4.13 in book

1. $\bar{x}_1 = 9.85$, $s_1^2 = 6.79$, $n_1 = 10$ | $\bar{x}_2 = 8.08$, $s_2^2 = 6.18$, $n_2 = 8$

a) Inference on Two chemical processes' variance — independent & normality assumed.

↳ Two sample variance test

$H_0: \sigma_1^2 = \sigma_2^2$ $H_a: \sigma_1^2 \neq \sigma_2^2$

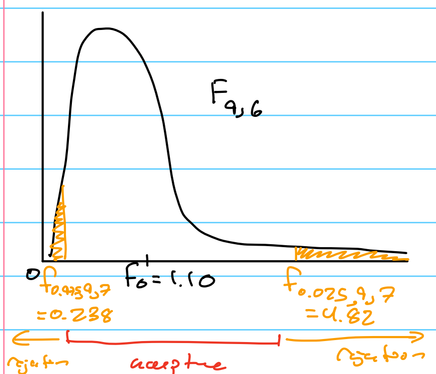
$F_0 = \frac{s_1^2}{s_2^2} = \frac{6.79}{6.18} = 1.10$

$D.F_1 = 10-1 = 9$

$D.F_2 = 8-1 = 7$

$\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$



Rejection Criteria:

$$\begin{cases} F_0 > f_{0.025,9,7} \\ F_0 < f_{0.975,9,7} = \frac{1}{f_{0.025,7,9}} \end{cases}$$

critical value

$f_{0.025,9,7} = 4.82$

$f_{0.975,9,7} = \frac{1}{4.20} = 0.238$

$F_0 < f_{0.025,9,7}$ & $F_0 > f_{0.975,9,7}$

Fail to reject H_0. We have insufficient evidence to reject the claim that the variances of the two chemical processes are different. We cannot conclude that the variance of the new purification unit differs from the old purification unit's variance.

b) Minitab P-value: 0.922 → Fail to reject as p-value $= 0.922 > \alpha = 0.05$

Test

Null hypothesis	$H_0: \sigma_1^2 / \sigma_2^2 = 1$
Alternative hypothesis	$H_1: \sigma_1^2 / \sigma_2^2 \neq 1$
Significance level	$\alpha = 0.05$

Test

Method	Statistic	DF1	DF2	P-Value
F	1.10	9	7	0.922

2. Changing catalyst => more variability in yield?

$H_0: \sigma_1^2 = \sigma_2^2$ $\quad H_a: \sigma_1^2 \neq \sigma_2^2$

a) Assume normality

Method

σ₁: standard deviation of Cat 1
σ₂: standard deviation of Cat 2
Ratio: σ₁/σ₂
F method was used. This method is accurate for normal data only.

Descriptive Statistics

Variable	N	StDev	Variance	95% CI for σ^2
Cat 1	10	3.444	11.864	(5.613, 39.541)
Cat 2	10	2.224	4.946	(2.340, 16.485)

Ratio of Variances

Estimated Ratio	95% CI for Ratio using F
2.39860	(0.596, 9.657)

Test

Null hypothesis $H_0: \sigma_1^2 / \sigma_2^2 = 1$
Alternative hypothesis $H_1: \sigma_1^2 / \sigma_2^2 \neq 1$
Significance level $\alpha = 0.05$

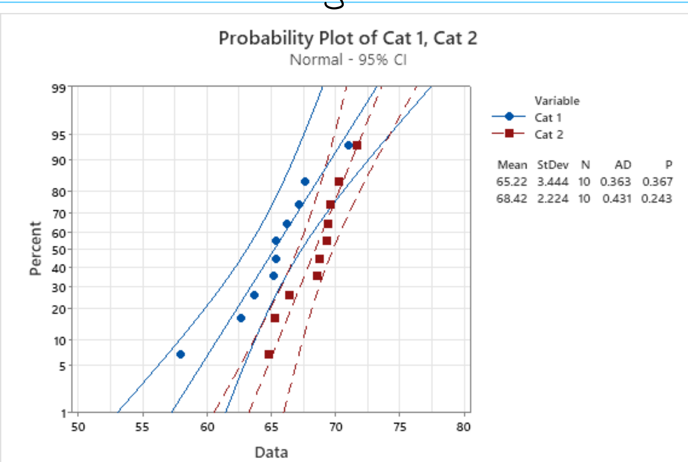
Method	Test Statistic	DF1	DF2	P-Value
F	2.40	9	9	0.209

P-value = 0.209 > α = 0.05. We have insufficient evidence to reject the claim that the variances of these two samples are equal. If we were to conduct a t-test, we could assume equal variances, as we have insufficient evidence to claim they are not equal.

This aligns with our rule of thumb:

$$\frac{11.864}{4.946} = 2.4 < 4 \checkmark$$

b) Check normality



Both samples have p-values > α, so there is insufficient evidence to reject the claim of normality for these data.

Normality assumption seems appropriate

4.23 in book

3. **Table 4E.4** Baked Density Data for Exercise 4.23

$a=4$

	Temperature (°C)	Density ← Carbon Anode Density					
1	500	41.8	41.9	41.7	41.6	41.5	41.7
2	525	41.4	41.3	41.7	41.6	41.7	41.8
3	550	41.2	41.0	41.6	41.9	41.7	41.3
4	575	41.0	40.6	41.8	41.2	41.9	41.5

Assume Equal Variance $n=6$

a. Does firing temperature in the ring furnace affect mean baked anode density?
b. Find the residuals for this experiment and plot them on a normal probability scale. Comment on the plot.
c. What firing temperature would you recommend using?

a) Multiple Population's Carbon Anode Density data at different temperatures → ANOVA

$H_0: \tau_1 = \tau_2 = \tau_3 = \tau_4 = 0$ $\quad H_a$: At least one $\tau_i \neq 0$ for some i

Table 4E.4 Baked Density Data for Exercise 4.23

	Temperature (°C)	Density						$y_{i\cdot}$ Totals	$\bar{y}_{i\cdot}$ Averages
$i=1$	500	41.8	41.9	41.7	41.6	41.5	41.7	250.2	41.7
$i=2$	525	41.4	41.3	41.7	41.6	41.7	41.8	249.5	41.583
$i=3$	550	41.2	41.0	41.6	41.9	41.7	41.3	248.7	41.45
$i=4$	575	41.0	40.6	41.8	41.2	41.9	41.5	248	41.333
								996.4	41.517

Eqns

→ $SS_T = \sum_{i=1}^{a}\sum_{j=1}^{n}(y_{ij}-\bar{y}_{\cdot\cdot})^2$

→ $SS_{treatments} = n\sum_{i=1}^{a}(\bar{y}_{i\cdot}-\bar{y}_{\cdot\cdot})^2$

→ $SS_E = SS_T - SS_{treatments}$

Totals

$y_{1\cdot} = 41.8 + \dots + 41.7 = 250.2$

$y_{2\cdot} = 41.4 + \dots + 41.8 = 249.5$

$y_{3\cdot} = 41.2 + \dots + 41.3 = 248.7$

$y_{4\cdot} = 41.0 + \dots + 41.5 = 248$

$y_{\cdot\cdot} = y_{1\cdot} + y_{2\cdot} + y_{3\cdot} + y_{4\cdot} = 996.4$

Averages

$\bar{y}_{1\cdot} = \frac{250.2}{6} = 41.7$

$\bar{y}_{2\cdot} = \frac{249.5}{6} = 41.583$

$\bar{y}_{3\cdot} = \frac{248.7}{6} = 41.45$

$\bar{y}_{4\cdot} = \frac{248}{6} = 41.333$

$\bar{y}_{\cdot\cdot} = \frac{996.4}{24} = 41.517$

$$SS_T = \sum_{i=1}^{4}\sum_{j=1}^{6}(y_{ij}-\bar{y}_{\cdot\cdot})^2 = (41.8-41.517)^2 + (41.9-41.517)^2 + \dots + (41.5-41.517)^2 = 2.5533$$

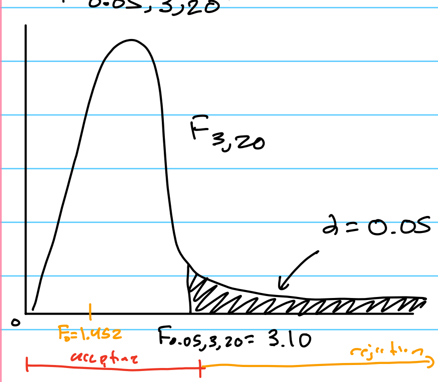
$$SS_{treatments} = 6\sum_{i=1}^{4}(\bar{y}_{i\cdot}-\bar{y}_{\cdot\cdot})^2 = 6\left[(41.7-41.517)^2 + (41.583-41.517)^2 + (41.45-41.517)^2 + (41.333-41.517)^2\right] = 0.4567$$

$$SS_E = 2.5533 - 0.4567 = 2.097$$

Test Statistics:
$$F_0 = \frac{SS_{treatments}/(a-1)}{SS_E/(a(n-1))} = \frac{0.4567/(4-1)}{2.097/(4(6-1))} = 1.452$$

Reject if $F_0 > F_{\alpha, a-1, a(n-1)}$

$F_{0.05,3,20} = 3.10$



$F_0 = 1.452 < F_{0.05,3,20} = 3.10$, fail to reject. We have insufficient evidence to accept the claim that baking temperature impacts the produced Carbon Anode's Density. We cannot conclude that firing temperature affects mean baked anode density.

b) Residuals

$E_{ij} = y_{ij} - \bar{y}_{i\cdot}$

residual obs avg

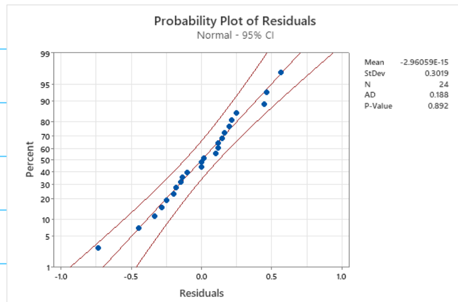
$E_{11} = y_{1} - \bar{y}_{1\cdot} = 41.8 - 41.7 = 0.1$

$E_{12} = y_{12} - \bar{y}_{1\cdot} = 41.9 - 41.7 = 0.2$

$\vdots$

$E_{46} = y_{46} - \bar{y}_{4\cdot} = 41.5 - 41.333 = 0.167$

Placed Residuals from Excel into One Column → Minitab Normal Prob Plot



A non-normality assumption is valid. P-value = 0.892 indicates that we cannot reject the normality assumption!

c) It does <u>not</u> matter. We have found that all temperatures produce products with the same specs, so I would recommend using the most time and cost effective temperature!

d) From Minitab

Method

Null hypothesis	All means are equal
Alternative hypothesis	Not all means are equal
Significance level	$\alpha = 0.05$

Equal variances were assumed for the analysis.

Factor Information

Factor	Levels	Values
Factor	4	500, 525, 550, 575

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Factor	3	0.4567	0.1522	1.45	0.258
Error	20	2.0967	0.1048		
Total	23	2.5533			

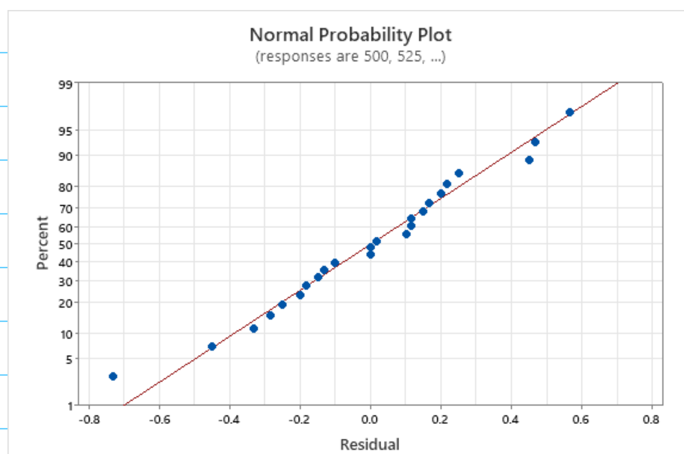
Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.323780	17.89%	5.57%	0.00%

Means

Factor	N	Mean	StDev	95% CI
500	6	41.7000	0.1414	(41.4243, 41.9757)
525	6	41.5833	0.1941	(41.3076, 41.8591)
550	6	41.450	0.339	(41.174, 41.726)
575	6	41.333	0.497	(41.058, 41.609)

Pooled StDev = 0.323780



p-value = $0.258 < 0.05$

We have insufficient evidence to accept the claim that the baking temperature has an effect on the density of produced carbon anodes.

4.25 in book

All calculations done in Minitab as problem states to do test in minitab!

4. **Table 4E.5** Radon Data for the Experiment in Exercise 4.25

Orifice Diameter	Radon Released (%)			
0.37	80	83	83	85
0.51	75	75	79	79
0.71	74	73	76	77
1.02	67	72	74	74
1.40	62	62	67	69
1.99	60	61	64	66

Assume Equal Variance

a) Hypothesis Test

$H_0: \tau_1 = \tau_2 = \dots = \tau_6 = 0$

$H_a:$ At least one $\tau_i \neq 0$

$SS_{Treatments} = 1133.4$

$SS_E = 132.3$

Reject if: $F_0 > F_{\alpha, a-1, a(n-1)}$

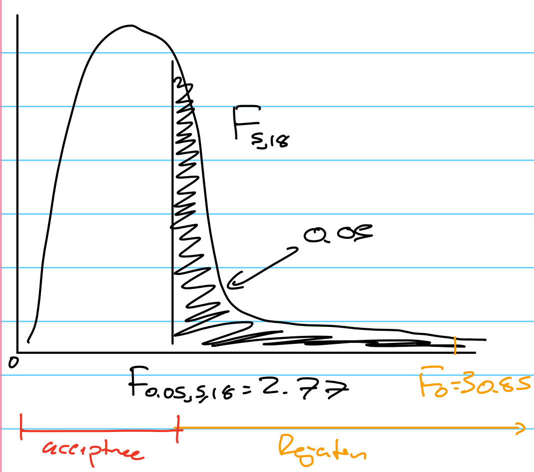
$\hookrightarrow F_{0.05,5,18} = 2.77$

a-1) Factors = 1 total (Orifice Diameter Category Only)

a-2) Levels = 6 levels (Orifice Diameter)

a-3) $$F_0 = \frac{SS_{Treatments}/(a-1)}{SS_E/a(n-1)} = \frac{1133.4/5}{132.3/18} = 30.85$$

Reference Distribution



Reject Ho. $F_0 = 30.85 > F_{0.05,5,18} = 2.77$. We have sufficient evidence to reject the claim that the treatments have no effect on result. At least one treatment (orifice diameter) affects the mean percentage of radon released.

b) Residuals

b-1) $E_{ij} = y_{ij} - \bar{y}_{i\cdot}$

$$\bar{y}_{1\cdot} = \frac{80+83+83+85}{4} = 82.75$$

$$\bar{y}_{2\cdot} = \frac{75+75+79+79}{4} = 77$$

$$\bar{y}_{3\cdot} = \frac{74+73+76+77}{4} = 75$$

$$\bar{y}_{4\cdot} = \frac{67+72+74+74}{4} = 71.75$$

$$\bar{y}_{5\cdot} = \frac{62+62+67+69}{4} = 65$$

$$\bar{y}_{6\cdot} = \frac{60+61+64+66}{4} = 62.75$$

$$E_{11} = y_{11} - \bar{y}_{1\cdot} = 80 - 82.75 = -2.75$$

$$E_{12} = y_{12} - \bar{y}_{1\cdot} = 83 - 82.75 = 0.25$$

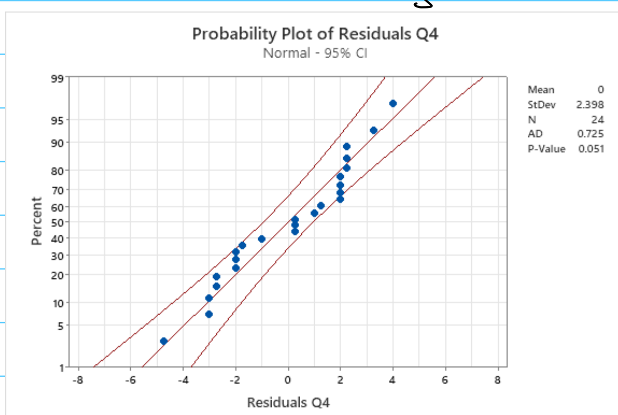
$$E_{13} = y_{13} - \bar{y}_{1\cdot} = 83 - 82.75 = 0.25$$

$$\vdots$$

$$E_{64} = y_{64} - \bar{y}_{6\cdot} = 66 - 62.75 = 3.25$$

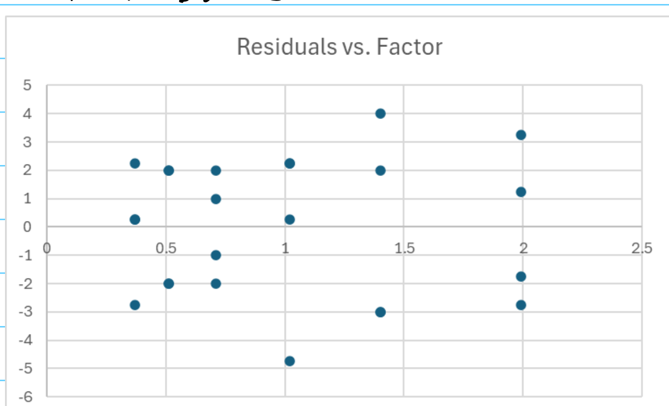
Remaining Residuals on 'Question 4' Sheet from Excel :)

b-2) Normal Probability Plot of Residuals



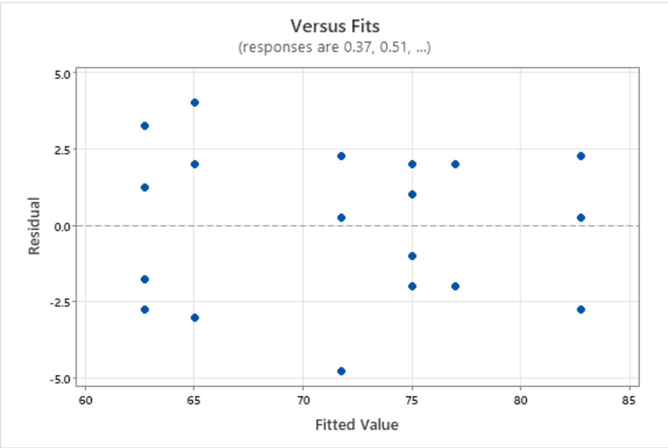
P-value = 0.051 < 0.05 = α, so we Fail to Reject the normality assumption. With linearity clearly visible and p-value being so high, normality is an OK assumption. This p-value is very close to α, so I would proceed with caution...

b-3) Residuals vs. Factor Levels



This graph checks equal variance at each factor level. The graph shows the residuals at each factor level being randomly, but evenly, scattered along each factor, so the equal variance assumption is ok here.

b-4) Residuals vs. Factor Levels



This also checks the equal variance assumption, but in such a way that it gives us 'next steps'. Here, the assumption is OK as the residuals are evenly spread about 0. If this wasn't true, we would either Redo Analysis with different assumptions or apply a data transformation!

c-1)

$$LSD = t_{\alpha/2, a(n-1)} \sqrt{\frac{2MSE}{n}}$$

$t_{0.025,18} = 2.101 \rightarrow LSD = 2.101\sqrt{2 \cdot \frac{6.625}{4}} = 3.82$

0.37 vs. 0.51 : $|82.75 - 77| = 5.75^*$ ← Yes! !!

0.37 vs. 0.71 : $|82.75 - 75| = 7.75^*$ ← Yes! !!

0.51 vs. 0.71 : $|77 - 75| = 2$ ← No! ?!

0.37 : A

0.51 : B

0.71 : B C

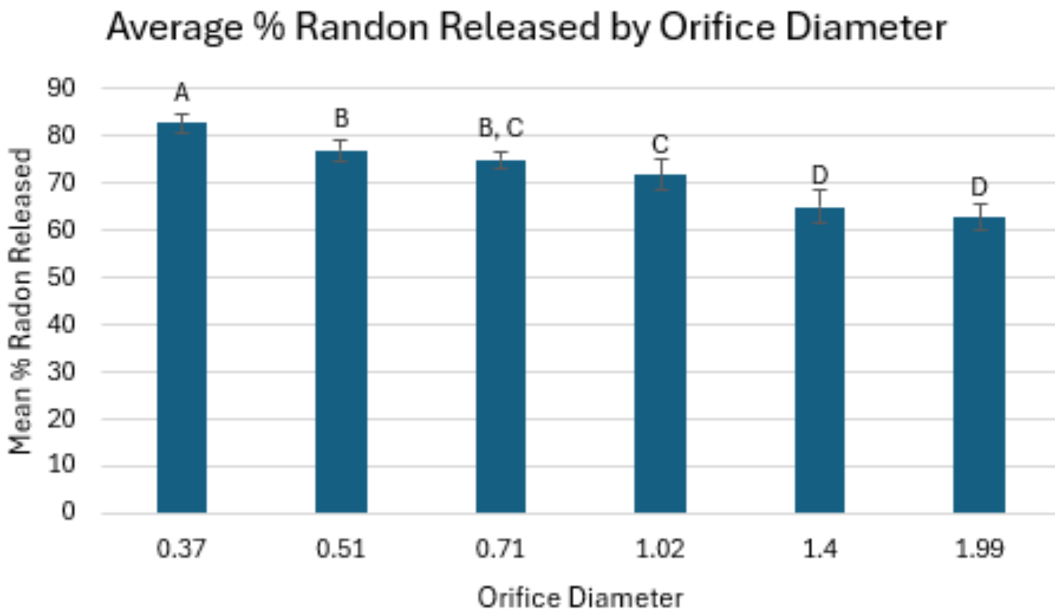
Minitab Output

Grouping Information Using the Fisher LSD Method and 95% Confidence

Factor	N	Mean	Grouping
0.37	4	82.75	A
0.51	4	77.00	B
0.71	4	75.000	B C
1.02	4	71.75	C
1.4	4	65.00	D
1.99	4	62.75	D

Means that do not share a letter are significantly different.

C-2)



All data are represented by the average ± standard error. ANOVA was performed to confirm statistical significance of Orifice Diameter differnce on the mean radon percentage released, followed by Fischer's LSD post hoc test to determine differences between treatment means using α=0.05. Bars that do not share a letter within a graph are statistically different.