BYOE Version 11

1. **(4 pts)** Compute the dot product and the cross product of $u = \langle 2, 5, -9 \rangle$ and $v = \langle 1, 2, 3 \rangle$.

2. Let $r(t) = \langle \exp 2t, \cos(4t), 5t^3 \rangle$ represent the position of a particle.

   (a) **(3 pts)** Determine the particle's velocity and acceleration functions.

   (b) **(2 pts)** Determine the particle's displacement on the interval $[0, 2\pi]$.

   (c) **(3 pts)** Determine the distance traveled by the particle on the interval $[0, 2\pi]$.

3. **(3 pts)** Find the unit tangent vector for the function $r(t) = \langle 6\cos(2t), \sin(13t), 5\cos(t) \rangle$.

4. Consider the lines $L_1(f) = \langle 3t - 7, 2t - 3, 4t + 5 \rangle$ and $L_2(t) = \langle t - 1, 3t - 5, 2t + 1 \rangle$.

   (a) **(2 pts)** Write the symmetric equations of each line, if possible. If this cannot be done, explain why.

   (b) **(3 pts)** Determine whether the lines intersect at a point, are parallel, skew, or the same line.

5. Let $f(x) = yx^4 - x^3 + 2xy^2 - 4y$.

   (a) **(2 pts)** Compute the gradient ∇f.

   (b) **(3 pts)** Compute the directional derivative $D_u f$ at point $P = (1, 3)$ in the direction of $u = \langle 3, 4 \rangle$.

6. **(7 pts)** Let $z = f(x, y) = x^2y - 2xy^2 - 8xy$, $v(s, t) = \langle 2s^2t, 3s^2 - t^2 \rangle$, and $(s, t) = (2, 3)$. Compute $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ using the Chain Rule.

7. **(3 pts)** Let $5 = z\exp(x^2) - 3y$ define z implicitly as a function of x and y. Compute $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial t}$ using implicit differentiation.

8. **(5 pts)** Given the function $f(x) = 2xy - 3x^2 + xy^2 + 3$, find the critical points and classify them with the Second Derivative Test.

9. Let R be the region between $x^2 + y^2 = 9$ and $x^2 + y^2 = 16$, restricted to $y \geq 0$.

   (a) **(2 pts)** Graph R and label the region carefully.

   (b) **(4 pts)** If R has density $\delta(x, y) = x^2 + y^2$, compute the mass of the region.

10. Let R be the region inside the rectangle from $(0, 0)$ to $(2, 3)$. Suppose R describes a lamina of density $\delta(x, y) = 3$.

    (a) **(4 pts)** Find the center of mass of R using double integration.

    (b) **(1 pts)** Briefly explain why your answer to (a) makes sense.

11. **(5 pts)** Compute the surface area of the function $f(x, y) = x^3 - 2x + 4y$ over the region bounded by $y = x^3$, $y = 2 - x^2$, $x = 0$, and $y = 0$.

12. **(2 pts)** Use cylindrical coordinates to prove that a cylinder with radius r and height h has a volume of $V = \pi r^2 h$.

13. **(2 pts)** Explain why it is necessary to insert an extra term in the integrand, such as r or $\rho^2 \sin\varphi$, when switching coordinate systems.

Total points: 60.