

MATH 321/421 PRACTICE EXAM

Directions: Show all of your work on each questions below. Proofs are to be written in full and grammatically correct sentences. The use of quantifiers in proofs such that $\forall, \exists, \implies$ in place of the written word will result in a point deduction from a problem for each use of such a quantifier. However, use of the notation $x \in A$ to denote an element x in a set A is perfectly acceptable. Each problem is worth 5 points.

1. (a) State the definition of what it means for a function $f: S \rightarrow \mathbb{R}$ to be continuous at a point $c \in S$.
(b) Prove that if c is a cluster point of S , then f is continuous at c if and only if $f(x)$ tends to $f(c)$ as $x \rightarrow c$.
(c) Is there a function $f: (0, 1) \rightarrow \mathbb{R}$ that is continuous at every irrational point in $(0, 1)$ and discontinuous at every rational number in $(0, 1)$? Explain.
2. Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges conditionally but not absolutely.
3. Give a detailed proof that a series $\sum_{n=1}^{\infty} x_n$ converges if and only if it is Cauchy.
4. Using the definition of continuity, prove that $f(x) = \sqrt{x}$ is continuous on $(0, \infty)$.