Concepts

Key Terminology

- **Six Sigma:** A data-driven management strategy
- **Quality:** Fitness for use
  - Two aspects:
    - a) Quality of design - high quality?
    - b) Quality of conformity - is the high quality consistent?
  - ∘ Characteristics or "dimensions of quality" that affect fitness for use as a product
    - (i) Performance — performs required job
    - (ii) Reliability — how often will it fail
    - (iii) Durability — lifetime → will it meet it
    - (iv) Serviceability — easy to repair
    - (v) Aesthetics — product's appearance
    - (vi) Features — what the product does
    - (vii) Perceived Quality — Reputation
    - (viii) Conformance to Standards
  - ∘ Producing a Service... list these dimensions of quality
    - (i) Responsiveness — time to reply to requests
    - (ii) Professionalism — Knowledge, timeliness
    - (iii) Attentiveness — Caring, personalized attention
- **Hidden Factory:** The part of the plant the customer doesn't see which deals w/ bad quality
- **Quality Engineering:** Consists of activities to ensure that ① that quality characteristics are at desired levels, and ② variability is at its minimum
  - will call c2qc
  - Sometimes called 'control to quality characteristic'
- **Quality Characteristics:** Important elements of a product/service to the customer
- **Variability:** Describes or quantifies differences within a set of data or parts or c2qc
- **Statistical Methods:** Analysis, interpretation, and presentation of numerical data
- **Specifications:** Desired measurements for quality characteristics. This is your target value
  - Often an upper and lower limit
- **Nonconformity:** Failure to meet spec
- **Defect:** Non conformity that limits use

Variability

- Quality is inversely proportional to variability
  - ∘ Variability will impact all quality dimensions
- Variability leads to greater cost & waste

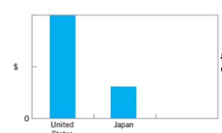


FIGURE 1.1 Warranty costs for transmissions.

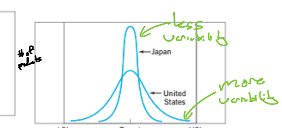


FIGURE 1.2 Distributions of critical dimensions for transmissions.

example to right examines domestic vs. imported car quality

- Why variability leads to more cost for a company

Quality Costs

Prevention Costs	Internal Failure Costs
Quality planning and engineering	Scrap
New products review	Rework
Product/process design	Retest
Process control	Failure analysis
Burn-in ← JR's company 20 yr test	Downtime
Training ← Personel	Yield losses
Quality data acquisition and analysis	Downgrading (off-specing)
Appraisal Costs	**External Failure Costs**
Inspection and test of incoming material	Complaint adjustment
Product inspection and test	Returned product/material
Materials and services consumed	Warranty charges
Maintaining accuracy of test equipment	Liability costs
	Indirect costs

- ∘ Products found in plant outside of specifications results in higher costs and waste
- ∘ Prevention & Appraisal costs are cheaper costs and result in more consistent products and cheaper costs down the line

Internal Failure & External Failure costs >> Prevention & appraisal costs!

Example: Cost of Variance ← Find manufacturing cost per good product

Illustrative Example: We are making 100 mechanical components per day where 75% of the parts conform to specifications, and the other 25% are non-conforming. Of the 25% of nonconforming, 60% can be reworked and the other 40% are scraped. Each part costs $20 to manufacture and an additional $4 to be reworked.

$$\frac{\text{cost}}{\text{good}} = \frac{\text{cost to make (\# made total)} + \text{cost to rework (\# reworked)}}{\text{\# of goods sold}}$$

$$= \frac{\$70(100) + \$4(100)(0.25 \cdot 0.60)}{70 + 15} = \boxed{\$22.89}$$

Now, the Benefit of Improvement ← Find manufacturing cost per good product

Illustrative Example: We improve the previous process such that now 95% of the parts conform to specifications, and the other **5% are non-conforming**. Similar to before, of the 5% of nonconforming, 60% can be reworked and the other 40% are scraped. Each part costs $20 to manufacture and an additional $4 to be reworked.

$$\frac{\text{cost}}{\text{good}} = \frac{\$70(100) + \$4(100)(0.05)(0.60)}{95 + 3}$$

$$= \boxed{\$20.53}$$

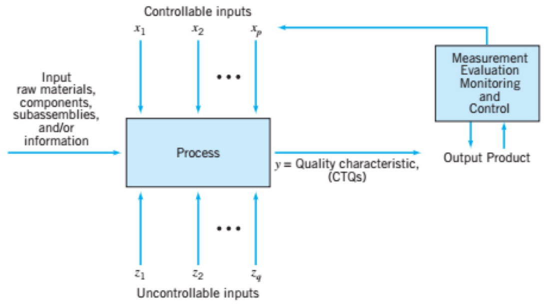
Quality Engineering

Process: A system of inputs and outputs

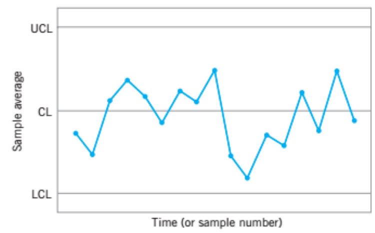
Example, Defining a process

Example: A hospital is trying to increase the quality of drug administration. To do this, it is considering providing patients with bar-coded wristbands to help guide workers. Your team is charged with studying the effects of bar-coding by carefully watching 250 episodes in which drugs are given to patients without bar coding and 250 episodes with bar coding. Every time a drug is administered you will check the amount, if any, of discrepancy between what was supposed to be given and what was given.

- Controllable input variable: barcoding
- Key output variable: right amount of drug



Control Charts: A process monitoring technique which tracks averages in a quality characteristic with time or sample number



- **Centerline:** where the characteristic should be if no abnormal sources of variation
- **Control Limit:** Defined by statistics. Alert the user to unusual variability
  - ↳ Different than spec limits

Designed experiments: discovers key variable that have a significant influence on quality characteristics in a process

Six Sigma

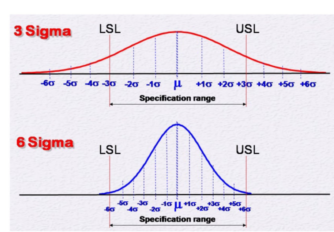
Broader Definition: Statistical based, data driven management approach and continuous quality improvement methodology for eliminating defects in a product, process, or service

↳ Reduces variability to a level where defects are unlikely

Three Elements of Six Sigma

(i) **Quality Planning:** Listening to the voice of your customer
(ii) **Quality Assurance:** Establishing a system to prevent quality issues from arising
(iii) **Quality Control & Improvement:** A set of specific steps and tools to ensure products meet requirements and are continuously improved

Name Meaning: Refers to the quality level target such that sigma (σ, or variance σ^2) should be small enough so that $6\cdot\sigma$ is within the specification limits



Spec. Limit	Percentage Inside Specs	ppm Defective
±1 Sigma	68.27	317300
±2 Sigma	95.45	45500
±3 Sigma	99.73	2700
±4 Sigma	99.9937	63
±5 Sigma	99.999943	0.57
±6 Sigma	99.9999998	0.002

(a) Normal distribution centered at the target (T)

Sigma Quality Level: The Probability that product/service is non-defective

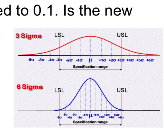
Why is Six Sigma Selected?

Suppose 100 parts: 3σ: $(.9973)^{100} = .7631$ is probability non-defective → ~24% defective
6σ: $(.9999998)^{100} = .99999998$ non-defective → ~.000002% defective

Some companies use other multiples of σ, depending on what required by company/customers

Example of Six Sigma (will be on HW)

Illustrative Example: Let's say you are making a part that has a radius of 10 mm. Specification limits are between 9 (LSL) and 11 (USL) mm. Before you applied quality improvement, the standard deviation (sigma – a measure of variability) was 1 mm, and after you applied improvements, the standard deviation (sigma) decreased to 0.1. Is the new process 6 sigma quality?



$$\frac{|\text{USL or LSL} - \text{target}|}{\sigma\,(\text{standard deviation})} = \frac{|9\,\text{mm} - 10\,\text{mm}|}{1\,\text{mm}} = 1 \quad \text{for old} \quad \text{"}1\sigma \text{ level"}$$

$$= \frac{|9\,\text{mm} - 10\,\text{mm}|}{0.1\,\text{mm}} = 10 \quad \text{for new} \quad \text{"}10\sigma \text{ level"}$$

Lean Manufacturing

Def: Another management philosophy aimed at eliminating waste (e.g., time and materials)

Process Cycle Efficiency (PCE): (Value added time) / (Process time)

↳ **Value added time** = time spent making the product more valuable

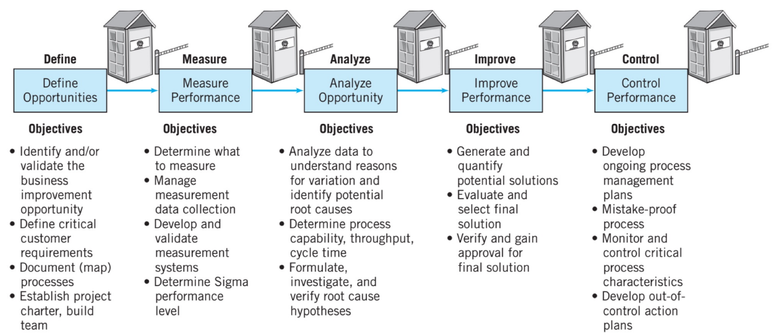
Process Cycle Time (PCT): (Number of items in progress) / (completion rate) ← backlog issues as an example

(number / unit time)

Why is a slow process (low PCE or high PCT) an expensive process?

- Hidden Factory →
  - (i) Customers don't like waiting
  - (ii) More handling = more personel
  - (iii) More opportunity for damage or loss
  - (iv) Inventory has to be higher
  - (v) More documentation

DMAIC Process



Def: A Project-based 5-step process and problem-solving procedure

↳ Define → Measure → Analyze → Improve → Control

Tollgates: where projects are reviewed to ensure on track

5 stages of DMAIC (will be on HW too)

(i) **Define Stage**

- **Main Obj:** Identify a project opportunity and verify that it represents a legitimate value opportunity or breakthrough
  - o **Breakthrough:** Major improvement to a process or product
  - o **Value Opportunity:** The financial opportunity at stake
- **Tools used**
  - o **SIPOC diagram:** High level map of process
    - → Suppliers: Those who provide information, material or other items that are worked on in the process
    - → Input: The actual information or material provided
    - → Process: The set of steps required to do the work
    - → Output: The product delivered to the customer
    - → Customers: who buys the product (internal or external) → Other depts in same company
- **Process Mapping:** Involves creating flow diagrams for systems
- **Value stream mapping:** A variant of process mapping where engineers focus on steps which could be eliminated or simplified to reduce waste

lean concept

- **Value stream:** The min amount of processing steps from raw material to customer

Example: Value Stream Mapping (Lawn Mowing)

{old}

Gather Tools (30 min) → Fuel Mowers (10 min) → Travel (30-60 min, variable) → Unload (10 min) → Talk to Customer (10-20 min, variable) → Care for Lawn (60-120 min, variable) → Bill Customer (10 min) → Gather Tools / Pack Up (10 min) → Drive to Next Customer (30-60 min, variable) → ...

Possible Improvements: Using customer's Lawnmower, storage SOP (leave on truck), prefuel, personel / interns, eliminate / reduce customer interactions, optimize route (graph shortest distance) / logistics

- **Pareto charting:** Plots frequency (# of parts) or cost (time/money) against causes
  - o **Pareto Rule:** 80% of the effects come from 20% of the causes

- Graph Pareto by Hand (Example)

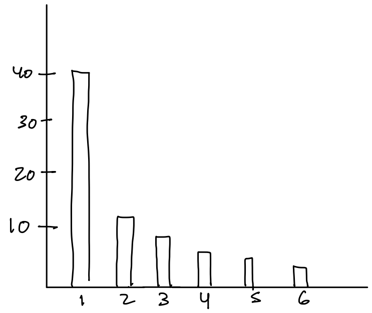
Often in order from largest cost to smallest

Cost or % defects is on y-axis, category is on the x-axis

Monthly Quality-Costs Information for Assembly of Printed Circuit Boards

	Type of Defect	Percentage of Total Defects	Scrap and Rework Costs
1	Insufficient solder	42%	$37,500.00 (52%)
2	Misaligned components	21	12,000.00
3	Defective components	15	8,000.00
4	Missing components	10	5,100.00
5	Cold solder joints	7	5,000.00
6	All other causes	5	4,600.00
	Totals	100%	$72,200.00

1000s of $



Can also do this with Lean concepts

Project Charter: Short document which describes the product.

Example to right, but varies accross companies

Business Case	Opportunity Statement
• This project supports the business quality goals, namely (a) reduce customer resolution cycle time by x% and (b) improve customer satisfaction by y%.	• An opportunity exists to close the gap between our customer expectations and our actual performance by reducing the cycle time of the customer return process.
Goal Statement	**Project Scope**
• Reduce the overall response cycle time for returned product from our customers by x% year to year.	• Overall response cycle time is measured from the receipt of a product return to the time that either the customer has the product replaced or the customer is reimbursed.
Project Plan	**Team**
• Activity — Start — End Define 6/04 6/30 Measure 6/18 7/30 Analyze 7/15 8/30 Improve 8/15 9/30 Control 9/15 10/30 Track Benefits 11/01	• Team Sponsor • Team Leader • Team Members

(ii) Measure Stage

- Objective: Evaluate & understand the current state of the process
- Baseline Performance: Quantify what the current process is performing at
- Measurement Capability: Quantify if the current measurement systems are adequate
- Tools Used: Charts to understand variability
  Histograms, boxplots, run charts, scatter diagrams

(iii) Analyze Stage

- Objective: Use data to begin to determine cause and effect relationships and sources of variability
- Common Causes of Variability: Internal Sources (noise)
- Assignable Causes of variability: External Sources
- Tools Used
  - Control Charts: Used to separate the common and assignable causes of variation
  - Hypothesis Testing & Confidence Intervals: Used to determine if different conditions produce significant results
  - Regression Analysis: Allows models of the process to be built
  - Failure Modes & Effects Analysis (FMEA): A method for identifying and prioritizing sources of variability, errors, or defects in a process or product
    - ↳ For each item, we determine:
      - (i) Likelihood smthg will go wrong | 1 (not likely) - 10
      - (ii) Ability to detect the failure or defect | 1 (likely) - 10
      - (iii) Severity | 1 (not severe) - 10

Risk Priority #: (i) × (ii) × (iii)

Example

1) Severity, 1 (not severe) – 10
2) Probability of occurrence, 1 (not likely)-10
3) Probability of detection, 1 (likely) -10

RPN = A X B X C

RPN = Risk Priority Number

FAILURE MODE & EFFECTS ANALYSIS (FMEA) — Date: 1/1/2018 — Revision: 1.3

Process Name: Left Front Seat Belt Install — Process Number: SBT 445

Failure Mode	A) Severity Rate 1-10 10=Most Severe	B) Probability of Occurence Rate 1-10 10=Highest Probability	C) Probability of Detection Rate 1-10 10=Lowest Probability	Risk Preference Number (RPN) AxBxC
1) Select Wrong Color Seat Belt	5	4	3	60
2) Seat Belt Bolt Not Fully Tightened	9	2	8	144
3) Trim Cover Clip Misaligned	2	3	4	24

(iv) Improve Stage

- Objective: Use creative thinking & info gathered previously to make specific suggestions that will have the desired effect on process. Confirmation is important
- Tools Used
  - Mistake Proofing (example)
  - Optimization
  - Pilot Testing
  - Confirmation Experiments
  - Documentation
    - (i) Solution obtained
    - (ii) Alternatives Considered
    - (iii) Confirmation tests
    - (iv) Risk Analysis

(v) Control Stage

- Objective: Complete all remaining work and hand off the improved process so gains in progress are institutionalized
- Tools Used
  - Control Charts
  - Transition Plan

Stats in Quality Characteristics

Statistics

Sample Mean: Average Value

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

where n is the number of data points in the sample and $x_1, x_2, x_3, \ldots x_n$ are data points

For this data set: 12, 12, 15, 18, 21, 22, 22, 23, 25, 25, 26, 27, 28, 29, 30, 30, 31

$$\bar{x} = \frac{12+12+15+\ldots+31}{17} = \frac{396}{17} = 23$$

Sample Variance: $\{(\text{sample units})^2\}$

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$

Standard Deviation: $s = \sqrt{s^2}$ $\{\text{sample units}\}$

For this data set: 12, 12, 15, 18, 21, 22, 22, 23, 25, 25, 26, 27, 28, 29, 30, 30, 31

$$s^2 = \frac{(12-23)^2 + (12-23)^2 + \ldots + (31-23)^2}{17-1} = 37 \qquad s = \sqrt{37} = 6.08$$

Illustrative Example

Sample 1	Sample 2
$x_1 = 1$	$x_1 = 1$
$x_2 = 3$	$x_2 = 5$
$x_3 = 5$	$x_3 = 9$
$\bar{x} = 3$	$\bar{x} = 5$

Sample 3
$x_1 = 101$
$x_2 = 103$
$x_3 = 105$
$\bar{x} = 103$

Sample 1: $s = 2$
Sample 2: $s = 4$
Sample 3: $s = 2$

Tools to Describe Data Numerically and Visualize Variability

↳ Often easier to arranged sorted (lowest → highest) data

- **Percentiles:** Tell us how much of the data is below a certain point

12	12	15	18	21	22	22	23	25	25	26	27	28	29	30	30	31	← Values
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	← Rank (1-indexed)

- Lower Quartile (Q_1 or 25^{th} perc.): Value with rank $= (n+1)(\frac{1}{4})$
  here: $\frac{17+1}{4}$ = rank of 4.5 → $\frac{18+21}{2} = \boxed{19.5}$
- Middle Quartile (Q_2, or median, or 50^{th} perc.): Value with rank $= (n+1)(\frac{1}{2})$
  here: $\frac{17+1}{2}$ = rank of 9 → $\boxed{25}$
- Upper Quartile (Q_3, or 75^{th} perc.): Value with rank $= (n+1)(\frac{3}{4})$
  here: $(17+1)(\frac{3}{4})$ = rank of 13.5 → $\boxed{28.5}$
- Interquartile Range: $Q_3 - Q_1$
  here: $Q_3 - Q_1 = 28.5 - 19.5 = 9$
- Whisker Limits: Max Limit $= Q_3 + 1.5(Q_3 - Q_1)$
  Min Limit $= Q_1 - 1.5(Q_3 - Q_1)$
  here: Max Limit = 43, Min Limit = 6

- Box Plot Features

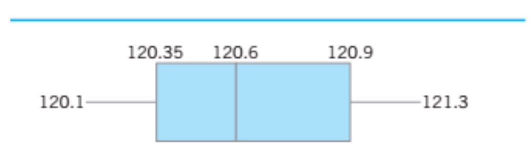
Displays 25^{th}, 50^{th} and 75^{th} quartiles (ends and middle of the box)

- 25^{th} and 75^{th} are the ends of the box
- Median (50^{th} percentile) is a single line through box
- The min and max of data (whiskers)
- Limits of whiskers (check) that the max and min do not go past

(points that fall outside this can be outliers, represented as single data points and the new min or max is the next data point within the limit) – limit formula on next slide

TABLE 3.4
Hole Diameters (in mm) in Wing Leading Edge Ribs

120.5	120.4	120.7
120.9	120.2	121.1
120.3	120.1	120.9
121.3	120.5	120.8



here:

Max= 31

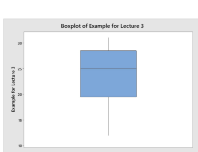
Upper quartile (Q3, 75^{th} percentile) = 28.5

Median is (Q2, 50^{th} percentile): = 25

Lower Quartile (Q1, 25^{th} percentile) = 19.5

min = 12

Whisker Limits:



- Histograms: Plots of frequencies of data occurring vs. the data sorted into bins/intervals
  - ↳ Bins: The range that data will be sorted into. # bins = ceil($\sqrt{n}$) ← round up. bin size = $\frac{\max - \min}{\# \text{ of bins}}$
    i. Need to determine the bin range for continuous data
    ii. Best if bins are the same size
    iii. Rule of thumb: number of bins = square root of number of observations (round up if not a whole number)

Note: for categorical data or discrete data, each value or category is its own bin

For this data set: 12, 12, 15, 18, 21, 22, 22, 23, 25, 25, 26, 27, 28, 29, 30, 30, 31

# of bins = $\sqrt{17} = 4.1 \rightarrow 5$

bin size = $\frac{31-12}{5} = 3.8$

Can be flexible w/ rounding but data must not overlap

Bin 1: min + bin size → 12+3.8 → [12, 15.8) Inclusive below, Exclusive above

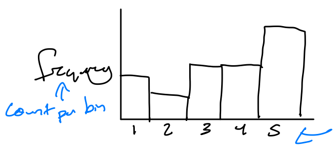
Bin 2: bin 1 end + bin size → 15.8+3.8 → [15.8, 19.6)

Bin 3: [19.6, 23.4)

Bin 4: [23.4, 27.2)

Bin 5: [27.2, 31]

↳ Final bin should be inclusive above



Usually should be bin limits

MINITAB: Graph > histogram > simple > add your column to graph variables
To change bins: double click on histogram bars (twice), select bins tab, change number of intervals to number of bins (Cut point is how we graphed in class)

Significant Digits / Figures: Numbers of digits contributing to measurement resolution

Rules: (i) Non-zero digits always significant
(ii) Any zeros between two significant digits are significant (leading zeros are not)
(iii) Trailing zeros in the decimal portion only are significant (unless indicated by a bar)

Examples:

How many sig. figs in each number?
0.00500 = 3
0.03040 = 4
102$\bar{0}$0 = 4

How would you write this # with the correct sig. figs?
0.899923 (report 2 sig. figs) 0.90
23000.0 (report 3 sig. figs) 23$\bar{0}$00

Will take away points for tons of sig figs! Just pay attention to them!

For your sample means: keep as many significant figures as the number with the lowest significant digits in your data set (same for median and quartiles)

For your standard deviation: report to the last *place* that is significant, but no more

Your Average and SD should have same # of places!

E.g. Sample mean = 4.09 Sample Standard Deviation = 0.00923. What do I report? 4.09 ± 0.01

Probability Distributions Introduction
- Sample: Collection of measurements from a larger source or population
- Population: All of the elements from a group
- Probability Distribution: Mathematical Model that relates value of a variable with its probability of occurrence
- Continuous Distribution: The variable is expressed on a continuous scale
- Discrete Distribution: The variable can only take on certain values
- Writing out Probability Statements
  - Discrete: The probability that x is equal to x_i ?

$$P\{x = x_i\} = p(x_i)$$

  - Continuous: The probability that x is between a and b?

$$P\{a \le x \le b\} = \int_a^b f(x)\,dx$$

- Calculating mean and variance

$$\mu = \begin{cases} \int_{-\infty}^{\infty} x f(x)\,dx, & x \text{ continuous} \\ \sum_{i=1}^{\infty} x_i p(x_i), & x \text{ discrete} \end{cases}$$

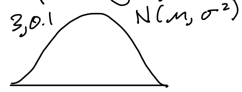
$$\sigma^2 = \begin{cases} \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx, & x \text{ continuous} \\ \sum_{i=1}^{\infty} (x_i-\mu)^2 p(x_i), & x \text{ discrete} \end{cases}$$

- Median: 50th percentile
- Mean: Central Tendency (average)
- Variance: Spread of the distribution

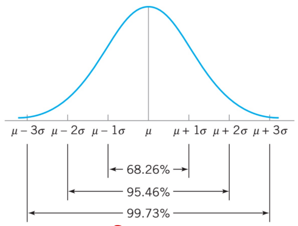
μ, σ^2 (population model params)
$\bar{x}, s^2$ (sample data notation)
Pay attention to this notation

The Normal Distribution

- For a random, normal variable x, the probability distribution of x is: $f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ $\quad -\infty < x < \infty$
  - ↳ Short hand $N(\mu, \sigma^2)$



  - ○ Continuous, symmetric, "bell-shaped"



Holds for any normally distributed variable 'x'

General Formula: $P\{a \le x \le b\} = \int_a^b f(x)\,dx$

In a Probability Equation:

$$P\{(\mu-\sigma) \le x \le (\mu+\sigma)\} = 0.68$$

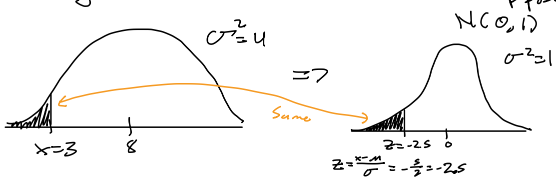
Due to symmetry:

$$P\{x > \mu\} = P\{x < \mu\} = 0.5$$

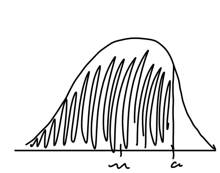
- ○ Getting these percentages: Set $\sigma = 1,\ \mu = 0$ so $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ $\quad \begin{cases} a = \mu - 1\sigma = -1 \\ b = \mu + 1\sigma = 1 \end{cases}$

$$\int_{-1}^{1} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\,dx = \text{(using mathematica) } 0.68$$

- Standardization: $N(\mu, \sigma^2) \Rightarrow N(0,1)$
- Standard Normal Distribution: $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$
- To standardize any random variable x: $z = \frac{x-\mu}{\sigma}$ $\quad$ Z is a standard normal variable
- Drawing out normal distributions: Suppose $N(8, 4)$



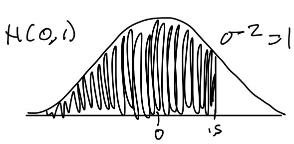
- Cummulative Normal Distribution: The probability that the variable x is less than or equal to some value a



↳ Standardization: $P\{x \le a\} = P\{z \le \frac{a-\mu}{\sigma}\}$ ← Go to Appendix II

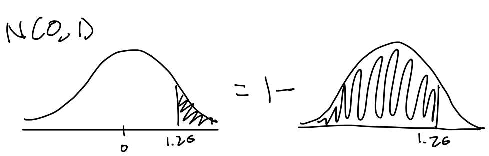
Example: Assume Z is a standard normal random variable, and you want to find the probability that Z is less than or equal to 1.5

$$P\{Z \le 1.5\} = 0.93$$



General Probability Demonstration

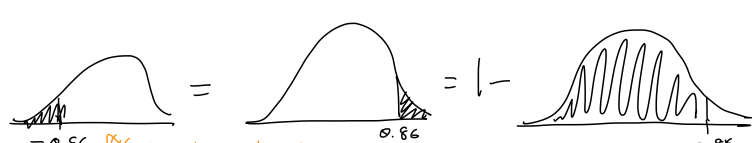
(i) $P(z > 1.26) = 1 - P(z < 1.26)$



$= 1 - 0.89616 = 0.1039$

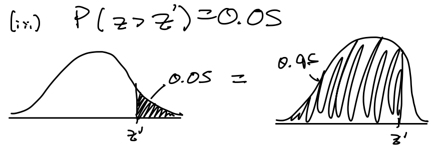
$\boxed{0.10}$

(ii) $P(z < -0.86) = P(z > 0.86) = 1 - P(z < 0.86)$



*(Negatives not in back of book)

(iii) $P(z > z') = 0.05$



Hypothesis Testing

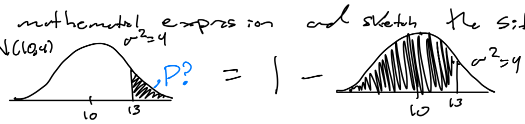
Example: Suppose the current measurements in a strip of wire are assumed to follow a normal distribution with a mean of 10 milliamps and a variance of 4 milliams². What is the probability that the measurement will exceed 13 milliamps?

1) Convert the word problem to mathematical expression and sketch the situation.

$\mu = 10$

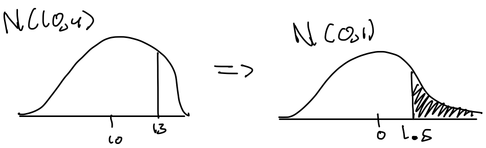
$\sigma^2 = 4$

$P(x \geq 13)$



3) Convert sketch and formula to standard normal (if using tables in back of book) and a cumulative normal distribution (so it is compatible with the table in the book and Minitab)

Minitab: Calc > Probability dist. > Normal



→ $P(z \geq 1.5)$

$z = \frac{x-\mu}{\sigma} = \frac{13-10}{2} = 1.5$

$1 - P(z < 1.5) = \boxed{0.07}$

→ what about the integral?

$$P(z \geq 1.5) = \int_{1.5}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz \quad \text{or} \quad P(x \geq 13) = \int_{13}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Checking for normality

→ How do we know if the normal distribution is a reasonable model for our data?

- Probability Plotting: Graphical method for determining whether sample data conform to a hypothesized distribution based on subjective visual examination.
- Constructing a normal prob. plot by hand: using

12, 12, 15, 18, 21, 22, 22, 23, 25, 25, 26, 27, 28, 29, 30, 30, 31 ← Data

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 ← Rank(j)

How-to (Steps)

1) Ranking data and ordering it ($x_1, x_2 \ldots x_j \ldots x_n$), j= the ranked value, n = total number of values
2) Calculating cumulative frequencies (j-0.5)/n for each data point

↳ tells us the % of data that lie below a certain ranked value

$3\,(n=10) = \frac{3-.5}{10} = 0.25$

3) Find a zj value for each rank using the cumulative frequency value is the probability

Find z_j s.t. $P(z < z_j) = \frac{j-0.5}{n}$. Tells us the value of z_j if the data are normally distributed

4) Plot the actual data (x-axis) vs. corresponding zj (y-axis)

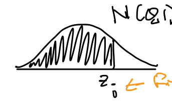
z_j ↑ data →

Normal Probability Plots in Minitab – Minitab: Graph > probability plot > in graph variables, choose column with desired data

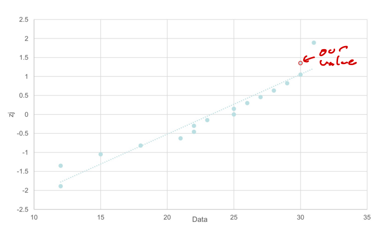
Note: each value is plotted versus the cumulative frequency with the y-axis is transformed so that the fitted distribution forms a straight line if normally distributed.

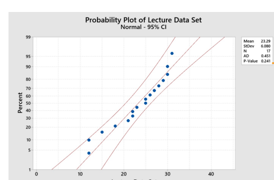
C. Freq $\left(\frac{j-0.5}{n}\right)$

For rank 16: $\frac{16-0.5}{17} = 0.91 \rightarrow z_j$: $P(z < z_j) = 0.91$



=> $z = 1.35$





P-value < α means not normal

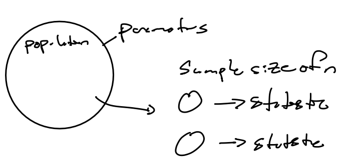
In literature,

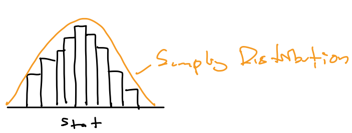
α = 0.05

or α = 0.01

Statistics Basically

- Statistical Inference: Drawing conclusions about a population based on a sample from that population (of them selected randomly)
- Statistics: Any function of sample data that doesn't contain unknown parameters
- Sampling Distributions: The probability distribution of a statistic





Suppose x is a normally distributed random variable w/ given μ and σ^2

→ If $x_1, x_2, \ldots x_n$ is a random sample of size n, the distribution of the sample mean is:

Normally distributed with a mean of μ and a variance of $\frac{\sigma^2}{n}$

$\mu_{\bar{x}} = \mu$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

$N\left(\mu, \frac{\sigma^2}{n}\right)$ shorthand

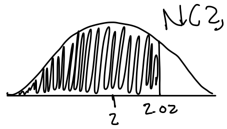
- **Central Limit Theorem:** Implies that the sample mean will be approximately normally distributed for large sample sizes, regardless of the distribution you are sampling from.
  - → "Large" Sample size: some suggest $n > 20$ (non-normal), $n = 3-4$ (close to normal)
- **Example**

Electrodes are manufactured with a population mean thickness of 2.0 mm and a standard deviation of 0.048 mm. Find the probability that a random sample of n = 16 electrodes will have a sample mean diameter less than or equal to 2.02 mm? (keep 2 sig figs)

↳ Central Limit Theorem Applies

$\bar{x} \Rightarrow N(\mu, \frac{\sigma^2}{n})$ $\quad n = 16, \mu = 2.0\,m, \sigma = 0.048\,mm, \sigma^2 = 0.0023$

$P(\bar{x} \leq 2.02)$ $\quad N(2, \frac{0.0023}{16} = 0.00014)$



$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$ New! Formulas

$\Rightarrow$ $N(0,1)$ $\quad z = 1.66$

$P(z \leq 1.66) = 0.95$

Hypothesis Testing

- **Parameters:** what probability distributions are described with
  - e.g. normal dist. is described with μ, σ
- **Statistical Hypothesis:** A statement about values of parameters of a probability distribution
- **Null Hypothesis:** States no statistical difference exists (H_0)
- **Alternative Hypothesis:** Contradicts to null hypothesis (H_1)

Example: Suppose we think the mean inside diameter of bearing is 1.500 in

$H_0: \mu = 1.500$ in $\quad H_1: \mu \neq 1.500$ two sided

$\begin{cases} \mu < 1.500 \\ \mu > 1.500 \end{cases}$ one-sided

- **Two outcomes of a hypothesis test:**
  - (i) Reject H_0 or (ii) Fail to reject H_0
- **Test Statistic:** Used to test a hypothesis and is computed from a random sample of the population
- **Reference Distribution (Null distribution):** The sampling distribution of the test statistic assuming that H_0 (the null hypothesis) is true
- **Critical Region:** Sets values of the test statistic that lead to the rejection of the null hypothesis
  - ↳ Can either fail to reject or reject H_0, critical region defines these cutoffs
- **Two types of errors**
  - (i) Type I errors: If the null hypothesis is rejected when it is true
    - ↳ Defined: $\alpha = P\{\text{type I error}\} = P\{\text{reject } H_0 \mid H_0 \text{ is true}\}$ This is specified
  - (ii) Type II errors: If the null hypothesis is not rejected when it is false
    - ↳ Defined: $\beta = P\{\text{type II errors}\} = P\{\text{fail to reject } H_0 \mid H_0 \text{ is false}\}$ Hard to control
  - ○ Type II errors are uncontrolled & unknown, so we try to minimize it / avoid it
- **Conclusions:** Must be worded very clearly
  - ○ Rejection of Null: A strong conclusion because we can control α to be small
  - ○ Failing to Reject: Weak conclusion because it is hard to control β

The Claim is the	The Decision is	The Conclusion is
Null Hypothesis	Fail to Reject the Null	The evidence is ***not sufficient*** to reject the claim.
	Reject the Null	The evidence is ***sufficient*** to reject the claim.
Alternative Hypothesis	Fail to Reject the Null	There is ***insufficient*** evidence to support the claim.
	Reject the Null	There is ***sufficient*** evidence to support the claim.

We never say accept the null hypothesis

One sample Z-test ← Inference on the Mean of a Population, Variance Known

Suppose x is a random variable with unknown mean μ and a known variance σ^2 and we wish to test the hypothesis that the population mean is equal to a standard value (μ_o).

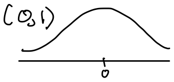
These are defined

→ Null Hypothesis: $\mu = \mu_0$

→ Alternative Hypothesis: $\mu \neq \mu_0$

→ Test Statistic: $Z_0 = \dfrac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$

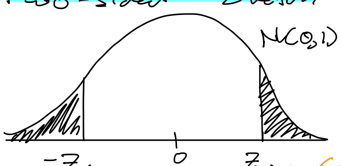
→ Reference Distribution: N(0,1)



→ Rejection Criterion (critical region): Reject H_0 if $|z_0| > Z_{\alpha/2}$

Assume 0.05 unless otherwise stated

◦ Two-sided Sketch



$-Z_{\alpha/2}$ 0 $Z_{\alpha/2}$ ← Critical Value

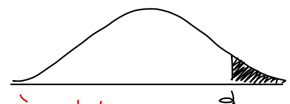
← 'acceptance' region → critical region

critical region: $Z_0 > Z_{\alpha/2}$ or $Z_0 < Z_{\alpha/2}$

◦ One-sided Test

$H_0: \mu = \mu_0$

$H_1: \mu > \mu_0$



α

'acceptance' region → critical region

General Procedure (for this class!)

(i) Parameter of Interest: From the problem context, identify known/unknown parameter of interest, the test you're going to do, and identify type of problem.

(ii) Null Hypothesis: State the null hypothesis H_0

(iii) Alternative Hypothesis: State the alternative hypothesis H_a

(iv) Test Statistic: Determine an appropriate test statistic & reference distribution

(v) Reject H_0 if: State rejection criterion for the null hypothesis. Should be drawn on plot w/ test statistic

(vi) Computations: Compute remaining sample quantities, substitute these into the equation for the test statistic, and compute that value. Compare to rejection criteria

(vii) Draw Conclusions: Decide whether or not H_0 should be rejected and report that in the problem context. Phrase appropriately!

Example

Example: You are testing the response time of a distributed computer system. The system manager wants to know whether the mean response time to a specific command exceeds 75 ms. Previous experience has determined the standard deviation is 8 ms. Use α = 0.05. A test is conducted executing the command 25 times, and the mean response time is 79.25 ms.

1) Parameter of Interest: μ, inferring mean of population, known variance, mean response time

↳ Test: One-sample z-test

2) $H_0: \mu = 75$ ms

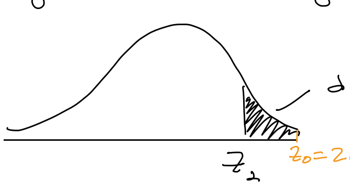
3) $H_1: \mu > 75$ ms (one-sided)

4) Test Statistic: $Z_0 = \dfrac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \dfrac{79.25 - 75}{\frac{8}{\sqrt{25}}} = 2.66$

Reference Dist: N(0,1)

5) Rejection criterion: Reject if $Z_0 > Z_\alpha = Z_{0.05}$

6)



'acceptance' region → critical region

$P(Z < Z_\alpha) = 0.95$

$Z_\alpha = 1.65$ critical value

7) Conclusion: $Z_0 > Z_\alpha$

Reject the null Hypothesis. Conclude that the mean response time exceeds 75 ms.

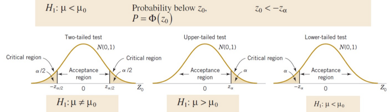
Summary of Z-tests

Testing Hypotheses on the Mean, Variance Known (Z-Tests)

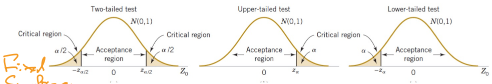
Null hypothesis: $H_0: \mu = \mu_0$

Test statistic: $Z_0 = \dfrac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$

Alternative Hypotheses	P-Value	Rejection Criterion for Fixed-Level Tests
$H_1: \mu \neq \mu_0$	Probability above $\lvert z_0 \rvert$ and probability below $-\lvert z_0 \rvert$, $P = 2\left[1 - \Phi(\lvert z_0 \rvert)\right]$	$z_0 > z_{\alpha/2}$ or $z_0 < -z_{\alpha/2}$
$H_1: \mu > \mu_0$	Probability above z_0, $P = 1 - \Phi(z_0)$	$z_0 > z_\alpha$
$H_1: \mu < \mu_0$	Probability below z_0, $P = \Phi(z_0)$	$z_0 < -z_\alpha$

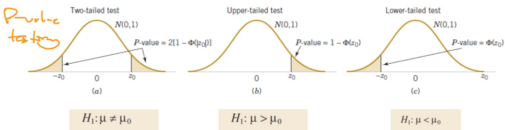


Fixed Significance Level testing



Critical Values Determined by H1 and α

P-value testing



$\Phi(z_0) = P(z < z_0)$

P-values determined by H1 and Test Statistic

Hypothesis Testing Involves

- Making statements about population probability distribution parameters (e.g. mean or variance)
- Testing the statements using a test statistic – calculated from a randomly selected sample
- Determining criteria for rejection of a null hypothesis - defined by Type I error level and H_1
- Remembering when making conclusions that the test is designed such that rejection of null is strong, not rejecting null – not so strong
- General: so many different hypothesis tests!!

P-values

P-value: The probability of obtaining a sample outcome at least as extreme as the one obtained given the value stated if the null hypothesis is true

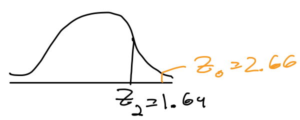
=> Reporting the compatibility of the data with the null hypothesis

Rejection Criterion: Reject H_0 if the p-value < α

Back to our example, but now with p-values!

The response time of a distributed computer system is an CTQ. The system manager wants to know whether the mean response time to a specific command exceeds 75 ms. Previous experience has determined the standard deviation is 8.0 ms. A test is conducted executing the command 25 times, and the mean response time is 79.25 ms. Compute the P-value appropriate for this situation.

Rejection Criterion: Reject H_0 if p-value < α



$$\text{P-value} = 1 - \Phi(z_0)$$
$$= 1 - P\{z < z_0\}$$

Z-test Assumes

- Cts data
- Normally dist. Data
- Random Samples
- Population Std dev is known

Confidence Intervals

The interval between two statistics that includes the true value of that parameter with some probability

→ 100(1−α)% confidence interval

$\mu: P(L \leq \mu \leq U) = 1 - \alpha$ where the statistics $\begin{cases} L = \text{Lower confidence Limit} \\ U = \text{Upper confidence Limit} \end{cases}$

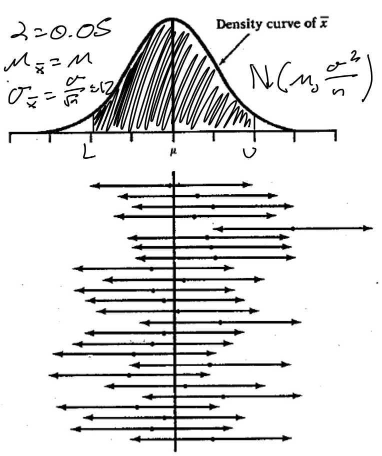
Example

$\mu = 90, \sigma = 36, n = 9$

$\alpha = 0.05$

$\mu_{\bar{x}} = \mu$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = 12$



Find L and U s.t.

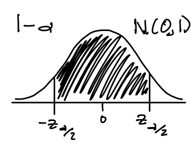
$P(L \leq 90 \leq \mu) = 0.95$

$\rightarrow L = 66.5, U = 113.5$

Assuming L, U are equidistant from the sample mean

→ L, U are 23.5 units from μ

Generally



$$P(L \leq \mu \leq U) = 1 - \alpha$$

$$P\left(-z_{\frac{\alpha}{2}} \leq \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \leq z_{\frac{\alpha}{2}}\right)$$

$$\rightarrow P\left(\underbrace{\bar{X} - z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}}_{L} \leq \mu \leq \underbrace{\bar{X} + z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}}_{U}\right) = 1 - \alpha$$

U includes infinity

One sided: $\mu \leq \bar{X} + z_\alpha \frac{\sigma}{\sqrt{n}}$ or $\bar{X} - z_\alpha \frac{\sigma}{\sqrt{n}} \leq \mu$

Recall

- $z_{\frac{\alpha}{2}}$ is value of the N(0,1) s.t. $P(z \geq z_{\frac{\alpha}{2}}) = \frac{\alpha}{2}$
- z_α is value of the N(0,1) s.t. $P(z \geq z_\alpha) = \alpha$

Actual Example/Application

We pull a sample of 20 electrodes from an assembly line of a process with known variance in electrode thickness to be 0.64 mm². The average thickness of the sample is 7.9 mm. Construct a 95% confidence interval.

$$P\left\{\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right\} = 1-\alpha$$

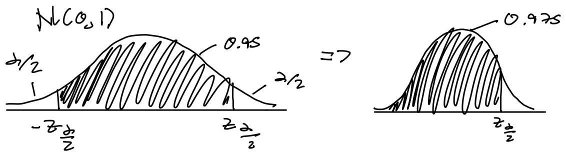
$n = 20$

$\sigma^2 = 0.64\ mm^2 \to \sigma = 0.8$

$\bar{x} = 7.9\ mm,\ \alpha = 0.05$

$L = 7.9 - 1.96\frac{0.8}{\sqrt{20}} = 7.5$

$U = 7.9 + 1.96\frac{0.8}{\sqrt{20}} = 8.3$



$z_{\alpha/2} = 1.96$ from tables

~~Often our result would be said like this:~~

~~*"We are 95% confident that the mean of the population is between 7.5 and 8.3*~~

Not this!

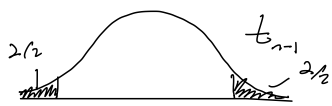
Perhaps a better description would be:

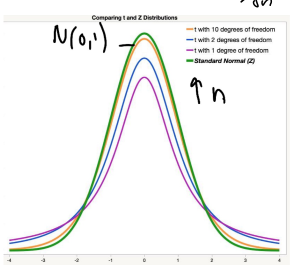
"In repeated sampling, this method will produce intervals that capture the true population mean about 95% of the time."

Use this! (might be on an exam!!)

One sample t-test

- Inference on the mean distribution, unknown variance.
- Suppose x is a normal random variable with unknown mean μ and unknown variance σ^2
- Null Hypothesis: $\mu = \mu_0$
- Alternative Hypothesis: $\mu \ne \mu_0$ Two-sided
- Test Statistic: $T_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$
- Reference distribution
  - → t-distribution w/ n-1 degrees of freedom
- Rejection Criterion: $|t_0| > t_{\alpha/2, n-1}$





For the t-distribution:

- Is a a continuous probability distribution of the z-score when the estimated standard deviation is used in the denominator rather than the true standard deviation.
- It is a type of normal distribution, except it accounts for the fact that as sample size increases, variability decreases.
- The t-distribution, like the normal distribution, is bell-shaped and symmetric about zero, but it has heavier tails, which means it tends to produce values that fall farther from its mean.
- As the sample size increases, the *t*-distribution becomes more similar to a normal distribution. *The shape depends on n!!* And this is why you MUST write down n each time you draw out a t-distribution.

Summary

Testing Hypotheses on the Mean of a Normal Distribution, Variance Unknown

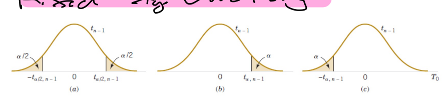
Null hypothesis: $H_0: \mu = \mu_0$

Test statistic: $T_0 = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$

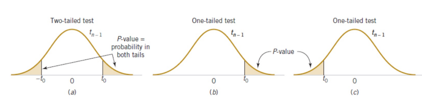
t dist with n-1 DOF

Alternative Hypotheses	P-Value	Rejection Criterion for Fixed-Level Tests
$H_1: \mu \ne \mu_0$	Probability above $\lvert t_0\rvert$ and probability below $-\lvert t_0\rvert$	$t_0 > t_{\alpha/2,n-1}$ or $t_0 < -t_{\alpha/2,n-1}$
$H_1: \mu > \mu_0$	Probability above t_0	$t_0 > t_{\alpha,n-1}$
$H_1: \mu < \mu_0$	Probability below t_0	$t_0 < -t_{\alpha,n-1}$

Fixed Sig Value Testing



P-Value



t-test Assumes

- Data are cts.
- Data are normally distributed
- Data are random samples

Example

Inference on the mean of a distribution, unknown variance

Rubber can be added to asphalt to reduce road noise. The viscosity (cP) of 15 specimens of asphalt is measured. To be suitable for use, the mean should be equal to 3200 cP. Test the hypothesis using alpha=0.05.

cP
3193
3124
3153
3145
3093
3466
3355
3079
3182
3227
3256
3332
3204
3282
3170

Use this data

n=15

1) Inference on mean, unknown variance, one-sample
   → One-sample t-test

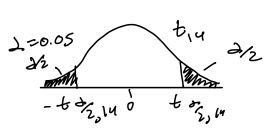
2) $H_0: \mu = 3200\ cP = \mu_0$ 3) $H_1: \mu \ne 3200\ cP$

4) $T_0 = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$, $\bar{x} = \frac{3193 + 3124 + \ldots + 3170}{15} = 3217$

Plug & Chug $= 0.64$

$s = \sqrt{\frac{(3193-3217)^2 + \ldots + (3170-3217)^2}{n-1}} = 106$

5) Reject if $\left\{\begin{array}{l} t_0 > t_{0.025,14} \\ t_0 < -t_{.025,14} \end{array}\right.$



$t_{\alpha/2} = 2.145$

7) Conclusion

(Opt 1) The evidence is not sufficient to reject the claim that the mean is equal to 3200cP

(Opt 2) There's insufficient evidence to support the mean not being equal to 3200cP.

CI for One-sample t-test

Suppose x is a normal random variable with unknown mean μ and unknown variance σ^2. From a sample of n observations, the sample mean $\bar{x}$ and sample variance s^2 are computed. Then:

$$\bar{x} - t_{\alpha/2,\, n-1}\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + t_{\alpha/2,\, n-1}\frac{s}{\sqrt{n}}$$ **Is a 100 (1-alpha)% two-sided CI**

where $t_{\alpha/2,\, n-1}$ is the value of the t distribution with n-1 degrees of freedom such that $P(t_{n-1} \ge t_{\alpha/2,\, n-1}) = \alpha/2$

$$-\infty \le \mu \le \bar{x} + t_{\alpha,n-1}\frac{s}{\sqrt{n}}$$ **Is a 100(1-alpha)% upper confidence bound**

where $t_{\alpha,\, n-1}$ is the percentage point of the t distribution with n-1 degrees of freedom such that $P(t_{n-1} \ge t_{\alpha,\, n-1}) = \alpha$

$$\bar{x} - t_{\alpha,n-1}\frac{s}{\sqrt{n}} \le \mu \le \infty$$ **Is a 100(1-alpha)% lower confidence bound**

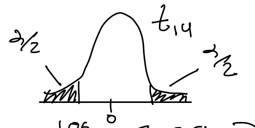
where $t_{\alpha,\, n-1}$ is the percentage point of the t distribution with n-1 degrees of freedom such that $P(t_{n-1} \ge t_{\alpha,\, n-1}) = \alpha$

→ For our previous example

$\bar{x} = 3217$

$s = 106$

$n = 15$



$t_{0.025,14} = 2.145$

$L = 3217 - 2.145\frac{106}{\sqrt{15}} = 3158$

$U = 3217 + 2.145\frac{106}{\sqrt{15}} = 3276$

$3158 \le \mu \le 3276$

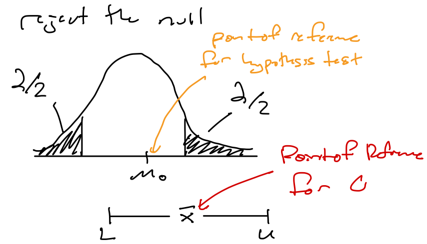
Minitab: Stat>Basic Stat>1-sample t-test, samples in columns (drop down), double click data, check hypothesis test select type and in options select alpha value

CI Relationship w/ Hypothesis Testing

- Both do make interfaces about a population parameter (but in different ways)
- A CI provides a range of values likely containing the true population parameter based on a set of data
- A hypothesis test determines whether a specific claim about the population parameter is supported by the data
- Difference in emphasis of the point of reference

Illustration

If the value in your Ho is in your CI, you likely cannot reject the null



Inference on the variance of Normal Dist. (1-variance test)

Suppose we wish to test the hypothesis that the variance of a normal population (σ^2) equals a specified value, say σ_0^2, or that the standard deviation of the population (σ) is equal to σ_0. Let $x_1, x_2, \ldots x_n$ be a random sample of n observations from this population to test:

Useful for making conclusions about the variability of your process

$H_0: \sigma^2 = \sigma_0^2$

$H_a: \sigma^2 \neq \sigma_0^2$

Test statistic: $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$

s = sample variance with n observations

Reference Distribution: Chi-Squared Distribution

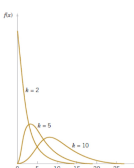
Which Hypothesis should I do?

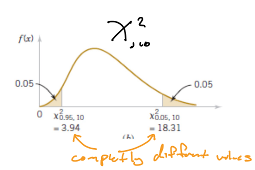
Test	# of Populations	Variance Known?	Parameter of interest?	Reference Dist.
One-sample Z-test	1	Yes	Mean	Standard normal
Two-sample Z-test	2	Yes	Difference between means	Standard normal
One-sample t-test	1	No	Mean	t-distribution
Two-sample (pooled) t-test (equal variance)	2	No	Means of two populations	t-distribution
Two-sample t-test (unequal variance)	2	No	Means of two populations	t-distribution
Paired t-test	2 (simplifies to 1)	No	Difference between paired means	t-distribution
1 Variance test	1	No	Variance of one population	chi-squared distribution

If both mean and variance of the population are known, its a probability problem

Chi-squared Distribution

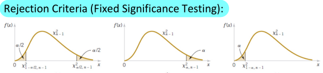
- χ_0^2 has a chi-squared distribution if the null is true
- Key Factors:
  - (i) Depends on n-1 degrees of freedom
  - (ii) Not symmetric
  - (iii) Not centered @ 0
  - (iv) Total Area under curve is still 1!





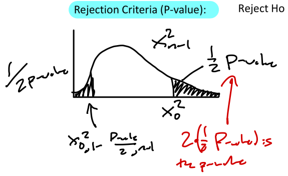
Rejections

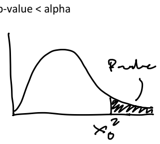
Rejection Criteria (Fixed Significance Testing):



$H_1: \sigma^2 \neq \sigma_0^2$	$H_1: \sigma^2 > \sigma_0^2$	$H_1: \sigma^2 < \sigma_0^2$
Reject Ho if:		
$\chi_0^2 > \chi^2_{\alpha/2,n-1}$ or $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$	$\chi_0^2 > \chi^2_{\alpha,n-1}$	$\chi_0^2 < \chi^2_{1-\alpha,n-1}$

Rejection Criteria (P-value): Reject Ho if p-value < alpha





Summary

Null hypothesis: $H_0: \sigma^2 = \sigma_0^2$

Test statistic: $\chi_0^2 = \dfrac{(n-1)S^2}{\sigma_0^2}$

Ref. Dist: Chi-Square

Alternative Hypothesis	Rejection Criteria
$H_1: \sigma^2 \neq \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha/2,n-1}$ or $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$
$H_1: \sigma^2 > \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha,n-1}$
$H_1: \sigma^2 < \sigma_0^2$	$\chi_0^2 < \chi^2_{1-\alpha,n-1}$

Check Normality! It's sensitive to this

Example

Inference on the variance of a distribution

Rubber can be added to asphalt to reduce road noise. The viscosity (cP) of 15 specimens of asphalt is measured. s is calculated to be 106. Let's say we are interested in knowing if the variance less than 30,000.

Use $\alpha = 0.05$

Data
3193
3124
3153
3145
3093
3466
3355
3079
3182
3227
3256
3332
3204
3282
3170

1) Parameter of interest: σ^2 → 1-variance test

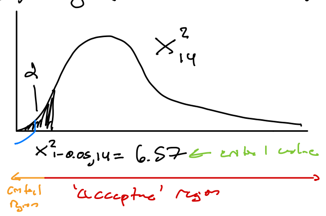
2) $H_0: \sigma_0^2 = 30{,}000\ cP^2$

3) $H_a: \sigma_0^2 < 30{,}000\ cP^2$

4) $$\chi_0^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(15-1)(106)^2}{30{,}000} = 5.2$$

Reference Distribution: χ^2_{14}

5) Rejection Criteria: Reject if $\chi_0^2 < \chi^2_{1-\alpha, n-1}$



Fixed-Significance Level

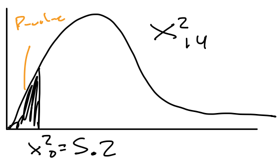
6) Computations: $5.2 < 6.57$

7) Conclusion: Reject H_0. We have evidence to reject the claim that the variance is 30,000 cP, having evidence that the viscosity test variance is less than 30,000 cP.

We accept H_1. ← Can accept H_1 but not accept H_0

p-value

Reject if: p-value $< \alpha$



5.2 lands between 0.990 and 0.975

→ p-value is between 1-tail

$(0.01 < p < 0.025)$ → both $< \alpha$ so reject H_0!

How do I do this in Minitab? Stat > basic stat > 1 variance

Confidence Intervals for Inference on the variance (χ^2)

Suppose x is a normal random variable with unknown mean μ and unknown variance σ². From a sample of n observations, the sample mean x̅ and sample variance s² are computed. Then:

$(100-\alpha)\%$ two-sided CI

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}$$

Where $X^2_{\alpha/2, n-1}$ denotes the percentage point of the chi-square distribution such that $P(X^2_{n-1} \ge X^2_{\alpha/2, n-1}) = \alpha/2$ and $X^2_{1-\alpha/2, n-1}$ denotes the percentage point of the chi-square distribution such that $P(X^2_{n-1} \ge X^2_{1-\alpha/2, n-1}) = 1-\alpha/2$

$(100-\alpha)\%$ upper bound

$$\sigma^2 \le \frac{(n-1)s^2}{\chi^2_{1-\alpha,n-1}}$$

Where $X^2_{1-\alpha, n-1}$ denotes the percentage point of the chi-square distribution such that $P(X^2_{n-1} \ge X^2_{1-\alpha, n-1}) = 1-\alpha$

$(100-\alpha)\%$ lower bound

$$\frac{(n-1)s^2}{\chi^2_{\alpha,n-1}} \le \sigma^2$$

Where $X^2_{\alpha, n-1}$ denotes the percentage point of the chi-square distribution such that $P(X^2_{n-1} \ge X^2_{\alpha, n-1}) = \alpha$

For our example (upper bound)

$$\sigma^2 \le \frac{(n-1)s^2}{\chi^2_{1-\alpha,n-1}} \rightarrow \sigma^2 \le \frac{(15-1)(106)^2}{\chi^2_{1-\alpha,n-1}}$$

$$\chi^2_{1-\alpha,n-1} = \chi^2_{0.95,14} = 6.57$$

$$\Rightarrow \sigma^2 \le \frac{(14)(106)^2}{6.57} = 23942\ cP^2$$

6-sigma Example/Application

Inference on the variance of a distribution

Rubber can be added to asphalt to reduce road noise. The viscosity (cP) of 15 specimens of asphalt is measured. The asphalt specification limit is 1100 cP away from the mean in either direction – what is the sigma level of this process given that we know $\sigma \le 173$ (from our hypothesis test) and we assume the mean of the population is on target?

Data
3193
3124
3153
3145
3093
3466
3355
3079
3182
3227
3256
3332
3204
3282
3170

$$\text{Sigma level} = \frac{|\text{spec limit} - \text{target}|}{\sigma} = \frac{1100}{173}$$

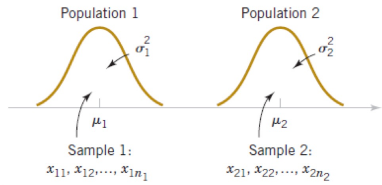
$= 6.35$ → at least 6.35

1-variance Test Assumptions

1. The data are continuous.
2. The data are randomly sampled.
3. The data are normally distributed.

Inference on means of two normally distributed populations

Let's say population 1 has a mean of μ_1 and a variance of σ_1^2 and population 2 has a mean of μ_2 and a variance of σ_2^2 and inferences are based on two random samples of size n_1 and n_2 respectively.



Want to know: is there a difference between their parameters

Two-Sample Z-test

Statistical inference on the difference in means $\mu_1 - \mu_2$ of the populations below with known variances of σ_1^2 and σ_2^2

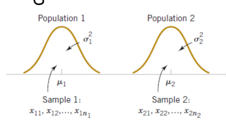
Null: $H_0: \mu_1 - \mu_2 = \cancel{\Delta_0}\ 0$

Alt: $H_1: \mu_1 - \mu_2 \neq 0$ (two-sided)

Test statistic: $Z_0 = \dfrac{\bar{x}_1 - \bar{x}_2 - \cancel{\Delta_0}\ 0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$

Reference Dist: $N(0,1)$

Rejection Criteria: Same as one-sample z-test



Summary

Ref: $N(0,1)$

Null hypothesis: $H_0: \mu_1 - \mu_2 = \Delta_0$

Test statistic: $$Z_0 = \frac{\bar{X}_1 - \bar{X}_2 - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \qquad (10\text{-}2)$$

Alternative Hypotheses	P-Value	Rejection Criterion for Fixed-Level Tests
$H_1: \mu_1 - \mu_2 \neq \Delta_0$	Probability above $\lvert z_0 \rvert$ and probability below $-\lvert z_0 \rvert$, $P = 2[1 - \Phi(\lvert z_0 \rvert)]$	$z_0 > z_{\alpha/2}$ or $z_0 < -z_{\alpha/2}$
$H_1: \mu_1 - \mu_2 > \Delta_0$	Probability above z_0, $P = 1 - \Phi(z_0)$	$z_0 > z_\alpha$
$H_1: \mu_1 - \mu_2 < \Delta_0$	Probability below z_0, $P = \Phi(z_0)$	$z_0 < -z_\alpha$

Assumptions

(1) $X_{11}, X_{12}, \ldots, X_{1n_1}$ is a random sample from population 1.
(2) $X_{21}, X_{22}, \ldots, X_{2n_2}$ is a random sample from population 2.
(3) The two populations represented by X_1 and X_2 are independent.
(4) Both populations are normal.
(5) The variances of the populations are known

Samples from two populations are independent if the samples selected from one population have no relationship with the samples selected from the other population.

Example

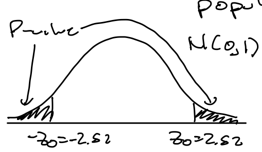
Paint Drying Time A product developer is interested in reducing the drying time of a primer paint. Two formulations of the paint are tested; formulation 1 is the standard chemistry, and formulation 2 has a new drying ingredient that should reduce the drying time. From experience, it is known that the standard deviation of drying time is 8 minutes, and this inherent variability should be unaffected by the addition of the new ingredient. Ten specimens are painted with formulation 1, and another 10 specimens are painted with formulation 2; the 20 specimens are painted in random order. The two sample average drying times are $\bar{x}_1 = 121$ minutes and $\bar{x}_2 = 112$ minutes, respectively. What conclusions can the product developer draw about the effectiveness of the new ingredient, using $\alpha = 0.05$?

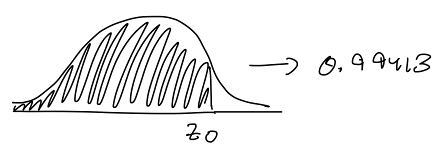
1) Parameter of interest is the difference in the mean drying time ($\mu_1 - \mu_2 = 0$), and we know the standard deviation is 8 min for both populations. → two-sample Z-test

2) $H_0: \mu_1 - \mu_2 = 0$ $(\mu_1 = \mu_2)$

3) $H_a: \mu_1 - \mu_2 \neq 0$ $(\mu_1 \neq \mu_2)$

4) $Z_0 = \dfrac{\bar{x}_1 - \bar{x}_2 - 0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \dfrac{121 - 112}{\sqrt{\frac{8^2}{10} + \frac{8^2}{12}}} = 2.52$





5) P-value: Reject H_0 if p-value < $\alpha = 0.05$

p-value $= 2(1 - 0.99143) = 0.012$

6) P-value $= 0.012 < \alpha = 0.05$

7) Reject H_0 and accept H_1 and conclude there is a difference btwn the drying time of the two

Confidence Intervals (do they cross zero?)

Confidence Intervals (do they cross zero!?)

100 (1-α)% two-sided confidence interval on $\mu_1 - \mu_2$

$$\bar{x}_1 - \bar{x}_2 - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \bar{x}_1 - \bar{x}_2 + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

where $Z_{\alpha/2}$ is the value of the N(0,1) such that $P(z \geq Z_{\alpha/2}) = \alpha/2$

$$\mu_1 - \mu_2 \le \bar{x}_1 - \bar{x}_2 + z_\alpha\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

100 (1-α)% one-sided upper confidence bound on $\mu_1 - \mu_2$ (interval includes - infinity)

$$\bar{x}_1 - \bar{x}_2 - z_\alpha\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2$$

100 (1-α)% one-sided lower confidence bound on $\mu_1 - \mu_2$ (interval includes + infinity)

where Z_α is the value of the N(0,1) such that $P(z \geq Z_\alpha) = \alpha$

Two-Sample t-test (variances unknown)

Case 1: $\sigma_1^2 = \sigma_2^2 = \sigma^2$ ← unknown values, but assumed the

Suppose we have two independent populations with unknown means μ_1 and μ_2 and equal variances $\sigma_1^2 = \sigma_2^2 = \sigma^2$ and we wish to know if the means are different:

Null: $H_0: \mu_1 - \mu_2 = \cancel{\Delta_0}^{\,0}$ $(\mu_1 = \mu_2)$

Alt: $H_a: \mu_1 - \mu_2 \neq \cancel{\Delta_0}^{\,0}$ $(\mu_1 \neq \mu_2)$

Test Statistic: $T_0 = \dfrac{\bar{X}_1 - \bar{X}_2 - \cancel{\Delta_0}^{\,0}}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

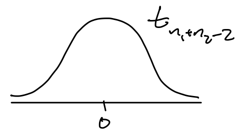
Square root of this

Pooling Sample Variance

$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}$$

Weighted average of two sample variances

Reference Distribution: t-dist with n_1+n_2-2 degrees of freedom

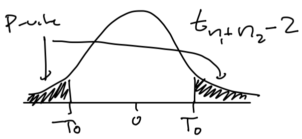


Rejection Criteria: See below exercise

Exercise: Sketch out Fixed rejection criteria & p-value for two-sided H_1

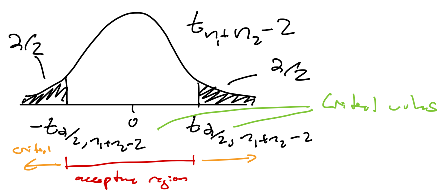
P-value:

Reject H_0 if p-value $< \alpha$



Fixed:

Reject if $T_0 > t_{\alpha/2, n_1+n_2-2}$ or $T_0 < t_{\alpha/2, n_1+n_2-2}$



Example

Yield from a Catalyst Two catalysts are being analyzed to determine how they affect the mean yield of a chemical process. Specifically, catalyst 1 is currently used; but catalyst 2 is acceptable. Because catalyst 2 is cheaper, it should be adopted, if it does not change the process yield. A test is run in the pilot plant

Observation Number	Catalyst 1	Catalyst 2
1	91.50	89.19
2	94.18	90.95
3	92.18	90.46
4	95.39	93.21
5	91.79	97.19
6	89.07	97.04
7	94.72	91.07
8	89.21	92.75
	$\bar{x}_1 = 92.255$	$\bar{x}_2 = 92.733$
	$s_1 = 2.39$	$s_2 = 2.98$

1) Interested in difference between 2 means, specifically yield of catalyst 1 and catalyst 2, variance unknown → two-sample t-test
↳ Assume equal variance ← rule of thumb later

2) $H_0: \mu_1 - \mu_2 = 0$

3) $H_a: \mu_1 - \mu_2 \neq 0$

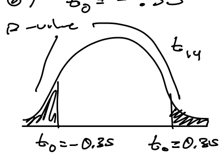
4) $t_0 = \dfrac{\bar{x}_1 - \bar{x}_2 - 0}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \dfrac{92.235 - 92.733}{2.7\sqrt{\frac{1}{8} + \frac{1}{8}}} = -0.35$

$S_p^2 = \dfrac{(n_1-1)s_1^2 - (n_2-1)s_2^2}{n_1+n_2-2} = \dfrac{7 \cdot 2.39^2 + 7 \cdot 2.98^2}{8+8-2} = 7.3$

$\rightarrow S_p = \sqrt{S_p^2} = \sqrt{7.3} = 2.7$

5) Reject if p-value $< \alpha = 0.05$

6) $t_0 = -.35$



$0.5 <$ p-value $< .8$ ← Can use range if tables don't have exact value

7) p-value $> \alpha$

Fail to reject H_0, we don't have enough evidence to say the yield from catalyst 1 is different from catalyst 2.

Paired t-test

A special case of the two-sample t-tests when observations of on the two populations are observed in pairs where each pair of observations is taken under homogenous conditions

To compare differences, we define Dj:

$D_j = X_{1j} - X_{2j}, \; j = 1, 2, \ldots, n.$ n = # of observations

where Dj's are assumed to be normally distributed with mean u_D and variance σ^2_D

n is the number of paired observations (j is the paired observation number)

1 and 2 represents the paired data we are comparing

$H_0: \mu_D = 0 = \cancel{\Delta_0}^{\,0}$

$H_a: \mu_D \neq \cancel{\Delta_0}^{\,0} = 0$

Test Statistic: $T_0 = \dfrac{\bar{D} - \Delta_0}{\frac{S_D}{\sqrt{n}}}$

where $\bar{D}$ = Sample average of n differences
S_D = Sample standard deviation of n differences

Reference Dist: t-dist with $n-1$ degrees of freedom

Rejection Criterion: Same as one sample t-test

Confidence Interval

$$\bar{d} - t_{\alpha/2, n-1}\, s_D/\sqrt{n} \le \mu_D \le \bar{d} + t_{\alpha/2, n-1}\, s_D/\sqrt{n}$$

Summary

Null hypothesis: $H_0: \mu_D = \Delta_0$ Ref: t-dist w/ n-1 dof

Test statistic: $T_0 = \dfrac{\bar{D} - \Delta_0}{S_D / \sqrt{n}}$ (10-24)

Alternative Hypotheses	P-Value	Rejection Criterion for Fixed-Level Tests
$H_1: \mu_D \neq \Delta_0$	Probability above $\lvert t_0 \rvert$ and probability below $-\lvert t_0 \rvert$	$t_0 > t_{\alpha/2,n-1}$ or $t_0 < -t_{\alpha/2,n-1}$
$H_1: \mu_D > \Delta_0$	Probability above t_0	$t_0 > t_{\alpha,n-1}$
$H_1: \mu_D < \Delta_0$	Probability below t_0	$t_0 < -t_{\alpha,n-1}$

Assumptions:
(1) Differences are continuous
(2) Differences are normally distributed
(3) Differences are obtained from a random sample
(4) Differences are obtained from homogonous pairs

Two Sample t-test (variances unknown & unequal)

Case 1: $\sigma_1^2 \neq \sigma_2^2$

Suppose we have two independent populations with unknown means μ_1 and μ_2 and unequal variances $\sigma_1^2 \neq \sigma_2^2$ and we wish to test if the means are different:

H_0, H_a are same as 2 sample t-test w/ equal variance

$$T_0^* = \frac{\bar{x}_1 - \bar{x}_2 - \cancel{\Delta_0}\,0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Reference Dist: t-dist with v degrees of freedom

where $v = \dfrac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(s_1^2/n_1\right)^2}{n_1-1} + \frac{\left(s_2^2/n_2\right)^2}{n_2-1}}$

If v is not an integer, round down

Example v calculation: $s_1 = 2.63$, $s_2 = 1.53$, $n_1 = n_2 = 10$ $\Big\}$ $v = 13.2 \to 13$

Summary: **two-sample t-test (unequal variance)**

Null hypothesis: $H_0: \mu_1 - \mu_2 = \Delta_0$. Ref.: t-dist with v dof

Test statistic: $T_0^* = \dfrac{\bar{X}_1 - \bar{X}_2 - \Delta_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$ (10-14)

Alternative Hypotheses	P-Value	Rejection Criterion for Fixed-Level Tests
$H_1: \mu_1 - \mu_2 \neq \Delta_0$	Probability above $\lvert t_0 \rvert$ and probability below $-\lvert t_0 \rvert$	$t_0 > t_{\alpha/2,\cancel{n_1+n_2-2}\,v}$ or ✓ $t_0 < -t_{\alpha/2,\cancel{n_1+n_2-2}\,v}$ ✓
$H_1: \mu_1 - \mu_2 > \Delta_0$	Probability above t_0	$t_0 > t_{\alpha,\cancel{n_1+n_2-2}\,v}$ ✓
$H_1: \mu_1 - \mu_2 < \Delta_0$	Probability below t_0	$t_0 < -t_{\alpha,\cancel{n_1+n_2-2}\,v}$ ✓

Note: replace n1+n2 with v in rejection criteria!

$$v = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{(s_1^2/n_1)^2}{n_1-1} + \frac{(s_2^2/n_2)^2}{n_2-1}}$$

If v is not an integer, round down to the nearest integer.

Checking in Minitab: stat -> basic stat -> 2 sample t -> input columns and in options deselect assume equal variance

Assumptions

(1) $X_{11}, X_{12}, \ldots, X_{1n_1}$ is a random sample from population 1.
(2) $X_{21}, X_{22}, \ldots, X_{2n_2}$ is a random sample from population 2.
(3) The two populations represented by X_1 and X_2 are independent.
(4) Both populations are normal.
(5) The variances of the populations are unequal

If you're in doubt, assume unequal

How do I know which t-test to use?

1) Are measurements being made on the exact same object? (Paired)
2) **If the ratio of the larger variance to the smaller variance is less than 4** then we can assume the variances are approximately equal (use pooled t-test) OR you could run a two-variance test (will learn about in the future!)

- Remember variance is standard deviation squared!!
- This rule of thumb can vary widely

↳ Use this for exam 1

Confidence Intervals

Equal Variance Two sided

$$\bar{x}_1 - \bar{x}_2 - t_{\alpha/2,n_1+n_2-2}\, s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_2 \le \bar{x}_1 - \bar{x}_2 + t_{\alpha/2,n_1+n_2-2}\, s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

Unequal Variance Two-sided

$$\bar{x}_1 - \bar{x}_2 - t_{\alpha/2,v} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \le \mu_1 - \mu_2 \le \bar{x}_1 - \bar{x}_2 + t_{\alpha/2,v} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

ECHE 313 Practice Exam 1 Solutions on canvas

Write your name:

Write the following statement, and sign your name acknowledging your agreement:

don't pay attention to this, the exam was take-home

“"I neither received nor gave external assistance on this exam "

The exam layout is such that all problems add up to 100 points. This point total is meant to guide you in time allocation for this test, will be adjusted such that this exam is 25% of your total class grade. You may use your class materials, class notes, homework, book, and Minitab or Excel to aid in the completion of the exam. The exam is designed to be completed in 1hr and 15 min, but you will have between 10 AM 2/27 to 10 AM 2/28 to complete the exam. All of the necessary tables are provided in the back of your book. Please show your work and all steps necessary to arrive at your answer. Note that it is expected you do this entire test by hand, using a calculator and the tables found in the back of your book. So while you are allowed to use Minitab if you wish to check your answers, it should not replace the work you have done by hand using the tables for your answers. Where applicable, it is best to write out the general formals you want to use first, then write in your subbed in values so it is clear. In addition, for this exam, we can use the rule of thumb that if the sample variance ratio (larger to smaller) is lower than 4, you can assume equal variance for a t-test (if needed). Also, when reporting numbers, keep the same number of decimals as the data you are working with unless there are specific directions on how many sig figs to report.

Good luck!

Premise: You are a process engineer working for a lean six-sigma pharmaceutical company that makes advanced protein-based biologics. Unless otherwise stated, your company uses α=0.01.

1) (12 points) You are finishing up a project where you focused on increasing process cycle efficiency and are ready to hand over the improved process.
   a. (2 points) What step of the DMAIC process are you in?

      Control

   b. (2 points) Name at least one tool that is used in this stage.

      Control chart or Transition Plan

   c. (4 points) Explain what process cycle efficiency is and how it contributes to lean manufacturing.

      Process Cycle Efficiency (PCE): $\frac{\text{value added time}}{\text{process time}}$

→ Value added time: Time spent making the product more valuable.
Thus, PCE is the function of time spent making a product more valuable.

→ Lean Manufacturing: A management philosophy aimed toward eliminating waste.
PCE can be used as a tool to measure and track wasted time in a process.

d. (4 points) Explain why a slow process is an expensive process with at least 3 specific reasons.

Slow processes are expensive because:

(i) Customers don't like waiting

(ii) More handeling = more personnel

(iii) More opportunities for damage or loss

(iv) Higher inventory

(v) More documentation

2) (39 points) Your next project is to determine if the bioreactor time can be reduced by getting a newer bioreactor system. You are able to demo a new unit and track the reactor time on 5 randomly selected batches in both the old and new bioreactors. You obtain the following data:

The following table shows the time it took in minutes for the batch to reach the designated a amount of bacteria:

$n=5$

Old Bioreactor Time (min) 1	New Bioreactor Time (min) 2
245	165
270	141
262	151
275	142
260	147

$$\bar{x}_1 = \frac{245+270+\dots+260}{5} = 262$$

$$s_1^2 = \frac{(245-262)^2+\dots+(260-262)^2}{5-1} = 132$$

$$\bar{x}_2 = \frac{165+141+\dots+147}{5} = 149$$

$$s_2^2 = \frac{(165-149)^2+\dots+(147-149)^2}{5-1} = 94$$

Ratio of s^2:

$$\frac{s_1^2}{s_2^2} = \frac{132}{94} = 1.4 < 4$$

$\frac{\text{bigger } s^2}{\text{Smaller } s^2}$

Use pooled !!

$$S_p = \sqrt{S_p^2} = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(5-1)(132)+(5-1)(94)}{5+5-2}} = 10.62$$

a. (3 points) List the *name* of the hypothesis test you would use

Unknown variance, 2-Sample t-test w/ equal variance (pooled t-test)

2-SampT Test w/ TI-84

b. (3 points) Name the parameter of interest in the context of the problem statement

Mean bioreactor time using an old reactor (μ_1) and new reactor (μ_2)

c. (2 points) Write out the correct null hypothesis for this test

$$H_0: \mu_1 - \mu_2 = 0 \quad (\text{or } \mu_1 = \mu_2)$$

d. (2 points) Write out the correct alternative hypothesis for this test

$$H_a: \mu_1 - \mu_2 \neq 0 \quad (\text{or } \mu_1 \neq \mu_2)$$

e. (7 points) Write the formula of the test statistic and calculate (report 3 sig figs)

$$T_0 = \frac{\bar{x}_1 - \bar{x}_2 - 0}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{262-149-0}{10.62\sqrt{\frac{1}{5}+\frac{1}{5}}} = 16.9$$

f. (4 points) Write out the correct rejection criteria for fixed significance level testing and the critical value (3 sig figs)

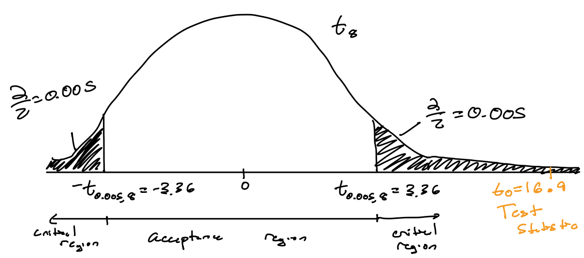
Reject if: $\begin{cases} t_0 > t_{\alpha/2, n_1+n_2-2} \\ t_0 < -t_{\alpha/2, n_1+n_2-2} \end{cases}$

Critical Values

± 3.36

$t_{\frac{0.01}{2}, 8} = 3.36$

g. (9 points) Sketch out your reference distribution making sure to fully define the distribution you are sketching on with the correct label, drawing it with the correct shape and where zero is. In addition, label 1) acceptance region, 2) the rejection region (also known as the critical region), 3) α, 4) the critical value and 5) the test statistic.



h. (3 points) State your conclusion and remember to word your conclusion appropriately in the context of the problem statement.

Reject H_0 and accept H_a to conclude that the new bioreactor significantly reduced process time.

i. (3 points) Explain what plot you would construct to check if the new bioreactor data are normally distributed (what feature(s) of the plot would you check?)

A normal probability plot; check for linearity

j. (3 points) Define in words what (α) is (what does it represent, what is it's definition?) and explain why it helps us make strong conclusions.

α is the probability of a type I error (rejecting the H_o when it is true). We can control the value of α, which means rejecting the H_o is strong because we know the probability we are wrong (α).

3) (32 points) To determine if the new bioreactor is suitable for the process, the team also needs to know if the new bio reactor produces a yield that is greater than 810 mg/L of product. Twelve random samples are taken from batches in the piolet run:

	Yield (mg/L)
	807
	825
	816
	805
	815
	820
	822
	817
	823
	817
	810
	812
Sample Ave.	815.75
Sample Standard Deviation:	6.31

a. (2 points) List the *name* of the hypothesis test you would use

One-Sample t-test T-test on TI-84

b. (3 points) Name the parameter of interest in the context of the problem statement

Mean Process yield (μ)

c. (2 points) Write out the correct null hypothesis for this test

$H_o: \mu = 810$ g/L

d. (2 points) Write out the correct alternative hypothesis for this test

$H_a: \mu > 810$ mg/L

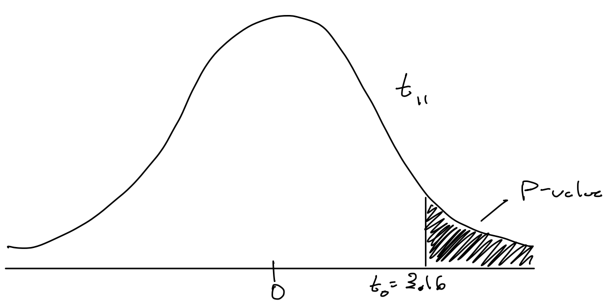
e. (3 points) Write the formula of the test statistic and calculate (report 3 sig figs)

$$T_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{815.75 - 810}{6.31/\sqrt{12}} = 3.16$$

f. (2 points) Write out the correct rejection criteria using the p-value method

Reject if p-value $< \alpha = 0.01$

g. (6 points) Sketch out your reference distribution making sure to fully define the distribution you are sketching on with the correct label, drawing it with the correct shape and where zero is. In addition, label 1) the test statistic and 2) the p-value.



h. (2 points) Find the p-value using the tables in the back of your book

$0.0025 < \alpha < 0.005$

closer to 0.005, still $< \alpha = 0.01$

i. (2 points) Explain what the p-value is in words (i.e. what is the definition of a p-value?)

The p-value is the probability of obtaining a sample outcome at least as extreme as the one obtained. OR you can think of it as indicating the compatibility with the Ho.

j. (3 points) State your conclusion and remember to word your conclusion appropriately in the context of the problem statement.

Reject H_0 and accept H_a. The process yield of the new bio reactors is greater than 810 mg/L.

k. (2 points) You calculate your 99% confidence interval and show it to your boss. Explain what the 99% confidence interval really represents generally, in words (Hint: do not just say it means you are 99% confident that the mean lies in the CI range). You do not need to actually calculate the CI to answer this question correctly.

In repeated samplings a 99% CI will capture the true mean of the population 99/100 times.

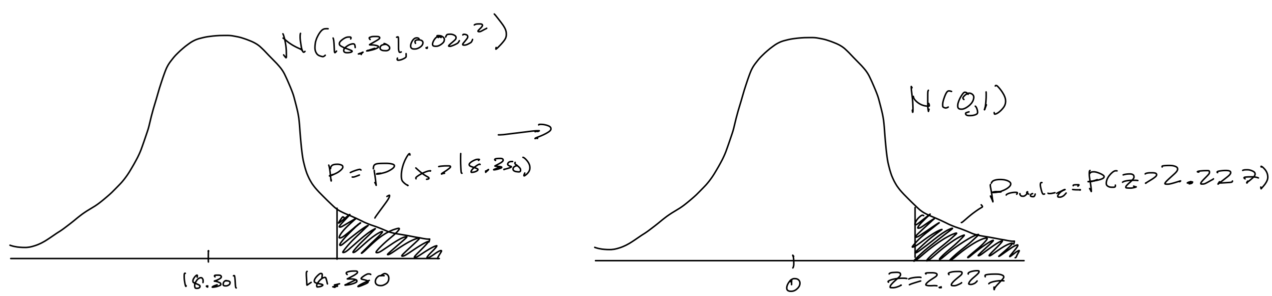
l. (3 points) If the manufacturer told you the standard deviation on the yield coming from these new units is known to be 4.00 mg/L, and you wanted to know if the yield was *different* than 810 mg/L, name the hypothesis test you would conduct, and if your alternative hypothesis would be 1 or 2 sided.

Population std dev known => One-Sample Z-test, two sided

4) (17 points) Your boss was so impressed with the projects you have been doing that she asked you to help the manufacturing floor with some questions regarding their packaging unit, which boxes up the biologics with instructional materials. Weight measurements are taken on each and every product automatically, thus it is known that the population mean μ is 18.301 g and the population standard deviation is σ is 0.022 g and is normally distributed. New weight restrictions are being instated and the manufacturing floor wants to know the probability that a product will have a weight above a new limit of 18.350 g.

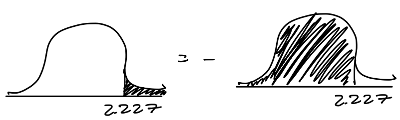
   a. (11 points) A write out the probability equation you wish to solve, including the integral (but you do not need to solve the integral just set it up). Also include a probability distribution sketch with the 1) probability you wish to find shaded in 2) the correct shape, 3) correct label of the distribution, 4) where 18.350g is and 5) where zero is.

$$P(x > 18.350\,g) = \int_{18.350}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$



b. (6 points) Solve the probability problem using the *tables in the back of the book*. Just like in class, sketches can help solve these problems, especially when using the tables. Report and appropriate number of sig figs based on the numbers given in the problem statement.

$$Z = \frac{x-\mu}{\sigma} = \frac{18.350 - 18.301}{0.022} = 2.227$$



$P(z > 2.227) = 1 - P(z \leq 2.227) = 1 - 0.987 = 0.013$

Overflow

Example

■ **TABLE 4.3** (n=8)

Paired Tensile Strength Data for Example 4.11

Specimen	Machine 1	Machine 2	Difference
1	74	78	−4
2	76	79	−3
3	74	75	−1
4	69	66	3
5	58	63	−5
6	71	70	1
7	66	66	0
8	65	67	−2

Get $\bar{D}$ and s_D from these differences

Mean difference: -1.38 $\bar{D}$

Standard deviation of differences: 2.67 s_D

Alpha = 0.05

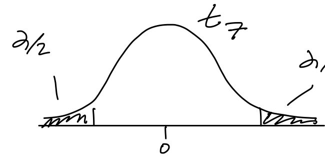
Do these machines yield the same tensile strength values on the same sample?

(i) Interested in the difference in mean tensile strengths measured by two devices, same samp → paired data → paired t-test

(ii) $H_0: \mu_D = 0$ (iii) $\mu_D \neq 0$

(iv) $t_0 = \frac{\bar{d}}{\frac{s_d}{\sqrt{n}}} = \frac{-1.38}{\frac{2.67}{\sqrt{8}}} = -1.46$

(v) Reject if $t_0 > t_{\alpha/2, n-1}$ or $t_0 < -t_{\alpha/2, n-1}$



$\rightarrow t_{\alpha/2, n-1} = 2.365$

Conclusion: Fail to reject null (H_0), we do not have sufficient evidence to reject the claim that the machines measure differently

Also remember the proper sketches for hypothesis testing should have:

1) The type of reference distribution (e.g., normal dist., t-distribution among others), draw the shape and write the defining parameters (e.g., μ, σ among others)
2) Show where the test statistic lies on your sketch
3) For solving using fixed significance level method – show the acceptance region, rejection region, alpha, as well as the critical value
4) For p-value method, shade in the p-value and report that value

HW1

1) Provide a definition of quality, and state what the two general aspects of quality are.
2) What are the four different types of quality costs? (List examples of each)
3) Describe what a quality engineering is
4) Define the "hidden factory"
5) In this problem, refer to the example problem in the lecture notes about using bar coding in a hospital as a basis. For this problem, another hospital decides to launch more thorough investigation. In addition to considering bar-coding, investigators consider 1) assigning fewer patients to each nurse, 2) setting a limit on how much time nurses can spend chatting with patients and 3) shortening the nurses shift hours. In addition to the correct dosage administration, they want to evaluate the job satisfaction of nurses. What are the controllable input variables, and key output variables in this new example?
6) Describe what a Control Chart is and how it helps Quality Engineers do their job
7) Define what Six Sigma is, the three elements for its successful execution and why it is called Six Sigma
8) You are working for a company that makes electrodes. One of the quality characteristics is the loading of catalyst (mg/cm^2). The target is 3 mg/cm^2 and acceptable limits are 2.4-3.6 mg/cm^2. With the current process, the standard deviation (remember this is sigma or a measure of variance) is 0.3 mg/cm^2. What sigma quality level is the current process at? What would the standard deviation need to be to reach a six sigma level performance?
9) Describe why slow processes are expensive and why lean focuses on eliminating waste by increasing process cycle efficiency and decreasing process cycle time
10) Describe the main objectives of the Define Step are and why it is a vital part of the DMAIC process.
11) Create an SIPOC diagram for the process of making a burger at a fast food restaurant (from making to delivery to customer)
12) Create a value stream map for the process of burger making above (include all steps – even those that might not be value added). List estimated process times and suggest ways to improve the process cycle time.

13) Use the following table of nonconformity counts and costs of non-fatal pacemaker failures, construct a) a Pareto chart of the count of non-conformities (count), and b) a cost Pareto chart of non-conformities. Make these charts by hand.

Category	Count	Cost
Battery life	12	113850
Irreg. heart beat	2	700
Electro. shielding	1	110000
Discomfort	1	350
Lethargy	1	350

14) Comment on which two non-conformities should be the subject of a process improvement effort in problem 13.
15) Use Mini-tab to generate the two graphs you sketched in problem 13:
    a. Open Mini-tab
    b. Label columns as "category" "Count" and "Cost"
    c. Input the correct values from the chart under each heading
    d. Go to Stat, Quality Tools, Pareto Chart
    e. Construct both graphs by defining the attribute data is the "category" column (to select, you have to double click and it will appear in the box, then define the frequencies as either "Count" or "Cost" depending on which graph you are making.
    f. Hint: you may need to select the box that says not to combine categories to ensure all of the categories make it onto the graphs
16) Describe what role a pareto chart can play lean manufacturing
17) Describe the objectives of the Measure, Analyze, Improve and Control Steps of the DMAIC process.
18) Use the following FMEA table constructed for a cookie baking process, which defect requires the most attention?

Defect	**Severity**	**Occurrence**	**Detection**	**RPN**
Burnt	6	2	1	
Too dry	4	2	2	
Too small	5	2	1	
Taste is off	9	2	4	
Freshness	8	3	4	
Number of chips is low	2	2	4	

ECHE313 Homework 1 - Due 1/23/25 **Trevor Swan (tcs94)**

1. Quality is simply a product's fitness for use. Its two aspects are:
(i) Quality of design - is a product high quality?
(ii) Quality of conformity - is the product consistently high quality?

2. (i) Prevention Costs - Product/Process design, Training personnel, Process control
(ii) Internal Failure Costs - Scrap, Rework/Retest, Failure Analysis
(iii) Appraisal Costs - Product inspection & testing, Maintaining accuracy of test equip.
(iv) External Failure Costs - Warranty charges, Liability costs, returned product/material

3. Quality Engineering is a set of activities that work to ensure that quality characteristics are kept at desired levels, and that variability is at its minimum.

4. The hidden factory is the part of a plant that the customer doesn't see which deals with bad quality.

5. Controllable Input Variables: Barcoding, # of patients per nurse, time limit on nurse-patient interactions, nurse shift length
Key output variables: Correct amount of drug/dosage, Job satisfaction of their employed nurses

6. Control charts are a form of monitoring which tracks averages in a quality characteristic with/over sample number/time. They are helpful as they indicate where a characteristic should be w/o abnormal variation and they alert users to unusual variation through statistically defined control limits.

7. Six Sigma is a statistical based, data driven management approach and cts. quality improvement methodology for eliminating defects in a product, process, or service. In short, it reduces variability to a level where defects are unlikely. Its three elements are: (i) Quality Planning - Listening to the voice of your customer
(ii) Quality Assurance - Establishing a system to prevent quality issues from arising
(iii) Quality Control & Improvement - A set of specific steps & tools to ensure products meet requirements or are improved continuously.
Its name refers to the quality level target such that std dev (σ) or variance (σ^2) is small enough to be within 6σ of the spec limit.

8. USL $= 2.4$ mg/cm^2
LSL $= 3.6$ mg/cm^2
Target $= 3$ mg/cm^2
$\sigma = 0.3$ mg/cm^3

a) $\dfrac{|2.4 \text{ mg/cm}^2 - 3 \text{ mg/cm}^2|}{0.3 \text{ mg/cm}^2} = 2$ for current process

b) $\dfrac{|2.4 \text{ mg/cm}^2 - 3 \text{ mg/cm}^2|}{\sigma \text{ mg/cm}^2} = 6 \Rightarrow \sigma = 0.1$ mg/cm^2 for 6σ process

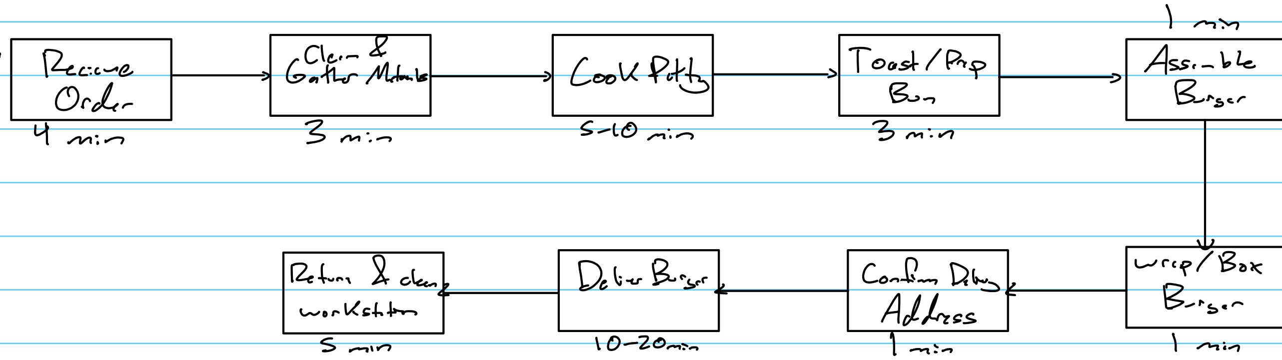
9. Slow processes are expensive as customers don't like waiting (complaint processing), more handling = more personnel & opportunity for damage/loss, more space has to be given to inventory, and more time has to be spent on documentation.
→ Increasing process cycle efficiency results in more product value compared to the same (or lower) process time, which results in higher profit.
→ Decreasing Process Cycle time means reducing the number of items in progress (i.e., the backlog), while increasing the completion rate. This results in more products being produced.
Lean focuses on both of these as higher PCE in combination with lower PCT results in more value coming from more sold products.

10. The design step is important because it helps identify a project opportunity and verify that it is a legitimate financial opportunity or may lead to major improvement to a process or product. Its main objectives are to:
(i) Identify and/or validate the business improvement opportunity
(ii) Define critical customer requirements
(iii) Document/map processes
(iv) Establish project charter & build team

11.

Suppliers	Inputs	Process	Output	Customer
Butcher/Processing Plant	Raw Burger	(i) Receive Cust. Order	Prepared Burger	Customers in-store
Bakery	Burger Buns	(ii) Gather Materials	Cust. Satisfaction	Customers online
Farms	Lettuce, Tomato, Onions	(iii) Cook Patty on Grill	Sales revenue	Delivery services
Cheese Supplier	Cheese slices	(iv) Toast & Prep Bun	Positive reputation	Event organizers
Condiment Company	Sauces/spreads	(v) Assemble Burger	Feedback	Other organizations
Packaging Company	Packaging materials	(vi) Wrap/Box burger	Taste	Caretakers
Equipment Company	cooking tools	(vii) Confirm Cust. Details	Warmness	Family Members
Employees	Employee Labor	(viii) Deliver Burger	Doneness	Significant Others
Utilities	Electricity	(ix) Return to clean		

12.



Areas for improvement:

(i) Keep all materials out and clean during the work-day

(ii) Toast Bun while Burger is cooking (towards end of grilling time)
↳ faster and better (warmer/fresher) product

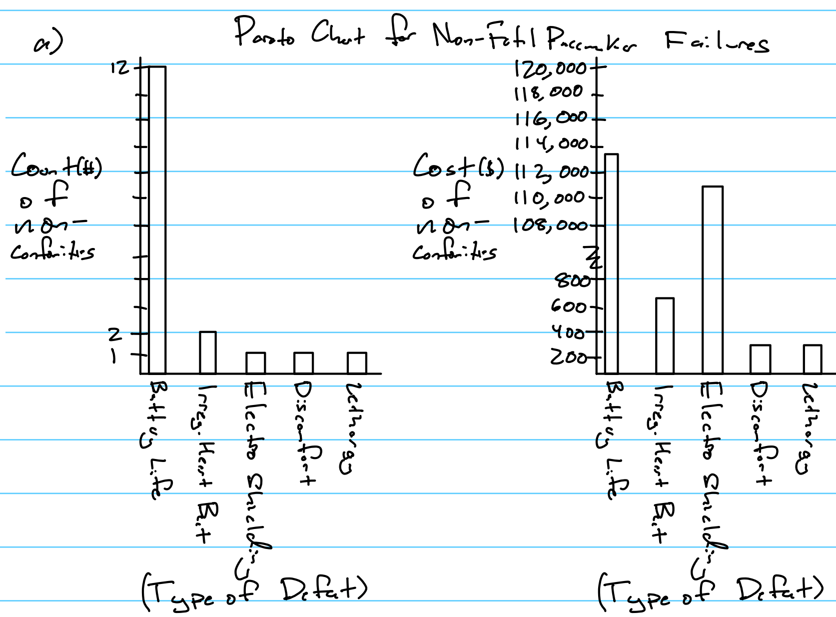
(iii) Confirm Delivery Address upon getting order

(iv) If performing multiple deliveries, make burgers in bulk and find a good route to deliver all burgers to customers quickly

13.

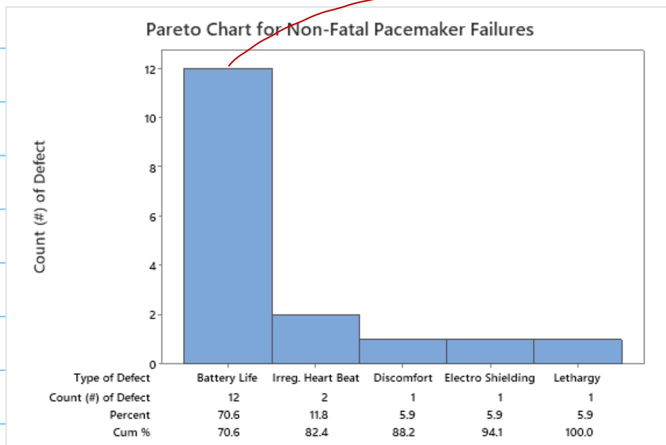
Category	Count	Cost
Battery life	12	113850
Irreg. heart beat	2	700
Electro. shielding	1	110000
Discomfort	1	350
Lethargy	1	350

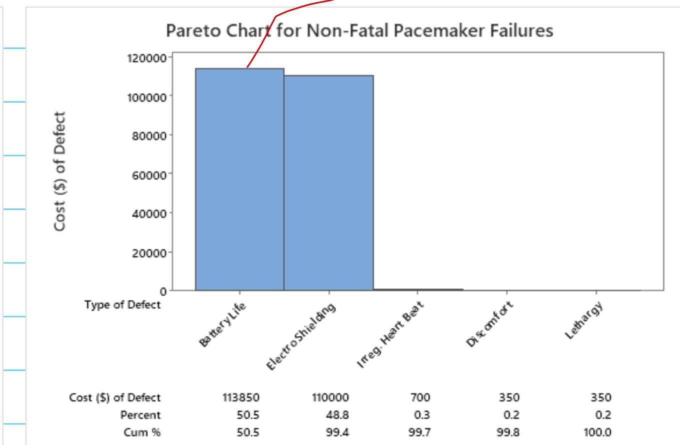
a)



14. While Battery Life and Irregular heart beat are most common, the frequency of the Irregular heart beat defect is not high enough to outweigh the cost of an Electro-shielding defect. As supported by the Cost vs. Defect pareto chart, Battery Life & Electro shielding should be the subject of a process improvement effort.

15.





16. Lean Manufacturing aims at eliminating waste (time or materials). A pareto chart shows opportunities of improvement based off the 80/20 rule (pareto rule). This suggests a few defect types could contribute to the majority of product rejections. Analyzing Pareto charts is done to reduce defects by highlighting significant areas of improvement, eliminating/reducing waste in the form of time wasting from dealing with bad product, and material wasting in the form of rejecting bad product.

17. Measure Objective: Evaluate & understand the current state of the process
Analyze Objective: Use data to begin to determine cause & effect relationships and sources of variability.
Improve Objective: Use creative thinking & info gathered previously to make specific suggestions that will have the desired effect on a process. Confirmation from others is an important aspect of this steps objective.
Control Objective: Complete all remaining work and hand off the improved process so gains in progress are realized/institutionalized.

18.

Defect	Severity	Occurrence	Detection	RPN
Burnt	6	2	1	12
Too dry	4	2	2	16
Too small	5	2	1	10
Taste is off	9	2	4	72
Freshness	8	3	4	96
Number of chips is low	2	2	4	16

Severity x Occurrence x Detection

Freshness has the highest RPN, and thus requires the most attention

HW2

1) Use data from problem 3.4 from your book to a) calculate the sample average, b) calculate the sample standard deviation c) explain how the sample mean and standard deviation describe different features of the data (in other words, define sample mean and standard deviation in words) and d) brainstorm some possible sources of variation for this data set. Assume your data set has numbers with 3 significant figures. (For those with the 7^{th} edition, use data set from problem 3.5)
2) For the same problem set in 3.4 (or 3.5 if using the 7^{th} edition) a) find the sample median of these data and b) answer how much could the largest temperature measurement increase without changing the sample median? c) explain what the sample median describes about the data (in other words, define median in words), d) find the median if one more value was added to the set: 952°F. Finally, for part e) include the output table from Minitab which includes the average, standard deviation, and median for the original data set in the book (the one with 9 observations). Assume this data set has numbers with 3 significant figures.
3) Problem 3.5 (3.8 in 7^{th} edition) from book and do a, b, and d (skip c) as well as these added parts: e) construct a box plot using the limits on whiskers talked about in class. Comment on the data trends you observe in this plot. f) construct a bar graph using excel (it will only have one bar) representing the average ± the standard deviation if the sample. Caption this figure appropriately!

For each part, show how you would calculate things by hand (can use ... abbreviation but write out the formulas with some data inserted to illustrate), draft up the graphs by hand first, then show the output from software *for each part*.

4) Assume that Z has a standard normal distribution. Use the table in Appendix II to solve each of the following below. Write out your <u>probability formulas used to solve the problem and draw the problem out pictorially</u> for full credit, and report 2 significant figures:

   a) $P(Z \leq 0.76)$
   b) $P(Z>1.1)$
   c) $P(Z \leq -0.91)$
   d) $P(-1.35 \leq Z \leq 0.25)$

   Find z such that:

   e) $P(Z>z) = 0.1$
   f) $P(Z>z) = 0.9$
   g) $P(-1.24<Z<z) = 0.8$
   h) $(-z<Z<z) = 0.95$

   i) g) Check at least one of these with minitab and show the output
   j) Write out the integral you are solving for part b (no need to integrate, but set it up)

5) The time until recharge for a battery in a laptop computer under common conditions is normally distributed with a mean of 260 minutes and a standard deviation of 50 minutes. Use the table in Appendix II to solve this problem.
   a) Let's say battery life is a critical quality characteristic and the customers really want more than 4 hours with a charge. What is the probability that the battery lasts more than four hours?
   b) What value of life in minutes is exceeded with 95% probability?
   c) Check your answer in Minitab and print the output

6) Problem 3.9 from your book (3.13 from the 7^{th} edition). In this problem, you will generate a normal probability plot "by hand" and by using Minitab. For the one "by hand" calculate the cumulative frequency (remember, the cumulative frequencies are really probabilities) for the data point with rank 1 and find z_1 using the Appendix II. Calculate the rest of the cumulative frequencies using Excel, and find corresponding zj using Minitab (you have to put the frequency values from Excel in as a column into Minitab then use the normal probability function to find the value of zj). Then, using the value of zj you obtained by Minitab, generate your normal probability plot in Excel. Remember to also input the data into Minitab directly and generate a Normal Probability Plot (this is much easier, of course) and remember to comment on if the normal assumption is reasonable.

3.4 in 8th Ed book

ECHE313 Homework 2 - Due 1/30/25 **Trevor Swan (tcs94)**

1. Data (°F): 953, 955, 948, 951, 957, 949, 954, 950, 959

a) Sample Average

$$\bar{x} = \frac{953+955+948+951+957+949+954+950+959}{9} = 952.89 \xrightarrow[\text{round}]{} \bar{x} = 953\,°F$$

b) Standard Deviation

$$S = \sqrt{\frac{\sum_i^n (x_i-\bar{x})^2}{n-1}} = \sqrt{\frac{(953-952.89)^2+(955-952.89)^2+(948-952.89)^2+(951-952.89)^2+(957-952.89)^2+(949-952.89)^2 \ldots}{9-1}}$$

$$\ldots \frac{(954-952.89)^2+(950-952.89)^2+(959-952.89)^2}{} = 3.7231 \xrightarrow[\text{round}]{} S = 3.72\,°F$$

c) The mean of 953 means that the average temperature reading was 953°F. Specifically, this is the measure of central tendency to describe the typical value or center. The standard deviation describes the variance of temperature readings in the data set. The value of 3.72 is the typical distance between each data point and the mean.

d) The data set describes furnace temperatures recorded on successive batches in a semi-conductor manufacturing process. Variations in this data set are most likely due to process-related variations (like control system errors, batch size, or calibrations), environmental variations, material variations (material properties and batch preparation changes), and human error (different operators and measurement intervals, and possible variations in measurement devices.

2. Data (°F): 953, 955, 948, 951, 957, 949, 954, 950, 959

Ranked: 948, 949, 950, 951, 953, 954, 955, 957, 959

Rank: 1 2 3 4 5 6 7 8 9

a) median = $(n+1)(\frac{1}{2}) = (9+1)(\frac{1}{2}) = 5 \Rightarrow$ median = 953 °F

b) The largest data point can increase by any amount without altering the median as median is determined by rank. The final value with rank 9 is going to remain rank 9 for any increase.

c) The median of a dataset describes the center value of the dataset. It is also defined as the 50th percentile. It is the exact middle of sorted data where ½ of the data is below the median and ½ of it is above the median. This value is more accurate for skewed data.

d) Data (°F): 953, 955, 948, 951, 957, 949, 954, 950, 959, 952

Ranked: 948, 949, 950, 951, 952, 953, 954, 955, 957, 959

Rank: 1 2 3 4 5 6 7 8 9 10

median = $(10+1)(\frac{1}{2}) = 5.5 \Rightarrow (952+953)(\frac{1}{2}) = 952.5$ °F

e) Minitab Output

Statistics Unrounded

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
Probelm 1	9	0	952.889	1.24102	3.72305	948	949.5	953	956	959

Statistics Rounded

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
Probelm 1	9	0	953	1.24	3.72	948	949.5	953	956	959

3.5 in 4th Ed book

3.

Table 3E.1 Electronic Component Failure Time

127 2	124 2	121 1	118 1
125 2	123 1	136 4	131 3
131 3	120 1	140 4	125 2
124 2	119 1	137 4	133 3
129 2	128 2	125 2	141 4
121 1	133 3	124 2	125 2
142 5	137 4	128 2	140 4
151 6	124 2	129 2	131 3
160 7	142 5	130 3	129 2
125 2	123 1	122 1	126 2

a) $\bar{x} = \dfrac{127+125+131+\dots+129+126}{40} = 129.98 \xrightarrow{\text{round}}$ $\bar{x} = 130\text{ hr}$

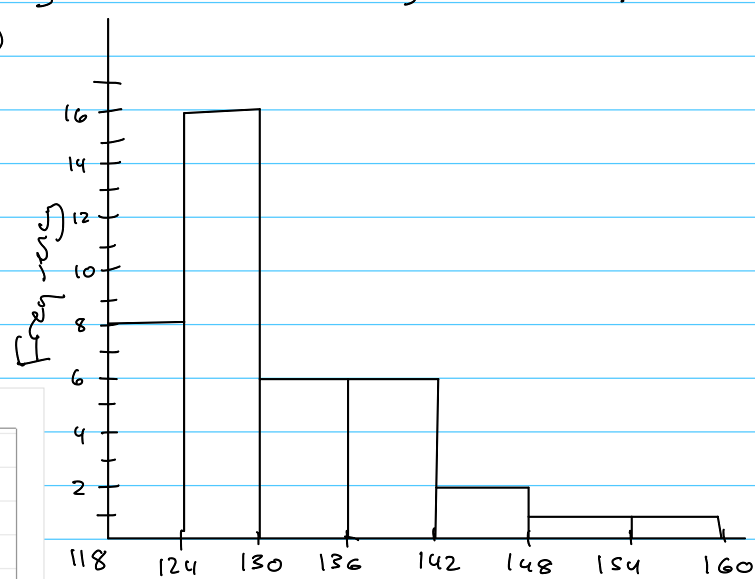
$S = \dfrac{(127-129.98)^2+(125-129.98)^2+\dots+(126-129.98)^2}{40-1} = 8.91 \xrightarrow{\text{round}}$ $S = 8.91\text{ hr}$

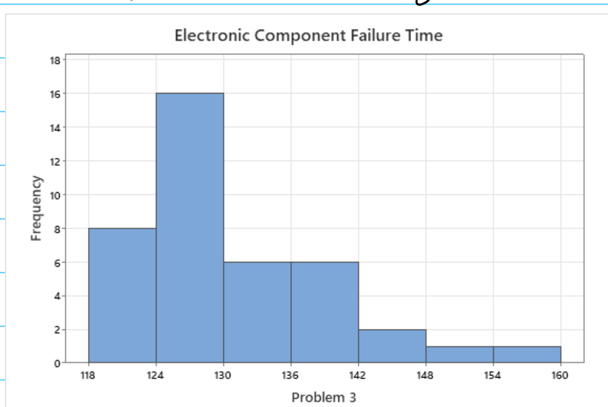
Statistics

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
Problem 3	40	0	129.975	1.40944	8.91408	118	124	128	135.25	160

b) # of bins $= \sqrt{40} = 6.32 \rightarrow 7$, min $= 118$, max $= 160$, bin size $= \dfrac{160-118}{7} = 6$

Counts	
8	bin 1: $118+6 \rightarrow [118, 124)$
16	bin 2: $124+6 \rightarrow [124, 130)$
6	bin 3: $130+6 \rightarrow [130, 136)$
6	bin 4: $136+6 \rightarrow [136, 142)$
2	bin 5: $142+6 \rightarrow [142, 148)$
1	bin 6: $148+6 \rightarrow [148, 154)$
1	bin 7: $154+6 \rightarrow [154, 160]$





(Skip c)

d)

118	1	128	21
119	2	129	22
120	3	129	23
121	4	129	24
121	5	130	25
122	6	131	26
123	7	131	27
123	8	131	28
124	9	133	29
124	10	133	30
124	11	136	31
124	12	137	32
125	13	137	33
125	14	140	34
125	15	140	35
125	16	141	36
125	17	142	37
126	18	142	38
127	19	151	39
128	20	160	40

Median (rank) $= \dfrac{n+1}{2} = \dfrac{40+1}{2} = 20.5$

Median $= \dfrac{128+128}{2} = 128$ median $= 128$ hr

Q1 (rank) $= \dfrac{n+1}{4} = \dfrac{40+1}{4} = 10.25$

Q1 $= \dfrac{124+124}{2} = 124$ Q1 $= 124$ hr

Q3 (rank) $= (n+1)\left(\frac{3}{4}\right) = (40+1)\left(\frac{3}{4}\right) = 30.75$

Q3 $= \dfrac{133+136}{2} = 134.5$ Q3 $= 134.5$ hr

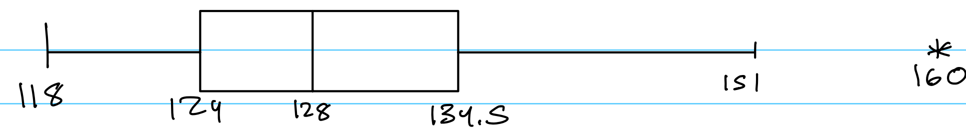
e) Min = 118

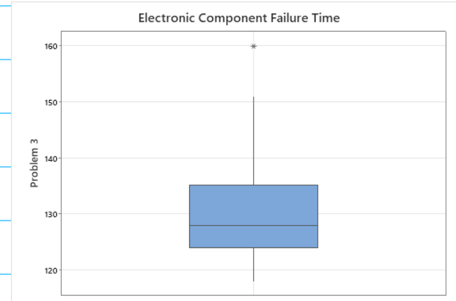
Q1 = 124

Q2 = 128

Q3 = 134.5

Max = 160





$UL = 134.5 + 1.5(134.5 - 124)$

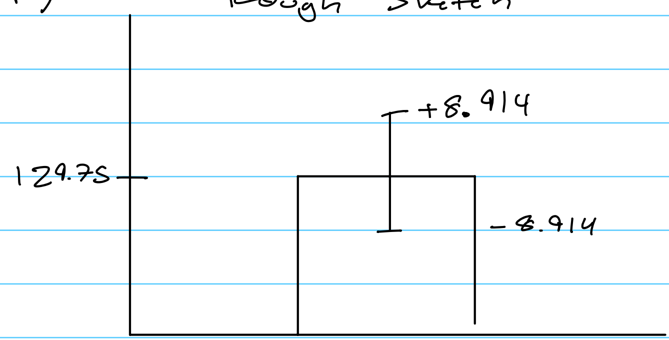
$= 150.25$ round to 151

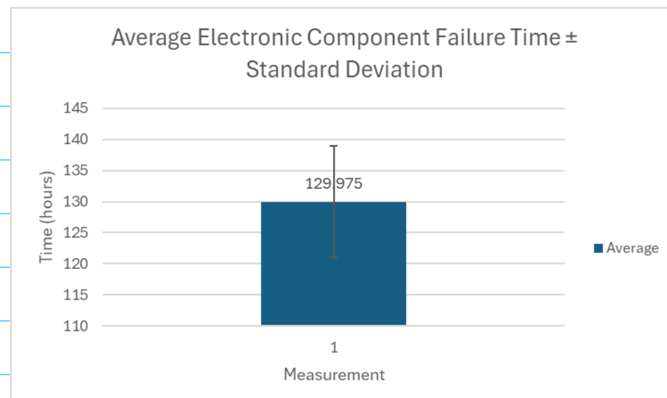
$BL = 124 - 1.5(134.5 - 124)$

$= 108.25$ do not use

From the box plot, it is clear that the data is skewed right. This means the tail to the right of the median is longer than the left tail. Due to the skewed nature, the median is less than the mean. There is an outlier of 160 in the data set! It is clearly not normally distributed.

f) Rough Sketch

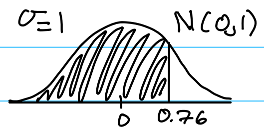




just said excel required for this problem

4. All values from the back of the book

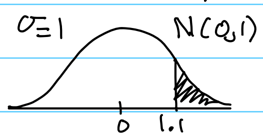
a) $P(Z \le 0.76)$



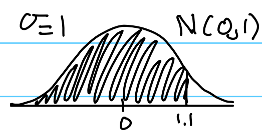
$= 0.77637$

0.78

b) $P(Z > 1.1)$



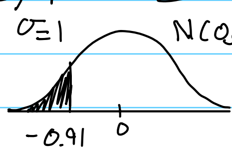
$= 1 - P(Z \le 1.1)$

$1 -$ 

$= 1 - 0.86433$

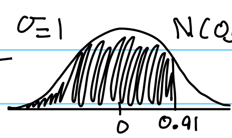
$= 0.1357$ 0.14

c) $P(Z \le -0.91)$



$= P(Z \ge 0.91) =$

$1 - P(Z \le 0.91)$

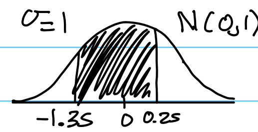
$1 -$ 

$= 1 - 0.81859$

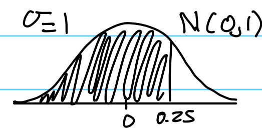
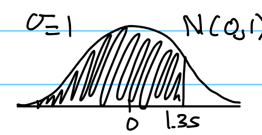
$= 0.1814$

0.18

d) $P(-1.35 \le Z \le 0.25) = P(Z \le 0.25) - P(Z \le -1.35)$



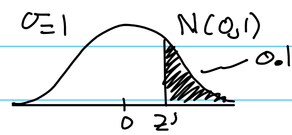
$= P(Z \le 0.25) - (1 - P(Z \le 1.35))$

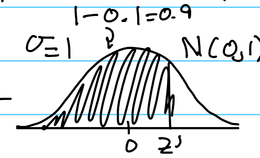
$\Rightarrow$  $- \left(1 -\right.$  $\left.\right)$

0.51

$= 0.59871 - (1 - 0.91149) = 0.5102$

e) $P(Z > z') = 0.1 \Rightarrow 0.9 = P(Z < z')$



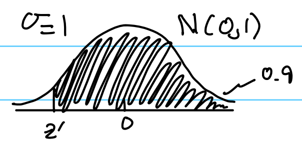
$\Rightarrow 1 -$ 

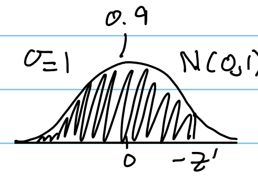
$1 - 0.1 = 0.9$

$z = 1.28$

value = 0.90320

f) $P(Z > z') = 0.9$

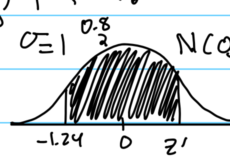


$\Rightarrow$ 

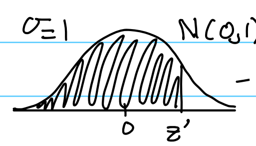
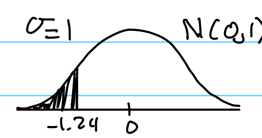
$-z = 1.3$

$z = -1.28$

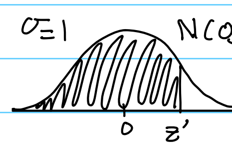
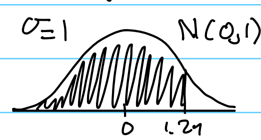
g) $P(-1.24 < Z < z') = 0.8$



$= P(Z \le z') - P(Z \le -1.24)$

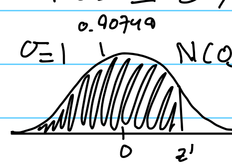
 $- ($  $)$

$\Rightarrow 0.8 = P(Z \le z') - (1 - P(Z \le 1.24))$

 $- (1 -$  $)$

$0.8 = P(Z \le z') - (1 - 0.89251)$

$\Rightarrow P(Z \le z') = 0.90749$



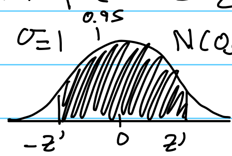
$z' = 1.33$

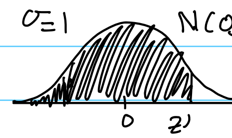
0.75 from 1.33
0.91 from 1.32

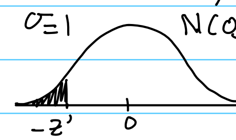
$z = 1.33$

value = 0.90824

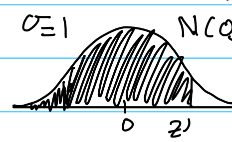
h) $P(-z' \le Z \le z') = 0.95 = P(Z \le z') - P(Z \le -z')$



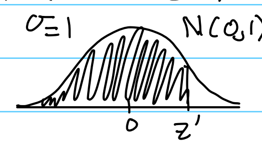




$= P(Z \le z') - (1 - P(Z \le z'))$



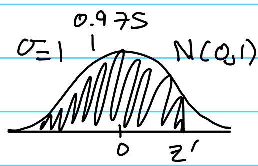
$- (1 -$



$)$

$\Rightarrow 0.95 = 2 \cdot P(Z \le z') - 1$

$P(Z \le z') = 0.975$

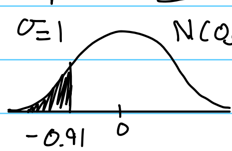


$z = 1.96$

value = 0.97500

i. Testing Minitab Output for part 'c', all others checked w/ calculator

$P(Z \le -0.91)$



Cumulative Normal $\Rightarrow$ Distribution

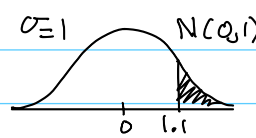
Full value	Rounded
0.181411	0.18

Normal with mean = 0 and standard deviation = 1

x	P(X ≤ x)
-0.91	0.181411

j. Integral for part b: $P(Z > 1.1)$

$$\boxed{P(z > 1.1) = \int_{1.1}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \, dz}$$

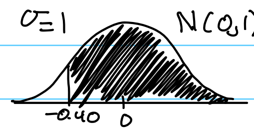


5. $\mu = 260$ minutes $\times \frac{1 \text{ hr}}{60 \text{ min}} = 4.3333$ hr Standardize to 0

$\sigma = 50$ minutes $\times \frac{1 \text{ hr}}{60 \text{ min}} = 0.8333$ hr Standardize to 1

a) $P(x \geq 4 \text{ hours})$

$z = \frac{x-\mu}{\sigma} = \frac{4-4.33}{0.8333} = -0.40$ so $P(z \geq -0.40)$
$= P(z \leq 0.40)$



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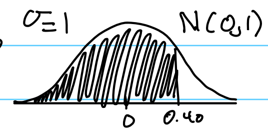
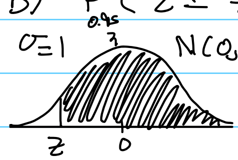
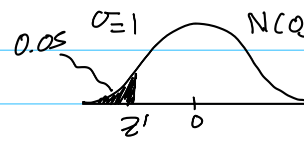
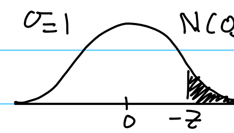


$\Rightarrow P(z \leq 0.40) = 0.65542$ 0.66

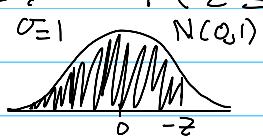
b) $P(Z \geq z) = 0.95 \Rightarrow 0.95 = 1 - P(Z \leq z) \Rightarrow 0.05 = P(z \geq -z)$







$\Rightarrow 0.95 = P(z \leq -z')$



$-z = 1.64$

$z = -1.64$

$-1.64 = \frac{x - 4.33}{0.8333}$

$x = 2.97$ hours

178.2 min, report to 178 min

c) $P(x>4)$, $N(4.33, 0.833)$

Normal with mean = 4.33333 and standard deviation = 0.833333

x	P(X ≤ x)
4	0.344578

For part c, the reported value is (1 − value)

$1 - 0.344578 = 0.655422$

Rounds to 0.66

$P(z \geq z') = 0.95$, $N(4.33, 0.833)$

Normal with mean = 4.33333 and standard deviation = 0.833333

P(X ≤ x)	x
0.05	2.96262

For part b, reporting 0.05, then convert hours to minutes

$2.96262 \times \frac{60 \text{ min}}{1 \text{ hr}} = 177.78$

Rounds to 178 min

6.

Table 3E.1 Electronic Component Failure Time

127	124	121	118
125	123	136	131
131	120	140	125
124	119	137	133
129	128	125	141
121	133	124	125
142	137	128	140
151	124	129	131
160	142	130	129
125	123	122	126

Rank 1

Rank 1 Value: 118, $n = 40$

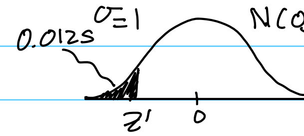
Cumulative Freq: $\frac{i-0.5}{n}$

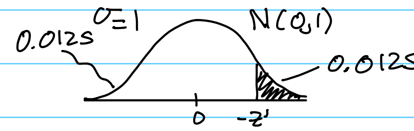
$\rightarrow \frac{1-0.5}{40} = 0.0125$

$P(z \le z_i) = 0.0125$

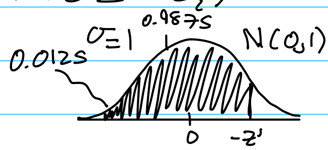
$= P(z \ge -z_i) = 0.0125$

$= 1 - P(z \le -z_i)$



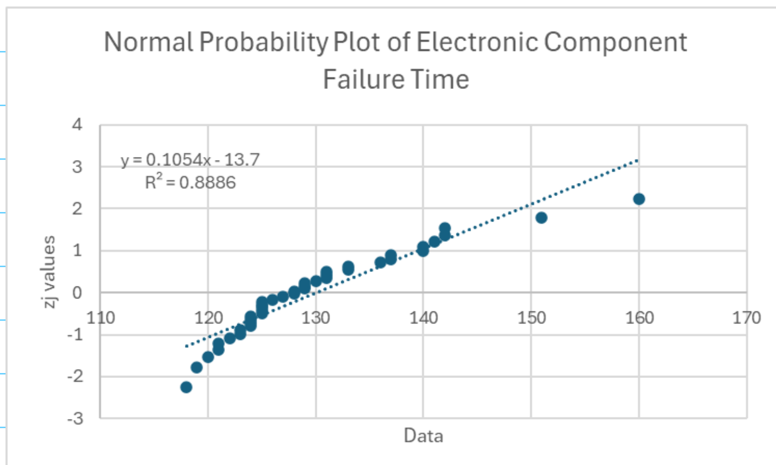


$P(z \le -z_i) = 0.9875$

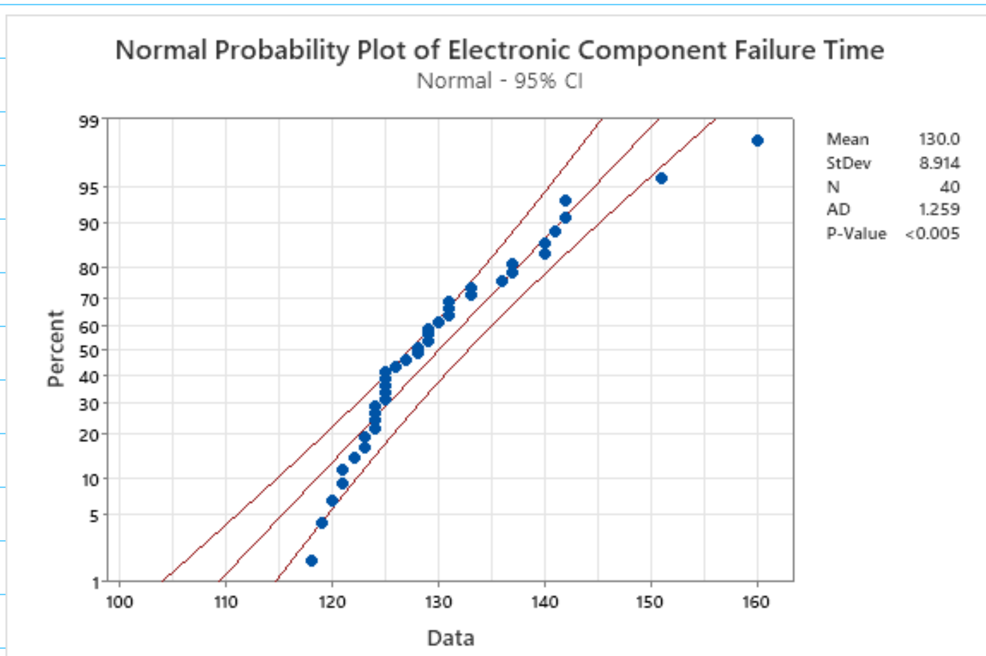


$\Rightarrow$ $-z_i = 2.24$

$z_i = -2.24$



The normal assumption is not confirmed or denied by this plot. Outliers and obviously non-linear data lead me to believe that the data is not normal, but minitab will confirm this.



The data is not normal within a 95% confidence Interval.

The p-value is reported as < 0.005, so we can not assume normality. Assuming $\alpha = 0.1$, p is much less than this!

HW3

1) The type of reference distribution (e.g., normal dist., t-distribution among others), draw the shape and write the defining parameters (e.g., μ, σ among others)
2) Show where the test statistic lies on your sketch
3) For solving using fixed significance level method – show the acceptance region, rejection region, alpha, as well as the critical value
4) For p-value method, shade in the p-value and report that value

1) PVC pipe is manufactured with a population mean diameter of 1.01 inch and a standard deviation of 0.00300 inch. Find the probability that a random sample of n = 9 sections of pipe will have a sample mean diameter greater than 1.009 inch and less than 1.012 inch. Report to 0.000 accuracy. Remember to provide a sketch of this probability problem as part of your answer.
2) Output from a software package follows (Z is the test statistic) – Report to 0.000 accuracy.

One-Sample Z:

Test of mu = 20 vs > 20

The assumed standard deviation = 0.75

Variable	N	Mean	SE Mean	Z	P
x	10	19.889	0.237	?	?

   a) Fill in the missing items (Z: the test statistic and P: the p-value). Calculate by hand. Make a sketch of the reference distribution (with its parameters defined), labeling the test statistic and the p-value.
   b) Provide a Minitab output which has these items (perform the same test that was in this chart)
   c) Is this a 1-sided or 2-sided test?
   d) Use the table in your book appendix, and the data in this table to construct a 95% 2-sided confidence interval on the mean. Comment on what the 95% CI really means (we are 95% confident that ...? And what does 95% confident mean?).
   e) What would the P-value be if the alternative hypothesis was H_1: $\mu \neq 20$? Calculate by hand, and provide a sketch of the reference distribution with the proper labels to illustrate your answer.
3) Medical researchers have developed a new artificial heart constructed of primarily titanium and plastic. The heart will last and operate almost indefinitely once it is implanted into a patient's body, but the battery pack needs to be recharged every four hours. A random sample of 50 battery packs is selected and subjected to a life test. The average life of the samples is 4.05 hours. Assume that batter life is normally distributed, with a standard deviation of 0.20 hours.
   a. Is there evidence to support the claim that the mean battery life of the population exceeds 4 hours? Remember to provide a sketch to illustrate your answer. Use α=0.05
   b. Construct a 1-sided, lower bound confidence interval on the mean with α=0.05 by hand, and explain what it represents. (Report this with to an accuracy of 0.00)
   c. Check these answers in Minitab and report the outputs.
4) Do problem 4.5 (a-b) in your book by hand (4.7 in the 7th edition), and then show a Minitab output for this test as part c (no need to do the part c in the book). Comment on what the p-value means in part b (i.e., what is the definition of a p-value). Assume 3 significant digits for reporting. Provide a sketch with the proper labels to illustrate your answer.
5) Explain the difference between type I and type II error, and their relationship to the convention that a rejection of the null hypothesis is considered a strong conclusion, but not rejecting is considered weak. Use the one sample z-test to explain, and drawings.

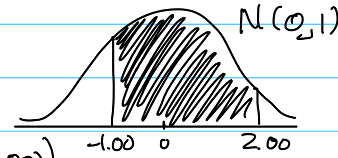
1. $\mu = 1.01\,in$ $\quad P(1.009\,in < \bar{x} < 1.012\,in) = ?$

$\sigma = 0.00300\,in$ $\quad \sigma_{\bar{x}} = \dfrac{0.00300\,in}{\sqrt{9}} = 0.00100\,in$ $\quad P(-1.00 \le Z \le 2.00)$

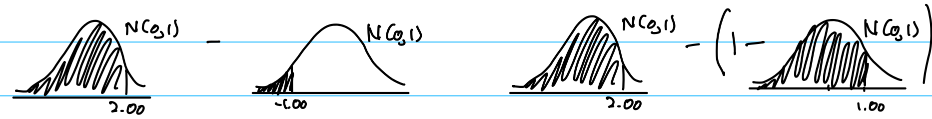
$n = 9$

$$Z_L = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{1.009 - 1.01}{0.00100} = -1.00$$

$$Z_u = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{1.012 - 1.01}{0.00100} = 2.00$$



$= P(z \le 2.00) - P(z \le -1.00) = P(z \le 2.00) - (1 - P(z \le 1.00))$



$= 0.97725 - (1 - 0.84134)$

$= \boxed{0.819}$

2.

One-Sample Z:					
Test of mu = 20 vs > 20					
The assumed standard deviation = 0.75					
Variable	N	Mean	SE Mean	Z	P
x	10	19.889	0.237	?	?

$Z = -0.468$

$P = 0.681$

a) Parameter of Interest: μ, Inferring pop. mean, known variance (std dev given)

$H_0: \mu = 20$ (One-Sample Z-test given)

$H_a: \mu > 20$ (one-sided)

Test Statistic: $Z_0 = \dfrac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \dfrac{19.889 - 20}{\frac{0.75}{\sqrt{10}}} = -0.468$

Reference Dist: $N(0,1)$

Reject if: $Z_0 > Z_\alpha = Z_{0.05}$

$\Rightarrow P(Z \le Z_\alpha) = 0.95$

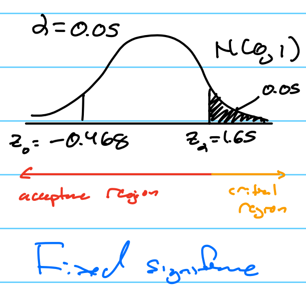
$\Rightarrow Z_\alpha = 1.65$

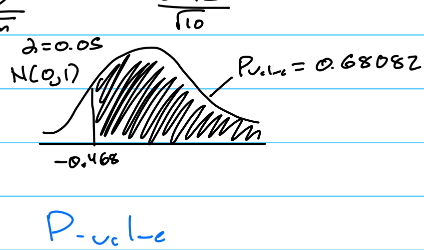
$\Rightarrow P_{value} = 1 - \Phi(Z_0) = 1 - P(Z \le Z_0)$

$= 1 - P(Z \le -0.468)$

$= 1 - (1 - P(Z \le 0.468))$

$= 1 - (1 - 0.68082) = 0.68082$





Since $z_0 < z_\alpha$ $(-0.468 < 1.65)$ and $P = 0.681 > 0.05$, we fail to reject H_0. There is insufficient evidence to deny the claim that the population mean is 20.

b) **Descriptive Statistics**

N	Mean	SE Mean	95% Lower Bound for μ
10	19.889	0.237	19.499

μ: population mean of Sample
Known standard deviation = 0.75

Test

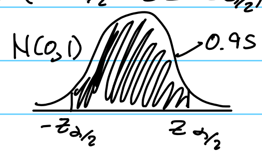
Null hypothesis	H_0: μ = 20
Alternative hypothesis	H_1: μ > 20

Z-Value	P-Value
-0.47	0.680

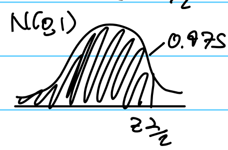
c) One-sided test as indicated in part (a)

d) 95% CI $\rightarrow \alpha = 0.05$

$P(-z_{\alpha/2} \le Z \le z_{\alpha/2}) = 0.95 \rightarrow P(Z \le z_{\alpha/2}) = 0.975 \rightarrow z_{\alpha/2} = 1.96$



$\rightarrow$



$\bar{x} = 19.889$

$\sigma = 0.75$

$n = 10$

$\text{Lower} = 19.889 - 1.96 \frac{0.75}{\sqrt{10}} = 19.424$

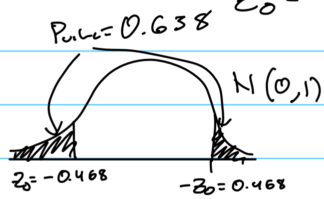
$\text{Upper} = 19.889 + 1.96 \frac{0.75}{\sqrt{10}} = 20.354$

$P\{19.424 \le \mu \le 20.354\} = 0.95$

In repeated sampling, this sampling method will produce intervals that capture the true population mean about 95% of the time.

e) Same setup as in part (c), but $H_a: \mu \neq 20$

Test statistic: $z_0 = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{19.889 - 20}{\frac{0.75}{\sqrt{10}}} = -0.468$ Reject if p-value $< \alpha = 0.05$



$$\text{P-value} = 2(1 - \Phi(|-0.468|))$$
$$= 2(1 - P(Z \le 0.468))$$
$$= 2(1 - 0.68082) = 0.63836$$

P-value = 0.638

Still fail to reject

3. a) Parameter of interest: μ, Inference about mean of population, known standard deviation

$H_0: \mu = 4$ hours   use $\alpha = 0.05$, One-sample Z-test

$H_a: \mu > 4$ hours

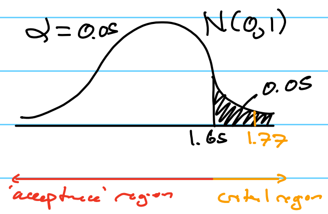
Given: $\bar{x} = 4.05$ hours, $\sigma = 0.20$ hours, $N = 50$ } normally distributed

Test Statistic: $Z_0 = \dfrac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \dfrac{4.05 - 4}{\frac{0.20}{\sqrt{50}}} = 1.77$

Ref. Dist: $N(0,1)$

Rejection Criteria: Reject if $Z_0 > Z_\alpha = Z_{0.05} = 1.65$

↳ value copied from Z-ca   critical value

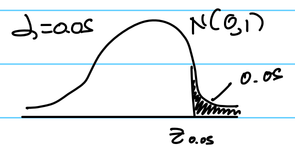


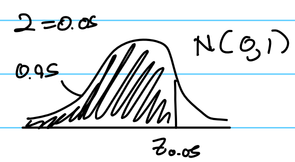
Conclusion: $Z_0 > Z_{0.05}$, $1.77 > 1.65 \rightarrow$ Reject H_0

We reject the null hypothesis. We have evidence to support the claim that the mean battery life of the population exceeds 4 hours.

b) One sided: $\bar{x} - Z_\alpha \frac{\sigma}{\sqrt{n}} \le \mu$, $\alpha = 0.05$

$Z_{0.05}: P(Z \ge Z_{0.05}) = 0.05 \rightarrow P(Z \le Z_{0.05}) = 0.95$





$Z_{0.05} = 1.65$

Lower $= 4.05 - 1.65 \dfrac{0.20}{\sqrt{50}} = 4.00$ 95% 1-sided CI: $4.00 \le \mu$

In repeated samples, we would expect similar methods to produce an interval that captures the true mean 90% of the time.

c)

Descriptive Statistics

N	Mean	SE Mean	95% Lower Bound for μ
50	4.0500	0.0283	4.0035

μ: population mean of Sample
Known standard deviation = 0.2

Test

Null hypothesis	H_0: μ = 4
Alternative hypothesis	H_1: μ > 4

Z-Value	P-Value
1.77	0.039

4. a) Parameter of interest: μ, inferring mean of population w/ known standard deviation

Diameter of Bearings (cm) Given:

$\sigma = 0.002$ cm

$\bar{x} = 8.25350$ cm

$N = 15$

$H_0: \mu = 8.25$ cm

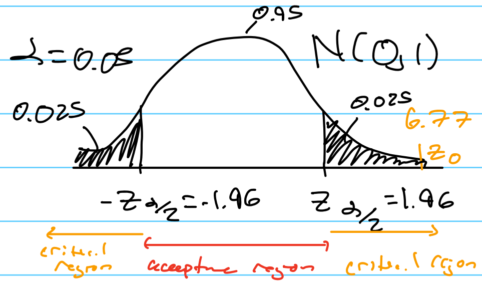
$H_a: \mu \neq 8.25$ cm

One-Sample Z-test

$\alpha = 0.05$

Test Statistic: $Z_0 = \dfrac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{N}}} = \dfrac{8.2535 - 8.25}{\frac{0.002}{\sqrt{15}}}$

$= 6.77$

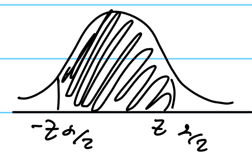


Reject if: $\begin{cases} Z_0 > Z_{\alpha/2} \\ Z_0 < -Z_{\alpha/2} \end{cases}$

$P(-Z_{\alpha/2} < Z < Z_{\alpha/2}) = 0.95$

$\Rightarrow P(Z < Z_{\alpha/2}) = 0.975$

$Z_{\alpha/2} = 1.96$



Conclusion: $Z_0 > Z_{\alpha/2} \rightarrow 6.77 > 1.96 \rightarrow$ Reject H_0

We reject the null hypothesis. We have sufficient evidence to reject the claim that the mean bearing diameter is 8.25 cm. We have evidence to support the claim that the mean bearing inside diameter is not 8.25 cm.

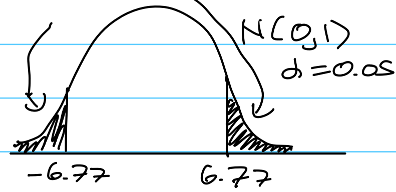
b) $P\text{-value} = 2(1 - \Phi(|Z_0|))$

$= 2(1 - P(Z < 6.77))$

$= 2(1 - 1)$ normalcdf(-99999, 6.77, 0, 1) = 1

Pvalue = 0 $= 0$

Had to use calculator as 6.77 exceeds table limits



What does the p-value mean

If the null hypothesis is true, where $\mu = 8.25$ cm, there is a 0 probability of obtaining another sample as extreme as the one obtained. This leads us to reject H_0.

Conclusion: Since $P\text{-value} < \alpha$ ($0 < 0.05$), we reject H_0. We have strong evidence to reject the claim that the mean bearing inner diameter is 8.25 cm. Our evidence supports the claim that the true mean diameter is really not 8.25 cm.

c) **Descriptive Statistics**

N	Mean	SE Mean	95% CI for μ
15	8.25350	0.00052	(8.25249, 8.25451)

μ: population mean of Sample
Known standard deviation = 0.002

Test

Null hypothesis	H_0: μ = 8.25
Alternative hypothesis	H_1: μ ≠ 8.25

Z-Value	P-Value
6.78	0.000

5. A type I error, defined α, is described by the probability of rejecting the null hypothesis when it's actually true. This is in comparison to a type II error, defined β, which is described by the probability of the null hypothesis not being rejected when it is actually false.

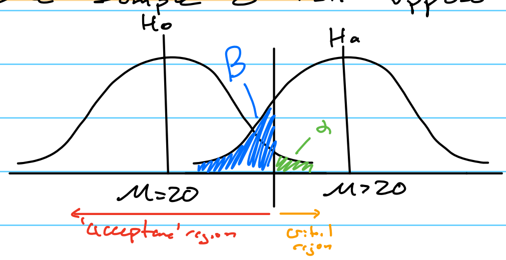
False Positive

→ Rejecting Ho is considered a strong conclusion as the rejection is based on α. If the p-value < α, then we reject Ho, and also minimize type I errors assuming α is small (done by convention). Since type I errors are controlled by α, it is ensured that the chance of rejecting a true null is low.

False Negative

→ Failing to reject is considered weak because it means p-value > α, which means we don't have evidence to reject Ho. This does not mean we can accept Ho as true. This could be due to small sample sizes, or other sampling errors. This probability is independent of α, and thus its risk is difficult to control.

One-sample Z-test: Suppose $H_0: \mu = 20$, $H_a: \mu > 20$



= type I error → Ho true but sample mean rejected

= type II error → Sample mean falls in acceptance region despite true mean being shifted. leads to type II error

HW4

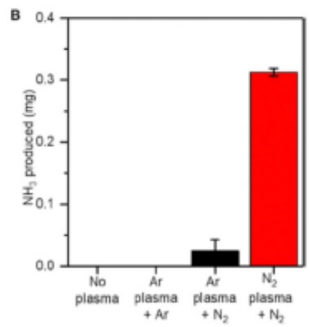
1) Do problem 4.8 in the book (4.11 in 7th edition). Do parts a and b by hand (write out how you would find average and standard deviation, but feel free to use the (….) notation frequently used in class.) For part c use Minitab to check the normality assumption and also check your answers in part a and b. Comment on the normality assumption. Use 4 significant figures.
2) Consider the following computer output (assume 3 significant figures).
   a. How many degrees of freedom would you use to find your critical value?
   b. Fill in the missing values by hand. You may find the bounds on the P-value using the table in the back of the book. What conclusions could you draw?
   c. If the hypothesis is H_0: $\mu = 12$ versus H_1: $\mu > 12$ would your conclusions change?

One-Sample T:

Test of mu = 12 vs not = 12

Variable	N	Mean	StDev	SE Mean	T	P
x	10	12.564	.936	0.296	?	?

3) This is real data an undergraduate in the CWRU chemical engineering department gathered for a paper in *Science Advances.* The student was measuring plasma-generated ammonia in a lab scale apparatus. They wanted to test the hypothesis the mean of the ammonia generated under the conditions (Ar plasma + N_2) is greater than zero. The following data for that group (black bar in graph) were obtained (in mg of ammonia made in 45 min): 0.007, 0.059, 0.023, 0.001, 0.049, 0.010 mg of ammonia. Describe the appropriate test to conduct for this hypothesis (write out H_o and H_1), and do the calculation in Minitab. Check the normality assumption in Minitab. Explain what conclusion you would come to after this analysis.



4) A research engineer for a tire manufacturer is investigating tire life for a new rubber compound and has built 16 tires and tested them to end-of-life in a road test. The sample mean and standard deviation are 60139.7 and 3645.94 kilometers.
   a. Can you conclude using $\alpha = 0.05$ that the standard deviation of tire life is less than 4000 kilometers? State any assumptions about the underlying distribution of the data and find the P-value for the test (estimate the range from the table in the back of the book).
   b. Construct a 95% upper bound confidence interval
   c. Show how you would solve this problem in Minitab
5) The output voltage of a power supply is assumed to be normally distributed. Sixteen observations taken at random on voltage are as follows: 10.35, 9.30, 10.00, 9.96, 11.65, 12.00, 11.25, 9.58, 11.54, 9.95, 10.28, 8.37, 10.44, 9.25, 9.38, 10.85.
   a. Test the hypothesis that the mean voltage equals 12V against a two-sided hypothesis assuming alpha =0.05 (do this in Minitab).
   b. Construct a 95% two-sided confidence interval on μ using Minitab.
   c. Test the hypothesis that $\sigma^2 = 11$ using alpha =0.05 (by hand, then check this in Minitab)
   d. Construct a 95% two-sided confidence interval on σ (by hand, and then check in Minitab)
   e. Does the assumption of normality seem reasonable for the output voltage? (use Minitab to answer)
   f. Assuming the specification limits are 10 volts over or under the target, use the CI for σ to determine if this is a “six sigma” process.

4.8 in textbook

1. Ten wafers

13.3987 $\times 1000 \mathring{A}$
13.3957
13.3902
13.4015
13.4001
13.3918
13.3965
13.3925
13.3946
13.4002
(units of $\times 1000 \mathring{A}$)

Sample Statistics $(n=10)$

$$\bar{x} = \frac{13.3987 + 13.3957 + \dots + 13.4002}{10} = 13.3962$$

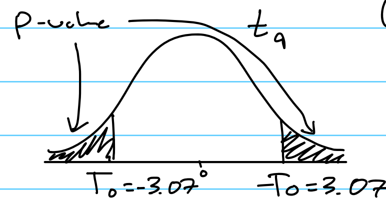
$$s = \sqrt{\frac{(13.3987-13.3902)^2 + \dots (13.4002-13.3902)^2}{10-1}} = 0.0039086$$

a) (i) Inference on mean thickness, variance unknown
↳ one-sample t-test w/ $\alpha = 0.05$

(ii) $H_0: \mu = 13.4 \times 1000 \mathring{A}$ (iii) $H_a: \mu \neq$

(iv) $T_0 = \dfrac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{13.3962 - 13.4}{\frac{0.0039086}{\sqrt{10}}} = -3.074$

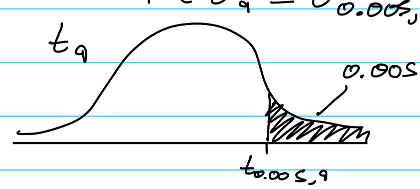
(v) Reject if p-value $< \alpha$



(vi) p-value $= 2(P\{t_9 > -T_0\})$
$= 2(P\{t_9 > 3.07\})$
$= 2((0.005, 0.01))$
p-value $\in (0.01, 0.02)$

(vii) P-value $\in (0.01, 0.02) < 0.05$ reject H_0.
We have sufficient evidence to reject the claim that the mean thickness is $13.4 \times 1000 \mathring{A}$. We can thus accept H_a, having evidence that the true mean is not $13.4 \times 1000 \mathring{A}$.

b) $t_{\alpha/2, 9}: P(t_9 \geq t_{\alpha/2, 9}) = \alpha/2$
$\rightarrow P(t_9 \geq t_{0.005, 9}) = 0.005$



$\Rightarrow t_{0.005, 9} = 3.250$

In repeated sampling, this method will produce intervals that capture the true mean about 99% of the time.

$$L = 13.3962 - 3.250 \frac{0.0039086}{\sqrt{10}} = 13.3922$$
$$U = 13.3962 + 3.250 \frac{0.0039086}{\sqrt{10}} = 13.4002$$

99% two-sided CI $13.39 \leq \mu \leq 13.40$

c) Based on the normal probability plot, the p-value is 0.711. This is much larger than any reasonable value of α, so we do not have any evidence to reject that these data are normally distributed.

1. c) Minitab output

Sample Data

Statistics

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1
Thickness Measurements (Ax10^3)	10	0	13.3962	0.0012360	0.0039086	13.3902	13.3923

Variable	Median	Q3	Maximum
Thickness Measurements (Ax10^3)	13.3961	13.4001	13.4015

Section a

Descriptive Statistics

N	Mean	StDev	SE Mean	95% CI for μ
10	13.3962	0.0039	0.0012	(13.3934, 13.3990)

μ: population mean of Thickness Measurements (Ax10^3)

Test

Null hypothesis H₀: μ = 13.4

Alternative hypothesis H₁: μ ≠ 13.4

T-Value	P-Value
-3.09	0.013

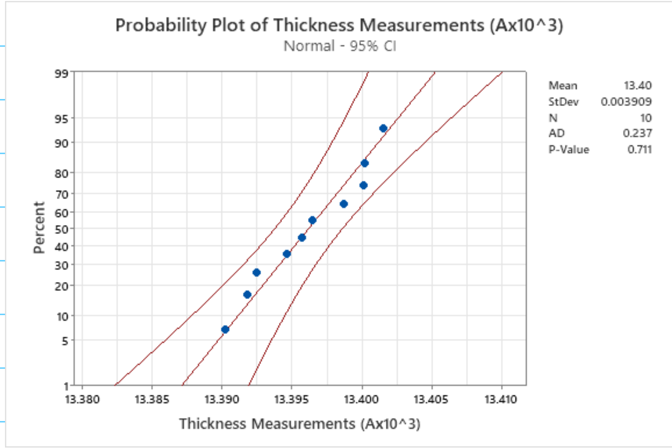
Section b

Descriptive Statistics

N	Mean	StDev	SE Mean	99% CI for μ
10	13.3962	0.0039	0.0012	(13.3922, 13.4002)

μ: population mean of Thickness Measurements (Ax10^3)

Section c



2.

One-Sample T:

Test of mu = 12 vs not = 12

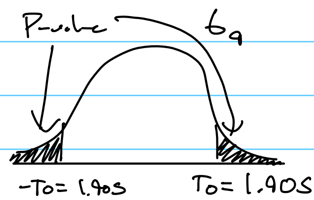
Variable	N	Mean	StDev	SE Mean	T	P
x	10	12.564	.936	0.296	?	?

a) DOF = N-1 = 10-1 = 9

t-dist w/ 9 DOF

b) $T_0 = \dfrac{\bar{x}-\mu}{\frac{s}{\sqrt{N}}} = \dfrac{12.564-12}{\frac{0.936}{\sqrt{10}}} = 1.905$

Reject if p-value < $\alpha = 0.05$

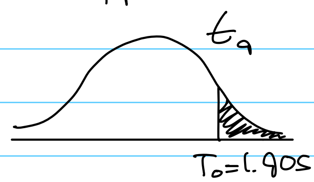


$$\text{P-value} = 2(P\{t_9 > T_0\})$$
$$= 2((0.025, 0.05))$$
$$\text{P-value} \in (0.05, 0.1)$$

$T_0 = 1.91$
P-value $\in (0.05, 0.1)$

The upper and lower bounds of my p-value exceed $\alpha = 0.05$, so we fail to reject H_0. We do not have sufficient evidence to reject the claim that the sample mean is 12.

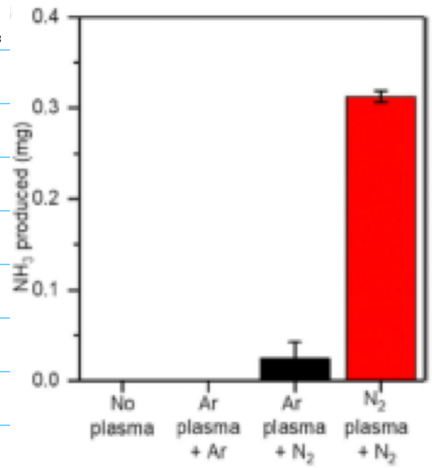
c) Suppose $H_0: \mu = 12$, $H_a: \mu > 12$



$$\text{P-value} = P\{t_9 > T_0\}$$
$$= (0.025, 0.05)$$
$$\text{P-value} \in (0.025, 0.05)$$

Yes, choosing this H_a yields a p-value whose bounds are both less than $\alpha = 0.05$. In this case, we reject H_0 and accept H_a, having sufficient evidence to accept the claim that the true mean exceeds 12.

3.



Data (in mg of Ammonia made in 45 min)

0.007, 0.059, 0.023, 0.001, 0.049, 0.010

Inference on mean amount of Ammonia, unknown variance of population → One-Sample t-test

H_0: $\mu = 0$ mg of Ammonia

H_a: $\mu > 0$ mg of Ammonia

Use t-distribution as t_5 (5 D.F)

Minitab Hypothesis test @ $\alpha = 0.05$

Descriptive Statistics

N	Mean	StDev	SE Mean	95% Lower Bound for μ
6	0.02483	0.02392	0.00977	0.00516

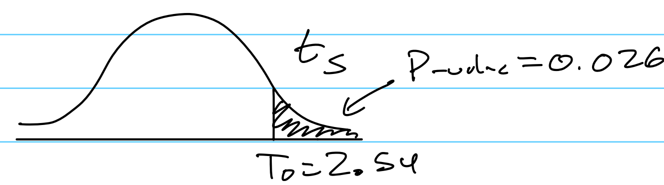
μ: population mean of mg of Ammonia

Test

Null hypothesis H_0: $\mu = 0$

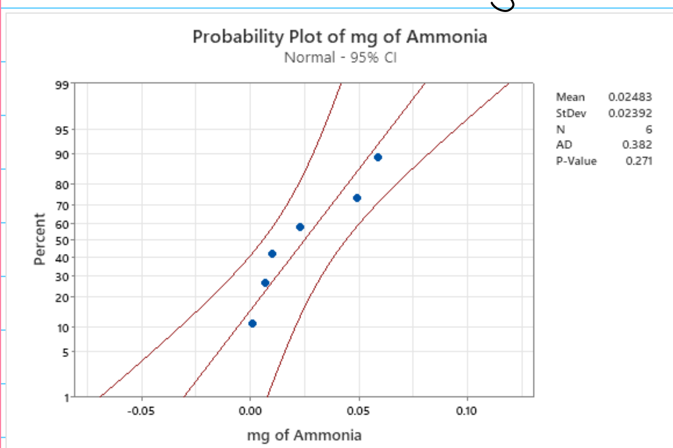
Alternative hypothesis H_1: $\mu > 0$

T-Value	P-Value
2.54	0.026



P-value = 0.026 < α = 0.05, so we reject H_0. We have sufficient evidence to accept H_a, in that the mean mg of Ammonia generated exceeds 0 mg.

Minitab Normal Probability Plot



P-value for normality test is 0.271 > 0.05. Therefore, we have insufficient evidence to reject the claim that the population is normally distributed. Seeing this, I would proceed with the t-test given normality is unable to be rejected.

4. Data (Tire Life in Km)

$\bar{x} = 60139.7$ Km

$s = 3645.94$ Km

$N = 16$

a) We are inferring about the variance of one population, so we will use the 1-variance test.

Parameter of Interest: σ^2

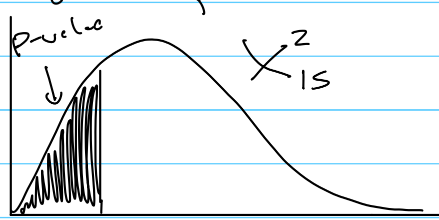
χ^2 tests are contingent on normality, I will assume that these data are roughly normal by the central limit theorem as N=16 tires is reasonably large. The χ^2 distribution is sensitive to this normality assumption.

$H_0: \sigma = 4000$ Km

$H_a: \sigma < 4000$ Km

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(16-1)(3645.94)^2}{(4000)^2} = 12.4623$$

Reject if p-value < $\alpha = 0.05$



$\chi_0^2 = 12.4623$

P-value $= 1 - P\{\chi^2_{15} > \chi^2_0 = 12.4623\}$

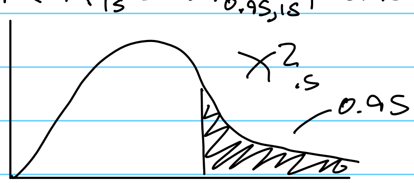
$= 1 - (0.950, 0.500)$

P-value $\in (0.05, 0.5)$

We found a p-value range of (0.05, 0.5) whose bounds exceed $\alpha = 0.05$. Therefore, we fail to reject H_0, having insufficient evidence to reject the claim that the std dev of the tires is 4000 Km.

b) $$\sigma \le \sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha,n}}} = \sqrt{\frac{(16-1)(3645.94)^2}{7.26}} = 5240.67 \text{ Km}$$

$P(\chi^2_{15} \ge \chi^2_{0.95,15}) = 0.95$



$\chi^2_{0.95,15} = 7.26$

$\sigma < 5240.67$ Km

In repeated sampling this method will produce intervals that capture the mean about 95% of the time.

5. Data (N=16) (Volts)

10.35	11.54
9.30	9.95
10.00	10.28
9.96	8.37
11.65	10.44
12.00	9.25
11.25	9.38
9.58	10.85

a-b) Inference on mean voltage, unknown variance → one sample t-test.

$H_0: \mu = 12V$ $H_a: \mu \neq 12V$

Descriptive Statistics

N	Mean	StDev	SE Mean	95% CI for μ
16	10.259	0.999	0.250	(9.727, 10.792)

μ: population mean of Output V of Power Supply

Test

Null hypothesis $H_0: \mu = 12$
Alternative hypothesis $H_1: \mu \neq 12$

T-Value	P-Value
-6.97	0.000

Reject H_0 => p-value = 0 < α = 0.05. We have sufficient evidence to reject the claim that the mean voltage output is 12V. We accept that it is not equal to 12V.

c) P-value = 0

b) $9.727 \leq \mu \leq 10.792$

↑ 95% CI on μ

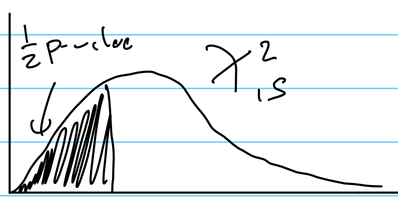
Interpreted simply, this method will produce intervals that capture the true mean 95% of the time.

c) Inference on one population's variance. 1-variance test using $\alpha = 0.05$

$H_0: \sigma^2 = 11\ V^2$ $H_a: \sigma^2 \neq 11\ V^2$

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(16-1)(0.999)^2}{11} = 1.361$$

Reject if p-value < α = 0.05



$\chi_0^2 = 1.361$

$$\text{P-value} = 2(1 - P\{\chi^2_{14} > 0.1237\})$$
$$= 2(1 - (1, 0.995))$$
$$\text{P-value} < 2(0.005)$$
$$\text{P-value} < 0.01$$

P-value < 0.01 < α = 0.05 so reject H_0. We have evidence to reject the claim that the voltage variance is $11\ V^2$. We accept the alternate hypothesis that it is different.

Descriptive Statistics

N	StDev	Variance	95% CI for σ using Bonett	95% CI for σ using Chi-Square
16	0.999	0.998	(0.749, 1.518)	(0.738, 1.546)

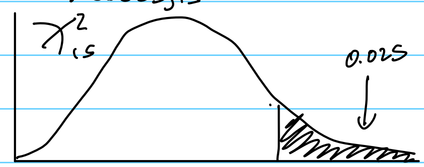
Test

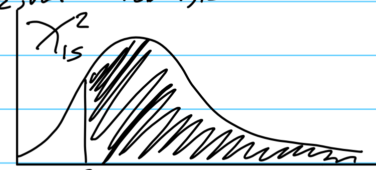
Null hypothesis $H_0: \sigma^2 = 11$
Alternative hypothesis $H_1: \sigma^2 \neq 11$

Method	Test Statistic	DF	P-Value
Bonett	—	—	0.000
Chi-Square	1.36	15	0.000

} agrees

d) $\chi^2_{\alpha/2,n-1} : \chi^2_{0.025,15} = 27.49$ $\chi^2_{1-\frac{\alpha}{2},n-1} : \chi^2_{0.975,15} = 6.27$





$\chi^2_{0.025,15} = 27.49$ $\chi^2_{0.975,15} = 6.27$

$$\text{Lower} = \frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}} = \frac{(16-1)(0.999)^2}{27.49} = 0.5446\ V^2 \rightarrow \sigma = 1.55\ V$$

$$\text{Upper} = \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2},n-1}} = \frac{(16-1)(0.999)^2}{6.27} = 2.388\ V^2 \rightarrow \sigma = 0.738\ V$$

$0.738 < \sigma < 1.55$

Descriptive Statistics

N	StDev	Variance	95% CI for σ using Bonett	95% CI for σ using Chi-Square
16	0.999	0.998	(0.749, 1.518)	(0.738, 1.546)

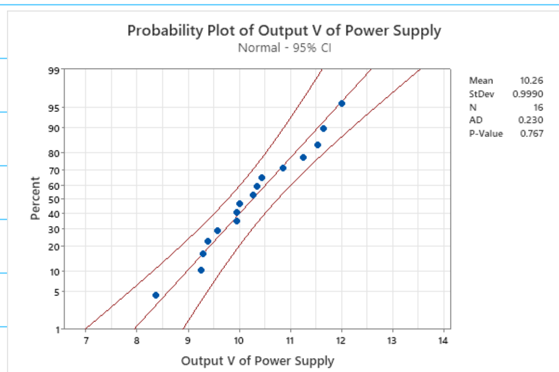
agrees

In repeated sampling, this method will result in intervals that capture the true mean 95% of the time.

Test

Null hypothesis $H_0: \sigma^2 = 11$
Alternative hypothesis $H_1: \sigma^2 \neq 11$

Method	Test Statistic	DF	P-Value
Bonett	—	—	0.000
Chi-Square	1.36	15	0.000

e) 

Minitab reports a p-value of 0.767 for normality of these data. Because 0.767 > α = 0.05, we do not have sufficient evidence to reject that these data are normally distributed. Thus, using χ^2 as a reference dist. is valid!!

f) Use $0.738 < \sigma < 1.55$ to get sigma level range

$$\text{Sigma level} = \frac{|USL - LSL|}{\sigma}$$

(i) $\frac{10}{0.738} = 13.55$

(ii) $\frac{10}{1.55} = 6.45$

Both bounds of our CI yield sigma levels greater than 6, so this is a six sigma process.

6.45 ← Upper bound

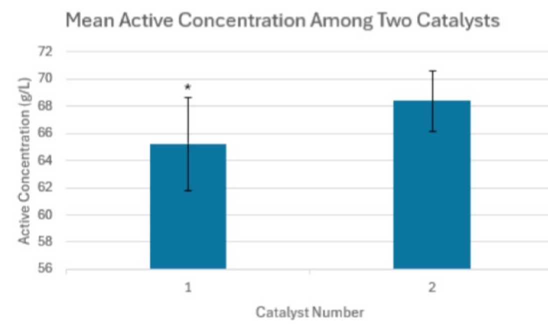
Upper bound of CI is greater than 6, so its a 6σ process.

HW5

1) The concentration of an active ingredient in a liquid laundry detergent is thought to be affected by the type of catalyst used in the process. The standard deviation of active concentration is known to be 3 g/L regardless of the catalyst type. 10 observations on concentration are taken with each catalyst, and the data follow:

Catalyst 1	Catalyst 2
57.9	66.4
66.2	71.7
65.4	70.3
65.4	69.3
65.2	64.8
62.6	69.6
67.6	68.6
63.7	69.4
67.2	65.3
71.0	68.8

a) Use Minitab to gather the descriptive statistics on these samples and determine if the normality assumption is appropriate.
b) Is there any evidence to indicate that the mean active concentrations depend on the choice of the catalyst? (use $\alpha = 0.05$)
c) Construct a 95% confidence interval for the difference between the means, and explain how this relates to the result in part b)
d) Construct a bar graph in Excel showing the results and the results of this hypothesis test and add a proper caption (see lecture 10)



Mean active concentration of liquid laundry detergent based on two different catalyst types. Data are represented as the mean ± standard deviation with n = 1. (*) represents p < 0.05 compared to the second catalyst's performance, using a two-sided, t-sample z-test in Minitab and by-hand with known population standard deviation of 3 g/L. We have sufficient evidence to reject the claim that the two catalysts yield the same concentrations, and accept that the concentration is dependent on catalyst choice.

2) Problem 4.11 (4.17 in edition 7) from your book a-c. Assume though instead of two technicians making measurements on the same part (as the problem statement implies) the data represent the surface finish of parts made in January (column to the left) and surface finish of parts made in February (column to the right), not technician 1 and technician 2. You want to know if there is a difference in the surface finish of these parts made in January vs. February. First though, construct a box plot and normal probability plot using Minitab to check if equal variance and normality are ok assumptions for this problem. You may use Minitab to calculate the average and standard deviation of your data as well, but the rest should be calculated by hand. Finally, use Minitab to check your final answer by running the appropriate hypothesis test.

3) Consider the following computer program output:

Two-Sample T-Test and CI

```
Sample    N      Mean     StDev   SE Mean
   1      15     54.73    2.13     0.55
   2      20     58.64    5.28     1.2
Difference = mu (1) - mu (2)
Estimate for difference: -3.91
95% upper bound for difference: ?
T-test of difference = 0(vs <): T-value =
-3.00 P-value = ? DF = ?
```

a) Fill in the missing values. Is this a one-sided or two sided test? Use upper and lower bounds to find the P-value.
b) What are your conclusions if α=0.05? α=0.01?
c) Suppose that the alternative hypothesis was $\mu_1 \neq \mu_2$, would your conclusions be different if α=0.05

4) Do problem 4.15 in your book (4.25 in the 7th edition). You may use Minitab to calculate the needed means and standard deviations but show the formulas in your answer for how you would obtain these (with some values subbed in) and the rest should be calculated by hand. Note that the inspectors are measuring the same part with the micrometer caliper, and the vernier caliper.

ECHE313 Homework 5 - Due 02/20/25 **Trevor Swan (tcs94)**

1.

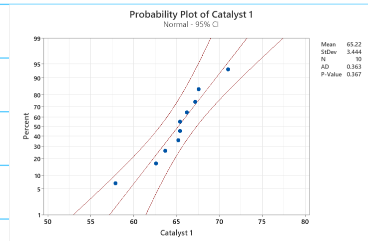
Catalyst 1	Catalyst 2
57.9	66.4
66.2	71.7
65.4	70.3
65.4	69.3
65.2	64.8
62.6	69.6
67.6	68.6
63.7	69.4
67.2	65.3
71.0	68.8

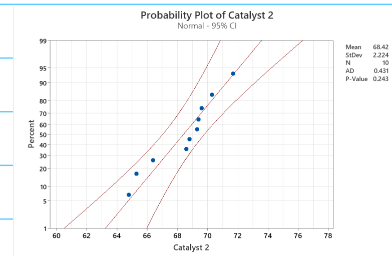
Population Std $= 3\ 9/2 = \sigma_1 = \sigma_2$

a)

Statistics

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
Catalyst 1	10	0	65.22	1.08922	3.44442	57.9	63.425	65.4	67.3	71
Catalyst 2	10	0	68.42	0.703294	2.22401	64.8	66.125	69.05	69.775	71.7





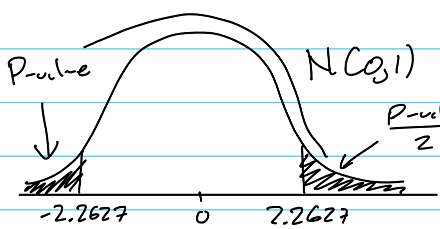
p-value > 0.05 for both catalysts' normality, so insufficient evidence to reject normality.

b) Inference on mean concentrations of two normally distributed populations w/ known variance

→ Two-sample Z $\quad H_0: \mu_1 = \mu_2 \quad H_a: \mu_1 \neq \mu_2$

$$Z_0 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n}}} = \frac{\cancel{68.42 - 65.22}\ \ 65.22 - 68.42}{\sqrt{\frac{3^2}{10} + \frac{3^2}{10}}} = \cancel{2.2627}\ \ -2.385$$

Reject if: P-value < $\alpha = 0.05$



$$P\text{-value} = 2(1 - P\{Z_0 < 2.2627\})$$
$$= 2(1 - 0.98809) = 0.02382$$

P-value = 0.022 $\alpha = 0.05$ ⇒ Reject H_0. We have sufficient evidence to reject the claim that the two catalysts yield the same concentrations. We accept that the catalyst choice impacts mean active concentrations.

c) $$\bar{x}_1 - \bar{x}_2 - Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \bar{x}_1 - \bar{x}_2 + Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$\text{Lower} = 68.42 - 65.22 - \cancel{2.2627}\ 2.385\sqrt{\frac{3^2}{10} + \frac{3^2}{10}} = \cancel{0.165}\ -5.83$$

$$\text{Upper} = 68.42 - 65.22 + \cancel{2.2627}\ 2.385\sqrt{\frac{3^2}{10} + \frac{3^2}{10}} = \cancel{6.23}\ -0.57$$

$\cancel{0.165 \le \mu_1 - \mu_2 \le 6.23}$

$-5.83 \le \mu_1 - \mu_2 \le -0.57$

In repeated sampling, this method will produce intervals that capture the true mean 95% of the time. As our interval does not cross 0, it supports our conclusion that the mean active concentration depends on catalyst choice.

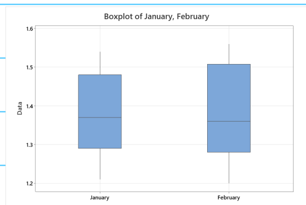
2\.

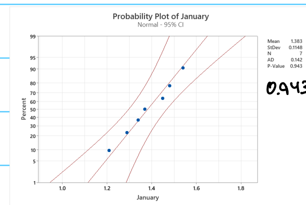
January	February
1.45	1.54
1.37	1.41
1.21	1.56
1.54	1.37
1.48	1.20
1.29	1.31
1.34	1.27
	1.35

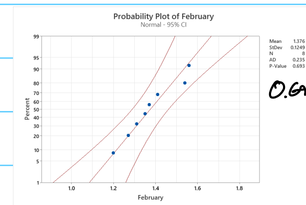
Problem 4.11, data btwn months, not techs

Statistics

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
January	7	0	1.38286	0.0434091	0.114850	1.21	1.29	1.37	1.48	1.54
February	8	0	1.37625	0.0441563	0.124893	1.2	1.28	1.36	1.5075	1.56







Distributed evenly and all about the same so assuming equivalence is reasonable

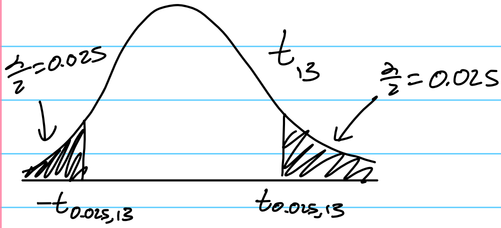
p-values both > 0.05, so we do not have evidence to reject that the data are normally distributed

a) Inference about mean surface finish, unknown variance, assume equal variance

$H_0: \mu_1 = \mu_2 \quad H_a: \mu_1 \neq \mu_2$

$$S_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(7-1)0.114850^2 - (8-1).124893^2}{7+8-2}} = 0.1204$$

$$\Rightarrow T_0 = \frac{1.38286 - 1.37625}{0.1204\sqrt{\frac{1}{7} + \frac{1}{8}}} = 0.106 \qquad DoF: 7+8-2=13$$



Reject if $\begin{cases} T_0 \geq t_{0.025,13} \\ T_0 \leq -t_{0.025,13} \end{cases}$

$t_{0.025,13} = 2.160$

$\Rightarrow$ Fail to Reject $0.106 < |2.160|$

Fail to reject H_0. We have insufficient evidence to claim that the mean surface finish varies between months of January and February

b) Based on our findings, I would conclude that the mean surface finish is independent of month, so work can be conducted in either Jan or Feb. If we rejected H_0, I would prevent work in the 'defect' month to prevent errors in products' surface finish.

c) $\bar{x}_1 - \bar{x}_2 - t_{\frac{\alpha}{2}, n_1+n_2-2}\, s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_2 \le$

$\bar{x}_1 - \bar{x}_2 + t_{\frac{\alpha}{2}, n_1+n_2-2}\, s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

$$\left.\begin{aligned} \text{Lower} &= 1.38286 - 1.37625 - 2.160\left(.1204\sqrt{\tfrac{1}{7} + \tfrac{1}{8}}\right) = -.128 \\ \text{Upper} &= 1.38286 - 1.37625 + 2.160\left(.1204\sqrt{\tfrac{1}{7} + \tfrac{1}{8}}\right) = .141 \end{aligned}\right\} -.128 \le \mu_1 - \mu_2 \le .141$$

In repeated sampling, this method will produce intervals that capture the true mean difference in surface finish 95% of the time. The interval crosses 0, so it supports our conclusion to fail to reject that the months are the same.

d) P-value $= 0.917 > \alpha = 0.05$, so fail to reject H_0. Conclusion is the same as stated in part (b), yay!

Test

Null hypothesis	H_0: $\mu_1 - \mu_2 = 0$
Alternative hypothesis	H_1: $\mu_1 - \mu_2 \neq 0$

T-Value	DF	P-Value
0.11	13	0.917

3.

Two-Sample T-Test and CI

Sample	N	Mean	StDev	SE Mean
1	15	54.73	2.13	0.55
2	20	58.64	5.28	1.2

Difference = mu (1) - mu (2)
Estimate for difference: -3.91
95% upper bound for difference: ?
T-test of difference = 0(vs <): T-value = -3.00 P-value = ? DF = ?

a) Two-Sample T-test

One-sided test:

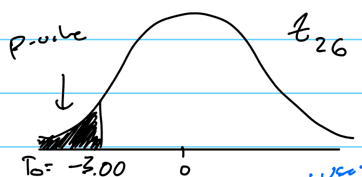
$H_0: \mu_1 - \mu_2 = 0$

$H_a: \mu_1 - \mu_2 > 0$

$$\frac{S_2^2}{S_1^2} = \frac{2.13^2}{5.28^2} = 6.14$$ Cannot use Pooled!

$$DF = \nu = \frac{\left(\frac{2.13^2}{15} + \frac{5.28^2}{20}\right)^2}{\frac{\left(\frac{2.13^2}{15}\right)^2}{15-1} + \frac{\left(\frac{5.28^2}{20}\right)^2}{20-1}} = 26.45 \rightarrow \boxed{DF = 26}$$

$\boxed{Upper = -1.69}$



P-value $= P\{t_{26} > 3.00\}$

btwn $(\overset{3.067}{0.0025}, \overset{2.779}{0.005})$

$\boxed{\text{P-value} \in (0.0025, 0.005)}$

$$Upper = \bar{x}_1 - \bar{x}_2 + \underset{\text{use t's dist.}}{t_{0.05,26}} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = 54.73 - 58.64 + 1.706\sqrt{\frac{2.13^2}{15} + \frac{5.28^2}{20}} = -1.69$$

b) $\alpha = 0.05 \rightarrow$ done above, p-value $\in (0.0025, 0.005) < 0.05 = \alpha$

$\alpha = 0.01 \rightarrow$ p-value $\in (0.0025, 0.005) < 0.01 = \alpha$

In both cases, p-value $< \alpha$, so we reject H_0. We have sufficient evidence to reject the claim that the mean difference is 0. We accept that it is less than 0 for both choices of α.

c) If $H_a: \mu_1 \neq \mu_2$, then p-value $\in 2(0.0025, 0.005)$

p-value $\in (0.005, 0.01)$

If $\alpha = 0.05$, the upper bound for our new interval is still less than alpha $(0.01 < 0.05)$, so we reject H_0 still. Our conclusion will be the exact same as found in (b).

4.

Inspector	Micrometer Caliper	Vernier Caliper	Difference
1	0.150	0.151	-0.001
2	0.151	0.150	0.001
3	0.151	0.151	0.000
4	0.152	0.150	0.002
5	0.151	0.151	0.000
6	0.150	0.151	-0.001
7	0.151	0.153	-0.002
8	0.153	0.155	-0.002
9	0.152	0.154	-0.002
10	0.151	0.151	0.000
11	0.151	0.150	0.001
12	0.151	0.152	-0.001

Statistics

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median
Micrometer Caliper	12	0	0.151167	0.0002410	0.0008348	0.15	0.151	0.151
Vernier Caliper	12	0	0.151583	0.0004680	0.0016214	0.15	0.15025	0.151
Caliper Difference	12	0	-0.0004167	0.0003786	0.0013114	-0.002	-0.00175	-0.0005

Variable	Q3	Maximum
Micrometer Caliper	0.15175	0.153
Vernier Caliper	0.15275	0.155
Caliper Difference	0.00075	0.002

Use $\alpha = 0.01$

Difference calculated as 'Micrometer' − 'Vernier'

$$\bar{D} = \frac{-.001 + 0.001 + .000 + \dots + 0.001 + (-.001)}{12} = -0.0004167$$

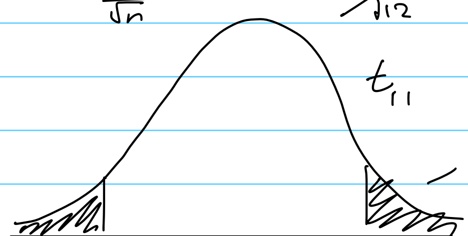
$$S_p = \sqrt{\frac{(-.001+0.0004167)^2 + (0.001+0.0004167)^2 + \dots + (-0.001+0.0004167)^2}{12}} = 0.0013114$$

Interested in the mean difference of diameter of metal rod measured by two different calipers. Same parts, so paired data → paired t-test

$H_0: \mu_D = 0 \qquad H_a: \mu_D \neq 0$

$$t_0 = \frac{\bar{D}}{\frac{S_p}{\sqrt{n}}} = \frac{-0.0004167}{0.0013114/\sqrt{12}} = -1.10$$

Reject if p-value $< \alpha = 0.01$



$$\text{P-value} = 2(P\{t_{11} \geq 1.10\})$$
$$= 2(\underset{0.700}{0.10}, \underset{1.372}{0.25})$$

$\Rightarrow$ P-value $\in (0.20, 0.50)$

P-value in range $(0.2, 0.5) < 0.01 = \alpha$. Fail to reject H_0. We have insufficient evidence to reject the claim that the mean difference in measured diameter of metal rods by Micrometer vs Vernier Calipers are not different. Our evidence does not allow us to say that there is a difference between the two!