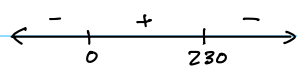


## Section 1.1 - 3, 5, 7, 13, 15, 17

3.  $\frac{dP}{dt} = 0.4P(1 - \frac{P}{230})$

a)  $0 = 0.4P(1 - \frac{P}{230})$

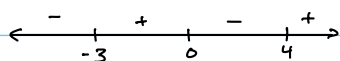
$P = 0 \text{ or } P = 230$

b) Increasing when  $0 < P < 230$ c) decreasing when  $P < 0$  or  $P > 230$ 

5.  $\frac{dy}{dt} = y^3 - y^2 - 12y = y(y^2 - y - 12)$

a)  $0 = y(y-4)(y+3)$

$y = 0, y = 4, \text{ and } y = -3$

b) Increasing on  $-3 < y < 0$  and  $y > 4$ c) decreasing on  $y < -3$  and  $0 < y < 4$ 

7.  $\frac{dr}{dt} = -\lambda r(t) \rightarrow r(t) = r_0 e^{-\lambda t}$

a)  $\hookrightarrow$  half life is 5230 years  
 $1 = 2e^{-5230\lambda} \quad \lambda = \frac{\ln(\frac{1}{2})}{-5230}$

$\ln(\frac{1}{2}) = \ln(e^{-5230\lambda}) \quad \lambda = 0.000133$

b) I-131 half life is 8 days

$1 = 2e^{-8\lambda} \quad \lambda = \frac{\ln(\frac{1}{2})}{-8}$

$\ln(\frac{1}{2}) = \ln(e^{-8\lambda}) \quad \lambda = 0.0866$

c) The exponential term must be unitless, so part a is in  $\text{years}^{-1}$ and part b is in  $\text{days}^{-1}$ d) Half life is not dependent on the amount of an isotope that is present to begin with. As proved by parts a and b, the values used to calculate  $\lambda$  are arbitrary

$\frac{dL}{dt} = K(1-L)$ ,  $K$  is a constant,  $L(t)$  is the fraction learned @ time  $t$

13.  $0 \leq L \leq 1$ , maximum value at  $L=0$

15.  $\frac{dL_A}{dt} = 2(1-L_A)$  and  $\frac{dL_B}{dt} = 3(1-L_B)^2$

u)  $\frac{dL_A}{dt}(0) = 2(1-0) = 2$ ,  $\frac{dL_B}{dt}(0) = 3(1-0)^2 = 3$

Beth's learning rate will be faster at 0 learned

b)  $\frac{dL_A}{dt}(\frac{1}{2}) = 2(1-\frac{1}{2}) = 1$ ,  $\frac{dL_B}{dt}(\frac{1}{2}) = 3(1-\frac{1}{2})^2 = 0.75$

Allie's Learning rate is faster at  $\frac{1}{2}$  learned

c)  $\frac{dL_A}{dt}(\frac{1}{3}) = 2(1-\frac{1}{3}) = \frac{4}{3}$ ,  $\frac{dL_B}{dt}(\frac{1}{3}) = 3(1-\frac{1}{3})^2 = \frac{4}{3}$

Both end Allie are at the same rate at  $\frac{1}{3}$  learned

17.  $\frac{dF}{dt} = KF(1 - \frac{F}{N})$

a)  $\frac{dF}{dt} = KF(1 - \frac{F}{N}) - 100$

b)  $\frac{dF}{dt} = KF(1 - \frac{F}{N}) - \frac{1}{3}F$

c)  $\frac{dF}{dt} = KF(1 - \frac{F}{N}) - \rho\sqrt{F}$

 $\rho$  is the proportionality coef.

# Section 1.2 - 1, 7, 13, 19, 23, 27, 35, 43

$$1. \frac{dy}{dt} = \frac{y+1}{t+1}$$

$$a) \text{ Bob: } y(t) = t \Rightarrow \frac{dy}{dt} = 1$$

$$\left. \frac{dy}{dt} \right|_{y=6} = \frac{t+1}{t+1} = 1 \quad \text{yes}$$

$$\text{Glen: } y(t) = 2t+1 \quad \frac{dy}{dt} = 2$$

$$\left. \frac{dy}{dt} \right|_{y=2t+1} = \frac{2t+2}{t+1} = 2 \quad \text{yes}$$

$$\text{Paul: } y(t) = t^2 - 2 \quad \frac{dy}{dt} = 2t$$

$$\left. \frac{dy}{dt} \right|_{y=t^2-2} = \frac{t^2-1}{t+1} = t-1 \quad \text{no}$$

$$b) \text{ They should've seen } y = -1$$

$$7. \frac{dy}{dt} = 2y+1 \quad \int \frac{dy}{2y+1} = \int dt$$

$$\frac{1}{2} \ln|2y+1| + C_1 = t + C_2 \quad 2y = Ce^{2t} - 1$$

$$\ln|2y+1| = 2t + C \quad y = \frac{1}{2}(Ce^{2t} - 1)$$

$$2y+1 = e^{2t+C}$$

$$13. \frac{dy}{dt} = \frac{t}{t^2+y^2} \quad y(t^2+1)dy = dt$$

$$\int y dy = \int \frac{t dt}{t^2+1} \quad u = t^2+1 \quad \frac{du}{dt} = 2t$$

$$\frac{1}{2} y^2 + C_1 = \frac{1}{2} \ln|t^2+1| + C_2$$

$$y^2 = \ln|t^2+1| + C$$

$$y = \pm \sqrt{\ln|t^2+1| + C}$$

$$19. \frac{dv}{dt} = t^2 v - 2 - 2v + t^2$$

$$\frac{dv}{dt} = t^2(v+1) - 2(v+1) + t^2 - 2(v+1)$$

$$\int \frac{dv}{v+1} = \int (t^2 - 2) dt$$

$$\ln|v+1| + C_1 = \frac{1}{3} t^3 - 2t + C_2$$

$$v+1 = e^{\frac{1}{3} t^3 - 2t + C} \quad v = Ce^{\frac{1}{3} t^3 - 2t} - 1$$

$$23. \frac{dw}{dt} = \frac{w}{t} \quad \int \frac{dw}{w} = \int \frac{dt}{t}$$

$$\ln|w| + C_1 = \ln|t| + C_2$$

$$\ln|w| = \ln|t| + C$$

$$w = e^{\ln t + C} \quad w = Ct \quad t \neq 0$$

$$27. \frac{dy}{dt} = -y^2, \quad y(0) = \frac{1}{2}$$

$$\int -\frac{dy}{y^2} = \int dt \quad \frac{1}{2} = \frac{1}{0+C}$$

$$y^{-1} + C_1 = t + C_2 \quad C = 2$$

$$y^{-1} = t + C$$

$$y = \frac{1}{t+C}$$

$$y = \frac{1}{t+2}$$

$$35. \frac{dy}{dt} = (y^2+1)t, \quad y(0) = 1$$

$$\int \frac{dy}{y^2+1} = \int t dt \quad 1 = \tan(\frac{1}{2}(0)^2 + C)$$

$$\tan^{-1}(y) + C_1 = \frac{1}{2} t^2 + C_2 \quad 1 = \tan(C)$$

$$\tan^{-1}(y) = \frac{1}{2} t^2 + C \quad C = \tan^{-1}(1) = \frac{\pi}{4}$$

$$y = \tan(\frac{1}{2} t^2 + C) \quad y = \tan(\frac{1}{2} t^2 + \frac{\pi}{4})$$

$$43. m \frac{dv}{dt} = mg - kv^2$$

$$a) \frac{dv}{dt} = g - \frac{Kv^2}{m}$$

$$\frac{dv}{dt} = g(1 - \frac{K}{gm} v^2)$$

$$\frac{dv}{dt} = g(1 - x^2 v^2) \quad \text{let } x = \sqrt{\frac{K}{gm}} \quad v = \sqrt{\frac{gm}{K}} \quad \text{1'hop 1}$$

$$\int \frac{dv}{1-x^2 v^2} = \int g dt$$

$$\frac{1}{(1-xv)(1+xv)} = \frac{A}{1-xv} + \frac{B}{1+xv}$$

$$1 = A(1+xv) + B(1-xv)$$

$$v = -\frac{1}{x} \cdot 1 = B(1+1) \therefore B = \frac{1}{2}$$

$$v = \frac{1}{x} \cdot 1 = A(1+1) \therefore A = \frac{1}{2}$$

$$\frac{1}{2} \int \frac{dv}{1-xv} + \frac{1}{2} \int \frac{dv}{1+xv} = \int g dt$$

$$-\frac{1}{2x} \ln|1-xv| + \frac{1}{2x} \ln|1+xv| + C_1 = gt + C_2$$

$$\frac{1}{x} (\ln|1+xv| - \ln|1-xv|) = 2gt + C_3$$

$$\ln \left| \frac{1+xv}{1-xv} \right| = 2gtx + C_4$$

$$\frac{1+xv}{1-xv} = e^{2gtx + C_4}$$

$$1+xv = (1-xv)(Ce^{2gtx})$$

$$xv = Ce^{2gtx} - xvCe^{2gtx} - 1$$

$$xv + xvCe^{2gtx} = Ce^{2gtx} - 1$$

$$v(x + Ce^{2gtx}) = Ce^{2gtx} - 1$$

$$v = \frac{Ce^{2gtx} - 1}{x + Ce^{2gtx}}$$

$$v = \frac{Ce^{\frac{2gt\sqrt{K}}{gm}} - 1}{\sqrt{\frac{K}{gm}} + C(\sqrt{\frac{K}{gm}})e^{\frac{2gt\sqrt{K}}{gm}}}$$