ECHE313 Homework 3 - Due 02/06/25

Trevor Swan (tcs94)

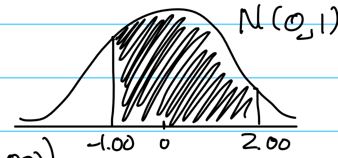
1. $\mu = 1.01 \text{ in}$ $\quad P(1.009 \text{ in} < \bar{x} < 1.012 \text{ in}) = ?$

$\sigma = 0.00300 \text{ in}$ $\quad \sigma_{\bar{x}} = \dfrac{0.00300 \text{ in}}{\sqrt{9}} = 0.00100 \text{ in}$ $\quad P(-1.00 \le Z \le 2.00)$

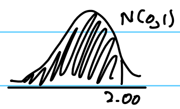
$n = 9$

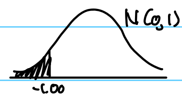
$$Z_L = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{1.009 - 1.01}{0.00100} = -1.00$$

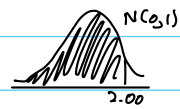
$$Z_u = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{1.012 - 1.01}{0.00100} = 2.00$$

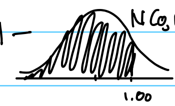


$= P(Z \le 2.00) - P(Z \le -1.00) = P(Z \le 2.00) - (1 - P(Z \le 1.00))$



$-$


$=$


$- \Big(1 -$


$\Big)$

$= 0.97725 - (1 - 0.84134)$

$= \boxed{0.819}$

2.

One-Sample Z:

Test of mu = 20 vs > 20

The assumed standard deviation = 0.75

Variable	N	Mean	SE Mean	Z	P
x	10	19.889	0.237	?	?

$Z = -0.468$

$P = 0.681$

a) Parameter of Interest: μ, Infer pop. mean, Known variance (std dev given)

$H_0: \mu = 20$ $\quad$ (One-Sample Z-test given)

$H_a: \mu > 20$ (one-sided)

Test Statistic: $Z_0 = \dfrac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \dfrac{19.889 - 20}{\frac{0.75}{\sqrt{10}}} = -0.468$

Reference Dist: $N(0,1)$

Reject if: $Z_0 > Z_\alpha = Z_{0.05}$

$\Rightarrow P(Z \le Z_\alpha) = 0.95$

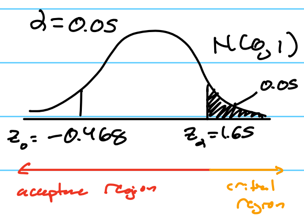
$\Rightarrow Z_\alpha = 1.65$

$\Rightarrow P_{value} = 1 - \Phi(Z_0) = 1 - P(Z \le Z_0)$

$= 1 - P(Z \ge -0.468)$

$= 1 - (1 - P(Z \le 0.468))$

$= 1 - (1 - 0.68082) = 0.68082$



Fixed significance

P-value

Since $Z_0 < Z_\alpha$ $(-0.468 < 1.65)$ and $P = 0.681 > 0.05$, we fail to reject H_0. There is insufficient evidence to deny the claim that the population mean is 20.

b)

Descriptive Statistics

N	Mean	SE Mean	95% Lower Bound for μ
10	19.889	0.237	19.499

μ: population mean of Sample
Known standard deviation = 0.75

Test

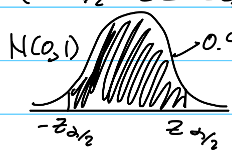
Null hypothesis	H_0: μ = 20
Alternative hypothesis	H_1: μ > 20

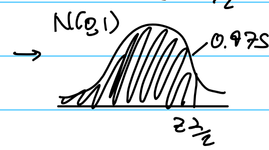
Z-Value	P-Value
-0.47	0.680

c) One-sided test as indicated in part (a)

d) 95% CI $\rightarrow \alpha = 0.05$

$P(-z_{\alpha/2} \le Z \le z_{\alpha/2}) = 0.95 \rightarrow P(Z \le z_{\alpha/2}) = 0.975 \rightarrow z_{\alpha/2} = 1.96$





$\bar{x} = 19.889$

$\sigma = 0.75$

$n = 10$

$\text{Lower} = 19.889 - 1.96 \frac{0.75}{\sqrt{10}} = 19.424$

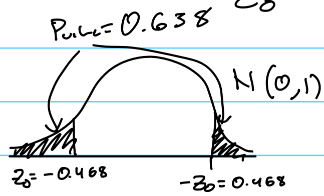
$\text{Upper} = 19.889 + 1.96 \frac{0.75}{\sqrt{10}} = 20.354$

$P\{19.424 \le \mu \le 20.354\} = 0.95$

In repeated sampling, this sampling method will produce intervals that capture the true population mean about 95% of the time.

e) Same setup as in part (c), but $H_a: \mu \ne 20$

Test statistic: $z_0 = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{19.889 - 20}{\frac{0.75}{\sqrt{10}}} = -0.468$ Reject if p-value $< \alpha = 0.05$



$\text{P-value} = 2(1 - \Phi(|-0.468|))$
$= 2(1 - P(Z \le 0.468))$
$= 2(1 - 0.68082) = 0.63836$

P-value = 0.638

Still fail to reject

3. a) Parameter of interest: μ, Inference about mean of population, known standard deviation

$H_0: \mu = 4$ hours   use $\alpha = 0.05$, One-sample Z-test

$H_1: \mu > 4$ hours

Given: $\bar{x} = 4.05$ hours

$\sigma = 0.20$ hours

$N = 50$

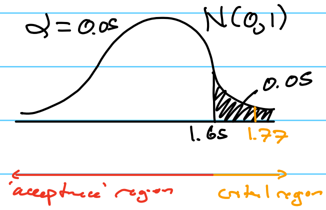
} normally distributed

Test Statistic: $Z_0 = \dfrac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \dfrac{4.05 - 4}{\frac{0.20}{\sqrt{50}}} = 1.77$

Ref. Dist: $N(0,1)$

Rejection Criteria: Reject if $Z_0 > Z_\alpha = Z_{0.05} = 1.65$

↳ value copied from Z-α   critical value

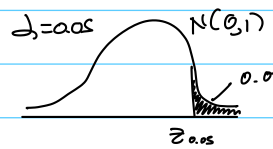


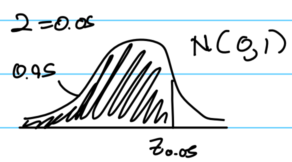
Conclusion: $Z_0 > Z_{0.05}$, $1.77 > 1.65 \rightarrow$ Reject H_0

We reject the null hypothesis. We have evidence to support the claim that the mean battery life of the population exceeds 4 hours.

b) One sided: $\bar{x} - Z_\alpha \frac{\sigma}{\sqrt{n}} \leq \mu$, $\alpha = 0.05$

$Z_{0.05}: P(Z \geq Z_{0.05}) = 0.05 \rightarrow P(Z \leq Z_{0.05}) = 0.95$





$Z_{0.05} = 1.65$

Lower $= 4.05 - 1.65 \dfrac{0.20}{\sqrt{50}} = 4.00$ 95% 1-sided CI: $4.00 \leq \mu$

In repeated sampling, we would expect similar methods to produce an interval that captures the true mean 90% of the time.

c)

Descriptive Statistics

N	Mean	SE Mean	95% Lower Bound for μ
50	4.0500	0.0283	4.0035

μ: population mean of Sample
Known standard deviation = 0.2

Test

Null hypothesis $H_0: \mu = 4$
Alternative hypothesis $H_1: \mu > 4$

Z-Value	P-Value
1.77	0.039

4. a) Parameter of interest: μ, inferring mean of population w/ known standard deviation

Diameter of Bearings (cm) Given:

$\sigma = 0.002$ cm

$\bar{x} = 8.2535$ cm

$N = 15$

$H_0: \mu = 8.25$ cm

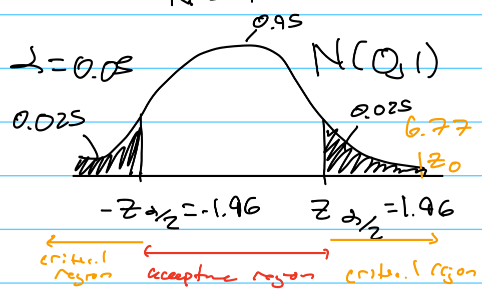
$H_a: \mu \neq 8.25$ cm

One-Sample Z-test

$\alpha = 0.05$

Test Statistic: $Z_0 = \dfrac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{N}}} = \dfrac{8.2535 - 8.25}{\frac{0.002}{\sqrt{15}}}$

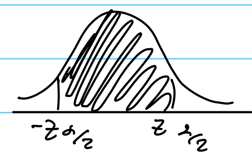
$= 6.77$



Reject if: $\begin{cases} Z_0 > Z_{\alpha/2} \\ Z_0 < -Z_{\alpha/2} \end{cases}$

$P(-Z_{\alpha/2} < Z < Z_{\alpha/2}) = 0.95$

$\Rightarrow P(Z < Z_{\alpha/2}) = 0.975$



$Z_{\alpha/2} = 1.96$

Conclusion: $Z_0 > Z_{\alpha/2} \rightarrow 6.77 > 1.96 \rightarrow$ Reject H_0

We reject the null hypothesis. We have sufficient evidence to reject the claim that the mean bearing diameter is 8.25 cm. We have evidence to support the claim that the mean bearing inside diameter is not 8.25 cm.

b) $P\text{-value} = 2(1 - \Phi(|Z_0|))$

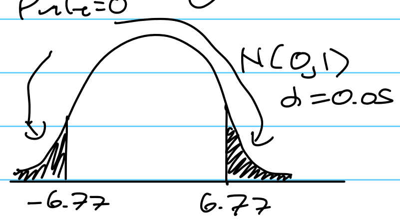
$= 2(1 - P(Z < 6.77))$

$= 2(1 - 1)$

normalcdf(-99999, 6.77, 0, 1) = 1

Had to use calculator as 6.77 exceeds table limits

Pvalue = 0 $= 0$



What does the p-value mean

If the null hypothesis is true, where $\mu = 8.25$ cm, there is a 0 probability of obtaining another sample as extreme as the one obtained. This leads us to reject H_0.

Conclusion: Since P-value < α ($0 < 0.05$), we reject H_0. We have strong evidence to reject the claim that the mean bearing inner diameter is 8.25 cm. Our evidence supports the claim that the true mean diameter is really not 8.25 cm.

c)

Descriptive Statistics

N	Mean	SE Mean	95% CI for μ
15	8.25350	0.00052	(8.25249, 8.25451)

μ: population mean of Sample
Known standard deviation = 0.002

Test

Null hypothesis	H_0: μ = 8.25
Alternative hypothesis	H_1: μ ≠ 8.25

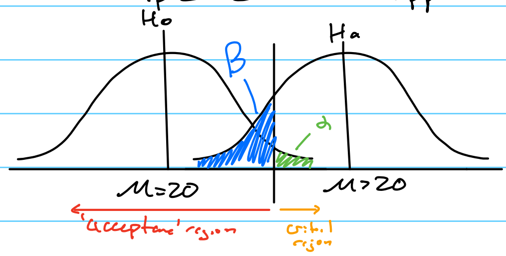
Z-Value	P-Value
6.78	0.000

5. A type I error, defined α, is described by the probability of rejecting the null hypothesis when it's actually true. This is in comparison to a type II error, defined β, which is described by the probability of the null hypothesis not being rejected when it is actually false.

False Positive → Rejecting H_0 is considered a strong conclusion as the rejection is based on α. If the p-value < α, then we reject H_0, and also minimize type I errors assuming α is small (done by convention). Since type I errors are controlled by α, it is ensured that the chance of rejecting a true null is low.

False Negative → Failing to reject is considered weak because it means p-value > α, which means we don't have evidence to reject H_0. This does not mean we can accept H_0 as true. This could be due to small sample sizes, or other sampling errors. This probability is independent of α, and thus its risk is difficult to control.

One-sample Z-test: Suppose $H_0: \mu = 20$, $H_a: \mu > 20$



= type I error → H_0 true but sample mean rejected

= type II error → sample mean falls in acceptance region despite true mean being shifted. leads to type II error