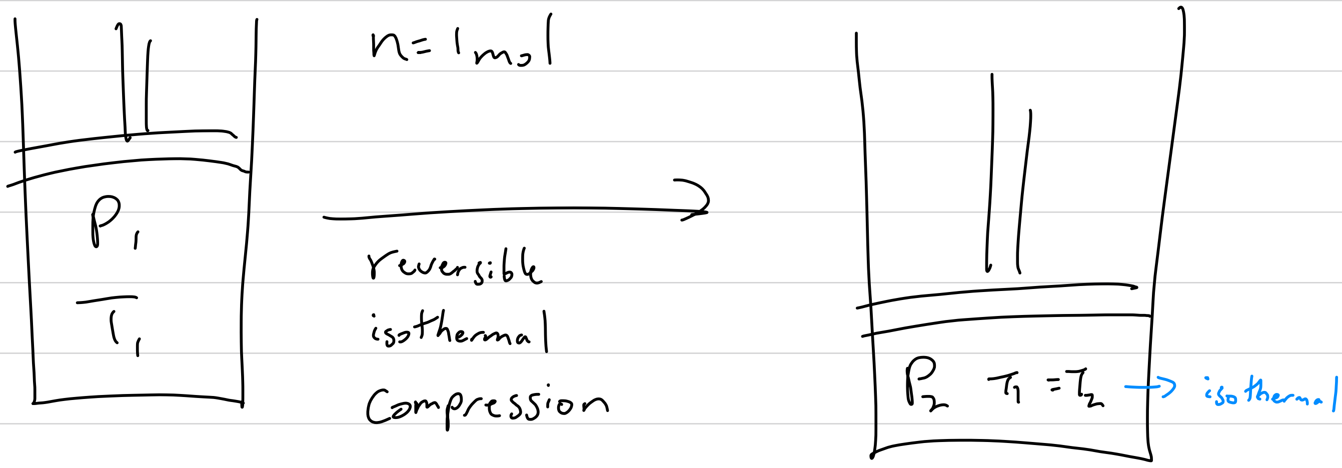
Homework #2

1)



→ Reversible ⇒ $P_E = P$

→ equation ⇒ $V_m = \frac{RT}{P} + b$

Work $W = -\int P_E dV = -\int P dV$

$$V_m = \frac{RT}{P} + b$$

$$\frac{V}{n} = \frac{RT}{P} + b$$

$$V = \frac{nRT}{P} + b$$

$$dV = d\left(\frac{nRT}{P} + b\right)$$

n, R, T, b are const.

$$dV = nRT\, d\left(\frac{1}{P}\right)$$

$$dV = nRT\left(-\frac{1}{P^2}\right)dP$$

$$W = -\int P dV$$

$$= -\int P\, nRT\left(-\frac{1}{P^2}\right)dP$$

$$= nRT\int_{P_1}^{P_2}\frac{1}{P}dP$$

$$W = nRT \ln\left(\frac{P_2}{P_1}\right)$$

OR:

$$v_m = \frac{RT}{P} + b$$

$$\Rightarrow P = \frac{RT}{v_m - b}$$

$$W = -\int P_E \, dv_m \underset{\uparrow}{=} -\int_{v_{m,1}}^{v_{m,2}} \frac{RT}{v_m - b} \, dv_m$$

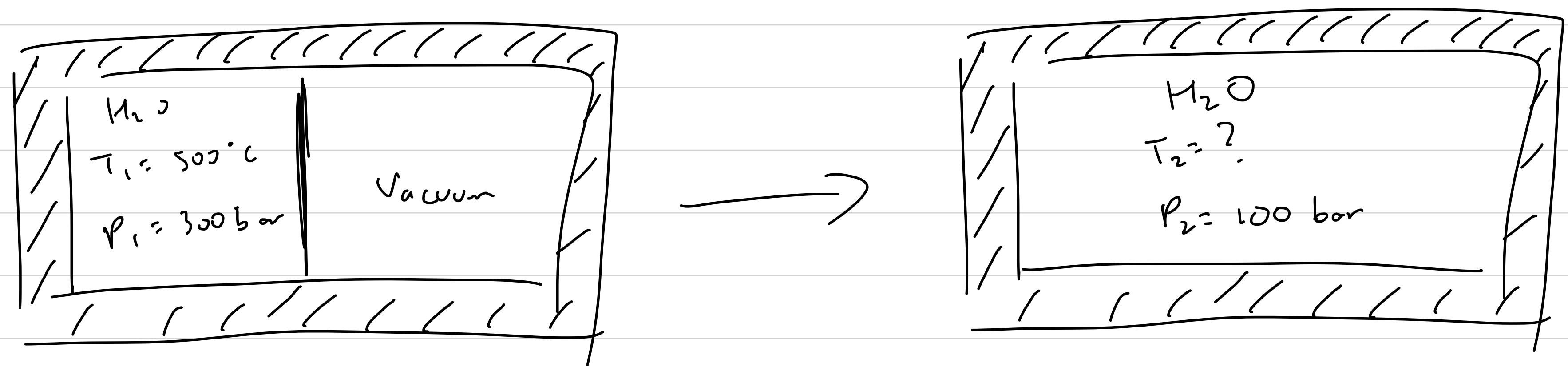
$$P_E = P$$

$$P = \frac{RT}{v_m - b}$$

$$W = -RT \ln\left(\frac{v_{m,2} - b}{v_{m,1} - b}\right)$$

$$W = RT \ln\left(\frac{P_2}{P_1}\right)$$

2)



Find: T_2, V

Insulated $\Rightarrow Q = 0$
Rigid system $\Rightarrow W = 0$
Closed system $\Rightarrow m_1 = m_2 = 1$ kg

$\Big\}$ Constant internal Energy $\rightarrow \Delta U = Q + W = 0$

From Table B.4 (30 MPa at 500°C)

$\hat{u}_1 = 2820.7$ kJ/kg

$\hat{u}_1 = \hat{u}_2 = 2820.7$ kJ/kg

Use Table B.4 to find $\hat{u} = 2820.7$ kJ/kg at 100 bar (10 MPa)

$\hat{u}_{low} = 2699.2$ kJ/kg $\quad T_{low} = 350°C$
$\hat{u}_2 = 2820.7$ kJ/kg $\quad T_2 = ?$
$\hat{u}_{High} = 2832.4$ kJ/kg $\quad T_{high} = 400°C$

$\Big\}$ Interpolate

Interpolation ex//

$$T_2 = T_{low} + (T_{High} - T_{low})\left(\frac{\hat{u}_2 - \hat{u}_{low}}{\hat{u}_{High} - \hat{u}_{low}}\right)$$

$$= 350 + (400 - 350)\left(\frac{2820.7 - 2699.2}{2832.4 - 2699.2}\right)$$

$$= 350 + (50)(0.912)$$

$$= 395.6°C$$

From Table:

$\hat{V}_{low} = 0.02242\ m^3/kg$

$\hat{V}_{High} = 0.02641\ m^3/kg$

} Interpolate

$$\hat{V}_2 = \hat{V}_{low} + (\hat{V}_{High} - \hat{V}_{low})\left(\frac{\hat{U}_2 - \hat{U}_{low}}{\hat{U}_{High} - \hat{U}_{low}}\right)$$

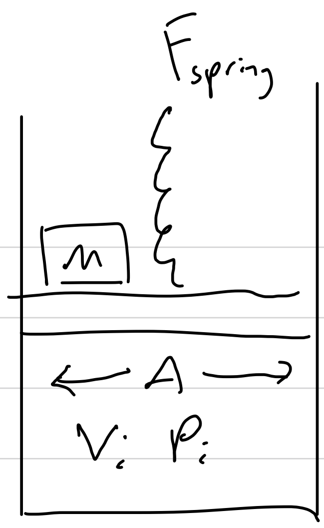
$$= 0.02242 + (0.02641 - 0.02242)(0.912)$$

$$= 0.02606\ m^3/kg$$

$$V = m\hat{V} = 1\ kg\ (0.02606\ m^3/kg)$$

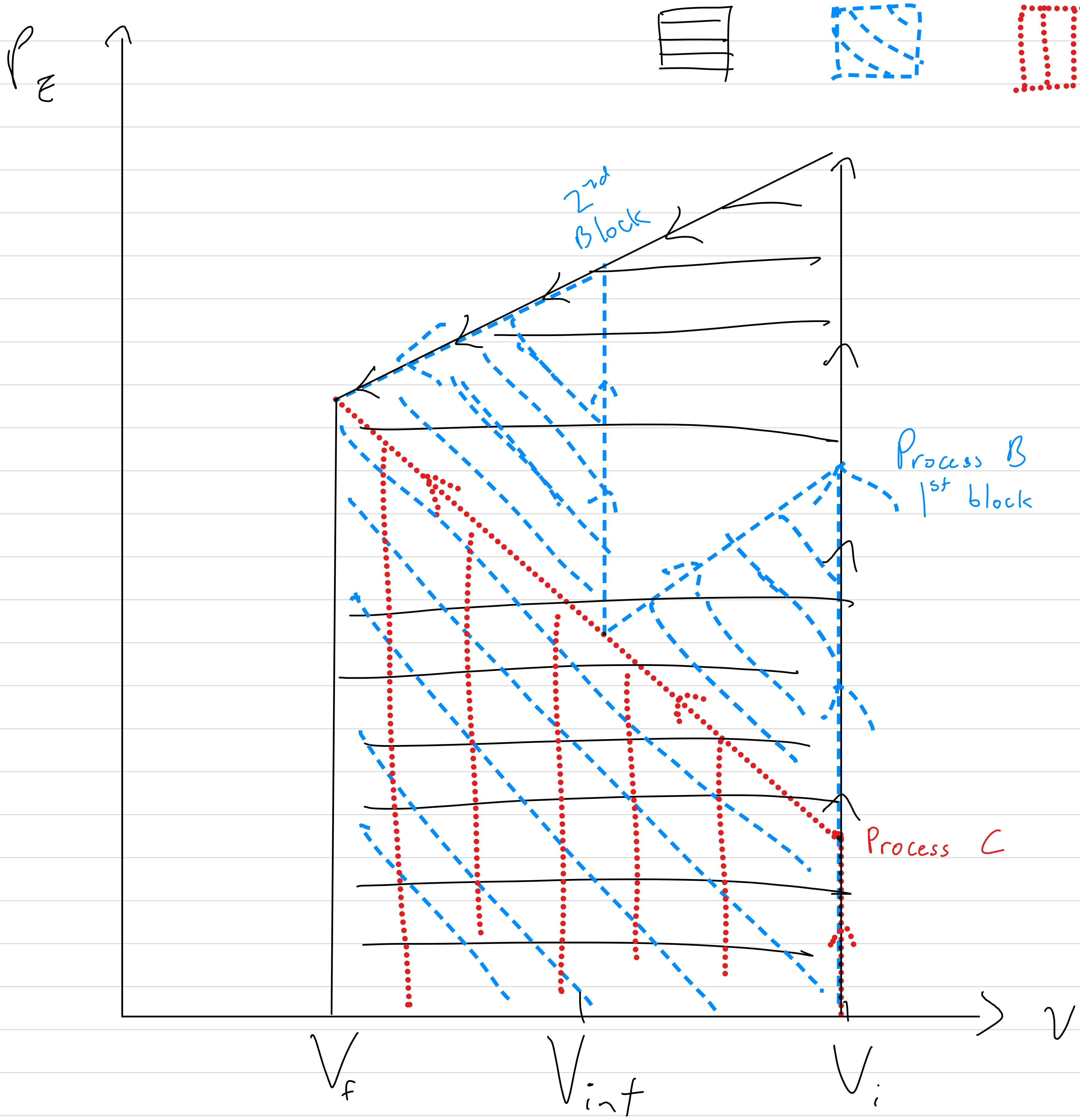
$$\boxed{V = 0.02606\ m^3}$$

3)



$A = 0.1 m^2$
$V_i = 0.1 m^3$
$P_i = 1$ bar

$|W_a| > |W_b| > |W_c|$



a) Hint: Find κ from force balance on final State $\quad V_f = 0.05 m^2$

$$\underbrace{P_f}_{\text{system}} = \underbrace{P_{E,f}}_{\text{external}} \quad \text{(mechanical equilibrium on final state)}$$

$$A P_f = A P_{E,f}$$
$$= A P_a + mg - F_{spring}$$
$$= A P_a + m \quad + \kappa x$$
$$A P_f = A P_a + mg + k\left(\frac{V_f - V_i}{A}\right)$$

$$\kappa\left(\frac{V_f - V_i}{A}\right) = A(P_f - P_a) - mg$$
$$\kappa = \left(\frac{A}{V_f - V_i}\right)\left[A(P_f - P_a) - mg\right]$$
$$= \left(\frac{0.1 m^2}{0.05 m^3 - 0.1 m^3}\right)\left[0.1 m^2\left(2 \times 10^5 - 10^5\right) - (2040 kg)(9.81 m/s^2)\right]$$

$$= 2.0 \times 10^4 \ Pa \cdot m^2 \ [N/m]$$

$$W = -\int_{V_i}^{V_f} P_E \, dV = -\int_{V_i}^{V_f}\left(\underline{P_a} + \underline{\frac{mg}{A}} + \kappa\left(\frac{V - V_i}{\underline{A^2}}\right)\right) dV$$

constants

$$= -\left(P_a + \frac{mg}{A} - \frac{kV_i}{A^2}\right)\int_{V_i}^{V_f} dV - \frac{k}{A^2}\int_{V_i}^{V_f} V\,dV$$

$$= -\left(P_a + \frac{mg}{A} - \frac{kV_i}{A^2}\right)\left(V_f - V_i\right) - \frac{k}{2A^2}\left(V_f^2 - V_i^2\right)$$

$$= -\left[\left(10^5 + \frac{2040(9.81)}{0.1} - \frac{(2\times10^4)(0.1)}{0.1^2}\right)\left(0.05 - 0.1\right)\right] - \left[\frac{5\times10^4}{2(0.1)^2}\left(0.05^2 - 0.1^2\right)\right]$$

$$= 5006.2 + 7500 = 12506$$

$$\boxed{W = 12.5 \ kJ}$$

b) $PV^c = \text{constant}$

First Compression (Intermediate)

$$P_c = P_a + \frac{mg}{A} + \frac{k(V - V_i)}{A^2}$$ where $m = 1020\ kg$, $k = 2 \times 10^4\ N/m$ (Same Spring Const.)

Second Compression

$$P_c = P_a + \frac{2mg}{A} + \frac{k(V - V_i)}{A^2}$$

Need c from $PV^c = \text{const.}$

$$P_i V_i^c = P_f V_f^c, \quad P_f = 2P_i$$

$$2 = \left(\frac{V_i}{V_f}\right)^c$$

$$\ln 2 = c \ln\left(\frac{V_i}{V_f}\right) \Rightarrow c = \frac{\ln(2)}{\ln\left(\frac{V_i}{V_f}\right)} = \frac{\ln(2)}{\ln\left(\frac{0.1}{0.05}\right)} = 1$$

2 equations, 2 unknowns

$$P_{int} V_{int}^c = P_i V_i^c$$

$$P_{int} = P_a + \frac{mg}{A} + \frac{k(V_{int} - V_i)}{A^2}$$

solve $P_{int} = 1.4 \times 10^5\ Pa$

$V_{int} = 0.071\ m^3$

from (a)

$$W_{int} = -\left(P_a + \frac{mg}{A} - \frac{kV_i}{A^2}\right)(V_{int} - V_i) - \frac{k}{2A^2}\left(V_{int}^2 - V_i^2\right)$$

$$\boxed{W_{int} = 5 \times 10^3\ J}$$

$$W_{int\ to\ final} = -\left(P_a + \frac{2mg}{A} - \frac{kV_i}{A^2}\right)(V_f - V_{int}) - \frac{k}{2A^2}\left(V_f^2 - V_{int}^2\right)$$

$$\boxed{W_{int\ to\ final} = 4.6 \times 10^3\ J}$$

$$\boxed{\text{Total Work} = 9.6\ kJ}$$

c) Least Amount of Work

$\Rightarrow$ Reversible Process

$P_E = P$ Since $PV^c = P_i V_i^c$

$W = -\int_{V_i}^{V_f} P\, \partial V$ when $P = P_i \left(\frac{V_i}{V}\right)^c = P_i V_i^c \left(\frac{1}{V}\right)^c$

$W = -P_i V_i^c \int_{V_i}^{V_f} V^{-c}\, \partial V$ Since $c = 1$

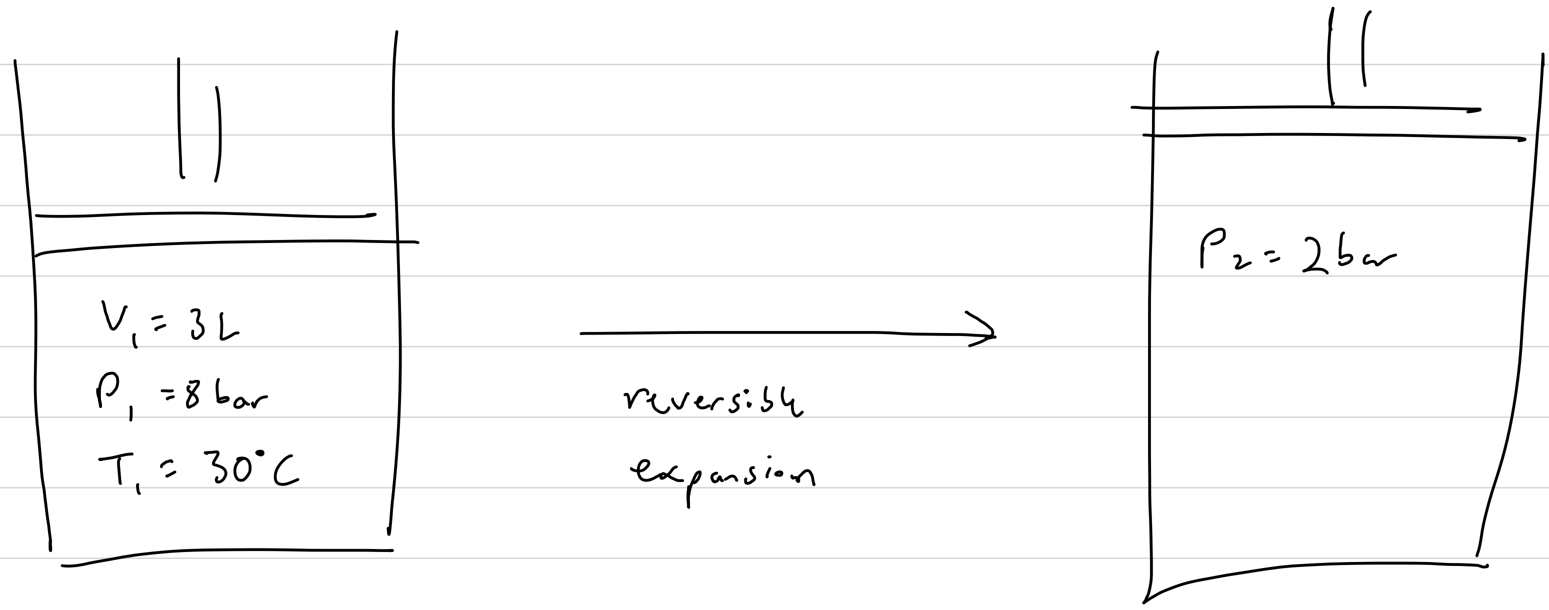
$= -P_i V_i \int_{V_i}^{V_f} \frac{\partial V}{V}$

$= -P_i V_i \ln\left(\frac{V_f}{V_i}\right)$

$= -(1 \times 10^5)(0.1) \ln\left(\frac{0.05}{0.1}\right)$

$\boxed{W = 6.9\,KJ}$

4)



a) Energy Balance $\Rightarrow$ $dU = \delta Q + \delta W$

$$dU = \delta Q - P_E dV \quad \rightarrow \text{Reversible}$$

Ideal Gas

$$dU = \delta Q - P dV$$

$$nC_{v,m}\, dT = \delta Q - P dV$$

b) Isothermal $\Rightarrow$ $dT = 0$

$\boxed{\therefore \Delta U = 0}$ (Ideal Gas)

from a)

$$0 = \delta Q - P dV$$

$$Q = \int_{V_1}^{V_2} P dV \qquad Q = -W$$

$$W = -\int_{V_1}^{V_2} P dV$$

ideal Gas $P = \frac{nRT}{V}$

$$= -nRT \int_{V_1}^{V_2} \frac{dV}{V}$$

$$= -nRT \ln\left(\frac{V_2}{V_1}\right) = nRT \ln\left(\frac{P_2}{P_1}\right)$$

$$n = \frac{P_1 V_1}{RT}$$

$$= P_1 V_1 \ln\left(\frac{P_2}{P_1}\right) = (8 \times 10^5\, Pa)(3 \times 10^{-3}\, m^3) \ln\left(\frac{2}{8}\right)$$

$$= -3327.11\ Pa\ m^3$$

$\boxed{W = -3.33\ kJ}$ $\boxed{Q = -W = 3.33\ kJ}$

c) Adiabatic $\Rightarrow Q = 0$

$\rightarrow$ Energy balance $\Rightarrow$ $\Delta U = \cancel{Q}^{0} + W$

$\Delta U = W$

- Process is expansion, so $\Delta V > 0$
- Work is < 0 since $W = -\int_{V_1}^{V_2} P\,dV$ and $V_2 > V_1$ $(\Delta V > 0)$
- $\Delta U < 0$ since $\Delta U = W$

- Since it is an ideal gas, $\Rightarrow U = f(T)$ and U increases w/ increasing temp.

- We know $\Delta U < 0$

  so the temp must <u>decrease</u> and <u>be less than 30°C</u>