ECHE 313 Final Exam

Name: KEY

The exam layout is such that all problems add up to 150 points. This point total will be adjusted such that this exam is 30% of your total class grade. Three sheets (can be double sided) of notes and a calculator are allowed for this exam. All of the necessary tables are provided. Please show your work and all steps necessary to arrive at your answer. Good luck!

1) **(8 points total) Describe the following:**
   a. (4 points) Three distinct ways extra variability in a process can lead to higher cost.

Variability can lead to:
1) increased internal costs (e.g. scrap, rework, retesting, failure analysis)
2) external cost increases (e.g. dealing with complaints, returned product, warranty or liability)
3) increased indirect cost (e.g. loss in reputation and sales)

   b. (4 points) At least three distinct reasons slow processes are expensive processes.

1.) Customers don't like waiting (lost revenue)
2.) More handeling requires more personell
3.) Longer storage means more chances for product to get damaged
4) Inventory is higher (old product needs to be discarded)
5) More documentation is needed the more it is handeled

2) **(10 Points) List the name of the statistical test that should be performed for each scenario below using the following word bank (names can be used more than once). Assume normal distributions for all data.**

Bank: One-sample Z-test, two-sample Z-test, one-sample t-test, two-sample t-test (equal variance), two-sample t-test (unequal variance), paired t-test, 1 variance test, 2 variance test

a. (2 points) Two machines are used for filling glass bottles with a soft-drink beverage. The filling processes have known standard deviations of $\sigma_1 = 0.010$ and $\sigma_2 = 0.015$, respectively. A random sample of bottles from machine 1 and bottles from machine 2 results in average net contents of $\overline{x}_1 = 2.04$, $\overline{x}_2 = 2.07$. We wish to test the hypothesis that that both machines fill to the same net contents.

Name of the test:

Two sample Z-test

b. (2 points) The inside diameters of bearings used in an aircraft landing gear assembly are known to have a standard deviation of $\sigma = 0.0002$ cm. A random sample of 15 bearings has an average inside diameter of 8.2535 cm. We wish to test the hypothesis that the mean inside bearing diameter is 8.25 cm.

Name of the test: One sample z-test

c. (2 points) The diameter of a metal rod is measured by 12 inspectors, each using both a micrometer caliper and a vernier caliper. The results are shown in the table to the right. We wish to test if there a difference between the mean measurements produced by the two types of caliper.

Inspector	Micrometer Caliper	Vernier Caliper
1	0.150	0.151
2	0.151	0.150
3	0.151	0.151
4	0.152	0.150
5	0.151	0.151
6	0.150	0.151
7	0.151	0.153
8	0.153	0.155
9	0.152	0.154
10	0.151	0.151
11	0.151	0.150
12	0.151	0.152

Name of the test: Paired t-test

d. (2 points) Sixteen observations of output voltage for a power supply are taken at random: 10.35, 9.30, 10.00, 9.96, 11.65, 12.00, 11.25, 9.58, 11.54, 9.95, 10.28, 8.37, 10.44, 9.25, 9.38, and 10.85. We wish to test the hypothesis that the mean voltage is 12 V.

Name of the test: One sample t-test

e. (2 points) Sixteen observations of output voltage for a power supply are taken at random: 10.35, 9.30, 10.00, 9.96, 11.65, 12.00, 11.25, 9.58, 11.54, 9.95, 10.28, 8.37, 10.44, 9.25, 9.38, and 10.85. We wish to test the hypothesis that $\sigma^2 = 11$.

Name of the test: 1 Variance test

3) (10 points) The time that your phone battery lasts under typical conditions is normally distributed with parameters μ = 9.5 hours and σ = 0.8 hours. What is the probability that your battery lasts more than 10 hours?

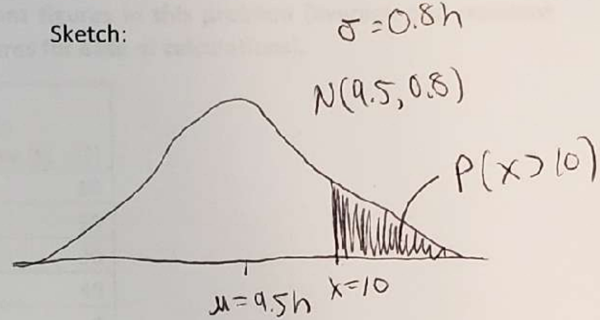
a. (5 points) Write out the probability equation for this problem (no need to normalize), and sketch out the problem using a normal probability sketch (label x=10, μ = 9.5, σ = 0.8 and P, the probability)

Probability equation:

We want to know

$P(x>10)$

Sketch:



b. (4 points) Convert the probability equation written in part (a) to a probability equation that can be solved using the standard normal distribution.

Show your work to convert equation: use $z = \frac{x-\mu}{\sigma}$, $x = z\sigma + \mu$

subbing

$x = z(0.8) + 9.5$

$P(x>10) = P(z(0.8) + 9.5 > 10)$

$= P(z(0.8) > 0.5)$

$= P(z > 0.625) = 1 - P(z \le 0.625)$

$= 1 - 0.734 = 0.266$

Write probability equation that can be solved using the standard normal distribution:

$1 - P(z \le 0.625) = \boxed{0.27}$

c. (1 point) Solve the probably equation

Write final answer: what is the probability that your phone battery lasts more than 10 hours?

4) (25 points) 11 operators on a manufacturing line have taken an online training module on lean manufacturing. The difference between the time taken to complete a task before the module and after the model (time before module – time after module) for the 11 operators was $\bar{x}$ = 15.45 min, and the standard deviation was σ = 14.40 min. You wish to know if there is enough evidence that the difference in time is non-zero at an alpha level = 0.05. You consider it possible that the video could increase or decrease the time to do a task. Keep 2 significant figures in this problem (average and standard deviation are shown with more figures for ease of calculations).

Operator	t1 (min)	t2 (min)	Difference in Process Time (t1 - t2)
1	60	50	10
2	80	50	30
3	80	50	30
4	90	50	40
5	100	100	0
6	90	90	0
7	80	70	10
8	80	70	10
9	100	70	30
10	90	80	10
11	80	80	0
			$\bar{x}$ = 15.45
			σ = 14.40

a) (2 point) Name the correct statistical test to conduct (use the word bank in in problem 2)

Name of test: Paired t-test (would also accept one sample t-test)

b) (4 points) Write the correct null and alternative hypotheses:

Null hypothesis: $H_0: \mu_D = \Delta_0 = 0$

Alternative hypothesis:

$H_1: \mu_D \neq 0.$

c) (5 points) Calculate the test statistic (keep two significant figures for the final answer)

Write out the general formula:

$$T_0 = \frac{\bar{D} - \Delta_0}{S_D/\sqrt{n}}, \quad \Delta_0 = 0$$

Sub in values and calculate:

$$T_0 = \frac{15.45 - 0}{14.40/\sqrt{11}} = \boxed{3.6}$$

d) (4 points) Write out the correct rejection criteria and the critical value that you will compare the test statistic to (keep two significant figures for the final answer)

Rejection criteria: Reject if:

$t_0 > t_{\alpha/2, n-1}$ or

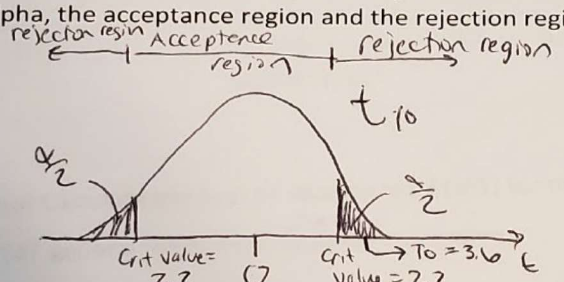
$t_0 < -t_{\alpha/2, n-1}$

Critical value:

$$t_{\alpha/2, n-1} = t_{\frac{0.05}{2}, 10} = t_{0.025, 10}$$

$$= 2.228 = 2.2$$

e) (8 points) Sketch out the reference distribution for this problem and label: the type of reference distribution used and any defining characteristics, the critical value, the test statistic, alpha, the acceptance region and the rejection region.

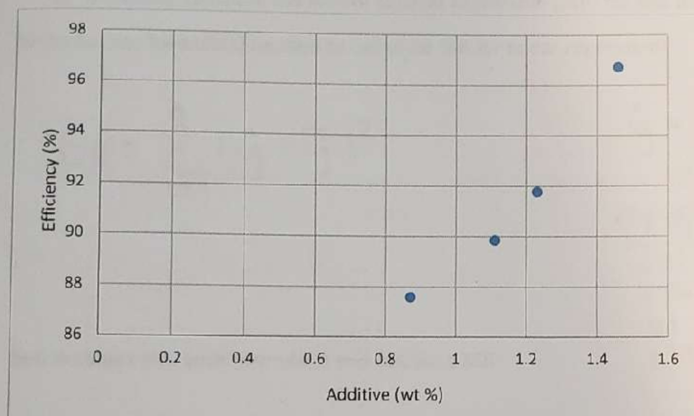


f) (2 points) Write your conclusions and comment on if you think the change in operator time to do a task is non-zero after watching the training video, and if you would recommend this video for the future.

We reject the null hypothesis (since $t_0 > t_{\alpha/2, n-1}$) and conclude that the change in operator time to do a task ~~ee~~ is no-zero after watching the video. In this case, the time significantly decreases so the video should be watched in the future.

5) **(35 points) You work for an electrode company and are investigating an ink additive. You obtain the following data by measuring the wt % of the ink additive and the ultimate performance (efficiency, %) of the resulting electrode. You wish to fit a simple linear regression model to this data, and determine if it is significant using ANOVA. You have already performed least squares regression to find that the slope (B1) is 15.47, and the intercept is (B0) is 73.42. Keep ~~4~~ 3 significant figures in your answers.**

Wt% Additive (x)	Efficiency % (y)
0.87	87.59
1.46	96.73
1.11	89.85
1.23	91.77



a) (6 points) Calculate the sum of squares total (SST) for this data set

Write out the SST general formula(s) for linear regression:

$$\bar{y} = 91.485$$

$$SS_T = \sum_{i=1}^{n} (y_i - \bar{y})^2 = (87.59 - 91.485)^2 + (96.73 - 91.485)^2 + (89.95 - 91.485)^2 + (91.77 - 91.485)^2$$

$$\boxed{SS_T = 45.4}$$

Sub in values into your formula(s) and calculate SST:

~~SSto~~ See prior page

b) (8 points) Calculate the sum of squares regression (SSR) for this data set

Write out the formula(s) needed to calculate SSR for linear regression:

$$SSR = \sum_{i=1}^{n} (\hat{y} - \bar{y})^2$$

$$\hat{y} = B_1 x + B_0$$
$$= 15.47x + 73.47$$

x	$\hat{y}$
0.87	86.93
1.46	96.06
1.11	90.64
1.23	92.50

$$\bar{y} = 91.49$$

Sub in values into your formula(s) and calculate SSR:

$$SSR = (86.93 - 91.49)^2 + (96.06 - 91.49)^2 + (90.64 - 91.49)^2 + (92.50 - 91.49)^2 = \boxed{43.4}$$

c) (4 points) Calculate SSE

Write out the formula(s) needed to calculate SSE for linear regression:

$$SS_T = SS_R + SS_E$$

Sub in values into your formulas and calculate SSE:

$$SS_E = SS_T - SS_R = 45.4 - 43.4 = \boxed{2.00}$$

d) (4 points) Write out the null and alternative hypothesis needed to test if the linear model is significant.

$H_0: \beta_1 = 0$

$H_1: B_1 \neq 0$

e) (7 points) Calculate the test statistic needed to test your hypothesis, and explain generally how the test statistic relates to the reference distribution

Write out the formula needed to calculate the test statistic (general):

$$F_0 = \frac{\frac{SS_R}{1}}{\frac{SS_E}{n-2}}$$

Sub in values into your formula and calculate the test statistic:

$$F_o = \frac{43.4}{\frac{2.04}{4-2}} = \boxed{\cancel{42.5}} \quad 43.4$$

How are the test statistic and the reference distribution related?

The test statistic will have a reference distribution when the null hypothesis is true.

f) (4 points) Based on your test statistic, find the p-value range – you can use your calculator to get a single value if you wish, and, explain what the p-value represents

F table $v_1 = 1$ $v_2 = 2$

P-value:

p-value @ $\alpha = 0.025 = 38.51$

" " $= 0.01 = 98.5$

$\boxed{0.01 < \text{p-value} < 0.025}$

The p-value is the probability of obtaining the sample data assuming the null is true

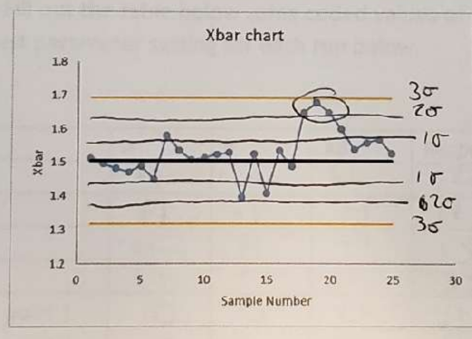
What does the p-value represent?

See previous page

6) **(11 points total) For the control chart below (showing 3σ control limit lines in yellow):**
a) (5 points) Use the criteria to determine if the process below is in control. State what, if any, rules have been violated.

Some Sensitizing Rules for Shewhart Control Charts

Standard Action Signal:	1. One or more points outside of the control limits	Western Electric Rules
	2. Two of three consecutive points outside the two-sigma warning limits but still inside the control limits	
	3. Four of five consecutive points beyond the one-sigma limits	
	4. A run of eight consecutive points on one side of the center line	
	5. Six points in a row steadily increasing or decreasing	
	6. Fifteen points in a row in zone C (both above and below the center line)	
	7. Fourteen points in a row alternating up and down	
	8. Eight points in a row on both sides of the center line with none in zone C	
	9. An unusual or nonrandom pattern in the data	



Is the process in control?

No, it violates rule 2 where circled + Rule 3, Rule 4

What, if any, rules have been violated?

maybe rule 9

b) (6 points) Explain what impact wide or narrow control limits have on type I and type II error, define type I and type II error as they pertain to control charts

Define type I and type II error:

Type I error (α): concluding the process is out of control when it is not

Type II error (β): claiming the process is in control when it's not

What impact does wide or narrow control limits have on type I and type II error?

If the limits are decreased (narrow) type I error will increase, type II will decrease

If the limits are increased (wide) type I error is decreased while type II is increased

7) **(27 points) Consider a 2^2 factorial experiment with four center points. The data are (1) = 21, a = 125, b = 154, ab = 352, and the responses at the center point are 168, 130, 162, 125. Use three significant figures in your final answers, and use alpha=0.05.**

a. (5 points) Fill out the table below using coded values of -1 for lowest setting, and 1 for highest parameter setting for each run below.

	A	B	AB	Response
(1)	-1	-1	+1	21
a	+1	-1	-1	125
b	-1	+1	-1	154
ab	+1	+1	+1	352
Center point 1	0	0	0	168
Center point 2	0	0	0	130
Center point 3	0	0	0	162
Center point 4	0	0	0	125

b. (10 points) Test for curvature only using pure error since the lowest ordered term here is a potentially significant two-way interaction. Write out all general formulas needed, sub in the correct values to calculate, and write out your conclusions.

To test for curvature: $F_0 = \dfrac{MS_{\text{pure quadradic}}}{MS_E}$

$$SS_{\text{pure quadradic}} = \frac{n_F n_c (\bar{y}_F - \bar{y}_c)^2}{n_F + n_c} = \frac{(4)(4)(163-146)^2}{4+4}$$

$$= 578$$

$n_F = 4 \quad n_c = 4$

$\bar{y}_F = 163$

$\bar{y}_c = 146$

Pure error

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{n \text{ centerpoints}} (y_i - \bar{y}_c)^2}{n_c - 1} = \frac{(168-146)^2 + (130-146)^2 + (162-146)^2 + (125-146)^2}{3}$$

$$= 479 = MS_E$$

$$F_0 = \frac{\frac{578}{1}}{479} = 1.21$$

$F_{crit} = F_{0.05,1,3} = 10.13$

Reject if $F_0 > F_{crit}$

$F_0 < F_{crit}$ so we fail to reject the null hypothesis. We do not have enough evidence that curvature is significant

c. (12 point) Test if Factor A is significant by using ANOVA and a value of 499.5 for MSE (which will have 4 degrees of freedom). Write out all general formulas needed, sub in correct values to calculate, and state your conclusion.

To test significance of factors:

$$F_0 = \frac{\frac{SS_{Factor}}{1}}{MS_E} \qquad SS_A = \frac{\text{Contrast } A^2}{n 2^k}$$

$$\text{Contrast } A = (-1)(21) + (125)(1) \mp (154)(1) (+1)(352) = 302$$

$n = 1 \quad k = 2$

$$SS_A = \frac{302^2}{2^2} = \frac{302^2}{4} = 22801$$

$$F_0 = \frac{22801}{499.5} = 45.6$$

$F_{crit} = F_{0.05,1,4} = 7.71$

Reject if $F_0 > F_{crit}$

We reject null hypothesis that A=0 and conclude Factor A is significant

d. (6 points) Let's say you perform the same analysis as in part c) for factor B and interaction AB and get a p-value of 0.001 and 0.103 for factor B and interaction AB respectively. You also calculate the coefficients of B and AB to be 90.0 and 23.5 respectively. Combined with your analysis in part c) write out a prediction model. What settings will give the highest response?

↳ grand average $B_0 = 154.6$ (average of all data points)

$$\hat{y} = Ax_1 + Bx_2 + B_0 \text{ (AB is not significant)}$$

$$A_{coeff} = \frac{\text{effect}}{2} = \frac{\frac{\text{contrast}}{n2^{k-1}}}{2} = \frac{\frac{302}{(1)(2^1)}}{2} = 75.5$$

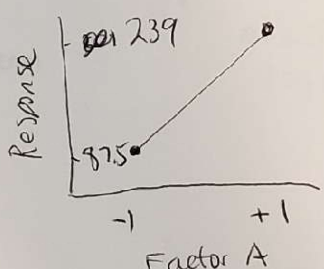
$$\hat{y} = 75.5x_1 + 90.0x_2 + 155$$

$$\hat{y}(1,0) = 75.5 + 155 = \boxed{230}$$

e) (5 points) Sketch out the main effect plot for A

Average at all +1 A points: 239

average at all -1 A points: 87.5



f) (7 points) Sketch out the interaction plot for AB and comment on if it agrees with the result of AB not being significant (what is it about the shape that supports AB not being significant?)

These lines do not intersect and so the interaction is likely not significant

8) **(6 points) Describe when fractional factorials might be useful, what is aliasing, and why aliasing may or may not be problematic**

Fractonal factorials are useful as the number of factors goes up, to reduce runs needed. In fractional factorials, main effets and interactions can be aliased with lower ordered effects (coupled). It can be problematic if both are significant, but often the terms are not. Also, if an effect is found to not be significant, a fractional factorial can collapse into a full factorial, avoiding aliasing all togeter.

HAVE A GOOD WINTER BREAK!