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tcs94

Section 1.3 - 1, 3, 7, 9, 15, 17

1\. $\frac{dy}{dt} = t^2 + t$

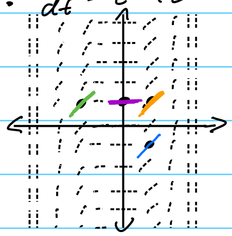
a) $(-2,1): y' = 2$

$(0,1): y' = 0$

$(1,-1): y' = 2$

$(1,1): y' = 2$

b/c) checked on desmos



3\. $\frac{dy}{dt} = 1 - 2y$

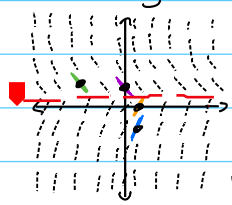
a) $(-2,1): y' = -1$

$(0,1): y' = -1$

$(1,-1): y' = 3$

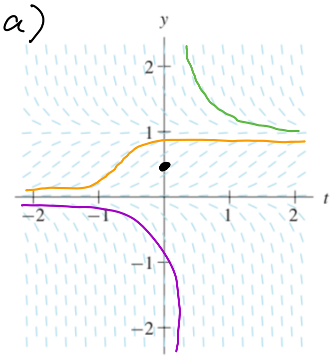
$(1,0): y' = 1$

b/c) checked on desmos



7\. $\frac{dy}{dt} = 3y(1-y)$

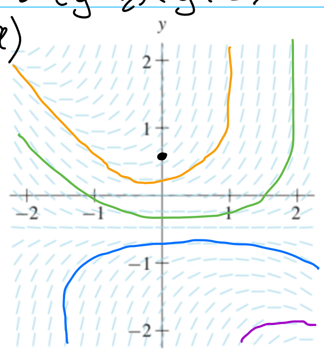
a)



b) $y(0) = \frac{1}{2}$ starts with a near-linear slope and approaches $y=1$, a solution to this DE, as t increases.

9\. $(y + \frac{1}{2})(y + t)$

a)



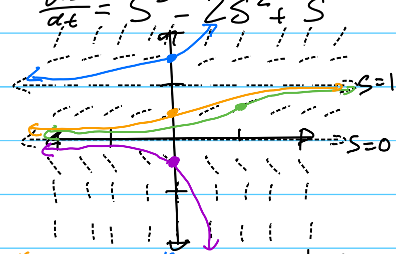
b) At the point $y(0) = \frac{1}{2}$, the curve solution will approach infinitely as t increases

15\. $\frac{dS}{dt} = S^3 - 2S^2 + S$

$S' = S(S^2 - 2S + 1)$
$= S(S-1)(S-1)$

$S = 0,\ S = 1$

Slopes along horizontal lines are identical



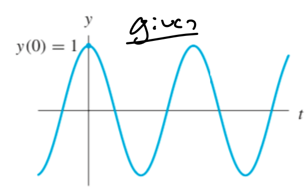
$S(0) = \frac{1}{2}$, $S(0) = \frac{3}{2}$ | $S'(-2) = -18$, $S'(-1) = -4$, $S'(1) = 0$, $S'(2) = 2$

$S(1) = \frac{1}{2}$, $S(0) = -\frac{1}{2}$ | $S'(-\frac{3}{2}) = -9\frac{3}{8}$, $S'(-\frac{1}{2}) = -1\frac{1}{8}$, $S'(\frac{1}{2}) = \frac{1}{8}$, $S'(\frac{3}{2}) = \frac{3}{8}$

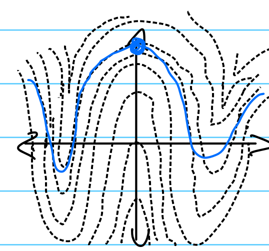
The curves do not pass equilibrium slns. or other curves

17\. $\frac{dy}{dt} = f(t)$

given



=>



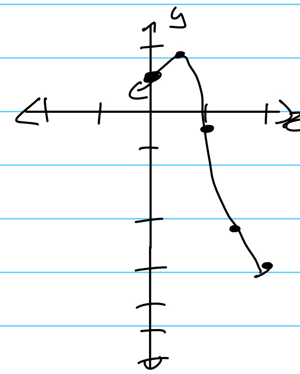
a) The function depends only on t, so the slope field will look identical to the curve given, with its slopes copied over vertical lines.

b) $y(0) = 2$ is shown in blue, it looks similar to given, just translated up by 1.

Section 1.4 - 3, 7, 15, 21

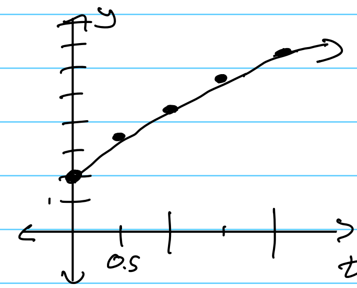
3. $\frac{dy}{dt} = y^2 - 4t$, $y(0)=0.5$, $0 \le t \le 2$, $\Delta t = 0.5$

k	t_k	y_k	$y_{k+1} = y_k + \Delta t(y'(t_k))$
0	0	0.5	$(0, 0.5)$: $y = 0.5 + 0.5(0.5^2 - 0) = 0.625$, $(0.5, 0.625)$
1	0.5	0.625	$(0.5, 0.625)$: $y = 0.625 + 0.5(.625^2 - 4(.5)) = -.18$, $(1, -.18)$
2	1	-.18	$(1, -.18)$: $y = -.18 + 0.5(-.18^2 - 4(1)) = -2.2$, $(1.5, -2.2)$
3	1.5	-2.2	$(1.5, -2.2)$: $y = -2.2 + 0.5(-2.2^2 - 4(1.5)) = -2.78$, $(2, -2.78)$
4	2	-2.78	



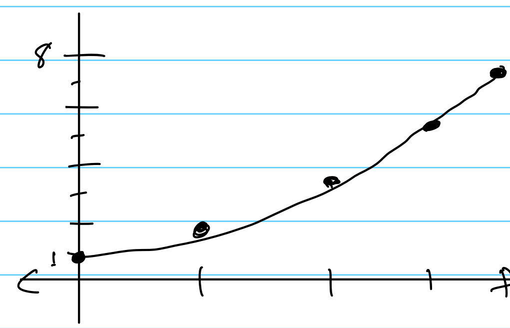
7. $\frac{dy}{dt} = e^{2/y}$, $y(0)=2$, $0 \le t \le 2$, $\Delta t = 0.5$

k	t_k	y_k	$y_{k+1} = y_k + \Delta t(y'(t_k))$
0	0	2	$(0, 2)$: $y = 2 + 0.5(e^{2/2}) = 3.36$, $(0.5, 3.36)$
1	0.5	3.36	$(0.5, 3.36)$: $y = 3.36 + 0.5(e^{2/3.36}) = 4.27$, $(1, 4.27)$
2	1	4.27	$(1, 4.27)$: $y = 4.27 + 0.5(e^{2/4.27}) = 5.07$, $(1.5, 5.07)$
3	1.5	5.07	$(1.5, 5.07)$: $y = 5.07 + 0.5(e^{2/5.07}) = 5.81$, $(2, 5.81)$
4	2	5.81	



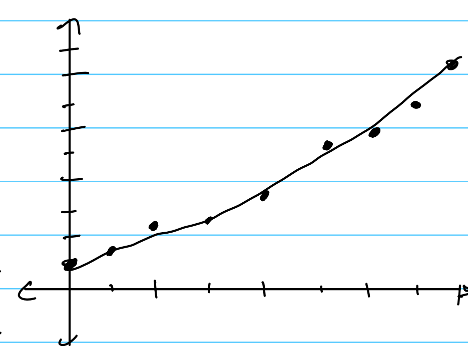
15. $\frac{dy}{dt} = \sqrt{y}$, $y(0)=1$, $0 \le t \le 4$, $\Delta t = 1$

k	t_k	y_k	
0	0	1	$(0, 1) = 1 + \sqrt{1} = 2$
1	1	2	$(1, 2) = 2 + \sqrt{2} = 3.41$
2	2	3.41	$(2, 3.41) = 3.41 + \sqrt{3.41} = 5.26$
3	3	5.26	$(3, 5.26) = 5.26 + \sqrt{5.26} = 7.56$
4	4	7.56	



$\frac{dy}{dt} = \sqrt{y}$, $y(0)=1$, $0 \le t \le 4$, $\Delta t = 0.5$

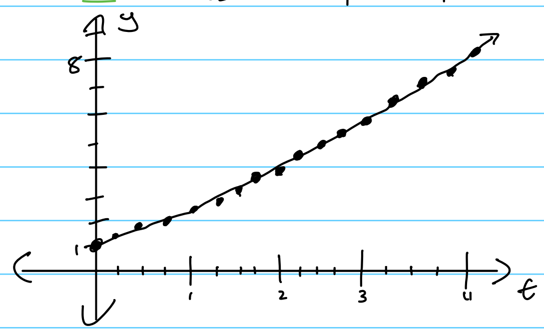
k	t_k	y_k	
0	0	1	$(0, 1)$: $1 + 0.5\sqrt{1} = 1.5$
1	.5	1.5	$(.5, 1.5)$: $1.5 + 0.5\sqrt{1.5} = 2.11$
2	1	2.11	$(1, 2.11)$: $2.11 + 0.5\sqrt{2.11} = 2.84$
3	1.5	2.84	$(1.5, 2.84)$: $2.84 + 0.5\sqrt{2.84} = 3.68$
4	2	3.68	$(2, 3.68)$: $3.68 + 0.5\sqrt{3.68} = 4.64$
5	2.5	4.64	$(2.5, 4.64)$: $4.64 + 0.5\sqrt{4.64} = 5.72$
6	3	5.72	$(3, 5.72)$: $5.72 + 0.5\sqrt{5.72} = 6.91$
7	3.5	6.91	$(3.5, 6.91)$: $6.91 + 0.5\sqrt{6.91} = 8.23$
8	4	8.23	



15. $\frac{dy}{dt} = \sqrt{y}$, $y(0)=1$, $0 \le t \le 4$, $\Delta t = 0.25$

k	t_k	y_k	k	t_k	y_k
0	0	1	9	2.25	4.32
1	.25	1.25	10	2.5	4.84
2	.5	1.53	11	2.73	5.39
3	.75	1.84	12	3	5.97
4	1	2.18	13	3.25	6.59
5	1.25	2.55	14	3.5	7.23
6	1.5	2.95	15	3.75	7.90
7	1.75	3.37	16	4	8.60
8	2	3.83			

$(0, 1)$: $1 + 0.25(\sqrt{1}) = 1.25$
$(.25, 1.25)$: $1.25 + 0.25(\sqrt{1.25}) = 1.53$
$(.5, 1.53)$: $1.53 + 0.25(\sqrt{1.53}) = 1.84$
$(.75, 1.84)$: $1.84 + 0.25(\sqrt{1.84}) = 2.18$
$(1, 2.18)$: $2.18 + 0.25(\sqrt{2.18}) = 2.55$
$(1.25, 2.55)$: $2.55 + 0.25(\sqrt{2.55}) = 2.95$
$(1.5, 2.95)$: $2.95 + 0.25(\sqrt{2.95}) = 3.37$
$(1.75, 3.37)$: $3.37 + 0.25(\sqrt{3.37}) = 3.83$
$(2, 3.83)$: $3.83 + 0.25(\sqrt{3.83}) = 4.32$
$(2.25, 4.32)$: $4.32 + 0.25(\sqrt{4.32}) = 4.84$
$(2.5, 4.84)$: $4.84 + 0.25(\sqrt{4.84}) = 5.39$
$(2.75, 5.39)$: $5.39 + 0.25(\sqrt{5.39}) = 5.97$
$(3, 5.97)$: $5.97 + 0.25(\sqrt{5.97}) = 6.59$
$(3.25, 6.59)$: $6.59 + 0.25(\sqrt{6.59}) = 7.23$
$(3.5, 7.23)$: $7.23 + 0.25(\sqrt{7.23}) = 7.90$
$(3.75, 7.90)$: $7.90 + 0.25(\sqrt{7.90}) = 8.60$



I would predict the actual solution to look like the one above because of the small step.

21. $\frac{dv_c}{dt} = V(t) - v_c / RC$ where $V(t) = 2\cos 3t$, $R = 4$, $C = 0.5$

$\frac{dv_c}{dt} = \frac{2\cos 3t - v_c}{4(0.5)}$, $v_c(0) = -2$, $0 \le t \le 10$, $\Delta t = 1$, let $v_c' = \frac{dv_c}{dt}$

k	t_k	v_{c_k}	k	t_k	v_{c_k}
0	0	-2	6	6	-0.51
1	1	0	7	7	0.41
2	2	-.99	8	8	-0.34
3	3	0.47	9	9	0.25
4	4	-0.68	10	10	-0.17
5	5	0.50			

$(0, -2) = -2 + v_c'(-2) = 0$
$(1, 0) = 0 + v_c'(0) = -.99$
$(2, -.99) = -.99 + v_c'(-.99) = 0.47$
$(3, 0.47) = 0.47 + v_c'(0.47) = -0.68$
$(4, -.68) = -0.68 + v_c'(-0.68) = 0.50$
$(5, .50) = 0.50 + v_c'(0.50) = -0.51$
$(6, -.51) = -0.51 + v_c'(-0.51) = 0.41$
$(7, 0.41) = 0.41 + v_c'(0.41) = -0.34$
$(8, -.34) = -0.34 + v_c'(-0.34) = 0.25$
$(9, 0.25) = 0.25 + v_c'(0.25) = -0.17$