ECHE363 Homework 4 - Due 02/17/25 Trevor Swan (tcs94)

1. Develop a general expression for Δs_m for an ideal gas that goes from (P_1, T_1) to (P_2, T_2), where the heat capacity is given by:

$$\frac{c_{p,m}}{R} = A + BT + CT^2 + \frac{D}{T}$$

where A, B, and C are constants.

$$dS_m = \frac{C_{p,m}}{T} dT - \frac{R}{P} dP$$

$$\Delta S_m = \int_{"1"}^{"2"} dS_m = \int_{T_1}^{T_2} \frac{C_{p,m}}{T} dT - \int_{P_1}^{P_2} \frac{R}{P} dP$$

$$\int_{T_1}^{T_2} \frac{R(A + BT + CT^2 + \frac{D}{T})}{T} dT = R\int_{T_1}^{T_2} \frac{A}{T} + B + CT + \frac{D}{T^2} dT$$

$$= R\left[A \ln T + BT + \frac{C}{2}T^2 - \frac{D}{T}\right]_{T_1}^{T_2}$$

$$= R\left(A \ln T_2 + BT_2 + \frac{C}{2}T_2^2 - \frac{D}{T_2} - \left(A \ln T_1 + BT_1 + \frac{C}{2}T_1^2 - \frac{D}{T_1}\right)\right)$$

$$= R\left(A \ln\left(\frac{T_2}{T_1}\right) + B(T_2 - T_1) + \frac{C}{2}(T_2^2 - T_1^2) - D\left(\frac{1}{T_2} - \frac{1}{T_1}\right)\right)$$

$$\int_{P_1}^{P_2} \frac{R}{P} dP = R \ln P \Big|_{P_1}^{P_2} = R \ln\left(\frac{P_2}{P_1}\right)$$

$$\therefore \Delta S_m = R\left(A \ln\left(\frac{T_2}{T_1}\right) + B(T_2 - T_1) + \frac{C}{2}(T_2^2 - T_1^2) - D\left(\frac{1}{T_2} - \frac{1}{T_1}\right)\right) - R \ln\left(\frac{P_2}{P_1}\right)$$

$$\Delta S_m = R\left[A \ln\left(\frac{T_2}{T_1}\right) + B(T_2 - T_1) + \frac{C}{2}(T_2^2 - T_1^2) - D\left(\frac{1}{T_2} - \frac{1}{T_1}\right) - \ln\left(\frac{P_2}{P_1}\right)\right]$$

2. a) Reversible Adiabatic Expansion

$$\Delta S = \int \frac{\cancel{\delta Q_{rev}}^{\,0}}{T} \text{ adiabatic} = 0 \qquad \boxed{\Delta S = 0}$$

There is no heat transfer to or from surroundings due to adiabatic nature, so the surroundings are not affected by heat on the entropy level.

b) Condensing 1 mol vapor (methylene) @ B.P. (111K); $\Delta h_{v,m} = 8.2 \frac{kJ}{mol}$

For condensation rxns, $Q = \Delta H$, but negative as heat of vaporization is given

$$dS_m = \int_{\text{"1"}}^{\text{"2"}} \frac{dQ_{rev}}{T} = \frac{Q_{rev}}{T} = \frac{-\Delta H_{v,m}}{T} = \frac{-8.2 \frac{kJ}{mol}}{111\,K} = -0.07387 \frac{kJ}{mol \cdot K}$$

← reference pt.

$\boxed{\Delta S = -0.07387 \frac{kJ}{K}}$ Negative makes sense as we are condensing the gas and here going from higher to lower entropy.

c) Liquid water (1 mol) from $100°C = 373K$ to $0°C = 273K$, for $c_p = 4.2 \frac{J}{g \cdot K}$

$C_{p,m} = \left(18 \frac{g}{1\,mol\,H_2O}\right)\left(4.2 \frac{J}{g \cdot K}\right) = 75.6 \frac{J}{mol \cdot K}$ ↳ Assume Incompressible

Choose P = constant, reversible, so $\delta q_{m,rev} = dh_m$ (1st Law)

$$\Delta S = \int dS = n \int \frac{\delta q_{m,rev}}{T} = n \int \frac{dh_m}{T} = n \int_{373K}^{273K} \frac{C_{p,m}}{T} dT = 1\,mol \int_{373}^{273} \frac{75.6}{T} dT$$

$$= 75.6 \ln\left(\frac{273}{373}\right) = \boxed{-23.6 \frac{J}{K}}$$

Negative makes sense, hotter = more excited, colder = less excited, going from hot to cold reduces order.

d) $T_{avg} = \frac{T_h + T_c}{2} = \frac{(200+273)+(100+273)}{2} = 423\,K$, $C_{p,m} = 24 \frac{J}{mol \cdot K}$

Incompressible; irreversible; isolated system

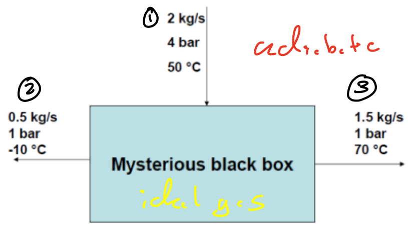
$$\Delta S_{h,m} = \int dS_m = \int \frac{\delta q_m}{T} \underset{\substack{\uparrow \\ \text{1st Law} \\ \text{const P}}}{=} \int_{T_h}^{T_{avg}} \frac{C_{p,m}}{T} dT = \int_{473}^{423} \frac{24}{T} dT = 24 \ln\left(\frac{423}{473}\right) = -2.68 \frac{J}{mol \cdot K}$$

$$\Delta S_{c,m} = \int_{T_c}^{T_{avg}} \frac{C_{p,m}}{T} dT = \int_{373}^{423} \frac{24}{T} dT = 24 \ln\left(\frac{423}{373}\right) = 3.02 \frac{J}{mol \cdot K}$$

$$\Delta S_m = \Delta S_{h,m} + \Delta S_{c,m} = -2.68 + 3.02 = \boxed{0.34 \frac{J}{mol \cdot K}}$$

Heat transfer from hot block to cold block will increase molecular motion randomness. This process is irreversible, and thus total entropy should increase.

3. Can it work? In other words, is $\Delta S_{univ} \geq 0$?



1st Law: $\frac{d}{dt}(U+E_k+E_p) = \dot{m}_{in}\hat{h}_{in} - \dot{m}_{out}\hat{h}_{out} + \cancel{\dot{W}_s} + \cancel{\dot{Q}}$ removing external parts, adiabatic

(open system, steady-state)

$$0 = \dot{m}_1\hat{h}_1 - (\dot{m}_2\hat{h}_2 + \dot{m}_3\hat{h}_3)$$

$$\rightarrow \dot{m}_1\hat{h}_1 = \dot{m}_2\hat{h}_2 + \dot{m}_3\hat{h}_3$$ won't be very useful

2nd Law: $\frac{dS_{univ}}{dt} = \cancel{\frac{dS}{dt}} + \dot{m}_{out}\hat{S}_{out} - \dot{m}_{in}\hat{S}_{in} - \cancel{\frac{\dot{Q}}{T_{surr}}}$

$$\frac{dS_{univ}}{dt} = \dot{m}_2\hat{S}_2 + \dot{m}_3\hat{S}_3 - \dot{m}_1\hat{S}_1$$

To work: $\dot{m}_2\hat{S}_2 + \dot{m}_3\hat{S}_3 \geq \dot{m}_1\hat{S}_1$

If we find entropy to decrease, the process is impossible. The gas goes from higher pressure and 50°C (lower entropy due to compression) to an expanded gas in the outputs. Stream 2 is cooled and expanded, while stream 3 is heated and expanded. To cool the 2nd stream, energy must be extracted externally, but there is no mechanism to do this without violating the second law. This suggests that the box will not function assuming it has no external parts and is adiabatic.

Quantitatively

$$\frac{dS_{univ}}{dt} = dS$$ ↑ Steady-State

Hypothetical 1 ($\dot{m} = 0.5\ \frac{kg}{s}$)

$P = 4\ bar \rightarrow 1\ bar$ ΔP

$T = 50°C \rightarrow -10°C$ ΔT

Hypothetical 2 ($\dot{m} = 1.5\ \frac{kg}{s}$)

$P = 4\ bar \rightarrow 1\ bar$ ΔP

$T = 50°C \rightarrow 70°C$ ΔT

$$\Delta S_1 = \int dS_m = \int_{T_1}^{T_2} \frac{C_{p,m}}{T}dT - \int_{P_1}^{P_2}\frac{R}{P}dP$$

$$= C_{p,m}\ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right) = C_{p,m}\ln\left(\frac{50+273}{-10+273}\right) - R\ln\left(\frac{4\ bar}{1\ bar}\right)$$

$$= 0.205\,C_{p,m} - 1.386R$$

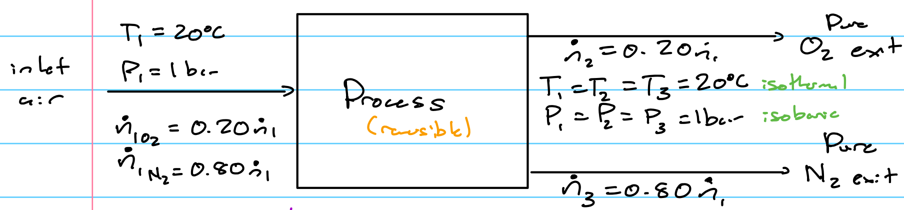
$$\Delta S_2 = C_{p,m}\ln\left(\frac{50+273}{70+273}\right) - R\ln\left(\frac{4}{1}\right) = -.06\,C_{p,m} - 1.386R$$

$$\int dS = \Delta S_{total} = \Delta S_1 + \Delta S_2 = (0.205\,C_{p,m} - 1.386R) + (-.06\,C_{p,m} - 1.386R)$$

$$= 0.145\,C_{p,m} - 2.772R$$

$C_{p,m} > 0$ and $R = 8.314\ \frac{J}{mol \cdot K}$. For $\frac{dS_{univ}}{dt} > 0$, $C_{p,m}$ must exceed $158.94\ \frac{J}{mol\,K}$, which is generally not the case for ideal gases, so the box will not function.

4. Minimum work required ≡ reversible so $\Delta S_{univ} = 0$



• Open system, assume steady-state

$O_2: \dot{n}_{1O_2} = \dot{n}_2$

$N_2: \dot{n}_{1N_2} = \dot{n}_3$

Tot: $\dot{n}_{1O_2} + \dot{n}_{1N_2} = \dot{n}_2 + \dot{n}_3$

• Will treat air as an ideal gas

1st Law: $Q - W = \cancel{\Delta H}^{0}$ Ideal gas, isothermal

$\rightarrow Q = W$

2nd Law: $dS = \frac{dQ_{rev}}{T}$ $\rightarrow$ Use 1st Law $= \frac{d\dot{W}}{T}$ $\Big\} \int_{in}^{out} d\dot{W} = \int_{in}^{out} T\,dS \Rightarrow W_{min} = T\Delta S$

Entropy of Mixing: $\Delta S_{mix} = -nR(x_1 \ln x_1 + x_2 \ln x_2)$
(googled)

$\rightarrow \Delta S_{m,mix} = -8.314 \frac{J}{mol \cdot K}(0.2\ln 0.2 + 0.8 \ln 0.8) = 4.16 \frac{J}{mol \cdot K}$

so, $W = T\Delta S_{mix} = (20°C + 273)(4.16 \frac{J}{mol K}) = 1218.48 \frac{J}{mol}$

$W_{min} = 1.22\,kJ$

Verify w/ Process Paths

O_2: (20°C, 0.20 bar partial P) $\rightarrow$ (20°C, 1 bar) reversible change in P

$\Delta S_m = R \ln\left(\frac{P_1}{P_2}\right) = 8.314 \frac{J}{mol \cdot K} \ln\left(\frac{0.2}{1}\right) = -13.38 \frac{J}{mol \cdot K}$

N_2: (20°C, 0.80 bar) $\rightarrow$ (20°C, 1 bar)

$\Delta S_m = R \ln\left(\frac{P_1}{P_2}\right) = 8.314 \frac{J}{mol \cdot K} \ln\left(\frac{0.8}{1}\right) = -1.855 \frac{J}{mol \cdot K}$

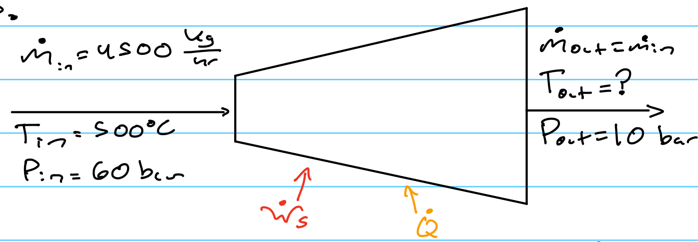
T_{surr} = Constant = 20°C = 283 K

$\Delta S_{surr} = y_{O_2}\Delta S_{m,O_2} + y_{N_2}\Delta S_{m,N_2} - \frac{Q_{real}}{T_{surr}} = 0$

$0.2 \cdot -13.38 + 0.80 \cdot -1.855 - \frac{Q_{real}}{293K} = 0 \rightarrow Q_{real} = -1218.88$

so, $\dot{W}_{min} = 1218 \frac{J}{s} \Rightarrow \dot{W}_{min} = 1.22\,kW$

S.



• open system: $\dot{m}_{in} = \dot{m}_{out} = 4500\ \frac{kg}{hr}$

↳ $\frac{dn}{dt} = 0$

a) 1st Law: $0 = \dot{m}\hat{h}_1 - \dot{m}\hat{h}_2 + \dot{W}_s + \cancel{\dot{Q}}^{\,0}$ adiabatic   • 60 bar = 6 MPa , 10 bar = 1 MPa

$\dot{W}_s = \dot{m}(\hat{h}_2 - \hat{h}_1)$ $\dot{m}$ is known

Steam @ (6 MPa, 500°C) => $\hat{h}_1 = 3422.1\ \frac{kJ}{kg}$

Steam @ (1 MPa, T_{out}) => $\hat{h}_2 = ?$ need to find $\dot{T}_1$

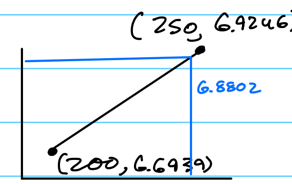
2nd Law: $\underbrace{\frac{d\cancel{S_{univ}}^{\,0}}{dt}}_{\text{to find max work}} = \underbrace{\frac{d\cancel{S}^{\,0}}{dt}}_{\text{steady state}} + \dot{m}(\hat{S}_2 - \hat{S}_1) - \frac{\cancel{\dot{Q}}^{\,0}}{T_{surr}}$ adiabatic

$0 = 0 + \cancel{\dot{m}}^{\,0}(\hat{S}_2 - \hat{S}_1) \Rightarrow \hat{S}_2 = \hat{S}_1$ isentropic

Steam @ (6MPa, 500°C) => $\hat{S}_2 = 6.8802\ \frac{kJ}{kg \cdot K} = \hat{S}_1$

Interpolate to get T_{out}:

$P_{out} = 1$ MPa, $\hat{S}_1 = 6.8801\ \frac{kJ}{kg \cdot K}$ } $T_{lower} = 200°C \rightarrow \hat{S}_{lower} = 6.6939\ \frac{kJ}{kg \cdot K}$; $T_{higher} = 250°C \rightarrow \hat{S}_{higher} = 6.9246\ \frac{kJ}{kg \cdot K}$



$$T_{out} = 200°C + \frac{(6.8802 - 6.6939)(250 - 200)}{(6.9246 - 6.6939)} = 240.38°C$$

$T_{out} = 240.38°C$

Interpolate to get $\hat{h}_2$ @ (1MPa, 240.38°C)

$T_{lower} = 200°C \rightarrow \hat{h}_{lower} = 2827.9\ \frac{kJ}{kg}$

$T_{higher} = 250°C \rightarrow \hat{h}_{higher} = 2942.6\ \frac{kJ}{kg}$

$$\hat{h} = 2827.9 + \frac{(240.38 - 200)(2942.6 - 2827.9)}{(250 - 200)} = 2920.53\ \frac{kJ}{kg}$$

$\dot{W}_s = 4500\ \frac{kg}{hr}\left(2920.53\ \frac{kJ}{kg} - 3422.1\ \frac{kJ}{kg}\right) = -2257065\ \frac{kJ}{hr} \cdot \frac{1\ hr}{3600\ s}$

$\rightarrow \dot{W}_s = -626.96\ \frac{kJ}{s}$ $\dot{W}_{max} = -626.96$ kW

b) $\dot{W}_{real} = 0.80 \cdot \dot{W}_{isentropic}$ $\qquad 4500 \frac{kg}{hr} = 1.25 \frac{kg}{s}$

$= 0.80 \cdot (-626.96) = -501.57 \text{ kW}$

$\dot{W}_{real} = \dot{m}(\hat{h}_2 - \hat{h}_1)$

$\hat{h}_2 = \frac{\dot{W}_{real}}{\dot{m}} + \hat{h}_1 = \frac{-501.57 \frac{kJ}{s}}{1.25 \frac{kg}{s}} + 3422.1 \frac{kJ}{kg} = 3020.844 \frac{kJ}{kg}$

$P_2 = 1 MPa$, $\hat{h}_2 = 3020.844 \frac{kJ}{kg}$ $\Big\}$ $T_{lower} = 250°C \rightarrow \hat{h}_{lower} = 2942.6 \frac{kJ}{kg}$; $T_{higher} = 300°C \rightarrow \hat{h}_{higher} = 3051.2 \frac{kJ}{kg}$

$T_{out} = 250°C + \frac{(3020.844 - 2942.6)(300-250)}{(3051.2 - 2942.6)} = 286.02°C$ $\qquad \boxed{T_{out} = 286.02°C}$

2nd Law: $\frac{dS_{univ}}{dt} = \dot{S} = \dot{m}(\hat{S}_2 - \hat{S}_1)$

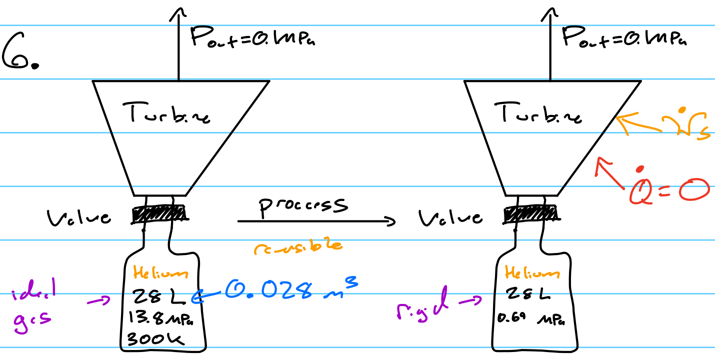
$P_2 = 1 MPa$, $T_2 = 286.02°C$ $\Big\}$ $T_{lower} = 250°C \rightarrow \hat{S}_{lower} = 6.9246 \frac{kJ}{kgK}$; $T_{higher} = 300°C \rightarrow \hat{S}_{higher} = 7.1228 \frac{kJ}{kgK}$

$\hat{S}_2 = 6.9246 + \frac{(286.02 - 250)(7.1228 - 6.9246)}{(300-250)} = 7.067 \frac{kJ}{kgK}$

$\dot{S} = 1.25 \frac{kg}{s}\left(7.067 \frac{kJ}{kgK} - 6.8801 \frac{kJ}{kgK}\right) = 0.2336 \frac{kJ}{sK}$

$\boxed{\dot{S} = 0.2336 \frac{kW}{K}}$

6.



- $C_{p,m} = 20.9 \frac{J}{mol \cdot K}$
- $C_{v,m} = C_{p,m} - R = 12.586 \frac{J}{mol \cdot K}$
- Open-System, transient

$$\frac{dn}{dt} = \cancel{\dot{n}_{in}}^{0} - \dot{n}_{out}$$ gas only flows out of tank

$$n_i = \frac{(13.8 \times 10^6 Pa)(28\ L)}{(8314.5 \frac{L \cdot Pa}{mol \cdot K})(300K)} = 154.9\ mol\ He \quad , \quad n_f = \frac{P_f V}{R T_f} = ?$$

Overall:

1st Law:
$$\frac{dU}{dt} = \cancel{\dot{n}_{in}}^{0}(h_m + e_{k,m} + e_{p,m})_{in} - \dot{n}_{out}(h_m + \cancel{e_{k,m} + e_{p,m}})_{out} + \cancel{\dot{Q}}^{0} + \dot{W}_s$$
only leaves — negligible — adiabatic

$$\frac{dU}{dt} = \dot{W}_s - \dot{n}_{out} h_{m,out} \qquad dU = dW + h_{m,out}\,dn$$
$$\rightarrow \Delta(nU_m) = W + h_{m,out}(n_f - n_i)$$
$$\frac{dU}{dt} = \frac{dW}{dt} + \frac{dn}{dt} h_{m,out} \qquad \Rightarrow W = \Delta(nU_m) - h_{m,out}(n_f - n_i)$$

2nd Law:
$$\frac{dS_{univ}}{\cancel{dt}}^{0} = \frac{dS}{dt} + \dot{n}_{out} S_{m,out} - \cancel{\dot{n}_{in}}^{0} S_{m,in} - \frac{\cancel{\dot{Q}}^{0}}{T_{surr}}$$
maximum — only exits — adiabatic

$$\frac{dS}{dt} = -\dot{n}_{out} S_{m,out} \Rightarrow \frac{dS}{dt} = \frac{dn}{dt} S_{m,out} \Rightarrow dS = S_{m,out}\,dn$$
$$\Delta S = S_{m,out}(n_f - n_i)$$

Turbine:

$\dot{n}_{in} = \dot{n}_{out}$ (steady-state)

1st Law:
$$\cancel{\frac{dU}{dt}}^{0} = \dot{n}_{in} h_{m,in} - \dot{n}_{out} h_{m,out} + \cancel{\dot{Q}}^{0} + \dot{W}_s$$
$$\dot{W}_s = \dot{n}(h_{m,out} - h_{m,tank})$$

2nd Law:
$$\cancel{\frac{dS_{univ}}{dt}}^{0} = \cancel{\frac{dS}{dt}}^{0} + \dot{n}_{out} S_{m,out} - \dot{n}_{in} S_{m,in} - \frac{\cancel{\dot{Q}}^{0}}{T_{surr}}$$
maximum — steady state — adiabatic

$$\Rightarrow \Delta S = \dot{n}(S_{m,out} - S_{m,in}) = 0$$

Tank:

1st Law:
$$\frac{dU}{dt} = \cancel{\dot{n}_{in}} h_{m,in} - \dot{n}_{out} h_{m,out} + \cancel{\dot{Q}}^{0} + \cancel{\dot{W}_s}^{0}$$
no work as no moving parts

$$\frac{dU}{dt} = \frac{dn}{dt} h_{m,tank} \rightarrow d(nU_m) = h_{m,tank}\,dn$$

2nd Law:
$$\frac{dS_{univ}}{\cancel{dt}}^{0} = \frac{dS}{dt} + \dot{n}_{out} S_{m,out} - \cancel{\dot{n}_{in}}^{0} S_{m,in} - \frac{\cancel{\dot{Q}}^{0}}{T_{surr}}$$
max — adiabatic

$$\frac{dS}{dt} = \frac{dn}{dt} S_{m,tank} \rightarrow \Delta S = S_{m,tank}(n_f - n_i)$$

Summary of Eqns

Overall:

$\dot{W} = \Delta(n U_m) - h_{m,out}(n_f - n_i)$ exiting enthalpy, tank's moles

$\Delta S_{univ} = 0$, $\Delta S = S_{m,out}(n_f - n_i)$

$h_{m,tank}$ ← outlet from tank / inlet to turbine

Turbine:

$\dot{W}_s = \dot{n}(h_{m,out} - h_{m,tank})$ exit enthalpy, tank enthalpy, $\dot{n}$ out from tank

$\Delta S_{univ} = 0$, $\Delta S = \dot{n}(S_{m,out} - S_{m,tank}) = 0$ gas exits system, gas exits tank

Tank:

$d(n U_m) = h_{m,tank}\, dn$

$\Delta S_{univ} = 0$, $\Delta S = S_{m,tank}(n_f - n_i)$ tank's moles

Isentropic for ideal gas as $\Delta S = 0$ overall:

$$T_{tank} = T_i\left(\frac{P_{tank}}{P_i}\right)^{R/c_p} = 300\text{K}\left(\frac{0.69}{13.8}\right)^{\frac{8.314}{20.9}} = 91.11\text{ K}$$

$$n_f = \frac{(0.69\times10^6\text{ Pa})(28\text{L})}{8314.4\frac{\text{L}\cdot\text{Pa}}{\text{mol}\cdot\text{K}}(91.11\text{K})} = 25.5\text{ mol}$$

$$T_{out} = T_{tank}\left(\frac{P_{out}}{P_{tank}}\right)^{R/c_p} = 91.11\left(\frac{0.1\text{MPa}}{0.69\text{MPa}}\right)^{\frac{8.314}{20.9}} = 42.25\text{ K}$$

Use overall balance:

$W = -{}^{*}h_{m,out}(n_f - n_i) + \Delta(n U)$

${}^{*}H = C_pT$ for an ideal gas, $U = C_vT$

$\dot{W} = -C_pT_{out}(n_f - n_i) + \Delta(nU_m) \rightarrow \Delta(nU_m) = n_fC_vT_f - n_iC_vT_i$

$= -C_pT_{out}(n_f - n_i) + n_fC_vT_f - n_iC_vT_i$

$= -C_pT_{out}(n_f - n_i) + C_v(n_fT_f - n_iT_i)$

$\Rightarrow \dot{W} = -20.9(42.25)(25.5 - 154.9) + 12.586(25.5(91.11) - 154.9(300))$

$= -441366.87\ \frac{\text{J}}{\text{s}} \rightarrow \dot{W} = -441.4\text{ kW}$

Very confused about this problem !!

7. Answer the following reflection questions (5 points):

   a. What about the way this class is taught is helping your learning?

   b. What about the way this class is taught is inhibiting your learning?

(A) I think the emphasis on examples is helping my learning a lot. A lot of the class seems to be focused on engaging discussion and comprehensive examples, which benefits me on the homework a lot. This emphasis makes it more simple to figure out how to solve homework problems, which is appreciated. It also seems to prepare me for exams in the class which is even more appreciated.

(B) I think it would help me if there was a slightly bigger emphasis on key concepts before or after each lecture. I find myself jumping around in my notes to piece together key concepts, as they are typically only presented in the examples and not explicitly. Everything else in the class is great, and I don't think theres anything other than that preventing/inhibiting my learning.