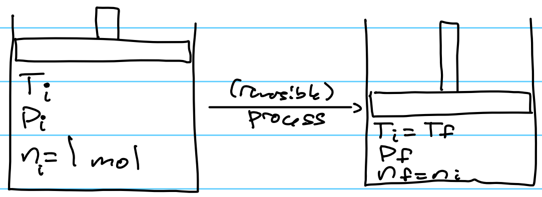
ECHE363 Homework 2 - Due 1/31/25 **Trevor Swan (tcs94)**

1. Find the work of a reversible, isothermal compression of 1 mole of gas, from pressure P_i to P_f, in a piston if $v_m = \frac{RT}{P} + b$, where b is a positive constant.



- Isothermal $\rightarrow T_i = T_f$
- Closed system: $m_i = m_f \rightarrow n_i = n_f$
- Reversible process: $P_E = P_{sys} = P$
- $V_m = \frac{RT}{P} + b$ for $b > 0$

PV work: $W = -\int_{V_0}^{V_f} P_E \, dV \xrightarrow{\text{reversible}} W = -\int_{V_0}^{V_f} P \, dV$

1. Solve V_m expression for P

$$V_m = \frac{RT}{P} + b \Rightarrow P = \frac{RT}{V_m - b}$$

2. Describe V based on moles given

$$V_m = \frac{V}{n} \Rightarrow V_m = \frac{V}{1\text{mol}} \Rightarrow V_m = V = \frac{RT}{P} + b \Rightarrow P = \frac{RT}{V - b}$$

3. Set up integral and solve (substitute $P = \frac{RT}{V-b}$)

$$W = -\int_{V_0}^{V_f} \frac{RT}{V-b} \, dV$$

$$= -RT \int_{V_0}^{V_f} \frac{1}{V-b} \, dV$$

$$= -RT \ln(V-b) \Big|_{V_0}^{V_f}$$

$$= -RT\left(\ln(V_f - b) - \ln(V_0 - b)\right)$$

$$= -RT \ln\left(\frac{V_f - b}{V_i - b}\right)$$

$$= -RT \ln\left(\frac{\frac{RT}{P_f} + b - b}{\frac{RT}{P_i} + b - b}\right) \longrightarrow \boxed{W = -RT \ln\left(\frac{P_i}{P_f}\right)}$$

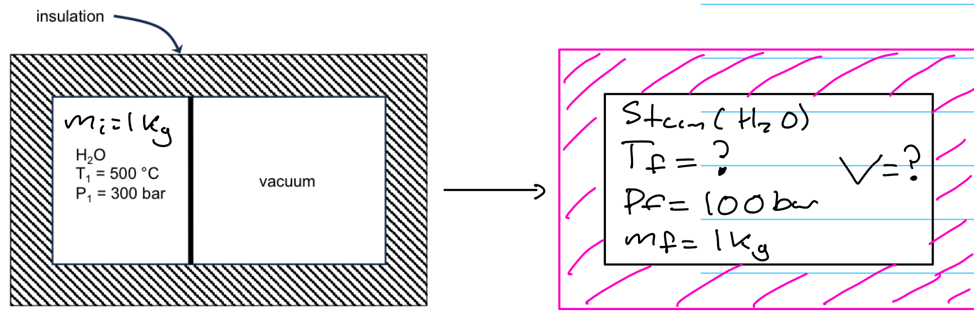
$P_i < P_f$, so $\ln\left(\frac{P_i}{P_f}\right) < 0$

$-RT \cdot (-) = (+)$

work being done on sys

which makes sense!

2. The insulated vessel shown below has two compartments separated by a membrane. On one side is 1 kg of steam at 500 °C and 300 bar. The other side is evacuated. The membrane ruptures and the steam fills the entire volume. The final pressure is 100 bar. Determine the final temperature of the steam and the volme of the vessel.



- Insulated: Adiabatic
- Closed System: $m_i = m_f = 1$ kg
- No work done on/by surroundings

Initial

Use B.4

$T_1 = 500°C > 374.14$ (T_c) → Superheated

$P_1 = 300$ bar $= 30$ MPa

$\hat{V} = 0.008679 \frac{m^3}{kg} \rightarrow V_1 = 0.008679\, m^3 = 8.679$ L (only 1 kg gas)

$\hat{u} = 2820.7 \frac{kJ}{kg}$

1st Law: $\Delta U = \cancel{Q}^{0} - \cancel{W}^{0}$ (well-insulated; no shaftwork, no PV work due to vacuum)

$$U_f - U_i = 0 \Rightarrow \hat{u}_f = \hat{u}_i = 2820.7 \frac{kJ}{kg}$$

Final

Interpolate

$P_f = 100$ bar $= 10$ MPa

$\hat{u}_f = 2820.7 \frac{kJ}{kg}$

$T_1 = 350 \rightarrow \hat{u}_1 = 2699.2 \frac{kJ}{kg}, \ \hat{V}_1 = 0.02242 \frac{m^3}{kg}$

$T_2 = 400 \rightarrow \hat{u}_2 = 2832.4 \frac{kJ}{kg}, \ \hat{V}_2 = 0.02641 \frac{m^3}{kg}$

$$2820.7 = \left(\frac{2832.4 - 2699.2}{400 - 350}\right)(T_f - 350°C) + 2699.2 \frac{kJ}{kg}$$

Corresponding $\hat{u}$ for T_f

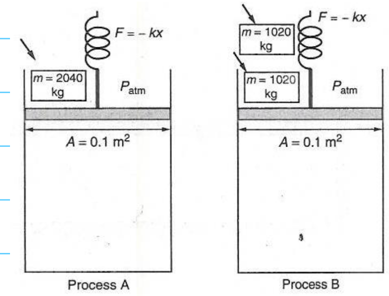
$$T_f = \frac{(2820.7 - 2699.2)(400 - 350)}{2832.4 - 2699.2} + 350 = 395.61\ °C$$

$$\hat{V} = \left(\frac{0.0261 - 0.02242}{400 - 350}\right)(395.61 - 350) + 0.02242 = 0.02578 \frac{m^3}{kg}$$

$$\Rightarrow V = \hat{V} \cdot 1\,kg = 0.02578\ m^3$$

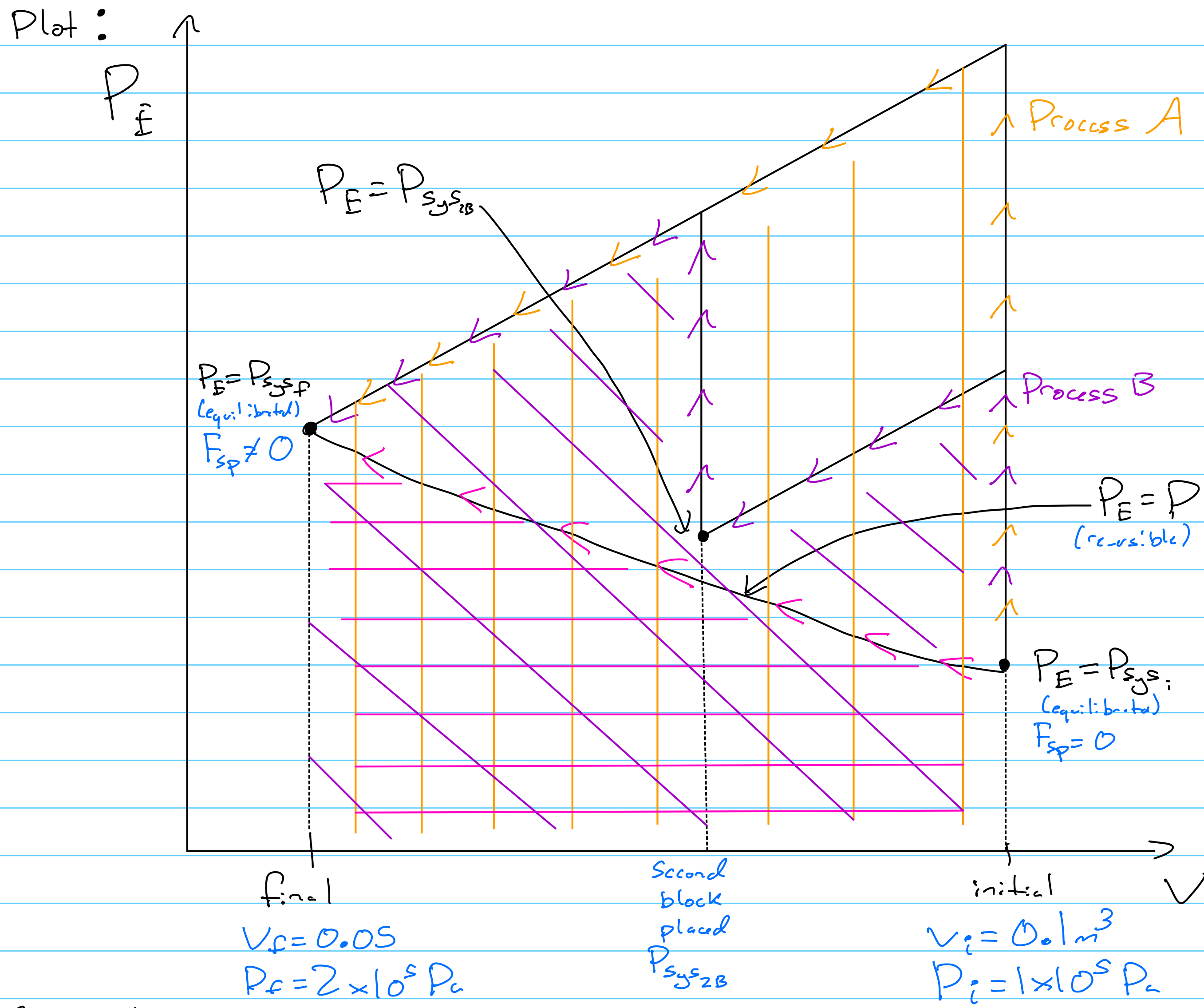
$$\boxed{\begin{array}{l} T_f = 395.61\ °C \\ V = 0.02578\ m^3 \end{array}}$$

3. Consider the piston–cylinder assembly containing a pure ideal gas shown below. The initial volume of the gas is 0.1 m^3, the initial pressure is 1 bar, and the cross-sectional area of the piston is 0.1 m^2. Initially the spring exerts no force on the very *thin* (i.e., massless) piston.



- Pure Ideal Gas: $PV = nRT$, $PV_m = RT$
- $V_1 = 0.1\ m^3$, $P_1 = 1\text{bar} = 1 \times 10^5\ Pa$
- Cross sectional Area of Piston $A = 0.1\ m^2$, $m = 0$
- Spring initially exerts no force on piston, i.e. $F = 0$

Plot:



Dotted lines used to align labels w/ points ☝

Some Notes...

At start and end of process, system is at eq so $P_{sys} = P_E$

Process A (irreversible)

a) Pressure spikes volume doesn't change due to spring, then P_E starts to decrease with volume linearly due to spring pressure.

Process B (irreversible)

b) Pressure initially spikes to half of A's pressure @ const V, then equilibrates, then jumps back up with other block and matches slope of A's line.

Process C (reversible)

c) Process is done reversibly, resulting in least amount of work done. Due to incremental changes in mass, $P_E = P_{sys}$ during full process, not just ends.

a) Find K, use $F_{sp}=0$ initially but not at P_f (load applied → F_{sp})

$P_{E_f} = P_{sys_f}$ → force balance

given as $F=-Kx$

$\rightarrow AP_{E_f} = AP_{sys_f}$

$x = \frac{V_f - V_i}{A} = \frac{0.05\,m^3 - 0.1\,m^3}{0.1\,m^2} = -0.5\,m$

$= F_{atm} + F_{block} - F_{spring}$

← $V_f - V_i$ aligns with $F=-Kx$ coordinate

$AP_{sys_f} = AP_{atm} + mg + Kx$

$$K = \left[A\left(P_{sys_f} - P_{atm}\right) - mg\right] \cdot \frac{1}{x} = \frac{\left[0.1\,m^2\left(2\times10^5\,Pa - 1\times10^5\,Pa\right) - 2040\,kg \cdot 9.81\,\frac{m}{s^2}\right]}{-0.5\,m}$$

$$= 20{,}024.8\ \frac{N}{m}$$

Determine Work

Generally, now turn back

$$P_E = P_{atm} + \frac{mg}{A} + \frac{k(V_f - V_i)}{A} \cdot \frac{1}{A} \qquad \frac{Kx}{A} = \frac{K}{A} \cdot \left(\frac{V_f - V_i}{A}\right)$$

$$W = -\int_{V_i}^{V_f} P_E\,dV = -\int_{V_i}^{V_f} P_{atm} + \frac{mg}{A} + \frac{k(V - V_i)}{A^2}\,dV$$

$$= -\int_{V_i}^{V_f} P_{atm} + \frac{mg}{A} - \frac{kV_i}{A^2} + \frac{K}{A^2}V\,dV$$

$$= -\left(P_{atm} + \frac{mg}{A} - \frac{kV_i}{A^2}\right)V\Big|_{V_i}^{V_f} - \frac{K}{2A^2}V^2\Big|_{V_i}^{V_f}$$

$$W = \left(1\times10^5 + \frac{2040 \cdot 9.81}{0.1} - \frac{20{,}024.8 \cdot 0.1}{0.1^2}\right)\left(0.1 - 0.05\right) - \frac{20{,}024.8}{2(0.1)^2}\left(0.05^2 - 0.1^2\right)$$

$$W = 12503.1\ N \cdot m \qquad W = 12503.1\ J$$

b) Told to assume $PV^c = \text{constant}$ for some constant c. Find c

$$P_i V_i^c = P_{2B} V_{2B}^c = P_f V_f^c = \text{constant}$$

$$\left.\begin{array}{l} P_f = 2\text{ bar} \\ P_i = 1\text{ bar} \end{array}\right\} P_f = 2P_i \quad \text{so,} \quad P_i V_i^c = 2P_i V_f^c$$ solve for c

$$\frac{V_i^c}{V_f^c} = \frac{2P_i}{P_i}$$

$$\left(\frac{V_i}{V_f}\right)^c = 2 \rightarrow c \ln\left(\frac{V_i}{V_f}\right) = \ln 2$$

$$c = \frac{\ln 2}{\ln\left(\frac{V_i}{V_f}\right)} = \frac{\ln 2}{\ln\left(\frac{0.1}{0.05}\right)} = 1$$

Determine P_{2B} and V_{2B}

$$P_{sys_{2B}} = P_{E_{2B}}$$

$$P_{2B} = P_{atm} + \frac{mg}{A} + \frac{k(V_{2B} - V_i)}{A^2}$$

$$P_{2B} V_{2B} = P_f V_f$$

$$V_{2B} = \frac{1}{P_{2B}} \cdot 2\times10^5 \cdot 0.05$$

$$P_{2B} = 1\times10^5 + \frac{1020 \cdot 9.81}{0.1} + 20024.8\left(\frac{\frac{10000}{P_{2B}} - 0.1}{0.1^2}\right)$$

$$P_{2B} = 200062 + 200248\left(\frac{100000}{P_{2B}} - 1\right)$$

$$\frac{P_{2B}^2}{P_{2B}} = -186 + 2\times10^{10} \cdot \frac{1}{P_{2B}}$$

$$\frac{P_{2B}^2 - 2\times10^{10}}{P_{2B}} = -186 \Rightarrow P_{2B}^2 + 186\,P_{2B} - 2\times10^{10}$$

$$P_{2B} = 141416 = 1.41\text{ bar} = 1.41\times10^5\,\text{Pa}$$

$$V_{2B} = \frac{1}{1.41} \cdot 2 \cdot 0.5 = 0.0709\text{ m}^3$$

Determining Work

$k = 20{,}024.8\ \frac{N}{m}$ Same spring

$V_i \rightarrow V_{2B}$ Two transitions 1020 kg brick placed both times $W_{tot} = W_{1B} + W_{2B}$

$V_{2B} \rightarrow V_f$

$$P_{E_{1B}} = P_{atm} + \frac{mg}{A} + k\left(\frac{V - V_i}{A^2}\right) = P_{atm} + \frac{mg}{A} - \frac{kV_i}{A^2} + \frac{k}{A^2}V$$ from V_i to V_{2B}

$$P_{E_{2B}} = P_{atm} + \frac{2mg}{A} + k\left(\frac{V - V_i}{A^2}\right) = P_{atm} + \frac{2mg}{A} - \frac{kV_i}{A^2} + \frac{k}{A^2}V$$ from V_{2B} to V_f

$$W_{1B} = -\int_{V_i}^{V_{2B}} P_{E_{1B}}\,dV = \int_{V_i}^{V_{2B}} P_{atm} + \frac{mg}{A} - \frac{kV_i}{A^2} + \frac{k}{A^2}V\,dV = -\left(P_{atm} + \frac{mg}{A} - \frac{kV_i}{A^2}\right)V\Big|_{V_i}^{V_{2B}} - \frac{k}{2A^2}V^2\Big|_{V_i}^{V_{2B}}$$

$$= \left(10^5 + \frac{1020 \cdot 9.81}{0.1} - \frac{20{,}024.8 \cdot 0.1}{0.1^2}\right)(0.1 - 0.0709) - \frac{20{,}024.8}{2(0.1)^2}\left(0.0709^2 - 0.1^2\right) = 4973.9\text{ J}$$

$$W_{1B} = -\int_{V_{2B}}^{V_f} P_{E_{2B}}\,dV = \int_{V_{2B}}^{V_f} P_{atm} + \frac{2mg}{A} - \frac{kV_i}{A^2} + \frac{k}{A^2}V\,dV = -\left(P_{atm} + \frac{2mg}{A} - \frac{kV_i}{A^2}\right)V\Big|_{V_{2B}}^{V_f} - \frac{k}{2A^2}V^2\Big|_{V_{2B}}^{V_f}$$

$$= \left(10^5 + \frac{2(1020)9.81}{0.1} - \frac{20{,}024.8 \cdot 0.1}{0.1^2}\right)(0.0709 - 0.05) - \frac{20{,}024.8}{2(0.1)^2}\left(0.05^2 - 0.0709^2\right) = 4617.35\text{ J}$$

$$W = 4973.9 + 4617.35 = 9591.25\text{ J}$$

$$W = 9591.25\text{ J}$$

c) Least amount of work is that with incremental changes in weights. This is a reversible process, which results in least work for closed system compression.

$n=1 \rightarrow P_i V_i = P_f V_f \longrightarrow$ Ideal gas Law (isothermal)

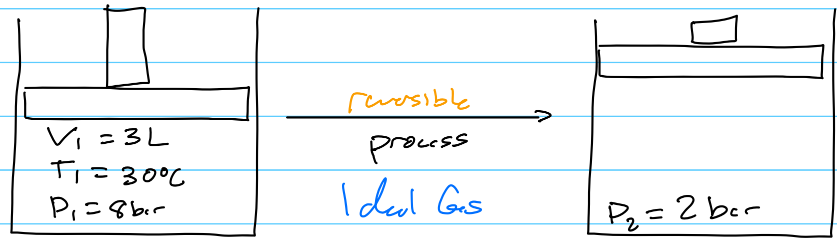
$$W = -nRT_1 \ln\left(\frac{V_f}{V_i}\right)$$

$$= P_i V_i \ln\left(\frac{V_i}{V_f}\right)$$

$$= (10^5\ Pa)(0.1\ m^3) \ln\left(\frac{0.1}{0.05}\right) = 6931.47\ J$$

$W = 6.93\ J$

4. Consider a piston–cylinder assembly that contains 3 L of an ideal gas at 30 °C and 8 bar. The gas reversibly expands to 2 bar.



- closed system
- reversible
- ideal gas
- no/negligible PE & KE

a. Write an energy balance for this process.

$$dU = \delta Q + \delta W$$

ideal gas $\rightarrow$ $n C_{v,m} dT = \delta Q - P_E dV \xrightarrow{rev} n C_{v,m} dT = \delta Q - P dV$

system pressure

$$\int n C_{v,m} dT = \int \delta Q - \int \frac{nRT}{V} dV$$

$$n C_{v,m} \Delta T = Q - nRT \ln\left(\frac{V_f}{V_i}\right)$$

b. Suppose that the process is done isothermally. What is the change in internal energy, ΔU, for the process? What is the work done, W, during the process? What is the heat transferred, Q?

$$\left.\begin{aligned}\Delta U &= n C_{v,m} \Delta T \\ &= n C_{v,m}(T_2 - T_1)\end{aligned}\right\} \text{Isothermal: } T_2 = T_1 \text{ so } \Delta U = 0$$

$P_1 V_1 = nRT_1$

$W = -nRT_1 \ln\left(\frac{V_f}{V_i}\right)$ Isothermal: $P_1 V_1 = P_2 V_2 \Rightarrow \frac{P_1}{P_2} = \frac{V_2}{V_1}$

$$W = -P_1 V_1 \ln\left(\frac{P_i}{P_f}\right)$$

$$= -(8 \times 10^5 Pa)(0.003 m^3) \ln\left(\frac{8}{2}\right) \quad 3.0 L \times \frac{1 m^3}{1000 L} = 0.003 m^3$$

$$= -3327.11$$

$W = -3.327 kJ$ negative bc of expansion

$0 = Q + W \rightarrow Q = 3.327 kJ$

c. If the process is done adiabatically (instead of isothermally), will the final temperature be greater than, equal to, or less than 30 °C? Explain.

Assuming expansion and $\Delta U = W$ due to $Q = 0$, $\Delta U < 0$ as $W < 0$. $\Delta U = n C_{v,m}(T_2 - T_1)$. Since $T_1 = 30°C$, T_2 must be less than this for ΔU to be negative as U is only a function of T for ideal gases.

Less than 30°C

5. Answer the following reflection questions (5 points):

   a. What about the way this class is taught is helping your learning?

   b. What about the way this class is taught is inhibiting your learning?

A. I think the conceptual discussions regarding multiple-choice style questions in class is helpful to my learning. It allows us to engage in conversation and also address common misconceptions as a class. I also like the emphasis on building things from the ground up, not just throwing equations at the class. This helps my understanding and promotes a deeper knowledge of the material. Finally, the use of examples in class is useful to see applications of the content being covered while also seeing the standard/expectations for the assignments and assessments.

B. I think the only thing inhibiting my learning is the difficultly spike between the in class examples and the homework. This week specifically, I felt the problems were a good survey of the *content* covering in the past week, but was not a good survey of that contents difficultly level. While I do firmly believe homework should be challenging, I found myself feeling lost on topics that I thought I should've know/were similar to the ones in class. The subtle nuances in the problems and their difficultly are great for homework, but I just feel my learning would be even more supported with more reflective in class examples (in terms of difficulty).