

Exam #2

November 3, 2023

Duration: 55 minutes; 10:30 – 11:25 am EDT

Any resources (book, notes, web, etc.) are allowed, but you are not allowed to talk with anyone during the exam. With submission of your answers, you implicitly affirm that all work is your own, without consultation of peers or others. Be sure to cite sources of information.

Submit your answers via Canvas to your recitation instructor by 11:25 am.

1a. Consider a composite of continuous and aligned Kevlar fibers in a polycarbonate matrix. The Kevlar loading is 45 vol. %. Compute the longitudinal modulus of elasticity of the composite. The moduli of elasticity of Kevlar and polycarbonate are 131 GPa and 2.4 GPa, respectively. (15 points)

$$\begin{aligned}
 E_f &= 131 \text{ GPa} & E_m &= 2.4 \text{ GPa} \\
 V_f &= 0.45 & V_m &= 1 - 0.45 = 0.55 \\
 E_{cl} &= E_m V_m + E_f V_f \\
 &= 2.4(0.55) + 131(0.45) = 60.27 \text{ GPa}
 \end{aligned}$$

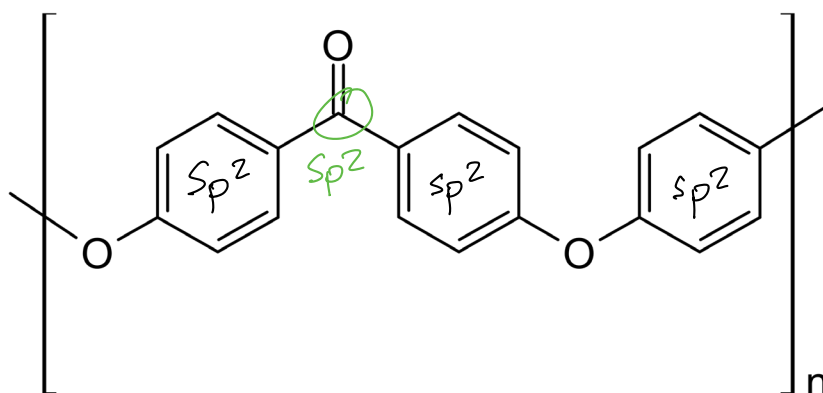
b. Why is Kevlar so much stiffer than polycarbonate? (5 points)

Kevlar is so much stiffer because its fibers are perfectly aligned and fully extended. Kevlar also has a lower density, and its higher modulus of 131 GPa causes its stiffness to be much higher due to the equation $\text{stiffness} = \frac{E}{\rho}$, where ρ is density.

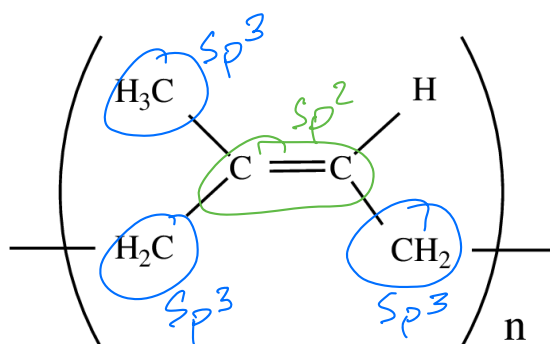
c. Comment on this statement regarding fiber composites: "the elastic modulus of the fiber should be much higher than that of the matrix." (5 points)

Ductile Materials should have a lower modulus so that they can withstand mechanical stress and dissipate it effectively. The fiber must be stiff to prevent tearing, which is characteristic of a higher modulus. The matrix must be ductile to prevent cracks from propagating and weakening the composite.

2a. The structures of two important polymers are shown below. Note the hybridization of every carbon atom in the repeat units. (15 points)



Poly(ether ether ketone) or PEEK

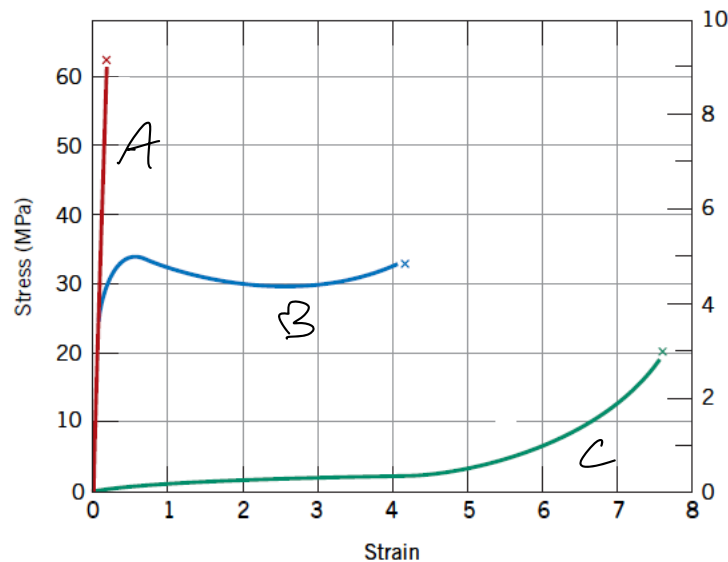


poly(cis-1,4-isoprene) or natural rubber

b. Natural rubber is typically reacted with sulfur to create occasional covalent crosslinks between chains. It is estimated that in a common rubber band, there are about 100 repeat units per crosslink. A stretched rubber band will retract quickly when strain is released, and it is said that the retraction is 'entropy-driven.' Explain. (10 points)

It is known that nature likes to be at high entropy. Rubber bands are at greatest entropy when they are not expanded because there are the greatest number of permutations in this state. When stretched, the chains are aligned more and have less possible permutations. This is not favorable, so the rubber band retracts quickly when strain is released. Thus, the retraction is entropy driven.

3. Stress-strain curves for three polymeric materials are shown below:



a. The three materials are an elastomer such as a rubber band, a semi-crystalline polymer such as HDPE, and a glassy polymer such as Plexiglass. Match the polymer type to the stress-strain curve and provide a brief justification for your choices. (15 points)

A (red) Plexiglass. Plexiglass is brittle in nature and fractures after little strain. This curve is characteristic of brittle materials.

B (blue) HDPE. HDPE is a ductile polymer because of its semi-crystalline domains. It isn't fully crystalline but it can withstand strain and dissipate it well. It will neck, which is only found in this graph.

C (green) Elastomer. Elastomers experience little stress with increased strain up until a point. This can be clearly observed by this curve.

b. Explain why 'dog-bone' specimens of the type shown below are typically used to obtain the curves above vs. rectangular samples. (10 points)



Having a narrow center region ensures that the fracture will occur in that region. This is because the force/unit area, or stress, will be greatest here as opposed to the ends. The taper ensures failure occurs in the tapered region whereas cylinders/rectangle samples could fail anywhere.

4a. A glass plate contains an atomic-scale surface crack. (Take the crack tip radius to be very small, approximately the diameter of an O^{2-} ion, or 0.264 nm.) Given that the crack is 1 μm long and the theoretical strength of the defect-free glass is 7.0 GPa, calculate the breaking strength of the plate. (15 points)

$$\sigma_m = 2\sigma_0 \left(\frac{a}{\rho_t} \right)^{1/2}$$

$1000 \text{ nm} = 1 \mu m$

$$\sigma_0 = 7.0 \text{ GPa}$$

$$\rho_t = 0.264 \text{ nm}$$

$$a = 1 \mu m \times \frac{1000 \text{ nm}}{1 \mu m} = 1000 \text{ nm}$$

$$\sigma_m = 2(7.0 \text{ GPa}) \left(\frac{1000 \text{ nm}}{0.264 \text{ nm}} \right)^{1/2}$$

$$= 861.64 \text{ GPa}$$

$$\sigma_0 = \frac{\sigma_m}{2 \left(\frac{a}{\rho_t} \right)^{1/2}} = \frac{7.0}{2 \left(\frac{1000}{0.264} \right)^{1/2}}$$

$$= 0.057 \text{ GPa}$$

b. Explain briefly how exchange of Na^+ with K^+ at the surface of a silica glass (think Gorilla Glass) provides a degree of resistance to damage from surface cracks. (10 points)

Na^+ acts as a network breaker in normal glass. When K^+ is used to replace the Na^+ network breakers, small cracks are squeezed shut. This is because K^+ ions are considerably larger than Na^+ ions. Replacing Na^+ with K^+ on the surface of the glass puts a strain on the network, causing compression that closes up surface cracks \rightarrow more resistant glass.