Homework #1

①

$$df = (A + Bx + Cx^2)\,dx + \frac{D}{y}\,dy$$

ⓐ $\left(\frac{\partial f}{\partial x}\right)_y$; $dy = 0$

$\Rightarrow df = (A + Bx + Cx^2)\,dx$ for $dy = 0$

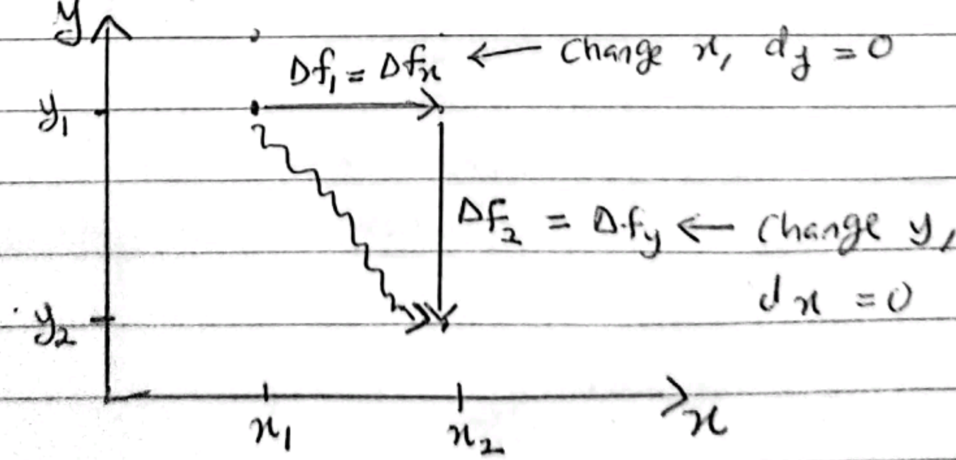
$\therefore \left(\frac{\partial f}{\partial x}\right)_y = A + Bx + Cx^2$

ⓑ Similarly: $\left(\frac{\partial f}{\partial y}\right)_x$; $dx = 0$

$\Rightarrow df = \frac{D}{y}\,dy$ for $dx = 0$

$\therefore \left(\frac{\partial f}{\partial y}\right)_x = \frac{D}{y}$

ⓒ $\Delta f = f(x_2, y_2) - f(x_1, y_1)$ state function, so:



$$\Delta f = \Delta f_x + \Delta f_y$$

$$= \int_{x_1}^{x_2} \left(\frac{\partial f}{\partial x}\right)_y dx + \int_{y_1}^{y_2} \left(\frac{\partial f}{\partial y}\right)_x dy.$$

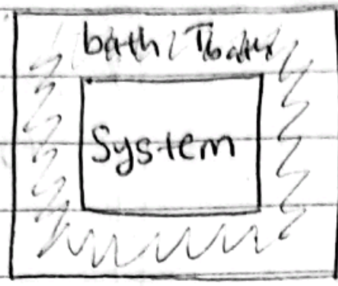
$$= \int_{x_1}^{x_2} (A + Bx + Cx^2)\, dx + \int_{y_1}^{y_2} \frac{D}{y}\, dy$$

$$= \left(Ax + \frac{Bx^2}{2} + \frac{Cx^3}{3}\right)\Bigg|_{x_1}^{x_2} + D\ln y \Bigg|_{y_1}^{y_2}$$

$$\therefore \Delta f = A(x_2 - x_1) + \frac{B}{2}(x_2^2 - x_1^2) + \frac{C}{3}(x_2^3 - x_1^3) + D\ln\left(\frac{y_2}{y_1}\right)$$

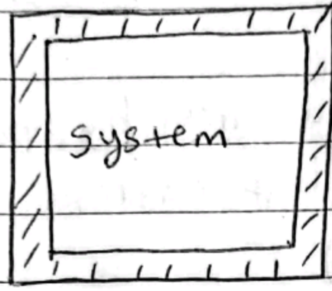
(2)

(a)



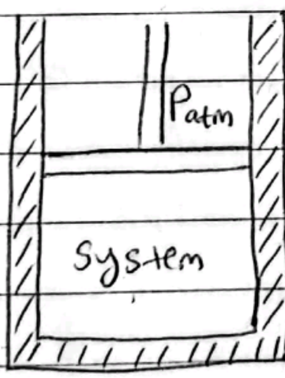
isothermal ($T = T_{bath}$)
(not adiabatic)

(b)



insulation
rigid (constant volume V), $W = 0$
adiabatic, $Q = 0$

(c) Option 1



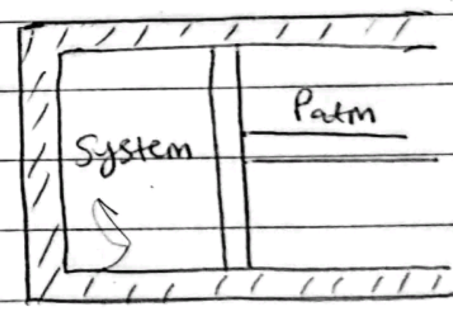
adiabatic, $Q=0$

$P_{sys} = P_{atm} + \underbrace{\frac{mg}{A}}$

gravitational force on piston per area

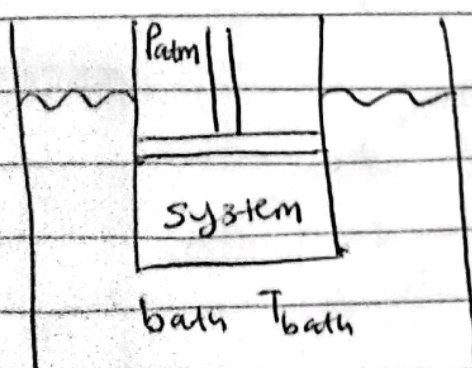
(A is cross sectional area of piston)

Option 2



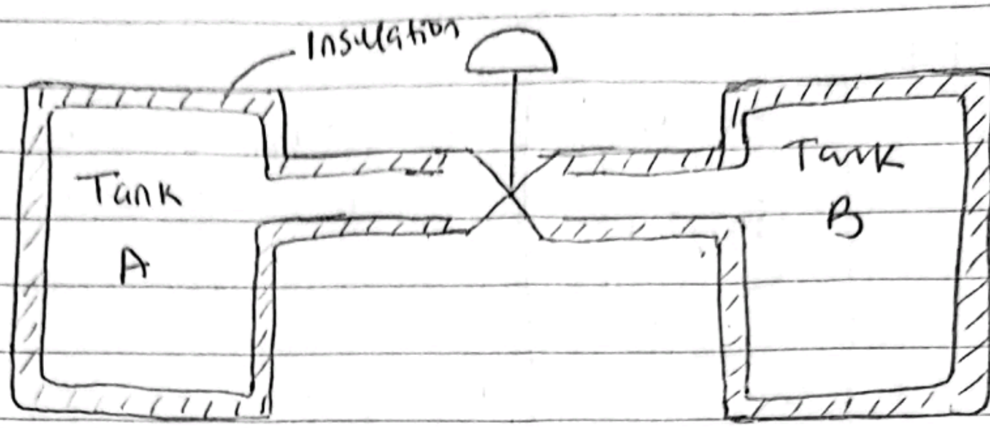
$P_{sys} = P_{atm}$
adiabatic, $Q = 0$

(d)



isothermal ($T = T_{bath}$)
(not adiabatic)

(e)



Short time: $P_A = P_B$ (flow driven by pressure gradient)

$T_A \neq T_B$ (heat conduction can be slow)

(long time: $T_A = T_B$)

③

ⓐ Extensive properties of subsystems can be summed:

$$V_1 = V_a + V_b$$

ⓑ $$\nu_{m,1} = \frac{V_a + V_b}{n_a + n_b} = \frac{n_a \nu_{m,a} + n_b \nu_{m,b}}{n_a + n_b}$$

Simplifying further:

$$\nu_{m,1} = \left(\frac{n_a}{n_a + n_b}\right) \nu_{m,a} + \left(\frac{n_b}{n_a + n_b}\right) \nu_{m,b}$$

$$\nu_{m,1} = x_a \nu_{m,a} + x_b \nu_{m,b}$$

where x_a, x_b are mole fractions:

$$x_a = \frac{n_a}{n_a + n_b}, \quad x_b = \frac{n_b}{n_a + n_b}$$

ⓒ $$K_1 = K_a + K_b$$

ⓓ $$K_{m,1} = \frac{K_a + K_b}{n_a + n_b} = \frac{n_a K_{m,a} + n_b K_{m,b}}{n_a + n_b}$$

$$K_{m,1} = \left(\frac{n_a}{n_a + n_b}\right) K_{m,a} + \left(\frac{n_b}{n_a + n_b}\right) K_{m,b}$$

$$K_{m,1} = x_a K_{m,a} + x_b K_{m,b}$$

4

$m = 2\,kg$
$P = 1\,bar$
$X_v = 0.45$
$V = ?$

$$\hat{V} = x_L \hat{V}_L + x_v \hat{V}_v$$

note : $\hat{V} = \frac{V}{m}$, $x_L = 1 - x_v$

$$V = m\left[(1-x_v)\hat{V}_L + x_v \hat{V}_v\right]$$

From Table B.2 :

$\hat{V}_L (P = 1\,bar) = 0.001043\ m^3/kg$

$\hat{V}_v (P = 1\,bar) = 1.694\ m^3/kg$

$V = 2\left[(1-0.45)\,0.001043 + 0.45 \times 1.694\right]$

$V = 2\left[5.7365 \times 10^{-4} + 0.7623\right]$

$\boxed{V = 1.5257\ m^3}$

(from Table B.2) $P = 1\,bar = 0.1\,MPa$

$T(P = 0.1\,MPa) = 99.62\,°C$

(Table B.2) $\begin{cases} \hat{S}_L = 1.3025\ kJ/kg\cdot K \\ \hat{S}_v = 7.3593\ kJ/kg\cdot K \end{cases}$

$$S = m\left[(1-x_v)\hat{S}_L + x_v \hat{S}_v\right]$$

$S = 2\left[(1-0.45)\,1.3025 + 0.45 \times 7.3593\right]$

$S = 2\left[0.716375 + 3.311685\right]$

$S = 8.05612\ (kJ/\cancel{kg}\cdot K)\ \cancel{kg}$

$S = 8.05612\ kJ/K$

(5)

Temperature = ?

Quality = ?

internal energy (u) = ?

$V = 1\ m^3$

$P = 1.6$ bar

$m = 4$ kg

$P = 1.6$ bar $= 0.16$ MPa

no value for 0.16 MPa, need to interpolate.

$P_{sat} = 0.15$ MPa (P_{low})

$T_{sat} = 111.37°C$

$\hat{V}_V = 1.1593\ m^3/kg$

$\hat{V}_L = 0.001053\ m^3/kg$

$\hat{U}_V = 2519.6$ KJ/kg

$\hat{U}_L = 466.92$ KJ/kg

$P_{sat} = 0.175$ MPa (P_{high})

$T_{sat} = 116.06$ °C

$\hat{V}_V = 1.0036\ m^3/kg$

$\hat{V}_L = 0.001057\ m^3/kg$

$\hat{U}_V = 2524.9$ KJ/kg

$\hat{U}_L = 486.78$ KJ/kg

find Temperature (T_{sat}) => assume T_{sat} varies linearly with P_{sat} between 0.15 and 0.175 MPa

from equations (1.19-1.20):

$$T_{sat}(P = 0.16\ MPa) = T(P_{low}) + \left(T(P_{high}) - T(P_{low})\right)\left(\frac{P - P_{low}}{P_{high} - P_{low}}\right)$$

$$T_{sat}(P = 0.16\ MPa) = 111.37°C + (116.06°C - 111.37°C)\left(\frac{0.16 - 0.15}{0.175 - 0.15}\right)$$

$$T = T_{sat}(P = 0.16\,MPa) = 113.25°C$$

find $\hat{V}_V$ and $\hat{V}_L$ $\Rightarrow$

$$\hat{V}_V(P_{sat} = 0.16\,MPa) = \hat{V}_V(P_{low}) + \left(\hat{V}_V(P_{high}) - \hat{V}_V(P_{low})\right)\left(\frac{P - P_{low}}{P_{high} - P_{low}}\right)$$

$$\hat{V}_V(P_{sat} = 0.16\,MPa) = 1.1593\,m^3/kg + \left(1.0036\,m^3/kg - 1.1593\,m^3/kg\right)\left(\frac{0.16 - 0.15}{0.175 - 0.15}\right)$$

$$\hat{V}_V(P_{sat} = 0.16\,MPa) = 1.0970\,m^3/kg$$

similarly for $\hat{V}_L$:

$$\hat{V}_L(P_{sat} = 0.16\,MPa) = 0.001053\,m^3/kg + \left(0.001057\,m^3/kg - 0.001053\,m^3/kg\right)\left(\frac{0.16 - 0.15}{0.175 - 0.15}\right)$$

$$\hat{V}_L(P_{sat} = 0.16\,MPa) = 0.0010546\,m^3/kg$$

find quality $\Rightarrow$ note $\hat{V} = V/m = \frac{1\,m^3}{4\,kg} = 0.25\,m^3/kg$

$$\hat{V} = x_V V_V + (1 - x_V) V_L$$

$$x_V = \frac{\hat{V} - \hat{V}_L}{\hat{V}_V - \hat{V}_L} = \frac{0.25\,m^3/kg - 0.0010546\,m^3/kg}{1.0970\,m^3/kg - 0.0010546\,m^3/kg}$$

$$\boxed{x_V = 0.23}$$

interpolate $\hat{U}_L + \hat{U}_V \Rightarrow$

$$\hat{U}_V(P_{sat} = 0.16\ MPa) = 2519.6\ kJ/kg + \left(2524.9\ \frac{kJ}{kg} - 2519.6\ \frac{kJ}{kg}\right)\left(\frac{0.16 - 0.15}{0.175 - 0.15}\right)$$

$$\hat{U}_V(P_{sat} = 0.16\ MPa) = 2521.72\ kJ/kg$$

$$\hat{U}_L(P_{sat} = 0.16\ MPa) = 466.92\ kJ/kg + \left(486.78\ \frac{kJ}{kg} - 466.92\ kJ/kg\right)\left(\frac{0.16 - 0.15}{0.175 - 0.15}\right)$$

$$\hat{U}_L(P_{sat} = 0.16\ MPa) = 474.86\ kJ/kg$$

$$\hat{U} = x_v \hat{U}_V + (1 - x_v)\hat{U}_L$$

$$\hat{U} = (0.23)(2521.72\ kJ/kg) + (1 - 0.23)(474.86\ kJ/kg)$$

$$= 945.63\ kJ/kg$$

$$U = \hat{U} m = (4\ kg)(945.63\ kJ/kg)$$

$$U = 3782.55\ kJ$$

(6)

(a) from Table B.1 $\rightarrow P(T_{sat} = 60°C) = 19.941$ kPa

(b) initially saturated liquid $\rightarrow V_i = m\hat{V}_L$

$V_i = (3\ kg)(0.001017\ m^3/kg)$

$V_i = 0.003051\ m^3$

(c) $\hat{V}_V = 7.671\ m^3/kg$ (from Table B-1)

$V = m_V\hat{V}_V + m_L\hat{V}_L$

$V = (2\ kg)(7.671\ m^3/kg) + (1\ kg)(0.001017\ m^3/kg)$

$V = 15.34\ m^3$

$\hat{U}_L\ (T_{sat} = 60°C) = 251.09\ kJ/kg$

$\hat{U}_V\ (T_{sat} = 60°C) = 2456.6\ kJ/kg$

initial:

$U_i = m\hat{U}_L = (3\ kg)(251.09\ kJ/kg)$

$U_i = 753.27\ kJ/kg$

final:

$U_f = m_L\hat{U}_L + m_V\hat{U}_V$

$U_f = (1\ kg)(251.09\ kJ/kg) + (2\ kg)(2456.6\ \frac{kJ}{kg})$

$U_f = 5164.29\ kJ/kg$

$\Delta U = U_f - U_i = 5164.29\ kJ/kg - 753.27\ kJ/kg$

$\boxed{\Delta U = 4411.0\ kJ}$

OR $\Delta\hat{U}_{LV} = \hat{U}_V - \hat{U}_L = 2456.6\ kJ/kg - 251.09\ kJ/kg = 2205.51\ kJ/kg$

$\Delta U = m\Delta\hat{U}_{LV} = (2\ kg)(2205.51\ kJ/kg)$

$\boxed{\Delta U = 4411.0\ kJ}$

(d) $\Delta \hat{U}_{LV} = 2205.5$ kJ/kg (from Table B.1)

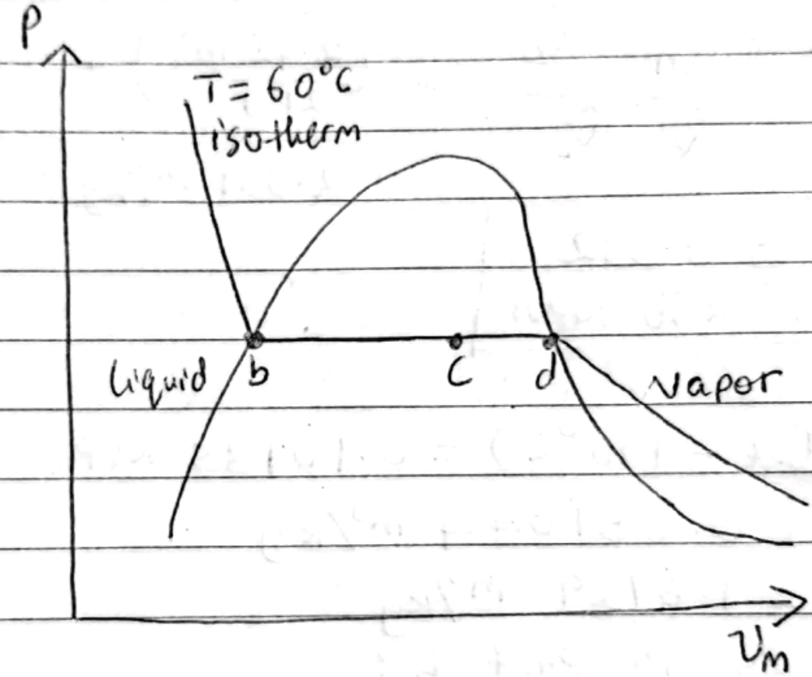
$\Delta \hat{h}_{LV} = 2358.5$ kJ/kg

for $\Delta m = 3$ kg

$\Delta U = (3\text{ kg})(2205.5 \text{ kJ/kg})$ $\boxed{\Delta U = 6616.5 \text{ kJ/kg}}$

$\Delta H = (3\text{ kg})(2358.5 \text{ kJ/kg})$ $\boxed{\Delta H = 7075.5 \text{ kJ/kg}}$

(e)



b ⇒ saturated liquid

c ⇒ 2 kg evaporated
(2/3 way across 2-phase line)

d ⇒ saturated vapour.

(7)

(a) from Table B.1 $\boxed{P(T_{sat}=90°C) = 70.139 \text{ kPa}}$

(b)

$$\boxed{\begin{array}{l} P = 70.139 \text{ kPa} \\ T = 90°C \\ m = 2 \text{ kg} \\ V = 2.42 \text{ m}^3 \end{array}}$$

$$\hat{V} = \frac{V}{m} = (1-X_v)\hat{V}_L + X_v \hat{V}_v$$

$$\left.\begin{array}{l} \hat{V}_L = 0.001036 \text{ m}^3/\text{kg} \\ \hat{V}_v = 2.361 \text{ m}^3/\text{kg} \end{array}\right\} \text{from Table B.1}$$

$$X_v = \frac{\frac{V}{m} - \hat{V}_L}{\hat{V}_v - \hat{V}_L} = \frac{\left(\frac{2.42 \text{ m}^3}{2 \text{ kg}}\right) - 0.001036 \text{ m}^3/\text{kg}}{2.361 \text{ m}^3/\text{kg} - 0.001036 \text{ m}^3/\text{kg}}$$

$$\boxed{\begin{array}{l} X_v = 0.512 \\ 51.2\% \text{ vapor} \end{array}}$$

(c) $P(T_{sat} = 100°C) = 0.10135 \text{ MPa}$

$\hat{V}_L = 0.001044 \text{ m}^3/\text{kg}$

$\hat{V}_v = 1.6729 \text{ m}^3/\text{kg}$

from eq. in part b:

$$X_v = \frac{\frac{V}{m} - \hat{V}_L}{\hat{V}_v - \hat{V}_L} = \frac{\left(\frac{2.42 \text{ m}^3}{2 \text{ kg}}\right) - 0.001044 \text{ m}^3/\text{kg}}{1.6729 \text{ m}^3/\text{kg} - 0.001044 \text{ m}^3/\text{kg}}$$

$$\boxed{\begin{array}{l} X_v = 0.723 \\ 72.3\% \text{ vapor} \end{array}}$$

initial :

$\hat{U}_L (T_{sat} = 90°C) = 376.82 \ KJ/kg$

$\hat{U}_v (T_{sat} = 90°C) = 2494.5 \ KJ/kg$

$\hat{h}_L (T_{sat} = 90°C) = 376.90 \ KJ/kg$

$\hat{h}_v (T_{sat} = 90°C) = 2660.1 \ KJ/kg$

$U_i = m[(1 - X_v)\hat{U}_L + X_v \hat{U}_v]$

$U_i = (2 \ kg)[(1 - 0.512)(376.82 \ KJ/kg) + (0.512)(2494.5 \ KJ/kg)]$

$U_i = 2922.14 \ KJ$

$H_i = m[(1 - X_v)\hat{h}_L + X_v \hat{h}_v]$

$H_i = (2 \ kg)[(1 - 0.512)(376.90 \ KJ/kg) + (0.512)(2660.1 \ KJ/kg)]$

$H_i = 3091.80 \ KJ$

final :

$\hat{U}_L (T_{sat} = 100°C) = 418.91 \ KJ/kg$

$\hat{U}_v (T_{sat} = 100°C) = 2506.5 \ KJ/kg$

$\hat{h}_L (T_{sat} = 100°C) = 419.02 \ KJ/kg$

$\hat{h}_v (T_{sat} = 100°C) = 2676.0 \ KJ/kg$

$U_f = (2 \ kg)[(1 - 0.723)(418.91 \ KJ/kg) + (0.723)(2506.5 \ KJ/kg)]$

$U_f = 3856.48 \ KJ$

$H_f = (2 \ kg)[(1 - 0.723)(419.02 \ KJ/kg) + (0.723)(2676.0 \ KJ/kg)]$

$H_f = 4101.63 \text{ kJ}$

$$\boxed{\begin{aligned} \Delta U &= U_f - U_i = 3856.48 \text{ kJ} - 2922.14 \text{ kJ} = 934.34 \text{ kJ} \\ \Delta H &= H_f - H_i = 4101.63 \text{ kJ} - 3091.80 \text{ kJ} = 1009.83 \text{ kJ} \end{aligned}}$$