Formula Sheet for Final Test (MATH224- Spring 2024)

- Euler's method for the system

$$\begin{aligned}\frac{dx}{dt} &= f(x,y)\\ \frac{dy}{dt} &= g(x,y)\end{aligned}$$

  - $x_{k+1} = x_k + \Delta t f(x_k, y_k)$
  - $y_{k+1} = y_k + \Delta t g(x_k, y_k)$

- Integrating Factors for $\dfrac{dy}{dt} + g(t)y = b(t)$
  - $\mu(t) = e^{\int g(t)\,dt}$
  - The general solution $y(t) = \dfrac{1}{\mu(t)} \displaystyle\int \mu(t)b(t)\,dt$

- $\mathscr{L}\left[\dfrac{d^2y}{dt^2}\right] = s^2\mathscr{L}[y] - sy(0) - y'(0)$.

- Euler's formula: $e^{a+ib} = e^a\cos(b) + ie^a\sin(b)$.

- Quadratic formula for $ax^2+bx+c=0$: $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

- $\mathbf{Y}(t) = k_1 e^{\lambda_1 t}\vec{v}_1 + k_2 e^{\lambda_2 t}\vec{v}_2$.

- $\vec{v}_1 = (A - \lambda I)\vec{v}_0$, and $\mathbf{Y}(t) = e^{\lambda t}\vec{v}_0 + te^{\lambda t}\vec{v}_1$.

- Simplified Method:
  - Case 1. If λ_1, λ_2 are distint real numbers,

$$\boxed{y(t) = k_1 e^{\lambda_1 t} + k_2 e^{\lambda_2 t}}$$

– Case 2. If $\lambda_1 = \lambda_2 = \lambda$,

$$\boxed{y(t) = k_1 e^{\lambda t} + k_2 t\, e^{\lambda t}}$$

– Case 3. If λ_1, λ_2 are complex numbers, that is, $\lambda_i = \alpha \pm i\beta$ for $i = 1, 2$,

$$\boxed{y(t) = k_1 e^{\alpha t} \sin(\beta t) + k_2 e^{\alpha t} \cos(\beta t)}$$

- Helpful Table:

$g(t)$	**Guess** $y_p(t)$	**NOTE**
$c:$ a constant	$C:$ also a constant	
$a + bt$	$A + Bt$	Even if $a = 0$, $y_p(t) = A + Bt$
$a + bt + ct^2$	$A + Bt + Ct^2$	
$a \sin(\omega t)$	$A \sin(\omega t) + B \cos(\omega t)$	
$b \cos(\omega t)$	$A \sin(\omega t) + B \cos(\omega t)$	
$a \sin(\omega t) + b \cos(\omega t)$	$A \sin(\omega t) + B \cos(\omega t)$	
$ce^{\lambda t}$	$Ce^{\lambda t}, Cte^{\lambda t}$ or $Ct^2 e^{\lambda t}$	Compare with $y_h(t)$ for the correct choice!

Table of Laplace transforms

Table 6.1
Frequently Encountered Laplace Transforms.

$y(t) = \mathcal{L}^{-1}[Y]$	$Y(s) = \mathcal{L}[y]$	$y(t) = \mathcal{L}^{-1}[Y]$	$Y(s) = \mathcal{L}[y]$
$y(t) = e^{at}$	$Y(s) = \dfrac{1}{s-a} \quad (s > a)$	$y(t) = t^n$	$Y(s) = \dfrac{n!}{s^{n+1}} \quad (s > 0)$
$y(t) = \sin \omega t$	$Y(s) = \dfrac{\omega}{s^2 + \omega^2}$	$y(t) = \cos \omega t$	$Y(s) = \dfrac{s}{s^2 + \omega^2}$
$y(t) = e^{at} \sin \omega t$	$Y(s) = \dfrac{\omega}{(s-a)^2 + \omega^2}$	$y(t) = e^{at} \cos \omega t$	$Y(s) = \dfrac{s-a}{(s-a)^2 + \omega^2}$
$y(t) = t \sin \omega t$	$Y(s) = \dfrac{2\omega s}{(s^2 + \omega^2)^2}$	$y(t) = t \cos \omega t$	$Y(s) = \dfrac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$
$y(t) = u_a(t)$	$Y(s) = \dfrac{e^{-as}}{s} \quad (s > 0)$	$y(t) = \delta_a(t)$	$Y(s) = e^{-as}$

Rules of Laplace transforms

Table 6.2
Rules for Laplace Transforms:
Given functions $y(t)$ and $w(t)$ with $\mathcal{L}[y] = Y(s)$ and $\mathcal{L}[w] = W(s)$ and constants α and a.

Rule for Laplace Transform	Rule for Inverse Laplace Transform
$\mathcal{L}\left[\dfrac{dy}{dt}\right] = s\mathcal{L}[y] - y(0) = sY(s) - y(0)$	
$\mathcal{L}[y + w] = \mathcal{L}[y] + \mathcal{L}[w] = Y(s) + W(s)$	$\mathcal{L}^{-1}[Y + W] = \mathcal{L}^{-1}[Y] + \mathcal{L}^{-1}[W] = y(t) + w(t)$
$\mathcal{L}[\alpha y] = \alpha\mathcal{L}[y] = \alpha Y(s)$	$\mathcal{L}^{-1}[\alpha Y] = \alpha\mathcal{L}^{-1}[Y] = \alpha y(t)$
$\mathcal{L}[u_a(t)y(t-a)] = e^{-as}\mathcal{L}[y] = e^{-as}Y(s)$	$\mathcal{L}^{-1}[e^{-as}Y] = u_a(t)y(t-a)$
$\mathcal{L}[e^{at}y(t)] = Y(s-a)$	$\mathcal{L}^{-1}[Y(s-a)] = e^{at}\mathcal{L}^{-1}[Y] = e^{at}y(t)$