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Group 4

Great work!

Worksheet 2 - MATH223

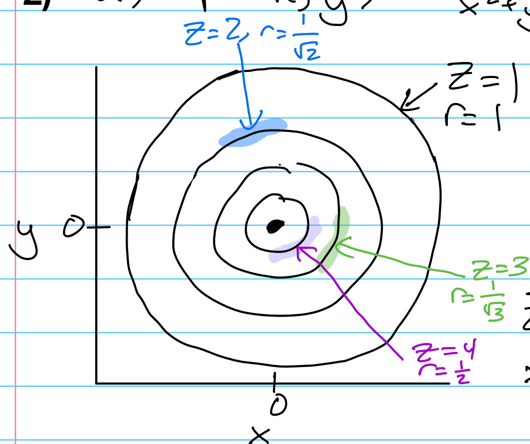
Nice description!

1) • Lake Erie must be "c" because it is abstract in nature and cannot be easily explained by a single equation

• $f_1(x,y) = y^2 - x$ corresponds to "b" due to the following: if x is constant, increasing y-values causes the function value, $f_1(x,y)$, to increase as well. Similarly, if y is held constant, increasing x-values causes $f_1(x,y)$ to decrease. Tracing these scenarios out on the given graphs reveals this relationship to hold true on "b".

• $f_2(x,y) = \frac{1}{x^2+y^2}$ corresponds to "c" and this can be proved by the same reasoning mentioned above. A more robust, yet less fun method, is the process of elimination. A graph on mathematica also reveals this correspondence.

2) a) $f(x,y) = \frac{1}{x^2+y^2}$, chosen z values: $z = \{1, 2, 3, 4$



• Centered @ (0,0)

• Radius decreases as z increases

$$Z = \frac{1}{x^2+y^2}$$

$$x^2+y^2 = \frac{1}{z}, \quad r = \frac{\sqrt{1}}{\sqrt{z}} = \frac{1}{\sqrt{z}}$$

$Z_0 = 1$, $r_0 = 1$

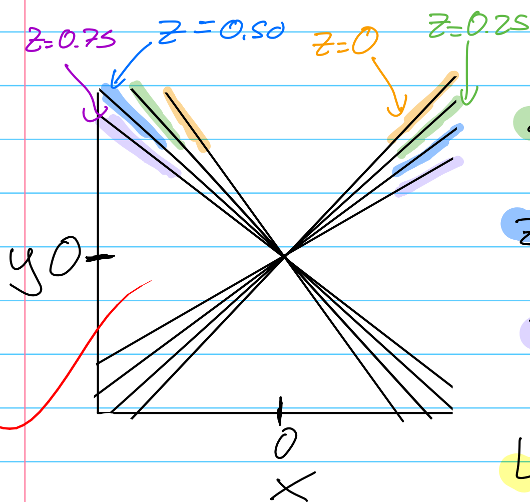
$Z_1 = 2$, $r_1 = \frac{1}{\sqrt{2}}$

$Z_2 = 3$, $r_2 = \frac{1}{\sqrt{3}}$

$Z_3 = 4$, $r_3 = \frac{1}{2}$

b) No, because the contour plot consists of circles that have constant, decreasing radii. The direction/angle you approach the origin does not effect the limit of f. Because the radius can never be equal to 0, the limit of f approaches infinity (or does not exist).

c) $g(x,y)=\frac{x^2-y^2}{x^2+y^2}$, chosen z values: $z=\{0, 0.25, 0.50, 0.75$



• Centered @ (0,0)

$z=\frac{x^2-y^2}{x^2+y^2}$, $z=0$, $0=\frac{x^2-y^2}{x^2+y^2}$ $\quad x^2-y^2=0$

$y^2=x^2 \Rightarrow y=\pm x$

$z=.25$, $0.25(x^2+y^2)=x^2-y^2$

$0.75x^2=1.25y^2$ $\quad y^2=\frac{0.75}{1.25}x^2$

$y=\pm .775x$

$z=0.50$, $0.50(x^2+y^2)=x^2-y^2$ $\quad y^2=\frac{0.50}{1.50}x^2$

$0.50x^2=1.50y^2$ $\quad y=\pm 0.577x$

$z=0.75$, $.75(x^2+y^2)=x^2-y^2$ $\quad y^2=\frac{0.28}{1.75}x^2$

$0.25x^2=1.75y^2$ $\quad y=\pm 0.378x$

Lines w/ tips highlighted the same color have the same z

d) Yes, the direction you approach the origin does effect the limit of g. If you approach from the y direction, where x is held constant, the limit is -1. When y is held constant and the origin is approached from the x direction, the limit 1. Due to the graph having two different limit values at the origin, the limit does not exist.

e) $f(x,y)=\frac{1}{x^2+y^2} \Rightarrow F(r,\theta)=\frac{1}{r^2}$

$g(x,y)=\frac{x^2-y^2}{x^2+y^2} \Rightarrow G(r,\theta)=\frac{r^2\cos^2\theta-r^2\sin^2\theta}{r^2}$

$G(r,\theta)=\cos^2\theta-\sin^2\theta$

F only has r in the polar equation, so this means limits are only determined by the radius. This supports the previous conclusion as when $r\to 0$, $f\to\infty$ (or DNE).

G is dependent on θ, so the limit of G at the origin is also dependent on θ. The limit changes based on the angle you approach the origin, so the limit does not exist.

3) Key Idea: Numerator and Denominator must be cts. and the denom must never equal 0 to be cts.

a) $f_1(x,y,z) = \frac{xy + z \cdot \sin y}{x^2+1}$

x, y, and z are all continuous input in the domain of f_1, so $xy + z\sin y$ must also be cts. because xy and $z\sin y$ are defined for all values of x, y, and z. x^2+1 is also a cts function because x itself is cts. Because x is squared and added to 1, the denom. can never equal zero. All criteria have been met ∴ f_1 is cts.

b) $f_2(x,y,z) = \exp(x^2-y^2)$

x and y are both continuous in the domain of f, so x^2 and y^2 are also cts for all x and all y. x^2-y^2 is also cts. because x^2 and y^2 are cts. e to the power of any real number yields an output, so it is cts for all x and y. All criteria have been met ∴ f_2 is cts.

c) $f_3(x,y,z) = \frac{x-1}{xy-y+xz^2-z^2}$

Denom: $y(x-1) + z^2(x-1) \Rightarrow (y+z^2)(x-1)$

f_3 can be proven discontinuous assuming x, y, and z are cts variables, which is true. When $x=1$, f_3 is undefined as it will be equal to $0/0$. Another discontinuity can be found when $x=0, y=0$, and $z=0$, where the output will be $-1/0$. This yields yet another divide by zero error. Finally, the graph is discontinuous for all y when $y=z^2$. Although only one instance of discontinuity was needed, these instances prove f_3 is dis cts.