

**Sample Exam #2**

**November, 2023**

**Duration: 55 minutes; 10:30 – 11:25 am EDT**

Any resources (book, notes, web, etc.) are allowed, but you are not allowed to talk with anyone during the exam. With submission of your answers, you implicitly affirm that all work is your own, without consultation of peers or others. Be sure to cite sources of information.

Submit your answers via Canvas to your recitation instructor by 11:25 am EDT

1a. From your book (15 points):

**15.10** For a continuous and oriented fiber-reinforced composite, the moduli of elasticity in the longitudinal and transverse directions are 33.1 and 3.66 GPa ( $4.8 \times 10^6$  and  $5.3 \times 10^5$  psi), respectively. If the volume fraction of fibers is 0.30, determine the moduli of elasticity of fiber and matrix phases.

**Longitudinal modulus** =  $E_{cl} = E_m(1 - V_f) + E_f V_f$

$$33.1 \text{ GPa} = E_m(1 - 0.30) + E_f(0.30)$$

**Transverse Modulus** =  $E_{ct} = \frac{E_m E_f}{(1 - V_f)E_f + V_f E_m}$

$$3.66 \text{ GPa} = \frac{E_m E_f}{(1 - 0.30)E_f + 0.30E_m}$$

**Solve simultaneous equations to give**

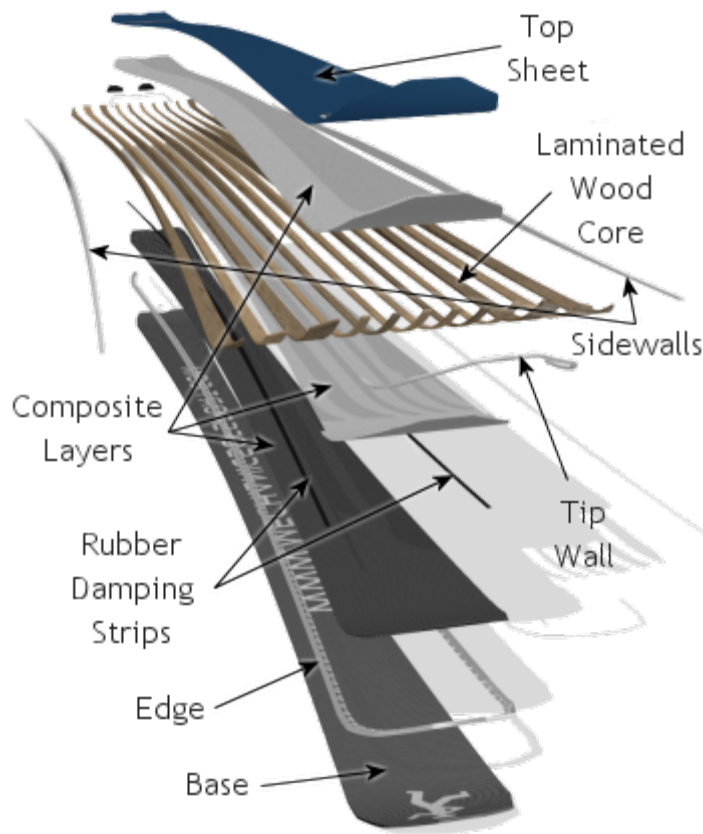
$$E_m = 2.6 \text{ GPa } (3.77 \times 10^5 \text{ psi})$$

$$E_f = 104 \text{ GPa } (15 \times 10^6 \text{ psi})$$

b. List three types of sports equipment that are composite materials. Select one example and cite the materials used for the matrix and dispersed phases and why these were chosen by engineers for the specific sporting good application. (10 points)

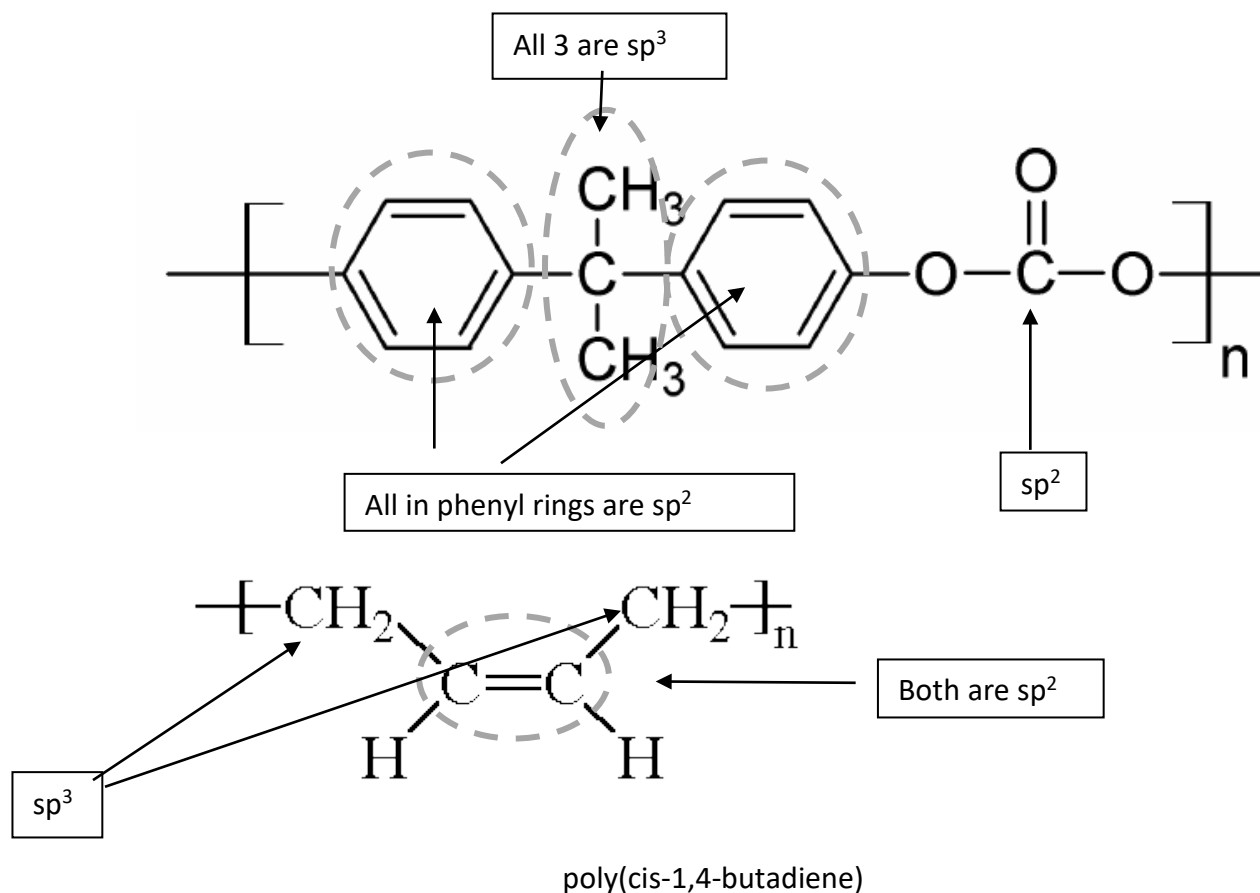
**Examples: tennis racket; golf club shaft; skis. Composites are attractive for each due to a combination of properties including light weight, stiffness, and strength.**

**Skis:** Composite layer typically fiberglass and resin, and offers a desirable combination of light weight, stiffness, and strength. Such a combination offers good performance vs cost for average skiers; for more advanced skiers, aramid or carbon fiber may be used.



[http://www.mechanicsofsport.com/skiing/equipment/skis/ski\\_construction.html](http://www.mechanicsofsport.com/skiing/equipment/skis/ski_construction.html)

2a. The structures of two important polymers are shown below. Note the hybridization of every carbon atom in the repeat units. (15 points)

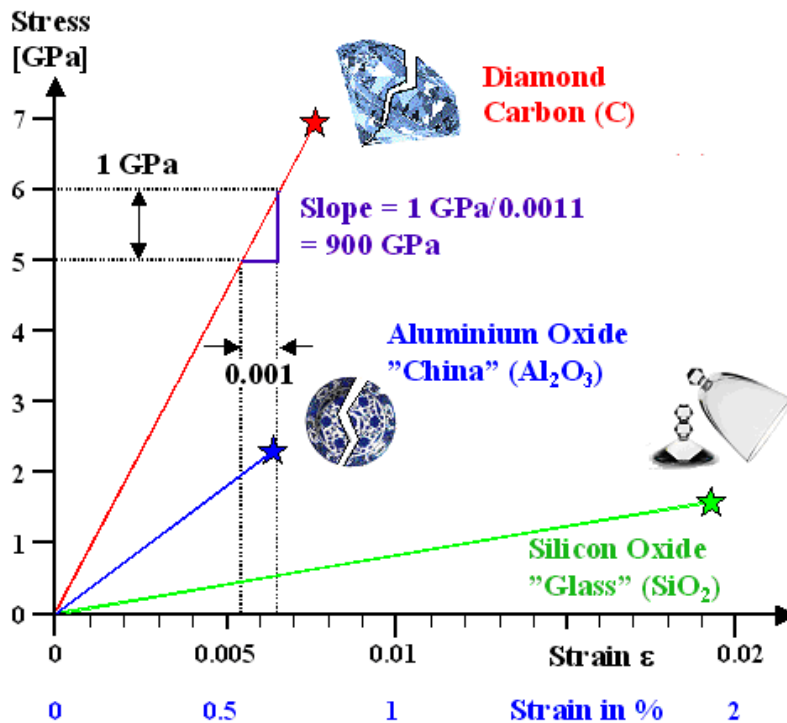


b. Both of the polymers in Part a are amorphous. The  $T_g$  of polycarbonate is about  $150^\circ\text{C}$ , whereas that of poly(cis-1,4-butadiene) is about  $-100^\circ\text{C}$ . Describe in your own words the physical (specifically, mechanical and optical) properties of each at room temperature. For poly(cis-1,4-butadiene), assume a small degree of crosslinking for your answer. (10 points)

**Polycarbonate with a  $T_g$  of  $150^\circ\text{C}$  is a rigid glass at room temperature (think eye glass lenses, and automobile headlight exteriors). In contrast, poly(cis-1,4-butadiene), with its low  $T_g$  and light crosslinking, will have the properties of an elastomer. The materials would show high and reversible extensibility (think rubber band).**

*A sidelight for fans of golf: virtually all golf balls have a core of lightly crosslinked poly(cis-1,4-butadiene). The reason is the very low  $T_g$  of about  $-100^\circ\text{C}$ . Upon rapid impact, the  $T_g$  is transiently raised (think time- or rate-dependence, or in other words viscoelasticity) and the rubbery nature of the core needs to be preserved. Such a low  $T_g$  ensures that.*

3. Stress-strain curves for three ceramic materials are shown below:



a. All show purely Hookean (linear stress-strain) behavior. Which of the three is the stiffest? Explain. Which of these would you label as brittle? Explain (15 points)

The stiffest is diamond, since the stress-strain plot has the highest slope. The slope is the Young's modulus or stiffness.

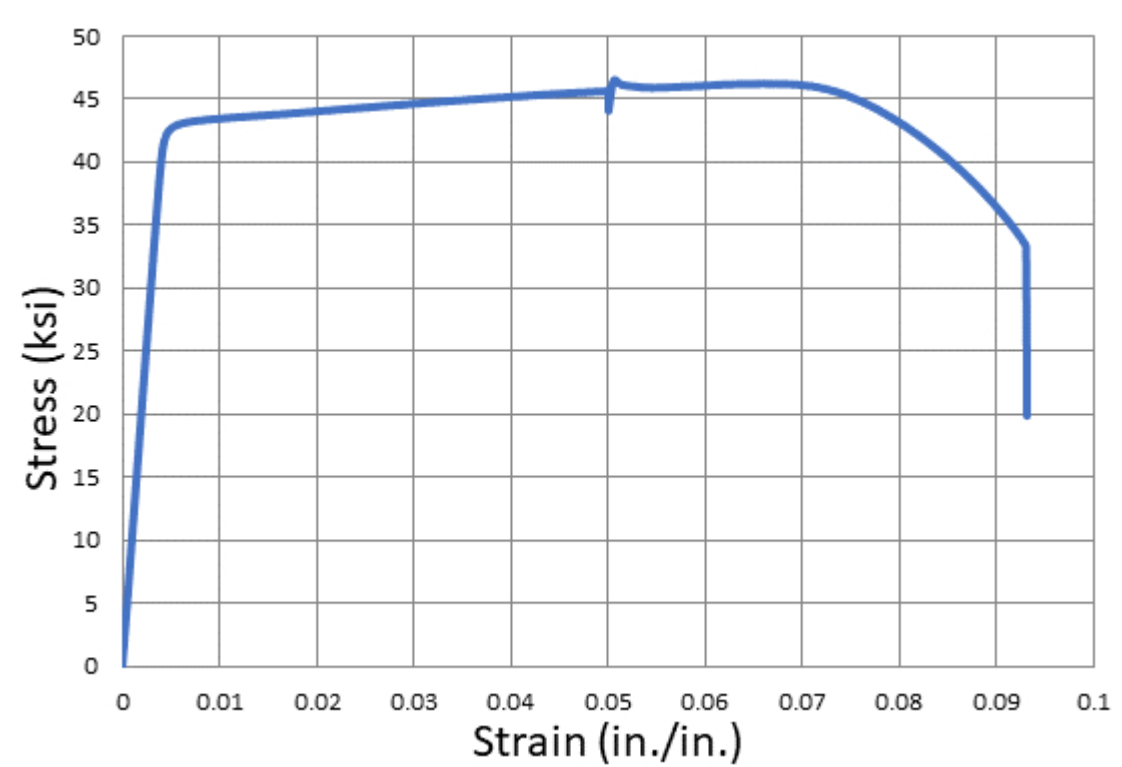
All three materials are brittle. Fracture occurs while still in the Hookean region for all three, at very low strains. This is a characteristic of brittle materials.

b. Silicon oxide (more correctly, silicon dioxide; "glass" above) is stronger in compression than in tension. Explain why. (10 points)

In tension, brittle materials suffer from amplification of applied stress at crack tips (stress concentrators). Therefore, brittle materials fail in tension at stresses well below those predicted for failure. In compression, surface and internal cracks are squeezed and therefore are not stress concentrators as they are in tension. Brittle materials such as concrete are always only used in compression.

4a. An engineering stress-strain curve (in tension) for aluminum is shown below. The data are rather old, and stress is reported in ksi (kilopound force per square inch). Convert ksi to MPa (1 ksi = 6.89 MPa) for your answers.

Use the curve to estimate (i) Young's modulus, (ii) the yield point, (iii) the ultimate tensile strength, and (iv) the strain at fracture. For (iv), elastic recovery can be ignored. (20 points)



**Young's modulus:** slope of linear portion. Choose for example 0.004 strain; here the stress is about 40 ksi, or 275 MPa. Divide by 0.004 to get 68.9 GPa. (A quick check via Google indicates that that number is very reasonable.)

**Yield point:** looks to be at 0.005 strain, corresponding to a stress of about 43 ksi or 296 MPa.

**Ultimate tensile strength:** the maximum load in an engineering stress-strain diagram; here about 46 ksi (0.07 strain), or about 317 MPa.

**Strain at fracture (without elastic recovery):** read directly from the graph to be a strain of approximately 0.92-0.93.

b. A cylindrical sample with a narrower center region (photo below) was used to take the data. Explain briefly why a narrower center, rather than a cylinder with a uniform cross-sectional area, is preferred for the tensile testing experiment. (5 points)



**The narrow center region ensures that force/unit area will be greatest there vs. the wider ends. This geometry gives more reliable data than a cylinder of uniform diameter, since the taper ensures that failure will occur in that region. With a uniform cylinder, deformation and failure could occur anywhere in the sample, including near the grips of the tensile-testing machine.**