ECHE 313 Test 1

Write your name:

Write the following statement, and sign your name acknowledging your agreement:

“"I neither received nor gave external assistance on this exam "

The exam layout is such that all problems add up to 100 points. This point total is meant to guide you in time allocation for this test, will be adjusted such that this exam is 25% of your total class grade. You may use your class materials, class notes, homework, book, and Minitab or Excel to aid in the completion of the exam. The exam is designed to be completed in 1hr and 15 min, but you will have between 10 AM 2/27 to 10 AM 2/28 to complete the exam. All of the necessary tables are provided in the back of your book. Please show your work and all steps necessary to arrive at your answer. Note that it is expected you do this entire test by hand, using a calculator and the tables found in the back of your book. So while you are allowed to use Minitab if you wish to check your answers, it should not replace the work you have done by hand using the tables for your answers. Where applicable, it is best to write out the general formals you want to use first, then write in your subbed in values so it is clear. In addition, for this exam, we can use the rule of thumb that if the sample variance ratio (larger to smaller) is lower than 4, you can assume equal variance for a t-test (if needed). Also, when reporting numbers, keep the same number of decimals as the data you are working with unless there are specific directions on how many sig figs to report.

Good luck!

Premise: You are a process engineer working for a lean six-sigma pharmaceutical company that makes advanced protein-based biologics. <u>Unless otherwise stated, your company uses α=0.01</u>.

1) (12 points) You are finishing up a project where you focused on increasing process cycle efficiency and are ready to hand over the improved process.
    a. (2 points) What step of the DMAIC process are you in?

    Control

    b. (2 points) Name at least one tool that is used in this stage.

    Control chart or transition plan

    c. (4 points) Explain what process cycle efficiency is and how it contributes to lean manufacturing.

    Process cycle efficiency (PCE): $\frac{\text{value added time}}{\text{process time}}$

    Value added time is time spent making the product more valuable.

Thus, PCE is the fraction of time spent making a product more valuable.

Lean manufacturing is a management philosophy aimed toward eliminating waste. PCE can be used as a tool to measure and track wasted time in a process.

d. (4 points) Explain why a slow process is an expensive process with at least 3 specific reasons.

Slow processes are expensive because:

1.) Customers don't like waiting
2.) More handeling = more personnel
3.) More opportunities for damage or loss
4.) Higher inventory
5.) More documentation

2) (39 points) Your next project is to determine if the bioreactor time can be reduced by getting a newer bioreactor system. You are able to demo a new unit and track the reactor time on 5 randomly selected batches in both the old and new bioreactors. You obtain the following data:

Ratio of s^2: $\frac{132}{94} = 1.41 < 4$ ✓ equal variance

The following table shows the time it took in minutes for the batch to reach the designated a amount of bacteria:

$n = 5$

Old Bioreactor Time (min) 1	New Bioreactor Time (min) 2
245	165
270	141
262	151
275	142
260	147

$\bar{x}_1 = \frac{245+270+\dots+260}{5} = 262$

$\bar{x}_2 = \frac{165+\dots+147}{5} = 149$

$s_1^2 = \frac{(245-262)^2+\dots+(260-262)^2}{5-1} = 132$

$s_2^2 = \frac{(165-149)^2+\dots+(147-149)^2}{5-1} = 94$

a. (3 points) List the *name* of the hypothesis test you would use

2-sample t-test, equal variance (pooled t-test)

b. (3 points) Name the parameter of interest in the context of the problem statement

The mean bioreactor time using an old reactor (μ_1) and new (μ_2)

c. (2 points) Write out the correct null hypothesis for this test

$H_0: \mu_1 - \mu_2 = 0$

d. (2 points) Write out the correct alternative hypothesis for this test

$H_1: \mu_1 - \mu_2 \neq 0$ $\quad H_1: \mu_1 - \mu_2 > 0$

Note: 2 sided is correct because could increase or decrease, but if you chose 1-sided this is how you should have done it

$$S_p = \sqrt{S_p^2} = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(5-1)(132) + (5-1)(94)}{5+5-2}} = 10.62$$

e. (7 points) Write the formula of the test statistic and calculate (report 3 sig figs)

$$T_0 = \frac{\bar{x}_1 - \bar{x}_2 - 0}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{262 - 194 - 0}{10.62\sqrt{\frac{1}{5}+\frac{1}{5}}} = \boxed{16.9}$$

f. (4 points) Write out the correct rejection criteria for fixed significance level testing and the critical value (3 sig figs)

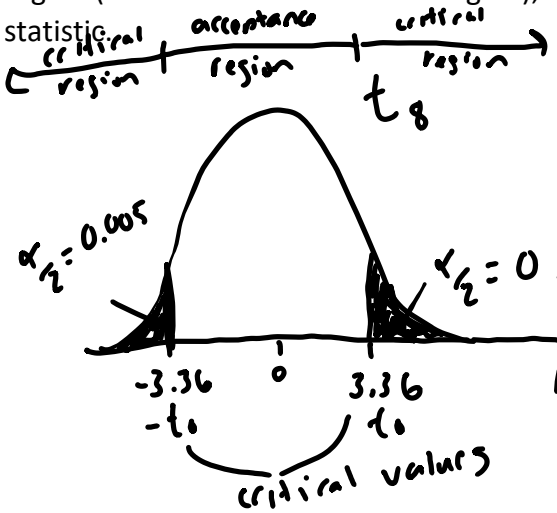
Reject if $t_0 > t_{\alpha/2, n_1+n_2-2}$ or if $t_0 < -t_{\alpha/2, n_1+n_2-2}$

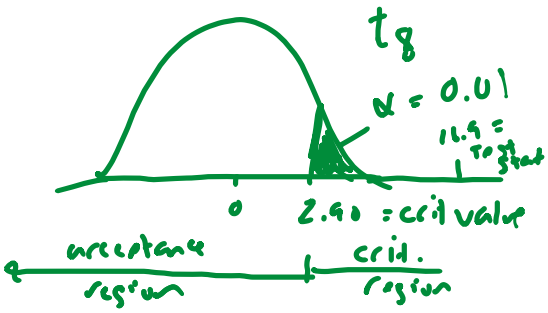
$t_{0.005, 8} = 3.36$

$\boxed{\pm 3.36}$ = critical values

Reject if $t_0 > t_{\alpha, n_1+n_2-2} = t_{0.01, 8} = 2.90$

g. (9 points) Sketch out your reference distribution making sure to fully define the distribution you are sketching on with the correct label, drawing it with the correct shape and where zero is. In addition, label 1) acceptance region, 2) the rejection region (also known as the critical region), 3) α, 4) the critical value and 5) the test statistic.





h. (3 points) State your conclusion and remember to word your conclusion appropriately in the context of the problem statement.

Reject H_0 and accept the H_1 to conclude that the new bio reactor significantly reduced process time.

i. (3 points) Explain what plot you would construct to check if the new bioreactor data are normally distributed (what feature(s) of the plot would you check?)

a normal probability plot, check for linearity

j. (3 points) Define in words what (α) is (what does it represent, what is it's definition?) and explain why it helps us make strong conclusions.

α is the probability of a type I error (rejecting the H_0 when it is true). We can control the value of α, which means rejecting the H_0 is strong because we know the probability we are wrong (α)

3) (32 points) To determine if the new bioreactor is suitable for the process, the team also needs to know if the new bio reactor produces a yield that is greater than 810 mg/L of product. Twelve random samples are taken from batches in the piolet run:

Yield (mg/L)
807
825
816
805
815
820
822
817
823
817
810
812

Sample Ave.	815.75
Sample Standard Deviation:	6.31

a. (2 points) List the *name* of the hypothesis test you would use

one-sample t-test

b. (3 points) Name the parameter of interest in the context of the problem statement

Mean process yeild (μ)

c. (2 points) Write out the correct null hypothesis for this test

$H_0: \mu = 810$ mg/L

d. (2 points) Write out the correct alternative hypothesis for this test

$H_1: M > 810$ mg/L

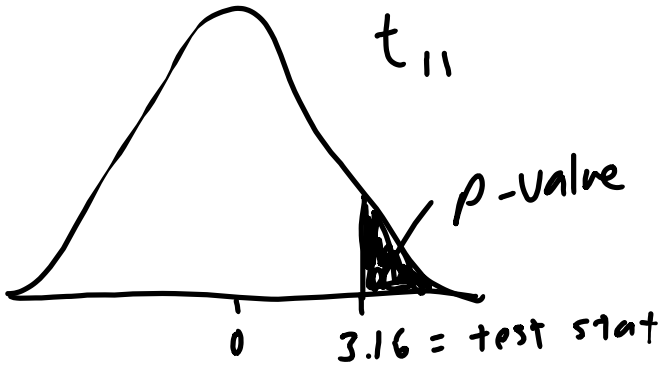
e. (3 points) Write the formula of the test statistic and calculate (report 3 sig figs)

$$T_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{815.75 - 810}{6.31/\sqrt{12}} = \boxed{3.16}$$

f. (2 points) Write out the correct rejection criteria using the p-value method

Reject if p-value $< \alpha = 0.01$

g. (6 points) Sketch out your reference distribution making sure to <u>fully define the distribution you are sketching on with the correct label</u>, drawing it with the <u>correct shape</u> and <u>where zero is</u>. In addition, label 1) the test statistic and 2) the p-value.



h. (2 points) Find the p-value using the tables in the back of your book

$0.0025 < \text{p-value} < 0.005$ ← close to this

i. (2 points) Explain what the p-value is in words (i.e. what is the definition of a p-value?)

The p-value is the probability of obtaining a sample outcome at least as extream as the one obtained. OR you can think of it as indicating the compatability with the H_0

j. (3 points) State your conclusion and remember to word your conclusion appropriately in the context of the problem statement.

Reject H_0 and accept H_1. The process yeild of the new bioreactor is greater than 810 mg/L.

k. (2 points) You calculate your 99% confidence interval and show it to your boss. Explain what the 99% confidence interval really represents generally, in words (Hint: do not just say it means you are 99% confident that the mean lies in the CI range). You do not need to actually calculate the CI to answer this question correctly.

In repeated sampling, a 99% CI will capture the true mean of the population 99/100 times.

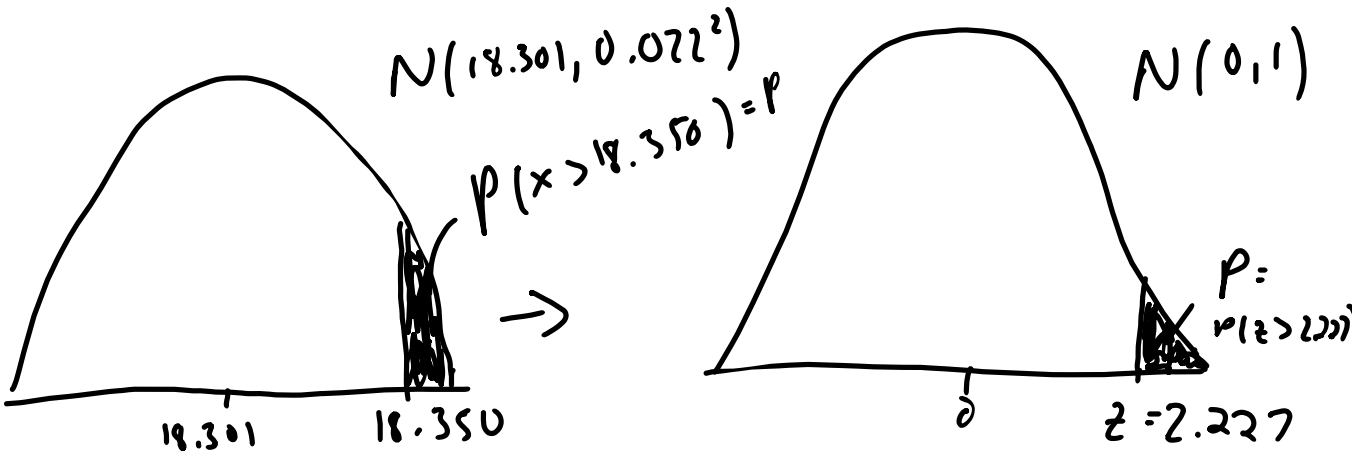
l. (3 points) If the manufacturer told you the standard deviation on the yield coming from these new units is known to be 4.00 mg/L, and you wanted to know if the yield was *different* than 810 mg/L, name the hypothesis test you would conduct, and if your alternative hypothesis would be 1 or 2 sided.

One sample Z test, two sided

4) (17 points) Your boss was so impressed with the projects you have been doing that she asked you to help the manufacturing floor with some questions regarding their packaging unit, which boxes up the biologics with instructional materials. Weight measurements are taken on each and every product automatically, thus it is known that the population mean μ is 18.301 g and the population standard deviation is σ is 0.022 g and is normally distributed. New weight restrictions are being instated and the manufacturing floor wants to know the probability that a product will have a weight above a new limit of 18.350 g.

a. (11 points) A write out the probability equation you wish to solve, including the integral (but you do not need to solve the integral just set it up). Also include a probability distribution sketch with the 1) probability you wish to find shaded in 2) the correct shape, 3) correct label of the distribution, 4) where 18.350g is and 5) where zero is.

$$P(x > 18.350\text{ g}) = \int_{18.350}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$



b. (6 points) Solve the probability problem using the *tables in the back of the book*. Just like in class, sketches can help solve these problems, especially when using the tables. Report and appropriate number of sig figs based on the numbers given in the problem statement.

$$z = \frac{x - \mu}{\sigma} = \frac{18.350 - 18.301}{0.022} = 2.22727$$

$$P(z > 2.227) = 1 - P(z \leq 2.227) = 1 - 0.987 = \boxed{0.013}$$

Bonus (1 point): Overall, what recommendation would you make to your boss regarding the new bioreactor? And what would you say to the manufacturing floor – should they be concerned about product being over the limit?

The new bioreactor produces a yield that is above the threshold value of 810 mg/L (so more product) in less time than the original bioreactor. It should be considered for purchase (a detailed cost analysis is warranted).

1.3% of the product is over the limit, which is a lot! The cost of having product rejected because of weight could be quite high. This would be a concern and should be investigated.