~~HW~~ 9

1) Problem 13.4

a) Do any of the tree factors affect tool life?

To solve this problem we use the equations in the book to calculate the effects, and SS for each factor and interaction. The table with −1, +1 is set up ~~as~~ based on the notation (e.g. for row a, we would have : $\begin{array}{ccc} A & B & C \\ +1 & -1 & -1 \end{array}$). for the Effects and SS :

$$\text{effect} = \frac{\text{Contrast}}{n 2^{k-1}} \qquad SS = \frac{\text{Contrast}}{n 2^{k}}$$

where $k = 3$ and $n = 2$ for this propblem

grand averge = 411.56 (average of ALL data points)

~~[illegible]~~

Note that interactions are generated by multiplying A x B colums for AB and so on.

Coefficenst are estimated ~~[illegible]~~ by taking $\frac{\text{effets}}{2}$

~~[illegible]~~

$SS_T = \sum(\text{data point} - \text{grand average})^2$ → summed over all data points

$SS_E = SS_T - SS_A - SS_B - SS_C - SS_{AB} - SS_{AC} - SS_{BC} - SS_{ABC}$

$= 26{,}312$

$DOF_E = DOF_T - DOF_{\text{for each factor}}$

$DOF_{Total} = 1 + n\,2^{k-1} = 15$

$15 - 7 = 8 = DOF_{error}$

Rewritten clearer

$DOF_{Total} = 1 + n\,2^{(k-1)} = 15$

$MS_E = \frac{SS_E}{DOF_E} = \frac{26{,}312}{8}$

$= 3289$

$F_0 = \frac{\frac{SS_{Factor}}{1}}{MS_E}$ (see sheet for all F_0's)

$F_{crit} = \left(F_{0.05,1,8}\right) = 5.32$

Reject null if $F_0 > 5.32$,

Factors B, C and AC are significant

We reject the null that they are zero, and conclude that B (metal hardness) and C (cutting angle) do affect tool life

Note: While A on its own isn't significant, it does have an interaction with C. Therefore A should be included in the model

Analysis of Variance

Source	DF	Seq SS	Contribution	Adj SS	Adj MS	F-Value	P-Value
Model	7	114888	85.36%	114888	16412.5	6.66	0.008
Linear	3	50317	37.39%	50317	16772.3	6.81	0.014
A	1	1332	0.99%	1332	1332.3	0.54	0.483
B	1	28392	21.10%	28392	28392.3	11.53	0.009
C	1	20592	15.30%	20592	20592.3	8.36	0.020
2-Way Interactions	3	59741	44.39%	59741	19913.6	8.09	0.008
A*B	1	506	0.38%	506	506.3	0.21	0.662
A*C	1	56882	42.26%	56882	56882.2	23.10	0.001
B*C	1	2352	1.75%	2352	2352.3	0.96	0.357
3-Way Interactions	1	4830	3.59%	4830	4830.2	1.96	0.199
A*B*C	1	4830	3.59%	4830	4830.2	1.96	0.199
Error	8	19700	14.64%	19700	2462.5		
Total	15	134588	100.00%				

	n	2	k	3			y...	413.125			DOF total	n2^k - 1	
	A	B	C	AB	AC	BC	ABC	Replicate 1	Replicate 2	SUM			
	-1	-1	-1	1	1	1	-1	221	311	532		15	DOF total
a	1	-1	-1	-1	-1	1	1	325	435	760		7	DOF factors
b	-1	1	-1	-1	1	-1	1	354	348	702		8	DOF error
ab	1	1	-1	1	-1	-1	-1	552	472	1024		SSE	19700
c	-1	-1	1	1	-1	-1	1	440	453	893		MSE	2462.5
ac	1	-1	1	-1	1	-1	-1	406	377	783			
bc	-1	1	1	-1	-1	1	-1	605	500	1105			
abc	1	1	1	1	1	1	1	392	419	811			
								SST		134587.8			
	-532	-532	-532	532	532	532	-532		36912.01563	10429.52			
	760	-760	-760	-760	-760	760	760		7766.015625	478.5156			
	-702	702	-702	-702	702	-702	702		3495.765625	4241.266			
	1024	1024	-1024	1024	-1024	-1024	-1024		19286.26563	3466.266			
	-893	-893	893	893	-893	-893	893		722.265625	1590.016			
	783	-783	783	-783	783	-783	-783		50.765625	1305.016			
	-1105	1105	1105	-1105	-1105	1105	-1105		36816.01563	7547.266			
	811	811	811	811	811	811	811		446.265625	34.51563			
Contrast	146	674	574	-90	-954	-194	-278						
Effect	18.25	84.25	71.75	-11.25	-119.25	-24.25	-34.75						
SS	1332.25	28392.25	20592.25	506.25	56882.25	2352.25	4830.25						
Coefficent	9.125	42.125	35.875	-5.625	-59.625	-12.125	-17.375						
Fo	0.541015	11.52985	8.362335	0.205584	23.09939	0.955228	1.961523						
Fcrit	F(0.05, 1, 8)		5.32										

Combination of factor levels?

Longest tool life

Regression Equation in Uncoded Units

C8 = 413.1 + 9.1 A + 42.1 B + 35.9 C - 59.6 A*C

$\hat{y} = 413.1 + 9.1A + 42.1B + 35.9C + 59.6AC$

max $\hat{y}$ would have low A (−1), high B (+1) and high C (+1)

So low A (cutting speed), high B (metal harness) and high C (cutting angle)

c) Is there a combo of A and C that gives good results regardless of B?

Yes, as long as C is high and A is low, the respond remains high (400+) see contour

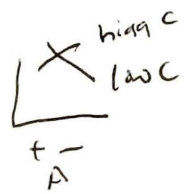
13.5 Model: $\hat{y} = 412 + 5.06A + 38.1B + 37.4C - 35.6AC$

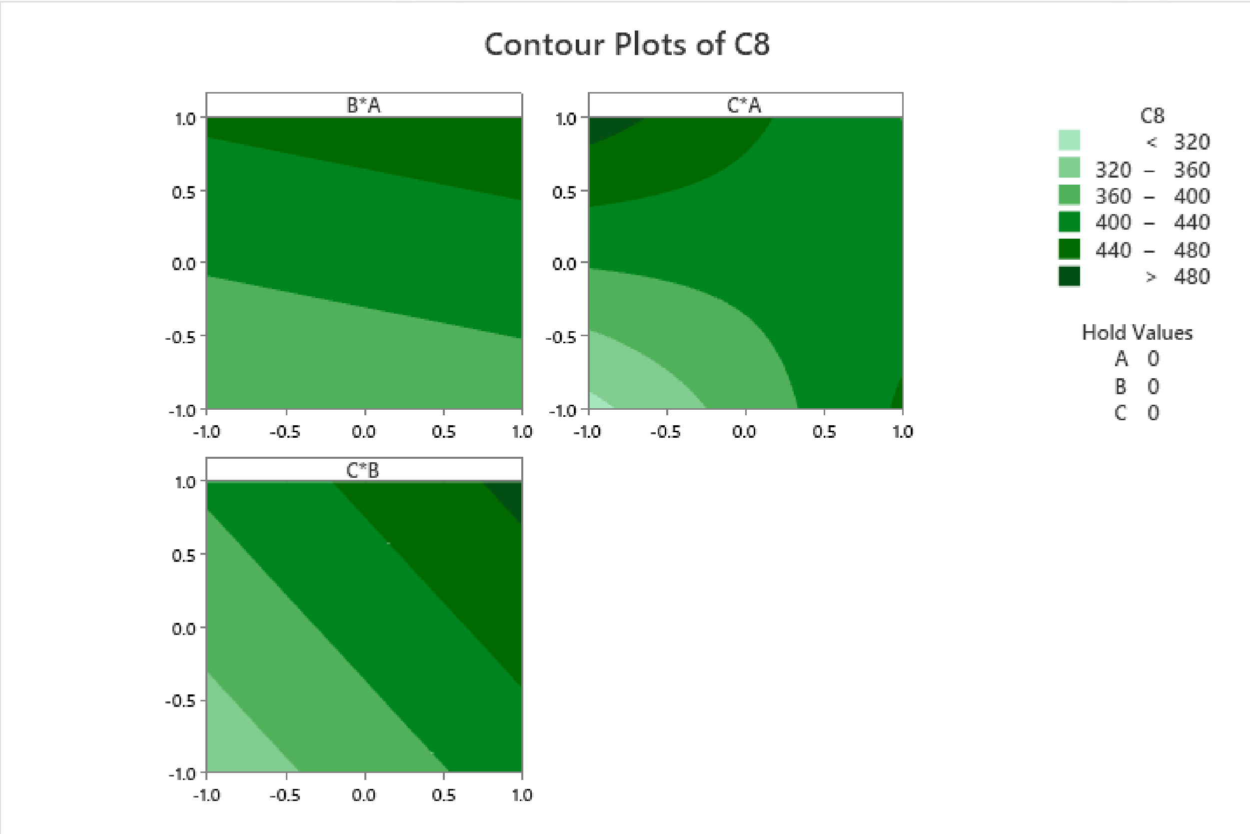
Residuals = data point − $\hat{y}$

How to calculate main effects and interactions plots in Excel?

1) Main effects: take average of all data points at high (+) or low (−) factors and plot: eg.

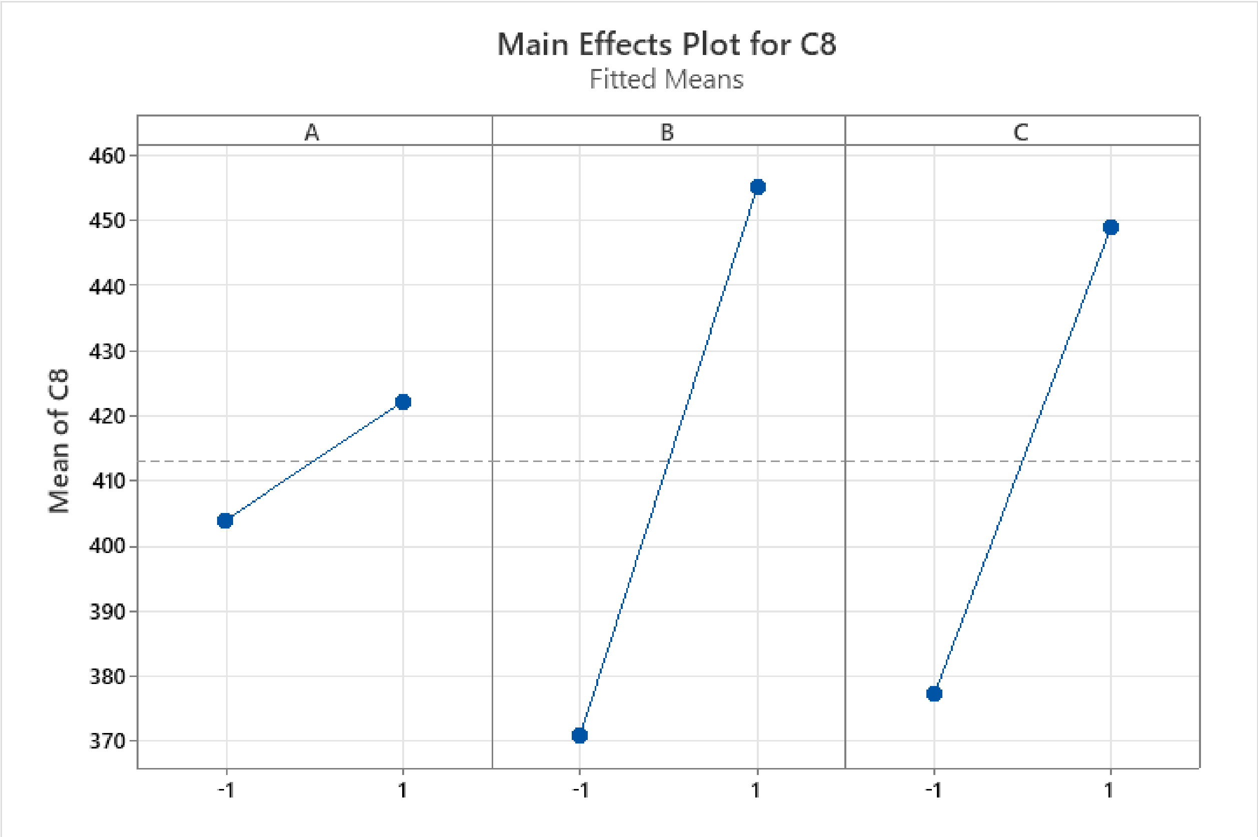
2) Interactions: take averages of all points at (++), (−−), (+−), (−+) for all three factors





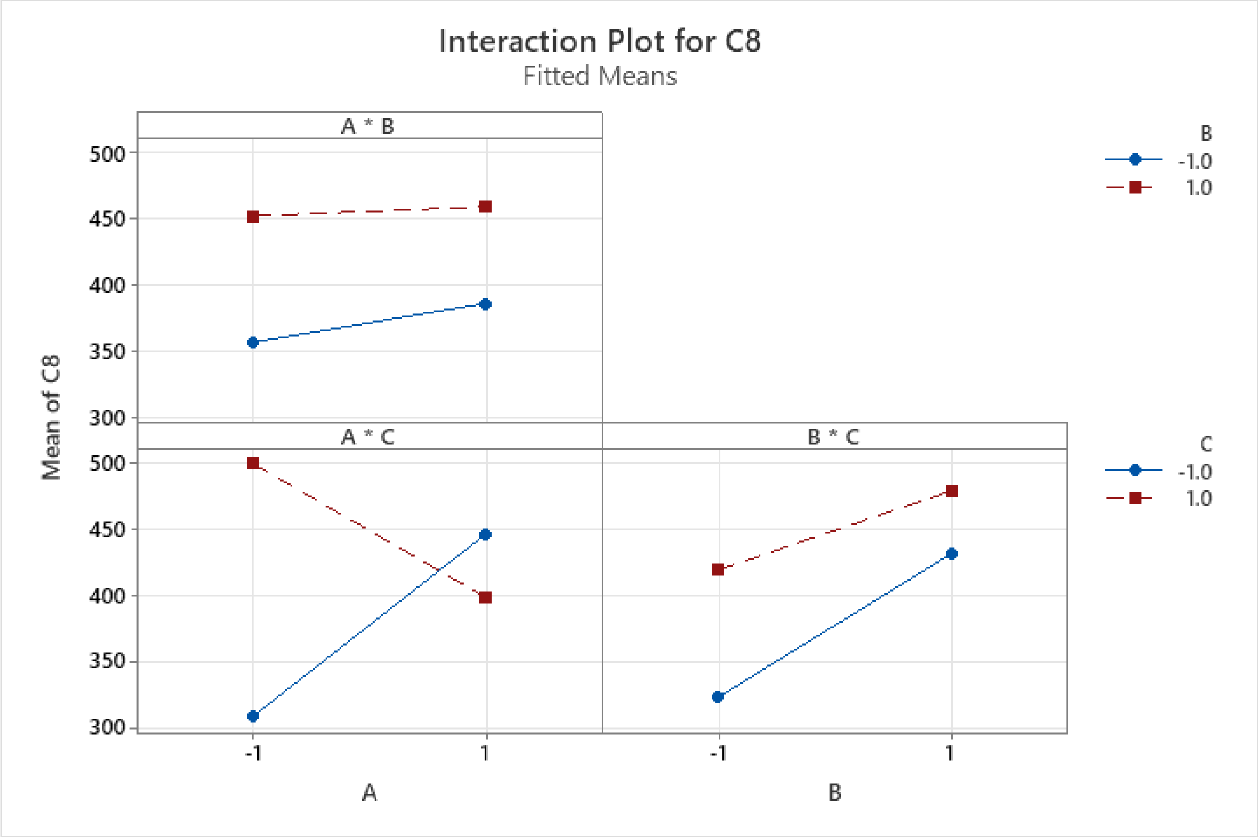
Contour Plots of C8
B*A
C*A
C*B
1.0
0.5
0.0
-0.5
-1.0
C8
< 320
320 – 360
360 – 400
400 – 440
440 – 480
> 480
Hold Values
A 0
B 0
C 0

c



Main Effects Plot for C8
Fitted Means
A
B
C
Mean of C8
460
450
440
430
420
410
400
390
380
370
-1
1
-1
1
-1
1

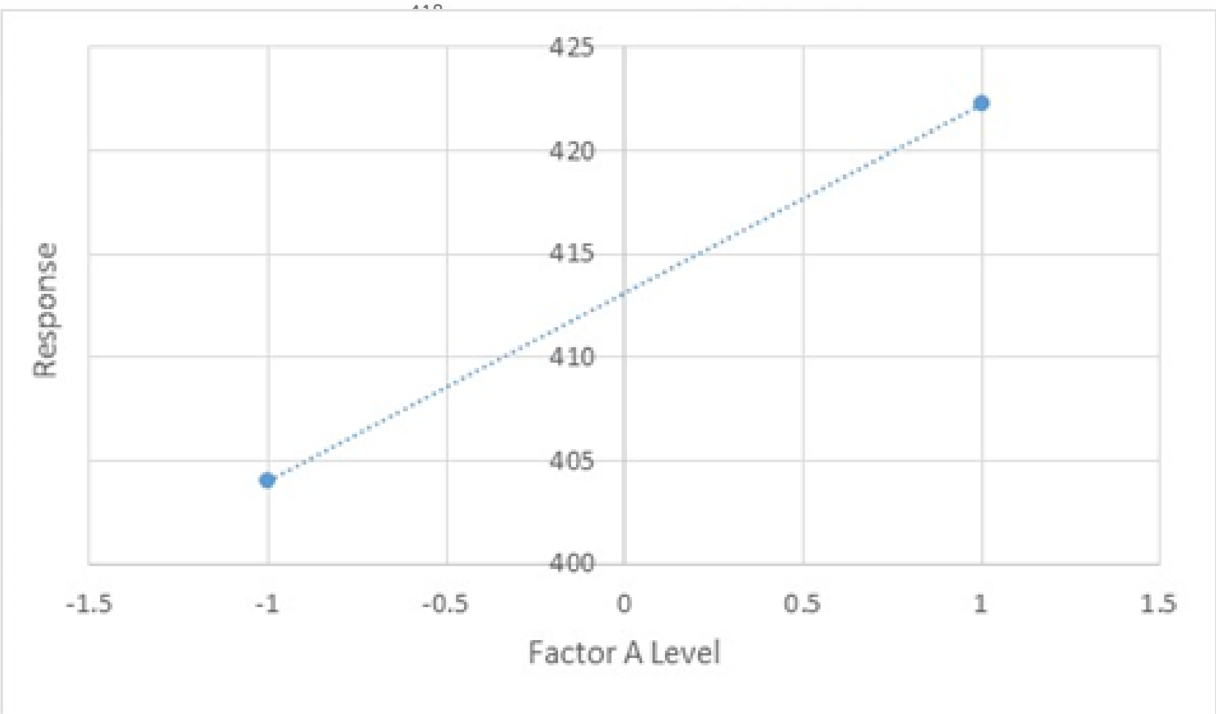
Means



Interaction Plot for C8
Fitted Means
A * B
A * C
B * C
Mean of C8
500
450
400
350
300
B
-1.0
1.0
C
-1.0
1.0
-1
1
A
B

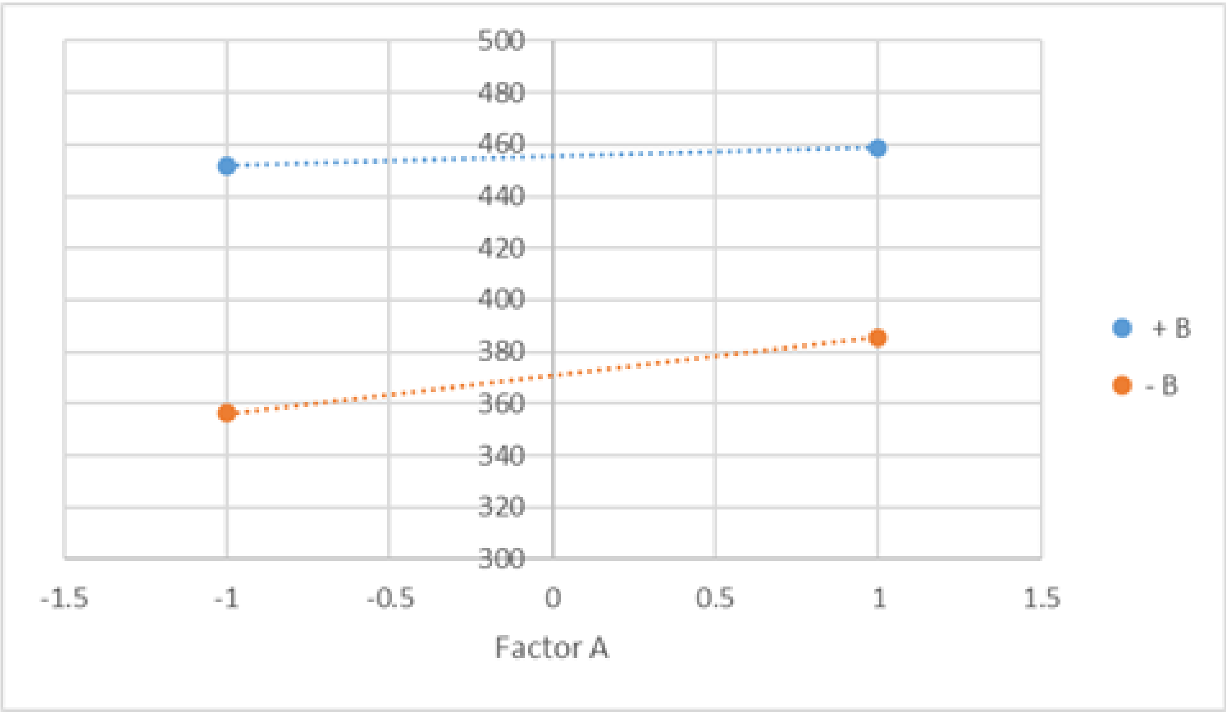
Main Effect Plot A

Low A	High A
-1	1
404	422.25



	High B	Low B
High A	458.75	385.75
Low A	451.75	356.25

We get these numbers by averaging all of the data points at: 1) low A, low B, 2) low A high B, 3) high A low b, 4) high a high b



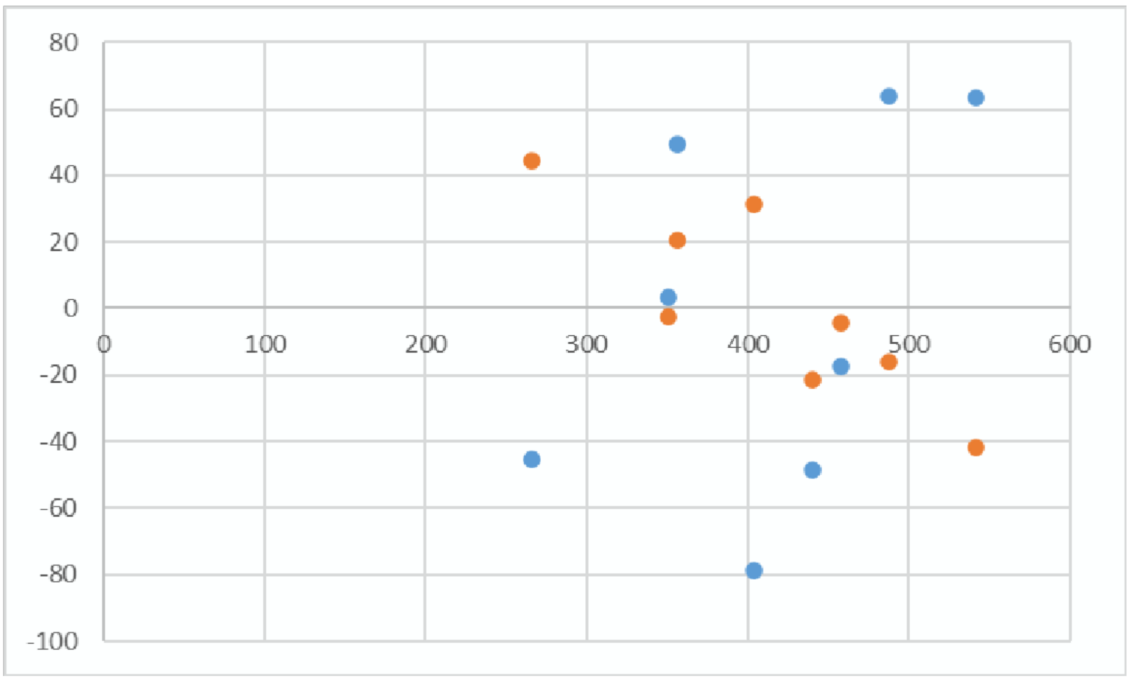
B.

1) residuals: data point − $\hat{y}$ for all data points

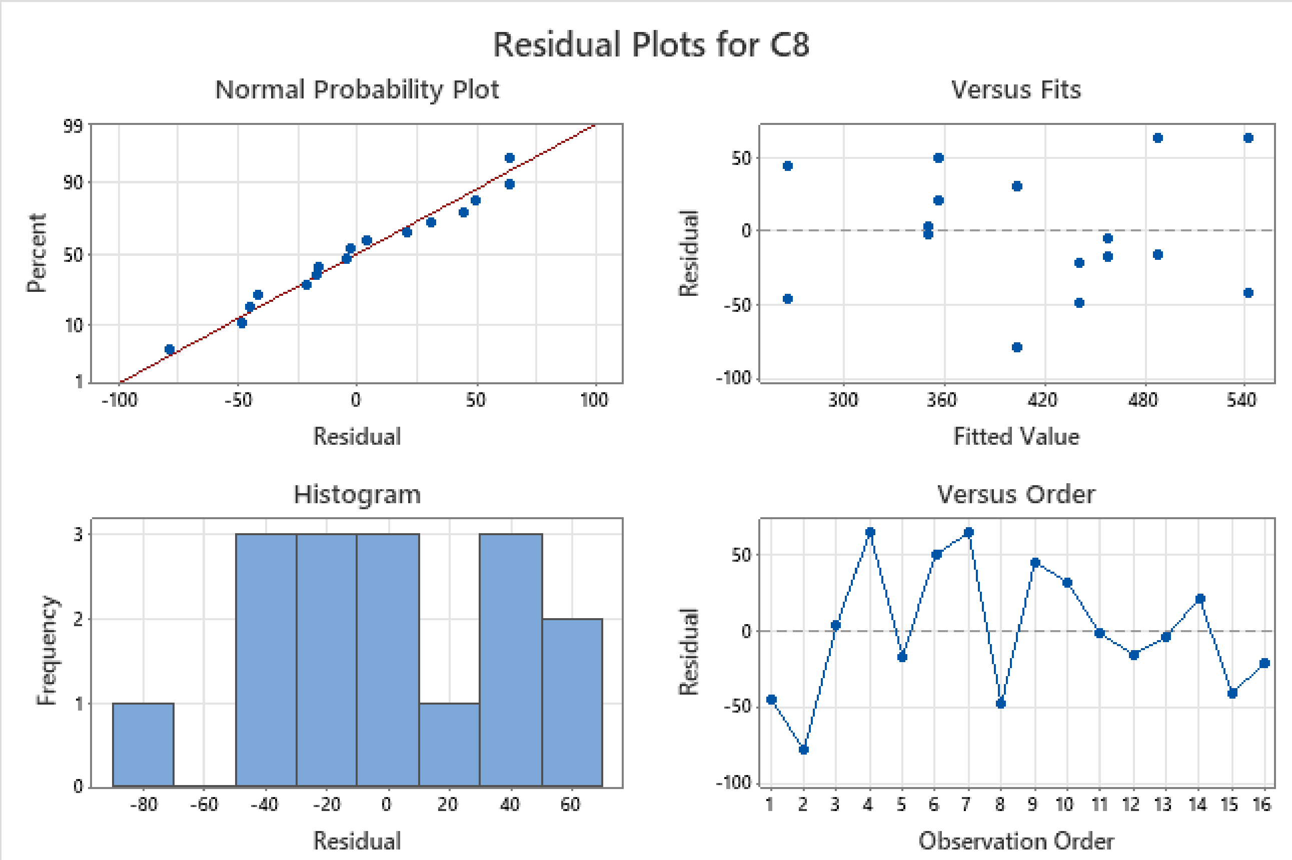
y_i

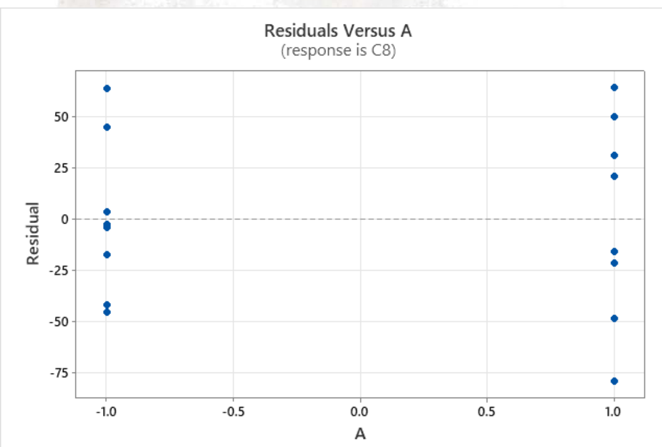
Predicted	Residuals:	
266.4	-45.4	44.6
403.8	-78.8	31.2
350.6	3.4	-2.6
488	64	-16
457.4	-17.4	-4.4
356.4	49.6	20.6
541.6	63.4	-41.6
440.6	-48.6	-21.6

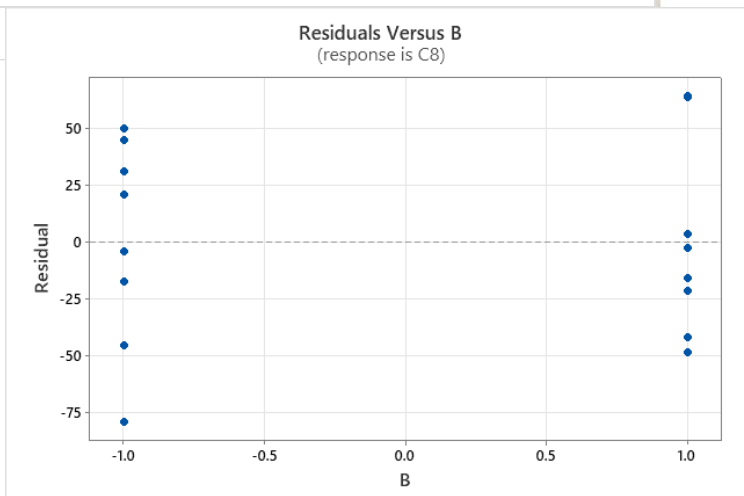
b)

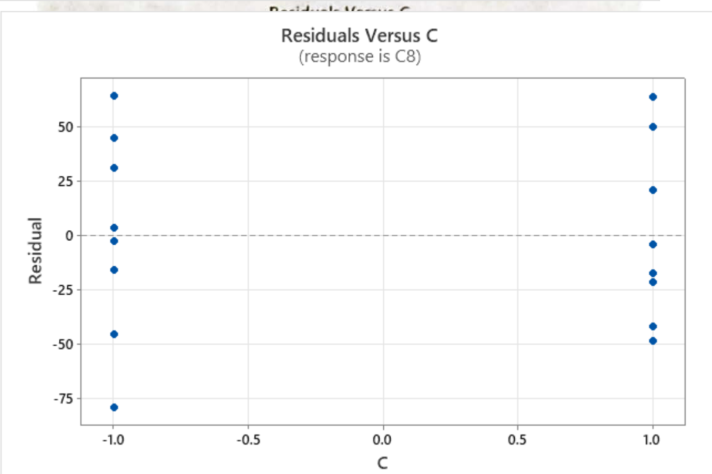


X axis = predicted, y axis = residuals


Residual Plots for C8
Normal Probability Plot
Percent
Residual
Versus Fits
Residual
Fitted Value
Histogram
Frequency
Residual
Versus Order
Residual
Observation Order

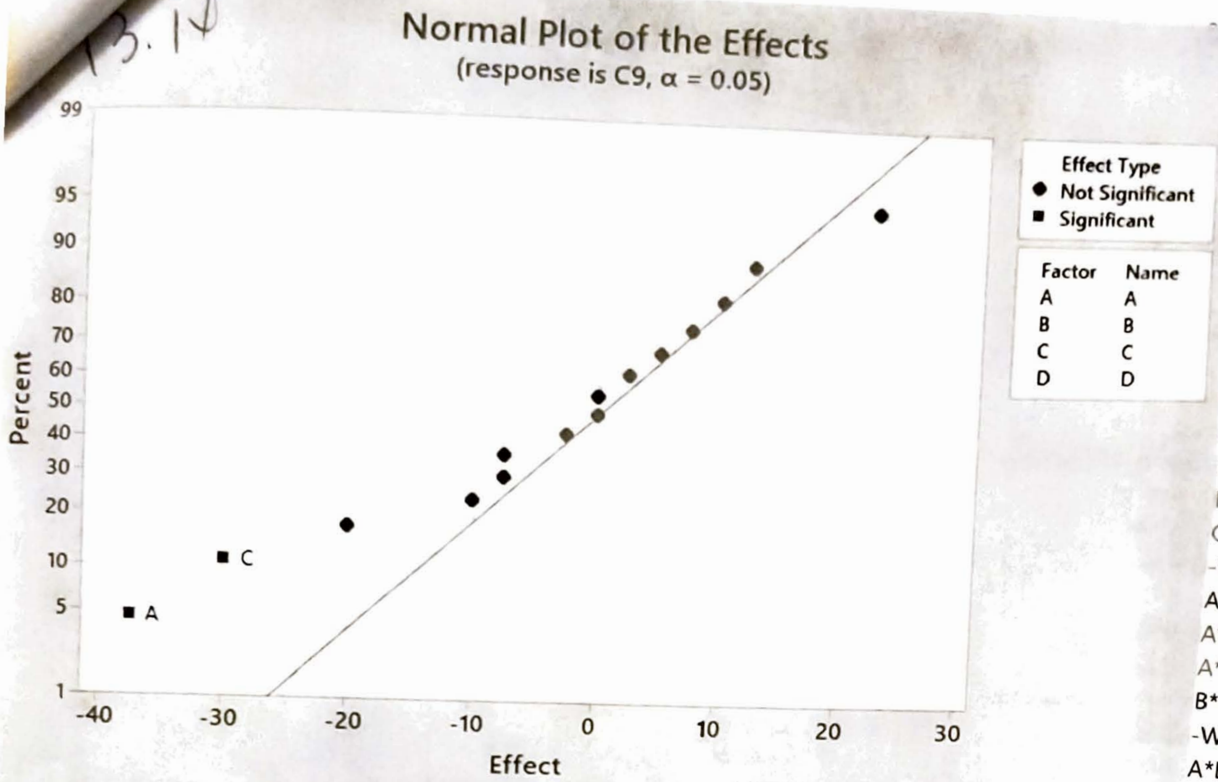

Residuals Versus A
(response is C8)
Residual
A


Residuals Versus B
(response is C8)
Residual
B


Residuals Versus C
(response is C8)
Residual
C

PROBLEM 3

3.14



Lenth's PSE = 11.25

alysis of Variance

ource	DF	Adj SS
odel	15	15100.0
Linear	4	9850.0
A	1	5625.0
B	1	400.0
C	1	3600.0
D	1	225.0
2-Way Interactions	6	4000.0
A*B	1	2025.0
A*C	1	25.0
A*D	1	100.0
B*C	1	1600.0
B*D	1	25.0
C*D	1	225.0
-Way Interactions	4	1250.0
A*B*C	1	225.0
A*B*D	1	0.0
A*C*D	1	400.0
B*C*D	1	625.0
-Way Interactions	1	0.0
A*B*C*D	1	0.0
or	0	*
otal	15	15100.0

The effects A and C are active

b)

For the model, only put the A and C terms in the model and run the analysis again. These are your results below. Note that they are pooling the non-significant effects to estimate error (this is how they are able to conduct ANOVA)

Analysis of Variance

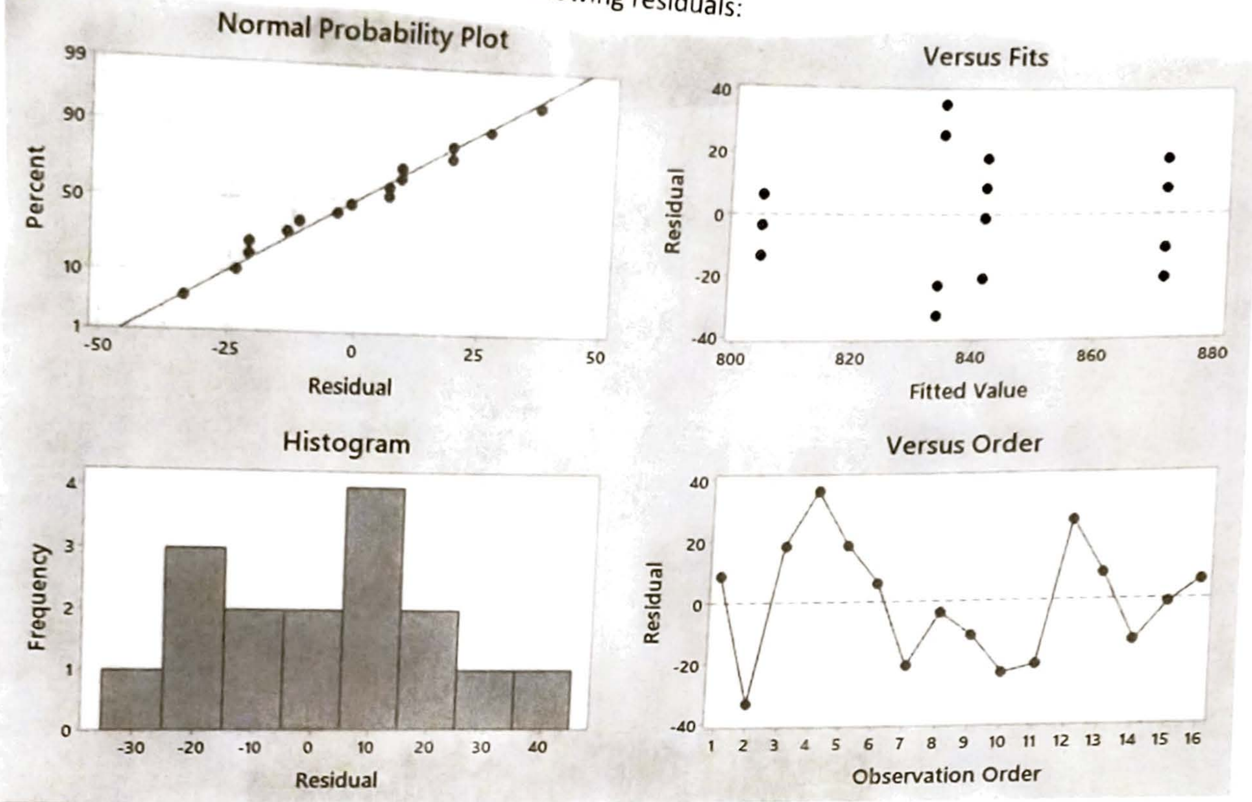
Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	2	9225	4612.5	10.21	0.002
Linear	2	9225	4612.5	10.21	0.002
A	1	5625	5625.0	12.45	0.004
C	1	3600	3600.0	7.97	0.014
Error	13	5875	451.9		
Total	15	15100			

Regression Equation in Uncoded Units

Response = 838 - 19 A - 15 C

NOTE: this is one form of model trimming - we just take out insignificant terms from the regression and run again. If you did the backward elimination in Minitab, you will get different answers - that is ok! At the end of the day, you just want to arrive at a model that will predict your response (and different methods might give you slightly different results)

With this re-run model we get the following residuals:



13.15

(a) $n_c = 4 \quad \bar{y}_c = 890$

$n_F = 16 \quad \bar{y}_f = 837.5$

$$SS_{\text{pure quadradic}} = \frac{n_F n_c (\bar{y}_F - \bar{y}_c)^2}{n_F + n_c} = \frac{(16)(4)(837.5-890)^2}{16+4}$$

$$= \frac{(16)(4)(2756)}{20} = 8820$$

$$\hat{\sigma}^2 = \frac{\sum_{i=17}^{20} (y_i - \overset{\bar{y}_c}{890})^2}{n_c - 1} = 1000 = \frac{3000}{3}$$

Pure error

SS: (from table in 13.14)

$= 1250$ (3 way + 4 way interactions SS)

DOF = 5

First we test if the 3 or 4 way terms are significant:

$$F_0 = \frac{\frac{1250}{5}}{1000} = 0.25$$

$$F_{crit} = F_{0.05,5,3} = 9.01$$

$F_0 < F_{crit}$ Do not reject null

thus, we can lump those terms into

$$SS_{residual} = 1250 + 3000 = 4250$$

$$MS_{residual} = \frac{4250}{3+5}$$

$$F_0 = \frac{\frac{8820}{1}}{\frac{4250}{8}} = 16.6$$

$$F_{crit} = F_{0.05,1,8} = 5.35$$

$F_0 > F_{crit}$

Reject Null and conclude there is quadradic curvature

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	10	13850.0	1385.00	0.95	0.533
Linear	4	9850.0	2462.50	1.70	0.234
A	1	5625.0	5625.00	3.87	0.081
B	1	400.0	400.00	0.28	0.612
C	1	3600.0	3600.00	2.48	0.150
D	1	225.0	225.00	0.15	0.703
2-Way Interactions	6	4000.0	666.67	0.46	0.822
A*B	1	2025.0	2025.00	1.39	0.268
A*C	1	25.0	25.00	0.02	0.898
A*D	1	100.0	100.00	0.07	0.799
B*C	1	1600.0	1600.00	1.10	0.321
B*D	1	25.0	25.00	0.02	0.898
C*D	1	225.0	225.00	0.15	0.703
Error	9	13070.0	1452.22		
Curvature	1	8820.0	8820.00	16.60	0.004
Lack-of-Fit	5	1250.0	250.00	0.25	0.915
Pure Error	3	3000.0	1000.00		
Total	19	26920.0			

The curvature is significant, and lack of fit (which is the terms we have excluded from the model) is not. This means that there is curvature, and the terms we have chosen to exclude are appropriate.

b) Since curvature is significant, you would have to Read the next chapter and add some points to your design to obtain a quadratic model. One example is a central composite experiment.

In summary → more designed experiments are needed!

). Fractional factorials are useful as the number of factors goes up. In fractional factorials, main effects and interactions can be aliased with lower order (likely not significant) effects.