This is a case of known population mean and standard deviation and finding the probabilities for a random sample is given by:

$\mu = 1.01$

$\sigma = 0.003$

$\bar{x}$ is $N(\mu, \sigma^2/n)$ b/c of the Central limit theorem

Problem 1

So for $n=9$, what is: $P(1.009 < \bar{x} < 1.012)$?

w/ a mean $= 1.01$

$\sigma_{\bar{x}} = \frac{\sigma^2}{n}$

If we wish to use table in back of book:
(good for test practice!)

standardizing gives:

$$\bar{z} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{\bar{x} - 1.01}{\frac{0.003}{\sqrt{9}}} \qquad \bar{z}(0.001) = \bar{x} - 1.01$$

$\bar{x} = \bar{z}(0.001) + 1.01$ and sub into:

$$\begin{aligned} P(1.009 < \bar{x} < 1.012) &= P(\bar{x} < 1.012) - P(x \leq 1.009) \\ &= P(\bar{z}(0.001) + 1.01 < 1.012) - P(\bar{z}(0.001) + 1.01 \leq 1.009) \\ &= P(\bar{z} < 2) - P(\bar{z} \leq -1) \\ &= P(\bar{z} < 2) - (1 - P(\bar{z} \leq 1)) \\ &= P(\bar{z} < 2) - (1 - 0.84134) \\ &= 0.97725 - (1 - 0.84134) \\ &= 0.97725 - 0.158655 = \boxed{0.819} \end{aligned}$$

If we used Minitab with no standardization

$$P(\bar{x} < 1.012) - P(x \leq 1.009)$$

$$0.97725 - 0.158655 = \boxed{0.819}$$

Problem 2

a) $H_0: \mu = 20$

$H_1: \mu > 20$

$\sigma = 0.75$

$n = 10$

$\bar{x} = 19.889$

$\mu_0 = 20$

$$z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{19.889 - 20}{0.75/\sqrt{10}} = -0.468$$

$$= \boxed{-0.47}$$

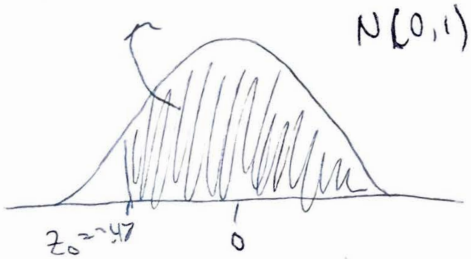
Probability above z_0:

Using back of book...

$$P = 1 - \Phi(z_0) = P(z > -0.47) = P(z < 0.47)$$

$N(0,1)$

because of symetry



$= 0.681$

$$\boxed{\text{P-Value} = 0.68}$$

The probability of obtaining z_0 if the null is true is large!

Descriptive Statistics

N	Mean	SE Mean	95% Lower Bound for μ
10	19.889	0.237	19.499

μ: mean of Sample
Known standard deviation = 0.75

Test

Null hypothesis H_0: $\mu = 20$
Alternative hypothesis H_1: $\mu > 20$

Z-Value	P-Value
-0.47	0.680

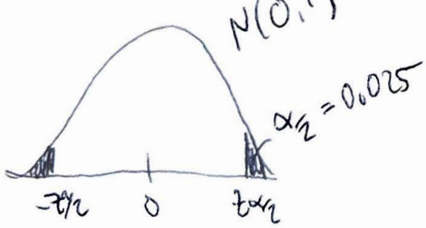
c) This is a 1-sided test

d) $$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$19.889 - 1.96\left(\frac{0.75}{\sqrt{10}}\right) \le \mu \le 19.889 + 1.96\left(\frac{0.75}{\sqrt{10}}\right)$$

$$\boxed{19 \le \mu \le 20}$$

N(0,1)



What does this really mean?

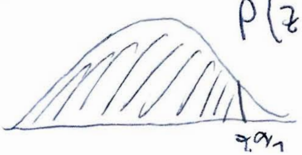
We are 95% confident the true mean lies between 19 and 20.... but what does 95% confident mean?

In repeated sampling, this method will produce intervals that capture the true population mean about 95% of the time.

Find $z_{\alpha/2}$:

The same value z gives the probability such that:

$$P(z \le z_{\alpha/2}) = 0.975$$

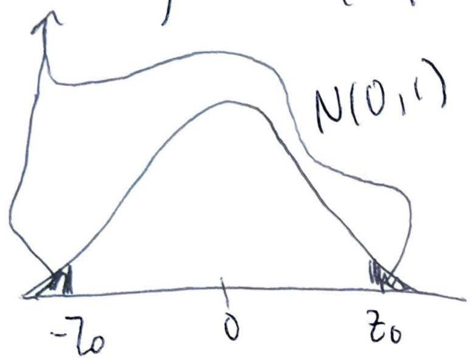


$$z_{\alpha/2} = 1.96$$

c) What would P-value be if $H_1: \mu \neq 20$?

$P = 2(1 - \Phi(|z_0|))$

Probability above $|z_0|$ and below $-|z_0|$



$= P(z < -0.47) + P(z > 0.47)$

$= [1 - P(z \leq 0.47)]2$

$= (1 - 0.68082)2$

$= 0.638$

$\boxed{\text{P-value} = 0.64}$

$n = 50 \quad \bar{x} = 4.05 \quad \sigma = 0.20hr \quad \alpha = 0.05$

Problem 3

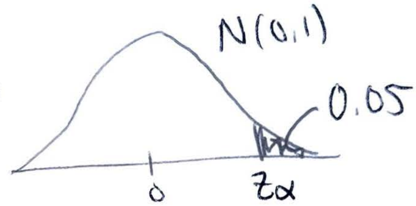
2) $H_0: \mu = 4hrs$

1) parameter of interest = battery life mean

3) $H_1: \mu > 4hrs$

4) $z_0 = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{4.05 - 4}{\frac{0.20}{\sqrt{50}}} = 1.768$ or 1.7

5) Reject H_0 if $z_0 > z_\alpha \rightarrow$



6) $z_0 = 1.7 > z_\alpha = 1.65$

$P(z > z_\alpha) = 0.05$
↓
same z_α as

$P(z \leq z_\alpha) = 0.95$

$z_\alpha = 1.65$ Appendix II

7) Reject null hypothesis and conclude that the mean battery life exceeds 4hrs

Problem 4

$\bar{x} - z_\alpha \dfrac{\sigma}{\sqrt{n}} \leq \mu$ lower confidence bound

$4.05 - 1.65 \dfrac{0.20}{\sqrt{50}} \leq \mu$

$4.0035 \leq \mu$

$\boxed{4.0 \leq \mu} \rightarrow$ we are 95% confident that the mean is greater than 4 !

Which means that if we repeated this test many times 95% of the time it will capture the true mean

) Minitab Output:

Descriptive Statistics

N	Mean	SE Mean	95% Lower Bound for μ
50	4.0500	0.0283	4.0035

μ: mean of Sample
Known standard deviation = 0.2

Test

Null hypothesis H_0: μ = 4
Alternative hypothesis H_1: μ > 4

Z-Value	P-Value
1.77	0.039

(6a) Hypothesis on the mean, variance known
(1-sample z-test)

Given: $\sigma = 0.002$cm $n = 15$ $\bar{x} = 8.2535$ cm

1) Parameter of interest is inside diameter of bearings (mean)

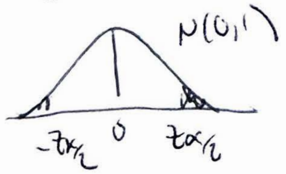
2) $H_0: \mu = 8.25$cm

3) $H_1: \mu \neq 8.25$cm

4) $z_0 = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{8.2535 - 8.25}{\frac{0.0020}{\sqrt{15}}} = 6.78$

5) Reject if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$

6) $z_{\alpha/2}$ → find for $\alpha = 0.05$



$P(Z \leq z) = 0.975$

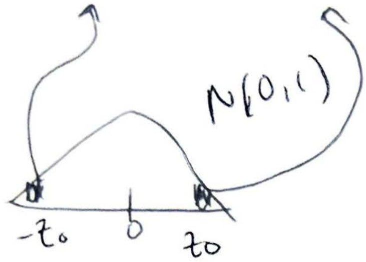
$z_{\alpha/2} = 1.96$

$z_0 = 6.78$
$z_{\alpha/2} = 1.96$
$z_0 > z_{\alpha/2}$

7) $z_0 > z_{\alpha/2}$ ∴ reject null hypothesis and conclude that $\mu \neq 8.25$ cm

value

$p = 2(1 - \Phi(|z_0|))$



Probability above $|z_0|$ and below $-|z_0|$

$= P(Z < 6.78) + P(Z > 6.78)$

$= [1 - P(Z \le 6.78)]2$

$= 1 - 1.00 = \boxed{0}$ or $\boxed{0.00}$

This is the probability of obtaining z_0 if the null hypothesis is true, which is very low. So the null hypothesis is very likely not true

6c) minitab:

Descriptive Statistics

N	Mean	SE Mean	95% CI for μ
15	8.25350	0.00052	(8.25249, 8.25451)

μ: mean of Sample
Known standard deviation = 0.002

Test

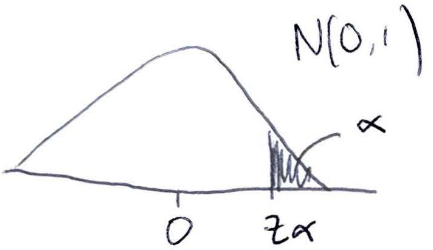
Null hypothesis	H_0: μ = 8.25
Alternative hypothesis	H_1: μ ≠ 8.25

Z-Value	P-Value
6.78	0.000

Problem 5

·) Type I error is the probability of rejecting the null hypothesis when it is true (α). We can control how much Type I error we have. Type II error is the probability of <u>not</u> rejecting the null hypothesis, when it is indeed false. We often don't have much control over type II error (β)

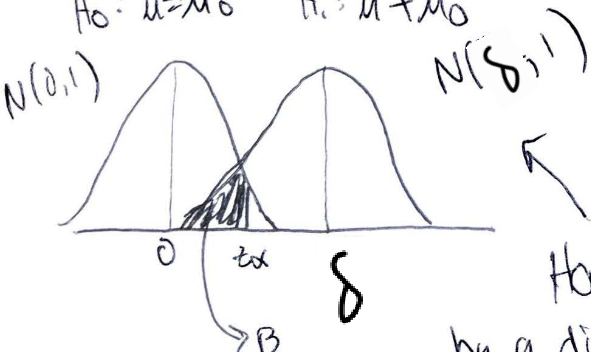
To illustrate: for a 1-sided z-test



upper tailed z-test

α - represents the chance that we have rejected the null hypothesis even though it is true. If true, the standardized data would be distributed according to $N(0,1)$, and there is a chance α we would get a value past the critical value z_α. We can set α aggressively to avoid this error and make it very unlikely.

$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$



However, if the null isn't true (represented by a dist. not around 0) then there is a chance (β) that we keep the null hypothesis even though it is false. We can control β by increasing n, but often this is hard to control.