Name Solution

Wednesday, February 28th

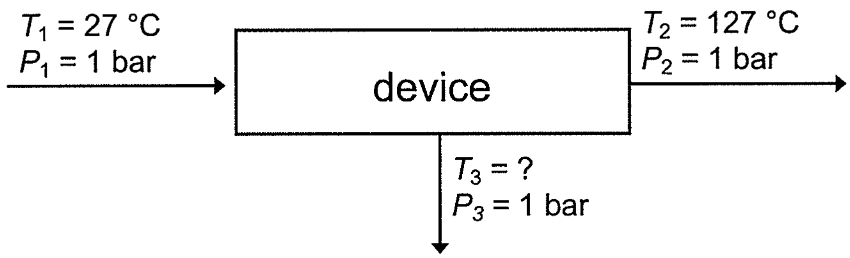
ECHE 363 – Thermodynamics of Chemical Systems
Midterm #1

Rules:

- 75 minutes total time. Once time is up, put aside answer sheets.
- Be sure to show all work to obtain maximum credit. This includes showing and simplifying any mass/mole, 1st Law, and 2nd Law balances that are needed to solve each problem.
- Closed book and no notes.
- Write your name on every page.
- Please only write on the front side of each page. Ask for additional paper if necessary.

Name ____

1. (25 points) Nitrogen gas at 27 °C in "stream 1" flows into a well-insulated device operating at steady state. There is no shaft work. Two-thirds of the nitrogen, by moles, exits at 127 °C and 1 bar in "stream 2". The remainder of the nitrogen exit through "stream 3" at an unknown temperature and 1 bar. Find the temperature of the nitrogen in the "stream 3" outlet. Assume ideal gas behavior, where nitrogen has $c_{p,m} = 29.1$ J/mol-K.



$$\text{mass balance: } \underbrace{\frac{dn}{dt}}_{\text{system}} = \overbrace{0}^{\text{steady state}} = \dot{n}_1 - \dot{n}_2 - \dot{n}_3$$

$$\dot{n}_1 = \dot{n}_2 + \dot{n}_3$$

$$\dot{n}_2 = \frac{2}{3}\dot{n}_1 \quad , \quad \dot{n}_3 = \frac{1}{3}\dot{n}_1$$

$$1^{st} \text{ Law: } \frac{d}{dt}\left(U + E_K + E_p\right) = \overbrace{0}^{\text{steady state}} = \dot{n}_1 h_{m,1} - \dot{n}_2 h_{m,2} - \dot{n}_3 h_{m,3}$$

$$+ \underbrace{\cancel{\dot{W}_s}}_{\substack{0 \\ \text{no shaft work}}} + \underbrace{\cancel{\dot{Q}}}_{0,\ \text{insulated}}$$

$$0 = h_{m,1} - \frac{2}{3} h_{m,2} - \frac{1}{3} h_{m,3}$$

$$0 = 2\underbrace{\left(h_{m,1} - h_{m,2}\right)}_{= \int_{T_2}^{T_1} C_{p,m}\, dT} + \underbrace{\left(h_{m,1} - h_{m,3}\right)}_{= \int_{T_3}^{T_1} C_{p,m}\, dT}$$

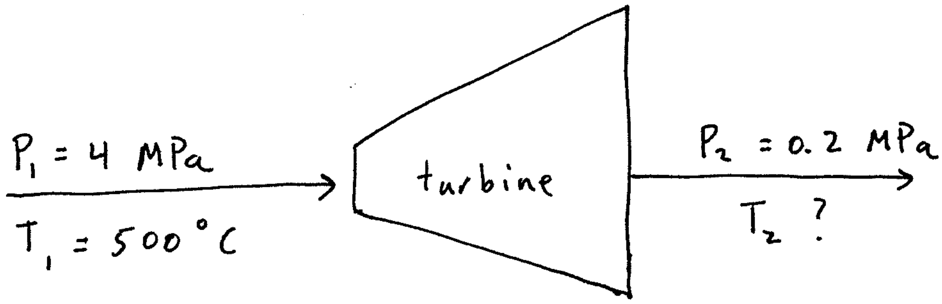
constant

$$0 = 2\,\cancel{c_{p/m}}\,(T_1 - T_2) + \cancel{c_{p/m}}\,(T_1 - T_3)$$

$$\boxed{T_3 = 3T_1 - 2T_2 = 100\ K = -173^\circ C}$$

Name ____

2. (40 points) Steam is fed to an adiabatic turbine at 4 MPa and 500 °C. It exits at 0.2 MPa. Assuming that the process occurs at steady state, what is the maximum possible work (per kg of steam) that can be extracted from the turbine?
   Hint: remember to determine the phase of both the input and output water streams.



mass balance: $\frac{dn}{dt} = 0 = \dot{n}_1 - \dot{n}_2 \longleftrightarrow \dot{n}_1 = \dot{n}_2 = \dot{n}$

steady state

$\dot{m}_1 = \dot{m}_2 = \dot{m}$

1^{st} Law: $\frac{d}{dt}(U + E_K + E_P) = 0 = \underbrace{\dot{m}_1 \hat{h}_1 - \dot{m}_2 \hat{h}_2}_{\text{ignore KE, PE}} + \dot{W}_s + \cancel{\dot{Q}}$

0 adiabatic

use MB $\longrightarrow$ $0 = \dot{m}(\hat{h}_1 - \hat{h}_2) + \dot{W}_s$

$\dot{W}_s = \dot{m}(\hat{h}_2 - \hat{h}_1)$

unknown

$\hat{h}_1$ can be looked up

Want $\dot{W}_{s,max}$ (reversible case, $\frac{dS_{univ}}{dt} = 0$)

2nd Law: $\frac{dS_{univ}}{dt} \overset{rev}{=} 0 = \underbrace{\cancel{\frac{dS}{dt}}}_{0,\ \text{steady state}} + \dot{m}_2 \hat{S}_2 - \dot{m}_1 \hat{S}_1 - \underbrace{\cancel{\frac{\dot{Q}}{T_{surr}}}}_{0,\ \text{adiabatic}}$

use MB $\longrightarrow$ $0 = \dot{m}(\hat{S}_2 - \hat{S}_1)$

$$\hat{S}_2 = \hat{S}_1$$

"1" is superheated, @ $T_1 = 500°C$
$P_1 = 4$ MPa

$\rightarrow \hat{S}_1 = 7.09$ kJ/kg-K
$\hat{h}_1 = 3445.2$ kJ/kg

Find where $\hat{S}_2 = \hat{S}_1 = 7.09$ kJ/kg-K
for $P_2 = 0.2$ MPa

@ $P_2 = 0.2$ MPa, $T^{sat} = 120.23$

$\hat{S}_L^{sat} = 1.53$ kJ/kg-K, $\hat{h}_L^{sat} = 504.68$ kJ/kg
$\hat{S}_V^{sat} = 7.1271$ kJ/kg-K, $\hat{h}_V^{sat} = 2706.6$ kJ/kg

$$\hat{S}_L^{sat} < \hat{S}_2 < \hat{S}_V^{sat}$$

$\therefore$ "2" is a saturated liq/vap mixture!

$$\hat{S}_2 = x \hat{S}_V^{sat} + (1-x)\hat{S}_L^{sat}$$

$$x = \frac{\hat{S}_2 - \hat{S}_L^{sat}}{\hat{S}_V^{sat} - \hat{S}_L^{sat}} = \frac{7.09 - 1.53}{7.1271 - 1.53} = 0.993$$

Back to 1st Law to find $\dot{W}_{s,max}$:

$$\dot{W}_s = \dot{m}(\hat{h}_2 - \hat{h}_1)$$

$$\frac{\dot{W}_s}{\dot{m}} = \hat{W}_s = \hat{h}_2 - \hat{h}_1$$

$$\uparrow$$

$$\hat{h}_2 = x \hat{h}_V^{sat} + (1-x)\hat{h}_L^{sat}$$

$$\hat{h}_2 = 2692.00 \text{ kJ/kg}$$

$$\hat{W}_s = 2692 - 3445.2$$

$$\boxed{\hat{W}_s = -753 \text{ kJ/kg}}$$

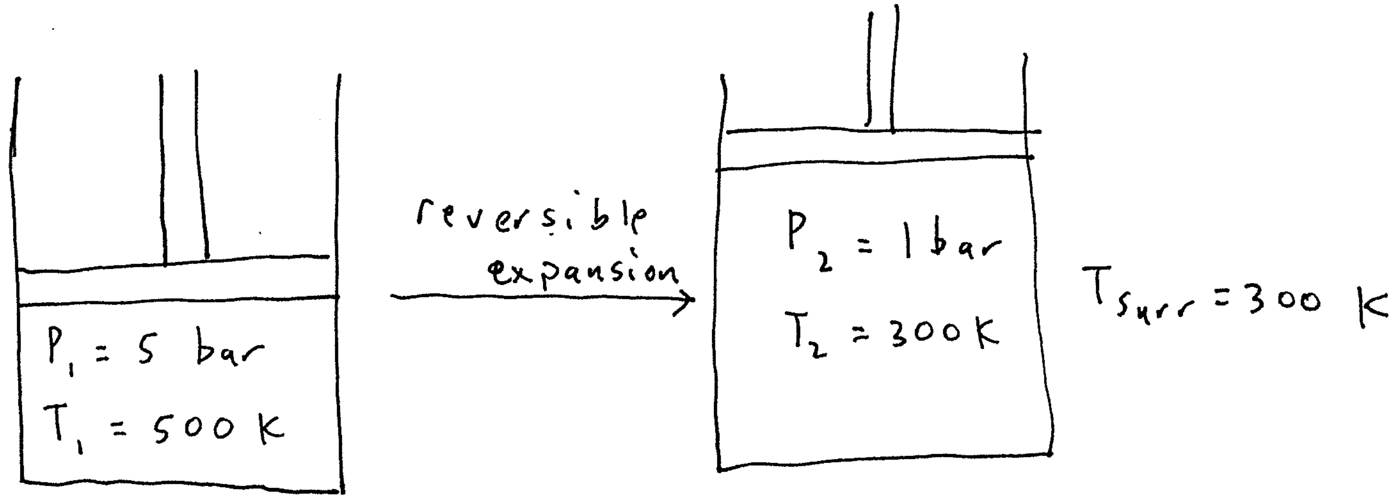
work extracted from turbine per mass of steam.

Name ______________________________

3. (35 points) One mole of an ideal gas contained within a piston–cylinder assembly has an initial pressure of $P_1 = 5$ bar and initial temperature of $T_1 = 500$ K. It undergoes a reversible expansion until it reaches a final pressure of $P_2 = 1$ bar and final temperature of $T_2 = 300$ K. The expansion is not adiabatic and thus heat transfer cannot be neglected. The surroundings have a temperature of $T_{surr} = 300$ K. The fluid has a constant-pressure heat capacity of:

   $c_{p,m}/R = A + BT$, where $A = 3.5$ and $B = 0.02\ \text{K}^{-1}$

   For the process described above, calculate: W, Q, ΔS_{sys}, ΔS_{surr}, ΔS_{univ}.



closed system, $n_1 = n_2 = n$

1st Law: $\Delta U = Q + W$ ← $= -\int_{V_1}^{V_2} P_E \, dV$

$\Delta U = n\int_{T_1}^{T_2} C_{v,m} \, dT$

Q: can't neglect Q!

$$\int_{T_1}^{T_2} C_{v,m} \, dT = Q - \int_{V_1}^{V_2} P_E \, dV$$

reversible, $P_E = P$

$$\int_{T_1}^{T_2} C_{v,m} \, dT = Q - \int_{V_1}^{V_2} P \, dV$$

P, V, T all changing

cannot evaluate this integral!

use 2nd Law:

$$\Delta S_{univ} = \Delta S + \Delta S_{surr}$$

$$\boxed{\Delta S_{univ} = 0}$$ reversible

$$\Delta S = n \Delta S_m = n \int_{\text{"1"}}^{\text{"2"}} dS_m$$

closed sys

$$\Delta S = n \left[\int_{T_1}^{T_2} \frac{C_{p,m}}{T} dT - \int_{P_1}^{P_2} \frac{R}{P} dP \right]$$

$$\Delta S = nR \left[\int_{T_1}^{T_2} \left(\frac{A}{T} + B \right) dT - \int_{P_1}^{P_2} \frac{1}{P} dP \right]$$

$$\Delta S = nR \left[A \ln\left(\frac{T_2}{T_1}\right) - B(T_2 - T_1) - R \ln\left(\frac{P_2}{P_1}\right) \right]$$

$$\boxed{\Delta S = -34.74 \text{ J/K}}$$

$$\boxed{\Delta S_{surr} = -\Delta S = 34.74 \text{ J/K}}$$

$$\Delta S_{surr} = -\frac{Q}{T_{surr}}$$

$$Q = -T_{surr}\,\Delta S_{surr}$$

$$\boxed{Q = -10421 \text{ J}}$$

1st Law: $W = \Delta U - Q$

$$W = n\int_{T_1}^{T_2} C_{v,m}\,dT - Q$$

$$W = nR\int_{T_1}^{T_2}\left(A + BT - 1\right)dT - Q$$

$$W = nR\left[(A-1)(T_2 - T_1) + \frac{B}{2}\left(T_2^2 - T_1^2\right)\right] - Q$$

$$\boxed{W = -7037.5 \text{ J}}$$