Name Solutions

Wednesday, February 22nd

ECHE 363 – Thermodynamics of Chemical Systems
Midterm #1

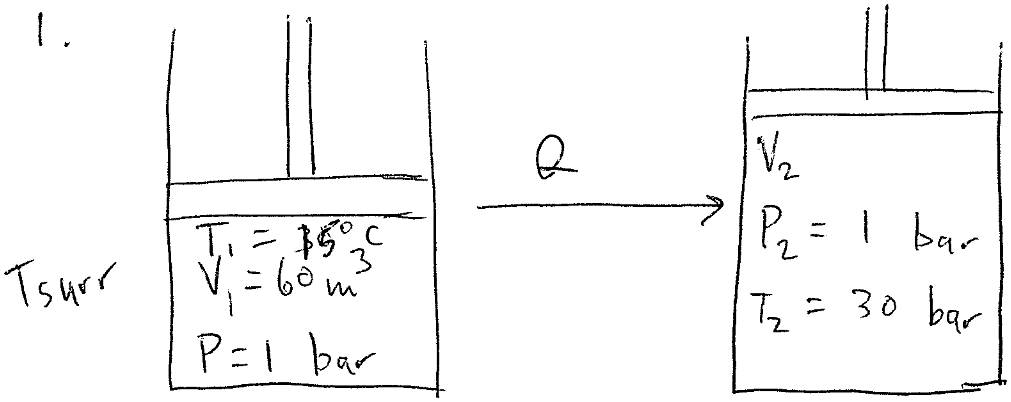
Rules:

- 75 minutes total time. Once time is up, put aside answer sheets.
- Be sure to show all work to obtain maximum credit.
- Closed book and no notes.
- Write your name on every page.
- Please only write on the front side of each page. Ask for additional paper if necessary.

Name ______________________________

1. (25 points) An ideal gas, with a temperature-independent $c_{p,m} = 3.5R$ at 15 °C has an initial volume of 60 m^3. It is heated at constant pressure ($P = 1$ bar) to 30 °C by heat transfer from a reservoir that is also at a pressure of 1 bar. Calculate the total ΔS, ΔS_{surr}, and ΔS_{univ} if:
   a. The surroundings are at 50 °C.
   b. The surroundings are at 20 °C.
   c. Which of the processes in (a) or (b) is thermodynamically possible?

1.



1st Law:

$$\Delta U = Q + W \qquad P_E = P = 1 \text{ bar}$$

$$\Delta U = Q - \int P_E \, dV$$

$$\Delta U = Q - P \int_{V_1}^{V_2} dV$$

$$\Delta U + P(V_2 - V_1) = Q$$

$$\rightarrow Q = \underbrace{(U_2 - U_1) + P(V_2 - V_1)}_{= H_2 - H_1}$$

$$Q = n \Delta H_m \underset{\substack{\uparrow \\ \text{ideal} \\ \text{gas}}}{=} n \int_{T_1}^{T_2} C_{p,m} \, dT$$

(Q needed for ΔS_{surr})

$$Q = \underset{\substack{\uparrow \\ = \frac{P_1 V_1}{R T_1}}}{n} C_{p,m} (T_2 - T_1)$$

$$Q = \frac{P_1 V_1}{RT_1} C_{p,m} (T_2 - T_1)$$

$$Q = \frac{(1 \times 10^5 \text{ Pa})(60 \text{ m}^3)}{(\cancel{8.314} \text{ J/mol-K})(15+273 \text{ K})} (3.5)(\cancel{8.314})(30-15)$$

$$Q = 1093700 \text{ J} = 1093.7 \text{ kJ}$$

a) $T_{surr} = 50°C = 323$ K

$$\Delta S = n\left[\int_{T_1}^{T_2} \frac{C_{p,m}}{T} dT - \underbrace{\int_{P_1}^{P_2} \frac{R}{P} dP}_{=0 \ (P \text{ const})}\right]$$

$$\Delta S = \left(\frac{P_1 V_1}{RT_1}\right) C_{p,m} \ln\left(\frac{T_2}{T_1}\right)$$

$$= \frac{(10^5)(60)}{(288)} (3.5) \ln\left(\frac{30+273}{15+273}\right)$$

$$\boxed{\Delta S = 3702.15 \text{ J/K}}$$

$$\boxed{\Delta S_{surr} = -\frac{Q}{T_{surr}} = -3386.1 \text{ J/K}}$$

$$\boxed{\Delta S_{univ} = \Delta S + \Delta S_{surr} = 316.1 \text{ J/K}}$$

b) $\boxed{\Delta S = 3702.15\ J/K}$ (same as a, state function!)

$$\boxed{\Delta S_{surr} = -\frac{Q}{\underset{\underset{20^\circ C}{\uparrow}}{T_{surr}}} = -3732.8\ J/K}$$

$$\boxed{\Delta S_{univ} = \Delta S + \Delta S_{surr} = -30.6\ J/K}$$

c) a is possible, $\Delta S_{univ} > 0$

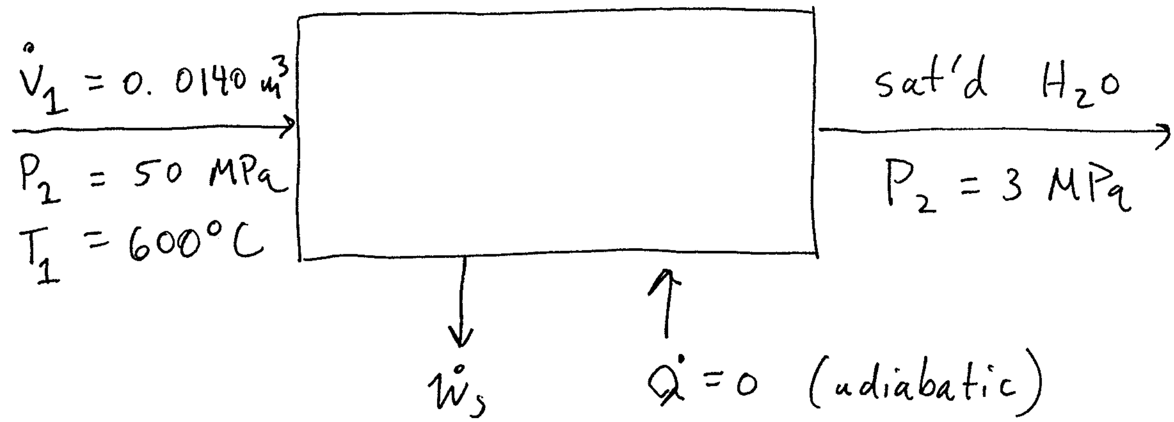
b is impossible, $\Delta S_{univ} < 0$

$\rightarrow$ b is inconsistent with second law!

Name ____

2. (40 points) Steam at 50.0 MPa, 600 °C and a volumetric flow rate of 0.0140 m^3/s is fed to an adiabatic turbine operating at steady state. The turbine exit stream is saturated water vapor at 3.00 MPa. Calculate the power (kW) produced by this turbine and its efficiency. You may assume that the exit stream for a process with 100% efficiency is also at 3.00 MPa.

2.



MB: (steady-state)

$$\frac{dm}{dt} = 0 = \underset{\substack{\uparrow \\ \text{in}}}{\dot{m}_1} - \underset{\substack{\uparrow \\ \text{out}}}{\dot{m}_2} \longrightarrow \dot{m}_1 = \dot{m}_2 = \dot{m}$$

1st Law:

$$\underset{\text{S.S.}}{0} = \dot{m}_1 \hat{h}_1 - \dot{m}_2 \hat{h}_2 + \underset{0}{\cancel{\dot{Q}}} + \dot{W}_s \leftarrow \text{power!}$$

$$\dot{W}_s = \dot{m}\left(\hat{h}_2 - \hat{h}_1\right)$$

$$\dot{W}_s = \left(\frac{\dot{V}_1}{\hat{V}_1}\right)\left(\hat{h}_2 - \hat{h}_1\right)$$

↑ given vol. flow rate in!

steam tables

in (1):
600°C, 50 MPa
(superheated)

$\hat{V}_1 = 0.0061123 \ m^3/kg$

$\hat{h}_1 = 3247.6 \ kJ/kg$

out (2):
3 MPa
(saturated vapor) ← $x = 1$

$\hat{h}_2 = 2804.1 \ kJ/kg$

$$\dot{W}_s = \left(\frac{\dot{V}_1}{\hat{V}_1}\right)\left(\hat{h}_2 - \hat{h}_1\right)$$

$$= \left(\frac{0.0140}{0.0061123}\right)\left(2804.1 - 3247.6\right)$$

$$\boxed{\dot{W}_s = -1015.8 \ kJ/s = -1015.8 \ kW}$$ ← real power

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Now, compare $\dot{W}_{s,real}$ to $\underbrace{\text{max (100\% eff) power}}_{\text{reversible case}}$

$\Downarrow$

$\Delta S_{univ} = 0$

Reversible case:

$$\frac{dS_{univ}}{dt} = \underset{\substack{\uparrow \\ \text{rev!}}}{0} = \underset{\substack{\downarrow \\ 0 \\ \text{S.S.}}}{\cancel{\frac{dS}{dt}}} + \dot{m}\underset{\substack{\uparrow \\ \text{different than} \\ \text{real out} \\ \text{conditions}}}{(\hat{S}_2 - \hat{S}_1)} - \cancel{\frac{\dot{Q}}{T_{surr}}}^{=0 \text{ adiabatic}}$$

$\rightarrow \hat{S}_2 = \hat{S}_1$ for reversible process

$\hat{S}_1 = \hat{S}(600°C, 50 \text{ MPa}) = 5.8177 \text{ kJ/kg-K}$

$\hat{S}_2 = \hat{S}_1$ at $P = 3$ MPa

$$\hat{S}_2^L(P^{sat} = 3 \text{ MPa}) < \hat{S}_2 < \hat{S}_2^V(P^{sat} = 3 \text{ MPa})$$

$\rightarrow$ out is V/L mixture

$\rightarrow$ need quality, x

$$x = \frac{\hat{S}_2 - \hat{S}_2^L}{\hat{S}_2^V - \hat{S}_2^L} = \frac{5.8177 - 2.6456}{6.1869 - 2.6456}$$

$$\underline{\underline{x = 0.896}}$$

Back to 1st law to calc $W_{s,rev}$:

$$\hat{h}_2 = x\,\hat{h}_2^V + (1-x)\,\hat{h}_2^L$$

rev case!

$$= (0.896)(2804.1) + (1-.896)(1008.41)$$

$$\hat{h}_2 = 2617.35 \text{ kJ/kg}$$

Now, calc $\dot{W}_{s,rev}$

$$\dot{W}_{s,rev} = \dot{m}\left(\hat{h}_2^{rev} - \hat{h}_1\right) \leftarrow 1^{st} \text{ law}$$

$$= \left(\frac{0.0140}{0.0061123}\right)(2617.35 - 3247.6)$$

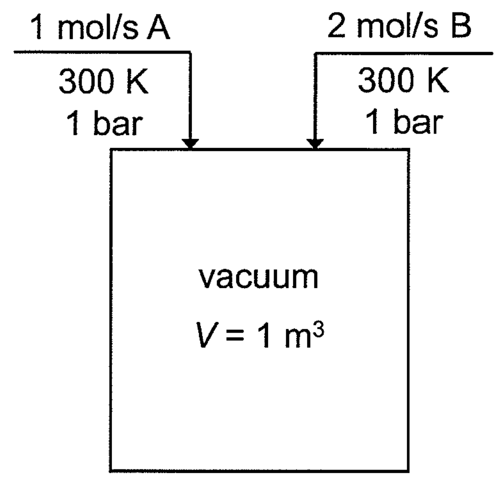
$$\boxed{\dot{W}_{s,rev} = -1443.6 \text{ kJ/s (kW)}}$$

efficiency:

$$\boxed{\eta = \frac{|\dot{W}_{s,real}|}{|\dot{W}_{s,rev}|} = \frac{1015.8}{1443.6} = 0.703 \qquad \sim 70\% \text{ efficiency}}$$

Name ____

3. (35 points) A rigid, 1 m^3 tank is initially evacuated. Two streams are then allowed to flow into the tank. One stream contains a gas (A) with a molar flow rate of 1 mol/s at $T_{A,in} = 300$ K and $P_{A,in} = 1$ bar. The other stream contains a different gas (B) with a molar flow rate of 2 mol/s at $T_{B,in} = 300$ K and $P_{A,in} = 1$ bar. Gas A has a constant $c^{A}_{p,m}$ of $5/2R$, and gas B has a different constant $c^{B}_{p,m}$ of $7/2R$. The two gases are flowed into the tank until it reaches a final pressure of 20 bar, whereupon the valve is closed. The temperature of the surroundings is 300 K. You may treat both "A" and "B" as an ideal gas throughout the entire process.



   a. Find the final temperature immediately after the valve is closed and the tank stops filling.
   b. Find the heat transferred (Q) and the final pressure after the tank is in storage for a long period of time, and comes to thermal equilibrium with the surroundings.

Mass balance: $\frac{dn}{dt} = \dot{n}_A + \dot{n}_B$

a)

$$\dot{n}_B = 2\dot{n}_A \;,\; dn_B = 2dn_A$$

1st Law: $\frac{dU}{dt} = \dot{n}_A h_{m,A,in} + \dot{n}_B h_{m,B,in} + \underbrace{\cancel{\dot{Q}}}_{\substack{0 \\ \text{adiabatic} \\ \text{(initially)}}} + \underbrace{\cancel{\dot{W}_s}}_{\substack{0 \\ \text{no work}}}$

OR: $dU = dn_A h_{m,A,in} + dn_B h_{m,B,in}$

$$\int_{\underbrace{init}_{=0}}^{fin} dU = \int_{\underbrace{init}_{=0}}^{fin} dn_A \left(h_{m,A,in} + 2 h_{m,B,in} \right)$$

constants!

$$U_f = n_{A,f} \left(h_{m,A,in} + 2h_{m,B,in} \right)$$

$$n_{A,f} U_{m,A,f} + n_{B,f} U_{m,B,f} = n_{A,f} \left(h_{m,A,in} + 2h_{m,B,in} \right)$$

$$\cancel{n_{A,f}} \left(U_{m,A,f} + 2U_{m,B,f} \right) = \cancel{n_{A,f}} \left(h_{m,A,in} + 2h_{m,B,in} \right)$$

$$0 = \left(h_{m,A,in} - \underbrace{U_{m,A,f}}_{= h_{m,A,f} - \bar{R}T_f \text{ (ideal gas)}} \right) + 2\left(h_{m,B,in} - \underbrace{U_{m,B,f}}_{= h_{m,B,f} - \bar{R}T_f} \right)$$

$$0 = \underbrace{\left(h_{m,A,in} - h_{m,A,f}\right)}_{\int_{T_f}^{T_{in}} C_{p,m}^{A}\,dT = C_{p,m}^{A}\left(T_{in}-T_f\right)} + 2\underbrace{\left(h_{m,B,in} - h_{m,B,f}\right)}_{\int_{T_f}^{T_{in}} C_{p,m}^{B}\,dT = C_{p,m}^{B}\left(T_{in}-T_f\right)} + 3RT_f$$

$$0 = C_{p,m}^{A}\left(T_{in} - T_f\right) + 2C_{p,m}^{B}\left(T_{in} - T_f\right) + 3RT_f$$

$$\left(C_{p,m}^{A} + 2C_{p,m}^{B} - 3R\right)T_f = \left(C_{p,m}^{A} + 2C_{p,m}^{B}\right)T_{in}$$

$$T_f = \left(\frac{C_{p,m}^{A} + 2C_{p,m}^{B}}{C_{p,m}^{A} + 2C_{p,m}^{B} - 3R}\right)T_{in}$$

$$T_f = \left[\frac{\left(5/2 + 14/2\right)\cancel{R}}{\left(5/2 + 14/2 - 3\right)\cancel{R}}\right]T_{in}$$

$$\boxed{T_f = \left(\frac{19}{13}\right)T_{in} = 438\ \text{K}}$$

b) Long time $(Q \neq 0)$,

let $T_1 = 438\text{ K}$ (T_f from part a)

$T_2 = T_{surr} = 300\text{ K}$

1^{st} law (closed): $\Delta U = Q + \underbrace{\cancel{W}}_{0 \text{ rigid}}$

$$\rightarrow Q = \Delta U = \Delta U_A + \Delta U_B$$

$$= n_A \int_{T_1}^{T_2} C_{v,m}^{A} + n_B \int_{T_1}^{T_2} C_{v,m}^{B}\, dT$$

A is $\frac{1}{3} n_{TOT}$ B is $\frac{2}{3} n_{TOT}$

$$= n_{TOT}\left[\left(\frac{1}{3}\right)\left(\frac{3}{2}R\right)(T_2 - T_1) + \left(\frac{2}{3}\right)\left(\frac{5}{2}R\right)(T_2 - T_1)\right]$$

ideal gas $\rightarrow C_{v,m}^{A} = C_{p,m}^{A} - R$ $\quad C_{v,m}^{B}$

(R's cancel) $\rightarrow$ $$= \frac{P_1 V_1}{T_1}\left[\left(\frac{1}{\cancel{3}}\right)\left(\frac{\cancel{3}}{2}\right)(T_2 - T_1) + \left(\frac{\cancel{2}}{3}\right)\left(\frac{5}{\cancel{2}}\right)(T_2 - T_1)\right]$$

$$= \frac{(20 \times 10^5\text{ Pa})(1\text{ m}^3)}{438\text{ K}}\left[\frac{1}{2}(300 - 438) + \frac{5}{3}(300 - 438)\right]$$

$$\boxed{\begin{aligned} Q &= -1365297\text{ J} \\ &= -1365\text{ kJ} \end{aligned}}$$

Find final pressure (P_2):

$n_1 = n_2$ (mole balance, closed sys)

$$\frac{P_1 \cancel{V_1}}{\cancel{R} T_1} = \frac{P_2 \cancel{V_2}}{\cancel{R} T_2} \leftarrow V_1 = V_2 \text{ (rigid)}$$

$$P_2 = \left(\frac{T_2}{T_1}\right) P_1 = \left(\frac{300}{438}\right)(20 \text{ bar})$$

$$\boxed{P_2 = 13.7 \text{ bar}}$$