.002

Use $\alpha = 0.01$

Difference calculated as 'Micrometer' − 'Vernier'

$$\bar{D} = \frac{-.001 + 0.001 + .000 + \dots + 0.001 + (-.001)}{12} = -0.0004167$$

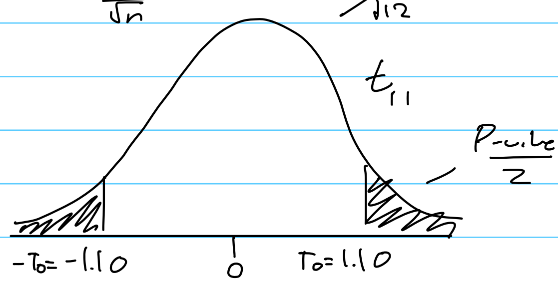
$$S_p = \sqrt{\frac{(-.001+0.0004167)^2 + (0.001+0.0004167)^2 + \dots + (-0.001+0.0004167)^2}{12}} = 0.0013114$$

Interested in the mean difference of diameter of metal rod measured by two different calipers. Same parts, so paired data → paired t-test

$H_0: \mu_D = 0 \qquad H_a: \mu_D \neq 0$

$$t_0 = \frac{\bar{D}}{\frac{S_p}{\sqrt{n}}} = \frac{-0.0004167}{0.0013114/\sqrt{12}} = -1.10$$

Reject if p-value < α = 0.01



$$\text{P-value} = 2(P\{t_{11} \geq 1.10\})$$
$$= 2(0.10, 0.25)$$

0.700 1.372

$\Rightarrow$ P-value $\in (0.20, 0.50)$

P-value in range (0.2, 0.5) < 0.01 = α. Fail to reject H_0. We have insufficient evidence to reject the claim that the mean difference in measured diameter of metal rods by Micrometer vs Vernier Calipers are not different. Our evidence does not allow us to say that there is a difference between the two!

Check w/ Minitab

Descriptive Statistics

Sample	N	Mean	StDev	SE Mean
Micrometer Caliper	12	0.151167	0.000835	0.000241
Vernier Caliper	12	0.151583	0.001621	0.000468

Estimation for Paired Difference

Mean	StDev	SE Mean	99% CI for μ_difference
-0.000417	0.001311	0.000379	(-0.001592, 0.000759)

μ_difference: population mean of (Micrometer Caliper - Vernier Caliper)

Test

Null hypothesis	H_0: μ_difference = 0
Alternative hypothesis	H_1: μ_difference ≠ 0

T-Value	P-Value
-1.10	0.295