3.4 in 8th Ed book

ECHE313 Homework 2 - Due 1/30/25 **Trevor Swan (tcs94)**

1. Data (°F): 953, 955, 948, 951, 957, 949, 954, 950, 959

a) Sample Average

$$\bar{x} = \frac{953+955+948+951+957+949+954+950+959}{9} = 952.89 \xrightarrow{\text{round}} \bar{x} = 953\,°F$$

b) Standard Deviation

$$S = \sqrt{\frac{\sum_{i}^{n}(x_i-\bar{x})^2}{n-1}} = \sqrt{\frac{(953-952.89)^2+(955-952.89)^2+(948-952.89)^2+(951-952.89)^2+(957-952.89)^2+(949-952.89)^2 \ldots}{9-1}}$$

$$\ldots \frac{(954-952.89)^2+(950-952.89)^2+(959-952.89)^2}{} = 3.7231 \xrightarrow{\text{round}} S = 3.72\,°F$$

c) The mean of 953 means that the average temperature reading was 953°F. Specifically, this is the measure of central tendency to describe the typical value or center. The standard deviation describes the variance of temperature readings in the data set. The value of 3.72 is the typical distance between each data point and the mean.

d) The data set describes furnace temperatures recorded on successive batches in a semi-conductor manufacturing process. Variations in this data set are most likely due to process-related variations (like control system errors, batch size, or calibrations), environmental variations, material variations (material properties and batch preparation changes), and human error (different operators and measurement intervals, and possible variations in measurement devices.

2. Data (°F): 953, 955, 948, 951, 957, 949, 954, 950, 959

Ranked: 948, 949, 950, 951, 953, 954, 955, 957, 959

Rank: 1 2 3 4 5 6 7 8 9

a) median $= (n+1)(\frac{1}{2}) = (9+1)(\frac{1}{2}) = 5 \Rightarrow$ median = 953 °F

b) The largest data point can increase by any amount without altering the median as median is determined by rank. The final value with rank 9 is going to remain rank 9 for any increase.

c) The median of a dataset describes the center value of the dataset. It is also defined as the 50th percentile. It is the exact middle of sorted data where ½ of the data is below the median and ½ of it is above the median. This value is more accurate for skewed data.

d) Data (°F): 953, 955, 948, 951, 957, 949, 954, 950, 959, 952

Ranked: 948, 949, 950, 951, 952, 953, 954, 955, 957, 959

Rank: 1 2 3 4 5 6 7 8 9 10

median $= (10+1)(\frac{1}{2}) = 5.5 \Rightarrow (952+953)(\frac{1}{2}) =$ 952.5 °F

e) Minitab Output

Statistics Unrounded

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
Probelm 1	9	0	952.889	1.24102	3.72305	948	949.5	953	956	959

Statistics Rounded

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
Probelm 1	9	0	953	1.24	3.72	948	949.5	953	956	959

3.5 in 6th Ed book

3.

Table 3E.1 Electronic Component Failure Time

127	2	124	2	121	1	118	1
125	2	123	1	136	4	131	3
131	3	120	1	140	4	125	2
124	2	119	1	137	4	133	3
129	2	128	2	125	2	141	4
121	1	133	3	124	2	125	2
142	5	137	4	128	2	140	4
151	6	124	2	129	2	131	3
160	7	142	5	130	3	129	2
125	2	123	1	122	1	126	2

a) $\bar{x} = \dfrac{127+125+131+\dots+129+126}{40} = 129.98 \xrightarrow{\text{round}}$ $\bar{x} = 130$ hr

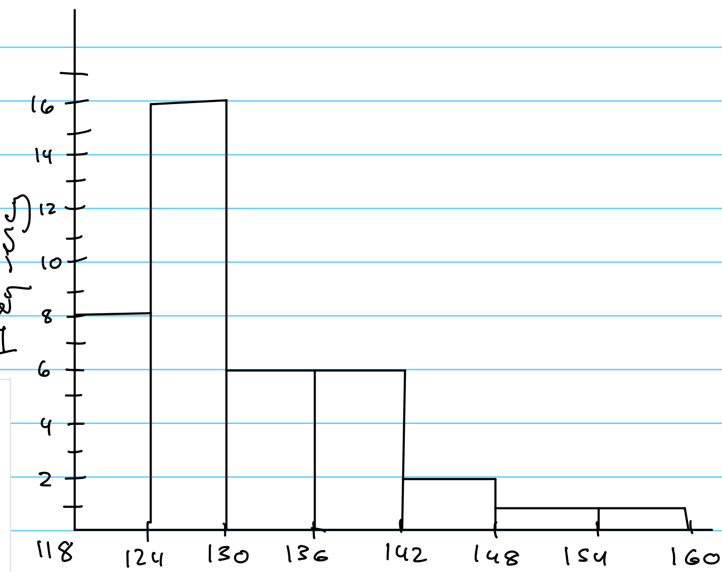
$S = \sqrt{\dfrac{(127-129.98)^2+(125-129.98)^2+\dots+(126-129.98)^2}{40-1}} = 8.91 \xrightarrow{\text{round}}$ $S = 8.91$ hr

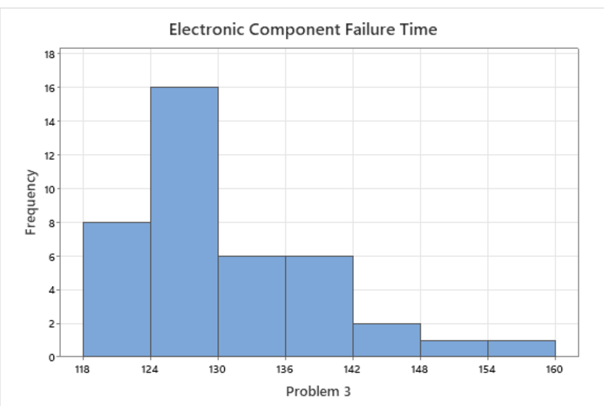
Statistics

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
Problem 3	40	0	129.975	1.40944	8.91408	118	124	128	135.25	160

b) # of bins $= \sqrt{40} = 6.32 \rightarrow 7$, min $= 118$, max $= 160$, bin size $= \dfrac{160-118}{7} = 6$

Counts	
8	bin 1: $118+6 \rightarrow [118, 124)$
16	bin 2: $124+6 \rightarrow [124, 130)$
6	bin 3: $130+6 \rightarrow [130, 136)$
6	bin 4: $136+6 \rightarrow [136, 142)$
2	bin 5: $142+6 \rightarrow [142, 148)$
1	bin 6: $148+6 \rightarrow [148, 154)$
1	bin 7: $154+6 \rightarrow [154, 160]$





(Skip c)

d)

118	1	128	21
119	2	129	22
120	3	129	23
121	4	129	24
121	5	130	25
122	6	131	26
123	7	131	27
123	8	131	28
124	9	133	29
124	10	133	30
124	11	136	31
124	12	137	32
125	13	137	33
125	14	140	34
125	15	140	35
125	16	141	36
125	17	142	37
126	18	142	38
127	19	151	39
128	20	160	40

Median (rank) $= \dfrac{n+1}{2} = \dfrac{40+1}{2} = 20.5$

Median $= \dfrac{128+128}{2} = 128$ median $= 128$ hr

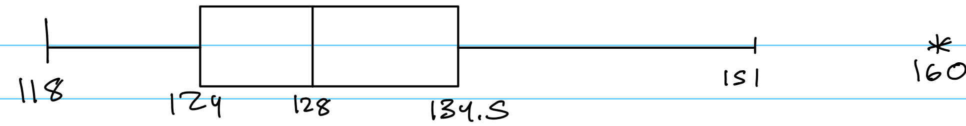
Q1 (rank) $= \dfrac{n+1}{4} = \dfrac{40+1}{4} = 10.25$

Q1 $= \dfrac{124+124}{2} = 124$ Q1 $= 124$ hr

Q3 (rank) $= (n+1)\left(\dfrac{3}{4}\right) = (40+1)\left(\dfrac{3}{4}\right) = 30.75$

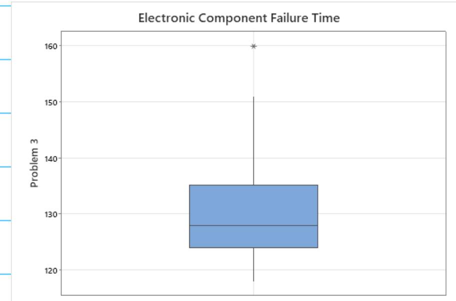
Q3 $= \dfrac{133+136}{2} = 134.5$ Q3 $= 134.5$ hr

e) Min = 118
Q1 = 124
Q2 = 128
Q3 = 134.5
Max = 160



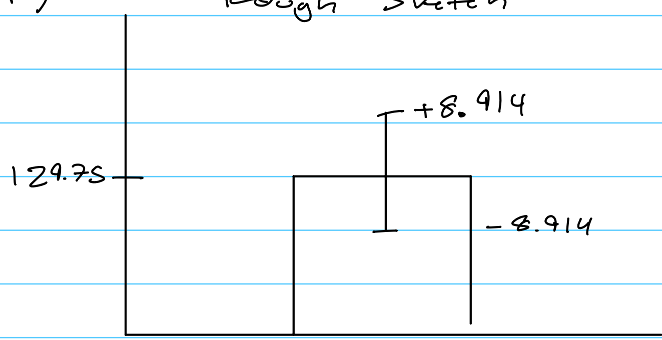
UL = 134.5 + 1.5(134.5 − 124)
= 150.25 round to 151

BL = 124 − 1.5(134.5 − 124)
= 108.25 don't use

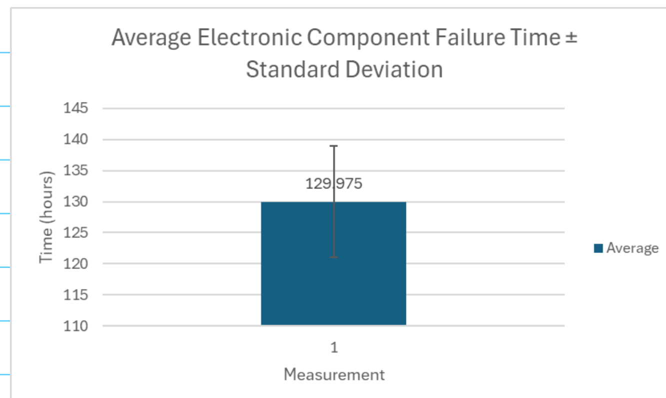


From the boxplot, it is clear that the data is skewed right. This means the tail to the right of the median is longer than the left tail. Due to the skewed nature, the median is less than the mean. There is an outlier of 160 in the data set! It is clearly not normally distributed.

f) Rough Sketch

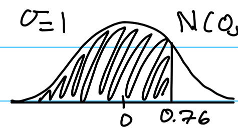


just said excel required for this problem



4. All values from the back of the book

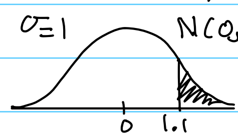
a) $P(z \le 0.76)$



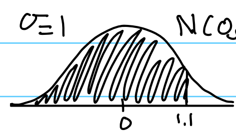
$= 0.77637$

0.78

b) $P(z > 1.1)$



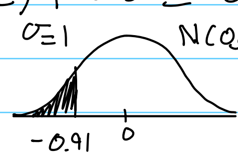
$= 1 - P(z \le 1.1)$

$1-$ 

$= 1 - 0.86433$

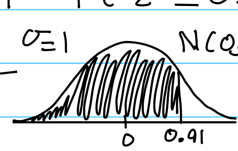
$= 0.1357$ 0.14

c) $P(z \le -0.91)$



$= P(z \ge 0.91) =$

$1 - P(z \le 0.91)$

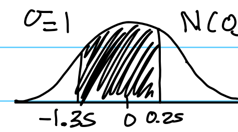
$1-$ 

$= 1 - 0.81859$

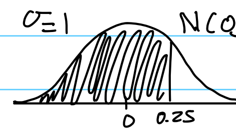
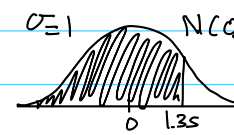
$= 0.1814$

0.18

d) $P(-1.35 \le z \le 0.25) = P(z \le 0.25) - P(z \le -1.35)$



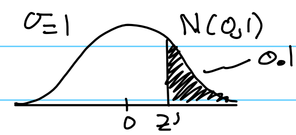
$= P(z \le 0.25) - (1 - P(z \le 1.35))$

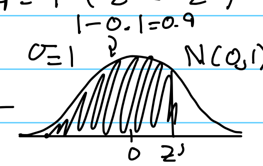
$\Rightarrow$  $- \Big(1 -$  $\Big)$

0.51

$= 0.59871 - (1 - 0.91149) = 0.5102$

e) $P(z > z') = 0.1 \Rightarrow 0.9 = P(z < z')$

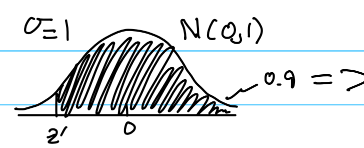
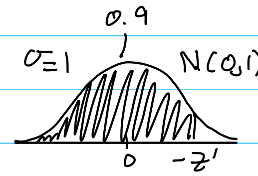


$\Rightarrow 1-$ 

$z = 1.28$

value = 0.90320

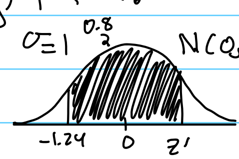
f) $P(z > z') = 0.9$

 $\Rightarrow$ 

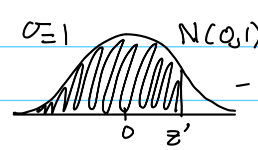
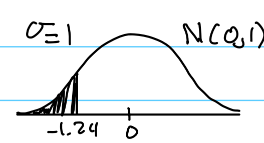
$-z = 1.3$

$z = -1.28$

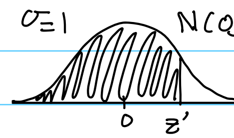
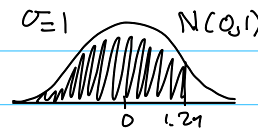
g) $P(-1.24 < z < z') = 0.8$



$= P(z \le z') - P(z \le -1.24)$

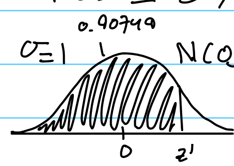
 $- \Big($  $\Big)$

$\Rightarrow 0.8 = P(z \le z') - (1 - P(z \le 1.24))$

 $- \Big(1 -$  $\Big)$

$0.8 = P(z \le z') - (1 - 0.89251)$

$\Rightarrow P(z \le z') = 0.90749$



$z' = 1.33$

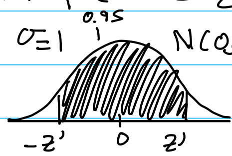
$z = 1.33$

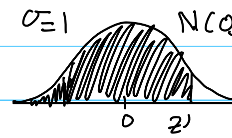
0.75 from 1.33

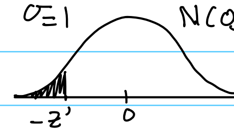
0.91 from 1.32

value = 0.90824

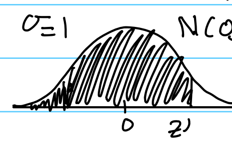
h) $P(-z' \le Z \le z') = 0.95 = P(Z \le z') - P(Z \le -z')$



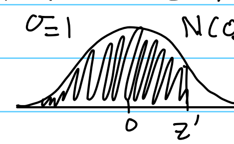




$= P(Z \le z') - (1 - P(Z \le z'))$



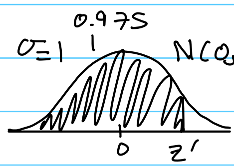
$- \Big(1 -$



$\Big)$

$\Rightarrow 0.95 = 2 \cdot P(Z \le z') - 1$

$P(Z \le z') = 0.975$

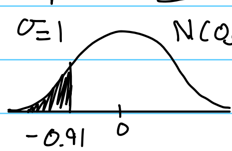


$z = 1.96$

value = 0.97500

i. Testing Minitab Output for part 'c', all others checked w/ calculator

$P(Z \le -0.91)$



Cumulative Normal $\Rightarrow$ Distribution

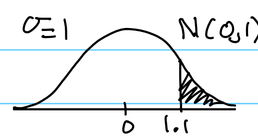
Full value	Rounded
0.181411	0.18

Normal with mean = 0 and standard deviation = 1

x	P(X ≤ x)
-0.91	0.181411

j. Integral for part b: $P(Z > 1.1)$

$$\boxed{P(Z > 1.1) = \int_{1.1}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}\, dz}$$

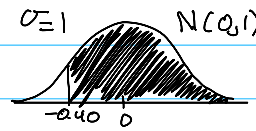


5. $\mu = 260$ minutes $\times \frac{1\text{ hr}}{60\text{ min}} = 4.3333$ hr   Standardize to 0

$\sigma = 50$ minutes $\times \frac{1\text{ hr}}{60\text{ min}} = 0.8333$ hr   Standardize to 1

a) $P(x \geq 4 \text{ hours})$

$z = \frac{x-\mu}{\sigma} = \frac{4-4.33}{0.8333} = -0.40$ so $P(z \geq -0.40)$
$= P(z \leq 0.40)$



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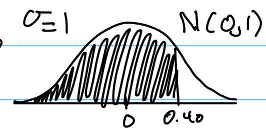
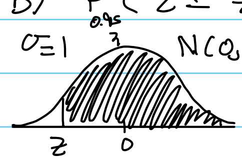
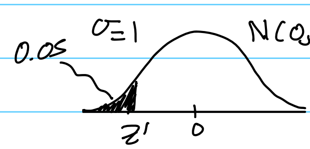
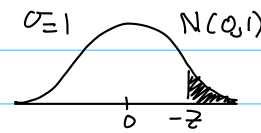


$\Rightarrow P(z \leq 0.40) = 0.65542$ 0.66

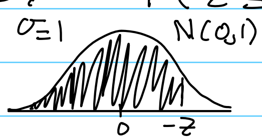
b) $P(Z \geq z) = 0.95 \Rightarrow 0.95 = 1 - P(Z \leq z) \Rightarrow 0.05 = P(Z \geq -z)$







$\Rightarrow 0.95 = P(Z \leq -z')$



$-z = 1.64$
$z = -1.64$

$-1.64 = \frac{x - 4.33}{0.8333}$
$x = 2.97$ hours

178.2 min, report to 178 min

c) $P(x > 4)$, $N(4.33, 0.833)$

Normal with mean = 4.33333 and standard deviation = 0.833333

x	P(X ≤ x)
4	0.344578

For part c, the reported value is (1 − value)

$1 - 0.344578 = 0.655422$

Rounds to 0.66

$P(Z \geq z') = 0.95$, $N(4.33, 0.833)$

Normal with mean = 4.33333 and standard deviation = 0.833333

P(X ≤ x)	x
0.05	2.96262

For part b, reporting 0.05, then convert hours to minutes

$2.96262 \times \frac{60\text{ min}}{1\text{ hr}} = 177.78$

Rounds to 178 min

6.

Table 3E.1 Electronic Component Failure Time

127	124	121	118
125	123	136	131
131	120	140	125
124	119	137	133
129	128	125	141
121	133	124	125
142	137	128	140
151	124	129	131
160	142	130	129
125	123	122	126

Rank 1

Rank 1 Value: 118, $n = 40$

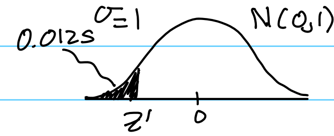
Cumulative Freq: $\frac{i-0.5}{n}$

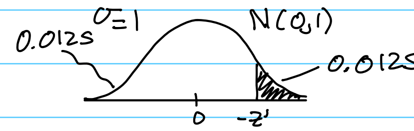
$\rightarrow \frac{1-0.5}{40} = 0.0125$

$P(z \le z_i) = 0.0125$

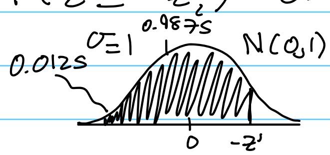
$= P(z \ge -z_i) = 0.0125$

$= 1 - P(z \le -z_i)$





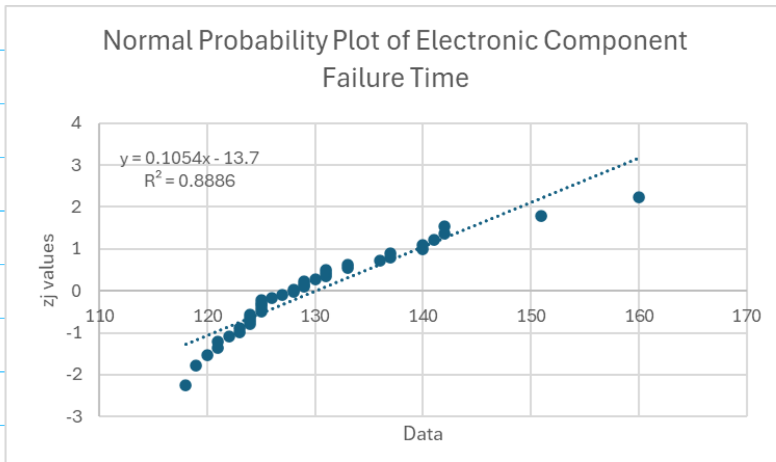
$P(z \le -z_i) = 0.9875$



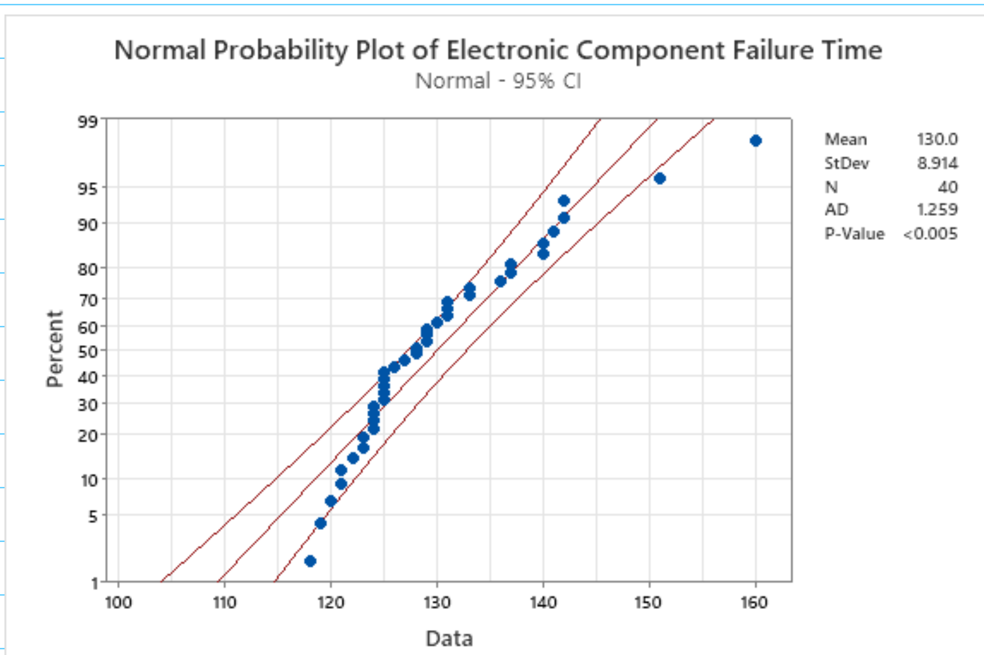
$-z_i = 2.24$

$\Rightarrow$

$z_i = -2.24$



The normal assumption is not confirmed or denied by this plot. Outliers and obviously non-linear data lead me to believe that the data is not normal, but minitab will confirm this.



The data is not normal within a 95% Confidence Interval.

The p-value is reported as < 0.005, so we can not assume normality. Assuming $\alpha = 0.1$, p is much less than this!