Homework 8

1)

Source	SS	DF	MS	F	P
A	0.322	1	0.322	0.0357	> 0.25
B	80.554	2	40.2771	4.462	0.025 < p-value < 0.05
AB	42.348	2	21.174	2.346	0.10 < p-value < 0.25
Error	108.327	12	9.027		
Total	231.551	17			

a) Fill in the blanks

$SS_{AB} = SS_{Total} - SS_E - SS_A - SS_B = 231.551 - 108.327 - 80.554 - 0.322$
$= 42.348$

DOF for A:

$a - 1 = 1 \quad \therefore \quad a = 2$

DOF for Error:

$ab(n-1) = 2(b)(n-1) = 2bn - 2b = 12$

$\therefore \quad 18 - 2b = 12$

$-2b = -6$

$b = 3 \quad \therefore$ DOF $b = b - 1 = 2$

DOF for Total:

$abn - 1 = 17 \quad \therefore \quad 2bn = 18$

DOF for AB:

$(a-1)(b-1) = 1 \times 2 = 2$

MS

$$MS_A = \frac{SS_A}{a-1} = \frac{0.322}{1} = \boxed{0.322}$$

$$MS_{AB} = \frac{SS_{AB}}{(a-1)(b-1)} = \frac{42.348}{2} = \boxed{21.174}$$

$$MS_E = \frac{SS_E}{ab(n-1)} = \frac{108.327}{12} = \boxed{9.027}$$

F

$$F_{OA} = \frac{MS_A}{MS_E} = \frac{0.322}{9.027} = \boxed{0.0357}$$

$$F_{OB} = \frac{MS_B}{MS_E} = \frac{40.2771}{9.027} = \boxed{4.462}$$

$$F_{OAB} = \frac{MS_{AB}}{MS_E} = \frac{21.174}{9.027} = \boxed{2.346}$$

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p-values

A $\Rightarrow$ $v_1 = 1$, $v_2 = 12$ — $\boxed{p\text{-value} > 0.25}$

B $\Rightarrow$ $v_1 = 2$, $v_2 = 12$ — $\boxed{0.025 < p\text{-value} < 0.05}$

AB $\Rightarrow$ $v_1 = 2$, $v_2 = 12$ — $\boxed{.10 < p\text{-value} < 0.25}$

How many levels in Factor B? $b = 3$ levels

c) How many replicates?

$b = 3$ and $2bn = 18$ (see part A)

$\therefore \quad 6n = 18 \qquad n = \frac{18}{6} = 3$ $\quad n = 3$

d) What conclusions?

Factor B significantly affects the response, whose A / AB do not (or we don't have enough evidence to say they do)

3)

$\tau_1 = \tau_2 = \tau_3 = 0$

$H_0: \tau_i = 0 \quad H_1: \tau_i \neq 0$

$\beta_1 = \beta_2 = 0$

$H_0: \beta_j = 0, \quad H_1: \beta_j \neq 0$

$H_0: (\tau\beta)_{ij} = 0, \quad H_1: (\tau\beta)_{ij} \neq 0$

$(\tau\beta)_{11} = (\tau\beta)_{12} = (\tau\beta)_{21} = (\tau\beta)_{22} = (\tau\beta)_{31} = (\tau\beta)_{32} = 0$

$a = 3$

$b = 2$

$n = 3$

$SS_T = SS_A + SS_B + SS_{AB} + SS_E$

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} (y_{ijk} - \bar{y}_{...})^2$$

$$= (280-263)^2 + (300-263)^2 + \dots$$

$$(235-263)^2 + (230-263)^2$$

$$SS_T = 16156$$

$$\bar{y}_{...} = \frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}}{abn} = \frac{47490}{(3)(2)(3)} = 263.33$$

$$SS_A = bn \sum_{i=1}^{a} (\bar{y}_{i..} - \bar{y}_{...})^2$$

$$\bar{y}_{i..} = \frac{\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}}{bn}$$

	$\bar{y}_{i..}$
1	260
2	273
3	256

$$SS_A = (2)(3)\left[(260-263)^2 + (273-263)^2 + (256.7-263)^2\right]$$

$$SS_A = 433.3$$

$$SS_B = an \sum_{j=1}^{b} (\bar{y}_{.j.} - \bar{y}_{...})^2$$

$$(3)(3)\left[(291.61-263)^2 + (235-263)^2\right]$$

$$SS_B = 14456$$

13.3

$$SS_{AB} = n \sum_{i=1}^{a} \sum_{j=1}^{b} (\bar{y}_{ij} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...})^2$$

$$= 3\left[(\underset{\text{cell ave}}{285} - \underset{\text{row ave}}{260} - \underset{\text{column ave}}{291.7} + \underset{\text{grand ave}}{263})^2 \right.$$

$$\left. + \text{for all cells} \ldots \right]$$

$$\boxed{SS_{AB} = 133.3}$$

$$SS_E = SS_T - SS_A - SS_B - SS_{AB}$$

$$= 633.3$$

	F_0	DOF (top)	DOF error	F_{crit}	Reject null?
A	$F_0 = \frac{\frac{SS_A}{DOF_A}}{\frac{SS_E}{DOF_E}} = \frac{\frac{9393}{2}}{\frac{6333}{12}} = \boxed{88}$	2	12	$F_{0.05,2,12} = 3.89$	yes
B	$F_0 = \frac{14450}{\frac{633.3}{12}} = \boxed{273.7}$	1		$F_{0.05,1,12} = 4.75$	yes
AB	$F_0 = \frac{\frac{133.3}{2}}{\frac{6333}{12}} = 1.26$	2		$F_{0.05,2,12} = 3.89$	no

Conclusions:

Glass type and phosphor type both had a significant effect on current necessary to obtain brightness level. There is not enough evidence to say there is a significant interaction between the two

		Factor B (Glass Type)					
		1			2		
Factor A (Phosphor Type)	1	280	290	285	230	235	240
	2	300	310	295	260	240	235
	3	290	285	290	220	225	230

y(bar)ij.	1	2	y(bar)i..
1	285	235	260
2	301.6667	245	273.3333
3	288.3333	225	256.6667
y(bar).j	291.6667	235	263.3333
	1	2	

SST **16150**

277.7777778	711.1111111	469.4444	1111.111	802.7778	544.4444
1344.444444	2177.777778	1002.778	11.11111	544.4444	802.7778
711.1111111	469.4444444	711.1111	1877.778	1469.444	1111.111

SSA **933.3333**

11.11111
100
44.44444

SSB **14450**

802.7778	802.7778

SSAB **133.3333333**

11.11111111	11.11111111
3.23117E-27	3.23117E-27
11.11111111	11.11111111

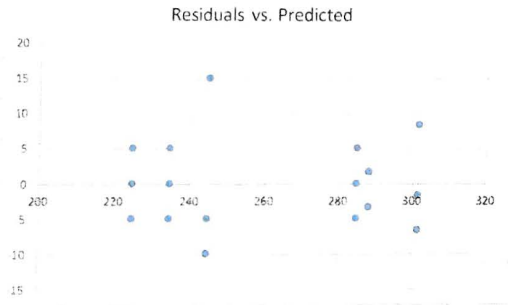
SSE **633.3333**

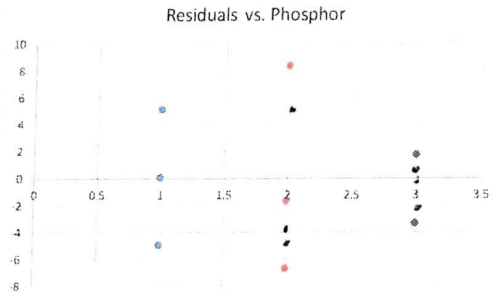
Residuals:

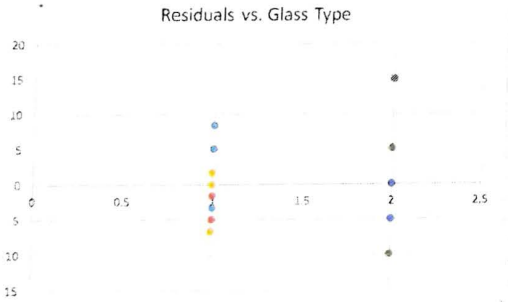
-5	5	0	-5	0	5
-1.66666667	8.333333333	-6.66667	15	-5	-10
1.666666667	-3.33333333	1.666667	-5	0	5

1	1	1
2	2	2
3	3	3

1	1	1	2	2	2
1	1	1	2	2	2
1	1	1	2	2	2







residuals look good!

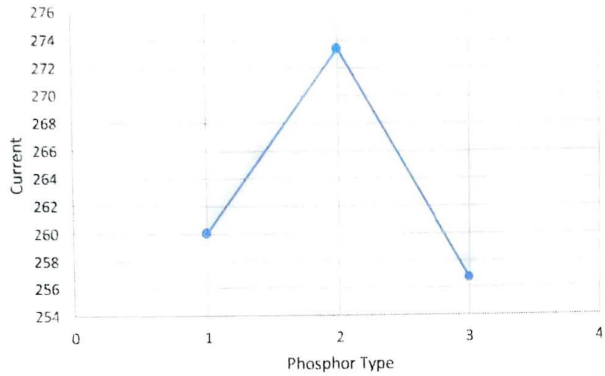
$$e_{ijk} = y_{ijk} - \hat{y}_{ijk}$$

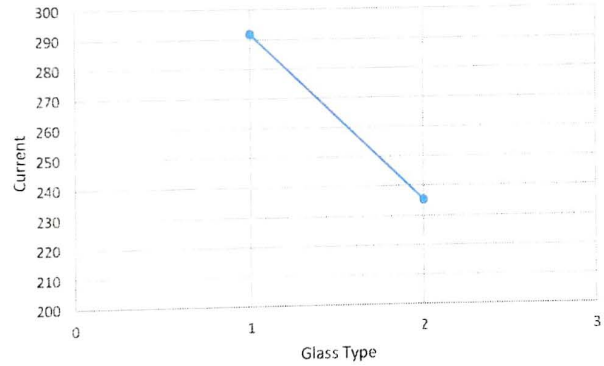
$$= y_{ijk} - \bar{y}_{ij.}$$

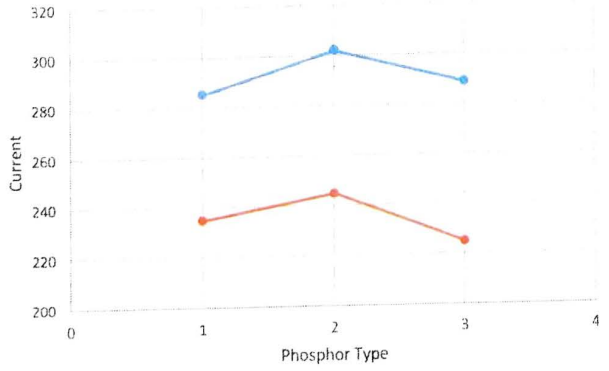
↑ cell average

_do for each data point

	Fo	DOF top	DOF error	Fcrit	Fo>Fcrit?
A	8.842105	2	12	3.89	*
B	273.7895	1		4.75	*
AB	1.263158	2		3.89	no





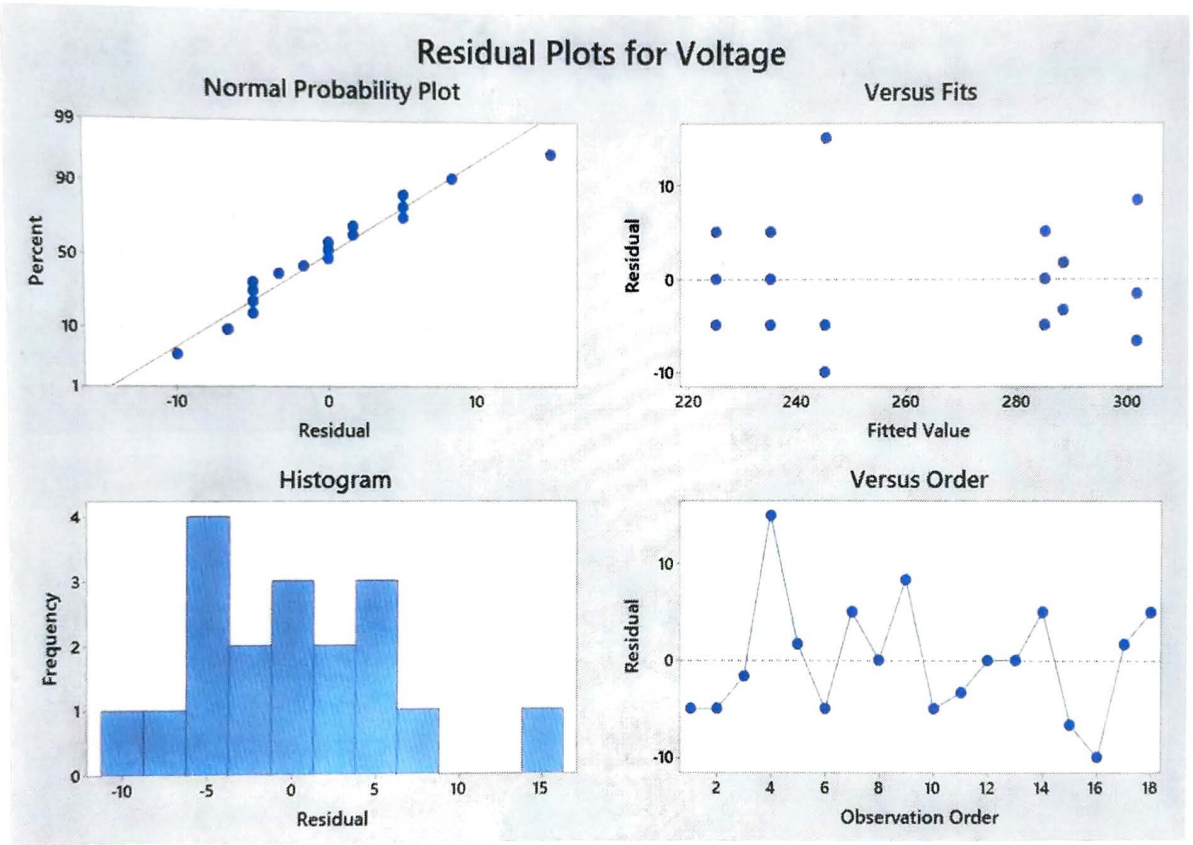


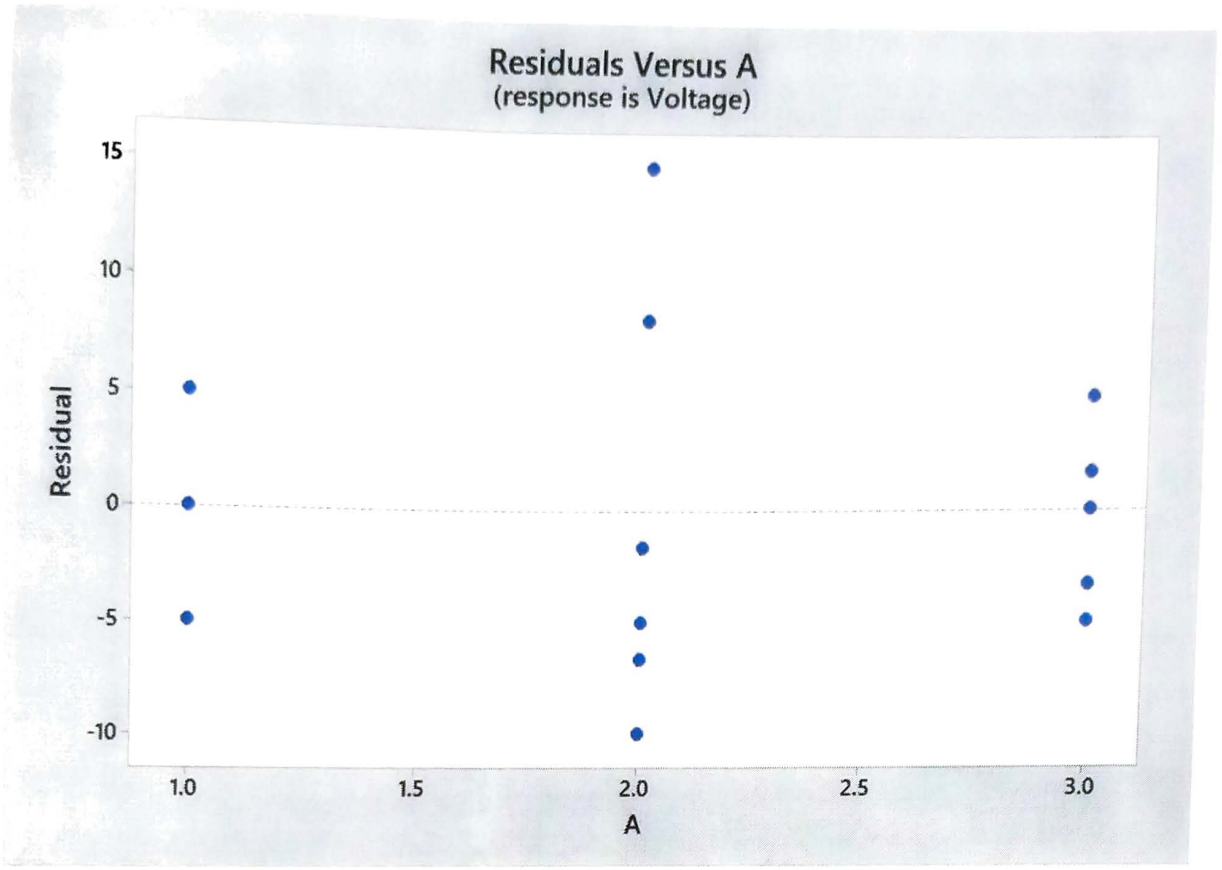
Factor Information

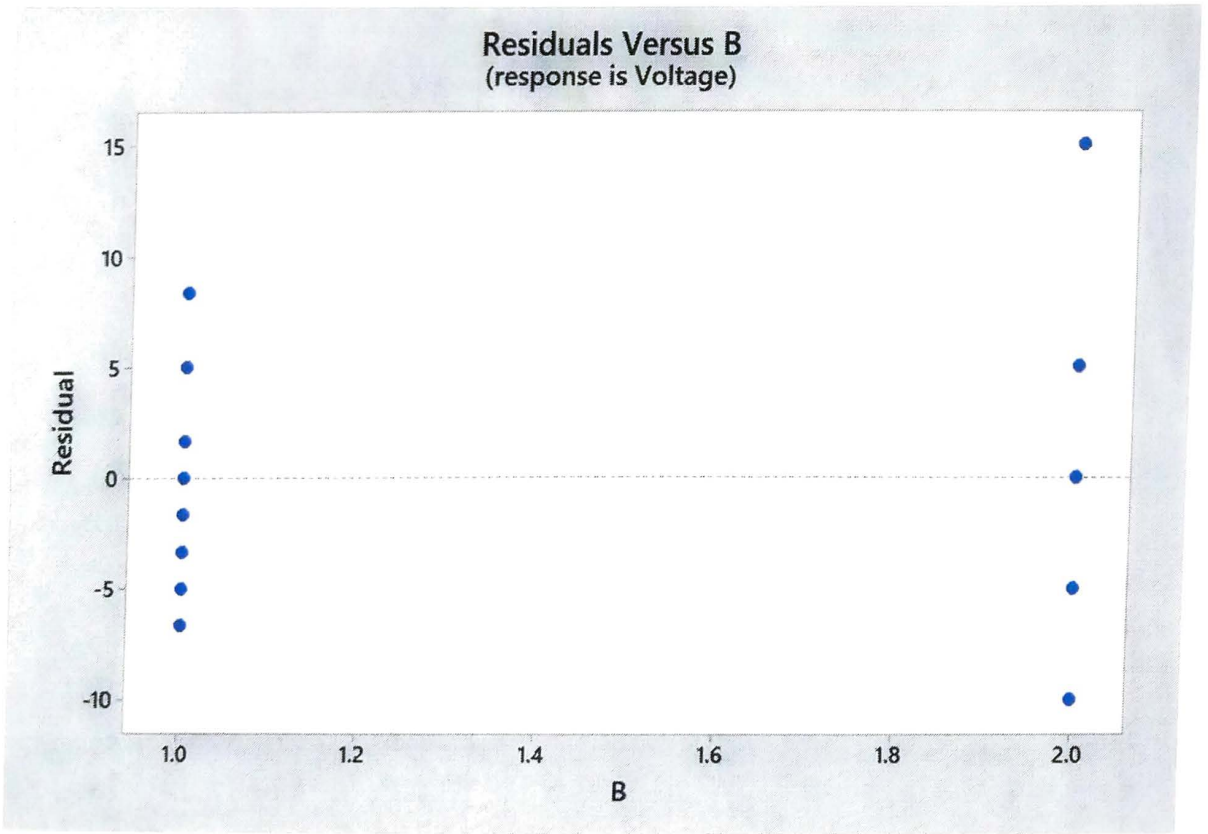
Factor	Levels	Values
A	3	1, 2, 3
B	2	1, 2

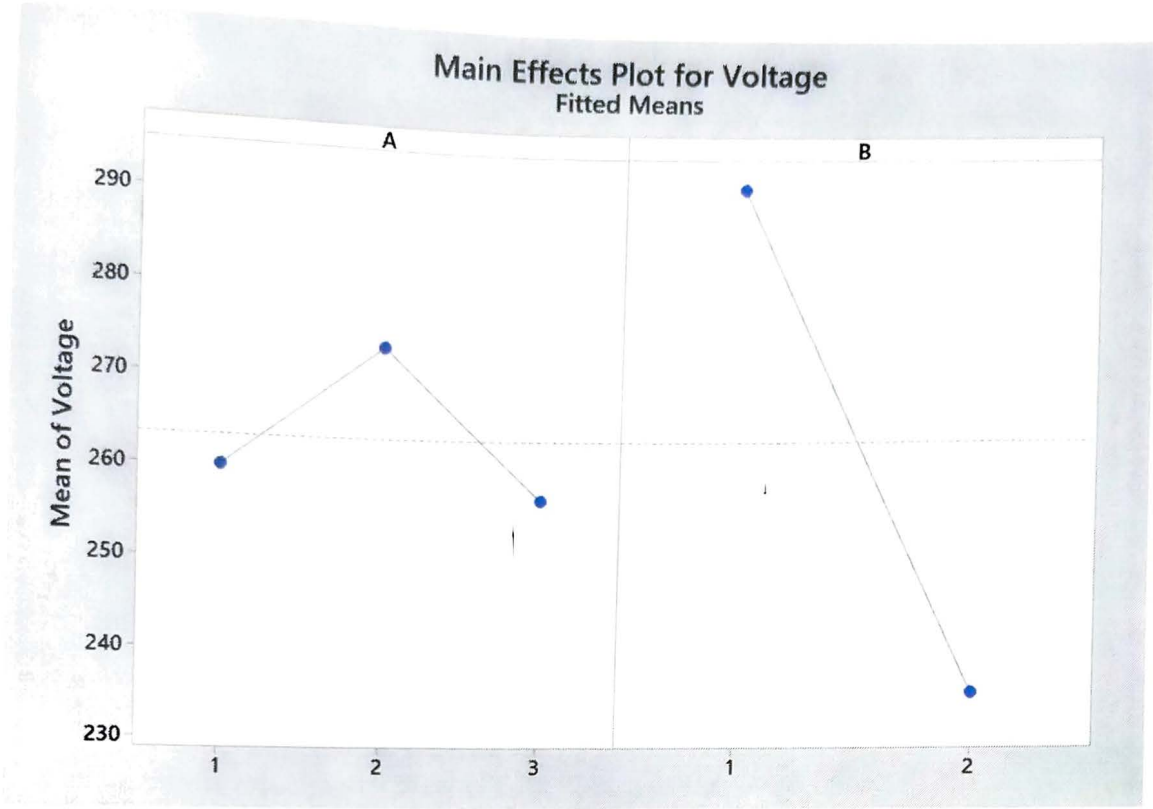
Analysis of Variance

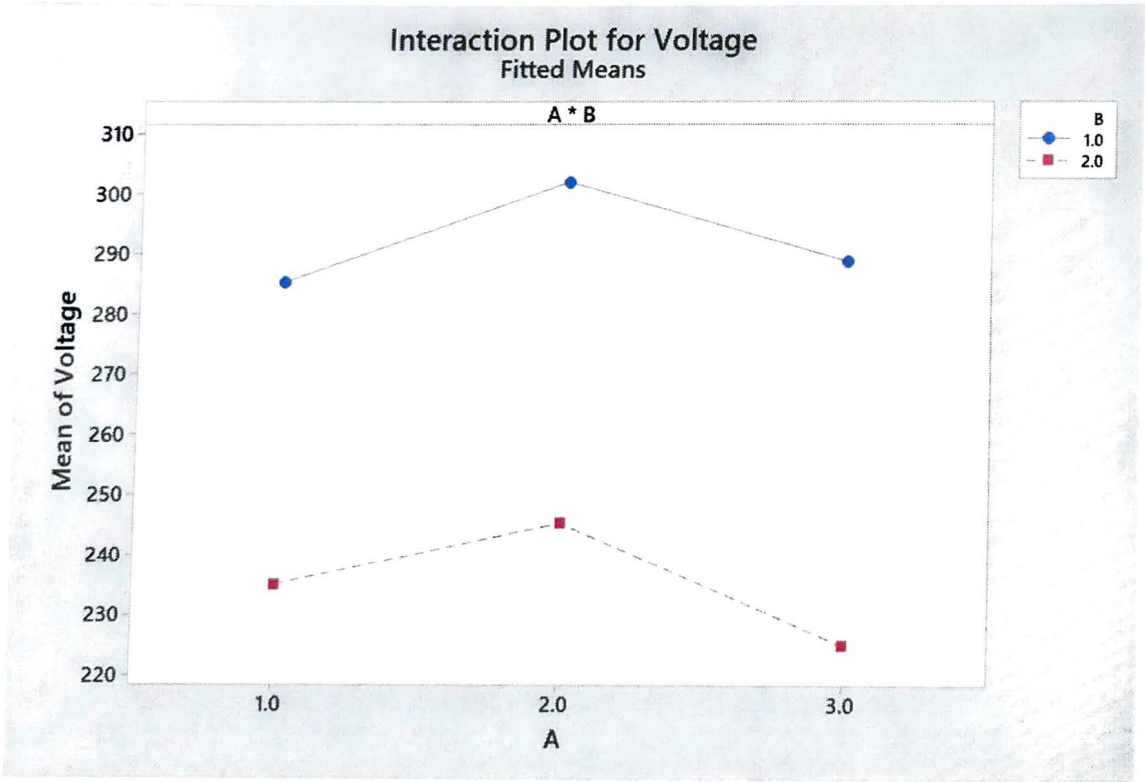
Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	5	15516.7	3103.3	58.80	0.000
Linear	3	15383.3	5127.8	97.16	0.000
A	2	933.3	466.7	8.84	0.004
B	1	14450.0	14450.0	273.79	0.000
2-Way Interactions	2	133.3	66.7	1.26	0.318
A*B	2	133.3	66.7	1.26	0.318
Error	12	633.3	52.8		
Total	17	16150.0			




Residuals Versus A
(response is Voltage)
Residual
15
10
5
0
-5
-10
1.0
1.5
2.0
2.5
3.0
A


Residuals Versus B
(response is Voltage)
Residual
15
10
5
0
-5
-10
1.0
1.2
1.4
1.6
1.8
2.0
B


Main Effects Plot for Voltage
Fitted Means
A
B
Mean of Voltage
290
280
270
260
250
240
230
1
2
3
1
2


Interaction Plot for Voltage
Fitted Means
A * B
B
1.0
2.0
Mean of Voltage
310
300
290
280
270
260
250
240
230
220
1.0
2.0
3.0
A