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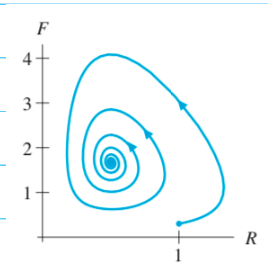
tcs94

Section 2.1: 1, 7, 9, 19

1. In either case, y is the number of predators and x is the number of prey.
(i) is large predators w/ small prey. Large y means large predators → Interactions do not benefit much ($\frac{xy}{20}$ term). Small x means small prey → Decrease rapidly ($-20xy$ term)
(ii) is small predators w/ large prey. Small y means small predators → Greatly benefited by large prey kills ($25xy$ term). Large x means large prey → Population not decreased by much ($\frac{xy}{100}$ term)

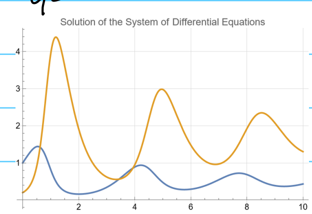
7.



The number of predators and prey will oscillate. As prey population decreases, predator population increases to a point, then it decreases allowing prey to grow. This repeats until equilibrium is reached around (0.5, 1.5)

$$\frac{dR}{dt} = 2\left(1-\frac{R}{3}\right)R - RF$$

$$\frac{dF}{dt} = -2F + 4RF$$



Assumptions were correct based off of Mathematica plot

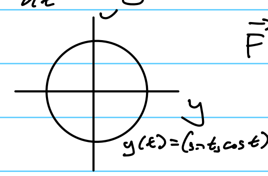
9. Because only prey is being hunted, add 2 to $\frac{dR}{dt}$ function in each system. $\frac{dF}{dt}$ is unchanged, so it is omitted: (i) $\frac{dR}{dt} = 2R - 1.2RF - 2$ (ii) $\frac{dR}{dt} = 2R\left(1-\frac{R}{2}\right) - 1.2RF - 2$

19. $\frac{d^2y}{dt^2} + y = 0$ $\quad y(0) = 0$, $y'(0) = v(0) = 1$

$$\begin{cases} \frac{dy}{dt} = v \\ \frac{dv}{dt} = -y \end{cases}$$

a) is $y(t) = \sin(t)$ a sln?
$y''(t) = -\sin(t)$
$-\sin(t) + \sin(t) = 0$ ✓

b)



$\vec{F}(y,v) = \begin{pmatrix} \frac{dy}{dt} \\ \frac{dv}{dt} \end{pmatrix} = \begin{pmatrix} v \\ -y \end{pmatrix}$

$\left.\begin{array}{l} v = -\cos t \\ y = \sin t \end{array}\right\} \vec{F} = (\cos t, -\sin t)$

c) Exact same curve as Figure 2.15.

d) Initial condition and parametrization are different.

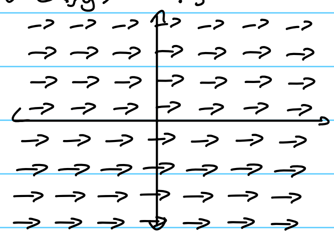
Note: I'm not an expert at mathematics, so my plots may be wrong in some parts.

Section 2.2: 1, 5, 9, 11, 21

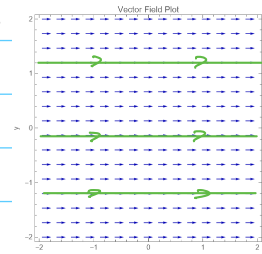
1. $\dfrac{dx}{dt} = 1$, $\dfrac{dy}{dt} = 0$

a) $V(x,y) = \langle 1, 0 \rangle$

b)



c)

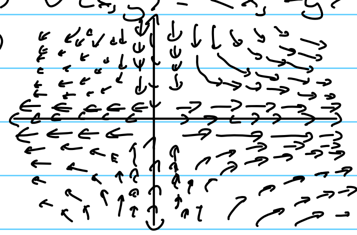


e) Solutions are horizontal and remain constant $\forall t$

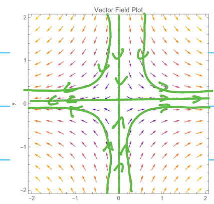
5. $\dfrac{dx}{dt} = x$, $\dfrac{dy}{dt} = -y$

a) $V(x,y) = \langle x, -y \rangle$

b)

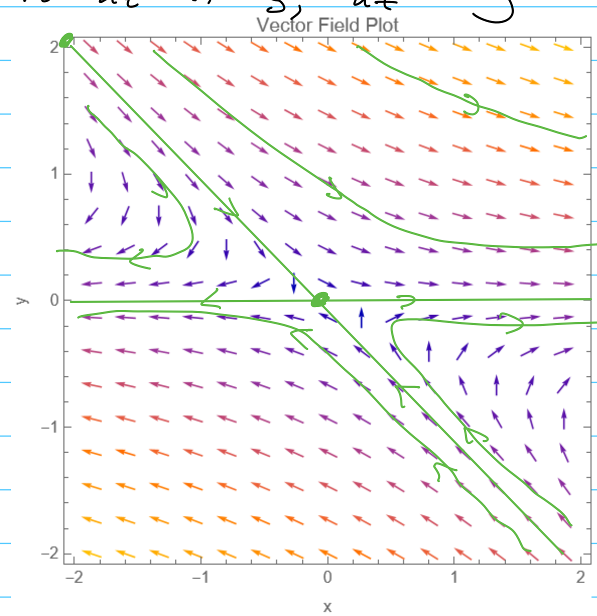


c)



e) Vertical line @ $x=0$ treats $(0,0)$ as a sink, but $y=0$ Horizontal is a source. Other solutions approach these lines as $t \to \infty$.

9. $\dfrac{dx}{dt} = x + 2y$, $\dfrac{dy}{dt} = -y$

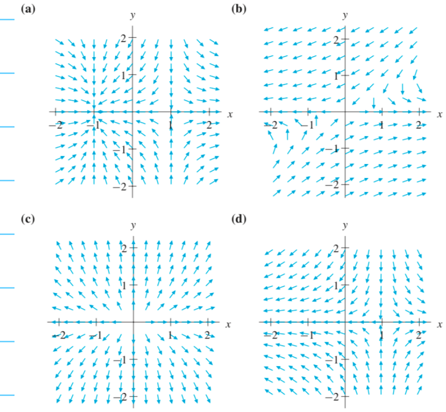


b) Along $y = -x$, All solutions tend to 0 as $t \to \infty$. Since $(2,-2)$ lies on this line, its solution curve tends to 0 as $t \to \infty$ as well.

11.

(i) $\dfrac{dx}{dt} = -x$, $\dfrac{dy}{dt} = y - 1$ (ii) $\dfrac{dx}{dt} = x^2 - 1$, $\dfrac{dy}{dt} = y$ (iii) $\dfrac{dx}{dt} = x + 2y$, $\dfrac{dy}{dt} = -y$ (iv) $\dfrac{dx}{dt} = 2x$, $\dfrac{dy}{dt} = y$

~~(v)~~ $\dfrac{dx}{dt} = x$, $\dfrac{dy}{dt} = 2y$ ~~(vi)~~ $\dfrac{dx}{dt} = x - 1$, $\dfrac{dy}{dt} = -y$ ~~(vii)~~ $\dfrac{dx}{dt} = x^2 - 1$, $\dfrac{dy}{dt} = -y$ (viii) $\dfrac{dx}{dt} = x - 2y$, $\dfrac{dy}{dt} = -y$



a) $\dfrac{dx}{dt} = 0$ @ $x = -1, 1$, y changes dir based on sign. must be vii

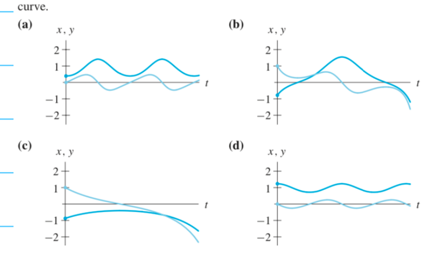
b) y behaves opposite its sign $(-y)$. Slope @ $(1,1) < 0$, must be viii

c) $(0,0)$ is a source. @ $(1,1)$, slope > 1. Must be v as $(1,1) \to \frac{2}{1}$

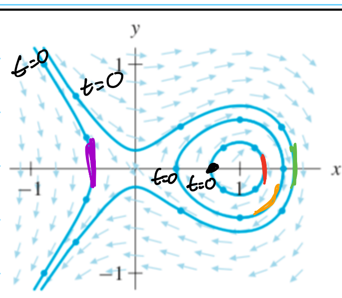
d) $\dfrac{dx}{dt} = 0$ @ $x = 1$ but not @ $x \neq 1$. Must be vi to satisfy.

21.

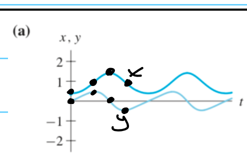
Match each solution curve with its corresponding pair of $x(t)$- and $y(t)$-graphs. Then on the t-axis mark the t-values that correspond to the distinguished points along the curve.



Times marked by "•" on graphs below

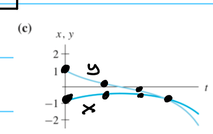


a)



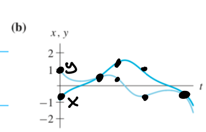
Orange
Almost a Circle

c)



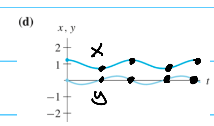
Purple
Diverges

b)



Green
Oval → Diverges

d)



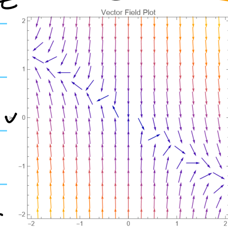
Red
Circle almost Perfectly

Section 2.3: 1, 3, 7

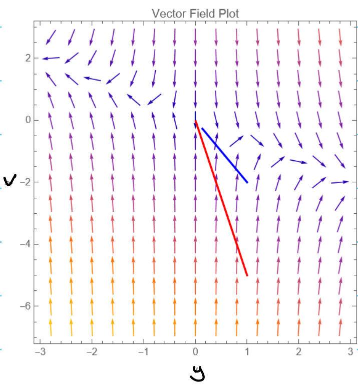
1. $\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 10y = 0$

$$\begin{cases} \frac{dy}{dt} = v \\ \frac{dv}{dt} + 7v + 10y = 0 \end{cases}$$

a)



Mathematica Plot



b) Let $y(t) = \alpha e^{st}$

$\therefore \alpha s^2 e^{st} + 7\alpha s e^{st} + 10\alpha e^{st} = 0$

$s^2 + 7s + 10 = 0$

$(s+5)(s+2) = 0 \rightarrow s=-5$ and $s=-2$

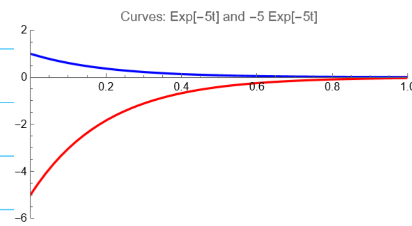
α is arbitrary here

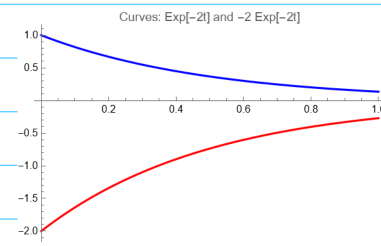
$\therefore y_1(t) = e^{-5t}$ and $y_2(t) = e^{-2t}$

$v_1 = \frac{dy_1}{dt} = -5e^{-5t}$ $\quad Y_1 = (e^{-5t}, -5e^{-5t})$

$v_2 = \frac{dy_2}{dt} = -2e^{-2t}$ $\quad Y_2 = (e^{-2t}, -2e^{-2t})$

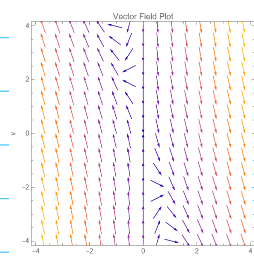
c) Plotting the solution curves

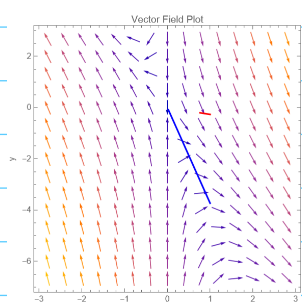




3. $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + y = 0$

$$\begin{cases} \frac{dy}{dt} = v \\ \frac{dv}{dt} + 4v + y = 0 \end{cases}$$





b) Let $y(t) = \alpha e^{st}$

$\alpha s^2 e^{st} + 4\alpha s e^{st} + \alpha e^{st} = 0$

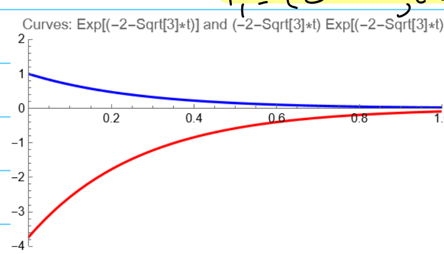
$s^2 + 4s + 1 = 0$

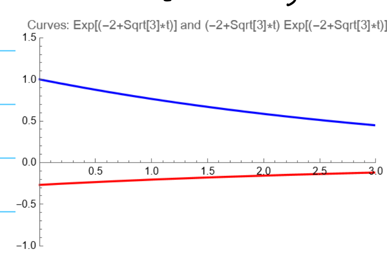
$s = \frac{-4 \pm \sqrt{16-4}}{2}, \quad s = \frac{-4 \pm \sqrt{12}}{2}$

$s = -2+\sqrt{3} \quad s = -2-\sqrt{3}$

$y_1(t) = e^{(-2+\sqrt{3})t}, \quad y_2(t) = e^{(-2-\sqrt{3})t}$

$Y_1 = (e^{(-2+\sqrt{3})t}, (-2+\sqrt{3})e^{(-2+\sqrt{3})t}) \quad Y_2 = (e^{(-2-\sqrt{3})t}, (-2-\sqrt{3})e^{(-2-\sqrt{3})t})$





7. $m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = 0$

a) Suppose $y_1(t)$ is a solution and $\alpha \in \mathbb{R}$

Let $y = \alpha y_1(t)$

Then: $m\frac{d^2\alpha y_1(t)}{dt^2} + b\frac{d\alpha y_1(t)}{dt} + k\alpha y_1(t) = 0$

$= \alpha\left(m\frac{d^2y_1}{dt^2} + b\frac{dy_1}{dt} + ky_1\right) = 0$

$= \alpha(0) = 0 \rightarrow 0=0$ ✓ as y_1 is a solution

b) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 0$ $\quad \begin{cases} \frac{dy}{dt} = v \\ \frac{dv}{dt} + 3v + 2y = 0 \end{cases}$

Let $y(t) = \alpha e^{st}$

$\therefore \alpha s^2 e^{st} + 3\alpha s e^{st} + 2\alpha e^{st} = 0$

$s^2 + 3s + 2 = 0$

$(s+2)(s+1) = 0 \rightarrow s = -2, -1$

$y_1(t) = e^{-2t}\alpha$ and $y_1(t) = e^{-t}\alpha$

c) Infinite Solutions for $\alpha \in \mathbb{R}$