

## Equations, Units, and Tables

### Midterm 2

First Law for closed systems:

$$dU + dE_K + dE_P = \delta Q + \delta W$$

$$\frac{dU}{dt} + \frac{dE_K}{dt} + \frac{dE_P}{dt} = \dot{Q} + \dot{W}$$

PV work:

$$\delta W_m = -P_E dv_m$$

First Law for open systems with one input and one output stream:

$$\begin{aligned} dU + dE_K + dE_P = & dn_{\text{in}} \left[ h_m + \frac{1}{2} |v|^2 (MW) + gz(MW) \right]_{\text{in}} \\ & - dn_{\text{out}} \left[ h_m + \frac{1}{2} |v|^2 (MW) + gz(MW) \right]_{\text{out}} \\ & + \delta Q + \delta W_s \end{aligned}$$

$$\begin{aligned} \frac{dU}{dt} + \frac{dE_K}{dt} + \frac{dE_P}{dt} = & \dot{n}_{\text{in}} \left[ h_m + \frac{1}{2} |v|^2 (MW) + gz(MW) \right]_{\text{in}} \\ & - \dot{n}_{\text{out}} \left[ h_m + \frac{1}{2} |v|^2 (MW) + gz(MW) \right]_{\text{out}} \\ & + \dot{Q} + \dot{W}_s \end{aligned}$$

Enthalpy definition:

$$h_m = u_m + Pv_m$$

Enthalpy of vaporization:

$$\Delta h_{m,\text{vap}}(T) = \Delta h_{m,\text{vap}}(T_0) + \int_{T_0}^T c_{p,m}^{\text{liq}} dT + \int_{T_0}^T c_{p,m}^{\text{vap}} dT$$

Property data for ideal gases:

$$du_m = c_{v,m} dT$$

$$dh_m = c_{p,m} dT$$

Carnot efficiency:

$$\eta = \frac{T_H - T_C}{T_H}$$

Entropy changes:

$$ds_m = \frac{\delta q_{\text{rev},m}}{T}$$

Differential entropy change for an ideal gas:

$$ds_m = c_{p,m} \frac{dT}{T} - R \frac{dP}{P}$$

Second Law for closed systems:

$$\frac{dS}{dt} - \frac{\dot{Q}}{T_{\text{surr}}} \geq 0$$

$$\Delta S - \frac{Q}{T_{\text{surr}}} \geq 0$$

Second Law for open systems with one input and one output stream:

$$dS + s_{m,\text{out}} dn_{\text{out}} - s_{m,\text{in}} dn_{\text{in}} - \frac{\delta Q}{T_{\text{surr}}} \geq 0$$

$$\frac{dS}{dt} + s_{m,\text{out}} \dot{n}_{\text{out}} - s_{m,\text{in}} \dot{n}_{\text{in}} - \frac{\dot{Q}}{T_{\text{surr}}} \geq 0$$

Fundamental Equation of Thermodynamics (where only work is PV work or shaft work)

$$du_m = Tds_m - Pdv_m$$

$$dh_m = Tds_m + v_m dP$$

$$dg_m = -s_m dT + v_m dP$$

$$da_m = -s_m dT - Pdv_m$$

Maxwell Relations (only work is PV work and shaft work)

$$\begin{aligned}\left(\frac{\partial T}{\partial v_m}\right)_{s_m} &= -\left(\frac{\partial P}{\partial s_m}\right)_{v_m} \\ \left(\frac{\partial T}{\partial P}\right)_{s_m} &= \left(\frac{\partial v_m}{\partial s_m}\right)_P \\ \left(\frac{\partial s_m}{\partial v_m}\right)_T &= \left(\frac{\partial P}{\partial T}\right)_{v_m} \\ -\left(\frac{\partial s_m}{\partial P}\right)_T &= \left(\frac{\partial v_m}{\partial T}\right)_P\end{aligned}$$

Selected real gas equations (others might need to be derived)

$$dh_m = c_{p,m} dT + \left[ v_m - T \left( \frac{\partial v_m}{\partial T} \right)_P \right] dP$$

$$du_m = c_{v,m} dT + \left[ T \left( \frac{\partial P}{\partial T} \right)_{v_m} - P \right] dv_m$$

$$ds_m = \frac{c_{p,m}}{T} dT - \left( \frac{\partial v_m}{\partial T} \right)_P dP$$

$$ds_m = \frac{c_{v,m}}{T} dT + \left( \frac{\partial P}{\partial T} \right)_{v_m} dv_m$$

Cyclic rule (triple product rule)

$$-1 = \left( \frac{\partial x}{\partial z} \right)_y \left( \frac{\partial y}{\partial x} \right)_z \left( \frac{\partial z}{\partial y} \right)_x$$

Clapeyron equation:

$$\frac{dP}{dT} = \frac{h_m^\alpha - h_m^\beta}{T(v_m^\alpha - v_m^\beta)}$$

General form of the Gibbs–Duhem equation:

$$K = \sum_i \bar{K}_i n_i$$

$$0 = \left( \frac{\partial K}{\partial T} \right)_{P, n_1, \dots, n_m} dT + \left( \frac{\partial K}{\partial P} \right)_{T, n_1, \dots, n_m} dP - \sum_i n_i d\bar{K}_i$$

Units and constants:

$$R = 8.314 \text{ J/(mol K)}$$

$$g = 9.8 \text{ m/s}^2$$

$$1 \text{ bar} = 10^5 \text{ Pa} = 10^5 \text{ kg}/(\text{m s}^2)$$

$$1 \text{ J} = 1 \text{ kg m}^2 \text{ s}^{-2}$$