7.11 in book   Example of Inferance on the mean, unknown variance

(one-sample t-test)

4) $n=10$ $\alpha = 0.05$

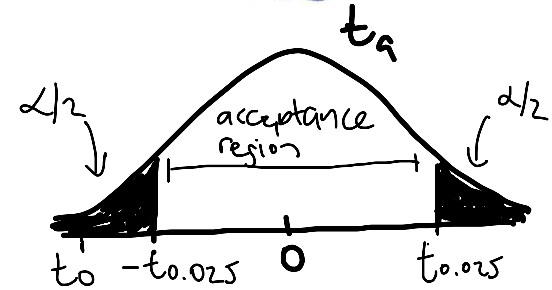
Problem 1

Data angstoms x1000
$$\begin{cases} 13.3987,\ 13.3957,\ 13.3902,\ 13.4015,\ 13.4001 \\ 13.3918,\ 13.3965,\ 13.3925,\ 13.3946,\ 13.4002 \end{cases}$$

1.) Parameter of interest is mean photoresist thichness

2) $H_0: \mu = 13.40 \times 1000\ \mathring{A}\ (\mu_0)$

3) $H_1: \mu \neq 13.40 \times 1000\ \mathring{A}$



4) $t_0 = \dfrac{\bar{x} - \mu_0}{S/\sqrt{n}} = \dfrac{13.3962 - 13.4}{\frac{0.00390862}{\sqrt{10}}} = -3.089$

$$\bar{x} = \frac{13.398 + 13.3957 \ldots 13.4002}{10} = 13.3962$$

$$S = \sqrt{\frac{(13.395-13.3962)^2 + (13.3957-13.3962)^2 \ldots + (13.4002-13.3962)^2}{9}}$$

$= 0.00390862$

5) Reject if $t_0 > t_{\alpha/2, n-1}$ or $t_0 < -t_{\alpha/2, n-1}$

6) $t_{\alpha/2, n-1} = t_{0.025, 9} = 2.262$ (App. IV)

$t_0 = -3.089$ $-t_{\alpha/2, n-1} = -2.262$

7) $\boxed{t_0 < -t_{\alpha/2, n-1} \text{ so reject null hypothesis and conclude } \mu \neq 13.40 \times 1000\ \mathring{A}}$

For two-sided 99% CI:

$\alpha = 0.01$

$$\bar{x} - t_{\alpha/2, n-1}\left(\frac{s}{\sqrt{n}}\right) \leq \mu \leq \bar{x} + t_{\alpha/2, n-1}\left(\frac{s}{\sqrt{n}}\right)$$

$t_{\frac{0.01}{2}, 9} = t_{.005, 9} = 3.2498$ (App. IV)

$$13.39618 - 3.2498\left(\frac{0.00391}{\sqrt{10}}\right) \leq \mu \leq 13.39618 + 3.2498\left(\frac{0.00391}{\sqrt{10}}\right)$$

$13.39216 \leq \mu \leq 13.4002$

$$\boxed{13.39 \leq \mu \leq 13.40}$$ −4 sig figs

ni tab:

Normality appears to be a good assumpion

Note: for practice, try making this plot by hand.

Descriptive Statistics

N	Mean	StDev	SE Mean	95% CI for μ
10	13.3962	0.0039	0.0012	(13.3934, 13.3990)

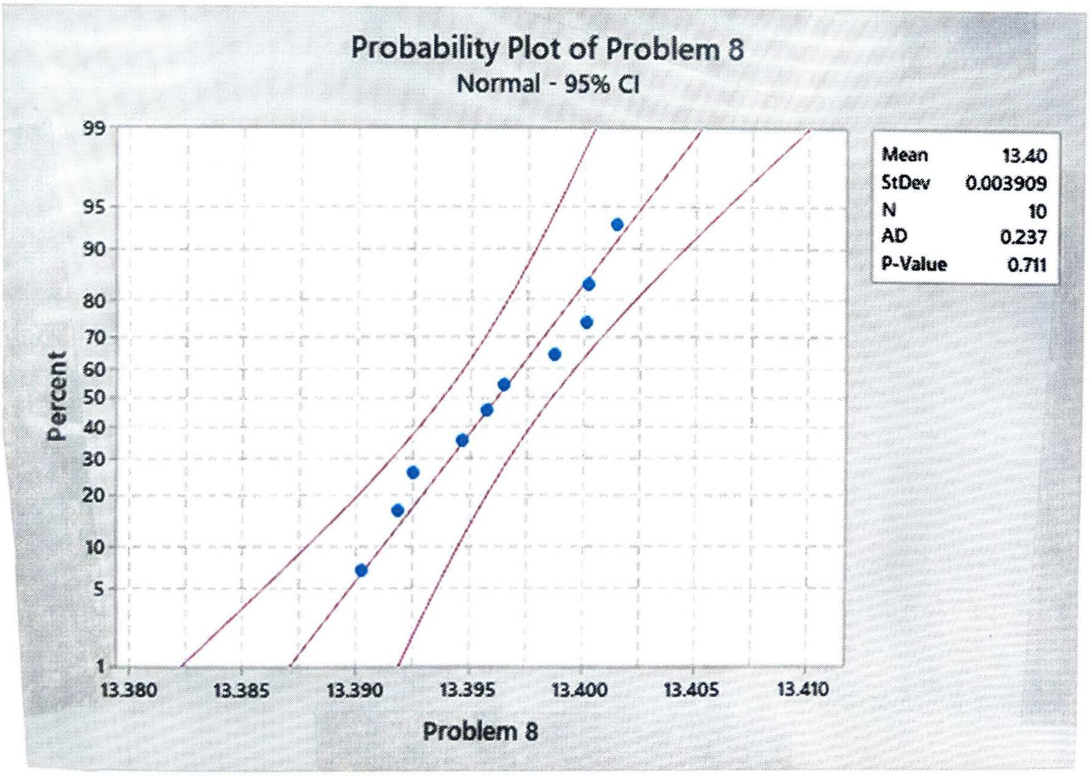
μ: mean of Problem 8

Test

Null hypothesis $H_0: \mu = 13.4$

Alternative hypothesis $H_1: \mu \neq 13.4$

T-Value	P-Value
-3.09	0.013



Problem 2

a) For this test $N=10$ so

$t_{\alpha/2, n-1}$ and $-t_{\alpha/2, n-1}$ are the critical values for a 2-sided test. The DOF are $10-1=\boxed{9}$

b) $S = S.E.\sqrt{n}$ ∴ $S = 0.296\sqrt{10} = \boxed{0.936}$

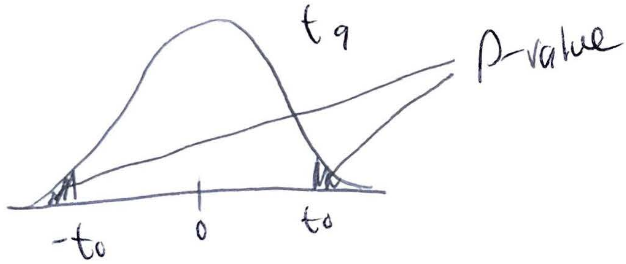
$$t_0 = \frac{\bar{x}-\mu_0}{S/\sqrt{n}} = \frac{12.564-12}{\frac{0.936}{\sqrt{10}}} = \boxed{1.91}$$

0.0100 - 0.0500 sig figs

p-value : $\boxed{0.1-0.05}$ ← $P(t<-t_0)+P(t>t_0)$

OR

$2(P(t>t_0))$



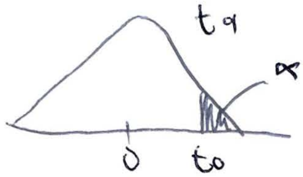
So find 1.91 in the 9 row of the table...
α is between 0.05 and 0.025

upper $= 2(0.05) = 0.1$
lower $= 2(0.025) = 0.05$

The conclusions depend on the level of Type I error you would be OK with. If $\alpha = 0.1$ we would reject the null hypothesis and say $\mu \neq 12$. If $\alpha = 0.05$, we could not reject the null - there is not enough evidence $\mu \neq 12$

using p-value, the probabilty of obtaining T_0 when the H_0 is true is not very large (5-10%)
However, threshhold values are often not above $\alpha = 0.05$
Conservatively, there isn't strong evidence to reject null.

C.) $H_0: \mu = 11.5$, $H_1: \mu > 12$
$T_0 = 1.91$ Pvalues:



p-value is between 0.025 and 0.05 (row 9, find 1.91 in Appendix)

In this case, the p-value indicates the probability of obtaining T_0 when H_0 is true is even less than in part b) (2.5-5%). Therefor we would have more evidence of rejecting H_0.
Since threshold values are typically $\alpha = 0.5$
we conclude to reject the null hypothesis and say $\mu > 12$

Problem 3

We are infering something about the mean of one sample, variance unknown:

One-sample t-test is good for this situation

where:

Minitab outputs:

$H_0: \mu = 0$

$H_1: \mu > 0$

Descriptive Statistics

N	Mean	StDev	SE Mean	95% Lower Bound for μ
6	0.02483	0.02392	0.00977	0.00516

μ: mean of Problem 10

Descriptive Statistics

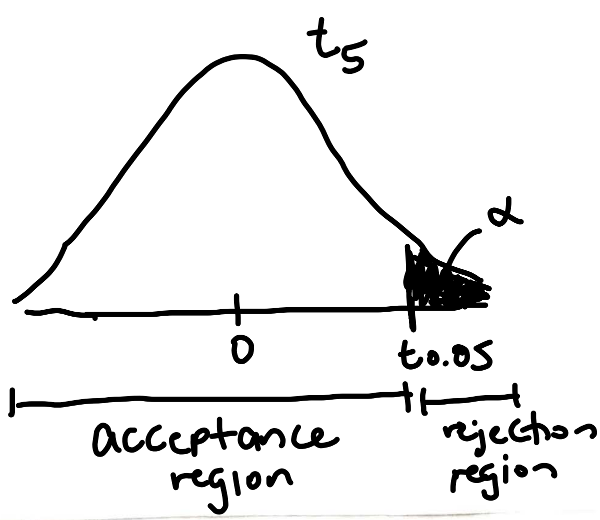
N	Mean	StDev	SE Mean	95% Lower Bound for μ
6	0.02483	0.02392	0.00977	0.00516

μ: mean of Problem 10

The normality assuption is appropriate here.

The p-value is $< \alpha = 0.05$ therefore we would reject the H_0 and conclude that $\mu > 0$, or

the mean at that condition is non-zero — ammonia is likely generated.



HW 4 Solutions

1) $n=16$ $\quad s = 3645.94$ kilometers $\quad \alpha = 0.05$

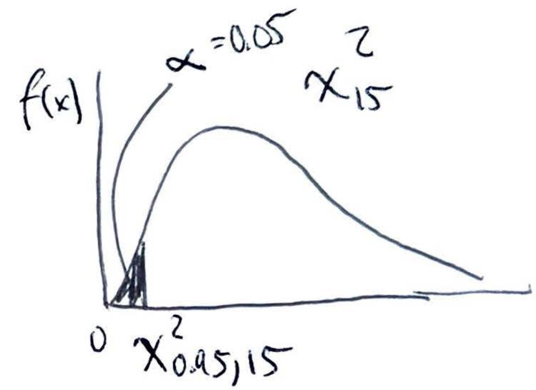
Problem 4

a. Assume a normal dist.

1) Parameter of interest is standard deviation

2) $H_0: \sigma^2 = \sigma_0^2 = 4000^2$

3) $H_1: \sigma^2 < \sigma_0^2 = 4000^2$



4) Test statistic: $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$

5) Reject H_0 if $\chi_0^2 < \chi^2_{1-\alpha,n-1} = \chi^2_{0.95,15} = 7.26$

or

Reject H_0 if p-value < 0.05

6) $\chi_0^2 = \dfrac{15(3645.94)^2}{4000^2} = 12.46$

p-value is between 0.05 and .50 (see Appendix III)

7) $\chi_0^2 > \chi^2_{1-\alpha,n-1}$

so fail to reject $H_0 \rightarrow$ we can't conclude that $\sigma^2 < \sigma_0^2 = 4000^2$

b)

95% CI upper bound $\sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha, n-1}} = \frac{15(3645.94)^2}{7.26}$

$\sigma^2 \leq 27464624$

$\boxed{\sigma^2 \leq 5240^2}$

This range is higher than 4000^2

c) Minitab Output

Descriptive Statistics

N	StDev	Variance	95% Upper Bound for σ using Chi-Square
16	3646	13292878	5240

Test

Null hypothesis H_0: σ = 4000
Alternative hypothesis H_1: σ < 4000

Method	Test Statistic	DF	P-Value
Chi-Square	12.46	15	0.356

7.15

Problem 5

a) This is a situation where we are inferring the mean with unknown variance. Thus - a one-sample t-test is appropriate.

Note: try aspects of this by hand for test practice!

Since p-value < 0.05, we reject the H_0 and conclude that $H_1: \mu \neq 12$ is true

Problem 2

Descriptive Statistics

N	Mean	StDev	SE Mean	95% CI for μ
16	10.259	0.999	0.250	(9.727, 10.792)

μ: mean of Problem 2

Test

Null hypothesis H_0: μ = 12
Alternative hypothesis H_1: μ ≠ 12

T-Value	P-Value
-6.97	0.000

b) From Minitab:

$$\boxed{9.73 \leq \mu \leq 10.8 \quad \text{(3 sig. figs)}}$$

Note: try doing this by hand for practice:

Eq. $\bar{x} - t_{\alpha/2, n-1}\left(\frac{s}{\sqrt{n}}\right) \leq \mu \leq \bar{x} + t_{\alpha/2, n-1}\left(\frac{s}{\sqrt{n}}\right)$

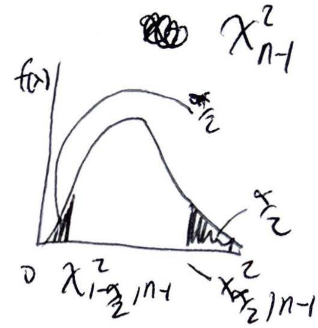
c.)

1) Parameters of interest is standard deviation

2) $H_0: \sigma^2 = 11 = \sigma_0^2$

3) $H_1: \sigma^2 \neq 11 = \sigma_0^2$

4) Test Statistic: $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$



5) Reject H_0 if

$\chi_0^2 > \chi^2_{\alpha/2, n-1}$ or

$\chi_0^2 < \chi^2_{1-\frac{\alpha}{2}, n-1}$

$\chi_{0.025, 15} = 27.49$

$\chi_{0.975, 15} = 6.27$

OR

if P-value < 0.05

a) $\chi_0^2 = \dfrac{15(0.999)^2}{11} = 1.36$

P-value less than 0.005

$\chi_0^2 < \chi_{1-\frac{\alpha}{2}, n-1}$ $\quad 1.36 < 6.27$

$\boxed{\text{So reject } H_0 \text{ and conclude } \sigma^2 \neq 11}$

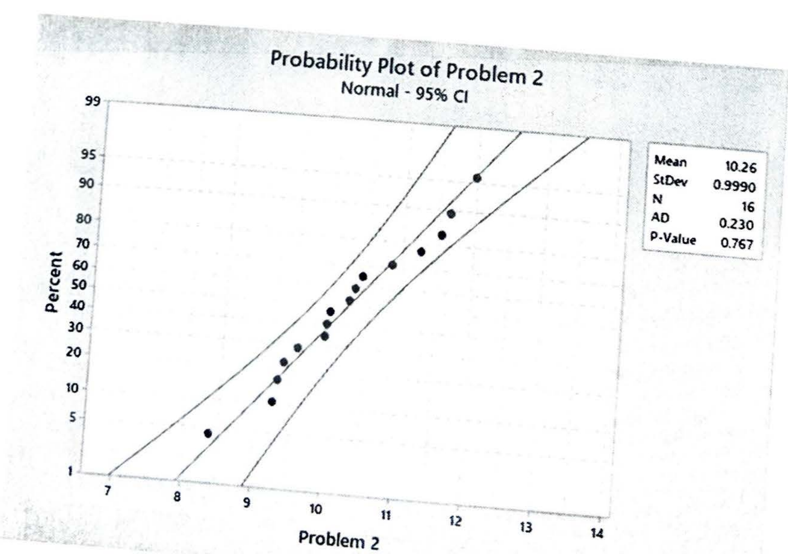
d) A 95% two sided CI:

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2,\,n-1}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi_{1-\alpha/2,\,n-1}}$$

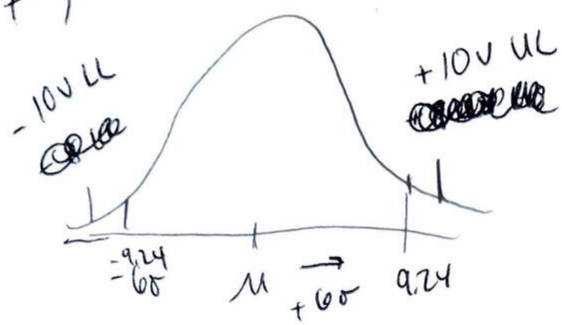
$$\frac{15(0.999)^2}{27.49} \le \sigma^2 \le \frac{15(0.999)^2}{6.27}$$

$$\boxed{0.544 \le \sigma^2 \le 2.38}$$

e) Normality seems appropriate:



f)



Use the CI as an estimate

$\sigma \sim$ $-$ 0.73

1.54

↳ take this as the extream

$\frac{10}{1.54} \sim 6.5\,\sigma$'s away from the mean

So yes this would likely be a 6σ process