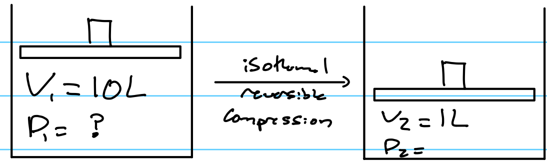
1.



$n = 2mol$
H_2O
$T = 1000K$

General Work Expression

$$\delta W = -P_E \, dV_m$$
$$= -P \, dV_m$$

(reversible => $P_E = P$)

$$\Rightarrow W = -\int_{V_{m_1}}^{V_{m_2}} P \, dV_m$$

Molar Volumes

$$V_{m_1} = \frac{10L}{2mol} = 0.005 \frac{m^3}{mol}$$

$$V_{m_2} = \frac{1L}{2mol} = 0.0005 \frac{m^3}{mol}$$

Mass of water

$$mass = 2mol \times \frac{18.02g}{1mol} \times \frac{1kg}{1000g}$$

a) Ideal gas law

$$W_m = -\int_{V_{m,1}}^{V_{m,2}} P \, dV_m = -\int_{V_{m,1}}^{V_{m,2}} \frac{RT}{V_m} dV_m = -RT \ln\left(\frac{V_{m,2}}{V_{m,1}}\right) = -\left(8.314 \frac{J}{mol\,K}\right)(1000K) \ln\left(\frac{0.0005}{0.005}\right)$$

$$W_m = 19143.69 \rightarrow W = nW_m = (2mol)\left(19143.69 \frac{J}{mol}\right) = 38287.4 \text{ J}$$

$$W = 38.3 \text{ kJ}$$

b) $$P = \frac{RT}{V_m - b} - \frac{a}{T^{1/2} V_m (V_m + b)} \qquad \begin{cases} a = 14.24 \dfrac{J K^{1/2} m^3}{mol^2} \\ b = 2.11 \times 10^{-5} \dfrac{m^3}{mol} \end{cases}$$

$$W = -n \int_{V_{m,1}}^{V_{m,2}} P \, dV_m = -n \int_{V_{m,1}}^{V_{m,2}} \frac{RT}{V_m - b} - \frac{a}{T^{1/2} V_m (V_m + b)} dV_m$$

$$= -n \left[RT \int_{V_{m,1}}^{V_{m,2}} \frac{1}{V_m - b} dV_m - \frac{a}{T^{1/2}} \int_{V_{m,1}}^{V_{m,2}} \frac{1}{V_m (V_m + b)} dV_m \right]$$

$$= -n \left[RT \ln\left(\frac{V_{m,2} - b}{V_{m,1} - b}\right) - \frac{a}{T^{1/2} b} \left(-\ln\left(\frac{V_{m,2} + b}{b}\right) + \ln\left(\frac{V_{m,1} + b}{b}\right) + \ln\left(\frac{V_{m,2}}{V_{m,1}}\right) \right) \right]$$

$$= -2 \left[8.314(1000) \ln\left(\frac{.0005 - 2.11 \times 10^{-5}}{.005 - 2.11 \times 10^{-5}}\right) - \frac{14.24}{2.11 \times 10^{-5} \sqrt{1000}} \left(-\ln\left(\frac{.0005 + 2.11 \times 10^{-5}}{2.11 \times 10^{-5}}\right) + \ln\left(\frac{.005 + 2.11 \times 10^{-5}}{2.11 \times 10^{-5}}\right) + \ln\left(\frac{.0005}{.005}\right) \right) \right]$$

$$= 37349.5$$

$$W = 37.4 \text{ kJ}$$

c) $\hat{V}_1 = V_{m,2} \cdot MW_{H_2O} = 0.005 \frac{m^3}{mol} \cdot \frac{1\,mol}{18.02\,g} \cdot \frac{1000\,g}{1\,kg} = .277 \frac{m^3}{kg}$

$\hat{V}_2 = V_{m,1} \cdot MW_{H_2O} = 0.0005 \frac{m^3}{mol} \cdot \frac{1\,mol}{18.02\,g} \cdot \frac{1000\,g}{1\,kg} = .0277 \frac{m^3}{kg}$

$T = 1000\,K = 727°C$

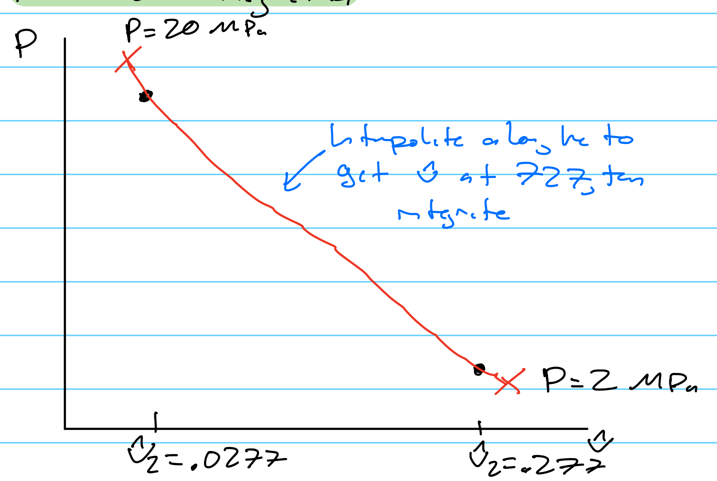
$\hat{V}_1$ lies somewhere in here

Pressure	T	$\hat{V}$ ($\frac{m^3}{kg}$)	Pressure	T	$\hat{V}$ ($\frac{m^3}{kg}$)
1.6 MPa	700°C	.27937	6 MPa	700°C	.07352
	800°C	.30859		800°C	.08160
1.8 MPa	700°C	.24818	7 MPa	700°C	.06283
	800°C	.27420		800°C	.06981
2 MPa	700°C	.22323	8 MPa	700°C	.05481
	800°C	.24668		800°C	.06097
2.5 MPa	700°C	.17832	9 MPa	700°C	.04857
	800°C	.14716		800°C	.05409
3 MPa	700°C	.14838	10 MPa	700°C	.04358
	800°C	.16414		800°C	.04859
3.5 MPa	700°C	.12649	12.5 MPa	700°C	.03460
	800°C	.14056		800°C	.03869
4 MPa	700°C	.11095	15 MPa	700°C	.028612
	800°C	.12287		800°C	.032096
4.5 MPa	700°C	.09847	17.5 MPa	700°C	.0243365
	800°C	.10911		800°C	.0273849
5 MPa	700°C	.08849	20 MPa	700°C	.0211311
	800°C	.09811		800°C	.0238532

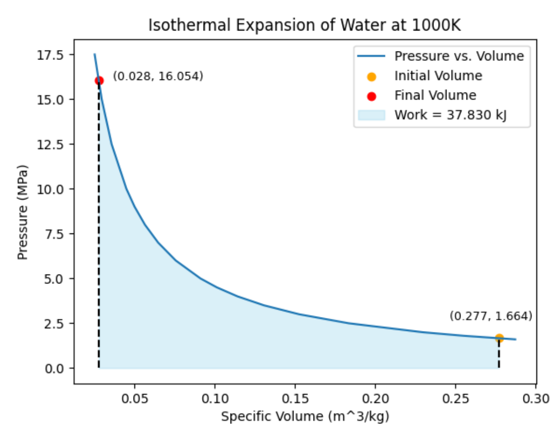
$\hat{V}_2$ lies somewhere in here

not needed data

Numeric Integration



Using Python



W = 37.83 kJ

d) Because the steam table contains specific data, I believe it is the most accurate and should be treated as absolute. The result obtained using the Redlich-Kwong EoS is closer than the ideal gas law. In usual applications, I would use an EoS over numeric integration as the EoS is much easier to use. That being said, it is nice to see tabulated data align with this equation of state, as sometimes there isn't a relation/equation we can use in an EoS. Ultimately, the tabulated data is the most accurate but most intensive to use, the EoS is easier but less-accurate, and the IGL is inaccurate for such applications and should not be used.

2. a) 30,000 units @ 43L containers, 12,400 kPa and \$6.10/kg N_2

Ideal Gas Law

T is not given, assume 298K

$PV = nRT$

\$1,103,472.43

$$n = \frac{PV}{RT} = \frac{(12{,}400{,}000\ Pa)(43 \cdot 10^{-3}\ m)}{(8.314\ \frac{J}{mol \cdot K})(298K)} = 215.2\ mol\ N_2$$

$$\text{Total Sales} = 215.2\ mol\ N_2 \cdot \frac{28.02g}{1\ mol\ N_2} \cdot \frac{1\ kg}{1000g} \cdot \frac{\$6.10}{1\ kg} \cdot 30{,}000\ units = 1103472.43$$

Redlich-Kwong EoS

$$P = \frac{RT}{\frac{V}{n} - b} - \frac{a}{T^{1/2}\frac{V}{n}\left(\frac{V}{n}+b\right)}$$

\$9,045,858.84

$$12{,}400{,}000\ Pa = \frac{(8.314)(298K)}{\frac{43\times10^{-3}m}{n} - 2.11\times10^{-5}\frac{m^3}{mol}} - \frac{14.24}{\sqrt{298}\left(\frac{43\times10^{-3}m}{n}\right)\left(\frac{43\times10^{-3}m}{n} + 2.11\times10^{-5}\frac{m^3}{mol}\right)}$$

Determine n w/ Wolfram => $n = 1764.13\ mol\ O_2$

$$\text{Total Sales} = 1764.13\ mol\ O_2 \cdot \frac{28.02g}{1\ mol\ O_2} \cdot \frac{1\ kg}{1000g} \cdot \frac{\$6.10}{1kg} \cdot 30{,}000\ units = 9045858.84$$

b) 30,000 units @ 43L containers, 15,000 kPa and \$4.00/kg N_2

Ideal Gas Law

T is not given, assume 298K

$PV = nRT$

\$2,253,555.11

$$n = \frac{PV}{RT} = \frac{(15{,}000{,}000\ Pa)(43 \cdot 10^{-3}\ m)}{(8.314\ \frac{J}{mol \cdot K})(298K)} = 260.34\ mol\ O_2$$

$$\text{Total Sales} = 260.34\ mol\ N_2 \cdot \frac{32.06g}{1\ mol\ N_2} \cdot \frac{1\ kg}{1000g} \cdot \frac{\$4.00}{1\ kg} \cdot 30{,}000\ units = 2253555.11$$

Redlich-Kwong EoS

$$P = \frac{RT}{\frac{V}{n} - b} - \frac{a}{T^{1/2}\frac{V}{n}\left(\frac{V}{n}+b\right)}$$

$$15{,}000{,}000\ Pa = \frac{(8.314)(298K)}{\frac{43\times10^{-3}m}{n} - 2.11\times10^{-5}\frac{m^3}{mol}} - \frac{14.24}{\sqrt{298}\left(\frac{43\times10^{-3}m}{n}\right)\left(\frac{43\times10^{-3}m}{n} + 2.11\times10^{-5}\frac{m^3}{mol}\right)}$$

Determine n w/ Wolfram => $n = 1765.15\ mol\ O_2$

$$\text{Total Sales} = 1765.15\ mol\ O_2 \cdot \frac{32.06g}{1\ mol\ O_2} \cdot \frac{1\ kg}{1000g} \cdot \frac{\$4.00}{1kg} \cdot 30{,}000\ units = 1527944 1.43$$

\$15,279,441.43

d) $P = \frac{RT}{v_m - b} - \frac{a}{T v_m^2}$ Solve for a and b in terms of T_c and P_c

$\left(\frac{\partial P}{\partial v_m}\right)_{T=T_c} = 0$ @ critical point, $\left(\frac{\partial^2 P}{\partial v_m^2}\right)_{T=T_c} = 0$ @ critical point

$$\left(\frac{\partial P}{\partial v_m}\right)_{T=T_c} = \frac{\partial}{\partial v_m}\left[\frac{RT}{v_{m,c} - b} - \frac{a}{T v_{m,c}^2}\right]_{T=T_c} = \frac{\partial}{\partial v_m}\left[\frac{RT_c}{v_{m,c} - b}\right] - \frac{\partial}{\partial v_m}\left[\frac{a}{T_c v_{m,c}^2}\right] = 0$$

$$\Rightarrow -\frac{RT_c}{(v_{m,c} - b)^2} + \frac{2a}{T_c v_{m,c}^3} = 0$$

$$\left(\frac{\partial^2 P}{\partial v_m^2}\right)_{T=T_c} = \frac{\partial}{\partial v_m}\left[-\frac{RT_c}{(v_{m,c} - b)^2} + \frac{2a}{T_c v_{m,c}^3}\right]_{T=T_c} = \frac{\partial}{\partial v_m}\left[-\frac{RT_c}{(v_{m,c} - b)^2}\right] + \frac{\partial}{\partial v_m}\left[\frac{2a}{T_c v_{m,c}^3}\right] = 0$$

$$\Rightarrow \frac{2RT_c}{(V_{m,c} - b)^3} - \frac{6a}{T_c V_{m,c}^4} = 0$$

(i) Solve $\left(\frac{\partial P}{\partial v_m}\right)_{T=T_c}$ for 'a'

$$-\frac{RT_c}{(v_{m,c} - b)^2} + \frac{2a}{T_c v_{m,c}^3} = 0 \rightarrow \frac{2a}{T_c v_{m,c}^3} = \frac{RT_c}{(v_{m,c} - b)^2} \rightarrow a = \frac{RT_c^2 V_{m,c}^3}{2(V_{m,c} - b)^2}$$

(ii) Substitute a into $\left(\frac{\partial^2 P}{\partial v_m^2}\right)_{T=C}$

$$\frac{2RT_c}{(V_{m,c} - b)^3} - \frac{6a}{T_c V_{m,c}^4} = 0 \rightarrow \frac{2RT_c}{(V_{m,c} - b)^3} - \frac{6}{T_c V_{m,c}^4}\left(\frac{RT_c^2 V_{m,c}^3}{2(V_{m,c} - b)^2}\right) = 0$$

(iii) Solve above expression for 'b' in terms of $v_{m,c}$

$$\frac{2RT_c}{(V_{m,c} - b)^3} - \frac{\cancel{6}\,3}{\cancel{T_c} V_{m,c}^{\cancel{4}}}\left(\frac{RT_c^{\cancel{2}} \cancel{V_{m,c}^3}}{\cancel{2}(V_{m,c} - b)^2}\right) = 0$$

$$\frac{2RT_c}{(V_{m,c} - b)^3} - \frac{3RT_c}{v_{m,c}(v_{m,c} - b)^2} = 0 \rightarrow \frac{2\cancel{RT_c}}{(V_{m,c} - b)^3} = \frac{3\cancel{RT_c}}{v_{m,c}(v_{m,c} - b)^2}$$

$$2 = \frac{3(v_{m,c} - b)^{\cancel{3}}}{v_{m,c}\cancel{(v_{m,c} - b)^2}} = \frac{3(v_{m,c} - b)}{v_{m,c}} \rightarrow 2v_{m,c} = 3v_{m,c} - 3b$$

$$3b = v_{m,c} \rightarrow b = \tfrac{1}{3} V_{m,c}$$

(iv) Substitute 'b' into 'a' to get 'a' in terms of $v_{m,c}$

$$a = \frac{RT_c^2 V_{m,c}^3}{2(V_{m,c} - b)^2} = \frac{RT_c^2 V_{m,c}^3}{2(v_{m,c} - \frac{1}{3}v_{m,c})^2} = \frac{RT_c^2 V_{m,c}^3}{2(\frac{2}{3} v_{m,c})^2} = \frac{RT_c^2 V_{m,c}^{\cancel{3}}}{2(\frac{4}{9} \cancel{V_{m,c}^2})} = \frac{9RT_c^2 V_{m,c}}{8}$$

(v) Use EoS and derived a and b values to get $V_{m,c}$ in terms of P_c, T_c

$$P_c = \frac{RT_c}{V_{m,c}-b} - \frac{a}{T_c V_{m,c}^2} \; ; \; a = \frac{9RT_c^2 V_{m,c}}{8}, \; b = \frac{1}{3}V_{m,c}$$

$$P_c = \frac{RT_c}{V_{m,c} - \frac{1}{3}V_{m,c}} - \frac{\left(\frac{9RT_c^{\cancel{2}}\cancel{V_{m,c}}}{8}\right)}{\cancel{T_c}\cdot V_{m,c}^{\cancel{2}}} = \frac{RT_c}{\frac{2}{3}V_{m,c}} - \frac{9RT_c^2}{8V_{m,c}} = \frac{\frac{3}{2}RT_c}{V_{m,c}} - \frac{\frac{9}{8}RT_c}{V_{m,c}}$$

$$P_c = \frac{\frac{3}{2}RT_c - \frac{9}{8}RT_c}{V_{m,c}} \rightarrow P_c = \frac{RT_c}{V_{m,c}}\left(\frac{3}{8}\right) \Rightarrow V_{m,c} = \frac{3}{8}\left(\frac{RT_c}{P_c}\right)$$

(vi) Substitute $V_{m,c}$ into 'a' and 'b' expressions

$$a = \frac{9RT_c^2}{8}\left(\frac{3}{8}\left(\frac{RT_c}{P_c}\right)\right) = \frac{27}{64}\left(\frac{R^2T_c^3}{P_c}\right) \rightarrow \quad a = \frac{27}{64}\left(\frac{T_c(RT_c)^2}{P_c}\right)$$

$$b = \frac{1}{\cancel{3}}\left(\frac{\cancel{3}}{8}\left(\frac{RT_c}{P_c}\right)\right) = \frac{1}{8}\left(\frac{RT_c}{P_c}\right) \rightarrow \quad b = \frac{1}{8}\left(\frac{RT_c}{P_c}\right)$$

4. Answer the following reflection questions (5 points):

   a. What about the way this class is taught is helping your learning?

   b. What about the way this class is taught is inhibiting your learning?

(A) This class has become one of my favorites this semester due to the lecture style as a whole. The use of class time, integrated discussion, and in depth examples all greatly benefit my learning. As the content gets more complex, I have found that the use of examples has been the most important driving factor for my learning. Moreover, asking the multiple choice questions in class builds my foundations up, and allows my conceptual knowledge to grow. The overall pace of the class and its way of handling difficult content through engaging discussion and the aforementioned use of examples helps my learning the most effectively.

(B) There is nothing currently inhibiting my learning in this class. A lot of the difficulty I found in the first half of the semester has been resolved due to a more firm grasp on foundational concepts. This class is well-taught and my learning is being supported very well.