Problem 1

a) Sample average: $\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$

n = 9

units °F

$$\bar{x} = \frac{953 + 955 + 948 + 951 + 957 + 949 + 954 + 950 + 959}{9}$$

$$\boxed{\therefore \bar{x} = 953\,°F}$$ 3 sig figs!

b) Standard deviation: $s = \sqrt{s^2} = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}$

$$s = \sqrt{\frac{(x_1-\bar{x})^2 + (x_2-\bar{x})^2 + \ldots (x_n-\bar{x})^2}{n-1}}$$

$$s = \sqrt{\frac{(953-953)^2 + (955-953)^2 + (948-953)^2 + (951-953)^2 + (957-953)^2 + (949-953)^2 + (954-953)^2 + (950-953)^2 + (959-953)^2}{9-1}}$$

$$\boxed{\therefore s = 4\,°F}$$ round to 1s place

c) sample mean is the average value of all of the values in the data set. It can be thought of as the balance point in a system of weights. sample standard deviation describes the amount of variability or scatter in the data set.

d) Possible sources of variation:
→ external temp, operators, load size

Problem 2

a) Sample median:
For an odd # of observations, median is: $\frac{(n-1)}{2}+1$ rank

$$\text{rank} = \frac{9-1}{2}+1=5$$

Rank:	1	2	3	4	5	6	7	8	9	
	948	949	950	951	$\boxed{953}$	954	955	957	959	°F

median

b) There is no limit to how high the largest # in the set can change because ranks would remain the same.

c) The sample median is the measure of central tendancy that exactly divides the data in 2 equal parts. Half of the data are above the median, and half below. Also known as 50th percentile.

d) If one more value was added to the data set, the set would have an even amount of observations.

→ add 952°F to data set

For an even # of observations:

median = average of $\frac{n}{2}$ and $\frac{n}{2}+1$

= average of rank: $\frac{10}{2}=5$ and $\frac{10}{2}+1=6$

Rank:	1	2	3	4	5	6	7	8	9	10	
	948	949	950	951	952	953	954	955	957	959	°F

$$\boxed{\text{median} = \frac{952+953}{2} = 952.5}$$

→ what to do with sig figs? It depends on the application. For example, if you're reporting to your boss who isn't interested in knowing how precise measurements are, rounding may be best. Otherwise, reporting the exact median may be important.

e) minitab:

Statistics

Variable	N	N*	Mean	SE Mean	StDev
Furnace Temp	9	0	952.89	1.24	3.72

Minimum	Q1	Median	Q3	Maximum
948.00	949.50	953.00	956.00	959.00

Problem 3

a) sample average: $\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n} = \frac{127+125+\ldots 126}{40}$

$\boxed{\bar{x} = 130}$

sample st. deviation:

$s = \sqrt{s^2} = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}}$

$= \sqrt{\frac{(127-130)^2+\ldots}{40-1}}$

$\boxed{s = 9}$

minitab:

Statistics

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
Failure Time	40	0	129.97	1.41	8.91	118.00	124.00	128.00	135.25	160.00

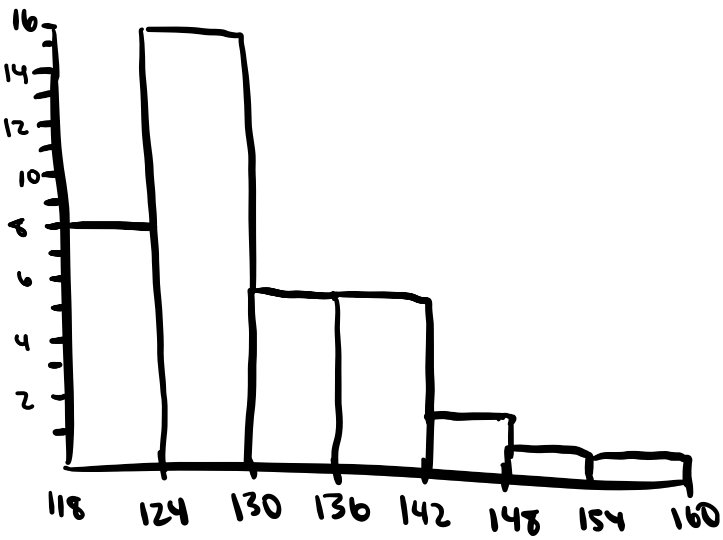
b) to construct histogram:

$$\text{\# of bins} \approx \sqrt{n} = \sqrt{40} = 6.2 \rightarrow 7 \quad \text{(round up)}$$

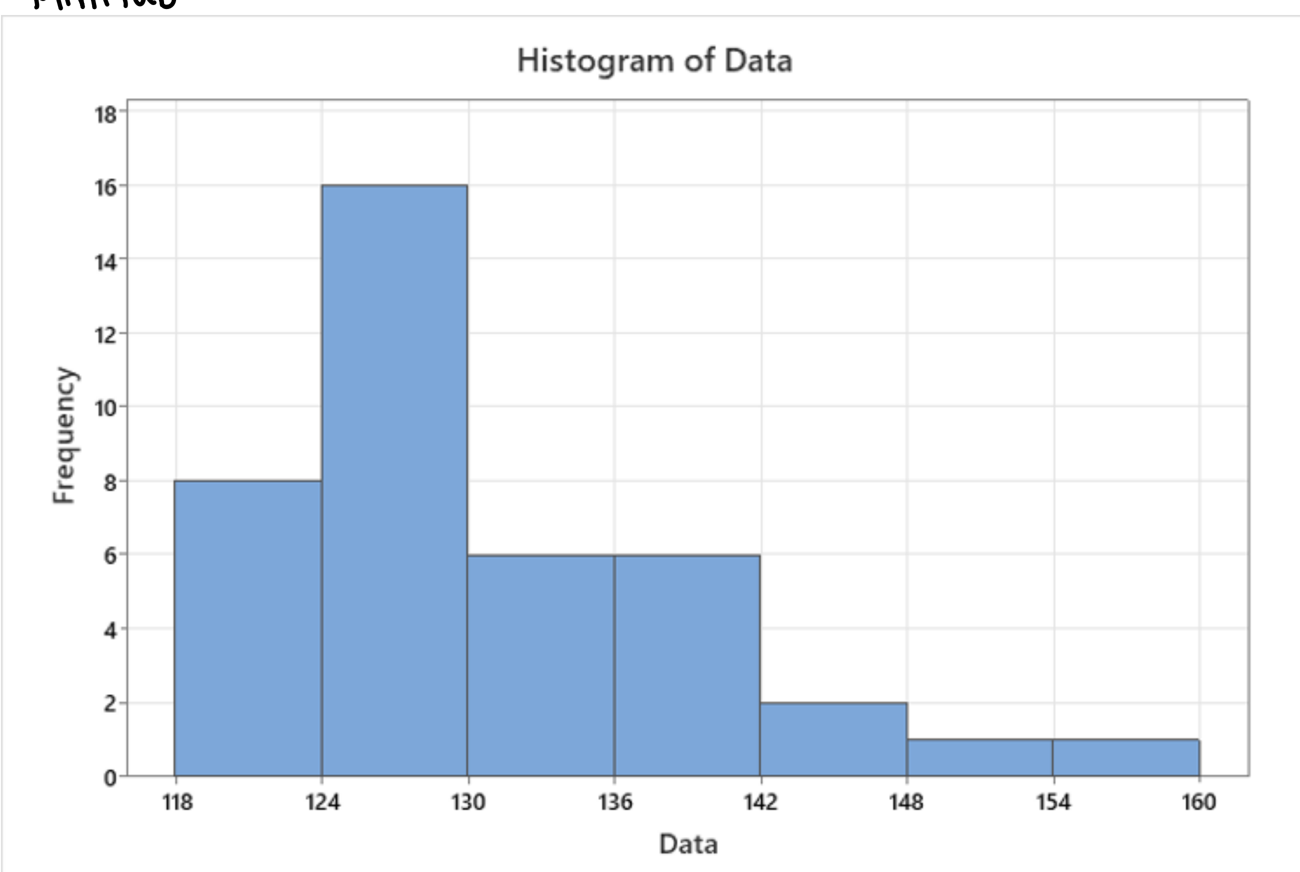
$$\text{Bin size} = \frac{\text{Max} - \text{Min}}{\sqrt{n}} = \frac{160 - 118}{7} = 6$$

Bins:	ranges:	frequencies
(1)	118 + 6 = 124	8
(2)	124 + 6 = 130	16
(3)	130 + 6 = 136	6
(4)	136 + 6 = 142	6
(5)	142 + 6 = 148	2
(6)	148 + 6 = 154	2 1
(7)	154 + 6 = 160	1

hand-drawn:



Minitab:



d) Find sample median and lower and upper quartiles:

For even # observations:

average of rank: $\frac{n}{2}$ and $\frac{n}{2}+1$

$\frac{40}{2}$ and $\frac{40}{2}+1$ so 20 & 21

$\boxed{\text{Median: } 128}$

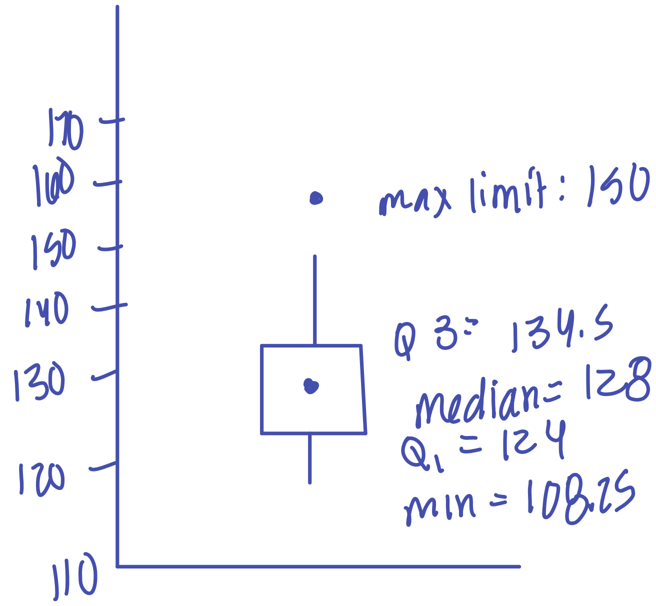
Quartiles are like medians of upper & lower halves of the data. So, since 1-20 is one half and 21-40 is the second, the medians will be averages of rank $\underline{10 \text{ and } 11}$ and $\underline{30 \text{ and } 31}$

124 124 133 136

$= \frac{133+136}{2}$

$\therefore$ $\boxed{Q_1 = 124 \quad Q_3 = 134.5}$

Draft of Box Plot:



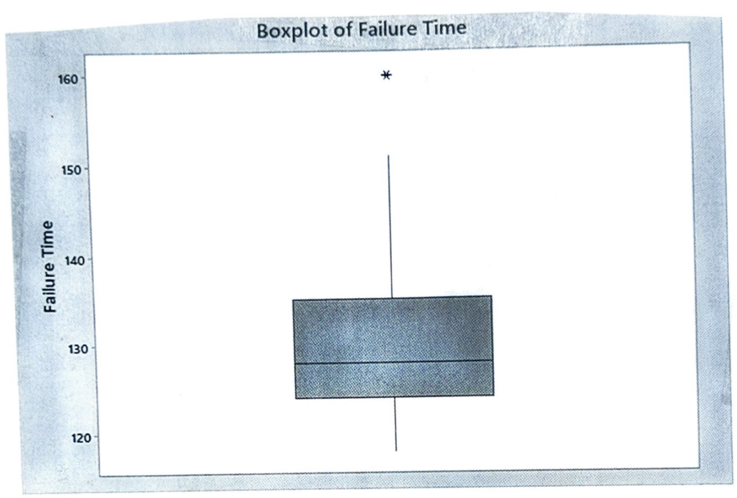
Note: The data points are more frequent at the lower end of this range. Since 160 lies outside the max limit, it is considered an outlier.

Limits: $1.5\,(Q3-Q1) = 15.75$

Max limit: $134.5 + 15.75 = \underline{150}$

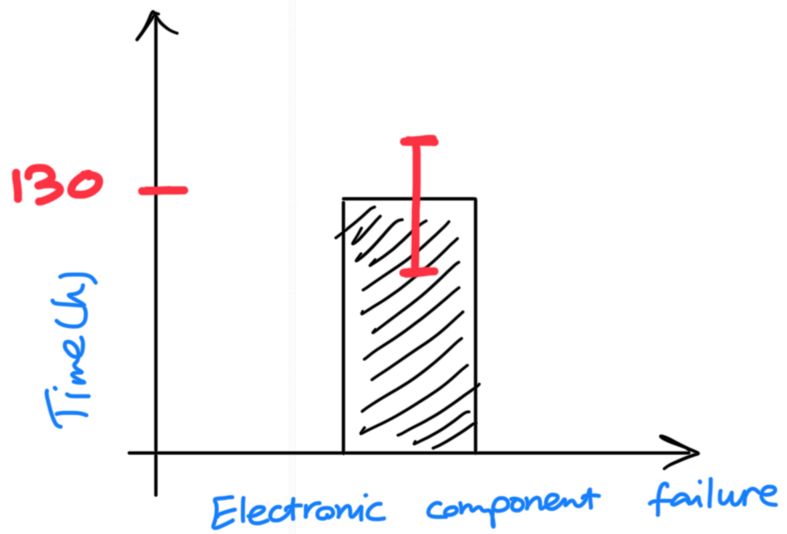
Min limit: $124 - 15.75 = \underline{108.25}$

Minitab:



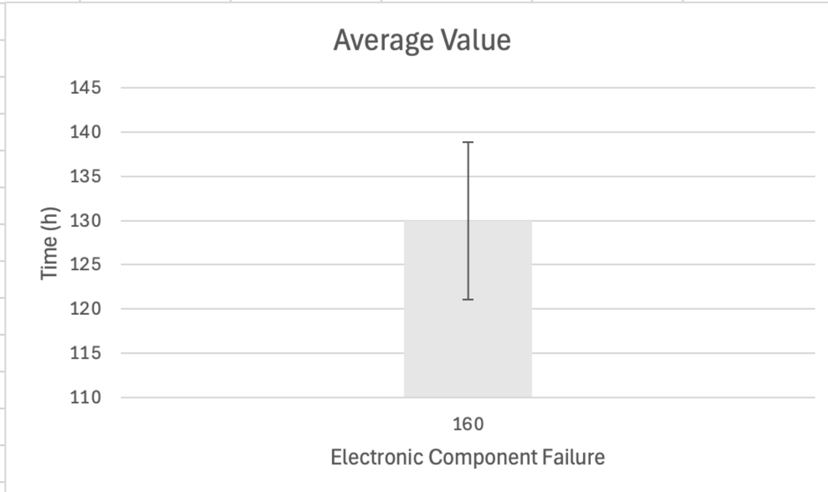
Q3 - part F.

Data	
160	
⋮	
118	
130	Average
7	STD



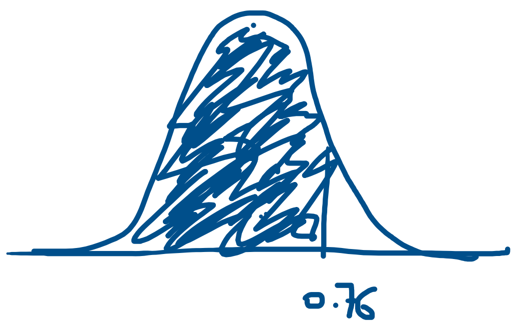
In excel :

Data		
160		
151		
142		
142		
141	Average	129.975
140	StD Dev	8.9140842
140		
137		
137		
136		
133		
133		
131		
131		
131		
130		
129		
129		
129		
128		
128		
127		
126		
125		
125		

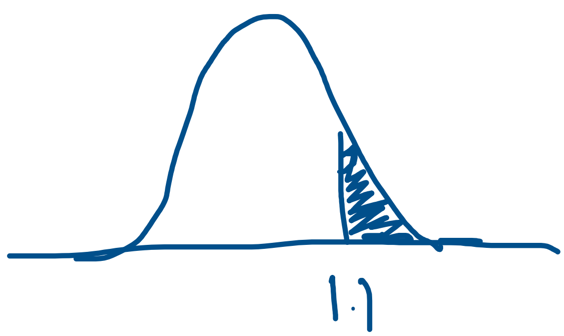


Problem 4)

a) $P(Z < 0.76) = 0.78$

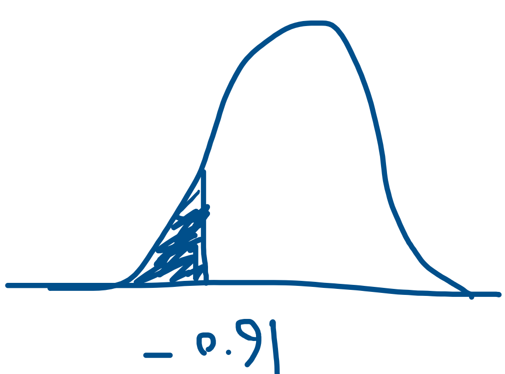


b) $P(Z > 1.1)$

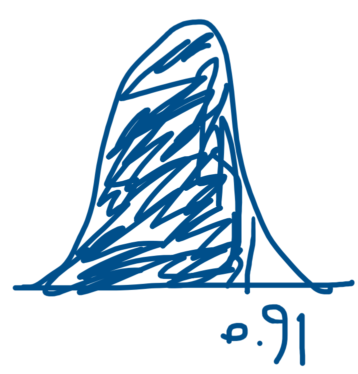


$= 1 - P(Z < 1.1) = 1 - 0.86433 = 0.1357$

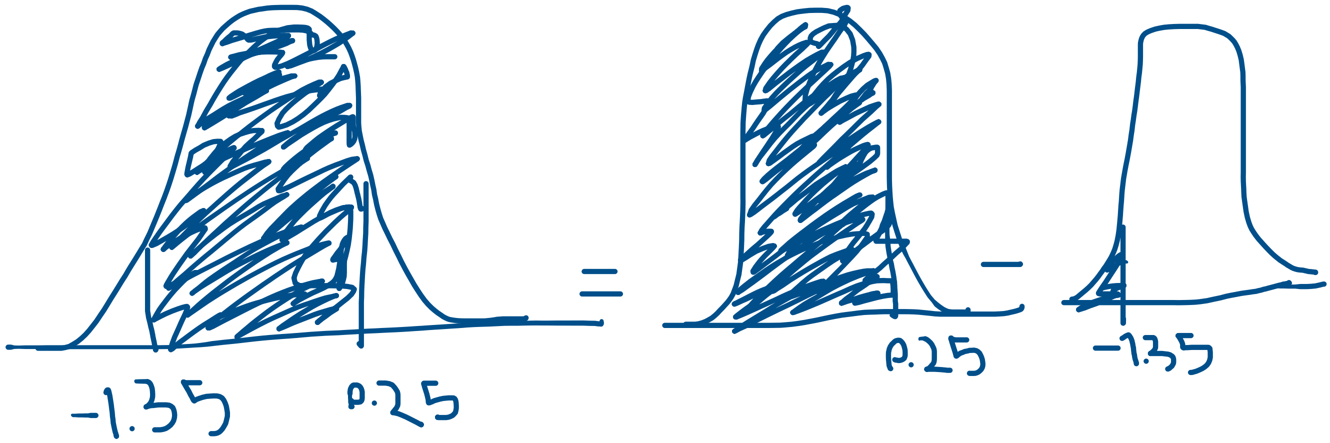
c) $P(Z < -0.91) = 1 - 0.81859 = 0.18141$



$= 1 -$

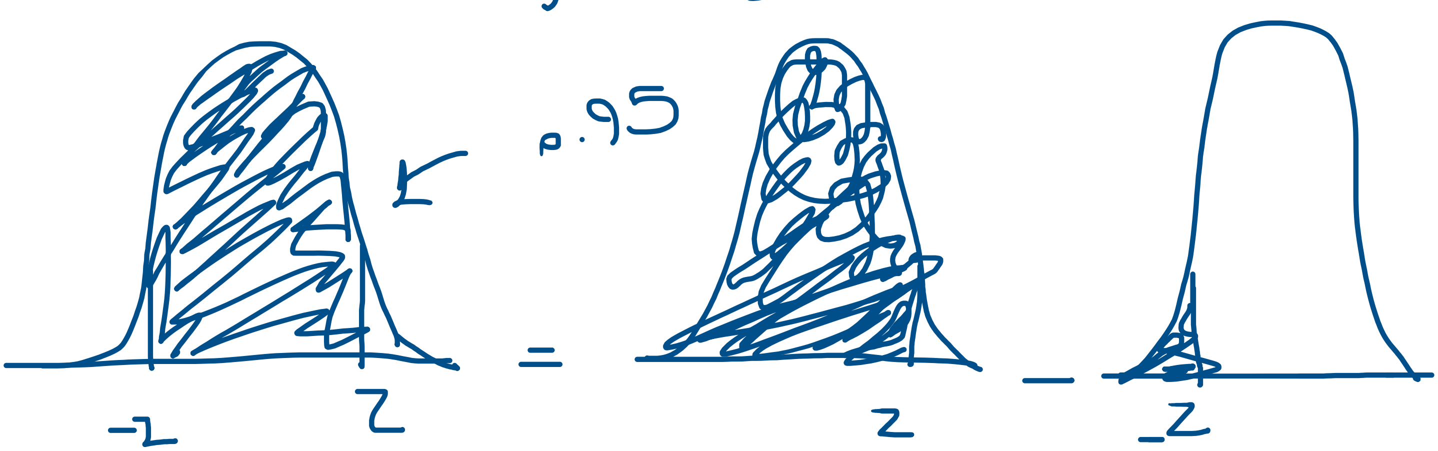


d) $P(-1.35 < Z < 0.25) =$



$0.59871 - (1 - 0.91149) = 0.5102$

h) $(-z < Z < z) = 0.95$



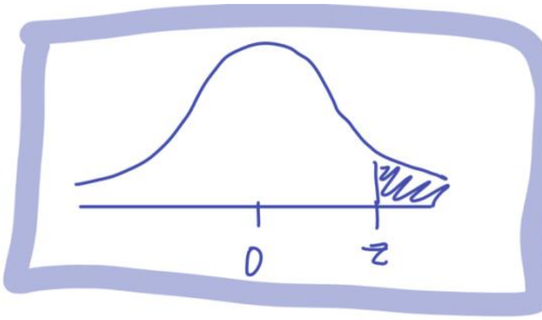
$P(Z < z) - (1 - P(Z < z))$

$= 2P(Z < z) - 1 = 0.95 \quad P(Z < z) = 0.975 \quad z = 1.96$

e) $P(Z > z) = 0.1$

$P(Z > z) = 1 (Z \leq z)$

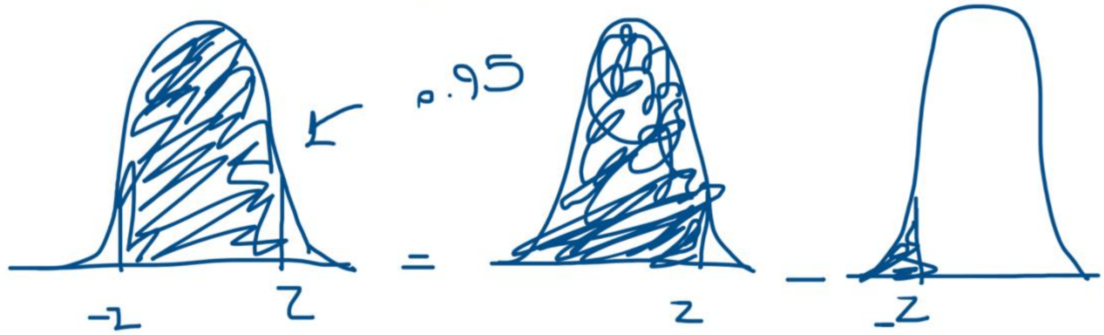
$= -P(Z - z) = -0.9$



$P(Z \leq z) = 0.9$

$\boxed{z = 1.3 \text{ (see a)}}$

h) $(-z < Z < z) = 0.95$

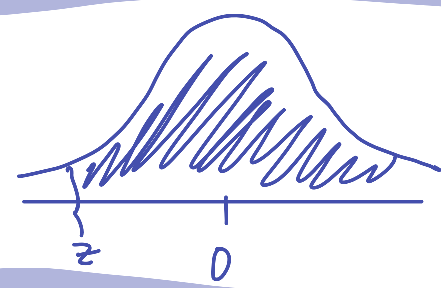


$P(Z < z) - (1 - P(Z < z)$

$= 2P(Z < z) - 1 = 0.95 \quad P(Z < z) = 0.975 \quad z = 1.96$

f)

$P(Z>z)=0.9$



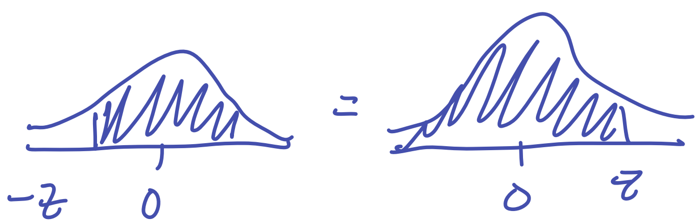
$1-P(Z\le z)=0.9$

$-P(Z\le z)=-0.1$

$P(Z\le z)=0.1$

$\boxed{z=-1.3}$

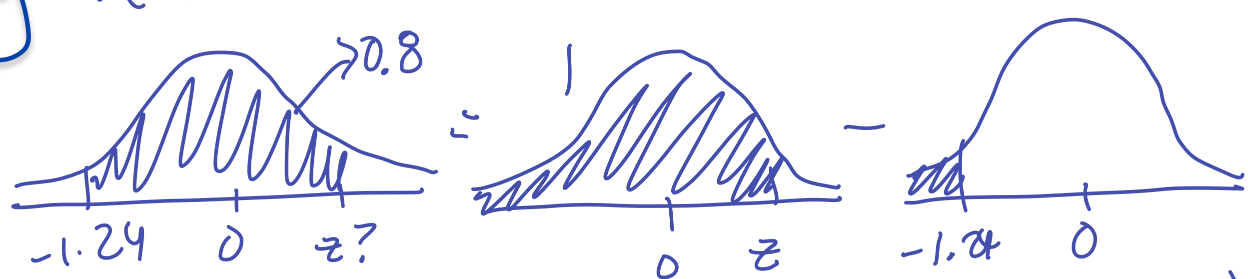
Since table only has positive values, use the rule:



$P(Z>z)=.9$

$P(Z<-z)=.9$

g) $P(-1.24<Z<z)=0.8$



$P(Z<z)$

$P(Z<-1.24)$

$=P(Z>1.24)$

$=1-P(Z\le 1.24)$

$=1-0.89251$

$\therefore\ 0.8 = P(Z<z) - 0.107$

$= 0.907 = P(Z<z)$

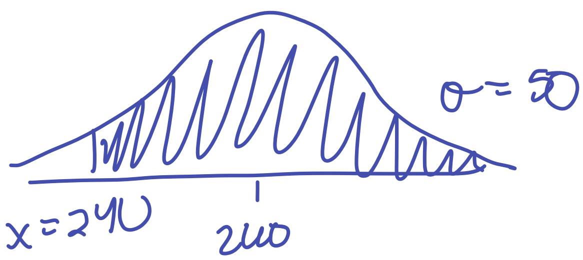
$\therefore\ \boxed{z = 1.3}$

ii) $P(z>1.1) = \int_{1.1}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}\,dx = 0.1357$

Problem 5

$\mu = 260$ min $\quad \sigma = 50$ min

a) $P(X > 4\text{ hr}) = P(X > 240\text{ min})$



$Z = \frac{X-\mu}{\sigma} = \frac{x-260}{50} \rightarrow 50z + 260 = x$

$P(X > 240) = P(50z + 260 > 240)$

$= P(50z > -20)$

$= P\left(z > \frac{-20}{50}\right)$

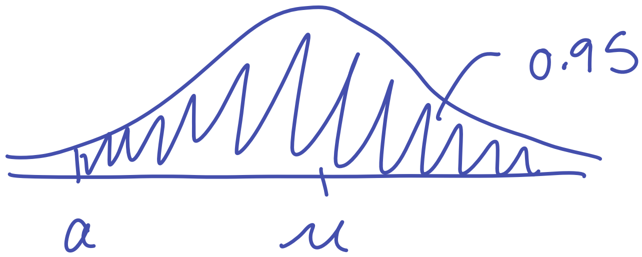
$= P(z > -0.4)$

$= P(z < 0.4) = 0.655$

$\boxed{\therefore\ P(z<0.4) = 0.66}$

b) $P(X > a) = 0.95$

where a is value to be exceeded



$$\underbrace{P\left(\frac{X-260}{50} > \frac{a-260}{50}\right)}_{\text{Need to transform both a and X}} = 0.95$$

$$P(Z > z) = 0.95 \quad , z = -1.61$$

$$P(Z < -z) = 0.95$$

$$z = -1.61 = \frac{X-260}{50} \qquad \boxed{X = 180 \text{ min}}$$

Minitab output

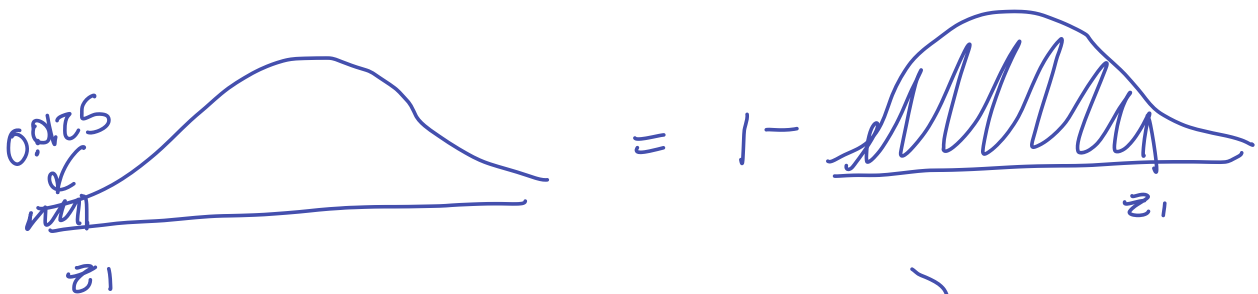
Problem 6

Rank 1: Cumulative Frequency $= \frac{j-0.5}{n} = \frac{1.5}{40}$

$\boxed{= 0.0125}$

Find z_1, $P(z < z_1) = 0.0125$

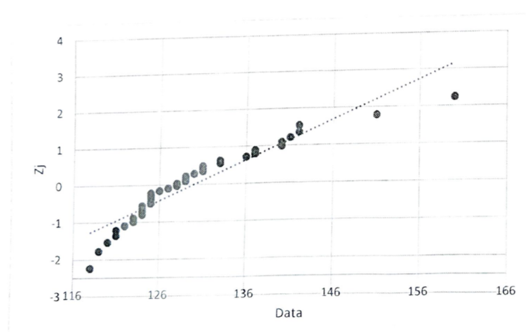
$P(z < z_1) = 0.0125$ or $1-(z < -z_1) = 0.0125$



$\therefore 0.9875 = (z < -z_1)$

$\boxed{z_1 = -2.24}$

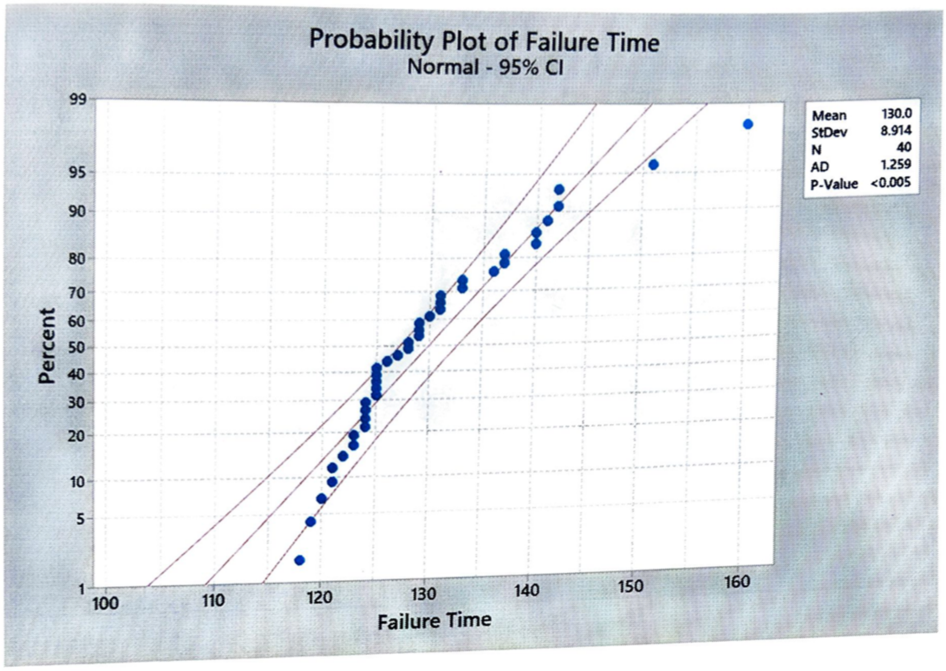
Plot in excel:



Normal with mean = 0 and standard deviation = 1

P(X ≤ x)	x
0.0125	-2.24140
0.0375	-1.78046
0.0625	-1.53412
0.0875	-1.35631
0.1125	-1.21334
0.1375	-1.09162
0.1625	-0.98423
0.1875	-0.88715
0.2125	-0.79778
0.2375	-0.71437
0.2625	-0.63566
0.2875	-0.56070
0.3125	-0.48878
0.3375	-0.41930
0.3625	-0.35178
0.3875	-0.28584
0.4125	-0.22112
0.4375	-0.15731
0.4625	-0.09414
0.4875	-0.03134
0.5125	0.03134
0.5375	0.09414
0.5625	0.15731
0.5875	0.22112
0.6125	0.28584
0.6375	0.35178
0.6625	0.41930
0.6875	0.48878
0.7125	0.56070
0.7375	0.63566
0.7625	0.71437
0.7875	0.79778
0.8125	0.88715
0.8375	0.98423
0.8625	1.09162
0.8875	1.21334
0.9125	1.35631
0.9375	1.53412
0.9625	1.78046
0.9875	2.24140

Plot in minitab:



This is not a good assumption (normality), because the high and low values are larger than expected.