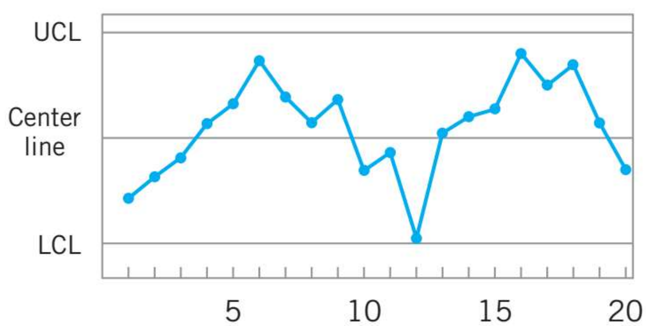
Homework 8

Due 4/3/25 by end of day

Directions: Reading the chapter 5 and 6 will help solve these problems along with content seen in class.

1) Most control charts choose 3 sigma levels for their control limits because it seems to be a good balance between type II and type II errors. Discuss the following:
    a. If narrower limits are chosen, what happens to the magnitude of type I and II error?
    b. What effect does the sigma level (high or low) have on alpha?
2) Laboratory glassware shipped from the manufacturer to Dr. Renner's Lab via an overnight package service has arrived damaged. Develop a cause-and-effect diagram that identifies and outlines the possible causes of this event. You won't necessarily be graded on the details, but include the major components of the "fishbone" diagram.
3) Sketch out diagrams and explain why it is important to control both process mean and variability (see Figure 6.1 in the book)
4) Problem 5.16, 5.17, 5.18 (5.19, 5.22, and 5.23in the 7th edition) – use the below control chart (you can cut and paste into as separate file and print it off as part of your answer). For 5.17 use Sensitizing Rules 5-10, and for 5.18 use the Western Electric Rules (1-4).



5) Problem 6.1
6) A hospital emergency department is monitoring the time require to admit a patient using $\bar{x}$ and R charts. The table below presents summary data for 20 subgroups of two patients each (time is in minutes)

Subgroup	Xbar	R
1	8.3	2
2	8.1	3
3	7.9	1
4	6.3	5
5	8.5	3
6	7.5	4
7	8	3
8	7.4	2
9	6.4	2
10	7.5	4
11	8.8	3
12	9.1	5
13	5.9	3
14	9	6
15	6.4	3
16	7.3	3
17	5.3	2
18	7.6	4
19	8.1	3
20	8	2

a) Use these data to determine the control limits for the $\bar{x}$ and R control charts for this patient admitting process.
b) Plot the preliminary data form the first 20 samples on the control charts that you set up in part (a). Is the process in statistical control?

7) A high-voltage power supply should have a nominal output voltage of 350V. A sample of four units is selected each day and tested for process-control purposes. The data shown in the table below give the difference between the observed reading on each unit and the nominal voltage times 10; that is xi = (observed voltage unit on i – 350)*10. Use Minitab to set up the $\bar{x}$ and R charts on this process. Is the process in statistical control?

Sample #	X1	X2	X3	X4
1	6	9	10	15
2	10	4	6	11
3	7	8	10	5
4	8	9	6	13
5	9	10	7	13
6	12	11	10	10

7	16	10	8	9
8	7	5	10	4
9	9	7	8	12
10	15	16	10	13
11	8	12	14	16
12	6	13	9	11
13	16	9	13	15
14	7	13	10	12
15	11	7	10	16
16	15	10	11	14
17	9	8	12	10
18	15	7	10	11
19	8	6	9	12
20	13	14	11	15

1. Control Charts

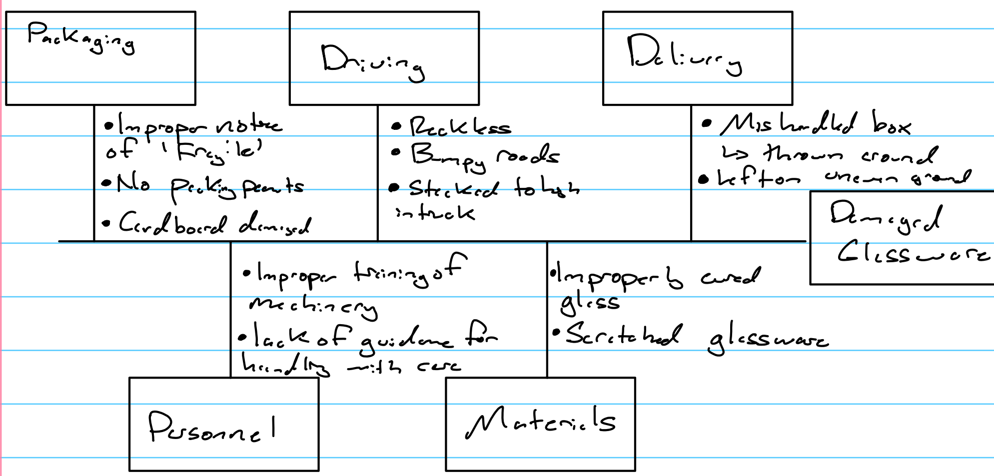
a) Effect of Narrow limits on mag. on type I & II Errors?

If we choose narrower limits, Type I error magnitude (false alarm) increases. Narrow limits → more points fall outside acceptable range → increased chance of false alarms. Type II error decreases too, as narrow limits make it more likely to detect small shifts, reducing β.

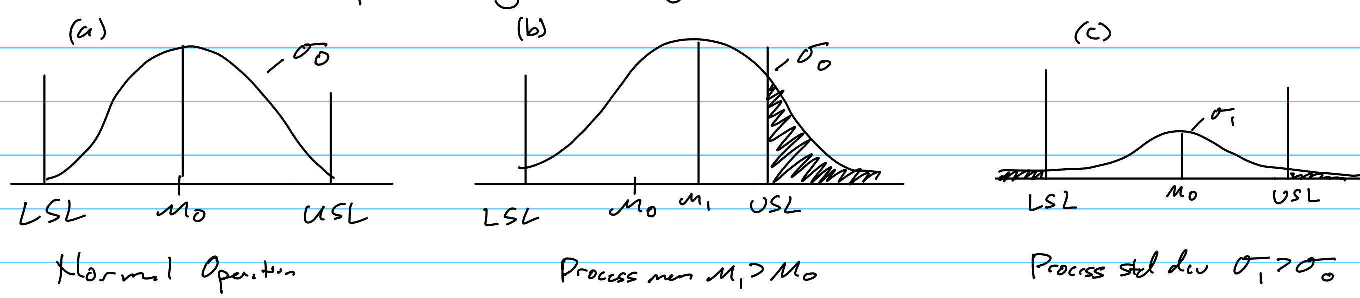
b) Effect of Sigma level on Alpha

A high sigma level increases the control limit range → harder for points to fall outside control limit range → reduces α, but increases β. Lower Sigma level results in tighter limits → more points fall outside → increasing α but reduces β.

2. Cause & Effect Diagram for Glassware Shipping Error

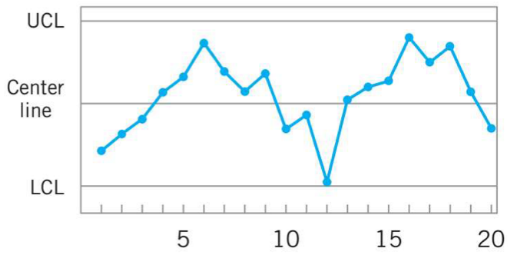


3. Sketches to explain why controlling mean & variability is needed



Controlling mean ensures process lies within specs as in (a), while preventing systematic errors which cause consistent deviation like in (b). Controlling variability reduces out-of-spec variability as in (a), preventing situations like in (c). Controlling both ensures high quality, consistent output within specs.

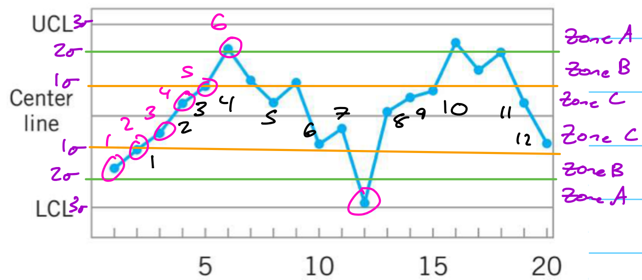
4.



a) Problem S.16

No, it does not appear random. The sharp dip in between two roughly random regions suggests assignable cause. The plot could also be viewed as sinusoidal, suggests a lack of randomness.

b) Problem S.17 – Sensitizing Rules (5-10)



5. Six points in a row steadily increasing or decreasing ✓

6. Fifteen points in a row in zone C (both above and below the center line) ✗

7. Fourteen points in a row alternating up and down ✗

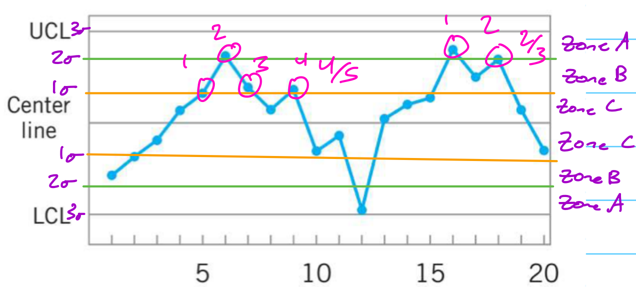
8. Eight points in a row on both sides of the center line with none in zone C ✗

9. An unusual or nonrandom pattern in the data ✓

10. One or more points near a warning or control limit ✓

Rules 5, 9, and 10 are violated on this control chart, as noted directly on the chart. This indicates potentially out of control conditions which should be investigated.

c) Problem S.18 – Western Electric Rules (1-4)



1. One or more points outside of the control limits ✗

2. Two of three consecutive points outside the two-sigma warning limits but still inside the control limits ✓

3. Four of five consecutive points beyond the one-sigma limits ✓

4. A run of eight consecutive points on one side of the center line ✗

Rules 2 and 3 are violated on this chart, noted directly on the chart. This indicates a potential out of control process or conditions which should be investigated.

5. Problem 6.1

A manufacturer of components for automobile transmissions wants to use control charts to monitor a process producing a shaft. The resulting data from 20 samples of 4 shaft diameters that have been measured are:

$$\sum_{i=1}^{20} \bar{x}_i = 10.275, \quad \sum_{i=1}^{20} R_i = 1.012$$

a. Find the control limits that should be used on the $\bar{x}$ and R control charts.

b. Assume that the 20 preliminary samples plot in control on both charts. Estimate the process mean and standard deviation.

a) $\bar{x}$ Chart

$UCL = \bar{\bar{x}} + A_2 \bar{R}$

$LCL = \bar{\bar{x}} - A_2 \bar{R}$

R Chart

$UCL = D_4 \bar{R}$

$LCL = D_3 \bar{R}$

Appendix ← n=20

$A_2 = 0.180$

$D_3 = 0.415$

$D_4 = 1.585$

$\bar{\bar{x}} = \frac{\bar{x}}{n} = \frac{10.275}{20} = 0.51375$

$\bar{R} = \frac{R}{n} = \frac{1.012}{20} = 0.0506$

$\bar{x} \begin{cases} UCL = 0.51375 + (0.180)(0.0506) = 0.523 \\ LCL = 0.51375 - (0.180)(0.0506) = 0.505 \end{cases}$

$R \begin{cases} UCL = (1.585)\,0.0506 = 0.0802 \\ LCL = (0.415)\,0.0506 = 0.0210 \end{cases}$

$\bar{x} \begin{cases} UCL = 0.523 \\ LCL = 0.505 \end{cases}$

$R \begin{cases} UCL = 0.0802 \\ LCL = 0.0210 \end{cases}$

b) $\mu \approx \bar{\bar{x}} = 0.51375$

Appendix n=20

$d_2 = 3.735$

$\hat{\sigma} \approx \frac{\bar{R}}{d_2} = \frac{0.0506}{3.735} = 0.0135$

Estimates $\begin{cases} \mu = 0.514 \\ \sigma = 0.014 \end{cases}$

6.

Subgroup	Xbar	R
1	8.3	2
2	8.1	3
3	7.9	1
4	6.3	5
5	8.5	3
6	7.5	4
7	8	3
8	7.4	2
9	6.4	2
10	7.5	4
11	8.8	3
12	9.1	5
13	5.9	3
14	9	6
15	6.4	3
16	7.3	3
17	5.3	2
18	7.6	4
19	8.1	3
20	8	2

a) $n=20$

$$\bar{\bar{X}} = \frac{\sum_{1}^{20} \bar{x}_i}{20} = \frac{8.3+8.1+7.9+\dots+8.1+8}{20} = 7.57$$

$$\bar{R} = \frac{\sum_{1}^{20} R}{20} = \frac{2+3+1+\dots+3+2}{20} = 3.15$$

Apendix: A_2 : 0.180

D_3 : 0.415

D_4 : 1.585

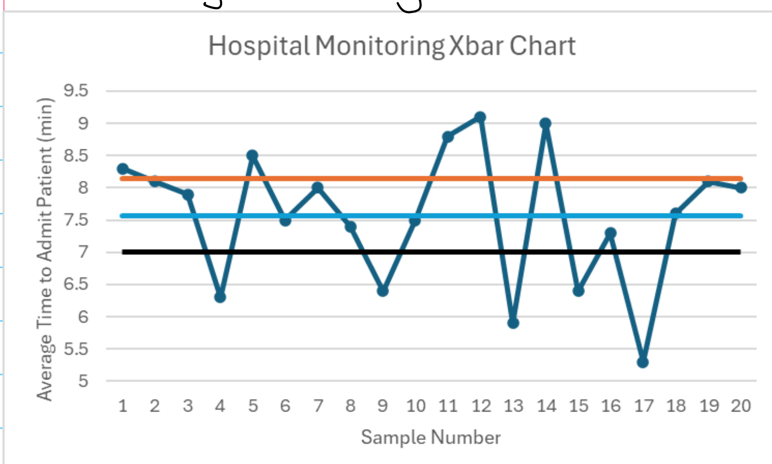
$\bar{X}$ $\begin{cases} UCL = 7.57 + 0.180(3.15) = 8.14 \\ LCL = 7.57 - 0.180(3.15) = 7 \end{cases}$

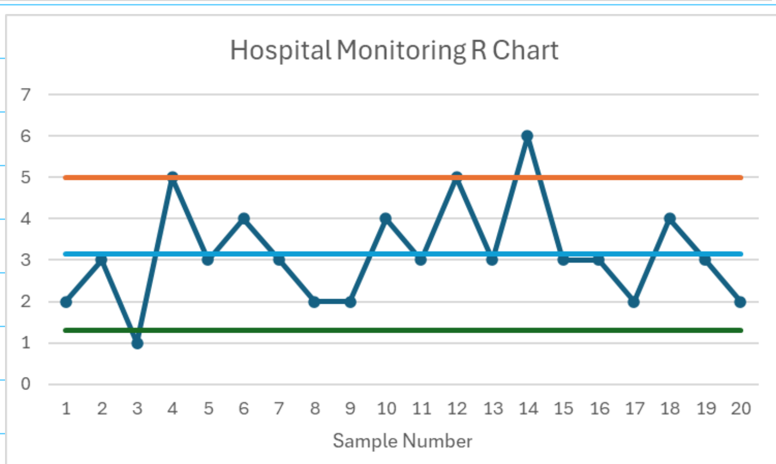
R $\begin{cases} UCL = 1.585(3.15) = 4.99 \\ LCL = 0.415(3.15) = 1.31 \end{cases}$

$\bar{X}$ $\begin{cases} UCL = 8.14 \\ LCL = 7 \end{cases}$

R $\begin{cases} UCL = 4.99 \\ LCL = 1.31 \end{cases}$

b) Plots generated using excel





No, both the xbar and R chart have points that lay outside the upper and lower control limits, indicating that the process is not in statistical control.

7. All points on the $\bar{x}$ and R charts are within the LCL and UCL shown below.