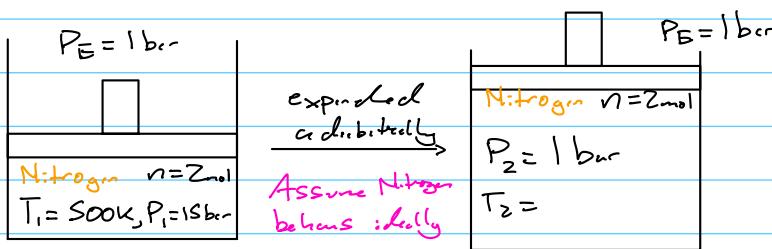


ECHE363 Homework 3 - Due 02/07/25

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- Remember
C_{p,m} = C_{v,m} + R*
1. Two moles of nitrogen are initially at 15 bar and 500 K in piston-cylinder assembly. The system is expanded adiabatically against a constant external pressure of 1 bar until it reaches mechanical equilibrium. Calculate the final temperature, the heat transferred, and the work of the system. You may assume ideal gas behavior for nitrogen. Use a temperature-dependent heat capacity for nitrogen. These data can be found in the appendices of the Koretsky textbook.

what Koretsky
gives
 $dU = nC_v dT$
Convert!
Use $P = 8.314 \frac{J}{mol\cdot K}$



- Closed system
- MB: $n_1 = n_2 = n = 2 \text{ mol}$
- Adiabatic expansion $\rightarrow Q = 0, \Delta U = 0$
- Constant external pressure
- $P_{ext} = 1 \text{ bar}$ at mechanical equilibrium

Koretsky Heat Capacity for Nitrogen ($C_p = A + BT + CT^2 + DT^{-2} + ET^3$ for T in K)

$$\begin{aligned} A &= 3.280 \\ B &= 0.593 \times 10^{-3} \\ D &= 0.04 \times 10^{-5} \end{aligned} \quad \left. \begin{aligned} C_{p,m} &= R \left(3.280 + \frac{0.593}{10^3} T + \frac{0.04}{10^5} T^{-2} \right) \\ &\Rightarrow C_{v,m} = R \left(3.280 + \frac{0.593}{10^3} T + \frac{0.04}{10^5} T^{-2} \right) - R \end{aligned} \right\} \text{ideal g.s.}$$

$$1^{\text{st}} \text{ Law: } dU = \cancel{dQ} + \cancel{dW} \rightarrow nC_v dT = -P_E dV$$

$$\Rightarrow \int_{T_1}^{T_2} nC_v dT = -P_E \int_{V_1}^{V_2} dV = -P_E (V_2 - V_1) = -P_E \left[\frac{nRT_2}{P_E} - \frac{nRT_1}{P_1} \right]$$

$$\rightarrow \int_{T_1}^{T_2} nC_v dT = nR \left[\frac{P_E T_1}{P_1} - T_2 \right]$$

$$\int_{T_1}^{T_2} nR \left(3.280 + \frac{0.593}{10^3} T + \frac{0.04}{10^5} T^{-2} - 1 \right) dT = nR \left[\frac{P_E T_1}{P_1} - T_2 \right]$$

$$\left[2.280T + \frac{0.593}{2(10^3)}T^2 - \frac{0.04}{10^5}T^{-1} \right]_{T_1}^{T_2} = \frac{P_E T_1}{P_1} - T_2$$

$$2.280(T_2 - T_1) + \frac{0.593}{2(10^3)}(T_2^2 - T_1^2) - \frac{0.04}{10^5}(T_2^{-1} - T_1^{-1}) = \frac{P_E T_1}{P_1} - T_2$$

$$2.280(T_2 - 500) + \frac{0.593}{2(10^3)}(T_2^2 - 500^2) - \frac{0.04}{10^5}(T_2^{-1} - 500^{-1}) - \frac{10^5 \cdot 500}{1.5 \times 10^6} + T_2 = 0$$

Plot w/ wolfram to get roots: $T_2 = \{-11.4128, 1\}$

Can solve numerically b.t.oof
alg.bra when we have a graph calc. b.
most reasonable

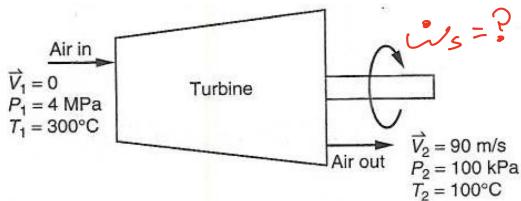
$$W = -P_E \left[\frac{nRT_2}{P_E} - \frac{nRT_1}{P_1} \right] = -10^5 \left[\frac{2 \cdot 8.314 \cdot 368.889}{10^5} - \frac{2 \cdot 8.314 \cdot 500}{1.5 \times 10^6} \right] = -5574.62 J$$

$$Q = 0$$

$$W = -5574.62 \text{ J}, T_2 = 368.889 \text{ K}$$

2.

Air enters a well-insulated turbine operating at steady state with negligible velocity at 4 MPa, 300 °C. The air expands to an exit pressure of 100 kPa. The exit velocity and temperature are 90 m/s and 100 °C, respectively. The diameter of the exit is 0.6 m. Determine the power developed by the turbine (in kW). You may assume air behaves like an ideal gas throughout the process.



$$\text{1st Law: } \frac{d}{dt} (\dot{m}(h + E_{int} + E_p)) = \dot{Q} + \dot{W}_s + \dot{m}_{in}(h_{in} + e_{kin,in} + e_{pot,in}) - \dot{m}_{out}(h_{out} + e_{kin,out} + e_{pot,out})$$

steady state adiabatic

• Open System

• Steady state? Given!

$$\rightarrow \dot{m}_{in} = \dot{m}_{out} = \dot{m}$$

• Adiabatic $\rightarrow \dot{Q} = 0$

• Ideal g.s behavior

• Exit diameter of 0.6 m

$$\dot{Q} = \dot{W}_s + \dot{m}(h_{in} - h_{out} - e_{kin,out})$$

$$\dot{m}(h_{out} - h_{in} + e_{kin,out}) = \dot{W}_s$$

$$\dot{m} \Delta h = \dot{W}_s$$

$$\dot{m} \left(\int_{T_1}^{T_2} C_p dT + \frac{1}{2} V_2^2 \right) = \dot{W}_s$$

$$\frac{C_p,m}{R} = A + BT + CT^2 + DT^{-2} + ET^3$$

$$A = 3.355$$

$$B = 0.575 \times 10^{-3}$$

$$D = -0.016 \times 10^{-5}$$

$$E = 0$$

$$\Rightarrow C_p,m = 6.314 \left(3.355 + 0.575 \times 10^{-3} T - 0.016 \times 10^{-5} T^{-2} \right)$$

$$\hat{C}_p = \frac{C_p,m}{M_w} = \frac{6.314 \left(3.355 + 0.575 \times 10^{-3} T - 0.016 \times 10^{-5} T^{-2} \right)}{28 \times 10^{-3} \text{ kg/mol}}$$

$$\frac{8.314}{28 \times 10^{-3}} \cdot \int_{300+273.15}^{100+273.15} \left(3.355 + 0.575 \times 10^{-3} T - 0.016 \times 10^{-5} T^{-2} \right) dT$$

$$= \frac{8.314}{28 \times 10^{-3}} \left[3.355 T + \frac{(0.575 \times 10^{-3})}{2} T^2 + \frac{0.016 \times 10^{-5}}{T} \right]_{273.15}^{373.15}$$

$$= \frac{8.314}{28 \times 10^{-3}} \left[3.355 (373.15) + \frac{(0.575 \times 10^{-3})}{2} (373.15)^2 + \frac{0.016 \times 10^{-5}}{373.15} \right] -$$

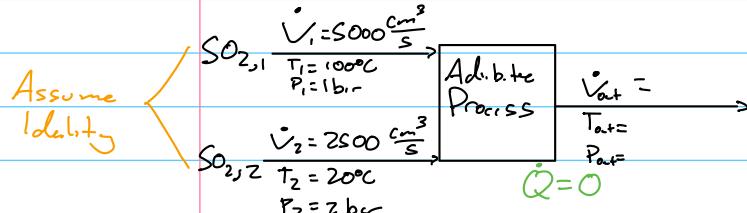
$$\left[3.355 (273.15) + \frac{(0.575 \times 10^{-3})}{2} (273.15)^2 + \frac{0.016 \times 10^{-5}}{273.15} \right] = -214951 \text{ J} \quad \left\{ \frac{\text{J}}{\text{kg}} \right\}$$

$$\dot{W}_s = 23.76 \frac{\text{kg}}{\text{s}} \left(-214951 \frac{\text{J}}{\text{kg}} + 4050 \frac{\text{J}}{\text{kg}} \right) = -5011007.76 \frac{\text{J}}{\text{s}}$$

$$\dot{W}_s = -5,011 \text{ kW} = -5,01 \text{ MW}$$

3. Sulfur dioxide (SO_2) with a volumetric flow rate of $5000 \text{ cm}^3/\text{s}$ at 1 bar and 100°C is mixed with a second SO_2 stream flowing at $2500 \text{ cm}^3/\text{s}$ at 2 bar and 20°C . The process occurs at steady state. You may assume ideal gas behavior and that the process is adiabatic. For SO_2 , take the heat capacity at constant pressure to be:

$$\frac{c_{p,m}}{R} = 3.627 + 5.324 \times 10^{-3}T \quad \text{for } T \text{ in K}$$



a) $PV = \dot{n}RT$, for $R = 8.314 \frac{\text{m}^3 \cdot \text{Pa}}{\text{mol} \cdot \text{K}}$

$$\dot{n}_1 = \frac{P_1 V_1}{R T_1} = \frac{(1 \times 10^5 \text{ Pa}) (0.005 \frac{\text{m}^3}{\text{s}})}{(8.314 \frac{\text{m}^3 \cdot \text{Pa}}{\text{mol} \cdot \text{K}})(100 + 273.15)} = 0.1612 \frac{\text{mol}}{\text{s}}$$

$$\dot{n}_2 = \frac{P_2 V_2}{R T_2} = \frac{(2 \times 10^5 \text{ Pa}) (0.0025 \frac{\text{m}^3}{\text{s}})}{(8.314 \frac{\text{m}^3 \cdot \text{Pa}}{\text{mol} \cdot \text{K}})(20 + 273.15)} = 0.2051 \frac{\text{mol}}{\text{s}}$$

• Open System

• Steady state? Given

$$\begin{cases} \dot{m}_{in} = \dot{m}_1 + \dot{m}_2 = \dot{m}_{out} = \dot{m} \\ \dot{n}_{in} = \dot{n}_1 + \dot{n}_2 = \dot{n}_{out} = \dot{n} \end{cases}$$

$$V_1 = 5000 \frac{\text{cm}^3}{\text{s}} \times \frac{1 \text{ m}^3}{1 \times 10^6 \text{ cm}^3} = 0.005 \frac{\text{m}^3}{\text{s}}$$

$$V_2 = 2500 \frac{\text{cm}^3}{\text{s}} \times \frac{1 \text{ m}^3}{1 \times 10^6 \text{ cm}^3} = 0.0025 \frac{\text{m}^3}{\text{s}}$$

$$P_1 = 1 \text{ bar} = 1 \times 10^5 \text{ Pa}$$

$$P_2 = 2 \text{ bar} = 2 \times 10^5 \text{ Pa}$$

$$\dot{n}_{out} = \dot{n}_1 + \dot{n}_2$$

$$= 0.1612 + 0.2051$$

$$= 0.3663$$

$$\boxed{\dot{n}_{out} = 0.3663 \frac{\text{mol}}{\text{s}}}$$

b) 1st Law:

$$\bullet \frac{d}{dt} (U + E_k + E_p) = 0 \quad \text{Steady-state given}$$

$$\rightarrow \dot{n}_1 h_{m,1} + \dot{n}_2 h_{m,2} = \dot{n}_{out} h_{m,out}$$

Choose $T = 273\text{K}$ as ref temp

$$\bullet \dot{W}_s = 0 \quad \text{no moving parts}$$

$$\Rightarrow \dot{n}_1 [\Delta h_{m,1}] + \dot{n}_2 [\Delta h_{m,2}] = \dot{n}_{out} [\Delta h_{m,out}]$$

$$\bullet \dot{e}_{p,m,in} = \dot{e}_{p,m,out} = 0 \quad \text{no position chgs}$$

$$\bullet \dot{e}_{k,m,in} = \dot{e}_{k,m,out} = 0 \quad \text{negligible}$$

$$T_{ref} = 273$$

$$\Delta h = \int_{T_{ref}}^T R (3.627 + 5.324 \times 10^{-3} T) dT$$

$$\rightarrow \Delta h_m = h(T) - h(T_{ref}) = R \left[3.627 (T - T_{ref}) + \frac{5.324}{2(10^3)} (T^2 - T_{ref}^2) \right]$$

$$\Delta h_{m,1} = 6.314 \frac{\text{J}}{\text{mol} \cdot \text{K}} \left[3.627 (373 - 273) + \frac{5.324}{2(10^3)} (373^2 - 273^2) \right] = 4445.206 \frac{\text{J}}{\text{mol}}$$

$$\Delta h_{m,2} = 6.314 \frac{\text{J}}{\text{mol} \cdot \text{K}} \left[3.627 (293 - 273) + \frac{5.324}{2(10^3)} (293^2 - 273^2) \right] = 853.630 \frac{\text{J}}{\text{mol}}$$

$$\Delta h_{m,out} = 6.314 \frac{\text{J}}{\text{mol} \cdot \text{K}} \left[3.627 (T_{out} - 273) + \frac{5.324}{2(10^3)} (T_{out}^2 - 273^2) \right] = [30.155 (T_{out} - 273) + 0.02213 (T_{out}^2 - 273^2)]$$

$$= 30.155 T_{out} - 8232.315 + 0.02213 T_{out}^2 - 16493.268$$

$$= 0.2213 T_{out}^2 + 30.155 T_{out} - 9881.642$$

$$\frac{\dot{m}_1 \cdot 4445.206 + \dot{m}_2 \cdot 853.630}{\dot{m}_{out}} = \dot{m}_{out} (0.02213 T_{out}^2 + 30.155 T_{out} - 4881.642)$$

$$\frac{\dot{m}_1 \cdot 4445.206 + \dot{m}_2 \cdot 853.630}{\dot{m}_{out}} = 0.1612 \cdot 4445.206 + 0.2051 \cdot 853.630$$

$$0.3663$$

$$= 2434.198$$

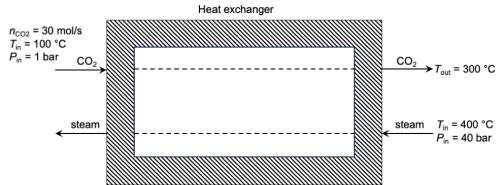
$$2434.198 = 0.02213 T_{out}^2 + 30.155 T_{out} - 4881.642$$

$$0.02213 T_{out}^2 + 30.155 T_{out} - 12315.8398 = 0$$

$$\overline{T}_{out} = 328.988 \text{ K} = 55.988^\circ\text{C}$$

$$T_{out} = 55.988^\circ\text{C}$$

4. You wish to heat a stream of CO₂ at pressure 1 bar, flowing at 30 mol/s, from 100 °C to 300 °C in a countercurrent heat exchanger. To do this task, you have been asked to use a stream of high-pressure steam available at 40 bar and 400 °C, as shown in the following figure. You may assume the pressure of each stream stays constant as it flows through the heat exchanger (i.e., neglect the pressure drop of the flowing streams). The entire system is well insulated, as shown. It is undesirable for the steam to condense in the heat exchanger tubes. What is the minimum volumetric flow rate of the steam inlet required (in m³/s) to keep it from condensing at the exit?



$$\text{1st Law: } \dot{Q}_{\text{CO}_2} = \dot{m}_{\text{CO}_2} (U + E_u + E_p) \quad (\text{CO}_2) \quad \text{std. state} \quad \text{normal p-ts}$$

$$\dot{Q}_{\text{CO}_2} = \dot{m}_{\text{CO}_2} \Delta h_m = \dot{m}_{\text{CO}_2} \int_{T_{in}}^{T_{out}} C_{p,m} dT$$

$$C_{p,m} = 8.314 \frac{\text{J}}{\text{mol}\cdot\text{K}} \left[5.457 + \frac{1.045}{10^3} T - \frac{1.157}{10^5} T^{-2} \right]$$

$$\int_{T_1}^{T_2} C_{p,m} dT = 8.314 \frac{\text{J}}{\text{mol}\cdot\text{K}} \left[5.457 T + \frac{1.045}{2 \cdot 10^3} T^2 + \frac{1.157}{10^5} T^{-1} \right]_{100+273.15}^{300+273.15}$$

$$= 8.314 \frac{\text{J}}{\text{mol}\cdot\text{K}} \left[5.457(300.15 - 273.15) + \frac{1.045}{2 \cdot 10^3} (300.15^2 - 273.15^2) + \frac{1.157}{10^5} (300.15^{-1} - 273.15^{-1}) \right]$$

$$= 8496.51 \frac{\text{J}}{\text{mol}}$$

$$\dot{Q}_{\text{CO}_2} = 30 \frac{\text{mol}}{\text{s}} \cdot 8496.51 \frac{\text{J}}{\text{mol}} = 264895 \text{ J/s} = 264.9 \text{ kW}$$

Steam

$$T_{in} = 400^\circ\text{C}$$

$$P_{in} = 40 \text{ bar} = 4 \text{ MPa}$$

$$\begin{aligned} \dot{h}_{in} &= 3213.5 \frac{\text{kJ}}{\text{kg}}, \dot{v}_{in} = 0.07311 \frac{\text{m}^3}{\text{kg}} \\ \dot{h}_{out} &= 2801.4 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

$$\text{1st Law: } -\dot{Q}_{\text{steam}} = \dot{m} (\dot{h}_{out, \text{steam}} - \dot{h}_{in, \text{steam}})$$

Same as CO₂, but neglect and use steam tables

$$\Rightarrow \dot{Q}_{\text{steam}} = \dot{m} (\dot{h}_{in, \text{steam}} - \dot{h}_{out, \text{steam}})$$

$$264.9 \frac{\text{kJ}}{\text{s}} = \dot{m} (3213.5 - 2801.4) \frac{\text{kJ}}{\text{kg}}$$

$$\dot{m} = 0.6549 \frac{\text{kg}}{\text{s}}$$

$$\dot{V} = \dot{m} \cdot \dot{v}_{in} = 0.6549 \frac{\text{kg}}{\text{s}} \cdot 0.07311 \frac{\text{m}^3}{\text{kg}} = 0.04808 \frac{\text{m}^3}{\text{s}}$$

• Open system

• Steady state, no accumulation

$$\dot{m}_{\text{CO}_2, in} = \dot{m}_{\text{CO}_2, out} = \dot{m}_{\text{CO}_2}$$

$$\dot{m}_{\text{steam}, in} = \dot{m}_{\text{steam}, out} = \dot{m}_{\text{CO}_2}$$

• Well-insulated outer system

$$\rightarrow \text{Heat lost from Steam} = \text{Heat gain for CO}_2$$

• Prevent Steam from condensing!

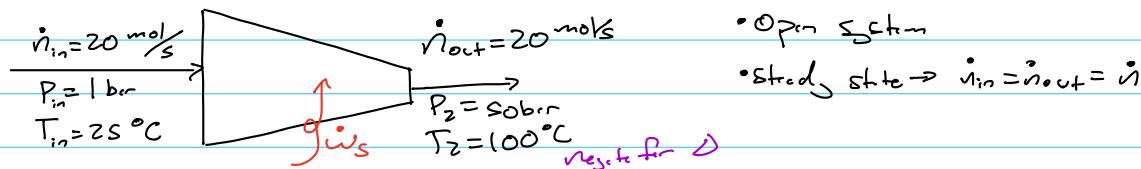
• Iso-baric stream lines

$$\dot{Q}_{\text{CO}_2} = \dot{m}_{\text{CO}_2} (\Delta h_m + \Delta E_{k,m} + \Delta E_{p,m})$$

negative no horiz. changes

- S. At steady state, an ideal gas enters a compressor with a molar flow rate of 20 mol/s. The inlet pressure is 1 bar and the inlet temperature is 25 °C. The gas exits at 50 bar and 100 °C. The ideal gas molar constant-pressure heat capacity is given by:

$$c_{p,m}/R = 3.60 + 0.500 \times 10^{-3}T \quad \text{for } T \text{ in K}$$



$$\text{a) 1st Law: } \frac{d}{dt}(U + E_k + E_p) = \dot{Q} + \dot{W}_s - \dot{m} (\Delta h_m + \Delta E_{k,m} + \Delta p_m)$$

Steady State adiabatic neg. for terms

$$\dot{W}_s = \dot{m} \Delta h_m = \dot{m} \int_{T_{in}}^{T_{out}} C_{p,m} dT = \dot{m} \int_{25+273}^{100+273} C_{p,m} dT$$

$$\int_{25+273}^{100+273} C_{p,m} dT = \int_{298}^{373} R \left(3.60 + \frac{0.500}{10^3} T \right) dT$$

$$= 8.314 \frac{\text{J}}{\text{mol} \cdot \text{K}} \left[3.60T + \frac{0.500}{2(10^3)} T^2 \right]_{298}^{373}$$

$$= 8.314 \frac{\text{J}}{\text{mol} \cdot \text{K}} \left[3.60(373 - 298) + \frac{0.500}{2(10^3)} (373^2 - 298^2) \right] = 2349.38$$

$$\Rightarrow \dot{W}_s = 20 \frac{\text{mol/s}}{\text{s}} \cdot 2349.38 \frac{\text{J}}{\text{mol}} = 46987.61 \frac{\text{J}}{\text{s}} \quad \dot{W}_s = 46.99 \text{ kW}$$

$$\text{b) 1st Law: } \dot{Q} + \dot{W}_s - \dot{m} \int_{T_{in}}^{T_{out}} C_{p,m} dT = 0$$

$$\dot{W}_s = 46.98 \text{ kW} - \dot{Q}$$

If the compressor loses heat to the surroundings, the work must be greater to maintain the same conditions as $\dot{Q} < 0$. If the compressor gains heat, power would be less ($\dot{Q} > 0$). We have no indication of if \dot{Q} is positive or negative as ΔT does not tell us how \dot{Q} behaves. We can say, with 100% certainty, that a non-adiabatic compressor will not have an equivalent power rating to an adiabatic one.

6. Answer the following reflection questions (5 points):

- a. What about the way this class is taught is helping your learning?
- b. What about the way this class is taught is inhibiting your learning?

A) In comparison to last week's homework and my response to that reflection question, I think this week was much better. The examples from class were more applicable to the homework, being more challenging and in depth. I think the heavy emphasis on examples is helping my learning a lot, as I have a much harder time understanding concepts when they are spoken to me compared to when I actually see how they are applied. I again want to emphasize how much the in class discussions help my learning, sparking ideas based on my classmates that I wouldn't have thought of on my own.

B) At this point, there is nothing really inhibiting my learning. One thing I would suggest is going through the property tables at least once when theres something new that we have to get. I found myself in this homework trying to find out what Cp value was in the Koretsky book, so maybe showing one actual derivation of the values would be more helpful than just writing some equation and then its answer. Other than that, there is nothing really inhibiting my learning, and this suggestion of mine is not very serious and does not inhibit my learning, but would help if it was implemented.