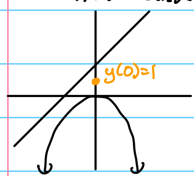
Section 1.5 - 3, 5, 9, 11

3. $\frac{dy}{dt} = f(t,y)$, f satisfies Uniqueness Thm. for all t in the ty-plane

- $y_1(t) = t+2$ for all t is a solution
- $y_2(t) = -t^2$ for all t is a solution
- Initial condition: $y(0)=1$



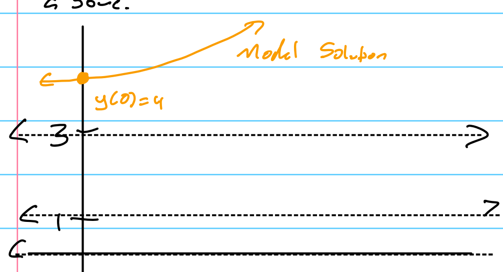
Because f(t,y) satisfies the Uniqueness theorem, solutions cannot cross, so the solution to this DE containing the IV $y(0)=1$ must satisfy the following: $y_2(t) < y(t) < y_1(t)$

or $-t^2 < y(t) < t+2$,

where y(t) is a solution containing $y(0)=1$.

5. $\frac{dy}{dt} = y(y-1)(y-3)$, $y(0)=4$

$\frac{dy}{dt}$ is a polynomial, so it and its partial derivatives are defined and continuous in the ty plane for all inputs. Existence thm. says there must be a solution containing the point $y(0)=4$. Uniqueness thm. says that the solution must be greater than $3 \, \forall t$, the closest equilibrium solution. We can also observe that the solution will always be increasing with this IV. These observations are guaranteed by the criteria presented above.



9. a) Show $y_1(t)=t^2$ and $y_2(t)=t^2+1$ are solutions to: $\frac{dy}{dt} = -y^2+y+2yt^2+2t-t^2-t^4$

$y_1(t)=t^2$: $y_1' = 2t$

$2t = \cancel{-t^4} + \cancel{t^2} + \cancel{2t^4} + 2t - \cancel{t^2} - \cancel{t^4}$

$2t \equiv 2t$ ∴ $y_1(t)=t^2$ is a solution

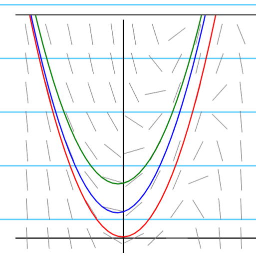
$y_2(t)=t^2+1$: $y_2'=2t$ and $y_2^2=t^4+2t^2+1$

$2t = \cancel{-t^4} - \cancel{2t^2} - \cancel{1} + \cancel{t^2} + \cancel{1} + \cancel{2t^4} + \cancel{2t^2} + 2t - \cancel{t^2} - \cancel{t^4}$

$2t \equiv 2t$ ∴ $y_2(t)$ is a solution

b) f(t,y) is cts everywhere, so we can draw a rectangle R of arbitrary length to contain a solution y(t). This uses existence thm. to prove that a solution y(t) exists and solves the DE. Because the IV presented lies between the functions y_1 and y_2, we know, by uniqueness thm, that the solution curve y(t) will not cross y_1 or y_2. This affirms the inequalities: $0 < y(0) < 1$

and $t^2 < y(t) < t^2+1$ for all t

c) 

The red graph is $y_1 = t^2$, green is $y_2 = t^2+1$. Notice how blue y(t) doesn't intersect either of the solutions

11. $\frac{dy}{dt} = \frac{y}{t^2}$

a) $y_1(t)=0$ ∴ $y_1'=0$

$0 = \frac{0}{t^2} \Rightarrow 0 \equiv 0$ yes, $y_1(t)$ is a solution

b) $\frac{dy}{dt} = \frac{y}{t^2} \Rightarrow \int \frac{dy}{y} = \int t^{-2}\,dt$

$\ln|y| = -t^{-1} + C$

$|y| = e^{-\frac{1}{t}} \cdot e^C$

$y = Ke^{-\frac{1}{t}}$, $K = \pm e^C$

undefined for $t = 0$

$y(t) = 0$ for $t \le 0$

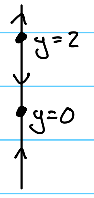
$y(t) = Ke^{-\frac{1}{t}}$ for $t > 0$

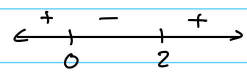
c) $y(t) = Ke^{-\frac{1}{t}}$ can never equal zero, but $y(t)=0$ can only equal zero, so all solutions are unique thus validating the uniqueness theorem.

Section 1.6 - 1, 5, 9, 15, 23, 25

1. $\frac{dy}{dt} = 3y(y-2)$

eq. solutions: $y=0$, $y=2$





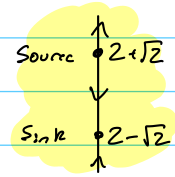
$y=2$ is a source

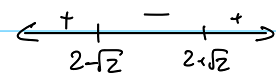
$y=0$ is a sink

23/25. $\frac{dy}{dt} = y^2 - 4y + 2$, $0 = y^2 - 4y + 2$

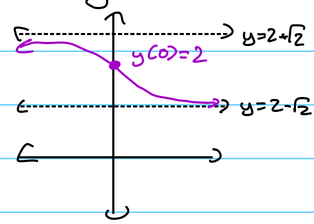
$y = \frac{4 \pm \sqrt{16-8}}{2}$

eq. sls: $y = 2 \pm \sqrt{2}$





23. $y(0) = 2$

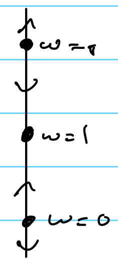


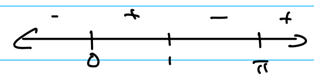
$\frac{dy}{dt}$ is negative at $y(0)=2$, so the long term behavior of solution curve will tend toward the closest eq solution less than it, in this case $y = 2-\sqrt{2}$

Uniqueness thm. guarantees that this solution cannot cross the others. $y = 2+\sqrt{2}$ will be tended towards as $t \to -\infty$

5. $\frac{dw}{dt} = (1-w)\sin(w)$

eq. solutions: $w = 1$, $w = \pi n$, $n \in \mathbb{Z}$



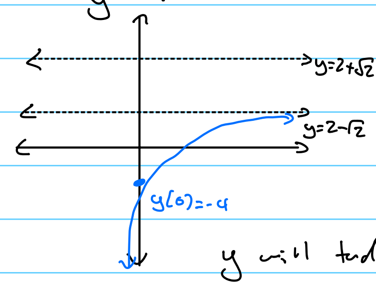


This behavior is observed for all $w = \pi n$

$w=1$ is a sink

$w=0$, $w=\pi$, and $w=2\pi n$ are sources

25. $y(0) = -4$

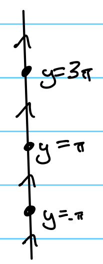


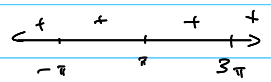
$\frac{dy}{dt}$ is positive at $y(0)=-4$, so as t increases, the solution curve will tend toward the next greatest eq. solution. It is $2-\sqrt{2}$ in this case.

y will tend towards $-\infty$ as $t \to -\infty$

9. $\frac{dy}{dt} = 1 + \cos(y)$

eq. solutions: $y = \pi n$, $n \in \mathbb{Z}$





$\left.\begin{array}{l} y = -\pi \\ y = \pi \\ y = 3\pi \end{array}\right\}$ all nodes? $y = \pi n$ for $n \in \mathbb{Z}$ and $n \neq 0$

15. $\frac{dy}{dt} = \cos y$; eq.: $y = \frac{\pi}{2} + \pi n$, $n \in \mathbb{Z}$

Phase line