WAVES-A

Interference of Light Waves

revised August, 2021

(*You will do two experiments; this one and the WAVES-B experiment. Sections will switch rooms and experiments half-way through the lab.*)

Learning Objectives:

During this lab, you will

1. estimate the uncertainty in directly measured quantities.
2. study interference and diffraction of light.

A. Introduction

Warning: Do not look directly into a laser beam; it can injure your eyes.

For Labs 7A and 7B you are to submit to Canvas one worksheet worth 30 points before you leave class. No paper is required.

19th century physicists were able to demonstrate that visible light has properties very similar to those of waves such as sound or water. In lab #7, you will investigate the wave properties of light by studying interference, diffraction and polarization.

It should be remembered that visible light occupies only a small portion of the electromagnetic (EM) spectrum. Visible light has wavelengths in the range 400 - 700 nm (*or frequencies of 7.5-4.3 $\times 10^{14}$ Hz and energies of 3.1 - 1.8 eV; note that the frequency and energy decrease as wavelength increases*). Interference *et. al.* can also be demonstrated in other parts of the electromagnetic spectrum. From short to long wavelength (*or high frequency & energy to low frequency & energy*), the electromagnetic spectrum encompasses gamma rays, x-rays, ultraviolet rays, visible light, infrared radiation, microwaves, FM radio and TV and longer AM radio waves.

If any of these EM waves interact with an object whose dimensions are comparable to the EM wavelength, interference effects become very obvious. The relevant size scale is roughly 0.1 nm for x-rays (*which show interference effects when interacting with atoms in a solid*), 500 nm for visible light (*the subject of this laboratory experiment*) and 1 m for radio waves (*which show interference from room or building-size objects*). The simplest electromagnetic wave is a linearly polarized plane wave. Such a wave, propagating in the *z*-direction, is illustrated in Figure l. The oscillating electric and magnetic fields of the wave are described by the equations

$$E_x = E_0\sin(kz-\omega t+\varphi) \qquad (1)$$

and

$$B_y = B_0\sin(kz-\omega t+\varphi) \qquad (2)$$

with $k = 2\pi/\lambda$, $\omega = 2\pi f$ and $c = \lambda f$, where λ is the wavelength, f is the frequency of the oscillation, φ is the phase of the wave, and c is the velocity of propagation of the electromagnetic wave, *i.e.*, the speed of light.

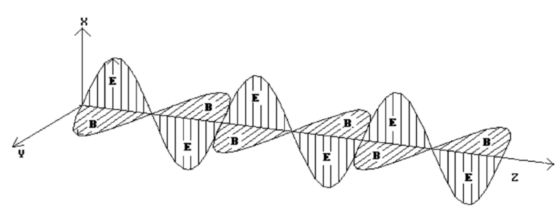

Figure 1: Plane wave.

All electromagnetic radiation travels through vacuum with speed $c = 2.9977\times10^8$ m/s. It follows that for waves propagating in free space, either f or λ alone is a sufficient characterization. Thus the expressions “10 megahertz radiation” is equivalent to “30 meter radiation”.

Radio waves and microwaves are usually *coherent*; that is, normal artificial sources of these waves produce signals with constant phases and frequencies. Such waves are also nearly monochromatic (a singular color in the visible spectrum); most of the energy is concentrated about a central wavelength. (*This is why a radio must be tuned very precisely for you to hear a broadcast station. A microwave oven works by being tuned to the vibrational frequency of water molecules.*) Most natural and artificial sources of visible light are not coherent but give a jumble of frequencies and phases. One normally must use a laser to produce convenient amounts of coherent visible light for studies of interference effects.

B.Interference Theory

Imagine taking a snapshot of the amplitude of a single EM plane wave traveling through empty space. A plot of this amplitude as a function of position would resemble one of the curves plotted in Figure 2. If a second plane wave of identical frequency and amplitude is sent along the same path as the first wave, the two will combine. The amplitude of this combined wave will depend on the relative phases $\Delta\varphi$ of the individual waves. If they are exactly in phase, they will combine to form a wave with twice their individual amplitudes. This is call constructive interference. (*Note that since the intensity of an EM wave is proportional to the square of its amplitude, this means that the combined intensity is **4 times** each individual intensity.*) If the waves are 180º out of phase, the two waves will exactly cancel. This is called destructive interference. Other phase differences will lead to intensities somewhere between 0 and 4 times the individual intensities.

It is very difficult to synchronize two independent sources so that their frequencies and phases are the same. So, to obtain two coherent waves, we use light from a single source and arrange to split the incident beam into two separate beams which travel different paths from the source to the detector. It is the difference in the lengths of these paths that leads to the phase difference $\Delta\varphi$. This phase difference will also depend on the placement of the detector. If the detector is a sheet of paper in the path of the light, there will be light and dark bands on the paper corresponding to constructive and destructive interference. These are caused by changes in the relative phases of the two beams as they make their way to different parts of the paper, with each beam following a slightly different path.

Constructive interference occurs if $\Delta\varphi=2\pi m$ where m is an integer; *i.e.,* constructive interference occurs when the path difference is an integral number of wavelengths (m = 0, 1, 2, 3...). This corresponds to a phase difference of some multiple of 360° degrees which is a full cycle, thus making the phases equal. Destructive interference occurs if $\Delta\varphi = 2\pi(m + ½)$; *i.e.*, the path difference is an odd number of half-wavelengths (m = 0, 1, 2, 3...).

B.1. Two-slit interference — Young’s Experiment

One way to split a beam is to pass it through a pair of slits. Thomas Young used this method in 1801 to demonstrate interference and establish the wave theory of light.

The light paths are illustrated in Figure

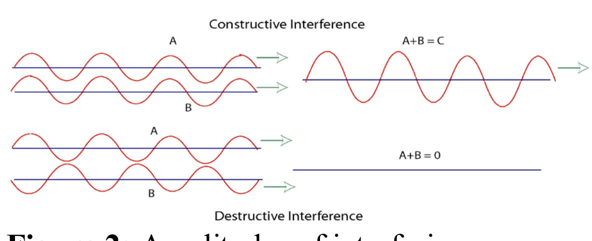

Figure 2: Amplitudes of interfering waves.

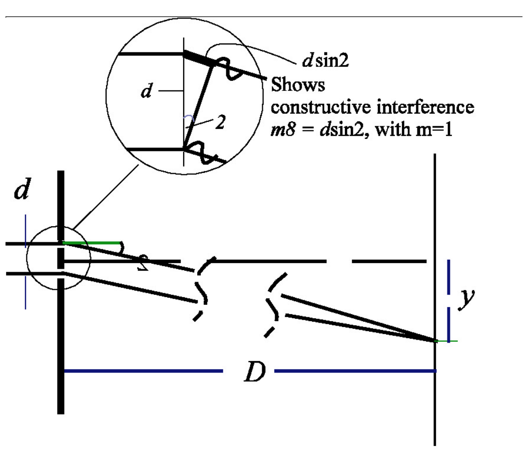

Figure 3: Young's two-slit experiment.

3. A monochromatic plane wave strikes two narrow slits separated by a distance d, creating two rays which strike a screen placed at a distance D from the slits at a height y (*measured from* $\theta = 0$). The two light paths from slit to screen are slightly different in length, giving rise to interference.

The inset in Figure 3 shows the region near the slits more clearly. The path difference for the two beams may be entirely attributed to the leg of the triangle of length $d\sin\theta$, since the two beams travel equal distances to the screen after the top beam traverses this first segment.

Constructive interference occurs if the path difference at some angle θ is equal to an integral number m of wavelengths. The number m is also called the *order,* as in *first order beam*, *second order beam*, etc. For $D >> d$, this requirement gives an intensity maximum when

$$d\sin\theta = m\lambda \text{ with } m = 0,1,2,... \quad (3)$$

Similarly, a minimum occurs when

$$d\sin\theta = (m+\tfrac{1}{2})\lambda \text{ with } m = 0,1,2,... \quad (4)$$

Note that smaller separations d give larger angles θ. For small θ, $\sin\theta \approx \theta \approx \tan\theta$. Since

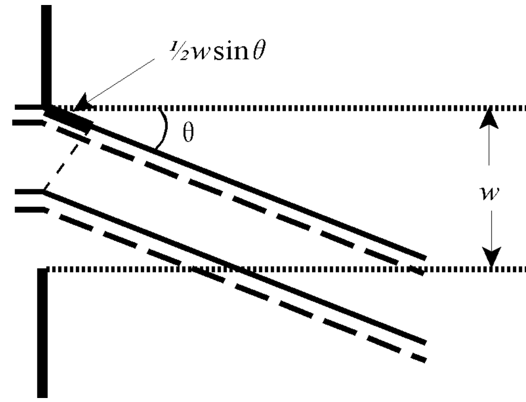

Figure 4: Single slit diffraction.

$\tan\theta = y/D$, we can write for the angular position of the maxima, $\theta \approx m\lambda/d$ or

$$y/D \approx m\lambda/d \quad (5)$$

with a similar expression for the minima.

B.2. Diffraction by a Single Slit

Light from a single slit or aperture also exhibits interference effects, referred to as *diffraction.* Figure 4 shows a plane wave striking a narrow slit of width w at normal incidence, illustrated by rays at the top and middle of the slit (*solid lines*). For $D >> d$, the ray from the middle of the slit will interfere destructively with the ray from the top of the slit if the path difference is half of a wavelength

$$\tfrac{1}{2}w\sin\theta = \tfrac{1}{2}\lambda. \quad (6)$$

The same condition applies to the dashed rays, each displaced by the same distance from the corresponding solid rays. Thus, at this angle, all rays passing through the top half of the slit interfere destructively with all rays passing through the bottom. If we divide the slit into quarters, we get cancellation of the top quarter by the next quarter, etc., and in general, we find destructive interference or diffraction *minima* if the path difference obeys the relation

$$½w\sin\theta = ½m\lambda \text{ or } w\sin\theta = m\lambda \quad (7)$$

with m = 1,2,3, ... There is a point of maximum intensity approximately, but not exactly, halfway between each pair of adjacent minima. For $D >> y$, we can use the small angle approximation to reduce Eq. 7 to

$$w\theta = m\lambda \qquad (8)$$

and the full angular width of the central maximum is given by

$$2\theta = 2\lambda/w \qquad (9)$$

Note that θ is proportional to $1/w$ so that light passing through a narrow aperture produces a wide central maximum. A wide aperture produces smaller diffraction effects. Diffraction is a limiting factor in determining the resolution of a camera lens or telescope and is a primary reason why bigger is usually better in the world of optics.

B.3. The Diffraction Grating

An array of many equally spaced slits forms a transmission *diffraction grating.* A reflection grating consists of a similar array of reflecting lines. The equations for the positions of the maxima and minima produced by a transmission grating are identical to those for the double slit system. The primary difference is that more slits means sharper lines. The width of a diffracted line roughly scales as $1/N$ where N is the number of slits. The equations for the reflection grating are similar.

C. Apparatus

You will use a He-Ne laser as a light source, shining it through various single and double-slit slides and a diffraction grating. Common He-Ne lasers emit red light at a wavelength of 632.8 nm. The slides are mounted to a translation stage designed for a microscope. *A photograph of this arrangement is posted on the lab web site and on Canvas.* The translation stage simplifies positioning the slits. The detector is just a piece of paper taped to a wall.

Don't bend over a table or in any way let a beam strike you in the eye and be careful as you move around the lab. The low-power lasers used in this experiment are not generally considered dangerous but the beam will appear very bright if it strikes your eye.

D. Measurements

Make sure that the laser beam is roughly perpendicular to the "wall". Set the slide holder at $D \approx 1$ m from the wall, measuring this distance with a meter stick. Ideally, **the mounted slides should be perpendicular to the beam**; this can be accomplished by adjusting the slide so that the light reflects from it back along the beam onto the laser. However, for safety reasons (*so that you don't blind someone with your laser*) you should mount your slide at an angle of 10-20 degrees off perpendicular to direct the reflected beam towards the floor. This will introduce only very small errors in your analysis. Your TA will help you with these adjustments if necessary.

Tape a blank sheet of paper to the "wall". You will mark positions of the maxima and/or minima on the paper, then remove the paper from the wall and make measurements on the paper. Do not mark the wall or leave any tape on the wall after you have finished your measurements.

Examine the three 35-mm slides. Note that the single-slit and double-slit slides are labeled and each includes 4 slits or pairs of slits. You should view all eight diffraction/interference patterns from the first two slides but will **take careful measurements from only 1 single slit and 1 pair of slits.** Slit widths w and, for the double slits,

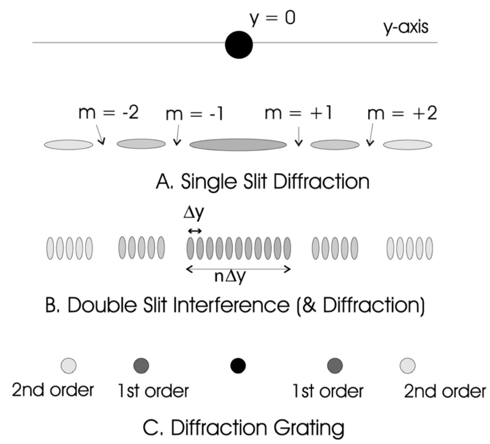

Figure 5: Diffraction patterns formed by: A-single slit; B-double slit; C-grating.

separations d are given on the slides in mm, you may assume these values are so precise that they make negligible contributions to any uncertainties in the experiment.

All images are formed symmetrically about $y = 0$ so that if there is a spot of maximum brightness at $+y$, there should be a corresponding bright spot at $-y$.

It can be confusing to sort out minima and maxima and single and double slit effects the first time you examine a diffraction pattern. Figure 5 may help you in this identification.

The single slit shows only diffraction effects. Fig 5 shows only four diffraction minima, labeled as $m = -2, -1, +1$ and $+2$, but you should see many more in the experiment. The smaller features in the double slit pattern are the interference effects; the overall modulation on the same scale as the single slit features are the diffraction effects caused by using slits to produce interference

For the single slit experiment, you will need to **measure the distance to each of the minima,** the darkest areas between brightly lit spots. Rather than take these measurements directly on the wall, we suggest that you **trace the spots on a piece of paper taped to the wall and then measure from this tracing. You should do the same thing for the double slit data**. The interference grating spots will actually be too far apart to use this technique (*they are not shown to scale in Fig. 5*) and will require direct measurement with a meter stick. You may want to take all of the data described below before analyzing any of it. This will make it possible to turn on the room lights earlier in the period, making it easier for everyone to work on the analysis.

D.1. Single Slits

Position the single-slit slide in the path of the laser beam. Starting with the smallest slit and using the screw adjustment on the slide holder to translate the slide, examine the diffraction image produced by each of the 4 slits. Note that the finest slit produces the biggest separation, as predicted by Eq. 9.

Select the "B" slit (w = 0.04 mm). **<u>Record and measure *y* for as many diffraction minima on both sides of the central maximum as you can see or fit on the paper</u>.** The minima are in the dark or unlit regions of the screen; you can pick the midpoints of each such region for your values of y. Note that the first-order minima come at $+y_1$ and $-y_1$ with a point of maximum intensity at y=0.

<u>Use all of your data to calculate your best estimate of λ.</u> To do this, use the fact that $D >> y$ to write $\sin\theta \approx \tan\theta = y/D$. Substituting this for $\sin\theta$ in Eq. 7, we get

$$y = m\lambda D/w \qquad (10)$$

To find λ, simply **use Origin to plot y_m vs. m**, with m = ±1, 2, 3, *etc.* for successive minima (*values of y*) and **calculate λ from the slope of this line.** (*You should already know D and w.*)

D.2. Double Slits

Because a double slit consists of 2 single slits of width w, with separation d (*d is* *necessarily greater than w*), the double slit pattern must include both two-slit interference and single-slit diffraction effects. In fact, multiple-slit interference effects could not occur without the spreading of the beams produced by single-slit diffraction, which causes light diffracted from each single slit to overlap with light from the other slit. The result is a modulation of the double slit pattern by the single slit pattern.

Examine the patterns produced by each of the 4 pairs of slits, then **select double slit "A"** (w=0.04 mm, d=0.25 mm). Examine the single-slit diffraction pattern from the double slit (*the broader features in the pattern*) just as you did for the single slit and **<u>enter in your worksheet the total distance between the first minima to the left and right of y=0</u>.**

The double-slit interference pattern produces the fine structure in the image. By fine structure we mean that the if we compare the single slit pattern to the double slit pattern we find that for each light band in the single slit pattern there are now a series of light and dark bands superimposed. **Find the mean separation Δy of maxima within the central diffraction maximum** (*find the average spacing of the bright dots in the central region*). The easiest way to do this is to **<u>measure the total size of the central maximum</u>** (***not the minima -*** *try to measure between the centers of the last clearly visible spots on the left and right edges*) **and divide by the <u>number of bright lines,</u> minus 1, to obtain the <u>average Δy</u> corresponding to Δm = 1. <u>Calculate λ from your data</u> and Eq. 5.** You do not need to plot your data as a function of m or make a linear fit. Simply **plug in your value of Δy for Δm = 1.** You know D and d so you can easily calculate λ. Note that we again use the small angle approximation.

D.3. The Diffraction Grating

Mount the high-resolution diffraction grating slide in the slide holder. Most of the gratings are labeled 600/mm. A few are labeled 13,400 with no units. This number is the number of lines per inch that are inscribed on the grating. 13,400 lines/inch converts to 528 lines/mm. **<u>Measure the positions of the first- and second-order maximum on either side of the spectrum and use this data to calculate λ</u>.** (*If one spot is obscured by a stand, a cabinet, etc. You may ignore it, but you should obtain at least 3 measurements.*) Note that because the angles are not small, you must **use Equation 3 for this calculation, applying it separately to each of your four values of y. This will leave you with four values of λ; take their average for your best estimate of λ.**

D.3.1. Diffraction Gratings & Colors

As you have just observed, a diffraction grating can cause large bends in the path of a beam of red light. Light of other colors or wavelengths would also be bent but at different angles given by Eq. 7. So a diffraction grating can serve as a prism, *dispersing* light across a detector with the position of the light depending on its wavelength. The smaller the spacing between lines in the grating, the larger this effect.

In the lab you will find some plastic sheet diffraction gratings. They are about 10×15 cm in size. They might look like clear plastic at first but actually have a spacing comparable to the gratings mounted in your slide holder. **Remove your grating from the holder and hold a plastic grating in its place.** Note the resulting spacing of the diffracted laser beams. (*No careful measurements are necessary.*)

Take a sheet to the lab next door and use it to examine the light from the fluorescent tubes in the ceiling, an

incandescent bulb and the mercury lamp set up in the lab. ***Do not stare directly at the mercury bulb!*** By tilting the grating with respect to your eye and/or the light source, you should be able to see a range of colors from the first two sources as the grating causes different colors to diffract at different angles. The mercury fixture is "purer" and consists of only a few bright colors. You should also be able to detect that the fluorescent fixture has several distinct bands of color rather than a smoothly changing spectrum. This is a property of the phosphors used in fluorescent bulbs and it makes most fluorescent light sources unsuitable for viewing paint colors, make-up, etc. since colors will look different compared to their appearance under sunlight.

The next time you are on the quad on a sunny day, observe the reflection diffraction grating on top of Crawford Hall. The grating consists of many closely spaced, precision-ruled lines on a metallic surface. There are two gratings, one facing south and another facing west. These gratings were installed in 1987 to celebrate the 100^{th} anniversary of the Michelson-Morley experiment.

At the right time on a sunny day it is possible to observe the entire spectrum in bright first order and pastel second order. If you are at a position where the angle of reflection to your eyes is equal to the angle of incidence with the sun (*i.e., the phase difference is zero for all wavelengths*), the grating looks like a highly polished silver mirror. Because sunlight produces a continuous spectrum of wavelengths, the color of light reflected from the grating changes depending on the position of the sun and where you happen to be standing.

This page intentionally left without useful information.