ECHE313 Homework 9 - Due 4/22/25 **Trevor Swan (tcs94)**

1. Problem 13.1

The following output was obtained from a computer program that performed a two-factor ANOVA on a factorial experiment.

Source	SS	DF	MS	F	P
A	0.322	1			
B	80.554		40.2771		
Interaction					
Error	108.327	12			
Total	231.551	17			

a. Fill in the blanks in the ANOVA table. You can use bounds on the *P*-values.
b. How many levels were used for factor B?
c. How many replicates of the experiment were performed?
d. What conclusions would you draw about this experiment?

a)

Source	SS	DF	MS	F	P	
A	0.322	1	0.322	.0357	(0.75, 0.90)	no!
B	80.554	2	40.2771	4.462	(0.025, 0.05)	yes! closer
Interaction	42.348	2	21.174	2.346	(0.10, 0.25)	no!
Error	108.327	12	9.027	—	—	no formula in notes so assume negligible :)
Total	231.551	17	—	—	—	

$$SS_T = SS_E + SS_A + SS_B + SS_{AB}$$
$$SS_{AB} = SS_T - SS_E - SS_A - SS_B$$
$$= 231.551 - 108.327 - 80.554 - 0.322 = 42.348$$

$DF_A = a-1 = 1 \rightarrow a=2$

$DF_B = b-1 \rightarrow MS_B = \frac{SS_B}{DF_B}$ $\quad DF_B = \frac{SS_B}{MS_B} = \frac{80.554}{40.2771} \approx 2 \rightarrow b-1=2 \quad b=3$

$DF_{AB} = (a-1)(b-1) = DF_A \cdot DF_B = 1 \cdot 2 = 2$

$MS_A = \frac{SS_A}{DF_A} = \frac{0.322}{1} = 0.322$ $\quad MS_E = \frac{SS_E}{DF_E} = \frac{108.327}{12} = 9.027$

$MS_{AB} = \frac{SS_{AB}}{DF_{AB}} = \frac{42.348}{2} = 21.174$

$F_{0_A} = \frac{MS_A}{MS_E} = \frac{0.322}{9.027} = 0.0357$

$F_{0_B} = \frac{MS_B}{MS_E} = \frac{40.2771}{9.027} = 4.462$

$F_{0_{AB}} = \frac{MS_{AB}}{MS_E} = \frac{21.174}{9.027} = 2.346$

P-value A D.F: (1, 12)

$F_{0.90,1,12} = \frac{1}{F_{0.10,12,1}} = .0165$

$F_{0.75,1,12} = \frac{1}{F_{0.25,12,1}} = 0.106$

P-value B D.F: (2, 12)

$F_{0.05,2,12} = 3.89$

$F_{0.025,2,12} = 5.10$ ← closer to this

P-value AB D.F: (2, 12)

$F_{0.25,2,12} = 1.56$

$F_{0.10,2,12} = 2.81$

b) Levels of Factor B $= b = 3$

3 Levels

c) Replicates: n $\quad n-1 = 6$

$DF_E = ab(n-1)$ $\quad n=7$ $\quad$ 7 replicates

$12 = (1)(2)(n-1)$

d) We conclude that method B has a significant impact on the results. There is not enough evidence to say there is an interaction between A and B, or that method A is significant.

2. Problem 13.2

An article in *Industrial Quality Control* (1956, pp. 5–8) describes an experiment to investigate the effect of glass type and phosphor type on the brightness of a television tube. The response measured is the current necessary (in microamps) to obtain a specified brightness level. The data are shown in Table 13E.1. Analyze the data and draw conclusions.

Table 13E.1 Data for Exercise 13.2

	Phosphor Type		
Glass Type	1	2	3
1	280	300	290
	290	310	285
	285	295	290
2	230	260	220
	235	240	225
	240	235	230

1) Parameter of Interest: Main effects and interactions of method of phosphor type and glass type.

2) Ho: $\tau_1 = \tau_2 = 0$

$\beta_1 = \beta_2 = \beta_3 = 0$

$(\tau\beta)_{11} = (\tau\beta)_{12} = (\tau\beta)_{13} = (\tau\beta)_{21} = (\tau\beta)_{22} = (\tau\beta)_{23}$

3) Ha: $\tau_i \neq 0$ for any $i \in [1,2]$

$\beta_j \neq 0$ for any $j \in [1,3]$

$(\tau\beta)_{ij} \neq 0$ for any $i \in [1,2], j \in [1,3]$

6) Calculations

$y_{ij\cdot}$

$y_{11\cdot} = 280 + 290 + 285 = 855$

$y_{12\cdot} = 300 + 310 + 295 = 905$

$y_{13\cdot} = 290 + 285 + 290 = 865$

$y_{21\cdot} = 230 + 235 + 240 = 705$

$y_{22\cdot} = 260 + 240 + 235 = 735$

$y_{23\cdot} = 220 + 225 + 230 = 675$

$\bar{y}_{ij\cdot}$

$\bar{y}_{11\cdot} = \frac{855}{3} = 285$

$\bar{y}_{12\cdot} = \frac{905}{3} = 301.67$

$\bar{y}_{13\cdot} = \frac{865}{3} = 288.33$

$\bar{y}_{21\cdot} = \frac{705}{3} = 235$

$\bar{y}_{22\cdot} = \frac{735}{3} = 245$

$\bar{y}_{23\cdot} = \frac{675}{3} = 225$

Problem Info

$a = 2$

$b = 3$

$n = 3$

$y_{i\cdot\cdot}$ & $\bar{y}_{i\cdot\cdot}$

$y_{1\cdot\cdot} = y_{11\cdot} + y_{12\cdot} + y_{13\cdot}$
$= 855 + 905 + 865 = 2625$

$\bar{y}_{1\cdot\cdot} = \frac{y_{1\cdot\cdot}}{bn} = \frac{2625}{9} = 291.67$

$y_{2\cdot\cdot} = y_{21\cdot} + y_{22\cdot} + y_{23\cdot}$
$= 705 + 735 + 675 = 2115$

$\bar{y}_{2\cdot\cdot} = \frac{y_{2\cdot\cdot}}{bn} = \frac{2115}{9} = 235$

$y_{\cdot\cdot\cdot}$ & $\bar{y}_{\cdot\cdot\cdot}$

$y_{\cdot\cdot\cdot} = y_{1\cdot\cdot} + y_{2\cdot\cdot} = 4710$

$\bar{y}_{\cdot\cdot\cdot} = \frac{y_{\cdot\cdot\cdot}}{abn} = \frac{4710}{18} = 263.33$

$y_{\cdot j\cdot}$ & $\bar{y}_{\cdot j\cdot}$

$y_{\cdot 1\cdot} = y_{11\cdot} + y_{21\cdot}$
$= 855 + 705 = 1560$

$\bar{y}_{\cdot 1\cdot} = \frac{y_{\cdot 1\cdot}}{an} = \frac{1560}{6} = 260$

$y_{\cdot 2\cdot} = y_{12\cdot} + y_{22\cdot}$
$= 905 + 735 = 1640$

$\bar{y}_{\cdot 2\cdot} = \frac{y_{\cdot 2\cdot}}{an} = \frac{1640}{6} = 273.33$

$y_{\cdot 3\cdot} = y_{13\cdot} + y_{23\cdot}$
$= 865 + 675 = 1540$

$\bar{y}_{\cdot 3\cdot} = \frac{y_{\cdot 3\cdot}}{an} = \frac{1540}{6} = 256.67$

Sum of Squares

$$SS_A = bn \sum_{i}^{a} (\bar{y}_{i..} - \bar{y}_{...})^2 = 3(3)\left[(291.67 - 263.33)^2 + (235 - 263.33)^2\right] = 14450$$

$$SS_B = an \sum_{i}^{b} (\bar{y}_{.j.} - \bar{y}_{...})^2 = 2(3)\left[(260 - 263.33)^2 + (273.33 - 263.33)^2 + (256.67 - 263.33)^2\right]$$
$$= 933.33$$

$$SS_{AB} = n \sum_{i}^{a} \sum_{j}^{b} (\bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...})^2$$
$$= 3\left[(285 - 291.67 - 260 + 263.33)^2 + (\text{all cells})\right] = 20400$$

$$SS_E = \sum_{i}^{a} \sum_{j}^{b} \sum_{k}^{n} (y_{ijk} - \bar{y}_{ij.})^2 = (280 - 285)^2 + (290 - 285)^2 + \dots + (230 - 225)^2$$
$$= 663.33$$

$$SS_T = SS_A + SS_B + SS_{AB} + SS_E$$
$$= 14450 + 933.33 + 133.33 + 663.33 = 16150$$

Degrees of Freedom

$DF_A = a - 1 = 1$

$DF_B = b - 1 = 2$

$DF_{AB} = (a-1)(b-1) = 2$

$DF_E = ab(n-1) = 12$

$DF_T = abn - 1 = 17$

Mean Squared Errors

$MS_A = \frac{SS_A}{DF_A} = \frac{14450}{1} = 14450$

$MS_B = \frac{SS_B}{DF_B} = \frac{933.33}{2} = 466.67$

$MS_{AB} = \frac{SS_{AB}}{DF_{AB}} = \frac{133.33}{2} = 66.67$

$MS_E = \frac{SS_E}{DF_E} = \frac{663.33}{12} = 52.78$

4) Test Statistic

F_0's

$F_{0A} = \frac{MS_A}{MS_E} = \frac{14450}{52.78} = 273.8$

$F_{0B} = \frac{MS_B}{MS_E} = \frac{466.67}{52.78} = 8.84$

$F_{0AB} = \frac{MS_{AB}}{MS_E} = \frac{66.67}{52.78} = 1.26$

5) Rejection Criteria

Critical F values

A: $F_{0.05,1,12} = 4.75$

B: $F_{0.05,2,12} = 3.89$

AB: $F_{0.05,2,12} = 3.89$

Reject A if $F_{0A} = 273.8 > F_{0.05,1,12} = 4.75$ yes!

Reject B if $F_{0B} = 8.84 > F_{0.05,2,12} = 3.89$ yes!

Reject AB if $F_{0AB} = 1.26 > F_{0.05,2,12} = 3.89$ no!

7) Conclusions (assuming assumptions are valid)

We conclude that glass type and phosphor methods have a significant impact on the brightness of a television tube. There is not enough evidence to say there is an interaction between glass and phosphor type.

8) Main effect and interaction plots/calculations

Use calculations from previous

$\bar{y}_{ij\bullet}$

$\bar{y}_{11\bullet} = \frac{855}{3} = 285$

$\bar{y}_{12\bullet} = \frac{905}{3} = 301.67$

$\bar{y}_{13\bullet} = \frac{865}{3} = 288.33$

$\bar{y}_{21\bullet} = \frac{705}{3} = 235$

$\bar{y}_{22\bullet} = \frac{735}{3} = 245$

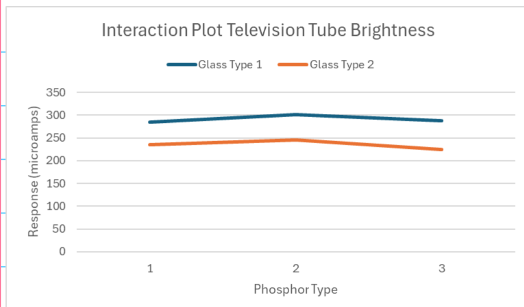
$\bar{y}_{23\bullet} = \frac{675}{3} = 225$

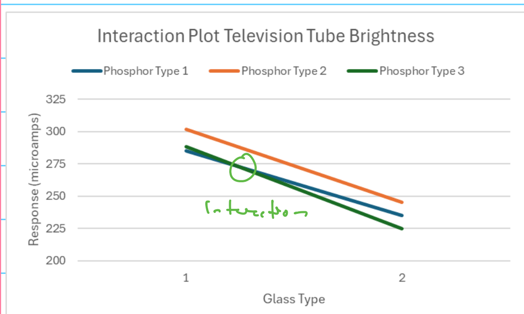
(i)

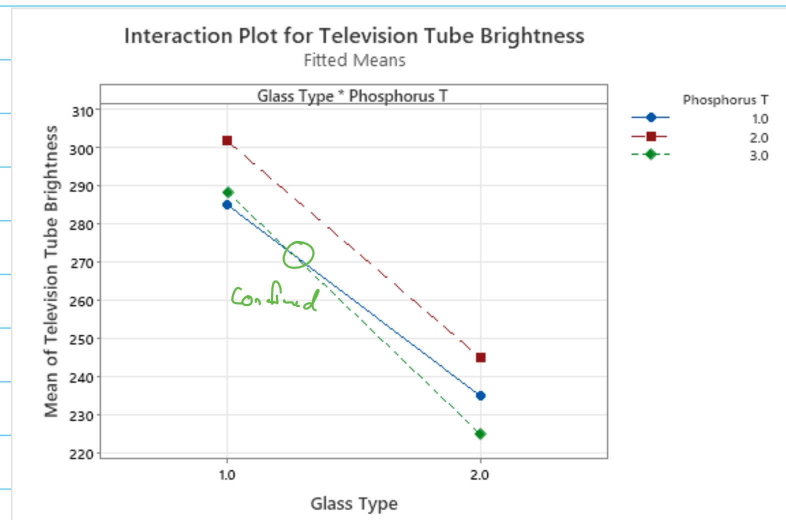
	Glass 1	Glass 2
Phosphor Type 1	285	235
Phosphor Type 2	301.67	245
Phosphor Type 3	288.33	225

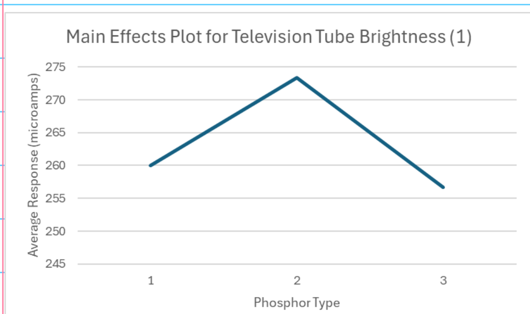
(ii)

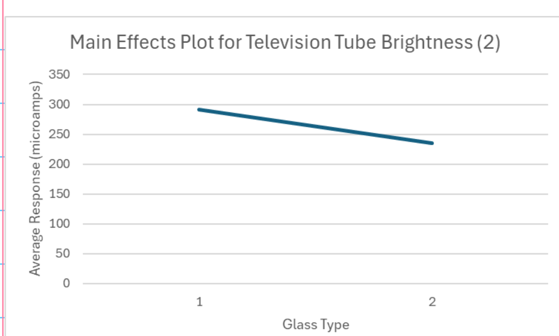
	PT 1	PT 2	PT 3
Glass 1	285	301.67	288.33
Glass 2	235	245	225

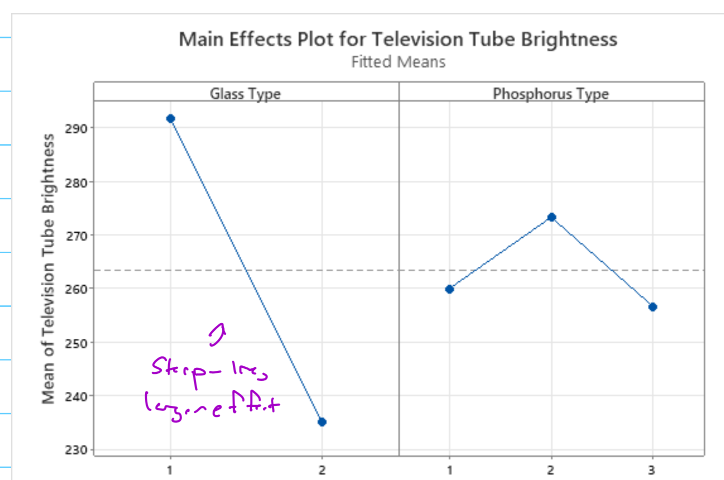












9) Checking Assumptions

Residuals: $\varepsilon_{ijk} = y_{ijk} - \bar{y}_{ij.}$

$\varepsilon_{111} = 280 - 285 = -5$

$\varepsilon_{112} = 290 - 285 = 5$

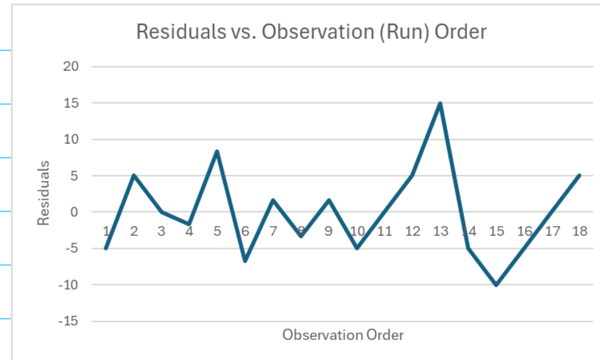
$\varepsilon_{113} = 285 - 285 = 0$

$\varepsilon_{121} = 300 - 301.67 = -1.67$

$\vdots$

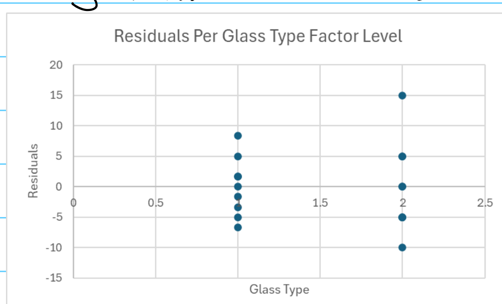
$\varepsilon_{233} = 230 - 225 = 5$

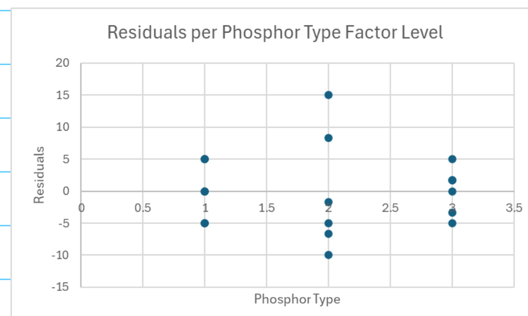
Residuals vs. Run Order



Residuals vs. Run order shows no obvious pattern → can claim that errors are independently distributed.

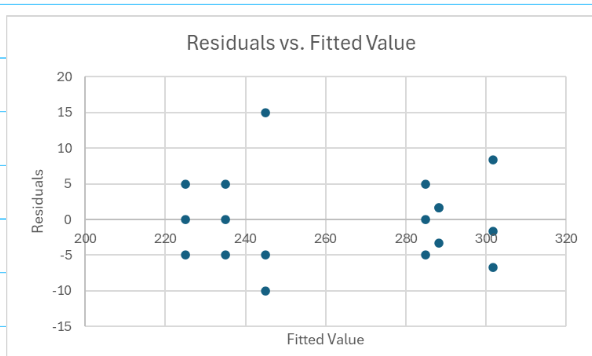
Using Residuals from above





Don't feel great about equivalence here

There seems to be a funnel shape in both plots indicating variance is not equal across factor levels, and thus data transformations may need to be done.



Fitted value plot seems reasonably scattered, so the equal variance assumption may not be entirely invalid.

Minitab output