Trevor Swan

Section 1.1 - 3, 5, 7, 13, 15, 17

3. $\frac{dP}{dt} = 0.4P(1-\frac{P}{230})$

a) $0 = 0.4P(1-\frac{P}{230})$

$P=0$ or $P=230$

$-$ $+$ $-$

$\longleftarrow 0 \longrightarrow 230 \longrightarrow$

b) Increasing when $0<P<230$

c) decreasing when $P<0$ or $P>230$

5. $\frac{dy}{dt} = y^3-y^2-12y = y(y^2-y-12)$

a) $0 = y(y-4)(y+3)$

$y=0$, $y=4$, and $y=-3$

$-$ $+$ $-$ $+$

$\longleftarrow -3 \longrightarrow 0 \longrightarrow 4 \longrightarrow$

b) Increasing on $-3<y<0$ and $y>4$

c) decreasing on $y<-3$ and $0<y<4$

7. $\frac{dr}{dt} = -\lambda r(t) \longrightarrow r(t) = r_0 e^{-\lambda t}$

a) ^{14}C half life is 5230 years,

$1 = 2e^{-5230\lambda}$ $\lambda = \frac{\ln(\frac{1}{2})}{-5230}$

$\ln(\frac{1}{2}) = \ln(e^{-5230\lambda})$ $\lambda = 0.000133$

b) I-131 half life is 8 days

$1 = 2e^{-8\lambda}$ $\lambda = \frac{\ln(\frac{1}{2})}{-8}$

$\ln(\frac{1}{2}) = \ln(e^{-8\lambda})$ $\lambda = 0.0866$

c) The exponential term must be unitless, so part a's is years^{-1}, and part b is in days^{-1}

d) Half life is not dependent on the amount of an isotope that is present to begin with. As proved by parts a and b, the values used to calculate λ are arbitrary

$\frac{dL}{dt} = K(1-L)$, K is a constant, $L(t)$ is the fraction learned @ time t

13. $0 \le L \le 1$, maximum value at $L=0$

15. $\frac{dL_A}{dt} = 2(1-L_A)$ and $\frac{dL_B}{dt} = 3(1-L_B)^2$

a) $\frac{dL_A}{dt}(0) = 2(1-0) = 2$, $\frac{dL_B}{dt}(0) = 3(1-0)^2 = 3$

Beth's learning rate will be faster at 0 learned

b) $\frac{dL_A}{dt}(\frac{1}{2}) = 2(1-\frac{1}{2}) = 1$, $\frac{dL_B}{dt}(\frac{1}{2}) = 3(1-\frac{1}{2})^2 = 0.75$

Ally's learning rate is faster at $\frac{1}{2}$ learned

c) $\frac{dL_A}{dt}(\frac{1}{3}) = 2(1-\frac{1}{3}) = \frac{4}{3}$, $\frac{dL_B}{dt}(\frac{1}{3}) = 3(1-\frac{1}{3})^2 = \frac{4}{3}$

Beth and Ally are at the same rate at $\frac{1}{3}$ learned

17. $\frac{dF}{dt} = KF(1-\frac{F}{N})$

a) $\frac{dF}{dt} = KF(1-\frac{F}{N}) - 100$

b) $\frac{dF}{dt} = KF(1-\frac{F}{N}) - \frac{1}{3}F$

c) $\frac{dF}{dt} = KF(1-\frac{F}{N}) - \rho\sqrt{F}$

ρ is the proportionality coef.

Section 1.2 - 1, 7, 13, 19, 23, 27, 35, 43

1. $\frac{dy}{dt} = \frac{y+1}{t+1}$

a) Bob: $y_1(t) = t \Rightarrow \frac{dy_1}{dt} = 1$

$\left.\frac{dy}{dt}\right|_{y=t} = \frac{t+1}{t+1} = 1$ yes

Glen: $y_2(t) = 2t+1 \quad \frac{dy_2}{dt} = 2$

$\left.\frac{dy}{dt}\right|_{y=2t+1} = \frac{2t+2}{t+1} = 2$ yes

Paul: $y_3(t) = t^2 - 2 \quad \frac{dy_3}{dt} = 2t$

$\left.\frac{dy}{dt}\right|_{y=t^2-2} = \frac{t^2-1}{t+1} = t-1$ no

b) They shouldn't seen $y=-1$

7. $\frac{dy}{dt} = 2y+1 \quad \int \frac{dy}{2y+1} = \int dt$

$\frac{1}{2}\ln|2y+1| + C_1 = t + C_2 \quad 2y = Ce^{2t} - 1$

$\ln|2y+1| = 2t + C \quad y = \frac{1}{2}(Ce^{2t} - 1)$

$2y+1 = e^{2t+C}$

13. $\frac{dy}{dt} = \frac{t}{t^2y+y} \quad y(t^2+1)dy = dt$

$\int y\,dy = \int \frac{t\,dt}{t^2+1} \quad u = t^2+1, \ \frac{du}{dt} = 2t$

$\frac{1}{2}y^2 + C_1 = \frac{1}{2}\ln|t^2+1| + C_2$

$y^2 = \ln|t^2+1| + C$

$y = \pm\sqrt{\ln|t^2+1| + C}$

19. $\frac{dv}{dt} = t^2v - 2 - 2v + t^2$

$\frac{dv}{dt} = t^2(v+1) - 2(v+1) = (t^2-2)(v+1)$

$\int \frac{dv}{v+1} = \int (t^2-2)\,dt$

$\ln|v+1| + C_1 = \frac{1}{3}t^3 - 2t + C_2$

$v+1 = e^{\frac{1}{3}t^3 - 2t + C} \quad v = Ce^{\frac{1}{3}t^3 - 2t} - 1$

23. $\frac{dw}{dt} = \frac{w}{t} \quad \int \frac{dw}{w} = \int \frac{dt}{t}$

$\ln|w| + C_1 = \ln|t| + C_2$

$\ln|w| = \ln|t| + C$

$w = e^{\ln t + C} \quad w = Ct$ $t \neq 0$

27. $\frac{dy}{dt} = -y^2, \ y(0) = \frac{1}{2}$

$\int -\frac{dy}{y^2} = \int dt \quad \frac{1}{2} = \frac{1}{0+C}$

$y^{-1} + C_1 = t + C_2 \quad C = 2$

$y^{-1} = t + C \quad y = \frac{1}{t+2}$

$y = \frac{1}{t+C}$

35. $\frac{dy}{dt} = (y^2+1)t, \ y(0) = 1$

$\int \frac{dy}{y^2+1} = \int t\,dt \quad 1 = \tan(\frac{1}{2}(0)^2 + C)$

$\tan^{-1}(y) + C_1 = \frac{1}{2}t^2 + C_2 \quad 1 = \tan(C)$

$\tan^{-1}(y) = \frac{1}{2}t^2 + C \quad C = \tan^{-1}(1) = \frac{\pi}{4}$

$y = \tan(\frac{1}{2}t^2 + C) \quad y = \tan(\frac{1}{2}t^2 + \frac{\pi}{4})$

43. $m\frac{dv}{dt} = mg - kv^2$

a) $\frac{dv}{dt} = g - \frac{kv^2}{m}$

$\frac{dv}{dt} = g(1 - \frac{k}{gm}v^2)$

$\frac{dv}{dt} = g(1 - x^2v^2)$ let $x = \sqrt{\frac{k}{gm}}$

$\int \frac{dv}{1-x^2v^2} = \int g\,dt$

$\frac{1}{(1-xv)(1+xv)} = \frac{A}{1-xv} + \frac{B}{1+xv}$

$1 = A(1+xv) + B(1-xv)$

$v = -\frac{1}{x}: \ 1 = B(1+1) \ \therefore \ B = \frac{1}{2}$

$v = \frac{1}{x}: \ 1 = A(1+1) \ \therefore \ A = \frac{1}{2}$

$\frac{1}{2}\int \frac{dv}{1-xv} + \frac{1}{2}\int \frac{dv}{1+xv} = \int g\,dt$

$-\frac{1}{2x}\ln|1-xv| + \frac{1}{2x}\ln|1+xv| + C_1 = gt + C_2$

$\frac{1}{x}(\ln|1+xv| - \ln|1-xv|) = 2gt + C_3$

$\ln\left|\frac{1+xv}{1-xv}\right| = 2gtx + C_4$

$\frac{1+xv}{1-xv} = e^{2gtx + C_4}$

$1 + xv = (1-xv)(Ce^{2gtx})$

$xv = Ce^{2gtx} - xvCe^{2gtx} - 1$

$xv + xvCe^{2gtx} = Ce^{2gtx} - 1$

$v(x + Cxe^{2gtx}) = Ce^{2gtx} - 1$

$v = \frac{Ce^{2gt(\sqrt{\frac{k}{gm}})} - 1}{\sqrt{\frac{k}{gm}} + C(\sqrt{\frac{k}{gm}})e^{2gt(\sqrt{\frac{k}{gm}})}}$

b) $\lim_{t\to\infty} \sqrt{\frac{gm}{k}} \frac{Ce^{2gt(\sqrt{\frac{k}{gm}})} - 1}{Ce^{2gt(\sqrt{\frac{k}{gm}})} + 1}$

$v = \sqrt{\frac{gm}{k}}$ l'hopital