

PATCH: a deep learning method to assess heterogeneity of artistic practice in historical paintings

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Abstract

The history of art has seen significant shifts in the manner in which artworks are created, driven by changes in technology and the art market. Thus, understanding creative processes is a central question in technical art history. In the Renaissance and Early Modern period, paintings were largely produced by master painters who directed workshops of apprentices and others who often contributed to the work produced. The masters varied significantly in both their artistic and managerial styles, meaning different combinations of artists and implements might be seen both between masters and within workshops or even individual canvases. Information about the way different workshops were managed and the processes by which artworks were created remains elusive, however. Machine learning methods have potential to unearth new information about artists' creative processes by extending the analysis of brushwork to a microscopic scale. Analysis of paintings created in a workshop setting, however, presents a challenge in that documentation of the artists and materials involved in individual works is sparse, meaning external examples are not available to train networks to recognize their contributions. Here we present a novel machine learning approach we call pairwise assignment training for classifying heterogeneity (PATCH) that is capable of identifying individual artistic practice regimes with no external training data, or "ground truth." The method achieves unsupervised results by supervised means, and outperforms both simple statistical procedures and unsupervised machine learning methods. We apply this method to two historical paintings by the Spanish Renaissance master, El Greco: *The Baptism of Christ* and *Christ on the Cross with Landscape*, and our findings regarding the former potentially challenge previous work that has assigned the painting to workshop members. Further, the results of our analyses create a measure of heterogeneity of artistic practice that can be used to characterize artworks across time and space.

Introduction

There are many ways to create a painting. The process can be collaborative or solitary, and there is no end to the styles, tools, materials and techniques that can be implemented. The evolution of what we call artistic practice—the technical, stylistic, material, and even physiological aspects of an artist's creative process—over time and its variation between artists remains a key element of

technical art history scholarship. For example, in the last two centuries, Western painters, aided (and in many ways, enabled) by concomitant changes in the tools of the trade such as the invention of the paint tube¹, have often chosen more solitary practice in the studio or even outdoors. From the late Middle Ages through the Early Modern period, on the other hand, painting was typically done in a bustling workshop². A master painter would establish a workshop and assemble a group of apprentices, journeymen and specialist painters and even collaborate with associates to help fulfill commissions and create smaller works to support the enterprise between larger contracts.

Workshop practice adds a second layer of complexity to artistic practice. Master painters varied considerably in their managerial styles. Some performed much of the work themselves, while others dictated the composition but left many elements to the hands of workshop members (e.g., van Dyck³). Still others involved apprentices in the conceptualization of a piece⁴. Often, there was a hierarchy, with young apprentices typically assigned more mundane tasks such as the mixing of paints and the construction of canvases and more senior workshop members directed to paint portions of larger works in the master's style⁵.

Because workshops varied so widely in their organization and management yet produced works with cohesive styles, deeper understanding of variation in artistic practice can shed light on workshop practice and vice versa. Research into these key aspects of Renaissance and Early Modern art production is hindered by a lack of surviving textual sources regarding the structure, size, and membership of individual workshops as well as their day-to-day operation and the variation in artistic practices⁵. While scholars have been able to reconstruct individual artists' workshops through a variety of methods, our collective understanding of the inner workings of workshops is still fragmentary⁶. Artistic practice varies widely, encompassing materials, methods, styles, and individual contributions. Understanding who interacted with individual paintings and how they did so—discerning the artists at work in a painting, the interplay of their individual “hands,” and the interaction of the artist's hand and eye with paint and brush⁷—is an important open question in art history⁸. Answering these questions requires new methods of analysis.

Machine learning has recently emerged as a promising complement to traditional art historical analyses in applications including preservation, conservation and the detection of forgeries^{9–12} as well as in fighting illegal trafficking in antiquities¹³. Much work involving style is focused on broad classification of works^{14,15}, such as the use of a neural network to arrange works of the Western canon in the correct chronological order¹⁶. Recently published work by some of the authors of the present study demonstrated the efficacy of ML for attribution of paintings by analyzing high-resolution topographic images of the surface texture of oil paintings by known authors (student painters) in a controlled experimental setting. The topographic images, created using optical profilometry, captured variations in the height of the paint with microscopic resolution. While existing methods of study and conservation can employ microscopic methods to small areas obtained using a variety of imaging techniques, optical profilometry is unique in that it creates a topographic image at microscopic resolution over the entire painting. Though time consuming, the method provides a wealth of data on the surface topography and even the topography of pigmented layers beneath a transparent varnish layer. In a supervised machine learning analysis, a convolutional neural network (CNN) was trained to sort microscopic surface textures among artists, assigning 1 cm² patches of paintings to the correct authors with ~95% accuracy¹⁷. Thus, the method has shown that microscopic analysis can complement traditional art historical analyses of macroscopic features.

But can deep learning on topographic images be used for such analyses when there is no ground truth to test against? When the number of artistic practice regimes in a given piece is unknown, known examples of the included artists' work may not exist, and knowledge of the painting practices employed is incomplete—as in a painting produced by a Renaissance or Early Modern

artist’s workshop? Here, we demonstrate a novel technique, to our knowledge, for ML-based attribution of paintings by unknown authors to the correct artist(s). We then combine this method with network analysis to identify artistic practice regimes (combinations of artists and materials) and create a measure of the heterogeneity of artistic practice (HAP) within a given painting or set of paintings. We apply this method to two important historical paintings by El Greco, one which is considered to be entirely by the master himself, and one previously thought to feature the work of members of his workshop.

ML methods can be divided into two classes based on the way the network is trained and the output of the network: supervised and unsupervised. In unsupervised learning, the machine analyzes individual objects (e.g., images or patches of an image) and creates classes based on the statistical properties of those objects. In supervised learning, the classes are already known, and the network learns by attempting to correctly classify objects in a training set, adjusting the weights of the interconnections between its nodes as it succeeds (or fails) in doing so. A successfully trained network will be able to assign novel objects to the correct classes. Supervised learning is thus particularly useful for image recognition tasks. The method employed in the previous publication described above¹⁷ was a standard application of supervised learning for image recognition to topographic images. The network was trained on known topographic images (in this case, two paintings by each artist) and then tested on novel objects (a third painting by each artist). The trained network was able to distinguish between four different artists, correctly assigning 1 cm² patches of each painting to its respective artist with 95% accuracy.

The Pairwise Assignment Training for Classifying Heterogeneity (PATCH) method described here uses supervised learning toward an unsupervised end. Rather than train a network to recognize individual classes, we test whether the network is *capable* of learning to distinguish between two objects and then use that information to build *post-hoc* classes. The PATCH method relies on the inability of the network to correctly sort patches of paintings or regions of a painting painted by the same artist. Given patches of two paintings by different painters—a different-artist pair—the network will learn to assign those patches to the correct painting with a high degree of accuracy, as demonstrated in our previous publication. But if the network is given patches of two paintings by the same artist—a same-artist pair—it will not be able to learn to assign those patches to the correct painting; it will perform no better than a coin flip. We then create a network of same-artist pairs and use network analysis to construct classes corresponding to artistic practice regimes. In this way, we obviate the need for separate ground truth information for individual artists and artistic practice regimes, enabling successful applications in contexts such as workshop paintings where the ground truth is not known with certainty.

Study 1: Method development

The PATCH method (Figure 1) consists of two phases. The first phase, pairwise assignment training, identifies pairs of paintings or pairs of regions of a single painting that are the “same,” that is, created by the same artist under the same conditions. The second phase, community finding, creates *post hoc* classes based on these pairings.

The first phase was developed on a set of paintings by known artists. A group of nine artists each individually painted three paintings, all using the same tools, materials, and subject (a photograph of lilies). The materials used for the paintings were described previously¹⁷. While the artists were instructed to realistically depict the subject, choices of style and technique (including the use of brushes and alteration of paint viscosity) were left up to the artists. One artist was only able to produce two paintings, and one painting was removed from the sample due to the optical profilometry data being corrupted, for a total of 25 paintings. Topographical information for each painting

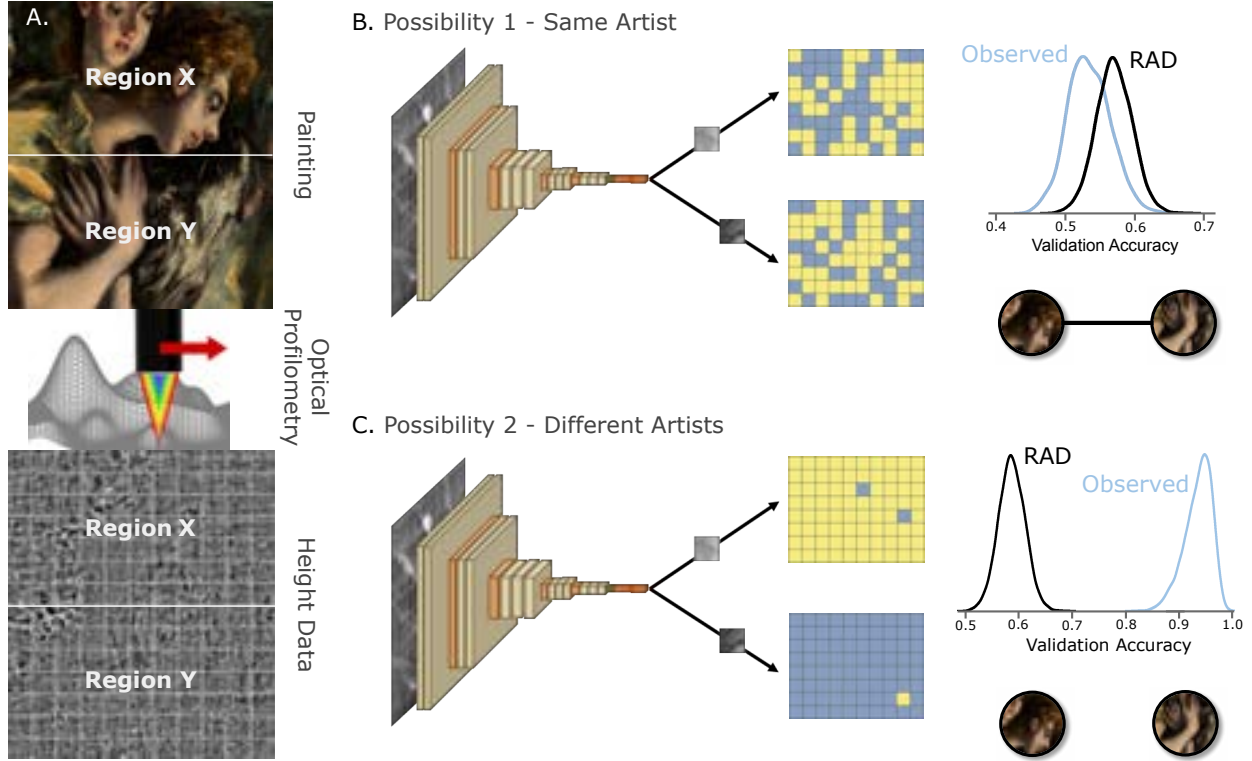


Figure 1: Overview of the PATCH method: A) Paint height data is gathered using optical profilometry, generating a 16-bit image file encoding the height values, which is subsequently divided into $1 \times 1 \text{ cm}^2$ patches. B, C) A CNN is repeatedly trained to sort patches of two images, recording its accuracy as a percentage of patches correctly sorted from the test set. This distribution of observed accuracies is compared to the random assignment distribution (RAD) of accuracies if the patches were sorted at random. If the observed accuracy is no better than the RAD accuracy, then the images were likely painted by the same artist, whereas if the observed accuracy is better than expected by chance, the images were likely painted by different artists.

was recorded using high resolution spectral confocal optical profilometry with a spatial resolution of 50 microns, and a height repeatability of 200 nm. The resulting data were processed to remove any large scale warping of the canvas by subtracting a mean-filtered version of the height map (with filter radius of 0.5 cm)¹⁷.

The network architecture chosen was VGG-16, implemented in TensorFlow and pretrained on the ImageNet dataset. Given the computational complexity of the training—the combinatorics of many region-to-region comparisons, repeated over many folds, described in detail below—the relatively light-weight but accurate VGG-16 architecture was ideal for the PATCH approach. Output from VGG-16 was pooled and flattened and passed through two dense layers with dropout rates of 0.25. The learning rate was set to 0.0001 and the batch size was set to 32 patches (see the *Supplementary Materials* for more information on hyperparameter optimization). In order to measure the ability of the network to learn to distinguish between two paintings or regions of a painting, we ran 26 training folds. Each fold comprised the following steps: 1 cm^2 patches from each painting/region were randomly selected, with replacement. Sampling with replacement creates a bootstrapping effect that drives down the success rate of same-artist pairs and drives up the success rate for different-artist pairs. In each fold, the network was trained for 25 epochs with validation set size set to 30% of patches, and the maximum validation accuracy (percentage of validation patches correctly identified) was recorded. At the end of 26 folds, the distribution of

	Precision	Recall	F ₁
Same-artist pairs	0.909	0.87	0.889
Different-artist pairs	0.989	0.993	0.991
Average	<i>0.949</i>	<i>0.931</i>	<i>0.940</i>

Table 1: Precision, recall and F₁ score for the pairwise assignment training process implemented on a group of paintings from a controlled experiment where the attribution was known. Same-artist and different-artist pairs are treated as “positives” in turn and the results are averaged.

the 26 maximum validation accuracies was compared to the distribution expected if the network were assigning patches at random.

If the network has failed to learn to assign objects to their correct classes, it should perform this task no better than if it were assigning them randomly. The prospect of randomly choosing from two options invokes the coin flip and, by extension, the binomial distribution. However, we are measuring the maximum validation accuracy from 25 training epochs in each of 26 folds. Thus, the distribution expected if the network fails to learn to distinguish between painters would be the random assignment distribution (RAD) $p_{n,k}^{\max}(m)$, the probability that you will see a maximum of m heads (i.e., correct assignments) over k repetitions (k epochs) of an experiment where you flip a fair coin n times (the number of patches in the test set). The *Supplementary Materials* contains an analytical derivation of this distribution.

Two decision criteria were selected to determine whether the distribution of maximum validation accuracies indicated that the network had failed to learn to distinguish between the input paintings: 1) if the observed mean had a z-score less than 2 relative to the RAD mean and 2) if the largest of the observed maximum validation accuracies (the right edge of the observed distribution) among the 26 folds was less than an empirically determined threshold based on finite sampling from RAD, plus 10% to account for the effects of bootstrapping (selection with replacement, see *Supplementary Information* for more details). Using these decision criteria, the accuracy in determining whether the artist was the same or different was exceptional. We identified same-artist pairs ($n = 23$) and different-artist pairs ($n = 277$) with 96.0% overall accuracy (87% for same and 99.3% for different). This translates to F₁ scores of 0.991 for different-artist pairs and 0.889 for same-artist pairs, an average of 0.940 (Table 1). The pairwise assignment training phase outperformed both an unsupervised method and a statistical method based on surface roughness (see details in SI Appendix C) in precision, recall and total F₁ for both same-artist and different-artist pairs.

While pairwise assignment training performs exceptionally well in identifying artists, situations where ground truth about the artistic practices employed in a particular painting is not available require some means to create the *post hoc* classes to which regions of a painting will be assigned. To this end, we use network analysis. We construct a network where same-artist pairs (or, for historical paintings, same-practice pairs) of paintings or regions are connected by an edge (lines connecting regions in Figure 3) and different-artist/practice pairs are unconnected.

The PATCH method is concerned with “sameness,” meaning we need a way to remove spurious edges from the network. To accomplish this, we employ an edge pruning process based on uniqueness. We assume that regions with a high degree (number of connections) have more common features and nodes with a low degree have more unique features. Therefore, a connection between regions with more unique features is more likely to be correct. We operationalize this assumption by assigning each edge a score based on the average uniqueness of its nodes, where uniqueness is defined as the percentage of nodes a node does not connect with: $W_{ij} = \frac{1}{2}((\max - \text{degree}_i)/\max + (\max - \text{degree}_j)/\max)$ where \max = the maximum possible degree

(i.e., number of connections) and degree = the degree of one of the two nodes in question. There were two erroneous same-artist pairs in our experimental dataset (9% of edges in the network), and these edges had the lowest uniqueness scores. Thus, we trim the 9% of edges (or more if there are ties) in a given network with the lowest uniqueness scores.

With potentially spurious edges removed, and the remaining edges weighted equally, we then employ a community finding algorithm to identify groups of regions that are the most similar. If a set of regions has a large number of internal links within the set, and fewer links to outside regions, it forms a “community” within the network^{18–20}, suggesting that the regions in a given community were likely painted under different circumstances (artists, materials, etc.) than those in other communities. To establish the degree of difference between communities, we can characterize the community structure of the network via measures such as modularity (Q)—the fraction of edges in the network that are internal to communities minus the mean fraction in a network with the same communities but where the edges are completely randomized¹⁹. We implemented the Louvain community finding algorithm²¹ in Gephi²². With the resolution set to 1.0, the algorithm returns the partition of the network with the maximum modularity. The Louvain algorithm returned a correct partition of the experimental dataset with $Q = 0.875$.

El Greco, *The Baptism*, and *Christ on the Cross*

El Greco, born Domenikos Theotokópoulos (1541–1614), is regarded as a pillar of the Renaissance in Spain and as an early progenitor of modernism²³. El Greco became a master of icon painting in Crete before journeying through Venice (where he may have worked with Titian) and Rome, eventually settling in Toledo, Spain^{24,25}. It was in Toledo that the master married Byzantine and Venetian motifs²⁶ and developed the peculiar style for which he is most widely recognized, with its distorted human figures, “expressive hands”²⁷, and “exploitation” of “pure colors to their limits”²⁸. Two examples of that style are *The Baptism of Christ* (1624, Hospital Tavera, Toledo, Esp., henceforth, *The Baptism*), shown in Figure 2A, and *Christ on the Cross with Landscape* (ca. 1600–1610, Cleveland Museum of Art, Cleveland, OH, henceforth *Christ on the Cross*), shown in Figure 2B.

These works were chosen for this analysis because of the difference in how art historians have characterized their authorship. *The Baptism* has long been thought to feature the work of El Greco and at least one other artist. Primary historical evidence indicates that El Greco began the painting under a contract with the Hospital Tavera but retained it at his death in 1614. It was delivered to the Hospital nearly a decade later²⁹, during which period art historians have proposed the painting was finished by workshop members, particularly the master’s son, Jorge Manuel²⁴. Previous art historical studies have attempted to attribute regions of *The Baptism* to the master and others (illustrated in Figure S2) largely through connoisseurship—visual analysis of artists’ styles and artistic choices³⁰. El Greco himself is proposed to have painted the entire top, with the possible exception of the robe of the angel on the right (which was not scanned for this study due to its poor condition)^{29,31}, as well as the angel in green (bottom left), with the exception of the wings. Jorge Manuel is proposed to have been painted John the Baptist (bottom right) and the adjacent figure in red. Lopera³¹ and Wethey²⁹ ascribe the image of Christ (bottom center) to Jorge Manuel and El Greco, respectively, suggesting that it could represent the work of both artists. Lopera believes that a third hand may have been involved in rendering the faces of the angels in the background on the bottom. The landscapes at the bottom were extended and “transformed into a river” by an unknown artist sometime between receipt of the painting and its installation in the epistle-side altarpiece ca. 1660, which was not an unusual practice at the time. Restoration of the landscape on the right at the very bottom was performed in 1936^{29,31,32}. In contrast, *Christ on the Cross* has

A.



B.



Figure 2: A) El Greco's *The Baptism of Christ*, begun in approximately 1608 and suspected to have been finished by his workshop before delivery to the Hospital Tavera in 1624. B) El Greco's *Christ on the Cross with Landscape*, ca. 1600–1610.

been entirely attributed to El Greco himself. It is generally accepted that El Greco created many works in their entirety without the aid of his workshop, including *Christ on the Cross*. Francis³³ states that most of El Greco's approximately twenty renditions of the Crucifixion were "by his own hand," and notes the "lightning dexterity of brushwork" in this particular example.

Study 2: Application to El Greco's *The Baptism*

Two large sections of *The Baptism* were scanned *in situ* using the Factum Foundation's Lucida scanner. The Lucida scanner is a non-contact laser triangulation scanner with 100-micron lateral resolution and is capable of acquiring scans *in situ* over a large area by stitching together 48 cm square tiles. The scanned regions used in this study are indicated in Figure 3. The height data was processed in the same way as in study 1, including correction for possible canvas warping.

Regions of the topographic image of *The Baptism* were selected by hand in order to minimize variation in subject matter (e.g., faces, robes) within each region and to constrain region size to between 180 and 540 cm². The lower bound was chosen because the method had been developed using 180-patch paintings in study 1 and the upper bound was selected to ensure

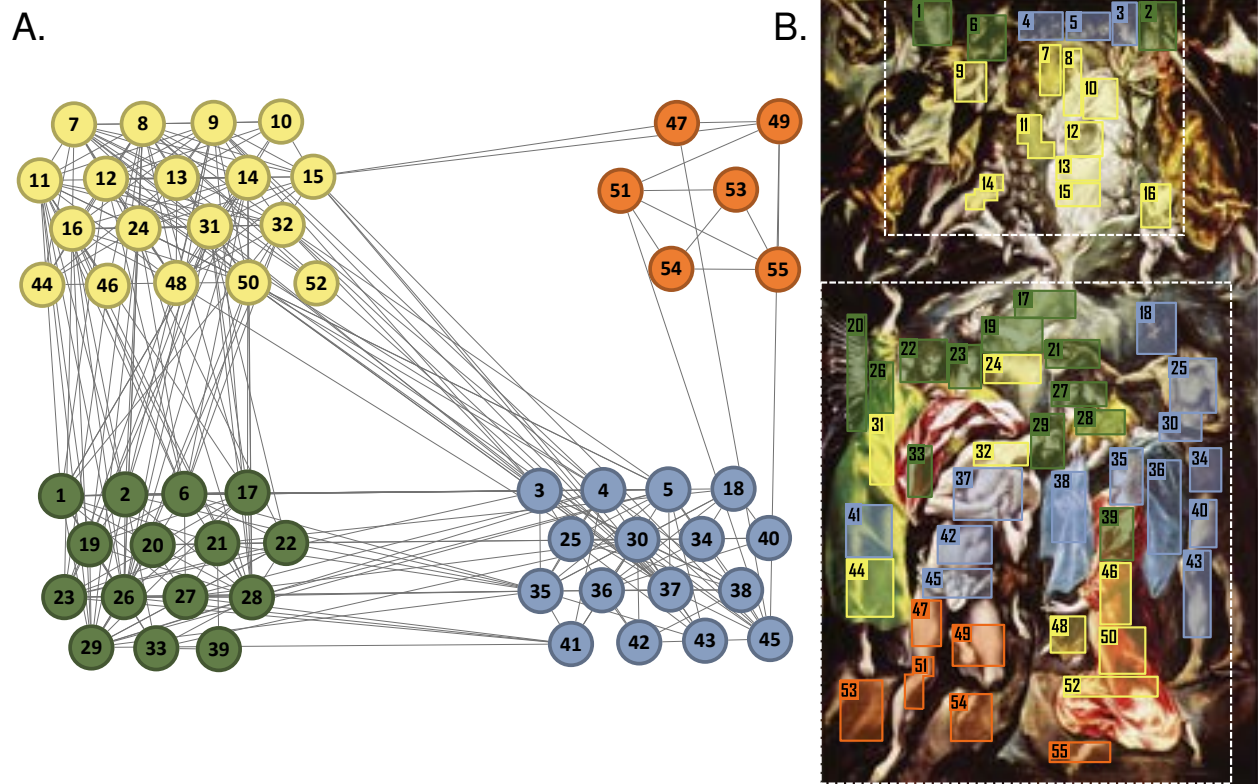


Figure 3: Results of PATCH analysis on El Greco's *The Baptism of Christ* (1624, Hospital Tavera, Toledo, Esp.). Areas that were scanned are outlined with dashed white lines. A) Network diagram showing the maximum modularity (Q) partition. Four communities are present. B) the communities mapped onto the corresponding regions of the painting that were analyzed.

adequate representation of patches when regions of significantly different size were compared. We avoided selecting areas with significant cracking or damage, including the robes of the large angels to the left and right of God, and the red swatch held by the “green angel” in the bottom half of the painting. Each fold proceeded as described in study 1, above. Where regions were of different sizes, the sample size was set equal to the number of patches in the smaller region.

Pairwise comparisons of all 55 regions (a total of 1485 pairwise tests) of *The Baptism* yielded a network with 356 edges, which was reduced to 314 edges after pruning. The Louvain community finding algorithm²¹ with resolution set to 1.0, returned a partition with $Q = 0.341$ that featured four communities (Figure 3a). A value of $Q \geq 0.3$ is often considered evidence of structure in real-world networks¹⁹.

The four communities are mapped onto the painting in Figure 3b. The first community overlaps much of the top portion of the painting, including the image of God the Father. The second overlaps the image of John the Baptist and Christ's torso and hands. The third covers much of the center of the painting, the face of Christ and the faces of many of the background angels. The final community is confined to Jesus' legs and the rocks at the bottom of the painting.

Study 3: Application to El Greco's *Christ on the Cross*

As in study 2, regions of the topographic image of *Christ on the Cross* were selected by hand in order to minimize variation in subject matter and to constrain region size. Areas with apparent

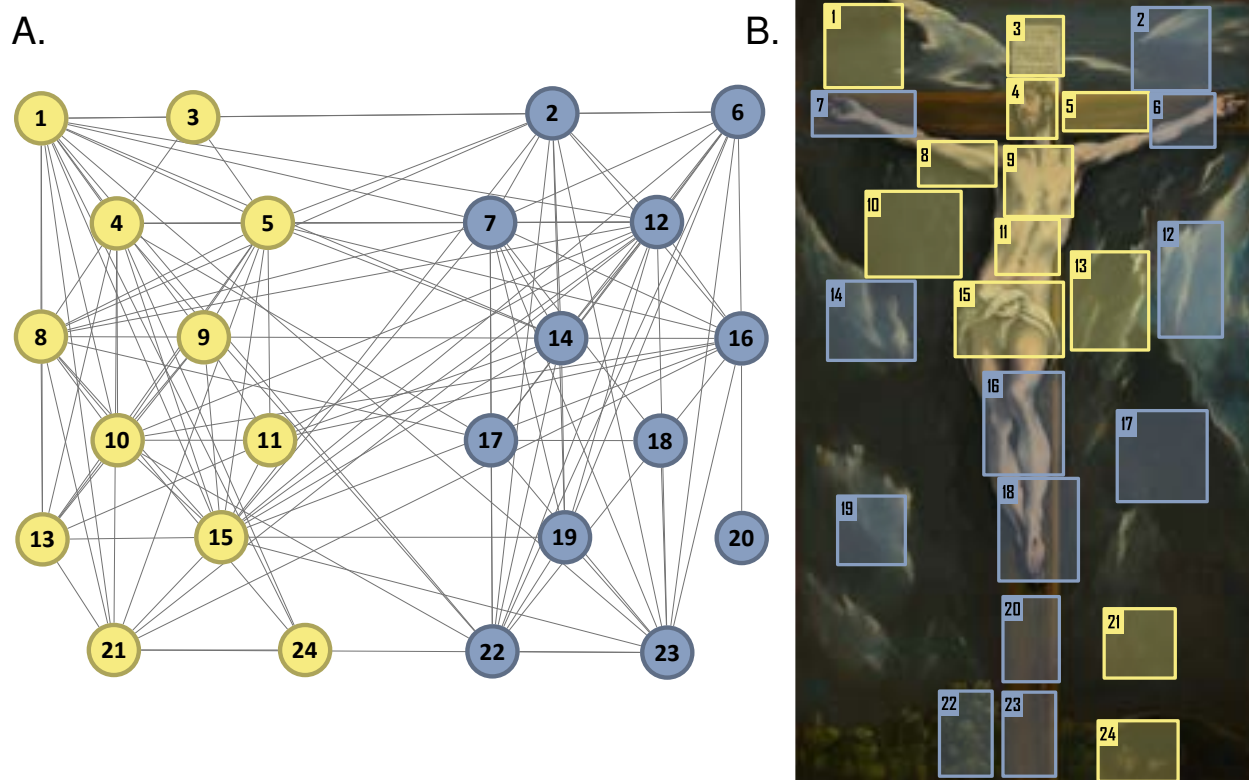


Figure 4: Results of PATCH analysis on El Greco's *Christ on the Cross with Landscape*. A) Network diagram showing the maximum modularity (Q) partition. Two communities are identified, though it should be noted that this is the minimum number of communities discoverable using a maximum-modularity partition. B) The communities mapped onto the corresponding regions of the painting that were analyzed.

cracking or damage, such as the background and trees on the lower left and the cross beam to the immediate left of Christ's face, were not included.

Pairwise comparison of all 24 regions (276 pairwise tests) yielded a network with 123 edges, which was reduced to 108 edges after pruning. The Louvain community finding algorithm²¹ at resolution 1.0 generated a partition with $Q = 0.231$ that featured two communities. Two is the smallest possible number of communities for a non-fully connected graph, as Q compares inter- and intra-community connections. Communities are mapped onto the painting in Figure 4. Constituent regions of both communities are not randomly distributed, with one community overlapping much of Christ's torso and face, while the other community overlaps Christ's legs, the lower portion of the cross and regions of the background.

Analysis of PATCH-generated communities

Further analysis of the community structure in the PATCH generated networks can provide additional insight into the community cohesiveness beyond the modularity index. We analyzed the average community degree, defined as the count of all edges between nodes within two communities. This is shown for all possible community pairs in Figure 5A and C, and within a single community (internal degree) or between nodes within and external to a given community (external degree) in Figure 5B and D. Each degree is normalized by the theoretical maximum degree based on the number of edges in the classification. The degree between community pairs and external

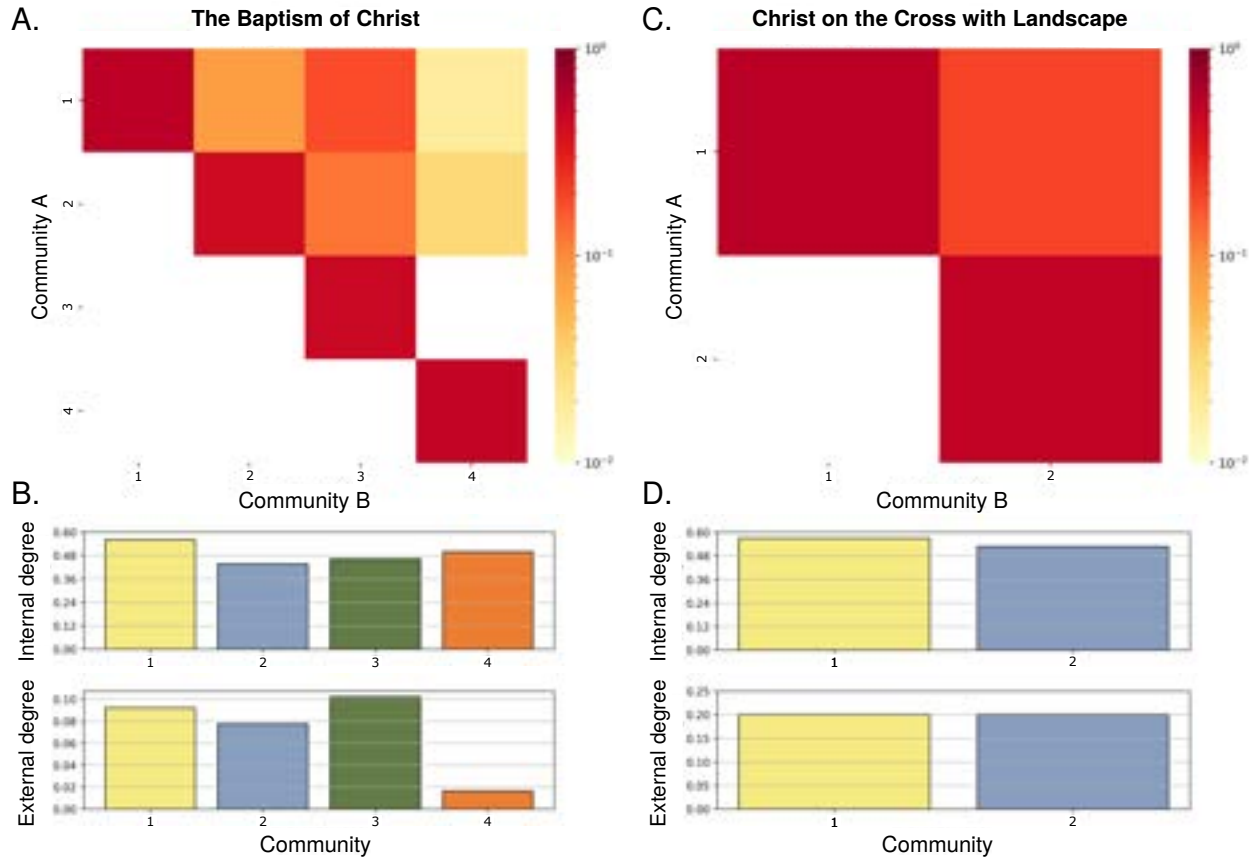


Figure 5: Analysis of the network community structure generated from the PATCH method for: A, B) El Greco's *The Baptism of Christ* and C, D) *Christ on the Cross with Landscape*. A, C) Normalized network degree between each pair of identified communities. The color scale is logarithmic. B, D) Normalized internal degree (considering all edges within a given community) and external degree (considering all edges between two distinct communities) averaged over all external community pairs in the network. Colors of the bars correspond to the communities in the network representations of Figures 3-4. Note that *Baptism* community 4 (orange) in panel B has significantly less external connectivity, and the external degree for the *Christ on the Cross* communities in panel D is significantly higher than for any *Baptism* community.

degree can be taken as a measure of the PATCH inter-community similarity between specific communities and between a community and all others, respectively. Likewise, the internal degree can be taken as a measure of intra-community cohesiveness.

In *The Baptism*, all of the connections between the first three communities are significantly stronger than those involving the fourth community, with the average external degree over four times larger. This indicates a significant isolation of the fourth community (colored orange in Figure 3, and represented by orange bars in Figure 5B), indicating a relatively strong disparity in the topographical data. The strongest connection is between communities 1 (yellow) and 3 (green) and the weakest is between 3 and 4, which has no connecting edges. Overall the 3rd community has a slightly greater connection with the remainder of the communities, indicating greater similarity. Interestingly, the internal degree is relatively similar for all communities, indicating that the differences within the observed topographical signal are evenly distributed across communities. A significant difference in the internal degree could, for instance, indicate a community with above or below average range of stylistic practice.

Although the external degree analysis is largely redundant in the case of *Christ on the Cross*, since only two communities are proposed, it should be noted that the external degree between the two communities is significantly larger than for any other two communities in *The Baptism* except the connection between communities 1 and 3. This indicates the two proposed communities are on average more closely connected than those proposed in *The Baptism*, which is reflected in the low modularity value. Additionally, the internal degree in each community is similar, which can indicate a similar intra-community stylistic distribution.

Discussion

To our knowledge, ours is the first study to demonstrate this learning method and apply it both to paintings from a controlled experiment and to historical paintings by a well-known artist with a workshop. Among AI/ML-based methods designed to aid in the analysis of historical paintings, PATCH is notable in that it requires no external, ground truth data. Networks trained on known examples can accomplish tasks associated with workshop practice and authorship with exceptional accuracy. For example, Ugail and colleagues recently used transfer learning to authenticate paintings by the Renaissance master Raphael (Raffello Sanzio)³⁴. By combining edge detection with typical feature extraction in a residual neural network, they were able to recognize works by Raphael with 98% accuracy. However, known examples of artists' work may not be available, as exemplified by the workshop context. Our method shows exceptional promise as a tool for ML analysis of complex historical works where known samples of work by the artists involved do not exist. Indeed, it may have extensive application in other image analysis tasks where there is little to no ground truth information available.

Further, the second phase of PATCH creates what we consider a measure of the heterogeneity of artistic practice (HAP) within a given dataset. Communities found within a dataset represent different artists, materials or both. The degree to which those communities are interconnected provides information about the possibility of shared authorship, materials, or other practices. A fully connected network with no distinct communities has $Q = 0$. As the number of communities increases and the interconnection between communities decreases, modularity increases toward 1 (a completely disconnected network has undefined modularity, but would effectively represent the maximum heterogeneity). The networks from our three studies provide empirical examples of lower and higher modularity networks. *Christ on the Cross* and *The Baptism* have Q values that cluster around 0.3, considered the threshold for evidence of structure, indicating low heterogeneity. The network for the experimental student paintings has much greater heterogeneity at $Q = 0.875$ (see Figure S3). The maximum modularity depends on the number of edges in the network³⁵, but also on the number of communities in the network. A network with two fully connected and completely disjoint communities has $Q = 0.5$, for example. As such, network modularity can function as a measure of absolute observed heterogeneity (Figure 6).

HAP informs interpretation of our analyses of *Christ on the Cross* and *The Baptism*. For the former, our analysis identifies two communities of regions, but with Q below the threshold generally accepted to indicate structure (0.231 vs 0.3). The regions in each community do cluster in space, however, which is worth considering. One community overlaps Christ's head and torso and surrounding areas. Conservation files generously provided by the Cleveland Museum of Art indicate that the figure of Christ (areas overlapped by the community in yellow in Figure 4) contains inpainting from previous conservation, which could create a distinguishable signal, though damage to the canvas is another, more likely culprit. Further mapping of damage and past conservation efforts will improve our understanding of this work. However, the results comport with the notion of *Christ on the Cross* as the work of a single artist with some variation introduced by artistic practice

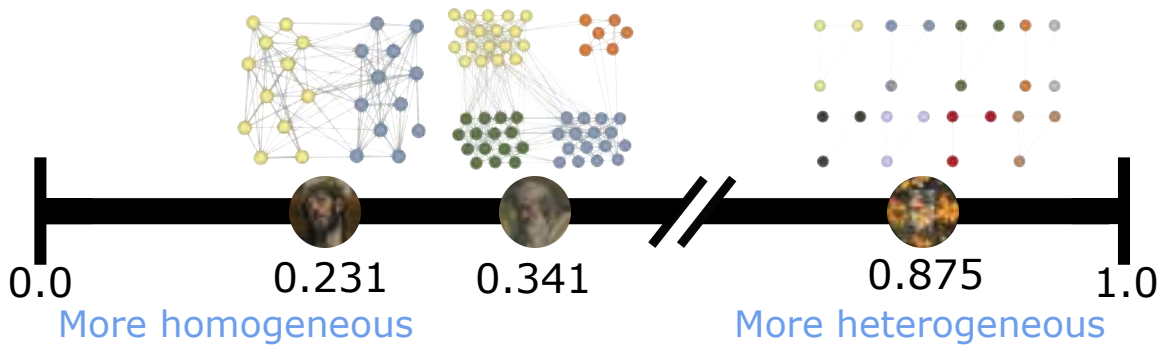


Figure 6: Networks and modularity (Q) values for *Christ on the Cross*, *The Baptism*, and the experimental student paintings along the axis of Q from least to most heterogeneous.

or possibly early conservation practice.

At first glance, the communities identified in our analysis of El Greco’s *Baptism* appear to roughly correspond to the proposed attributions by previous authors discussed above. The community in yellow in Figure 3 overlaps much of the top of the painting, previously attributed to El Greco, while the community in blue overlaps John the Baptist and the figure in red, assigned to Jorge Manuel. We also find a combination of communities in the face and torso of Christ, which was attributed to both artists. A single, small and particularly unique community (in orange in Figure 3) overlaps the landscape at the bottom of the painting and Christ’s legs.

Overall, however, the PATCH findings suggest a different story underlying the existing art historical interpretation. The partition of *The Baptism* has $Q = 0.341$, which is only slightly above the threshold for evidence of structure. Nearly a third of all edges in the network are interconnections between three of the four communities we have identified. The yellow and green groups have 49 intercommunity edges, blue and green 30, and blue and yellow 22. If the blue, green and yellow communities indeed represented three distinct artists working with the same or similar materials, we would expect the communities to be more insular and the modularity of the network to be substantially higher, similar to what we observed in our controlled test. Yet the modularity of *The Baptism* is much closer to that of *Christ on the Cross* ($Q = 0.231$), which appears to feature one artist. It is worth noting that, as shown in the analysis of student paintings, the CNN is particularly proficient at recognizing different-artist pairs and is more likely to incorrectly label a same-artist pair as different than the converse. Thus, the intercommunity edges in *The Baptism* network that survived our initial pruning are not liable to be spurious.

This suggests that some unifying factor connects these three communities. One possibility is a single artist working with different brushes. In El Greco’s time, brushes were made with either bristles (which are coarse) or hair (which is fine and soft). Each served distinct technical purposes and often came in material-specific shapes³⁶. Variation could arise from using the hair/bristles of different species in the same type of brush. Key sources were hog bristles (especially white hog) and “minever” (gray squirrel tail fur)³⁷, though dog and mongoose often replaced minever due to its rarity^{36,38}. Hog’s bristles likewise may have been replaced with ox³⁹. Such changes could affect paint application. For example, hog’s bristles are naturally flagged (the ends of the hairs are split), allowing them to pick up more paint⁴⁰. Bristle substitution could decrease brush capacity. Further experimentation to understand the effect of materials on PATCH analysis is needed.

The evidence could also indicate the work of one artist with changing style or technique over time. Interestingly, Biannuci and colleagues⁴¹ present evidence that El Greco suffered a series of ischemic events, one in the 1590s and a second in 1608 (the year the contract for the altarpieces

for the Hospital Tavera was initiated), “resulting in progressive disabilities with fluctuating course characterized by temporary improvements and worsening before his death.” The regions that were previously identified as the work of different individual painters could conceivably represent variation in the master’s individual style (however this may have arisen) over the course of his final years. Investigation of changes in style and technique and their effect on PATCH analysis will be an important direction for future research, as well.

PATCH has the capability to make a substantial contribution to research as a complement to existing art historical methods. The accuracy of the method has been demonstrated in the validation on student paintings. Our analyses of *Christ on the Cross* and *The Baptism* show the true potential of PATCH by contributing important new information to art historical scholarship on the creation of objects in El Greco’s workshop in Toledo.

In the analysis of *The Baptism*, the fact that the orange group, which overlaps a region known to have been altered after the delivery of the painting, is so disconnected from the remainder of the communities speaks to the method’s capability of recognizing a unique contribution or substantial departure from the technique used in the rest of the painting. That the other three communities roughly correspond to art historical attributions suggests PATCH possesses a kind of semantic fidelity⁴²—our method recognizes areas that appear different to the trained eye of an art historian. However, the connections between those areas revealed by PATCH call this attribution into question. Traditional attribution studies rely on the inspection of painters’ unconscious tendencies and individual styles. For example, in attributing the painting of John the Baptist to El Greco’s son, Wethey²⁹ cites the “hardness” of the details. Computer vision allows us to view the surface of a painting at a different scale. Previous analyses indicate that features as small as the diameter of a single paint brush bristle (length scales of 0.2–0.4 mm) may be integral to the network’s identification of an artist¹⁷. By analyzing spatial correlations at this microscopic scale, the network may be revealing aspects of the physics of paint application and of the physiology of hand movements. PATCH adds micro-scale features to the macro (brushstrokes) and meta-scale (historical and material) data, allowing for a full-scale analysis and the discovery of heretofore unseen evidence of the processes by which paintings were created.

Future research with PATCH will focus on teasing apart the influences of materials and artists on the heterogeneity of artistic practice. Expansion of PATCH applications to include *inter*-painting comparisons and other art historical applications will be important as well. However, uses of PATCH are not limited to those contexts: it provides a general-purpose approach to clustering data that can be encoded in the form of image-like arrays, as occurs in a wide variety of applications across the social and natural sciences. Underlying the effectiveness of pairwise assignment training is the power of convolutional neural networks to find extremely subtle differences among such data points, and the counter-intuitively useful information gained when the network consistently fails to succeed in this task. As such, PATCH can be deployed where supervised learning is impossible (because the ground truth is unknown) and when alternative unsupervised approaches are ineffective.

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Supplemental Materials:

“PATCH: a deep learning method to assess heterogeneity of artistic practice in historical paintings”

Appendix A: Analytical derivation of the random assignment distribution

In order to determine whether the network has learned to correctly sort patches of both images, we generated the distribution of correct assignments expected if the network were assigning each patch to a class at random (as though it were simply flipping a coin). The network is trained for 26 folds of 25 epochs each. In each epoch, the network is trained on 70% of the patches and tested on the remaining 30% (there are equal numbers of patches from each image). The percentage of this test set that is assigned correctly is called the “validation accuracy.” At the end of each fold, the maximum validation accuracy is recorded, the dataset is resampled, and a new fold begins. After all folds are completed, we have a distribution of 26 maximum validation accuracies that represent how well the network has learned to distinguish between the two images in each of its attempts. The derivation of this expected distribution is as follows:

The goal is to find the distribution $p_{n,k}^{\max}(m)$, the probability that you will see a maximum of m heads over k repetitions (“ k epochs”) of an experiment where you flip a fair coin n times. We start with the distribution of finding m heads in a single experiment, which is the binomial distribution for a fair coin,

$$p_n(m) = 2^{-n} \binom{n}{m}, \quad (1)$$

where $\binom{n}{m} \equiv \frac{n!}{m!(n-m)!}$ is the binomial coefficient. We also define a tail distribution, which is the cumulative probability that you would find m or more coin flips in a single experiment:

$$p_n^{\text{tail}}(m) = \sum_{j=m}^n p_n(j) = \binom{n}{m} {}_2F_1(m, n+1, m+1; -1). \quad (2)$$

Here ${}_2F_1(a, b, c; z)$ is a hypergeometric function, defined by the series:

$${}_2F_1(a, b, c; z) = \sum_{j=0}^{\infty} \frac{(a)_j (b)_j}{(c)_j} \frac{z^j}{j!}, \quad (3)$$

where $(x)_j \equiv x(x+1) \cdots (x+j-1)$ for $j > 0$ and $(x)_0 \equiv 1$. There are other equivalent ways of expressing the tail probability of a binomial distribution (using for example incomplete beta functions), but all of them are similarly complicated.

If we repeat the experiment k times, the probability that every single experiment had fewer than m heads is:

$$U_{n,k}(m) = (1 - p_n^{\text{tail}}(m))^k. \quad (4)$$

The difference between consecutive values of this probability, $U_{n,k}(m+1) - U_{n,k}(m)$, is the probability that we would find a maximum exactly equal to m over k repetitions. Hence we get our final answer:

$$p_{n,k}^{\max}(m) = U_{n,k}(m+1) - U_{n,k}(m). \quad (5)$$

It is easy to check that this probability is properly normalized:

$$\sum_{m=0}^n p_{n,k}^{\max}(m) = U_{n,k}(n+1) - U_{n,k}(0) = 1. \quad (6)$$

Hyperparameter	Min search val.	Max search val.	Max posterior	Final optimization
Learning rate	0.0001	0.02	0.0025	0.0001
Dense layers	1	4	1	1
Max nodes	25	200	97	64
Dropout rate	0.0	0.25	0.20	0.25
L2 reg. rate	0.0001	0.13	0.0007	0.001

Table S1: Optimization ranges for the searched hyperparameters.

In practice, we compared the mean and maximum of the observed distribution to the random assignment distribution derived above. However, the theoretical maximum of the distribution is $m = n$. Given enough tries (infinite sampling), a random assignment engine would eventually achieve perfect accuracy. Thus, we developed an empirical “maximum” value based on finite sampling using the image size of our student paintings (180 patches, 360 total, $n = 108$ patches in the test set). We created a simple coin-flipping script that gathered the maximum “validation accuracies” (i.e., percentage of flips that land heads) for 25x108 flips and ran 250,000 iterations of this process. This generated a maximum value of $m = 80$ “correct assignments,” or 74.074%. The probability of 80 correct assignments is 0.000004676581 (truncated for clarity). As the distribution is a probability mass function based on discrete events, for tests with larger numbers of patches, we used the number of correct assignments with a probability closest to this probability, even if it was less probable.

Appendix B: Hyperparameter Optimization

The model hyperparameters for the neural network artistic practice discriminator model are optimized with a commercially available Bayesian hyperparameter optimization package built around the tree parzen optimization algorithm¹. The optimization loss is based on the model performance after initial training on a restricted subset of the training data, because using the full dataset for this initial optimization step is computationally infeasible. In particular each model was trained for: 6 folds, 10 epochs, and patch size 200 on a reduced student painting dataset consisting of 5 paintings with 4 same artist combinations and 6 different artist combinations. Training loss was calculated as the RMSE distance between the predicted artist similarity and the ground truth, averaged over all painting combinations. The restricted training scheme was employed to reduce the total optimization time on finite computational resources. The considered hyperparameters and ranges are shown in Table S1. The model structure consists of a variable number of dense, fully-connected layers following the VGG16 model, where the number of nodes per layer decreases linearly from a variable maximum at the first dense layer to 2 at the output layer. Maximum posterior estimation was performed on the generated set of all trained models (130 instances) to predict the optimal hyperparameter set. Final optimization of the model shown in the manuscript utilized a full training scheme on the full dataset, as described in the manuscript, and is performed by hand. This is necessary since the full training scheme leads to unreasonably long convergence time in the optimization algorithm. The final hyperparameters align closely with the maximum posterior estimation result, as shown in Figure S1. Note that the largest deviation between the optimized and maximum posterior hyperparameters occurs for the learning rate, which is expected since the longer number of epochs in the full training process allows the model to converge with a smaller learning rate.

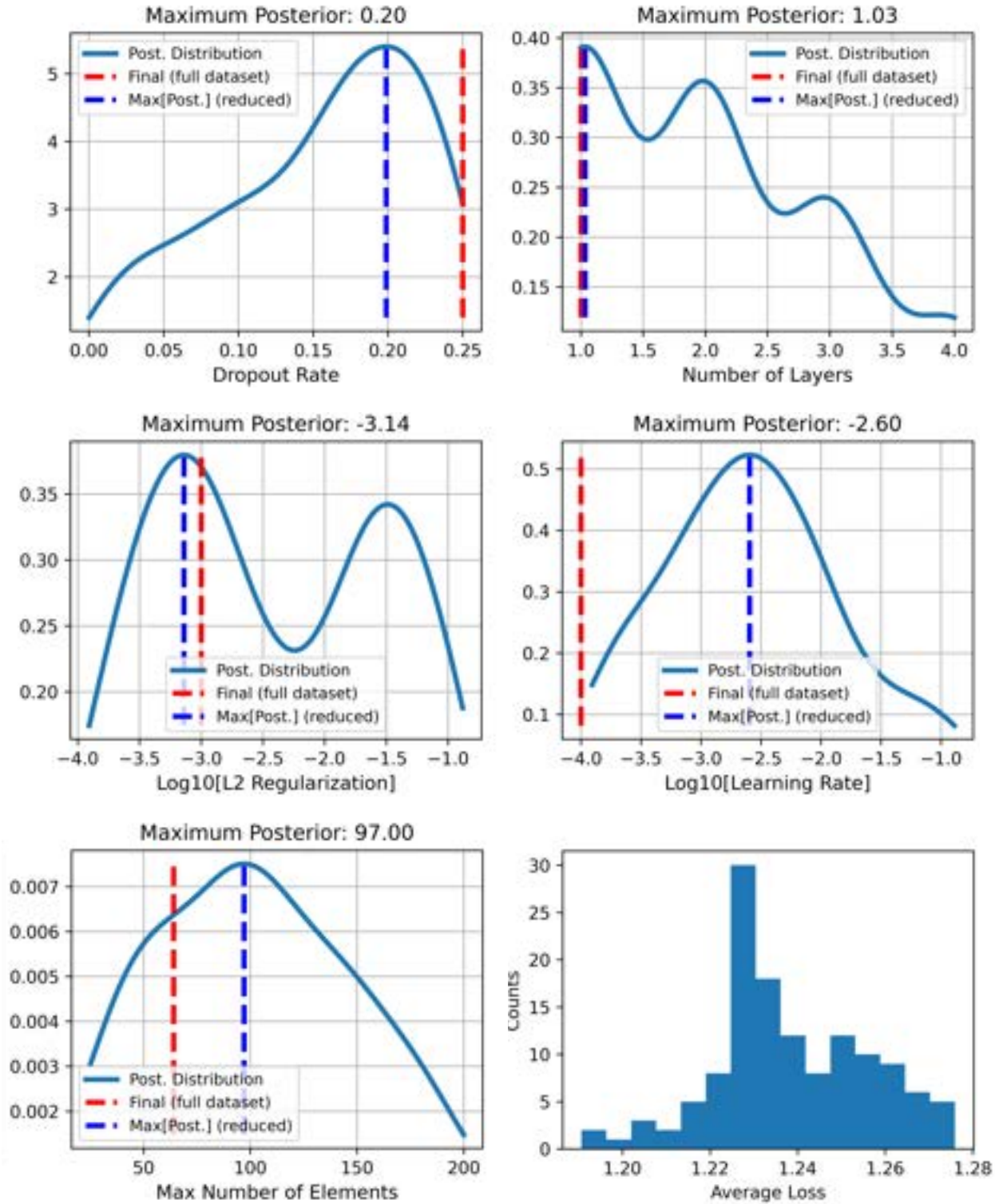


Figure S1: Results of the hyperparameter optimization on a reduced dataset compared to the final model trained on the full dataset, shown in the manuscript. The posterior distribution over all optimization training is shown for each hyperparameter (blue curve), with the maximum posterior from the optimization process (blue dashed line) and the final model hyperparameters (red dashed line) indicated for comparison. The final model hyperparameters align closely with the optimization maximum posterior in most cases. The distribution of the loss for all trained model instances in the optimization is shown in the bottom right panel.

Appendix C: Comparison to alternative methods

Surface roughness method

Pairwise comparison of large numbers of regions via ML is computationally expensive, so a simpler method would be preferable if it were equally accurate. One simple way of distinguishing between regions of a painting using topography would be to compare the “roughness” of the surfaces. We applied a roughness-based method to the student paintings to determine its accuracy relative to the PATCH method. We used the standard deviation of height values as a measure of roughness, and calculated the standard deviation of height values for each 1x1 cm² patch of each student painting. We then compared paintings pairwise using the pairwise Wilcoxon rank sum test implemented in R using the stats package. A significant result on the pairwise Wilcoxon test means the distributions (specifically the medians) are significantly different, which in this context would indicate a different-artist pair. Thus, same-artist pairs would be tests where the null hypothesis could not be rejected. Using the Bonferroni correction for multiple tests with $\alpha=0.01$, any pairwise comparison with a p -value greater than 0.00003 was considered a same-artist pair. This method had an overall accuracy of 81.0%, identifying 19 of 23 same-artist pairs (82.6%) and 220 of 277 different artist pairs (79.4%), a significant reduction in accuracy relative to PATCH. This translates to F_1 scores of 0.384 for same-artist pairs and 0.878 for different-artist pairs, an average of 0.631 (Table S2). We conclude that PATCH substantially outperforms roughness for identification of painters. This also indicates that the network is not simply sorting patches of images based on their roughness.

	Precision	Recall	F_1
Same-artist pairs	0.250	0.826	0.384
Different-artist pairs	0.982	0.794	0.878
Average	<i>0.616</i>	<i>0.810</i>	<i>0.631</i>

Table S2: Precision, recall, and overall F_1 for the surface roughness method.

Unsupervised method

As described in the main text, PATCH achieves unsupervised ends by supervised means. How would PATCH fair against fully unsupervised learning on the surface topography, using clustering of dimension-reduced feature vectors for each patch? To facilitate this comparison, we developed an unsupervised pairwise region analysis method, in the spirit of the PATCH approach. We used the student paintings for validation, dividing them up into 1x1 cm² (200 x 200 px) patches, giving 180 patches per painting. We then took every possible pair of paintings, to determine whether each pair could be distinguished (marked as “different artist”) or not (marked as “same artist”). For a given pair of paintings (360 patches total) we ran each patch through the following unsupervised workflow:

- *Feature extraction:* each patch was converted to a feature vector using a 2D Fourier transform, which created a Fourier domain 200x200 patch with the origin in the center. We took the absolute values of the upper half of the domain (the bottom was identical by symmetry), leaving out the 13 central columns. These columns correspond to the largest wavelengths (comparable to the patch size), and deemphasizing them puts the focus on brushstroke details at smaller wavelengths. We flattened the absolute values into a vector of size 17578. Other parameter choices gave comparable or worse final performance. We also tried an

alternative feature extractor based on the VGG16 convolutional neural network (CNN) that had been trained to classify height data in our earlier student painting controlled experiment², with the bottom layers of the network cut out. This performed worse than the Fourier feature extractor, so we stayed with the latter.

- *Dimension reduction*: to allow eventual clustering of the feature vectors, we used the UMAP dimension reduction algorithm³, with default parameters, to map the full feature vectors onto 4-dimensional vectors.
- *Clustering*: Finally, we used the flat clustering module of HDBSCAN⁴ (minimum cluster size = 25, cluster selection method = ‘leaf’) to force the algorithm to group the 4-dimensional vectors into two clusters.

If there was a statistically significant difference in the numbers of patches assigned to each cluster among the two paintings (chi-squared test with $p < 0.05$), then the pair of paintings were marked as different artist. For example, if the labeled patches in one painting were mostly assigned to cluster 1, and the labeled patches in the other painting to cluster 2, this would be taken as evidence that different artists were involved. Otherwise (i.e. if cluster assignments were fairly evenly distributed among the two paintings) the pair was marked as same artist. The algorithm occasionally failed to find two clusters at all, which we also treated as evidence that the pair was made by the same artist. The final results were quite similar to the surface roughness approach described above: the method correctly identified 18 out of 23 same-artist pairs (78.3%), and 225 out of 277 different-artist pairs (81.2%), yielding F_1 scores of 0.387 and 0.888 for the same-artist and different-artist cases respectively. Changing the HDBSCAN parameters like minimum cluster size and the cluster selection method did not improve the results, and in all cases the approach performed significantly worse than the PATCH method.

	Precision	Recall	F_1
Same-artist pairs	0.257	0.783	0.387
Different-artist pairs	0.978	0.812	0.888
Average	<i>0.618</i>	<i>0.797</i>	<i>0.637</i>

Table S3: Precision, recall, and overall F_1 for the unsupervised method.

Appendix D: Additional Figures



Figure S2: Art historical attributions of portions of the painting. Areas attributed to El Greco are overlaid in purple, Jorge Manuel in gold, both artists in red, and a third hand or hands in light blue.

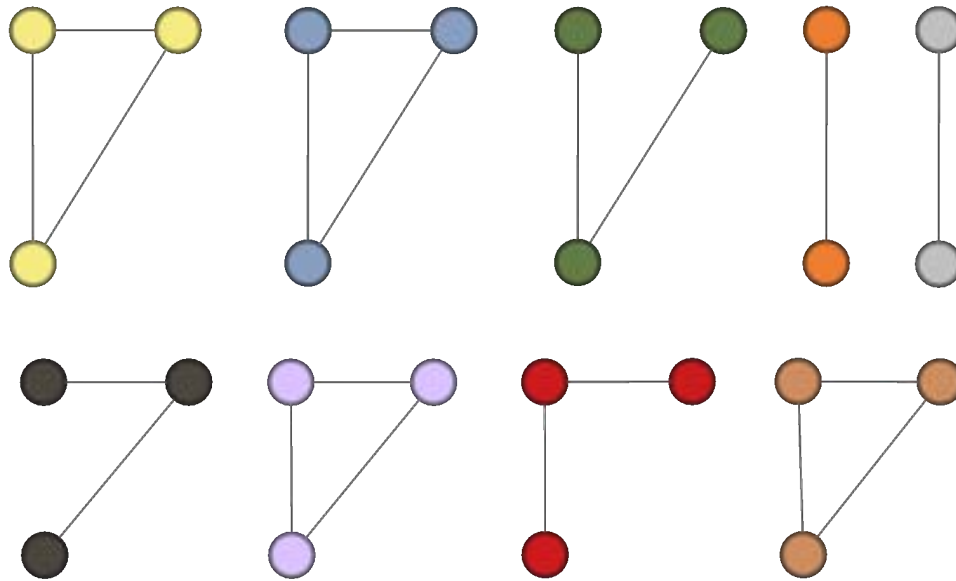


Figure S3: Network diagram for the student paintings with spurious edges trimmed. Nine disjoint communities are identified, corresponding to the individual artists.

Supplementary References

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