Note: not all the lables are on the diagrams in this version of the solutions (a new version will be posted later!)

see next page be a 6σ process

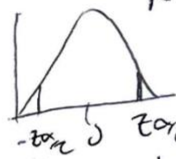
Problem 6

3.) From Minitab: $\bar{x}_1 = 65.22$ $\bar{x}_2 = 68.42$ $n_1 = 10 = n_2$

$\sigma_1 = 3 = \sigma_2$

1. The parameter of interest is difference b/t two population means

Note: p-value also can be used!



$\downarrow$ -1.96 $\quad$ $\downarrow$ 1.96

2. $H_0: \mu_1 - \mu_2 = 0$

3. $H_1: \mu_1 - \mu_2 \neq 0$

4. Test statistic: $z_0 = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$

$z_{\alpha/2} = z_{0.025}$

same as:

$P(z < z_{\alpha/2}) = 0.975$

5. Reject H_0 if $z_0 > z_{\alpha/2}$

or if $z_0 < -z_{\alpha/2}$

6. $z_0 = \dfrac{65.22 - 68.42}{\sqrt{\frac{3^2}{10} + \frac{3^2}{10}}} = -2.385$

$z_0 < -z_{\alpha/2}$

7. $z_0 < -1.96$, so reject null hypothesis and conclude $\mu_1 \neq \mu_2$
the mean active concentrations ~~do~~ depend on choice of catalyst

c) $\bar{X}_1 - \bar{X}_2 - Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \bar{X}_1 - \bar{X}_2 + Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

$65.22 - 68.42 - 1.96\sqrt{\frac{3^2}{10} + \frac{3^2}{10}} \le \mu_1 - \mu_2 \le 65.22 - 68.42 + 1.96\sqrt{\frac{3^2}{10} + \frac{3^2}{10}}$

$-3.20 - 1.96(1.34) \le \mu_1 - \mu_2 \le -3.20 + 1.96(1.34)$

$$\boxed{-5.83 \le \mu_1 - \mu_2 \le -0.57}$$

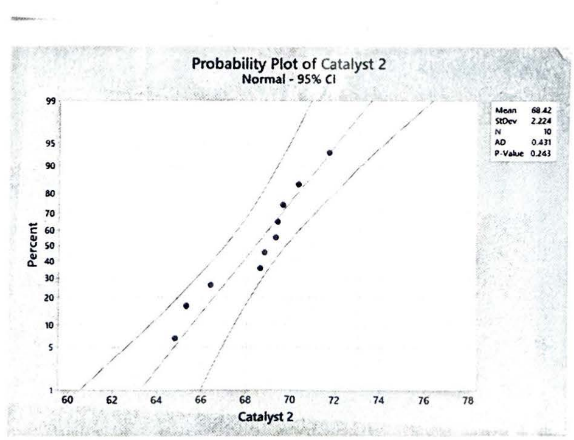
Sig figs b/c of rules of sub/addition

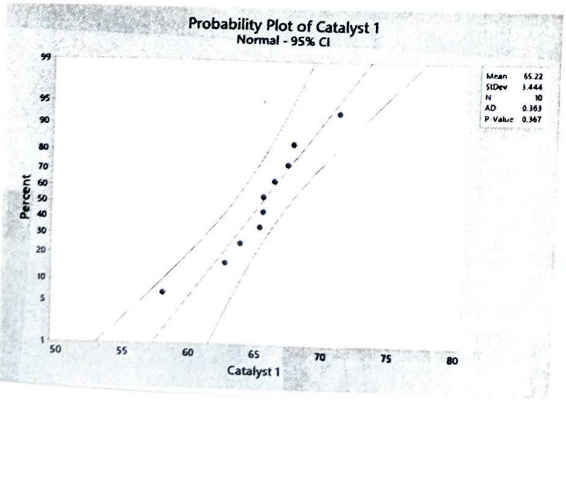
Notice that the range of the difference never crosses 0. So we are 95% confident that $\mu_1 - \mu_2$ isn't 0

Statistics

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
Catalyst 1	10	0	65.22	1.09	3.44	57.90	63.43	65.40	67.30	71.00
Catalyst 2	10	0	68.420	0.703	2.224	64.800	66.125	69.050	69.775	71.700

a) Minitab output

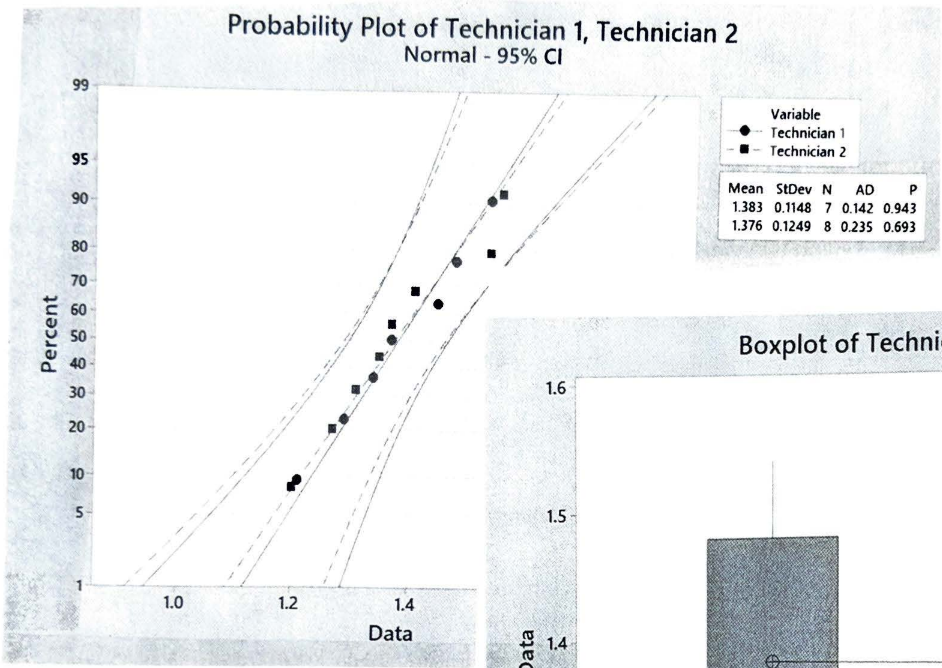


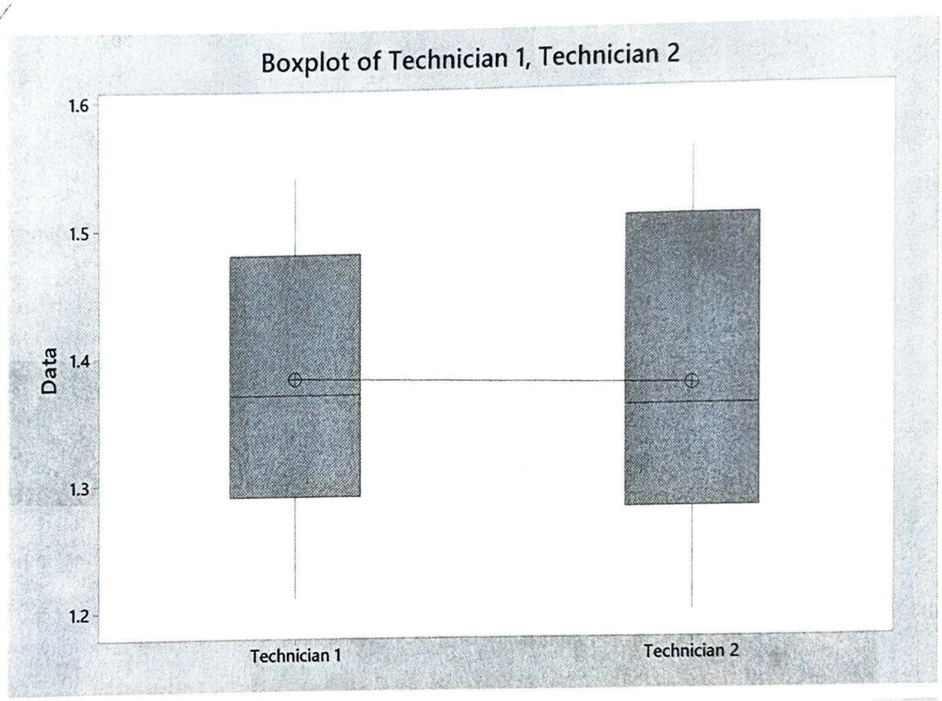


Check assumptions:

The data appear to have ~ similar variances and are normally distributed

Therefore, we can do a t-test with equal variance





a.) $\alpha = 0.05$, Technician 1 : $n = 7$ $\bar{x}_1 = 1.383$, $s_1 = 0.115$

Technician 2 : $n = 8$ $\bar{x}_2 = 1.376$, $s_2 = 0.125$

Step 1) Parameter of interest : mean surface finish measured by two technicians - we are interested in determining if $\mu_1 - \mu_2 = 0$

Step 2) Null hypothesis: $H_0 : \mu_1 - \mu_2 = 0$

Step 3) Alternative hypothesis: $H_1 : \mu_1 - \mu_2 \neq 0$

Step 4) Test statistic: $t_0 = \dfrac{\bar{x}_1 - \bar{x}_2 - 0}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$

Step 5) Rejection Criteria: Reject H_0 if $|t_0| > t_{\alpha/2, n_1+n_2-2}$

Step 6) Computations: $S_p = \sqrt{\dfrac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}} = \sqrt{\dfrac{(7-1)0.115^2+(8-1)(0.125^2)}{7+8-2}}$

$= 0.1204$

$t_0 = \dfrac{1.383-1.376}{0.1204\sqrt{\frac{1}{7}+\frac{1}{8}}} = 0.11$

$t_{\alpha/2, n_1+n_2-2} = t_{0.025,13} = 2.160$ (Apendix IV)

$|t_0| < t_{\alpha/2, n_1+n_2-2}$

Step 7) Conclusions: Because $|t_0| < t_{\alpha/2, n_1+n_2-2}$ we can't reject H_0, and there is not enough evidence to conclude there is a difference between measurements made by technician 1 and 2

Note: you can use P-values as well.

for $t_0 = 0.11$ with 13 dof, p-valu is > 0.40 using the table

$\therefore$ p-value > 0.05 so fail to reject null

b) It means we do not have sufficient evidence to conclude that the technician affects the mean. Because the p-valu is really high, it is likely this is true, however, we must always be careful about failing to reject the null → type II error (β) is often not controlled and so it is a weak conclusion.

If the null had been rejected, this would cause alarm, and we should investigate why the means are different between the two (what are they doing differenty?)

c) $n_1=7$, $\bar{x}_1=1.383$, $s_1=0.115$, $n_2=8$, $\bar{x}_2=1.376$, $s_2=0.125$

$S_p=0.120 \quad \underbrace{t_{\alpha/2, n_1+n_2-2} = t_{0.025,13} = 2.160}_{\text{Found in part a}}$

$$(\bar{x}_1-\bar{x}_2) - t_{\alpha/2, n_1+n_2-2}\, S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \leq \mu_1-\mu_2 \leq (\bar{x}_1-\bar{x}_2) + t_{\alpha/2, n_1+n_2-2}\, S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

Subbing...

$$(1.383-1.376) - 2.1604(0.120)\sqrt{\tfrac{1}{7}+\tfrac{1}{8}} \leq \mu_1-\mu_2 \leq (1.38-1.376) + 2.1604\sqrt{\tfrac{1}{7}+\tfrac{1}{8}}$$

$$-0.127 \leq \mu_1-\mu_2 \leq 0.141$$

Since the CI includes 0, there is not enough evidence to conclude there is a difference in measurements obtained by the two technicians

...initab:

Method

μ_1: mean of Technician 1
μ_2: mean of Technician 2
Difference: $\mu_1 - \mu_2$

Equal variances are assumed for this analysis.

Descriptive Statistics

Sample	N	Mean	StDev	SE Mean
Technician 1	7	1.383	0.115	0.043
Technician 2	8	1.376	0.125	0.044

Estimation for Difference

Difference	Pooled StDev	95% CI for Difference
0.0066	0.1204	(-0.1280, 0.1412)

Test

Null hypothesis $H_0: \mu_1 - \mu_2 = 0$
Alternative hypothesis $H_1: \mu_1 - \mu_2 \neq 0$

T-Value	DF	P-Value
0.11	13	0.917

5. This is a one-sided t-test

Was it done assuming unequal variances?

T_0^* in that case is: $T_0^* = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{54.73 - 58.64}{\sqrt{\dfrac{2.13^2}{15} + \dfrac{5.28^2}{20}}} = -3.00$

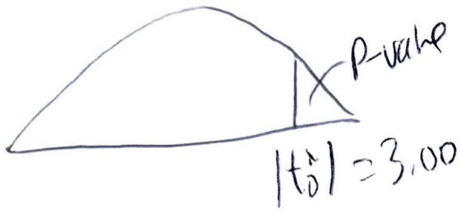
So the DoF are given by:

$$v = \frac{\left[\left(\frac{s_1^2}{n_1}\right) + \left(\frac{s_2^2}{n_2}\right)\right]^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$$

$$v = \frac{\left(\frac{2.13^2}{15} + \frac{5.28^2}{20}\right)^2}{\frac{\left(\frac{2.13^2}{15}\right)^2}{14} + \frac{\left(\frac{5.28^2}{20}\right)^2}{19}} = \boxed{26}$$

P-value : Since $t_0^* = -3.00$, and DoF = 26

appendix gives: $\boxed{\text{Between } 0.005 \text{ and } 0.0025 \text{ for p-value}}$

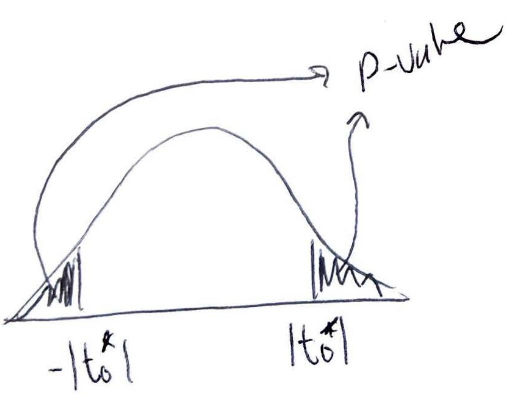


b) Since p-value < α in both cases, we would reject the null hypothesis and conclude that $H_1: \mu_1 - \mu_2 < 0$

c) $H_1: \mu_1 \neq \mu_2$

This would only increase the p-value × 2 making the range between 0.01 and 0.005

Since p-value is still $< \alpha = 0.05$, we would keep the same conclusion,



6) Problem 4.25

$$\bar{D} = -0.000417 = \sum_{i=1}^{n} \frac{D_i}{n} = \frac{D_1 + D_2 \cdots}{12}$$

$$S_D = 0.001311 = \sqrt{\frac{(D_1-\bar{D})^2 + (D_2-\bar{D})^2 \ldots (D_{12}-\bar{D})^2}{12-1}}$$

1. Parameter of interest: the difference in mean measurements made by two devices by the same operator on the same sample

2. Null hypothesis $H_0: \mu_d = 0$

3. Alternative hypothesis $H_1: \mu_d \neq 0$

4. Test statistic $t_0 = \dfrac{\bar{d}}{S_d/\sqrt{n}}$

5 Rejection Criteria: $t_0 > t_{\alpha/2, n-1}$ or $t_0 < -t_{\alpha/2, n-1}$

5. Computations:

$$t_0 = \frac{-0.000417}{\frac{0.001311}{\sqrt{12}}} = -1.10$$

$$t_{\alpha/2, n-1} = t_{.005, 11} = 3.106$$

6.) Conclusions

t_0 is not $> t_{\alpha/2, n-1}$ or $< -t_{\alpha/2, n-1}$

Fail to reject the null hypothesis. There is not enough evidence to say there is a difference in the mean measurements produced by these devices.

mini tab

Problme 6

Descriptive Statistics

Sample	N	Mean	StDev	SE Mean
Micrometer Caliper	12	0.151167	0.000835	0.000241
Vernier Caliper	12	0.151583	0.001621	0.000468

Estimation for Paired Difference

Mean	StDev	SE Mean	99% CI for μ_difference
-0.000417	0.001311	0.000379	(-0.001592, 0.000759)

μ_difference: mean of (Micrometer Caliper - Vernier Caliper)

Test

Null hypothesis H_0: μ_difference = 0

Alternative hypothesis H_1: μ_difference ≠ 0

T-Value	P-Value
-1.10	0.295

Descriptive Statistics

N	Mean	StDev	SE Mean	99% CI for μ
12	-0.000417	0.001311	0.000379	(-0.001592, 0.000759)

μ: mean of Difference

Test

Null hypothesis H_0: $\mu = 0$

Alternative hypothesis H_1: $\mu \neq 0$

T-Value	P-Value
-1.10	0.295