ECHE313 Homework 4 - Due 02/13/25 **Trevor Swan (tcs94)**

4.8 in textbook

1. Temperatures

13.3987 $\times 1000 \mathring{A}$
13.3957
13.3902
13.4015
13.4001
13.3918
13.3965
13.3925
13.3946
13.4002
(units of $\times 1000 \mathring{A}$)

Sample Statistics $(n=10)$

$$\bar{x} = \frac{13.3987 + 13.3957 + \ldots + 13.4002}{10} = 13.3962$$

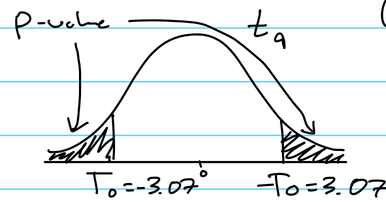
$$s = \sqrt{\frac{(13.3987-13.3902)^2 + \ldots (13.4002-13.3902)^2}{10-1}} = 0.0039086$$

a) (i) Inference on mean thickness, variance unknown
↳ one-sample t-test w/ $\alpha = 0.05$

(ii) $H_0: \mu = 13.4 \times 1000 \mathring{A}$ (iii) $H_a: \mu \neq$

(iv) $T_0 = \dfrac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{13.3962 - 13.4}{\frac{0.0039086}{\sqrt{10}}} = -3.074$

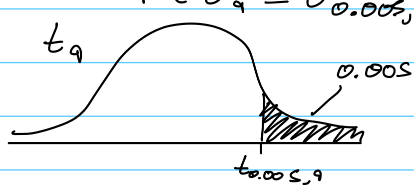
(v) Reject if p-value $< \alpha$



(vi) p-value $= 2\left(P\{t_9 > -T_0\}\right)$
$= 2\left(P\{t_9 > 3.07\}\right)$
$= 2\left((0.005, 0.01)\right)$
p-value $\in (0.01, 0.02)$

(viii) P-value $\in (0.01, 0.02) < 0.05$ reject H_0.
We have sufficient evidence to reject the claim that the mean thickness is $13.4 \times 1000 \mathring{A}$. We can thus accept H_a, having evidence that the true mean is not $13.4 \times 1000 \mathring{A}$.

b) $t_{\alpha/2, 9}: P(t_9 \geq t_{\alpha/2, 9}) = \alpha/2$
$\rightarrow P(t_9 \geq t_{0.005, 9}) = 0.005$



$\Rightarrow t_{0.005, 9} = 3.250$

In repeated sampling, this method will produce intervals that capture the true mean about 99% of the time.

$$L = 13.3962 - 3.250 \frac{0.0039086}{\sqrt{10}} = 13.3922$$

$$U = 13.3962 + 3.250 \frac{0.0039086}{\sqrt{10}} = 13.4002$$

99% two-sided CI $13.39 \leq \mu \leq 13.40$

c) Based on the normal probability plot, the p-value is 0.711. This is much larger than any reasonable value of α, so we do not have any evidence to reject that these data are normally distributed.

1. c) Minitab output

Sample Data

Statistics

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1
Thickness Measurements (Ax10^3)	10	0	13.3962	0.0012360	0.0039086	13.3902	13.3923

Variable	Median	Q3	Maximum
Thickness Measurements (Ax10^3)	13.3961	13.4001	13.4015

Section a

Descriptive Statistics

N	Mean	StDev	SE Mean	95% CI for μ
10	13.3962	0.0039	0.0012	(13.3934, 13.3990)

μ: population mean of Thickness Measurements (Ax10^3)

Test

Null hypothesis H_0: $\mu = 13.4$
Alternative hypothesis H_1: $\mu \neq 13.4$

T-Value	P-Value
-3.09	0.013

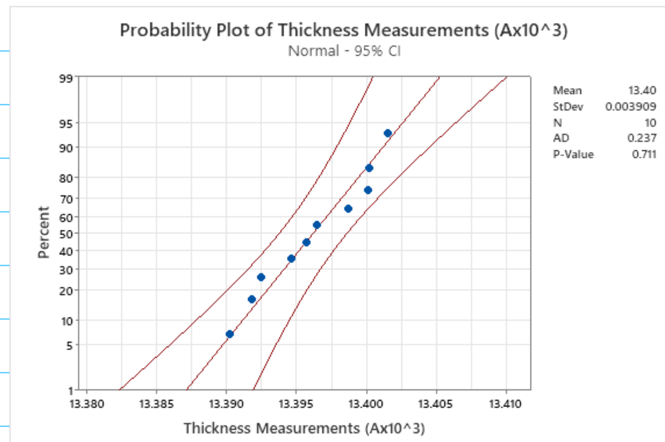
Section b

Descriptive Statistics

N	Mean	StDev	SE Mean	99% CI for μ
10	13.3962	0.0039	0.0012	(13.3922, 13.4002)

μ: population mean of Thickness Measurements (Ax10^3)

Section c



2.

One-Sample T:

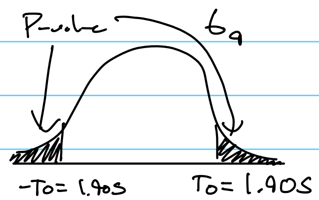
Test of mu = 12 vs not = 12

Variable	N	Mean	StDev	SE Mean	T	P
x	10	12.564	.936	0.296	?	?

a) $DOF = N-1 = 10-1 = 9$

t-dist w/ 9 DOF

b) $T_0 = \dfrac{\bar{x}-\mu}{\frac{s}{\sqrt{N}}} = \dfrac{12.564-12}{\frac{0.936}{\sqrt{10}}} = 1.905$ Reject if p-value $< \alpha = 0.05$

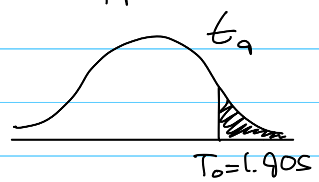


$P\text{-value} = 2(P\{t_9 > T_0\})$
$= 2((0.025, 0.05))$
$P\text{-value} \in (0.05, 0.1)$

$T_0 = 1.91$
$P\text{-value} \in (0.05, 0.1)$

The upper and lower bounds of my p-value exceed $\alpha = 0.05$, so we fail to reject H_0. We do not have sufficient evidence to reject the claim that the sample mean is 12.

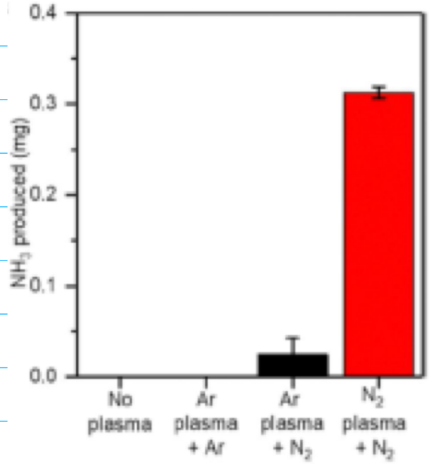
c) Suppose $H_0: \mu = 12$, $H_a: \mu > 12$



$P\text{-value} = P\{t_9 > T_0\}$
$= (0.025, 0.05)$
$P\text{-value} \in (0.025, 0.05)$

Yes, choosing this H_a yields a p-value whose bounds are both less than $\alpha = 0.05$. In this case, we reject H_0, and accept H_a, having sufficient evidence to accept the claim that the true mean exceeds 12.

3.



Data (in mg of Ammonia made in 45 min)

0.007, 0.059, 0.023, 0.001, 0.049, 0.010

Inference on mean amount of Ammonia, unknown variance of population → One-Sample t-test

$H_0: \mu = 0$ mg of Ammonia

$H_a: \mu > 0$ mg of Ammonia

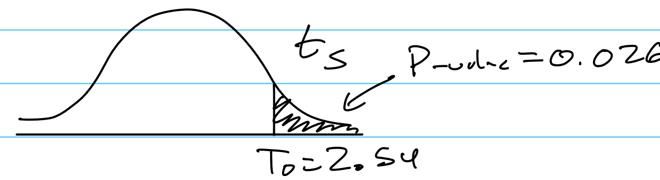
Use t-distribution as t_5 (5 D.F)

Minitab Hypothesis test @ $\alpha = 0.05$

Descriptive Statistics

N	Mean	StDev	SE Mean	95% Lower Bound for μ
6	0.02483	0.02392	0.00977	0.00516

μ: population mean of mg of Ammonia



Test

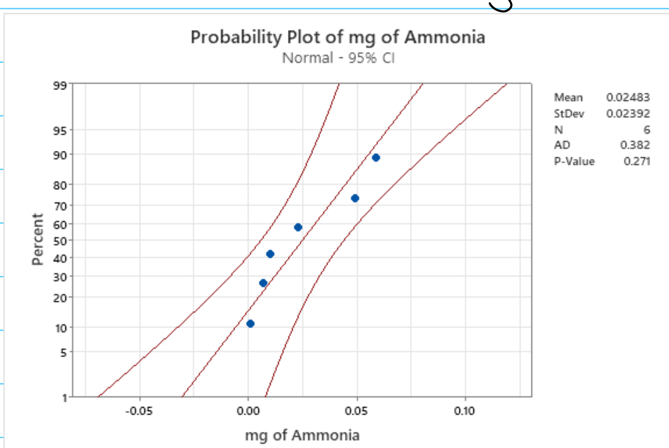
Null hypothesis $H_0: \mu = 0$

Alternative hypothesis $H_1: \mu > 0$

T-Value	P-Value
2.54	0.026

P-value = 0.026 < α = 0.05, so we reject H_0. We have sufficient evidence to accept H_a, in that the mean mg of Ammonia generated exceeds 0 mg.

Minitab Normal Probability Plot



P-value for normality test is 0.271 > 0.05. Therefore, we have insufficient evidence to reject the claim that the population is normally distributed. Seeing this, I would proceed with the t-test given normality is unable to be rejected.

4. Data (Tire Life in Km)

$\bar{x} = 60139.7$ Km

$s = 3645.94$ Km

$N = 16$

a) We are inferring about the variance of one population, so we will use the 1-variance test.

Parameter of Interest: σ^2

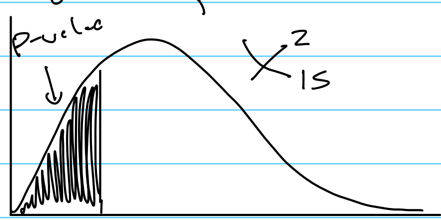
χ^2 tests are contingent on normality, I will assume that these data are roughly normal by the central limit theorem as N=16 tires is reasonably large. The χ^2 distribution is sensitive to this normality assumption.

$H_0: \sigma = 4000$ Km

$H_a: \sigma < 4000$ Km

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(16-1)(3645.94)^2}{(4000)^2} = 12.4623$$

Reject if p-value < $\alpha = 0.05$



$\chi_0^2 = 12.4623$

P-value $= 1 - P\{\chi^2_{15} > \chi_0^2 = 12.4623\}$

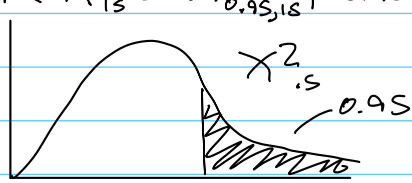
$= 1 - (0.950, 0.500)$

P-value $\in (0.05, 0.5)$

We found a p-value range of (0.05, 0.5) whose bounds exceed $\alpha = 0.05$. Therefore, we fail to reject H_0, having insufficient evidence to reject the claim that the std dev of the tires is 4000 Km.

b) $$\sigma \le \sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha,n}}} = \sqrt{\frac{(16-1)(3645.94)^2}{7.26}} = 5240.67 \text{ Km}$$

$P(\chi^2_{15} \ge \chi^2_{0.95,15}) = 0.95$



$\chi^2_{0.95,15} = 7.26$

$\sigma < 5240.67$ Km

In repeated sampling this method will produce intervals that capture the mean about 95% of the time.

c) Minitab: Stat → Basic Stat → 1 variance − choose "sample std dev", input 16 for size, 3645.94 for std dev → hypothesis test on → Value = 4000 for std dev. → Options choose 95% → Std dev < hypo std. dev → OK → OK

Descriptive Statistics

N	StDev	Variance	95% Upper Bound for σ using Chi-Square
16	3646	13292878	5240

Test

Null hypothesis $H_0: \sigma = 4000$
Alternative hypothesis $H_1: \sigma < 4000$

Method	Test Statistic	DF	P-Value
Chi-Square	12.46	15	0.356

5. Data (N=16) (Volts)

10.35	11.54
9.30	9.95
10.00	10.28
9.96	8.37
11.65	10.44
12.00	9.25
11.25	9.38
9.58	10.85

a-b) Inference on mean voltage, unknown variance → One sample t-test.

$H_0: \mu = 12V$ $H_a: \mu \neq 12V$

Descriptive Statistics

N	Mean	StDev	SE Mean	95% CI for μ
16	10.259	0.999	0.250	(9.727, 10.792)

μ: population mean of Output V of Power Supply

Test

Null hypothesis $H_0: \mu = 12$
Alternative hypothesis $H_1: \mu \neq 12$

T-Value	P-Value
-6.97	0.000

Reject H_0 => p-value $= 0 < \alpha = 0.05$. We have sufficient evidence to reject the claim that the mean voltage output is 12V. We accept that it is not equal to 12V.

c) P-value = 0

b) $9.727 \leq \mu \leq 10.792$

↑ 95% CI on μ

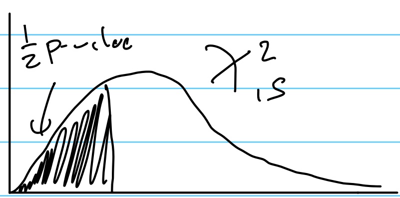
Interpreted simply, this method will produce intervals that capture the true mean 95% of the time.

c) Inference on one population's variance. 1-variance test using $\alpha = 0.05$

$H_0: \sigma^2 = 11 \ V^2$ $H_a: \sigma^2 \neq 11 \ V^2$

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(16-1)(0.999)^2}{11} = 1.361$$

Reject if p-value $< \alpha = 0.05$



$\chi_0^2 = 1.361$

$$\text{P-value} = 2(1 - P\{\chi^2_{14} > 0.1237\})$$
$$= 2(1 - (1, 0.995))$$
$$\text{P-value} < 2(0.005)$$
$$\text{P-value} < 0.01$$

P-value $< 0.01 < \alpha = 0.05$ so reject H_0. We have evidence to reject the claim that the voltage variance is $11 \ V^2$. We accept the alternate hypothesis that it is different.

Descriptive Statistics

N	StDev	Variance	95% CI for σ using Bonett	95% CI for σ using Chi-Square
16	0.999	0.998	(0.749, 1.518)	(0.738, 1.546)

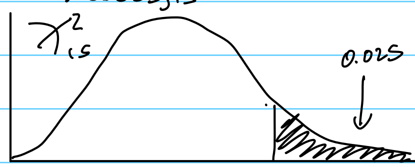
Test

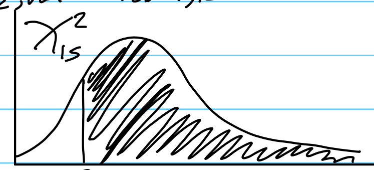
Null hypothesis $H_0: \sigma^2 = 11$
Alternative hypothesis $H_1: \sigma^2 \neq 11$

Method	Test Statistic	DF	P-Value
Bonett	—	—	0.000
Chi-Square	1.36	15	0.000

} agrees

d) $\chi^2_{\alpha/2,n-1}: \chi^2_{0.025,15} = 27.49$ $\quad$ $\chi^2_{1-\frac{\alpha}{2},n-1}: \chi^2_{0.975,15} = 6.27$





$$\text{Lower} = \frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}} = \frac{(16-1)(0.999)^2}{27.49} = 0.5446\ V^2 \rightarrow \sigma = 1.55\ V$$

$$\text{Upper} = \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2},n-1}} = \frac{(16-1)(0.999)^2}{6.27} = 2.388\ V^2 \rightarrow \sigma = 0.738\ V$$

$0.738 < \sigma < 1.55$

Descriptive Statistics

N	StDev	Variance	95% CI for σ using Bonett	95% CI for σ using Chi-Square
16	0.999	0.998	(0.749, 1.518)	(0.738, 1.546)

agrees

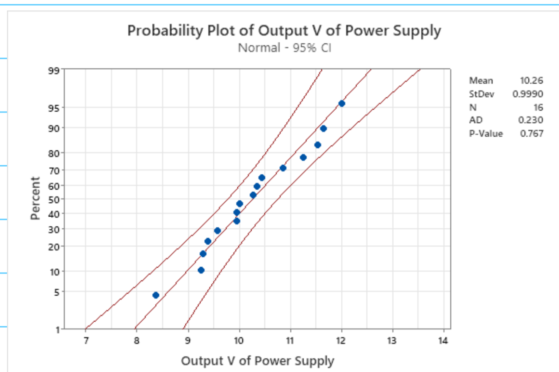
Test

Null hypothesis $H_0: \sigma^2 = 11$
Alternative hypothesis $H_1: \sigma^2 \neq 11$

Method	Test Statistic	DF	P-Value
Bonett	—	—	0.000
Chi-Square	1.36	15	0.000

In repeated sampling, this method will result in intervals that capture the true mean 95% of the time.

e)



Minitab outputs a p-value of 0.767 for normality of these data. Because 0.767 > α = 0.05, we do not have sufficient evidence to reject that these data are normally distributed. Thus, using χ^2 as a reference dist. is valid!!

f) Use $0.738 < \sigma < 1.55$ to get sigma level range

$$\text{Sigma level} = \frac{|USL - LSL|}{\sigma}$$

(i) $\frac{10}{0.738} = 13.55$

(ii) $\frac{10}{1.55} = 6.45$

6.45 ↖ Upper bound

Both bounds of our CI yield sigma levels greater than 6, so this is a six sigma process.

Upper bound of CI is greater than 6, so it's a 6σ process.